IMC−PID Controller Design for Improved Disturbance Rejection of Time-Delayed Processes

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The IMC−PID tuning rules demonstrate good set-point tracking but sluggish disturbance rejection, which becomes severe when a process has a small time-delay/time-constant ratio. In this study, an optimal internal model control (IMC) filter structure is proposed for several representative process models to design a proportional−integral−derivative (PID) controller that produces an improved disturbance rejection response. The simulation studies of several process models show that the proposed design method provides better disturbance rejection for lag-time dominant processes, when the various controllers are all tuned to have the same degree of robustness according to the measure of maximum sensitivity. The robustness analysis is conducted by inserting a perturbation in each of the process parameters simultaneously, with the results demonstrating the robustness of the proposed controller design with parameter uncertainty. A closed-loop time constant λ guideline is also proposed for several process models to cover a wide range of θ/τ ratios.

1. Introduction

The proportional−integral−derivative (PID) control algorithm is widely used in process industries because of its simplicity, robustness, and successful practical application. Although advanced control techniques can provide significant improvements, a well-designed PID controller has proved to be satisfactory for a large number of industrial control loops. The well-known internal model control−PID (IMC−PID) tuning rules have the advantage of only using a single tuning parameter to achieve a clear tradeoff between closed-loop performance and robustness to model inaccuracies. The IMC−PID controller provides good set-point tracking but has a sluggish disturbance response, especially for the process with a small time-delay/time-constant ratio.[1−9] However, because disturbance rejection is much more important than set-point tracking for many process control applications, a controller design that emphasizes disturbance rejection rather than set-point tracking is an important design problem that has been the focus of renewed research recently.

The IMC−PID tuning methods by Rivera et al.,[1] Morari and Zafiriou,[6] Horn et al.,[3] Lee et al.,[4] and Lee et al.[9] and the direct synthesis methods by Smith et al.[10] (DS) and Chen and Seborg[5] (DS-d) are examples of two typical tuning methods based on achieving a desired closed-loop response. These methods obtain the PID controller parameters by computing the controller that gives the desired closed-loop response. Although this controller is often more complicated than a PID controller, the controller form can be reduced to that of either a PID controller or a PID controller cascaded with a first- or second-order lag by some clever approximations of the dead time in the process model.

Several workers (Morari and Zafiriou,[6] Lee et al.,[4] Lee et al.,[9] Chien and Fruehauf,[2] Horn et al.,[3] Chen and Seborg,[5] and Skogestad[8]) have reported that the suppressing load disturbance is poor when the process dynamics are significantly slower than the desired closed-loop dynamics.

In fact, regarding the disturbance rejection for lag-time dominant processes, the well-known old design method by Ziegler and Nichols[11] (ZN) shows better performance than the

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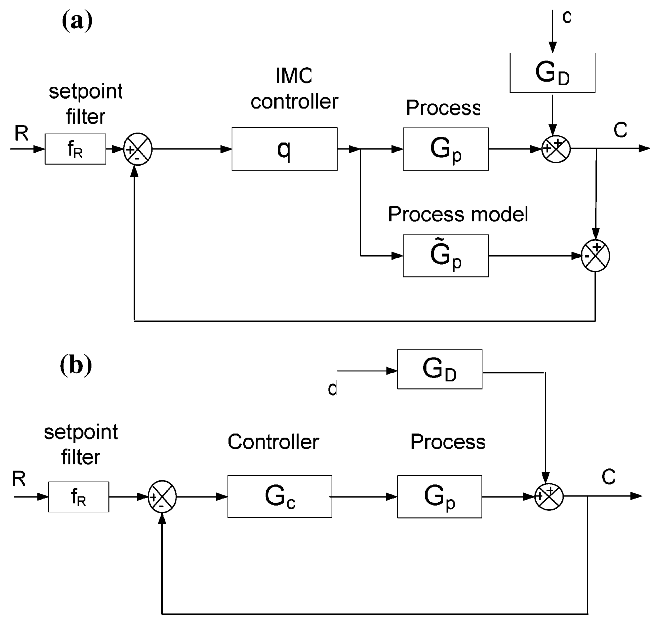

Figure 1. Block diagram of (a) IMC and (b) classical feedback control structures.

IMC−PID design methods based on the IMC filter $f = 1/(\lambda s + 1)^r$. Horn et al.[3] proposed a new type of IMC filter which includes a lead term to cancel out the process dominant poles. On the basis of this filter, they developed an IMC−PID tuning rule that leads to the structure of a PID controller with a second-order lead-lag filter. The performance of the resulting controller showed a clear advantage over those based on the conventional IMC filter. Chen and Seborg[5] proposed a direct synthesis design method to improve disturbance rejection for several popular process models. To avoid excessive overshoot in the set-point response, they utilized a set-point weighting factor. In order to improve the set-point performance by including a set-point filter, Lee et al.[4] proposed an IMC−PID controller based on both the filter suggested by Horn et al.[3] and a two-degree-of-freedom (2DOF) control structure. Lee et al.[9] extended the tuning method to unstable processes such as first- and second-order delayed unstable process (FODUP and SODUP) models and for the set-point performance 2DOF control structure proposed. Skogestad[8]

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Table 1. IMC−PID Controller Tuning Rules

process	K_C	τ_I	τ_D	β
$\dfrac{K\,e^{-\theta s}}{\tau s+1}$	$\dfrac{\tau_I}{K(3\lambda-2\beta+\theta)}$	$(\tau+2\beta)-\dfrac{(3\lambda^2-\theta^2/2+2\beta\theta-\beta^2)}{(3\lambda-2\beta+\theta)}$	$\dfrac{(2\tau\beta+\beta^2)-\dfrac{(\lambda^3+\theta^3/6-\beta\theta^2+\beta^2\theta)}{(3\lambda-2\beta+\theta)}}{\tau_I}-\dfrac{(3\lambda^2-\theta^2/2+2\beta\theta-\beta^2)}{(3\lambda-2\beta+\theta)}$	$\beta=\tau\left[1-\left(\left(1-\dfrac{\lambda}{\tau}\right)^3 e^{-\theta/\tau}\right)^{1/2}\right]$
$\dfrac{K\,e^{-\theta s}}{s}$	$\dfrac{\tau_I}{\psi K(3\lambda-2\beta+\theta)}$	$(\psi+2\beta)-\dfrac{(3\lambda^2-\theta^2/2+2\beta\theta-\beta^2)}{(3\lambda-2\beta+\theta)}$	$\dfrac{(2\psi\beta+\beta^2)-\dfrac{(\lambda^3+\theta^3/6-\beta\theta^2+\beta^2\theta)}{(3\lambda-2\beta+\theta)}}{\tau_I}-\dfrac{(3\lambda^2-\theta^2/2+2\beta\theta-\beta^2)}{(3\lambda-2\beta+\theta)}$	$\beta=\psi\left[1-\left(\left(1-\dfrac{\lambda}{\tau}\right)^3 e^{-\theta/\psi}\right)^{1/2}\right]$
$\dfrac{K\,e^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)}$	$\dfrac{\tau_I}{K(4\lambda-\beta_1+\theta)}$	$(\tau_1+\tau_2+\beta_1)-\dfrac{(6\lambda^2-\theta^2/2+\theta\beta_1-\beta_2)}{(4\lambda-\beta_1+\theta)}$	$\dfrac{(\tau_1\tau_2+(\tau_1+\tau_2)\beta_1+\beta_2)-\dfrac{(4\lambda^3+\theta^3/6-\beta_1\theta^2/2+\theta\beta_2)}{(4\lambda-\beta_1+\theta)}}{\tau_I}-\dfrac{(6\lambda^2-\theta^2/2+\theta\beta_1-\beta_2)}{(4\lambda-\beta_1+\theta)}$	$\beta_1=\dfrac{\tau_1^{\,2}\left[\left(1-\dfrac{\lambda}{\tau_1}\right)^4 e^{-\theta/\tau_1}-1\right]-(\tau_2)^2\left[\left(1-\dfrac{\lambda}{\tau_2}\right)^4 e^{-\theta/\tau_2}-1\right]}{(\tau_2-\tau_1)}$ $\beta_2=\tau_2^{\,2}\left[\left(1-\dfrac{\lambda}{\tau_2}\right)^4 e^{-\theta/\tau_2}-1\right]+\tau_2\beta_1$
$\dfrac{K\,e^{-\theta s}}{s(\tau s+1)}$	$\dfrac{\tau_I}{\psi K(4\lambda-\beta_1+\theta)}$	$(\psi+\tau+\beta_1)-\dfrac{(6\lambda^2-\theta^2/2+\theta\beta_1-\beta_2)}{(4\lambda-\beta_1+\theta)}$	$\dfrac{(\psi\tau+(\tau+\psi)\beta_1+\beta_2)-\dfrac{(4\lambda^3+\theta^3/6-\beta_1\theta^2/2+\theta\beta_2)}{(4\lambda-\beta_1+\theta)}}{\tau_I}-\dfrac{(6\lambda^2-\theta^2/2+\theta\beta_1-\beta_2)}{(4\lambda-\beta_1+\theta)}$	$\beta_1=\dfrac{\psi^2\left[\left(1-\dfrac{\lambda}{\psi}\right)^4 e^{-\theta/\psi}-1\right]-(\tau)^2\left[\left(1-\dfrac{\lambda}{\tau}\right)^4 e^{-\theta/\tau}-1\right]}{(\tau-\psi)}$ $\beta_2=\tau^2\left[\left(1-\dfrac{\lambda}{\tau}\right)^4 e^{-\theta/\tau}-1\right]+\tau\beta_1$
$\dfrac{K\,e^{-\theta s}}{\tau s-1}$	$-\dfrac{\tau_I}{K(3\lambda-2\beta+\theta)}$	$(-\tau+2\beta)-\dfrac{(3\lambda^2-\theta^2/2+2\beta\theta-\beta^2)}{(3\lambda-2\beta+\theta)}$	$\dfrac{(-2\tau\beta+\beta^2)-\dfrac{(\lambda^3+\theta^3/6-\beta\theta^2+\beta^2\theta)}{(3\lambda-2\beta+\theta)}}{\tau_I}-\dfrac{(3\lambda^2-\theta^2/2+2\beta\theta-\beta^2)}{(3\lambda-2\beta+\theta)}$	$\beta=\tau\left[\left(\left(1+\dfrac{\lambda}{\tau}\right)^3 e^{\theta/\tau}\right)^{1/2}-1\right]$
$\dfrac{K\,e^{-\theta s}}{(\tau s-1)(as+1)}$	$-\dfrac{\tau_I}{K(4\lambda-2\beta+\theta)}$	$(a-\tau+2\beta)-\dfrac{(6\lambda^2-\theta^2/2+2\beta\theta-\beta^2)}{(4\lambda-2\beta+\theta)}$	$\dfrac{(-a\tau-2\beta(-a+\tau)+\beta^2)-\dfrac{(4\lambda^3+\theta^3/6-\beta\theta^2+\beta^2\theta)}{(4\lambda-2\beta+\theta)}}{\tau_I}-\dfrac{(6\lambda^2-\theta^2/2+2\beta\theta-\beta^2)}{(4\lambda-2\beta+\theta)}$	$\beta=\tau\left[\left\{\left(\dfrac{\lambda}{\tau}+1\right)^4 e^{\theta/\tau}\right\}^{1/2}-1\right]$

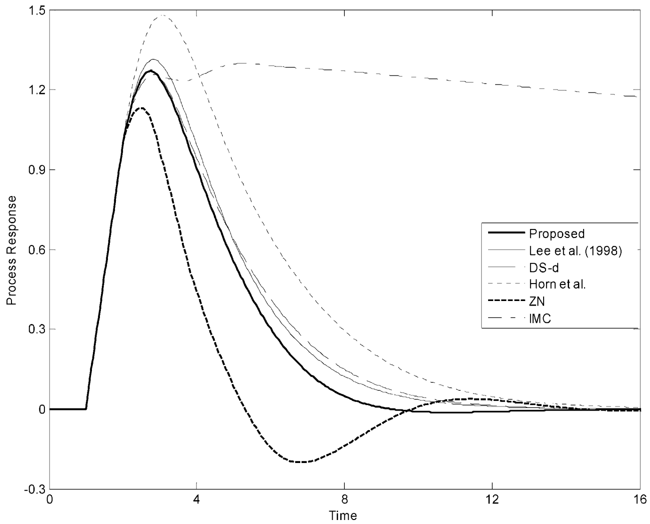

Figure 2. Simulation results of PID controllers for unit step disturbance (example 1).

