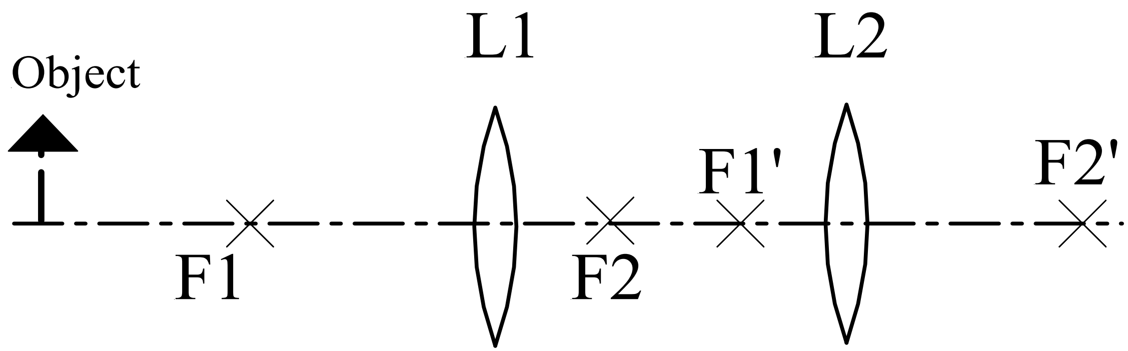
Imaging with two lenses

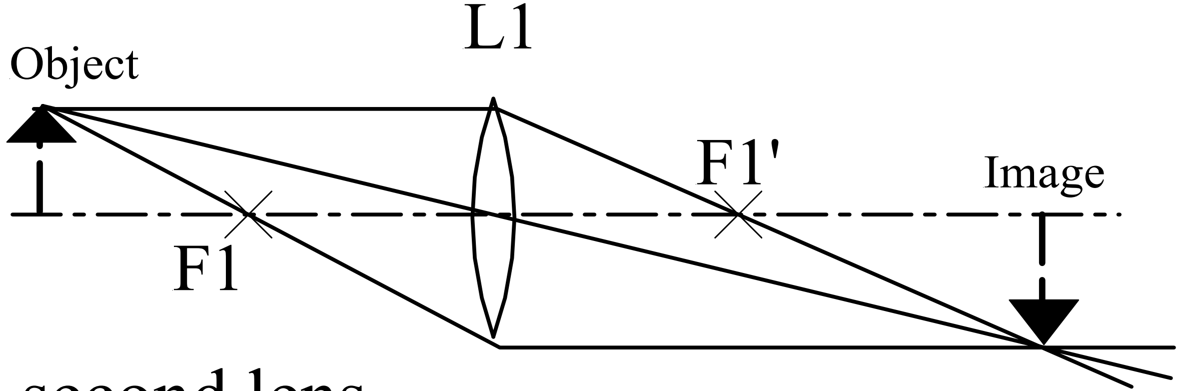
- Graphical methods
  - Parallel-ray method, find intermediate image, use as object for next lens
  - Virtual objects
  - Oblique-ray method, lens-to-lens, no need to find intermediate image
- Mathematical methods
  - Find intermediate image, use as object for next lens
  - lens-to-lens (sequential raytracing)-later in semester
- Combinations of thin lenses
  - In contact
  - Separated

Example, two separated positive lenses



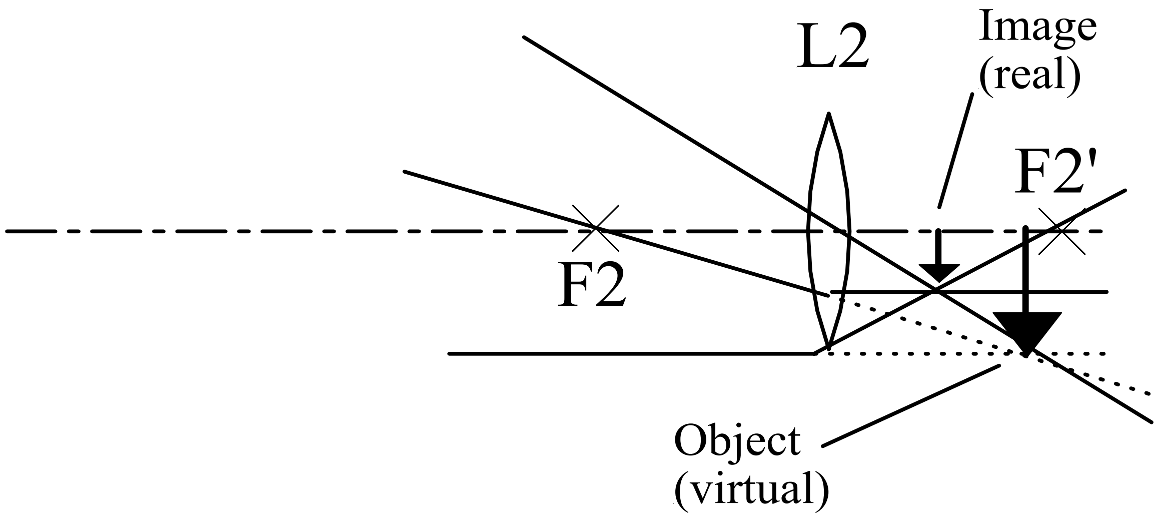
- Needed information
  - focal lengths of lenses
  - location of lenses
  - location of object

