IMPERFECT INFORMATION

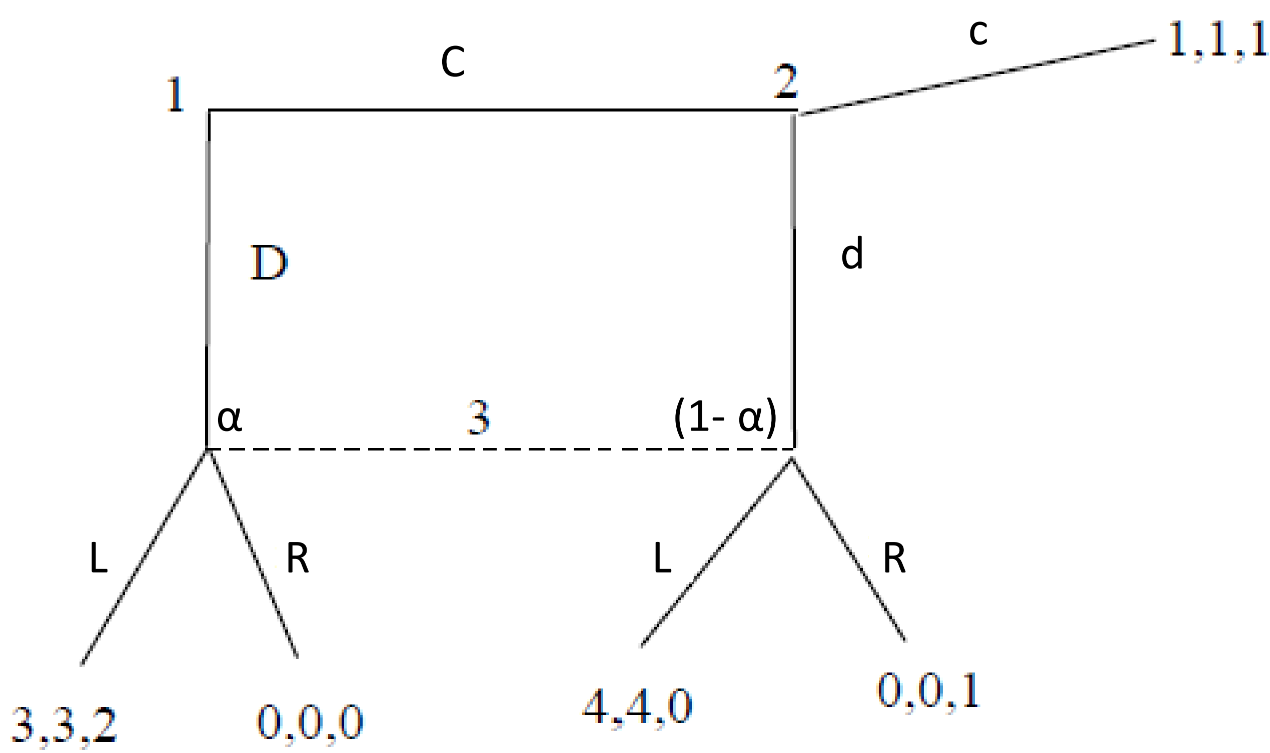
(SIGNALING GAMES AND APPLICATIONS)

Equilibrium concepts

Concept	Best responses	Beliefs
Nash equilibrium	On the equilibrium path	Implicit (on path only)
Subgame perfect equilibrium	In every proper subgame	Implicit (on path and off path)
Perfect Bayesian equilibrium	At every information set given (some) beliefs	Determined by Bayes' Rule on the path of play