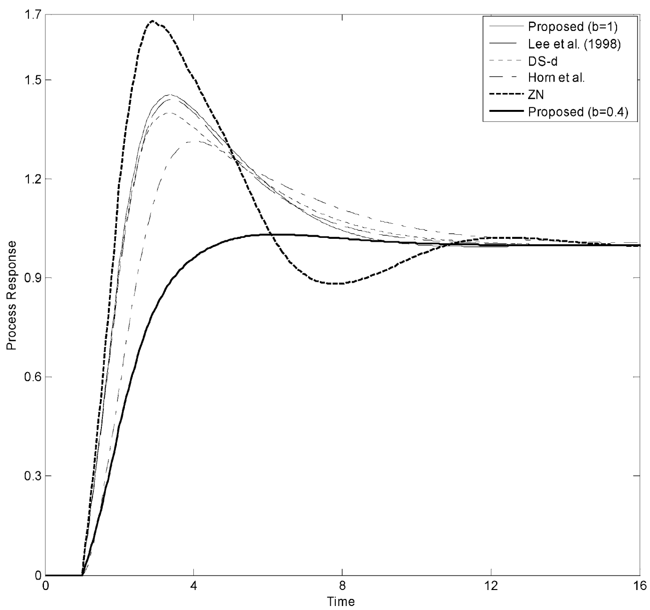

Figure 3. Simulation results of PID controllers for unit step set-point change (example 1).

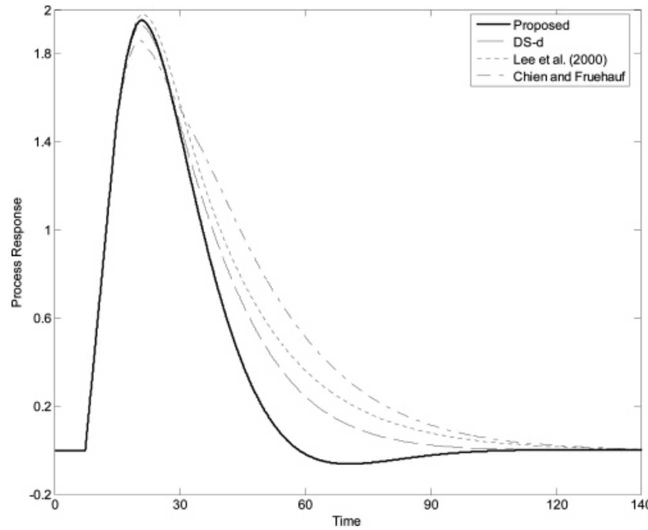

Figure 4. Simulation results of PID controllers for unit step disturbance (example 2).

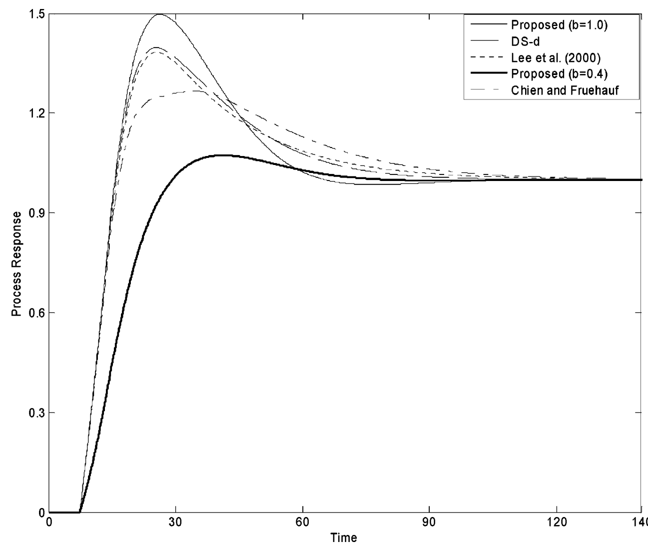

Figure 5. Simulation results of PID controllers for unit step set-point change (example 2).

proposed a model reduction technique to reduce the higher-order process model to a lower-order model and also developed the SIMC−PID rule for improved disturbance rejection in several lag-time dominant processes.

It is clear that, in the IMC−PID approach, the performance of the PID controller is mainly determined by the IMC filter structure. In most previous works of IMC−PID design, the IMC filter structure has been designed to be just as simple as to satisfy a necessary performance of the IMC controller. For example, the order of the lead term in the IMC filter is designed to be small enough to cancel out the dominant process poles, and the lag term is simply set to make the IMC controller proper. However, the performance of the resulting PID controller is based not only on the IMC controller performance but also on how closely the PID controller approximates the ideal controller equivalent to the IMC controller, which mainly depends on the structure of the IMC filter. Therefore, for the IMC−PID design, the optimum IMC filter structure has to be selected considering the performance of the resulting PID controller rather than that of the IMC controller.

In the present study, we have the following objectives:

(1) Determine the optimum IMC filter, which gives the best performance of the resulting PID controller (according to Skogestad,[8] “best” could, for example, mean it minimizes the integrated absolute error (IAE) with a specified value of the sensitivity peak, Ms) for several representative process models. This filter is used to design a PID controller for disturbance rejection and the corresponding analytical IMC−PID tuning rule.

(2) Provide the guidelines for selection of a closed-loop time constant λ for several process models to cover a wide range of θ/τ.

(3) Conduct a robust study by inserting a perturbation uncertainty in each parameter simultaneously (for the worst-case model). Several illustrative examples are included to demonstrate the superiority of the proposed tuning methods.

2. IMC−PID Approach for PID Controller Design

Figure 1a shows the block diagrams of IMC control, where G_P is the process, $\tilde{G}_P$ is the process model, and q is the IMC

Table 2. PID Controller Setting for Example 1 ($\theta/\tau = 0.01$) (FOPDT)

						set point					disturbance				
tuning methods	λ	K_c	τ_I	τ_D	Ms	TV	IAE	ISE	ITAE	overshoot	TV	IAE	ISE	ITAE	overshoot
proposed ($b = 1.0$)	1.51	0.827	3.489	0.356	1.94	1.67	3.08	1.86	8.22	1.45	1.96	4.30	3.74	15.91	1.26
proposed ($b = 0.4$)	1.51	0.827	3.489	0.356	1.94	0.79	2.37	1.79	3.72	1.03	1.96	4.30	3.74	15.91	1.26
DS-d[5]	1.2	0.828	4.051	0.353	1.94	1.57	3.06	1.77	8.69	1.40	1.88	4.89	4.08	20.04	1.27
Lee et al.[4]	1.32	0.810	3.928	0.307	1.94	1.54	3.09	1.83	8.47	1.44	1.87	4.85	4.26	19.29	1.31
Horn et al.[3] [a]	1.68	15.146	100.5	0.497	1.94	1.62	3.56	1.95	23.23	1.31	1.69	6.64	6.41	30.85	1.47
ZN[11]		0.948	1.99	0.498	2.29	3.25	3.58	2.21	11.07	1.67	3.04	3.22	2.18	12.7	1.17
IMC[1]	0.85	0.744	100.5	0.498	1.94	1.18	2.11	1.46	15.29	1.0	1.58	84.47	77.74	3634	1.29

[a] $G_c = K_c(1 + 1/\tau_I s + \tau_D s)(1 + cs + ds^2)/(1 + as + bs^2)$ where $a = 100.2127$, $b = 21.2687$, $c = 4.2936$, and $d = 0$.

Table 3. Robust Analysis for Example 1 (FOPDT), $G_P = G_D = 120\ e^{-1.2s}/(80s + 1)$

	set point					disturbance				
tuning methods	TV	IAE	ISE	ITAE	overshoot	TV	IAE	ISE	ITAE	overshoot
proposed ($b = 1.0$)	6.36	5.47	3.50	27.77	2.12	7.33	6.31	6.39	36.03	1.95
DS-d[5]	6.20	5.23	3.24	26.46	2.06	7.19	6.28	6.50	35.53	1.96
Lee et al.[4]	5.93	5.61	3.49	30.0	2.09	7.00	6.80	7.07	40.39	2.1
Horn et al.[3]	4.44	5.15	2.92	35.98	1.84	5.91	7.76	9.52	45.09	2.23
ZN[11]	24.21	12.89	8.01	166.2	2.56	24.13	12.50	8.36	170.8	1.83
IMC[1]	4.58	3.26	1.81	17.53	1.43	6.15	84.59	79.07	3616	1.92

Table 4. PID Controller Setting for Example 2 (Distillation Column Model)

						set point					disturbance				
tuning methods	λ	K_c	τ_I	τ_D	Ms	TV	IAE	ISE	ITAE	overshoot	TV	IAE	ISE	ITAE	overshoot
proposed ($b = 1.0$)	11.3	0.531	24.533	2.467	1.90	1.128	24.04	14.65	495.4	1.49	1.99	49.19	66.86	1366	1.95
proposed ($b = 0.4$)	11.3	0.531	24.533	2.467	1.90	0.58	18.41	13.48	243.8	1.07	1.99	49.19	66.86	1366	1.95
DS-d[5]	9.15	0.543	31.15	2.558	1.90	1.0	23.28	13.36	508.3	1.39	1.84	57.47	73.35	1794	1.93
Lee et al.[9]	11	0.536	35.137	2.286	1.90	0.95	23.46	13.14	550.2	1.38	1.78	65.35	81.95	2292	1.98
Chien and Fruehauf[2]	15.28	0.526	37.96	3.339	1.90	0.99	23.68	12.62	630	1.27	1.81	71.88	86.57	2716	1.86

Table 5. Robust Analysis for Example 2 (Distillation Column Model), $G_P = G_D = 0.24\ e^{-8.88s}/s$

	set point					disturbance				
tuning methods	TV	IAE	ISE	ITAE	overshoot	TV	IAE	ISE	ITAE	overshoot
proposed($b = 1.0$)	1.89	27.81	20.30	565.4	1.88	3.27	49.86	88.05	1335	2.54
DS-d[5]	1.80	25.38	17.79	499.5	1.77	3.09	57.47	90.24	1793	2.52
Lee et al.[9]	1.74	25.85	17.56	549.3	1.75	3.06	65.34	99.4	2288	2.57
Chien and Fruehauf[2]	1.62	25.14	15.23	580.9	1.58	2.90	71.69	98.48	2683	2.43

controller. In the IMC control structure, the controlled variable is related as

$$C = \frac{G_P q}{1 + q(G_P - \tilde{G}_P)} f_R R + \left[\frac{1 - \tilde{G}_P q}{1 + q(G_P - \tilde{G}_P)}\right] G_D d \quad (1)$$

For the nominal case (i.e., $G_P = \tilde{G}_P$), the set-point and disturbance responses are simplified to

$$\frac{C}{R} = \tilde{G}_P q f_R \quad (2)$$

$$\frac{C}{d} = [1 - \tilde{G}_P q] G_D \quad (3)$$

In the classical feedback control structure shown in Figure 1b, the set-point and disturbance responses are represented by

$$\frac{C}{R} = \frac{G_c G_P f_R}{1 + G_c G_P} \quad (4)$$

$$\frac{C}{d} = \frac{G_D}{1 + G_c G_P} \quad (5)$$

where G_c denotes the equivalent feedback controller.

According to the IMC parametrization (Morari and Zafiriou[6]), the process model $\tilde{G}_P$ is decomposed into two parts,

$$\tilde{G}_P = P_M P_A \quad (6)$$

where P_M and P_A are the portions of the model inverted and not inverted, respectively, by the controller (P_A is usually a nonminimum phase and contains dead times and/or right half plane zeros); $P_A(0) = 1$.