Parallel-ray method - step 1



- Ignore second lens
- Trace at least two of the rays shown from tip of object
  - tip of image found from intersection of rays
  - exactly as we did in previous module
  - NOTE: all rays from tip of object intersect at tip of image we have chosen three only because they are easy to trace!
  - Image is real in this case, but method is exactly the same if it is virtual WHENEVER NEEDED YOU MAY EXTEND OBJECT-SPACE OR IMAGE-SPACE RAYS TO INFINITY IN EITHER DIRECTION

Parallel-ray method - step 2



- Image from step 1 becomes Object for step 2
  - After producing this object, lens1 one can be ignored
  - object can be real or virtual (virtual in this case)
  - object can be real even if image from lens 1 is virtual
- Trace any two of the three rays shown through tip of object to find tip of final image
  - Final image may be real or virtual (real in this case)

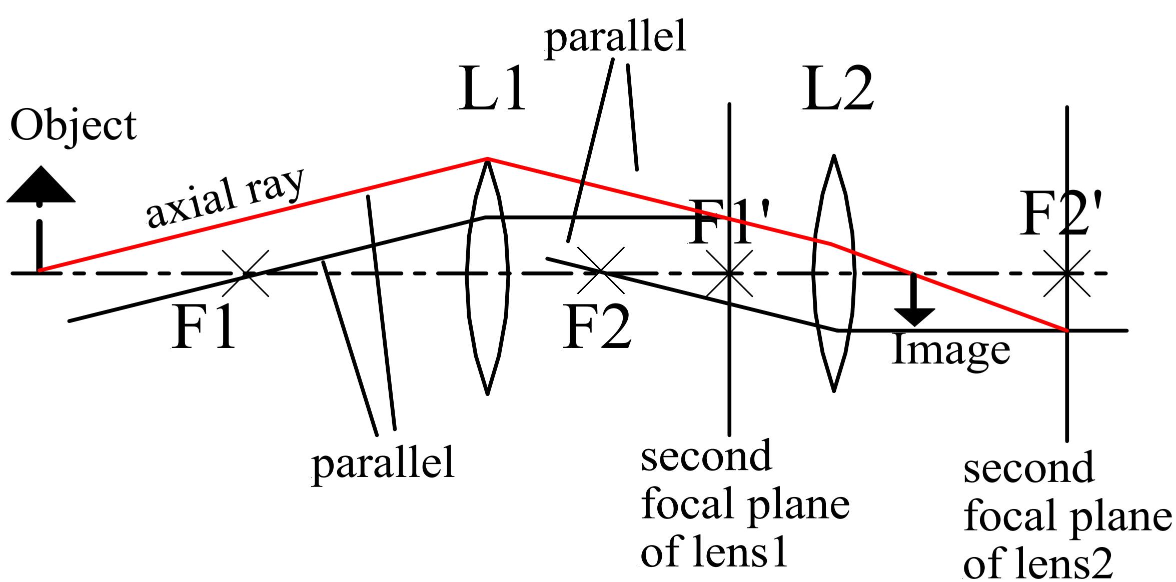
Reminder about real and virtual objects and images

Remember: For purposes of calculations, light travels from left to right (not necessarily in real life, of course)