$$\mathrm{PBE} \subseteq \mathrm{SPE} \subseteq \mathrm{NE}$$

Selten's Horse



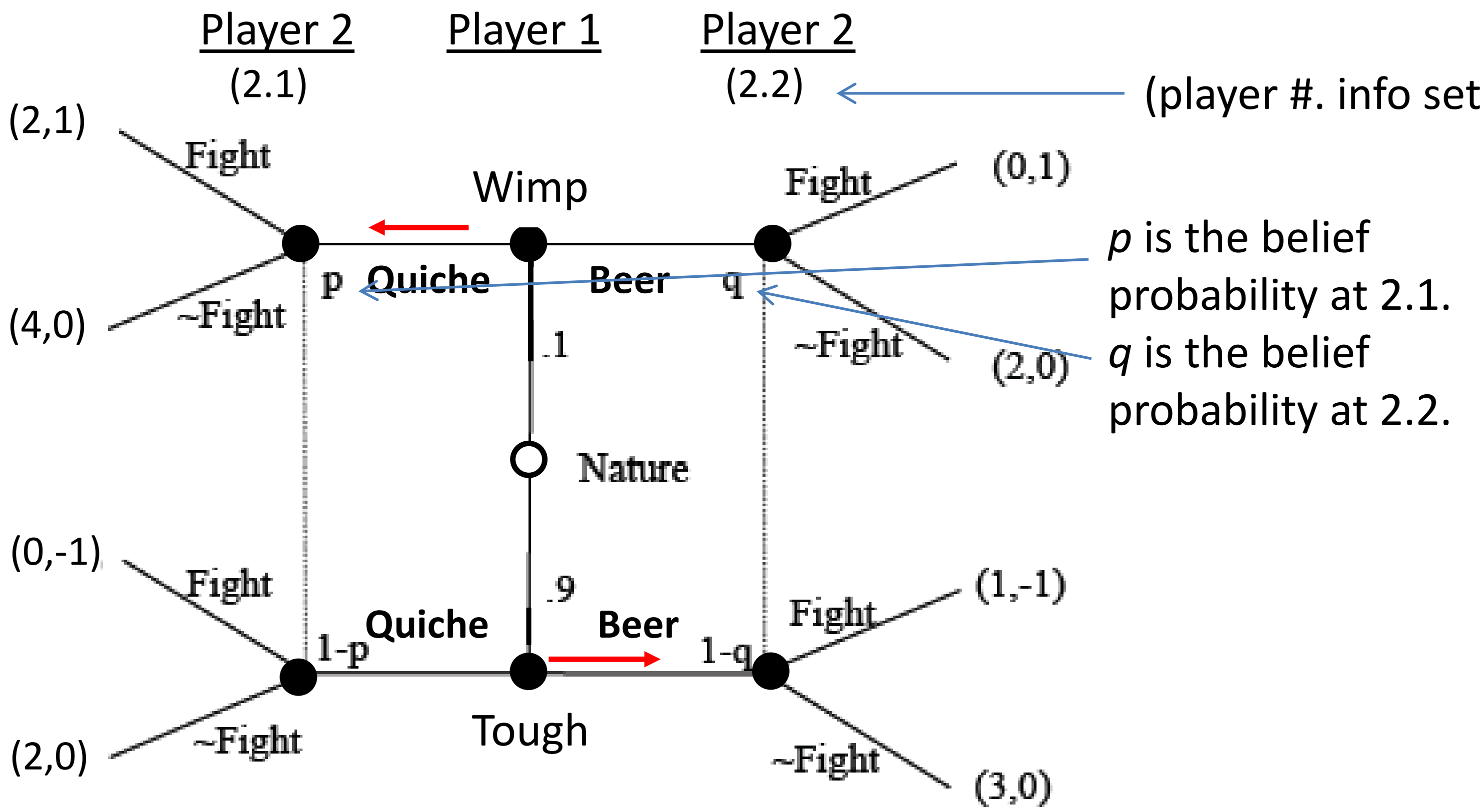
Signaling games

- An important class of games of incomplete information with asymmetric information about player types.
- Basic structure
  - Nature chooses player types.
  - The informed player (i.e., the one that knows their own type) observes Nature's choice and chooses an action.
  - The uninformed player observes the informed player's action (but NOT her type) and thinks something about her type based on her actions.
- Applications

Crisis bargaining	Candidate entry/deterrence
Informational committees	Bureaucratic delegation
Lobbying	Reputations
Education	Product quality