The IMC controller is designed by

$$q = P_M^{-1} f \quad (7)$$

The ideal feedback controller that is equivalent to the IMC controller can be expressed in terms of the internal model, $\tilde{G}_P$, and the IMC controller, q:

$$G_c = \frac{q}{1 - \tilde{G}_P q} \quad (8)$$

Since the resulting controller does not have a standard PID controller form, the remaining issue is to design the PID controller that approximates the equivalent feedback controller most closely. Lee et al.[4] proposed an efficient method for converting the ideal feedback controller G_c to a standard PID

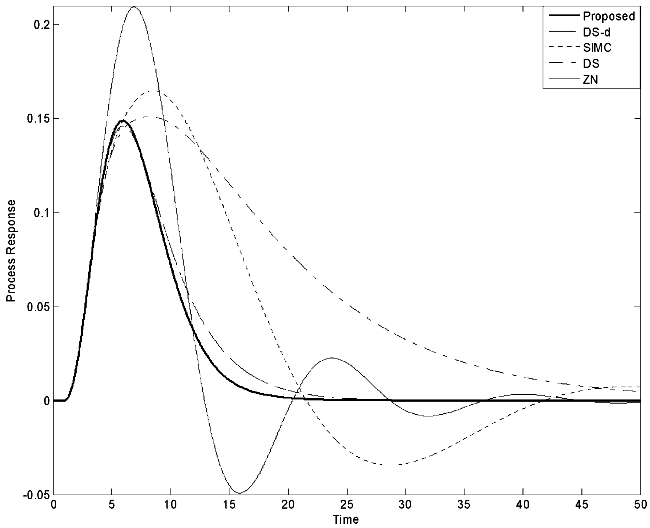

Figure 6. Simulation results of PID controllers for unit step disturbance (example 3).

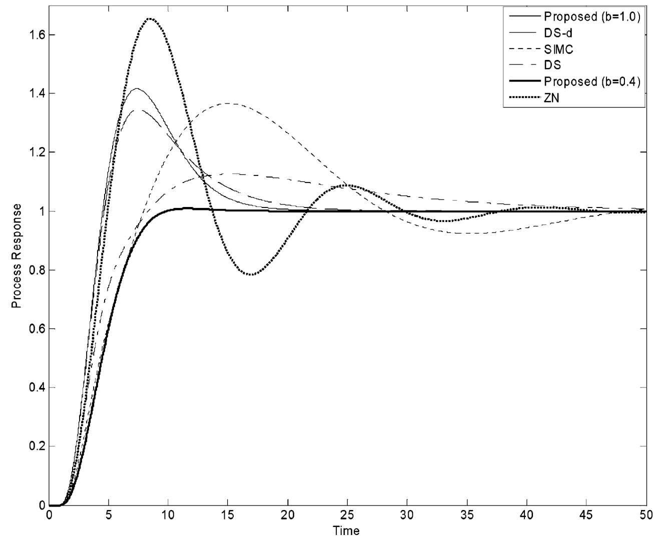

Figure 7. Simulation results of PID controllers for unit step set-point change (example 3).

controller. Since G_c has an integral term, it can be expressed as

$$G_c = \frac{g(s)}{s} \tag{9}$$

Expanding G_c in Maclaurin series in s gives

$$G_c = \frac{1}{s}\left(g(0) + g'(0)s + \frac{g''(0)}{2}s^2 + ...\right) \tag{10}$$

The first three terms of the above expansion can be interpreted as the standard PID controller, which is given by

$$G_c = K_c\left(1 + \frac{1}{\tau_I s} + \tau_D s\right) \tag{11}$$

where

$$K_c = g'(0) \tag{12a}$$

$$\tau_I = g'(0)/g(0) \tag{12b}$$

$$\tau_D = g''(0)/2g'(0) \tag{12c}$$

3. IMC−PID Tuning Rules for Typical Process Models

This section proposes the tuning rules for several typical time-delayed process models.

3.1. First-Order Plus Dead Time Process (FOPDT). The most commonly used approximate model for chemical processes is the FOPDT model as given below,

$$G_P = G_D = \frac{K\,e^{-\theta s}}{\tau s + 1} \tag{13}$$

where K is the gain, τ is the time constant, and θ is the time delay. The optimum IMC filter structure is found as $f = (\beta s + 1)^2/(\lambda s + 1)^3$. The resulting IMC controller becomes $q = (\tau s + 1)(\beta s + 1)^2/K(\lambda s + 1)^3$. Therefore, the ideal feedback controller, which is equivalent to the IMC controller, is

$$G_c = \frac{(\tau s + 1)(\beta s + 1)^2}{K[(\lambda s + 1)^3 - e^{-\theta s}(\beta s + 1)^2]} \tag{14}$$

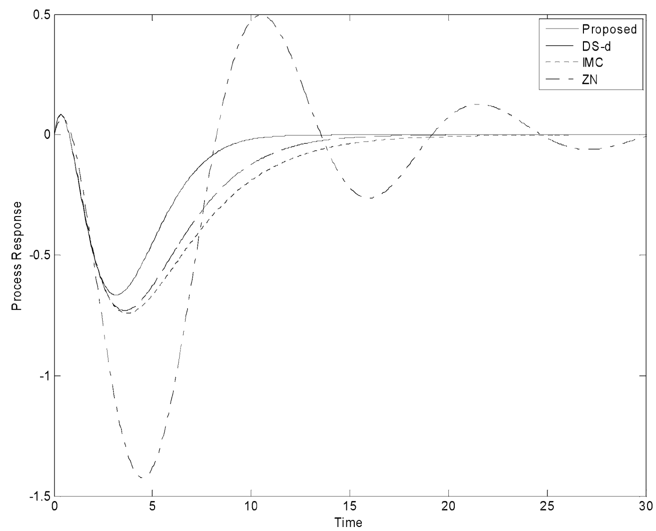

Figure 8. Simulation results of PID controllers for unit step disturbance (example 4).

The analytical PID formula can be obtained from eq 12 as

$$K_C = \frac{\tau_I}{K(3\lambda - 2\beta + \theta)} \tag{15a}$$

$$\tau_I = (\tau + 2\beta) - \frac{(3\lambda^2 - \theta^2/2 + 2\beta\theta - \beta^2)}{(3\lambda - 2\beta + \theta)} \tag{15b}$$

$$\tau_D = \frac{(2\tau\beta + \beta^2) - \dfrac{(\lambda^3 + \theta^3/6 - \beta\theta^2 + \beta^2\theta)}{(3\lambda - 2\beta + \theta)}}{\tau_I} - \frac{(3\lambda^2 - \theta^2/2 + 2\beta\theta - \beta^2)}{(3\lambda - 2\beta + \theta)} \tag{15c}$$

The value of the extra degree of freedom β is selected so that it cancels out the open-loop pole at $s = -1/\tau$ that causes the sluggish response to the load disturbance. Thus, β is chosen so that the term $[1 - Gq]$ has a zero at the pole of G_D. That is, we want $[1 - Gq]|_{s=-1/\tau} = 0$ and $[1 - (\beta s + 1)^2\, e^{-\theta s}/(\lambda s +$

Table 6. PID Controller Setting for Example 3 (SOPDT)

						set point					disturbance				
tuning methods	λ	K_c	τ_I	τ_D	Ms	TV	IAE	ISE	ITAE	overshoot	TV	IAE	ISE	ITAE	overshoot
proposed ($b = 1.0$)	1.6	6.415	6.859	1.9798	1.87	11.59	5.66	3.36	28.50	1.41	1.83	1.06	0.11	7.90	0.14
proposed ($b = 0.4$)	1.6	6.415	6.859	1.9798	1.87	4.86	4.72	3.63	13.17	1.0	1.83	1.06	0.11	7.90	0.14
DS-d[5]	2.4	6.384	7.604	2.0977	1.87	10.82	5.67	3.22	31.19	1.34	1.71	1.19	0.12	9.77	0.14
SIMC[8]	0.43	3.496	5.72	5.0	1.87	5.514	10.08	4.77	133.6	1.36	1.47	2.50	0.26	38.50	0.16
DS[10]	0.5	5.0	15.0	3.33	1.91	7.18	6.53	3.28	68.19	1.12	1.42	3.0	0.31	49.47	0.15
ZN[11]		4.72	5.83	1.46	2.26	12.75	8.73	4.88	78.02	1.65	2.71	1.79	0.23	18.9	0.21

Table 7. Robust Analysis for Example 3 (SOPDT), $G_P = G_D = 2.4\ e^{-1.2s}/(8s + 1)(4s + 1)$

	set point					disturbance				
tuning methods	TV	IAE	ISE	ITAE	overshoot	TV	IAE	ISE	ITAE	overshoot
proposed ($b = 1.0$)	41.88	5.11	2.95	29.45	1.46	6.41	1.09	0.11	8.61	0.17
DS-d[5]	47.93	5.14	2.87	33.12	1.38	7.40	1.21	0.19	10.49	0.17
SIMC[8]	50.36	8.71	3.99	104.9	1.37	14.19	2.31	0.25	31.78	0.18
DS[10]	82.83	6.34	2.99	76.93	1.22	16.50	3.00	0.30	49.41	0.17
ZN[11]	14.90	6.00	3.88	30.23	1.68	3.09	1.29	0.22	8.69	0.24

Table 8. PID Controller Setting for Example 4 (Level Control Problem)

						set point					disturbance				
tuning methods	λ	K_c	τ_I	τ_D	Ms	TV	IAE	ISE	ITAE	overshoot	TV	IAE	ISE	ITAE	overshoot
proposed ($b = 1.0$)	0.935	−1.456	4.195	1.250	1.94	4.70	3.087	1.90	9.64	1.40	3.43	2.96	1.38	12.11	−0.66
proposed ($b = 0.4$)	0.935	−1.456	4.195	1.250	1.94	1.49	2.536	1.96	4.14	1.01	3.43	2.96	1.38	12.11	−0.66
DS-d[5]	1.6	−1.25	5.3	1.45	1.93	3.70	3.42	1.91	13.57	1.32	3.14	4.32	2.17	22.49	−0.72
IMC[1]	1.25	−1.22	6.0	1.5	1.95	3.58	3.505	1.88	15.49	1.28	3.10	5.00	2.48	29.53	−0.74
ZN[11]		−0.752	3.84	0.961	2.77	2.89	7.401	4.09	59.91	1.79	4.03	9.65	7.64	84.17	−1.42

$1)^3]|_{s=-1/\tau} = 0$. The value of β after some simplification becomes

$$\beta = \tau\left[1 - \left(\left(1 - \frac{\lambda}{\tau}\right)^3 e^{-\theta/\tau}\right)^{1/2}\right] \tag{16}$$

3.2. Delayed Integrating Process (DIP).

$$G_P = G_D = \frac{K\,e^{-\theta s}}{s} \tag{17}$$

The delayed integrating process (DIP) can be modeled by considering the integrator as a stable pole near zero. This is necessary because it is not possible to apply the aforementioned IMC procedure for DIP, since the term of β disappears at $s = 0$. Usually, the controller based on the model with a stable pole near zero can give a more robust closed-loop response than that based on the model with an integrator or unstable pole near zero, as suggested by Lee et al.[9] Therefore, DIP can be approximated to FOPDT as shown below,

$$G_P = G_D = \frac{K\,e^{-\theta s}}{s} = \frac{K\,e^{-\theta s}}{s + 1/\psi} = \frac{\psi K\,e^{-\theta s}}{\psi s + 1} \tag{18}$$

where ψ is an arbitrary constant with a sufficiently large value. Accordingly, the optimum filter structure for DIP is the same as that for the FOPDT model, i.e., $f = (\beta s + 1)^2/(\lambda s + 1)^3$.