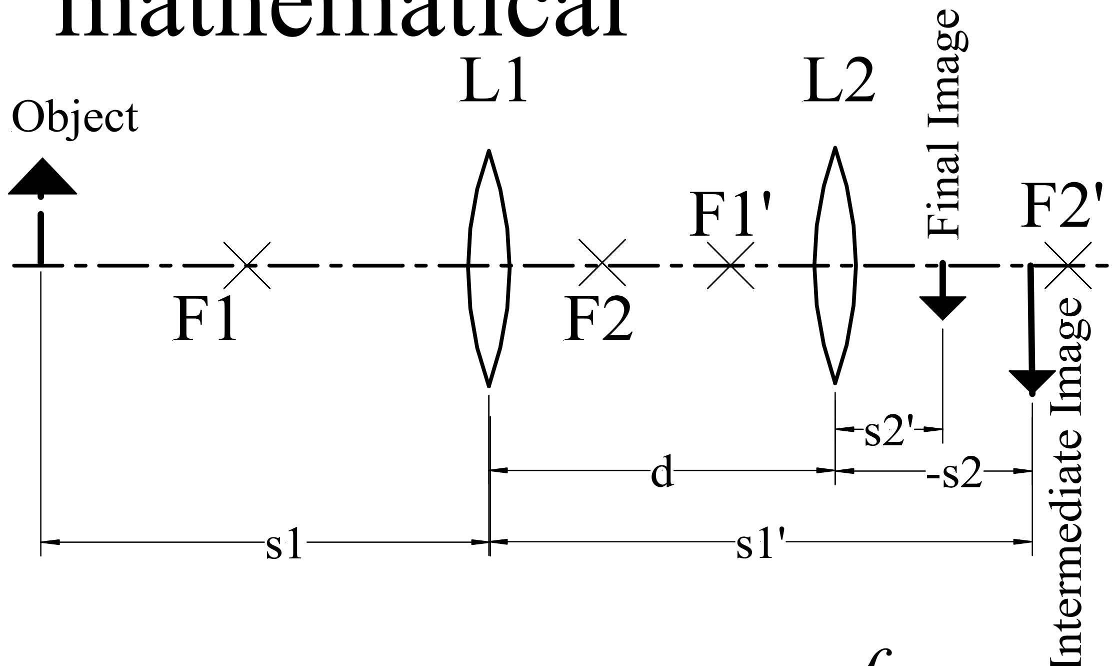
- Positive distances correspond to real objects and real images
  - Object distance positive when it is to the left of the lens
  - Image distance positive when it is to the right of the lens
  - This is the common situation for a single positive image forming an image on a screen, Examples- viewgraph machine, camera, eye, etc.
- Negative distances correspond to virtual objects and virtual images
  - Object to the right or image to the left of the lens

Oblique-ray method

- Is it really necessary to find the image due to the first lens?
- Any ray can be traced through the lens system using the oblique-ray method
- For example, trace the axial ray
  - point of intersection with axis in image space gives image location



Imaging through multiple lenses - mathematical



Given, f_1, f_2, d, and s_1 find s_2'

- Apply imaging formula to first lens to find image in that lens $$s_1' = \frac{s_1 f_1}{s_1 - f_1}$$
- Find object distance for second lens (negative means virtual object) $$s_2 = d - s_1'$$
- Use imaging formula again to find final image $$s_2' = \frac{s_2 f_2}{s_2 - f_2}$$

Magnification with multiple lenses

- The magnification is by definition the image size divided by the object size
- For the second lens in a system the object size is the image size for the first lens

$$y_2 = y_1' = M_1 y_1$$

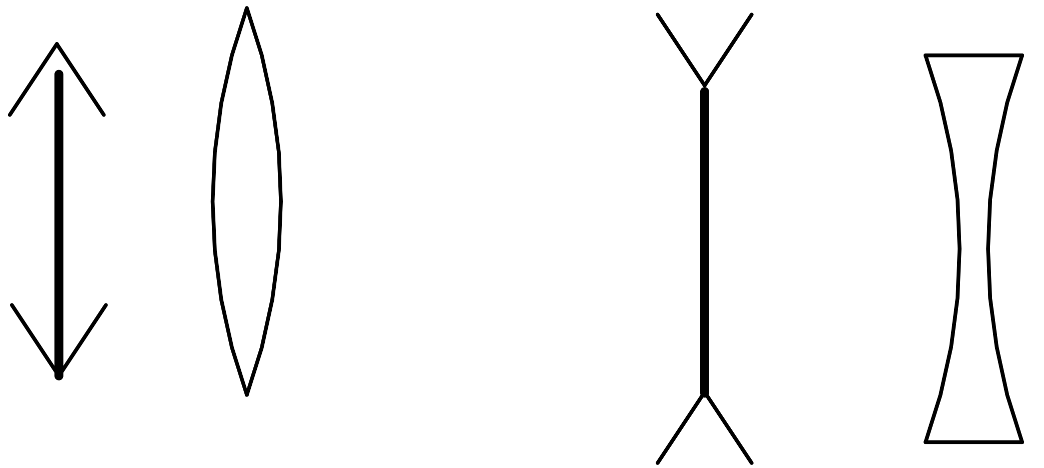
- The image size after the second lens is found by multiplying the second lens magnification by the size of the object for the second lens

$$y_2' = M_2 y_2 = M_1 M_2 y_1$$

- The system magnification is the final image size divided by the original object size