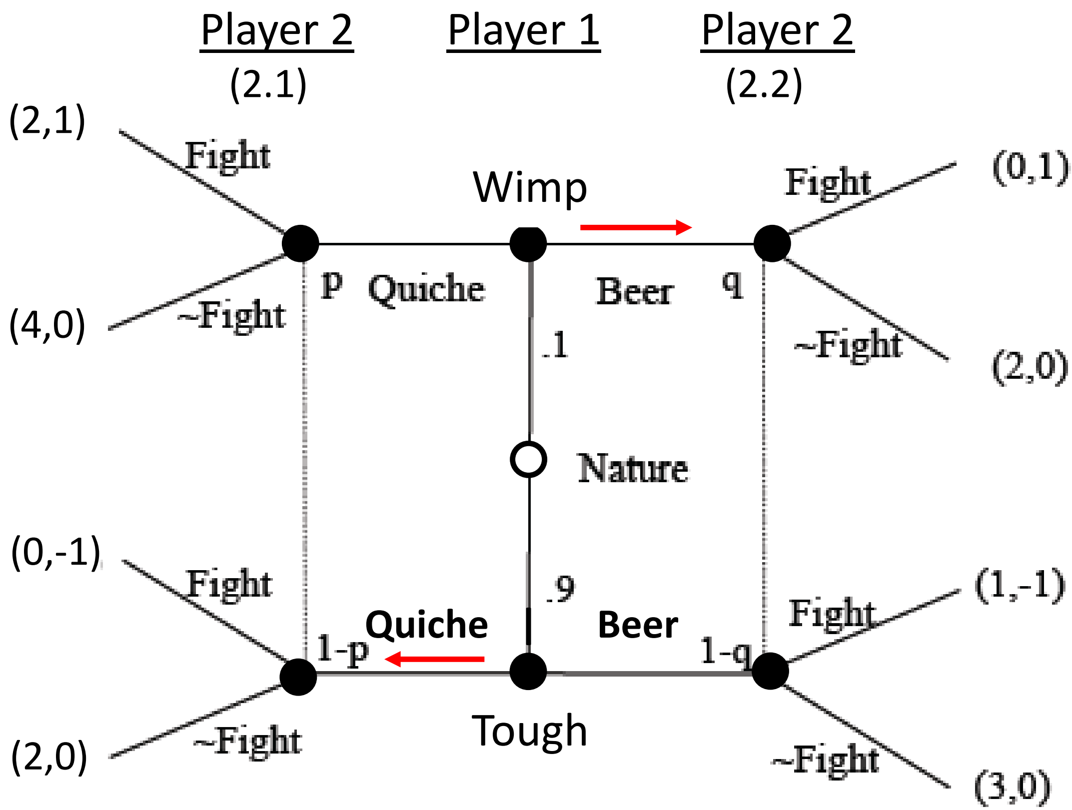
Beer or Quiche?

Beer or Quiche?



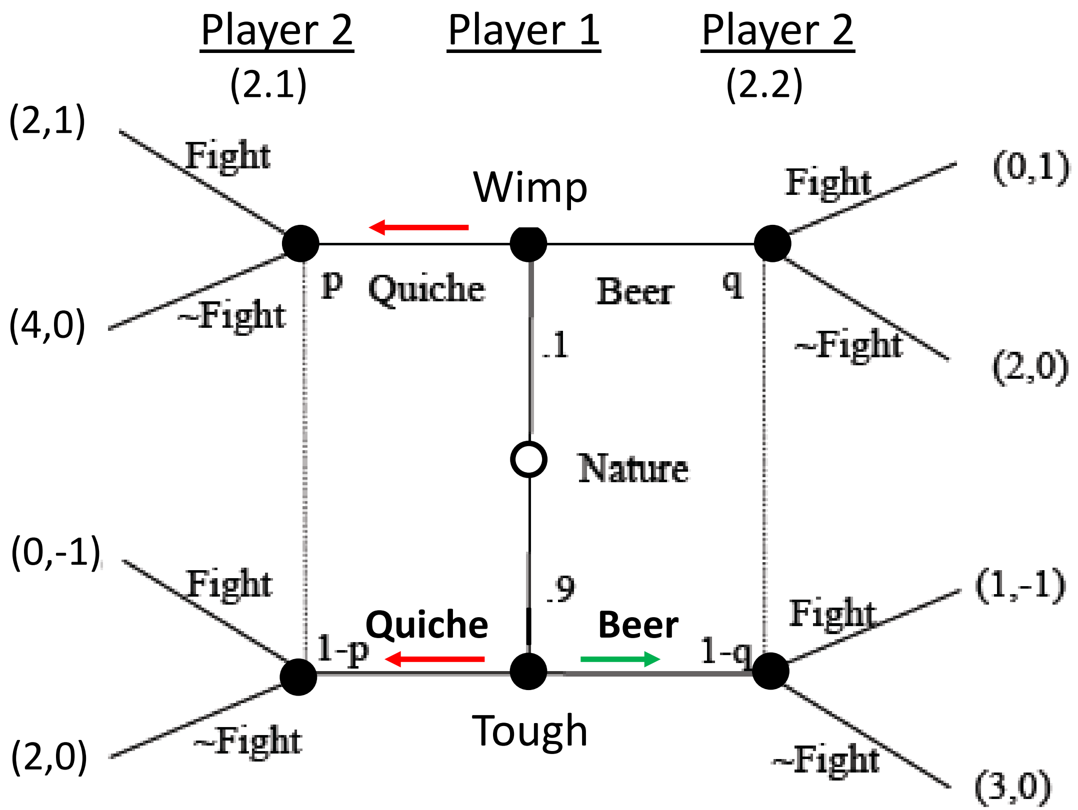
Separating with Q^wB^T: Given player 1's play, it must be that p=1 and q= 0 (step 2). Thus, player 2 plays fight at 2.1 and ~fight at 2.2 (step 3). Player 1 has no unilateral incentive to deviate under either type (step 4). Hence, PBE = {Q^wB^T; (fight| p=1, ~fight| q=0)}.