Therefore, the resulting IMC controller becomes $q = (\psi s + 1)(\beta s + 1)^2/K\psi(\lambda s + 1)^3$, and the ideal feedback controller is $G_c = (\psi s + 1)(\beta s + 1)^2/K\psi[(\lambda s + 1)^3 - e^{-\theta s}(\beta s + 1)^2]$. The resulting PID tuning rules are listed in Table 1.

3.3. Second-Order Plus Dead Time Process (SOPDT). Consider a stable SOPDT system:

$$G_P = G_D = \frac{K\,e^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)} \tag{19}$$

The optimum IMC filter structure is found as $f = (\beta_2 s^2 + \beta_1 s + 1)/(\lambda s + 1)^4$. The IMC controller becomes $q = (\tau_1 s + 1)(\tau_2 s + 1)(\beta_2 s^2 + \beta_1 s + 1)/K(\lambda s + 1)^4$, and the ideal feedback controller equivalent to the IMC controller is $G_c = (\tau_1 s + 1)(\tau_2 s + 1)(\beta_2 s^2 + \beta_1 s + 1)/K[(\lambda s + 1)^4 - e^{-\theta s}(\beta_2 s^2 + \beta_1 s + 1)]$. The resulting PID tuning rules are listed in Table 1.

3.4. First-Order Delayed Integrating Process (FODIP). Consider the following FODIP system:

$$G_P = G_D = \frac{K\,e^{-\theta s}}{s(\tau s + 1)} \tag{20}$$

The above process can be approximated as the SOPDT model, and it becomes

$$G_P = G_D = \frac{K\,e^{-\theta s}}{s(\tau s + 1)} = \frac{\psi K\,e^{-\theta s}}{(\psi s + 1)(\tau s + 1)} \tag{21}$$

where ψ is an arbitrary constant with a sufficiently large value. Thus, the optimum IMC filter is the same as that for the SOPDT, $f = (\beta_2 s^2 + \beta_1 s + 1)/(\lambda s + 1)^4$. Therefore, the resulting IMC controller becomes $q = (\tau s + 1)(\psi s + 1)(\beta_2 s^2 + \beta_1 s + 1)/K\psi(\lambda s + 1)^4$. The resulting PID tuning rules are listed in Table 1.

3.5. First-Order Delayed Unstable Process (FODUP). One of the most popular unstable processes with time delay is the FODUP:

$$G_P = G_D = \frac{K\,e^{-\theta s}}{\tau s - 1} \tag{22}$$

The optimum IMC filter is found to be $f = (\beta s + 1)^2/(\lambda s + 1)^3$. Therefore, the IMC controller becomes $q = (\tau s - 1)(\beta s + 1)^2/K(\lambda s + 1)^3$, and the ideal feedback controller is $G_c = (\tau s - 1)(\beta s + 1)^2/K[(\lambda s + 1)^3 - e^{-\theta s}(\beta s + 1)^2]$. The resulting PID tuning rules are given in Table 1.

3.6. Second-Order Delayed Unstable Process (SODUP). The process model is

$$G_P = G_D = \frac{K\,e^{-\theta s}}{(\tau s - 1)(a s + 1)} \tag{23}$$

Table 9. Robust Analysis for Example 4 (Level Control Problem), $G_P = G_D = -1.92(-0.6s + 1)/s(2.4s + 1)$

tuning methods	set point					disturbance				
	TV	IAE	ISE	ITAE	overshoot	TV	IAE	ISE	ITAE	overshoot
proposed ($b = 1.0$)	17.89	2.318	1.61	8.49	1.26	13.23	3.13	1.30	12.41	−0.59
DS-d[5]	13.85	2.74	1.63	12.22	1.23	11.95	4.47	2.06	22.68	−0.67
IMC[1]	14.70	2.85	1.63	14.04	1.21	12.93	5.14	2.39	29.55	−0.68
ZN[11]	2.59	4.49	2.98	19.07	1.70	3.71	6.47	6.40	31.58	−1.49

The optimum IMC filter is found to be $f = (\beta s + 1)^2/(\lambda s + 1)^4$. The IMC controller is $q = (\tau s - 1)(as + 1)(\beta s + 1)^2/K(\lambda s + 1)^4$. The resulting PID tuning rules are shown in Table 1.

4. Performance and Robustness Measure

In this study, the performance and robustness of the control system are evaluated by the following indices.

4.1. Integral Error Criteria. Three popular performance indices based on integral error are used to evaluate the performance: integral of the absolute error (IAE), integral of the squared error (ISE), and integral of the time-weighted absolute error (ITAE).

$$\mathrm{IAE} = \int_0^\infty |e(t)|\, \mathrm{d}t \tag{24}$$

$$\mathrm{ISE} = \int_0^\infty e(t)^2\, \mathrm{d}t \tag{25}$$

$$\mathrm{ITAE} = \int_0^\infty t|e(t)|\, \mathrm{d}t \tag{26}$$

where the error signal $e(t)$ is the difference between the set point and the measurement. The ISE criterion penalizes larger errors, whereas the ITAE criterion penalizes long-term errors. The IAE criterion tends to produce controller settings that are between those for the ITAE and ISE criteria.

4.2. Overshoot. Overshoot is a measure of how much the response exceeds the ultimate value following a step change in set point and/or disturbance.

4.3. **Maximum Sensitivity (Ms) to Modeling Error.** To evaluate the robustness of a control system, the maximum sensitivity, Ms, which is defined by $\mathrm{Ms} = \max |1/[1 + G_P G_c(i\omega)]|$, is used. Since Ms is the inverse of the shortest distance from the Nyquist curve of the loop transfer function to the critical point (−1,0), a small value indicates that the stability margin of the control system is large. Typical values of Ms are in the range of 1.2−2.0.[12] For fair comparison, throughout all our simulation examples, all the controllers compared were designed to have the same robustness level in terms of the maximum sensitivity, Ms.

4.4. Total Variation (TV). To evaluate the manipulated input usage, we compute TV of the input $u(t)$, which is the sum of all its moves up and down. If we discretize the input signal as a sequence $[u_1, u_2, u_3, ..., u_i, ...]$, then $\mathrm{TV} = \sum_{i=1}^{\infty} |u_{i+1} - u_i|$ should be as small as possible. TV is a good measure of the smoothness of a signal.[8]

4.5. Set-Point and Derivative Weighting. The conventional form of the PID controller that is used for the simulation in this study is given as

$$G_{\mathrm{PID}} = K_c\left(1 + \frac{1}{\tau_I s} + \tau_D s\right) \tag{27}$$

A more widely accepted control structure that includes set-point weighting and derivative weighting is given by Åström and Hägglund.[13] The PID controller after set-point and derivative weighting becomes

$$u(t) = K_c\left([br(t) - y(t)] + \frac{1}{\tau_I}\int_0^t [r(t) - y(t)]\, \mathrm{d}\tau + \tau_D \frac{\mathrm{d}[cr(t) - y(t)]}{\mathrm{d}t}\right) \tag{28}$$

where b and c are additional parameters. The integral term must be based on error feedback to ensure the desired steady state. The controller given by eq 28 has a structure with two degrees of freedom. The set-point weighting coefficient b is bounded by $0 \le b \le 1$, and the derivative weighting coefficient c is also bounded by $0 \le c \le 1$. The overshoot for set-point changes decreases with decreasing b.

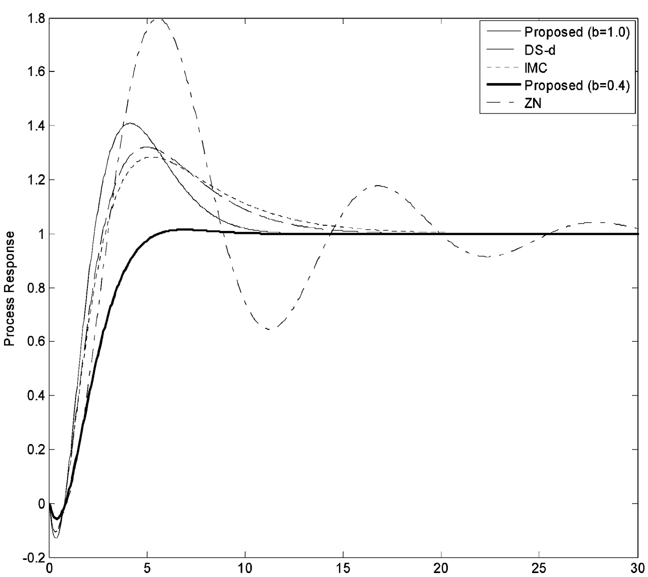

Figure 9. Simulation results of PID controllers for unit step set-point change (example 4).

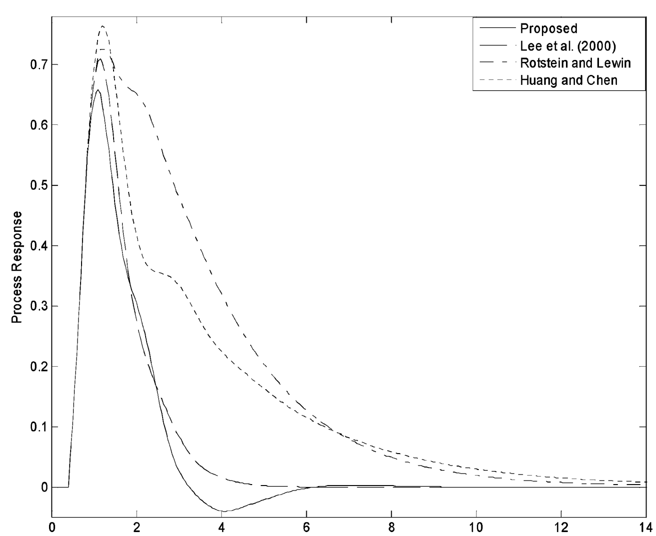

Figure 10. Simulation results of PID controllers for unit step disturbance (example 5).

Table 10. PID Controller Setting for Example 5 (FODUP)

						set point					disturbance				
tuning methods	λ	K_c	τ_I	τ_D	Ms	TV	IAE	ISE	ITAE	overshoot	TV	IAE	ISE	ITAE	overshoot
proposed ($b = 1.0$)	0.63	2.573	2.042	0.207	3.08	9.58	2.12	1.56	3.17	2.06	3.71	0.92	0.37	1.55	0.65
proposed ($b = 0.1$)	0.63	2.573	2.042	0.207	3.08	2.44	1.30	0.91	1.27	1.07	3.71	0.92	0.37	1.55	0.65
Lee et al.[9]	0.5	2.634	2.519	0.154	3.03	9.13	2.09	1.68	2.79	2.21	3.54	0.96	0.44	1.50	0.71
De Paor and O Malley[16]		1.459	2.667	0.25	4.92	11.01	8.74	6.94	57.66	2.53	7.02	5.96	3.49	39.14	1.13
Rotstein and Lewin [17]		2.250	5.760	0.20	2.48	6.32	3.74	2.28	11.24	1.82	3.03	2.56	1.18	8.19	0.72
Huang and Chen[14]		2.636	5.673	0.118	3.21	9.14	3.28	1.96	9.86	2.19	3.62	2.15	0.80	7.57	0.76

Table 11. Robust Analysis for Example 5 (FODUP), $G_P = G_D = 1.2\ e^{-0.48s}/1.2s - 1$

	set point					disturbance				
tuning methods	TV	IAE	ISE	ITAE	overshoot	TV	IAE	ISE	ITAE	overshoot
proposed ($b = 1.0$)	14.90	2.11	1.89	2.95	2.45	5.63	0.86	0.42	1.42	0.76
Lee et al.[9]	17.49	2.51	2.27	4.31	2.58	6.49	1.03	0.51	1.98	0.82
De Paor and O Malley[16]	7.38	5.62	4.33	23.86	2.31	4.79	3.95	2.45	17.23	1.08
Rotstein and Lewin[17]	7.97	3.48	2.01	10.89	2.05	3.69	2.56	1.09	9.28	0.83
Huang and Chen[14]	18.29	3.24	2.41	9.71	2.49	6.90	2.15	0.84	8.35	0.86

The controllers obtained for different values of b and c respond to disturbance and measurement noise in the same way as a conventional PID controller, i.e., different values of b and c do not change the closed-loop response for disturbances.[5] Therefore, the same PID tuning rules developed for this study are also applicable for the modified PID controller in eq 28. However, the set-point response does depend on the values of b and c. In this study, the coefficient c was fixed as $c = 1$ for all simulation examples, while the set-point filter $f_R = (b\tau_I s + 1)/(\tau_I\tau_D s^2 + \tau_I s + 1)$ was used with $0 \le b \le 1$.