$$M_{system} = M_1 M_2$$

Symbols for thin lenses

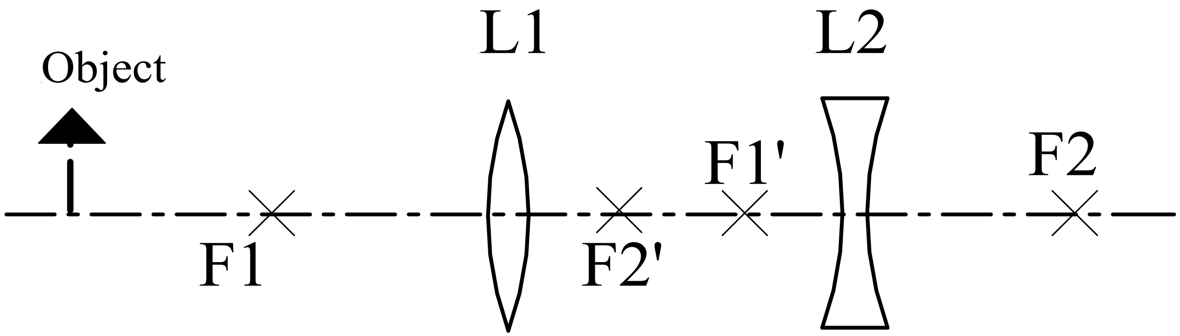


- Arrows symbolize the outline of the glass at the edge

Raytracing through several lenses

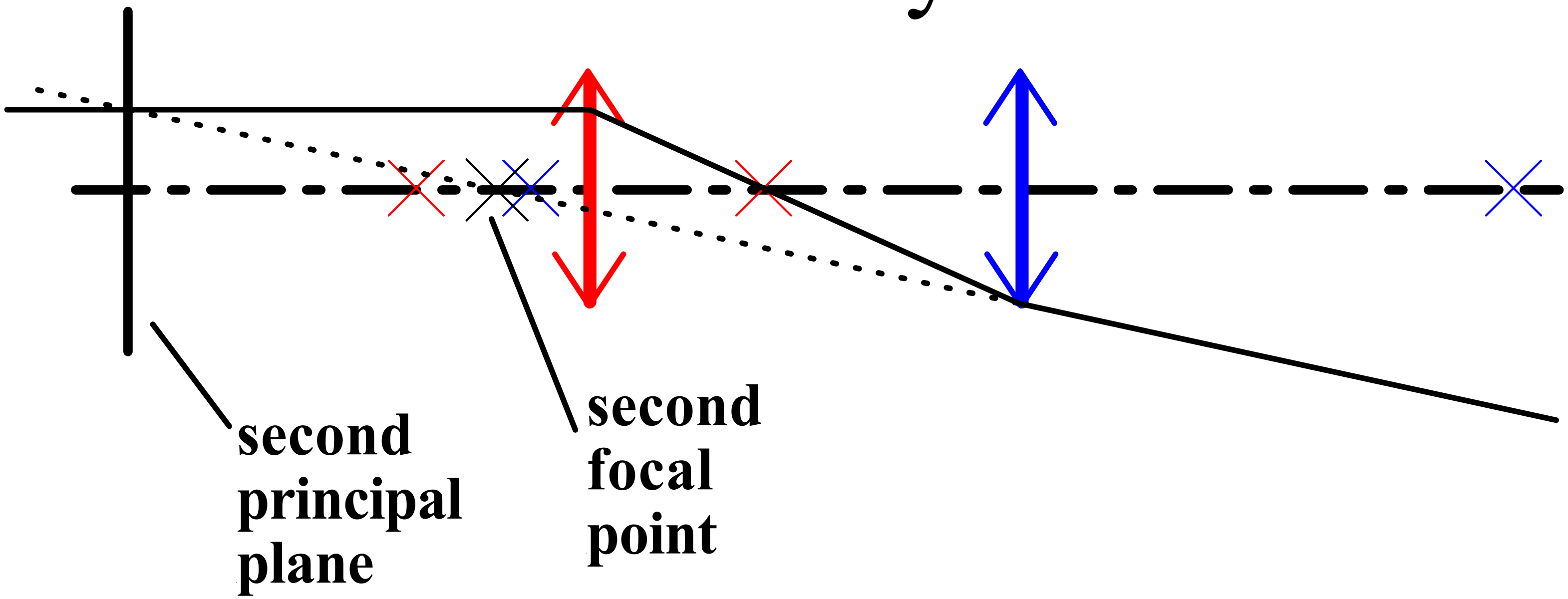
- Go through lenses in the order the light strikes them
  - This is true even if there are mirrors in the system!
- For mathematical raytracing be careful of sign convention
  - many possibilities of focal lengths and spacings but sign convention covers them all

Special considerations for negative lenses – there aren’t any



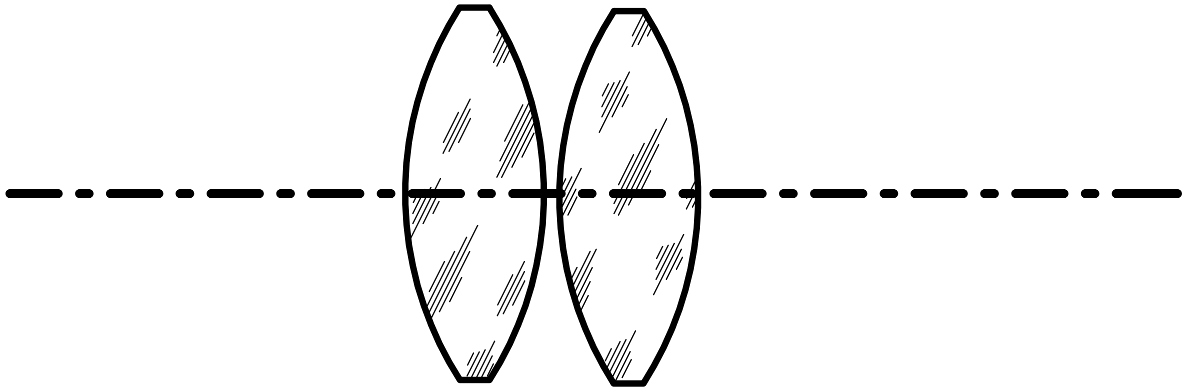
- Note that the prime on focal points for negative lens have reversed
- Primary and secondary focal points on opposite sides compared to positive lens