Beer or Quiche?



Separating with $B^W Q^T$: Given player 1's play, it must be that p=0 and q= 1. Thus, player 2 plays ~fight at 2.1 and fight at 2.2 (step 3). But then player 1 would prefer Q^W to B^W. Therefore, $B^W Q^T$ is not a PBE.

Beer or Quiche?

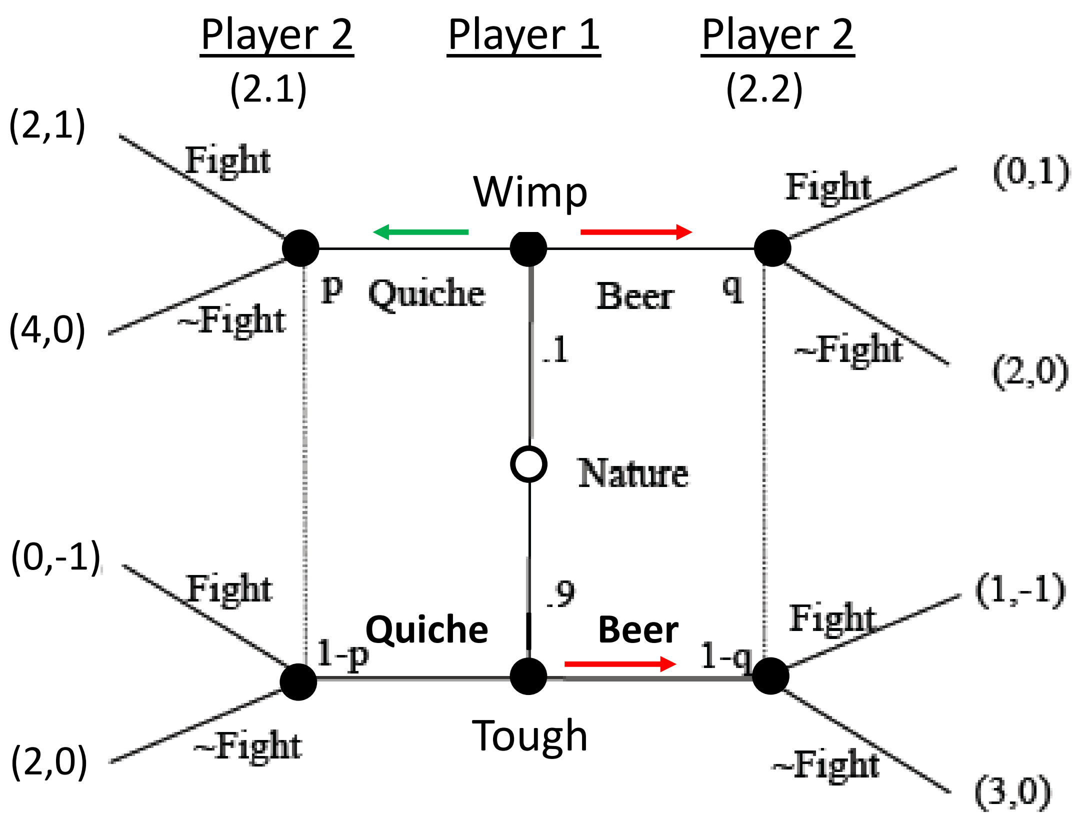


Pooling with Q^wQ^T: Bayes rule requires p =.1. Hence player 2 expects:

.1(1) + .9(-1) = -.8 if fight,
0 if ~fight,
and he does not fight.