5. Simulation Results

This section demonstrates the simulation study for the six different kinds of process models, which are widely used in process industries and have also been studied by other researchers. In every simulation study, different performance and robustness matrices have been calculated and compared with other existing methods.

5.1. Example 1: FOPDT. The following FOPDT model (Chen and Seborg[5]) has been considered.

$$G_P = G_D = \frac{100\ e^{-1s}}{100s + 1} \tag{29}$$

A unit step disturbance is acting at the plant input, and the corresponding simulation result is shown in Figure 2. All the tuning methods except the ZN method[11] were adjusted to have the same robustness level as Ms = 1.94 by varying the λ value. The performance matrix for the disturbance rejection is listed in Table 2. The proposed method was compared with other existing IMC−PID methods (Horn et al.,[3] Lee et al.,[4] and Rivera et al.[1]), the direct synthesis method (Chen and Seborg[5]), and the ZN method.[11] The performance matrix containing IAE, ISE, and ITAE shows that the proposed method performs better than the other IMC−PID methods. IAE, ISE, ITAE, and overshoot values are minimized for the ZN method. However, the ZN method gives the Ms value of 2.29. The proposed method has better performance indices than the ZN method when Ms = 2.29.

The resulting output response when the unit step changes are introduced into the set point is shown in Figure 3. It is clear that, under the 1DOF control structure, any controller with good disturbance rejection essentially accompanies an excessive overshoot in the set-point response. To avoid this waterbed effect, a 2DOF control structure is used. The corresponding response is shown in the same figure. For the 1DOF case, the output response for the proposed method has a larger overshoot than all the other methods except the ZN method, but the settling time is faster for the proposed method than all the other methods. The overshoot for the proposed controller can be eliminated without affecting the disturbance response by setting $b = 0.4$ in the 2DOF controller, as suggested by Åström and Hägglund[13] and Chen and Seborg.[5] On the basis of the comparison of the output response and the value of the performance matrices listed in Table 2, it can be concluded that the proposed controller shows the best performance.

The robustness of the controller is evaluated by inserting a perturbation uncertainty of 20% in all three parameters simultaneously to obtain the worst-case model mismatch, i.e., $G_P = G_D = 120\ e^{-1.2s}/(80s + 1)$, as an actual process. The simulation results for the proposed and other tuning rules are given in Table 3 for both the set-point and the disturbance rejection. The error integral values for the disturbance rejection of the proposed method and that of Chen and Seborg[5] (DS-d) appear to be almost similar, whereas DS-d has slightly better performance. For the servo response, the IMC has a clear advantage, and the proposed, including the DS-d and those by Lee et al.[4] and Horn et al.,[3] give almost similar robust performances.

5.2. Example 2: DIP (Distillation Column Model). The distillation column model studied by Chien and Fruehauf[2] and

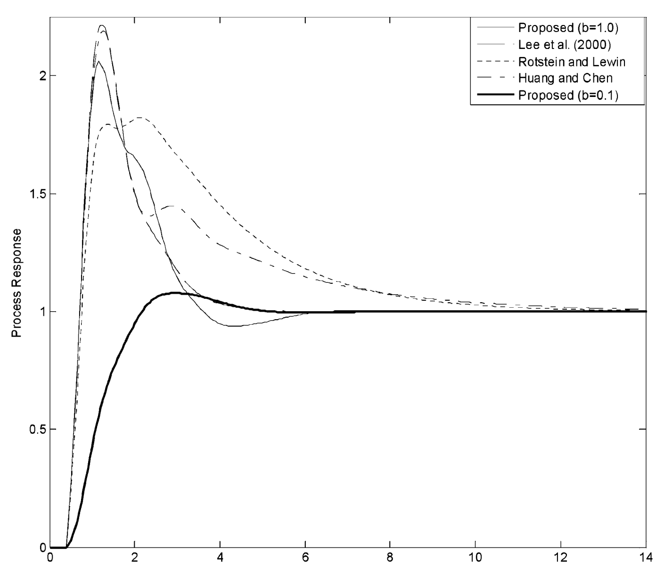

Figure 11. Simulation results of PID controllers for unit step set-point change (example 5).

Table 12. PID Controller Setting for Example 6 (SODUP)

						set point					disturbance				
tuning methods	λ	K_c	τ_I	τ_D	Ms	TV	IAE	ISE	ITAE	overshoot	TV	IAE	ISE	ITAE	overshoot
proposed ($b = 1.0$)	0.938	7.017	5.624	1.497	4.35	36.44	5.35	3.59	24.84	1.89	5.21	0.85	0.11	4.85	0.20
proposed ($b = 0.3$)	0.938	7.017	5.624	1.497	4.35	12.27	3.49	2.58	9.46	1.05	5.21	0.85	0.11	4.85	0.20
Lee et al.[9]	1.20	7.144	6.696	1.655	4.34	36.45	5.19	3.03	24.93	1.71	5.12	0.95	0.10	5.76	0.19
Huang and Chen [14]		6.186	7.17	1.472	3.63	26.04	5.57	3.70	25.95	1.85	4.25	1.16	0.17	7.04	0.23
Huang and Lin[15]		3.954	4.958	2.074	2.18	11.99	8.99	5.55	73.31	1.86	3.16	2.19	0.38	20.13	0.29
Poulin and Pomerleau[18]		3.050	7.557	2.07	1.86	8.71	11.0	6.98	105.8	1.88	3.00	3.81	0.97	40.02	0.40

Table 13. Robust Analysis for Example 6 (SODUP), $G_P = G_D = 1.2\ e^{-0.6s}/(5s - 1)(2s + 1)(0.5s + 1)$

	set point					disturbance				
tuning methods	TV	IAE	ISE	ITAE	overshoot	TV	IAE	ISE	ITAE	overshoot
proposed ($b = 1.0$)	167.03	13.54	5.88	285.7	2.04	23.56	1.95	0.15	41.29	0.22
Lee et al.[9]	248.09	15.19	5.45	446.6	1.84	34.50	2.26	0.15	63.86	0.21
Huang and Chen[14]	65.05	7.915	4.32	75.7	1.96	10.48	1.44	0.18	14.24	0.25
Huang and Lin[15]	13.34	7.303	4.55	46.38	1.82	3.43	1.78	0.34	13.33	0.31
Poulin and Pomerleau[18]	8.25	8.69	5.45	64.61	1.79	2.90	3.12	0.84	26.52	0.41

Chen and Seborg[5] was considered. The distillation column separates a small amount of a low-boiling material from the final product. The bottom level of the distillation column is controlled by adjusting the steam flow rate. The process model for the level control system is represented as the following DIP model, which can be approximated by the FOPDT model as

$$G_P = G_D = \frac{0.2\ e^{-7.4s}}{s} = \frac{20\ e^{-7.4s}}{100s + 1} \quad (30)$$

The method proposed by Chen and Seborg,[5] Lee et al.,[9] and Chien and Fruehauf[2] was used to design the PID controllers, as shown in Figure 4 and Table 4. $\lambda = 11.3$ was chosen for the proposed method, $\lambda = 9.15$ was chosen for Chen and Seborg,[5] $\lambda = 11.0$ was chosen for Lee et al.,[9] and $\lambda = 15.28$ was chosen for Chien and Fruehauf,[2] resulting in Ms = 1.90. Figure 4 shows the output response, where the proposed tuning rule results in the least settling time. Chien and Fruehauf's method[2] has the slowest response and requires the highest settling time. The performance matrix in Table 4 shows that the proposed method has the lowest error integral value while Chien and Fruehauf's[2] method has the highest value at an equal robustness level. On the basis of Table 4 and Figure 4, it is clear that the proposed method performs better than the other conventional methods.

The simulation results for the unit set-point change are shown in Figure 5. The overshoot for the proposed method is large, but the settling time is minimized for the 1DOF controller. The overshoot can be minimized by using the 2DOF controller with $b = 0.4$. The results in Table 4 and Figure 4 show the superior performance of the proposed method.

The robustness of the controller is evaluated by inserting a perturbation uncertainty of 20% in both parameters simultaneously. The worst plant–model mismatch case after 20% perturbation is $G_P = G_D = 0.24\ e^{-8.88s}/s$. The simulation results for all tuning rules are given in Table 5. It is clear that the proposed method has better performance for disturbance rejection followed by DS-d and that by Lee et al.[9] The set-point responses of the DS-d and the methods by Lee et al.[9] and Chien and Fruehauf[2] are almost similar and superior to the proposed method.

5.3. Example 3: SOPDT. Consider the SOPDT model described by Chen and Seborg:[5]

$$G_P = G_D = \frac{2\ e^{-1s}}{(10s + 1)(5s + 1)} \quad (31)$$

The proposed, DS-d (Chen and Seborg[5]), ZN, SIMC (Skogestad[8]), and DS (Smith et al.,[10] Seborg et al.[7]) methods were used to design the PID controller. The DS and IMC–PID (Rivera et al.[1]) methods give the exact same tuning formula for the SOPDT process. The parameters of the PID controller settings for the DS and ZN methods were taken from Chen and Seborg.[5] All the other methods were adjusted to have the equal Ms value of 1.87 for a fair comparison. Figure 6 shows the output response for all tuning methods mentioned above. The proposed method has a similar overshoot to that of the DS-d method, while the ZN tuning method gives the highest peak. The settling time is the lowest for the proposed method, whereas the DS method shows the slowest response. Apart from the DS-d method, all the other tuning methods have either higher overshoot or slower response for disturbance rejection.

The simulation results for the unit set-point change are shown in Figure 7. The overshoot for the ZN method is the largest followed by that of the proposed method, but the settling time is the lowest for the proposed method among all the 1DOF controllers. The overshoot can be minimized by using the 2DOF controller with $b = 0.4$. On the basis of the performance shown in the above figures and the performance matrix in Table 6, the proposed method has the best performance.

To evaluate the robust performance, the worst plant–model mismatch case was considered as $G_P = G_D = 2.4\ e^{-1.2s}/(8s + 1)(4s + 1)$ by inserting a perturbation uncertainty of 20% in all

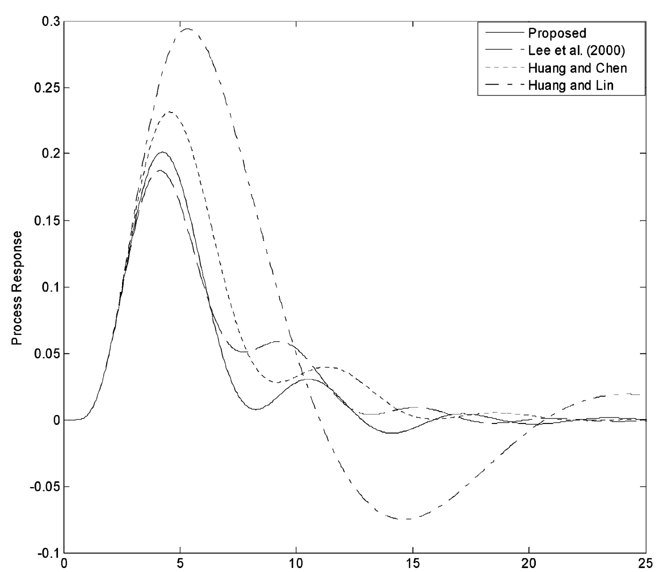

Figure 12. Simulation results of PID controllers for unit step disturbance (example 6).