Principal planes and focal lengths of multi-lens systems



- Defined exactly the same as for thick lens

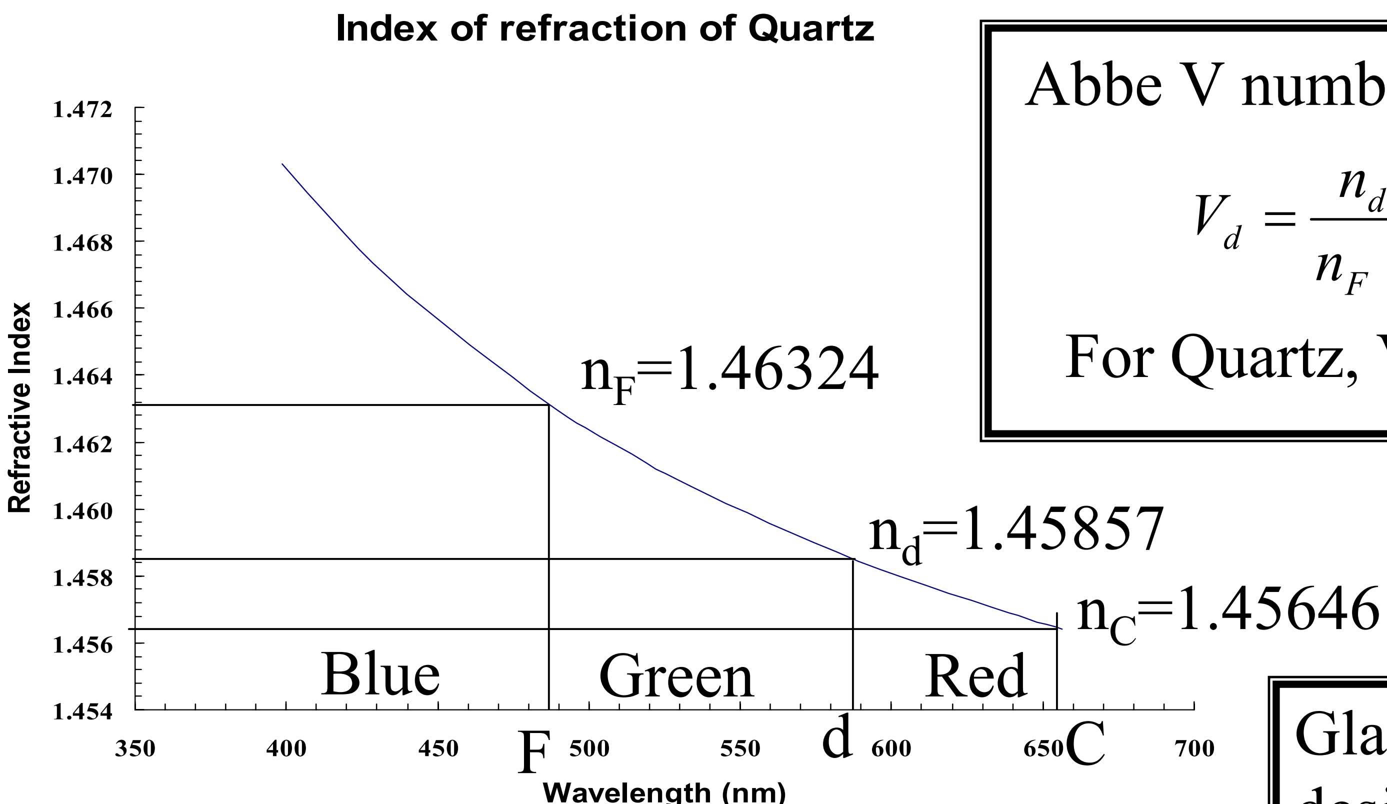
Thin lenses in contact



$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \qquad P = P_1 + P_2$$

- Can be applied to multiple thin lenses in contact as well
- As always, be careful with sign convention