Because player 1 would prefer B^T to Q^T, Q^wQ^T is not a PBE.

Beer or Quiche?



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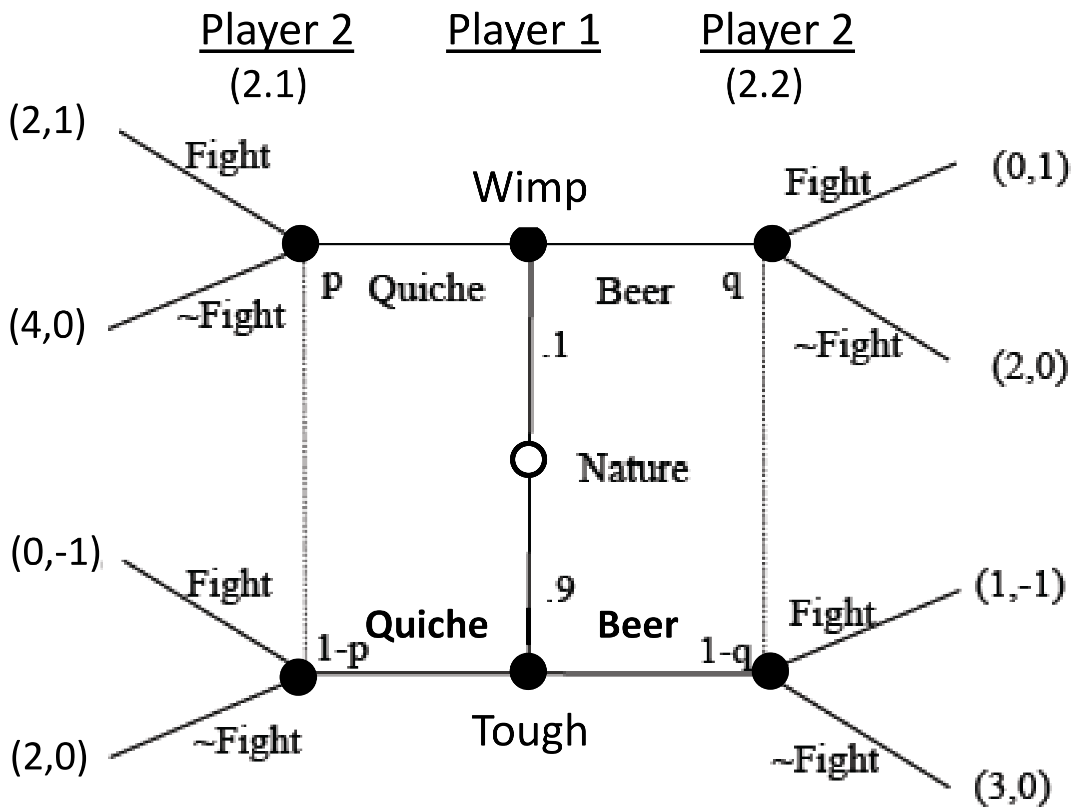
Beer or Quiche?



Note: with different payoffs, or different probabilities on player types, you might get different conclusions.

Conclude (in pure strategies):

If you see quiche, fight; If you see beer, don't fight.

The action signals the player type.

Gilligan and Krehbiel

- A **Simplified version** of the Gilligan and Krehbiel Model.
- Basic structure
  - Two players: floor median and a committee.
  - Floor median choses open or closed rule.
  - The committee proposes a bill $\hat{p} \in R$.
  - The floor median observes the committee's report, and then adopts a bill $p \in R$.
    - Note, $\hat{p}$ may or may not equal p.
  - The policy outcome *x* resulting from this interaction is a function of the bill adopted *p*, and a random shock $\omega \in [0, 1]$ that is uniformly distributed in the [0,1] interval. Specifically, $x = p + \omega$.
    - Note, without subscript x is the policy outcome. With subscript it will be an ideal point.