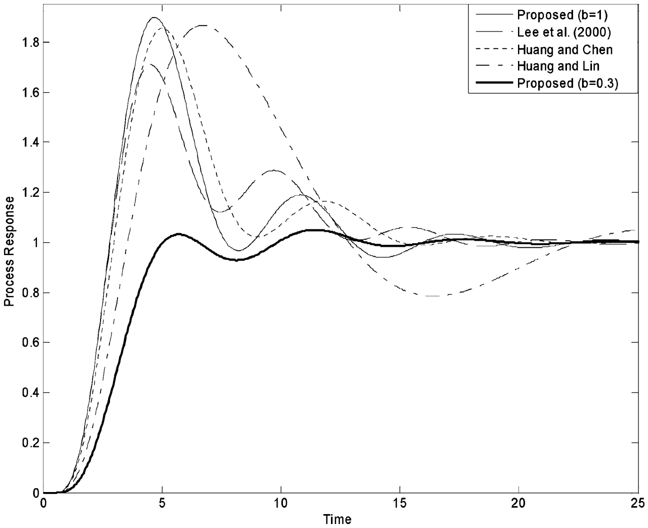

Figure 13. Simulation results of PID controllers for unit step set-point change (example 6).

three parameters simultaneously. The simulation results for all tuning rules are given in Table 7. When compared to the other methods, the error integral values for the proposed method are the best for both set-point and disturbance rejection, and the overshoot for the proposed method is similar to those by the DS-d and DS methods. The ZN method has a higher overshoot than the other methods.

5.4. Example 4: FODIP (Reboiler Level Model). Consider a level-control problem proposed by Chen and Seborg.[5] It is an approximate model for a liquid level in the reboiler of a steam-heated distillation column, which is to be controlled by adjusting the control valve on the steam line. The process model is given by

$$G_P = G_D = \frac{-1.6(-0.5s + 1)}{s(3s + 1)} \tag{32}$$

This kind of "inverse response time constant" (negative numerator time constant) can be approximated as a time delay such as $(-\theta_0^{inv}s + 1) \approx e^{-\theta_0^{inv}}$. This is reasonable since an inverse response has a deteriorating effect on control, similar to that of a time delay.[8]

Therefore, the above model can be approximated as

$$G_P = G_D = \frac{-1.6\ e^{-0.5s}}{s(3s + 1)} \tag{33}$$

The above process model can be treated as FODIP, and the tuning parameters can be estimated by the analytical rule proposed in Table 1.

Figure 8 shows the output response of the proposed tuning method and its comparison with the DS-d, IMC, and ZN methods. The PID controller settings for all the other methods were taken from Chen and Seborg.[5] The value $\lambda = 0.935$ was selected for the proposed method, which gives Ms = 1.94. From the figure, the proposed output response has a small overshoot and a fast settling time, followed by the DS-d and IMC methods, while the ZN method has a very aggressive response with significant overshoot and oscillation that takes a long time to settle. The performance values are also listed in Table 8 which indicates the clear advantage of the proposed method over other tuning rules.

The output responses for the unit set-point change are shown in Figure 9. The overshoot by the proposed method with the 1DOF structure is somewhat large but shows a fast settling time before reaching its final value. The overshoot can be drastically minimized by using the 2DOF controller. It is apparent from Figure 8 and Table 8 that the proposed method is superior over the other tuning methods for the disturbance rejection.

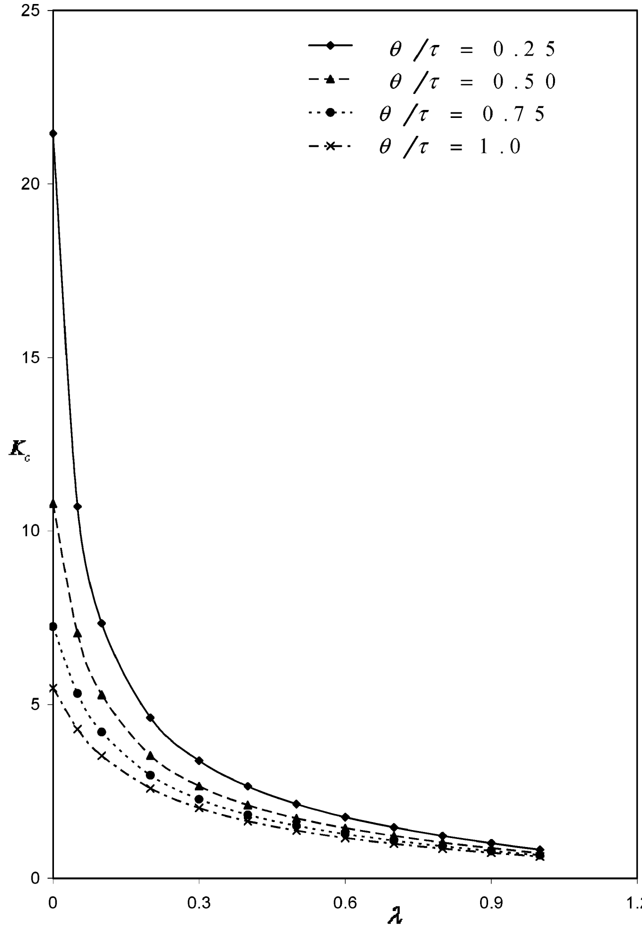

Figure 14. Proportional gain (K_c) setting for different λ values.

The robustness of the controller is evaluated considering the worst case under a 20% uncertainty in all three parameters as $G_P = G_D = -1.92(-0.6s + 1)/s(2.4s + 1)$. The simulation results for all tuning rules are given in Table 9. The error integral and overshoot values for the proposed method prove to be the best. The overshoot and IAE of the ZN method is the highest among the other tuning methods.

5.5. Example 5: FODUP. The following FODUP (Huang & Chen[14] and Lee et al.[9]) was considered:

$$G_P = G_D = \frac{1\ e^{-0.4s}}{s - 1} \tag{34}$$

The closed-loop time constant $\lambda = 0.63$ is selected for the proposed tuning method for Ms = 3.08. Setting $\lambda = 0.5$ results in the same value of Ms = 3.08 for Lee et al.'s[9] method, thus providing a fair comparison. The controller setting parameters for the other existing methods were taken from Lee et al.[9] Figure 10 shows the output response of all tuning methods, with the proposed method showing the clear advantage. In Table 10, the performance and robustness matrices are listed for all tuning rules, and the proposed method shows a clear advantage over the other methods.

The response for the unit set-point change of the 1DOF controller is shown in Figure 11 where every 1DOF controller shows a significant overshoot. By using the 2DOF controller,

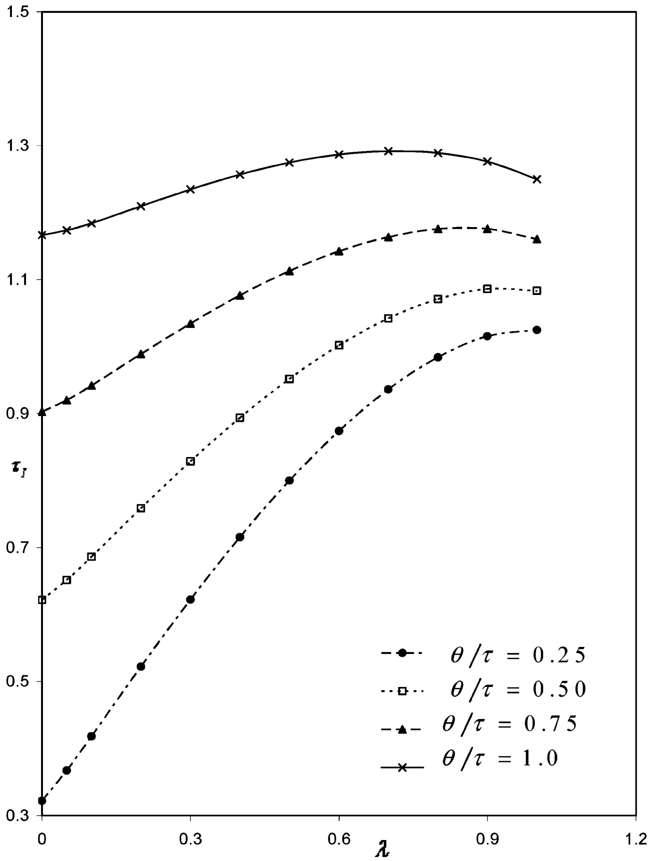

Figure 15. Integral time constant (τ_I) setting for different λ values.

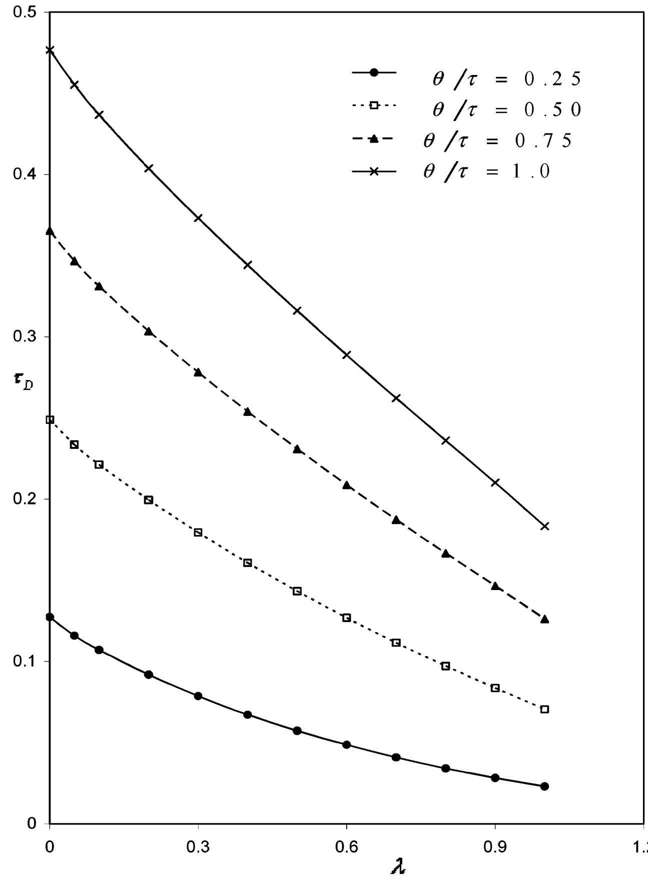

Figure 16. Derivative time constant (τ_D) setting for different λ values.

the overshoot can be minimized, as shown in the response of the proposed method by setting $b = 0.1$. The performance shown in Figures 10 and 11 and Table 10 demonstrates the clear advantage of the proposed method over the other methods.

For the robustness study, the controller is evaluated by inserting a perturbation uncertainty of 20% to all three parameters and finding the actual process as $G_P = G_D = 1.2\ e^{-0.48s}/(1.2s - 1)$. The simulation results of the model mismatch for all the tuning rules are given in Table 11. In the model mismatch case, as seen from the performance matrix in Table 11, the proposed method gives a superior performance over all the other methods both for set-point and disturbance rejection.

5.6. Example 6: SODUP. The following unstable process was considered for the present study (Huang and Chen[14] and Lee et al.[9]):

$$G_P = G_D = \frac{1\ e^{-0.5s}}{(5s-1)(2s+1)(0.5s+1)} \tag{35}$$

The above model was approximated to the SODUP model by Huang and Chen[14] and was also used by Lee et al.[9]

$$G_P = G_D = \frac{1\ e^{-0.939s}}{(5s-1)(2.07s+1)} \tag{36}$$

The value $\lambda = 1.2$ was used for the Lee et al. method,[9] which has Ms = 4.35. For the proposed tuning rule, the value $\lambda = 0.938$ was adjusted to give Ms = 4.35 for the fair comparison with Lee et al.[9] The other methods by Huang and Chen[14] and Huang and Lin[15] were also included in the simulation, and the controller parameters were obtained from Lee et al.[9] Figure 12 shows the output response of the proposed tuning method compared to the other existing tuning rules. The proposed tuning method has a fast settling time compared to all the other existing methods. Lee et al.'s[9] results have a smaller peak, but it is slower and more oscillatory. In Table 12, the controller setting parameters and performance indices are given, where the proposed method shows a clear advantage over the others.