Dispersion-Abbe V number



Abbe V number-Definition

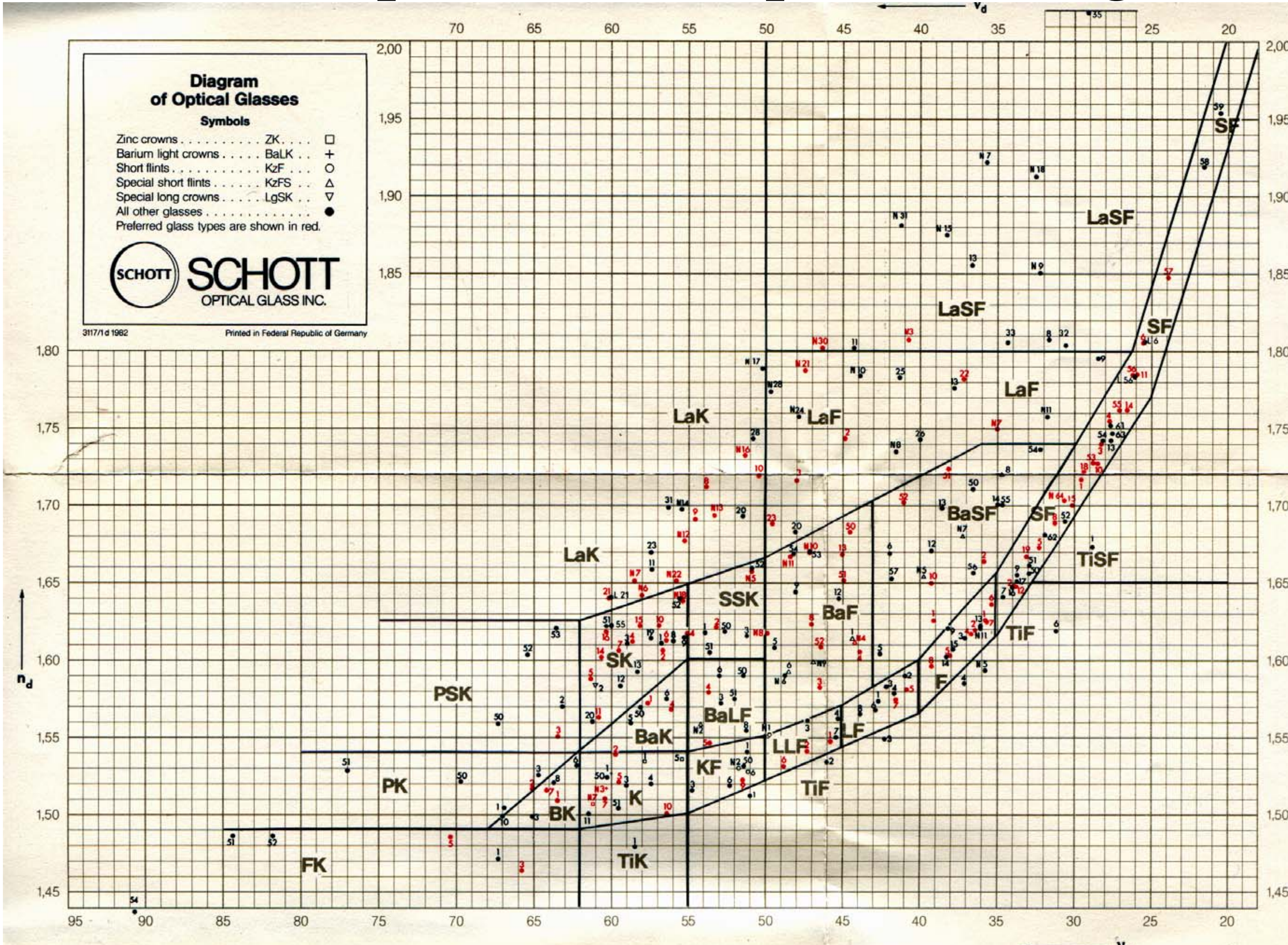
$$V_d = \frac{n_d - 1}{n_F - n_C}$$

For Quartz, V_d=67.6

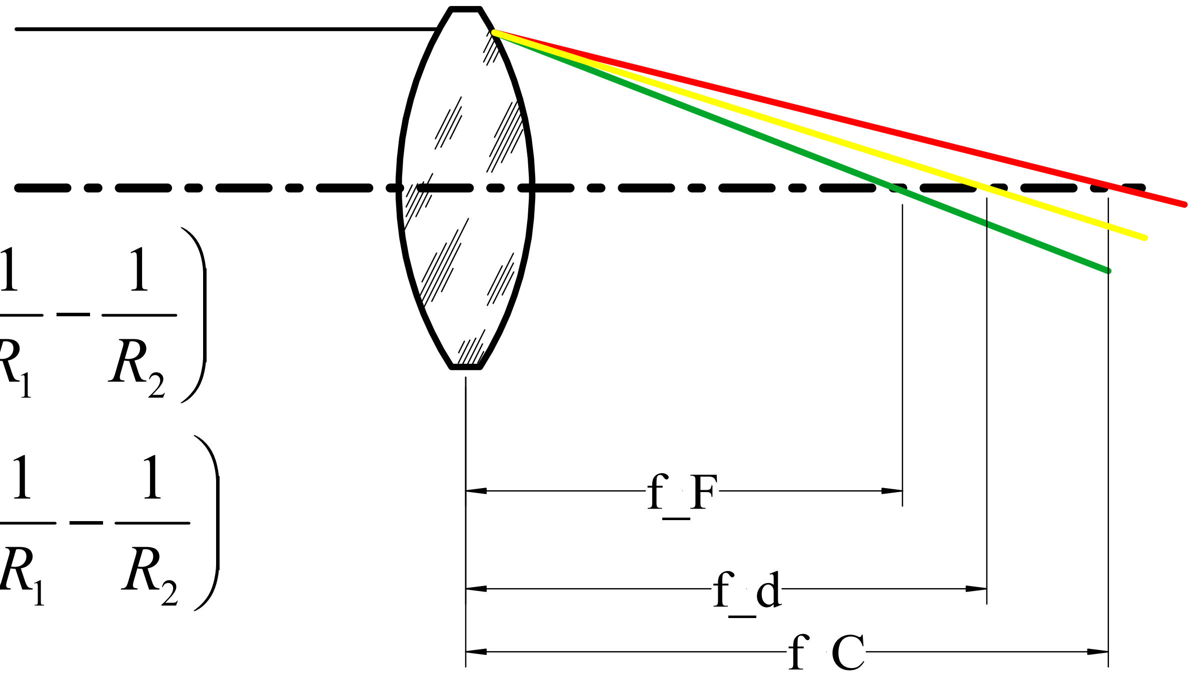
Glass designation
Quartz, 459676
BK7, 517642

- Index of refraction changes with wavelength
- Usually decreases for longer wavelengths
- Small, but important effect

Glass map - index/dispersion of glasses



Chromatic aberration of thin lenses



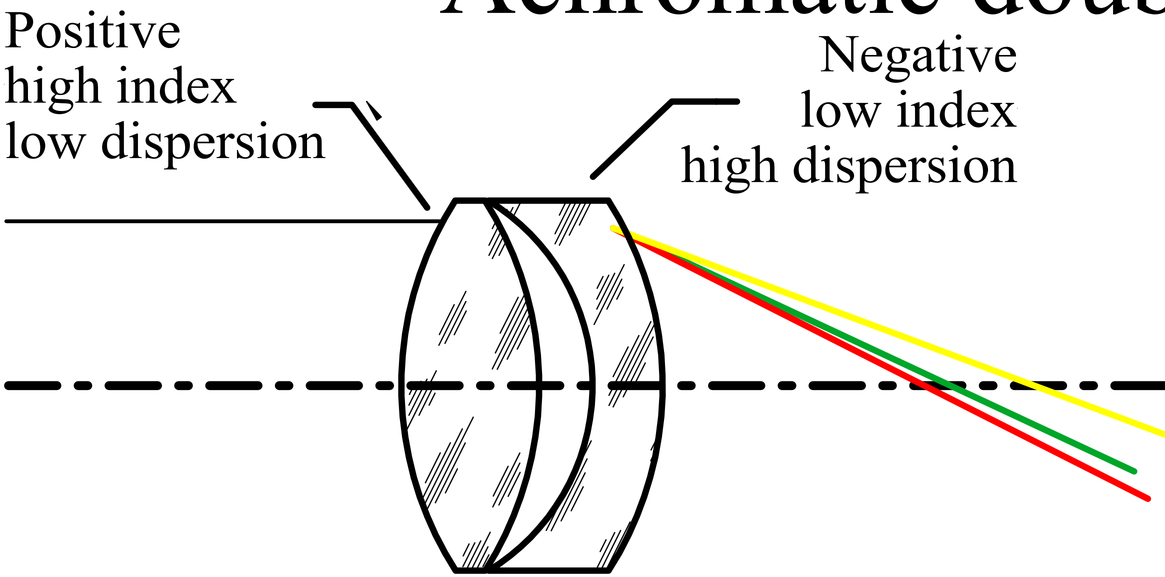
$$P_F = (n_F - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$P_C = (n_C - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$P_F - P_C = (n_F - n_c)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{(n_F - n_C)P_d}{n_d - 1} = \frac{P_d}{V_d}$$

- Called longitudinal chromatic aberration
  - This is the only chromatic aberration possible in a single thin lens with the stop at the lens