Figure 13 compares the output responses for the unit set-point change. It is clear that the 2DOF controller can improve the set-point response by eliminating the overshoot. Table 13 shows performance index values, with 20% uncertainty in gain and dead time. The proposed method shows better response both for set-point and disturbance rejection when compared with Lee et al. method.[9] Because of the different Ms bases in the nominal case, it is difficult to get the fair comparison; the Huang and Lin[15] method has more robust performance followed by that of Huang and Chen,[14] as the performance indices in Table 13 show.

6. Discussion

6.1. Effect of λ on the Tuning Parameters. The proposed IMC-based PID tuning method has a single tuning parameter, λ, that is related to the closed-loop performance as well as the robustness of the control system. It is important to analyze the effect of λ on the PID parameters, K_c, τ_I, and τ_D. Consider a FOPDT model given by

$$G_P = G_D = \frac{1\ e^{-\theta s}}{1s+1} \tag{37}$$

The PID parameters were calculated using the proposed method for different closed-loop time constant λ values for each case of $\theta/\tau = 0.25$, 0.5, 0.75, and 1.0.

Figure 14 shows the K_c variation with λ for different values of the θ/τ ratio. As the θ/τ ratio decreases, the effect of λ on K_c is severe. From the above figure, it is clear that K_c becomes

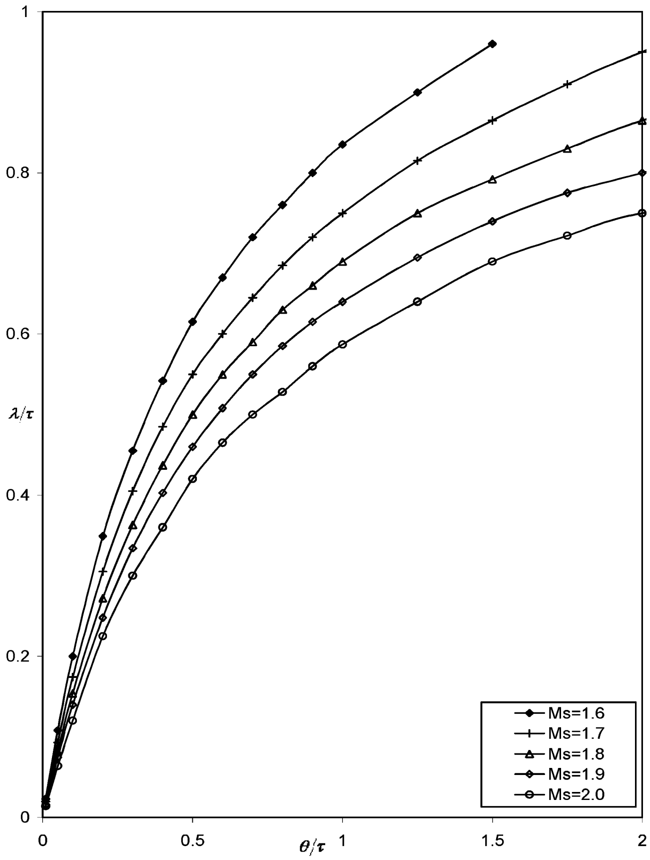

Figure 17. λ guidelines for FOPDT.

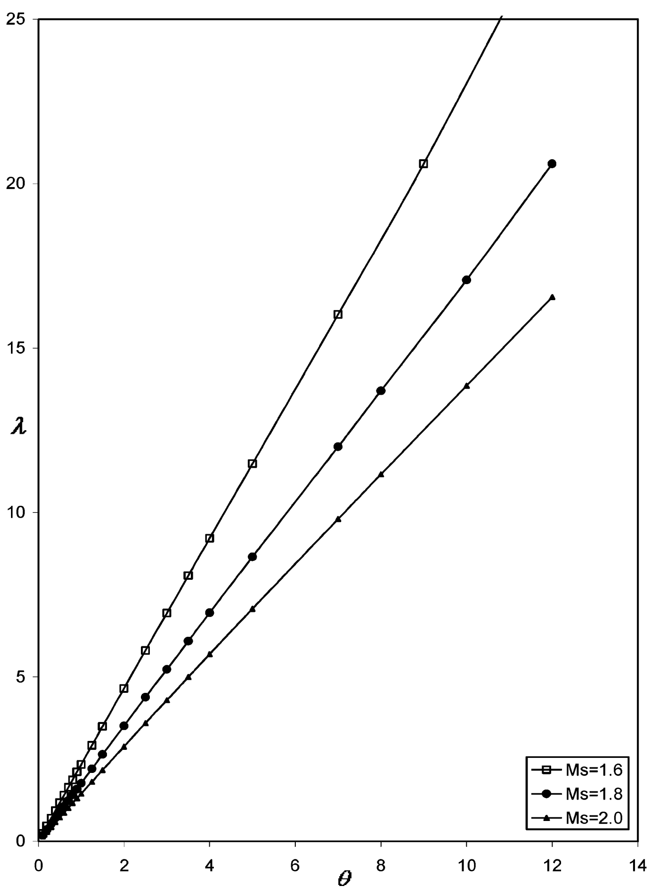

Figure 18. λ guidelines for DIP.

less sensitive to λ with increasing θ/τ. As the value of λ increases sufficiently, K_c variation is significantly reduced.

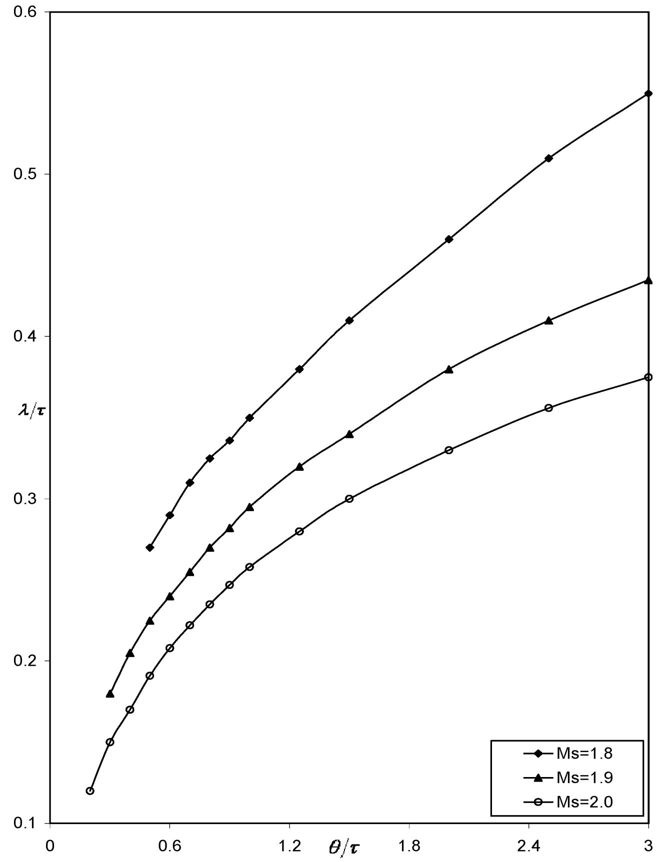

Figure 19. λ guidelines for SOPDT.

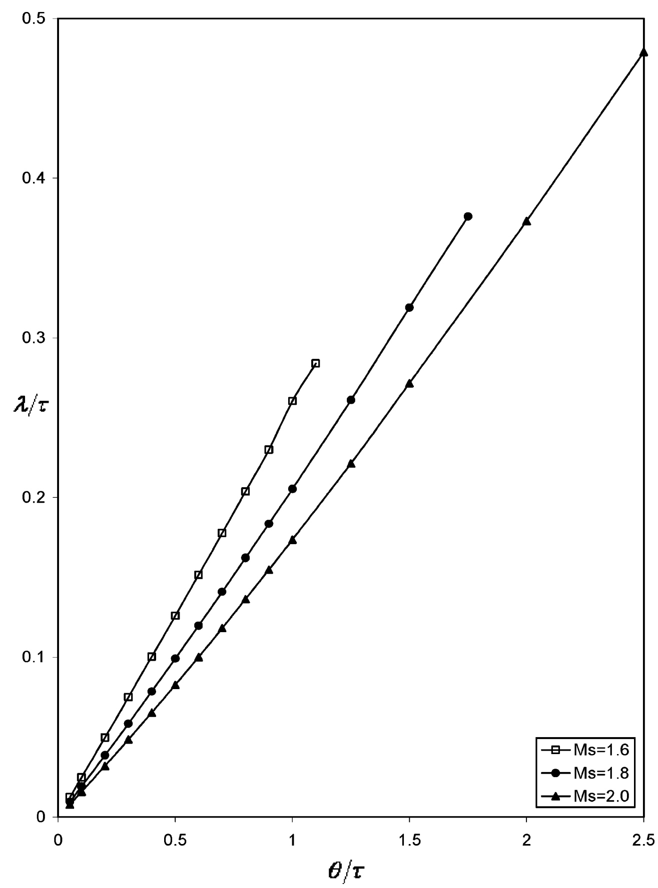

Figure 20. λ guidelines for FODIP.

Figure 15 shows the variation of τ_I with λ. The τ_I value increases initially with increasing λ for different θ/τ ratios, but

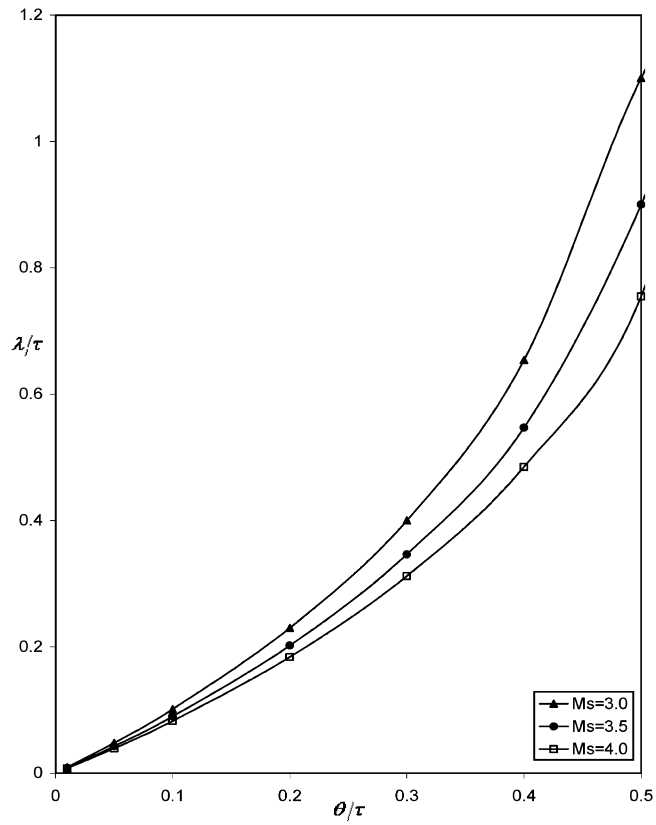

Figure 21. λ guidelines for FODUP.