Achromatic doublets



To achieve the desired focal length requires

$$P_d = P_{+d} + P_{-d}$$

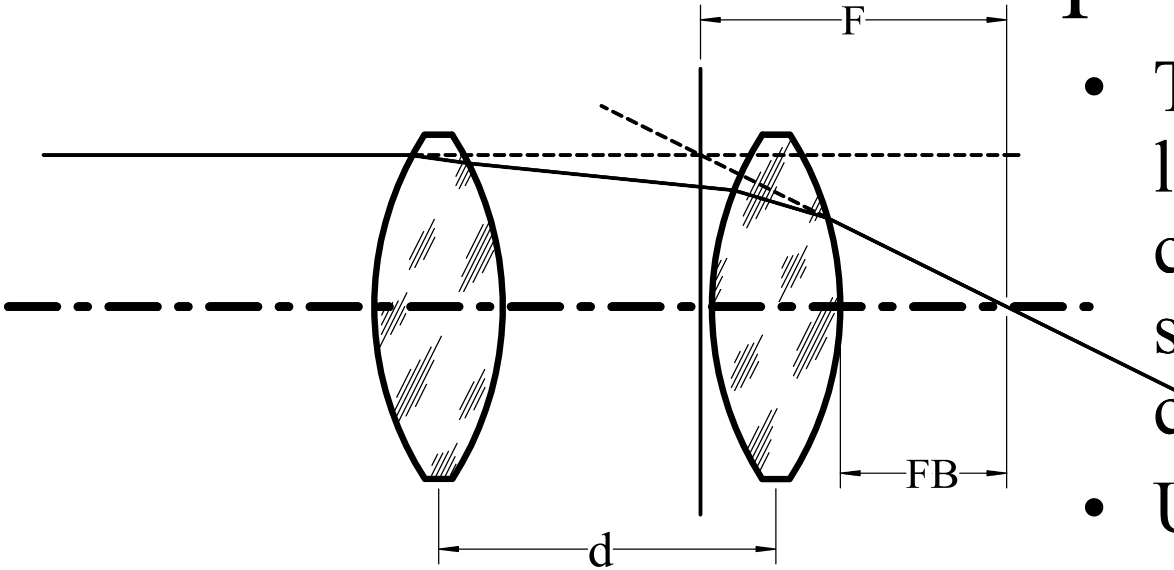
Many choices of powers satisfy this

We can also choose the powers to make chromatic zero

$$P_F - P_C = 0 = \left(P_{+F} - P_{+C}\right) + \left(P_{-F} - P_{-C}\right) = \frac{P_{+d}}{V_{+d}} + \frac{P_{-d}}{V_{-d}}$$

- Two radii used to get each power, one can be freely chosen
- One is usually chosen to make inner surfaces have the same curvature (cemented doublet)
- Remaining radius chosen to minimize spherical aberration

Thin lenses separated



- Two positive thin lenses are weaker in combination when separated than when in contact
- Use of these equation in solving homework problems is discouraged

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$P = P_1 + P_2 - dP_1 P_2$$

Back focal length

$$F_B = \left(1 - \frac{d}{f_1}\right)\frac{1}{P} = \left(1 - \frac{d}{f_1}\right)F$$

Second lens to principal plane $F_B - F = -\frac{P_1}{P} d$

Principal plane locations for thin lens combinations

First principal plane

Second principal plane

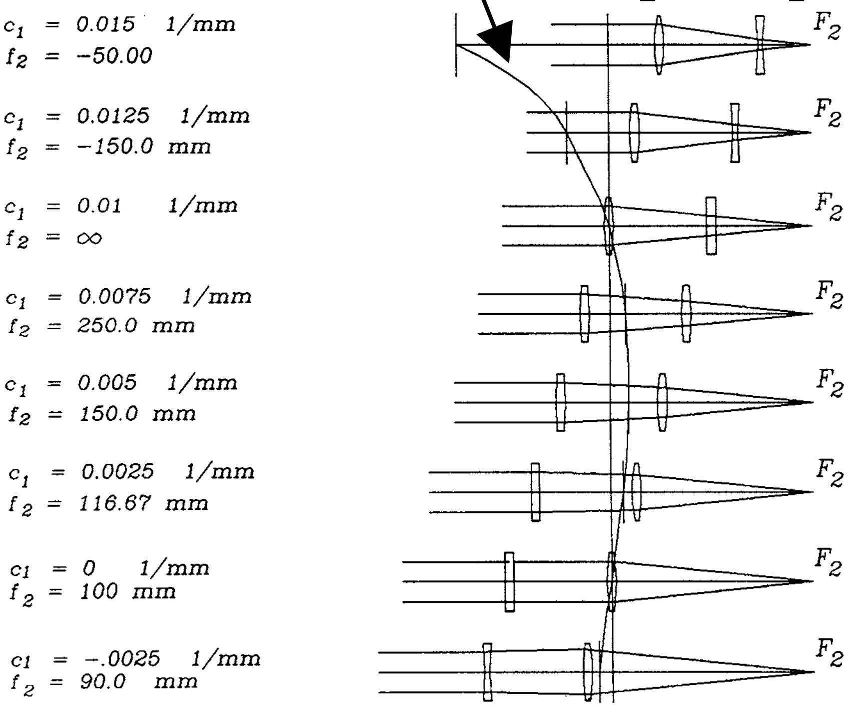


Figure 4.10.- Position of the principal planes for a system of two separated thin lenses

The position of the principal planes for several lens combinations is illustrated in Fig. 4.10. Many interesting properties may be noticed by a close examination of this figure.

- c_1 is power of first lens
- f_2 is focal length of second lens
- Focal length of combination is kept constant at 100 mm