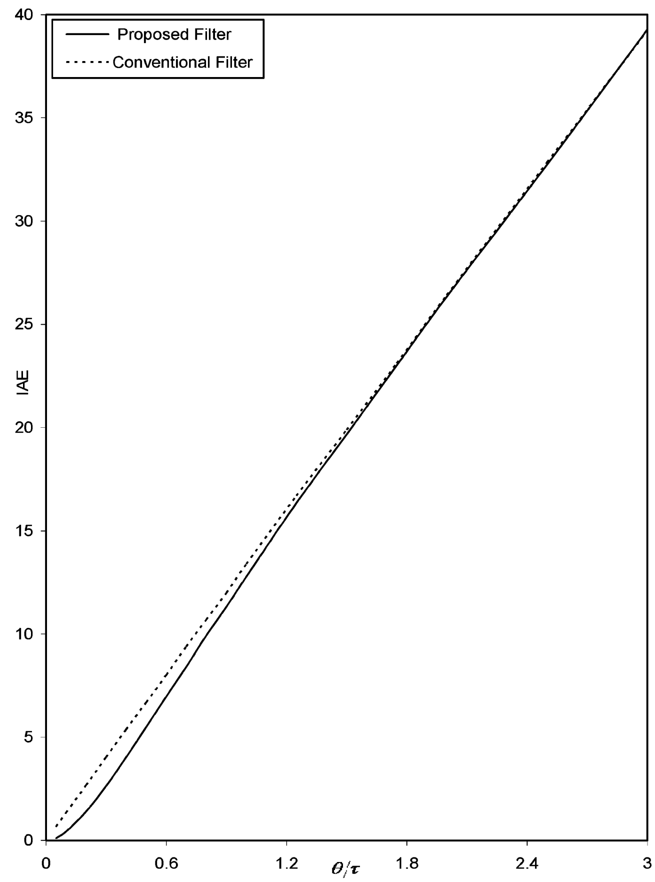

Figure 22. Performance of the proposed filter vs the conventional filter.

as the θ/τ ratio increases from 0.25 to 1.0, the variation of τ_I comparatively decreases, as clearly indicated by the $\theta/\tau = 1.0$ line. The trend of τ_I increasing with λ reverses after a specific λ value for each θ/τ ratio. For the above-mentioned θ/τ ratios, τ_I starts decreasing with increasing λ after some significantly bigger value of λ. It is important to note that, even for large values of λ, τ_I has a positive value.

Figure 16 shows the variation of τ_D with λ for different θ/τ ratios. The value of τ_D decreases with increasing λ at different θ/τ ratios, maintaining a positive value.

6.2. λ Guideline for PID Parameter Tuning. Since the closed-loop time constant λ is the only user-defined tuning parameter in the proposed tuning rule, it is important to have some λ guidelines to provide the best performance with a given robustness level. Figure 17 shows the plot of λ/τ vs θ/τ ratios for different Ms values for the FOPDT model. Figure 18 shows the λ guideline plot for the DIP model, where the desired λ can be calculated for a given θ value at different Ms values. Figure 19 shows the λ guidelines for the SOPDT model. It is necessary to describe the model reduction technique proposed by Skogestad.[8] Using Skogestad's[8] half rule, we can convert a SOPDT model into a FOPDT model and then obtain the desired λ/τ value from the plot in Figure 19 for the SOPDT model. Although this model reduction technique introduces some modeling error, it is within the acceptable limit. Figures 20 and 21 show the λ guideline plots for the FODIP and FODUP models, respectively.

6.3. Beneficial Range of the Proposed Method. The load performance by the proposed PID controller is superior as lag time dominates, but the superiority to that based on the conventional filter diminishes as dead time dominates. In the case of a dead-time dominant process (i.e., $\theta/\tau \gg 1$), the filter time constant should be chosen as $\lambda \approx \theta \gg \tau$ for stability. Therefore, the process pole at $-1/\tau$ is not a dominant pole in the closed-loop system. Instead, the pole at $-1/\lambda$ determines the overall dynamics. Thus, introducing the lead term $(\beta s + 1)$ into the filter to compensate the process pole at $-1/\tau$ has little impact on the speed of the disturbance-rejection response. Furthermore, the lead term generally increases the complexity of the IMC controller, which in turn degrades the performance of the resulting controller by causing a large discrepancy between the ideal and the PID controllers. It is also important to note that, as the order of the filter increases, the power of the denominator term $(\lambda s + 1)$ also increases, which causes an unnecessarily slow output response. As a result, in the case of a dead-time dominant process, a conventional filter without any lead term could have an advantage. Figure 22 compares the IAE values of the load responses by the PID controllers based on the proposed filter and the conventional filter for the process model $G_P = G_D = 10\ e^{-\theta s}/(1s + 1)$. The plot clearly indicates that the IAE gap between the proposed and conventional filters decreases as the θ/τ ratio increases. Furthermore, it is observed that both the conventional and proposed filter based PID controllers have similar robustness in the dead time dominant process.

6.4. Optimum Filter Structure for IMC−PID Design. One common problem with the conventional IMC−PID approaches is that the IMC filter is usually selected based on the resulting IMC performance, while the ultimate goal of the IMC filter design is to obtain the best PID controller. In the conventional approach for the filter design, it is assumed that the best IMC controller results in the best PID controller. However, since all the IMC−PID approaches utilize some kind of model reduction techniques to convert the IMC controller to the PID controller, an approximation error necessarily occurs. Therefore, if the IMC filter structure causes a significant error in conversion to the PID controller, although it gives the best IMC performance, the resulting PID controller could have poor control performance. The performance of the resulting PID controller depends

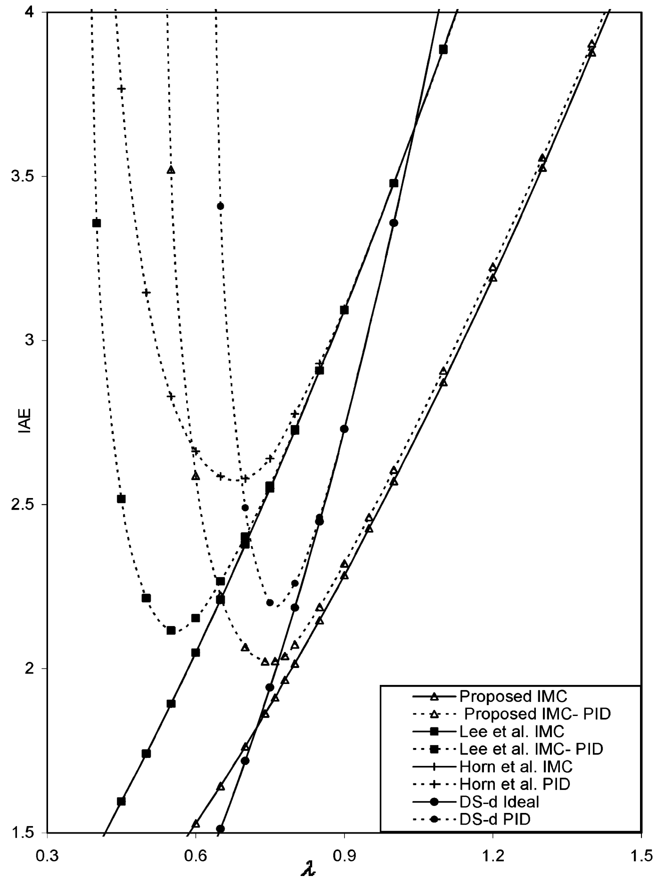

Figure 23. Plot of λ vs IAE for different tuning rules for FOPDT.

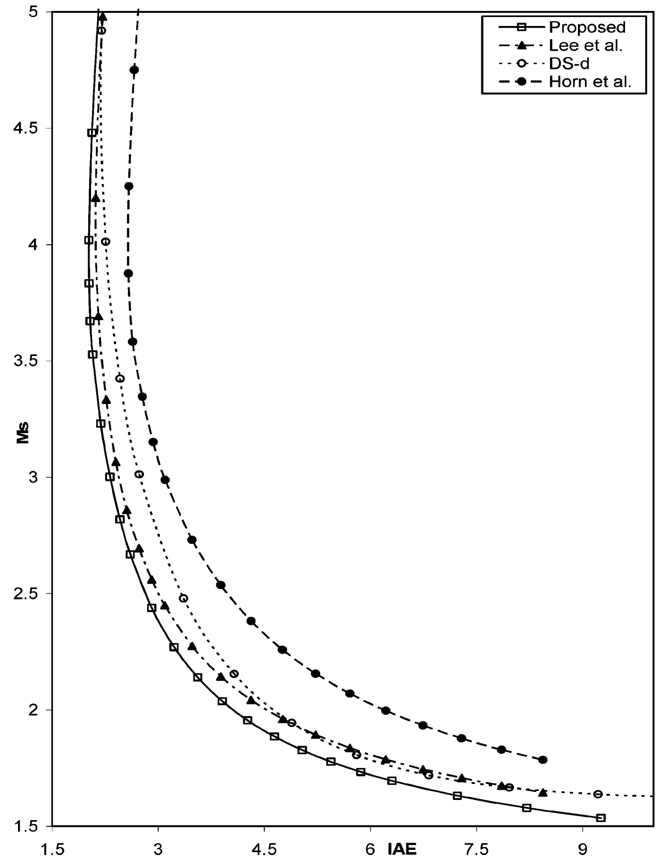

Figure 24. Plot of Ms vs IAE for different tuning rules for FOPDT.

on both the conversion error and the dead-time approximation error, which is also directly related to the filter structure and the process model. Therefore, there exists an optimum filter structure for each specific process model that gives the best PID performance. For a given filter structure, as λ decreases the discrepancy between the ideal and the PID controller increases and the nominal IMC performance improves. This indicates that an optimum λ value also exists that balances these two effects to give the best performance. Therefore, the best filter structure as defined in this paper is that gives the best PID performance for the optimum λ value.

To find the optimum filter structure, we evaluate the IMC filters with the structure of $(\beta s + 1)^r/(\lambda s + 1)^{r+n}$ for the first-order models and $(\beta_2 s^2 + \beta_1 s + 1)^r/(\lambda s + 1)^{2r+n}$ for the second-order models, where r and n are varied from 0 to 2, respectively. Our investigation shows that a high-order filter structure generally gives a better PID performance than a low-order filter structure. For example, for an FOPDT model, it is found that the high-order filter, $f(s) = (\beta s + 1)^2/(\lambda s + 1)^3$, provides the best disturbance rejection in terms of IAE. On the basis of the optimum filter structures, we derived the PID controller tuning rules for several representative process models, which are listed in Table 1.

Figure 23 shows the variation of IAE with λ for several tuning methods for the FOPDT model studied in the earlier sections (in example 1). The tuning rules proposed by Horn et al.[3] and Lee et al.[4] are based on the same filter $f(s) = (\beta s + 1)/(\lambda s + 1)^2$. Horn et al.[3] use a 1/1 Pade approximation for the dead time for calculating β and also the PID parameters. Lee et al.[4] obtained the PID parameters using a Maclaurin series approximation. Since both methods use the same IMC filter structure, the IMC controllers of Lee et al.[4] and Horn et al.[3] coincide with each other, as seen in Figure 23. Because of the approximation error in $e^{-\theta s}$ when calculating PID, the performance of the Lee et al.[4] method shows a clear advantage over the Horn et al.[3] method. It is clear from this figure that, down to some optimum λ value, the ideal (or IMC) and the PID controllers have no significant difference in performance, and after some minimum IAE point, the gap rises sharply toward unstable limits. The smallest IAE value can be achieved by the proposed tuning method, while the Horn et al.[3] tuning method shows the worst performance. It is also apparent that, for the case of model mismatch where a large λ value is required, the proposed method provides the best performance.

It is also worthwhile to visualize the performance and robustness of the controller design. The Ms and IAE are well-known indices for robustness and performance, respectively. Figure 24 shows the plot of Ms vs minimum IAE for the different tuning methods for the FOPDT model used in example 1. The figure clearly illustrates that, for a constant Ms value, the PID controller by the proposed method always produces a lower IAE value than those by the other tuning rules.

7. Conclusions

Optimum IMC filter structures were proposed for several representative process models to improve the disturbance rejection performance of the PID controller. On the basis of the proposed filter structures, tuning rules for the PID controller were derived using the generalized IMC−PID method. The simulation results demonstrate the superiority of the proposed method when the various controllers are all tuned to have the same degree of robustness in terms of maximum sensitivity. The proposed method becomes more beneficial as the process is lag-time dominant. The robustness analysis was conducted by inserting a 20% perturbation in each of the process parameters in the worst direction, with the results demonstrating the robustness of the proposed method against the parameter uncertainty. The closed-loop time constant λ guidelines were also proposed for several process models over a wide range of θ/τ ratios.

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