Gilligan and Krehbiel

- Preferences
  - The floor median's ideal point: $X_f = 0$.
  - The committee median's ideal point: $X_c > 0$.
  - Both players have quadratic utility:

$$U_f(p) = -(0 - x)^2 = -(p + \omega)^2$$

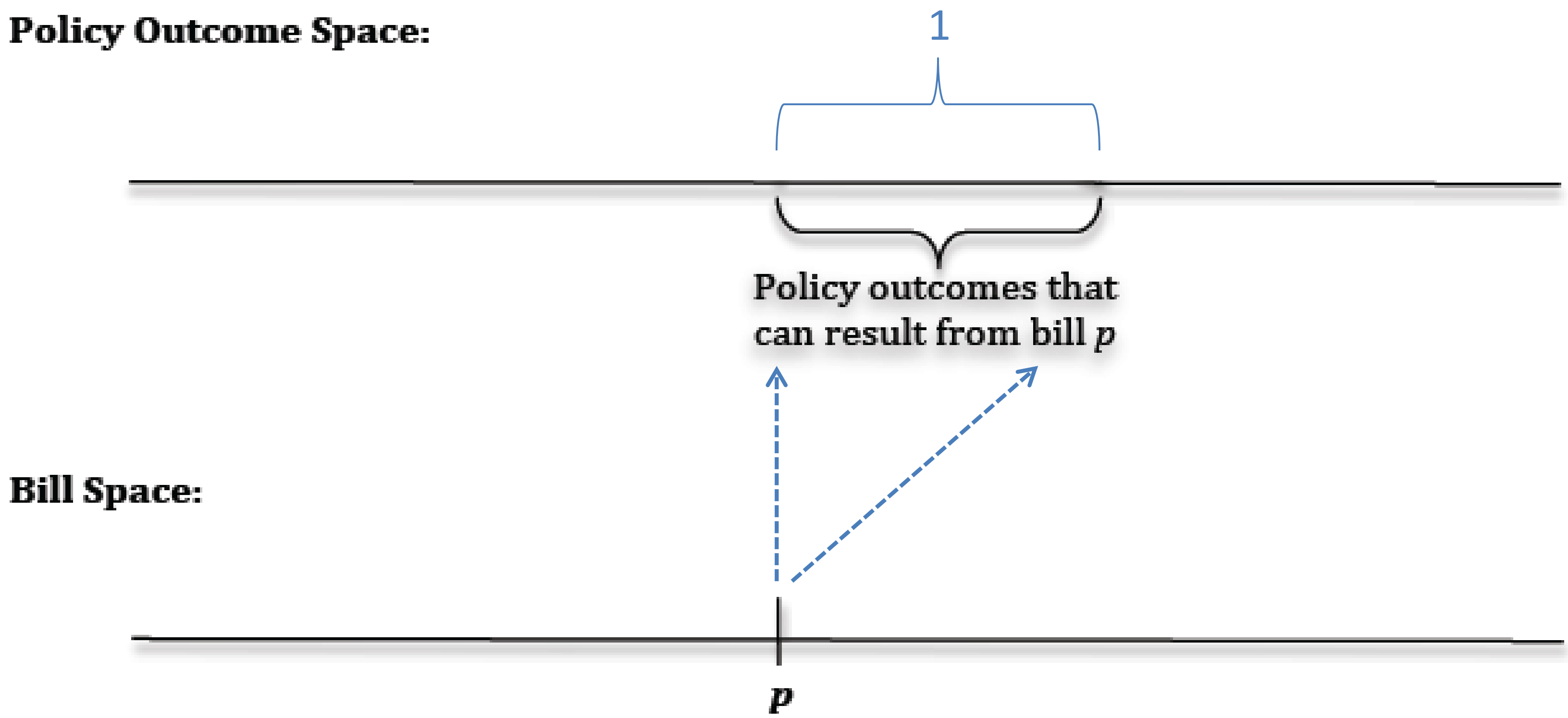
$$U_c(p) = -(X_c - x)^2 = -(X_c - p - \omega)^2$$

For why, recall: $x = p + \omega$.

Gilligan and Krehbiel

- Policy Expertise
  - The committee is made up of policy experts, who are perfectly informed about the shock ω before making their decision. That is, the committee knows the value of ω that has been realized.
  - The floor median knows only the distribution of ω, but not its value.
    - Ex: a committee approving building projects for bridges knows that ω=10 million will be added to the expense, so they request $\hat{p} = 190$ million for bridges if they want an outcome of x = 200 million.
    - Unlike the actual paper, we will assume the committee specializes (i.e. choses to have ω revealed) and there is no cost for finding this value.

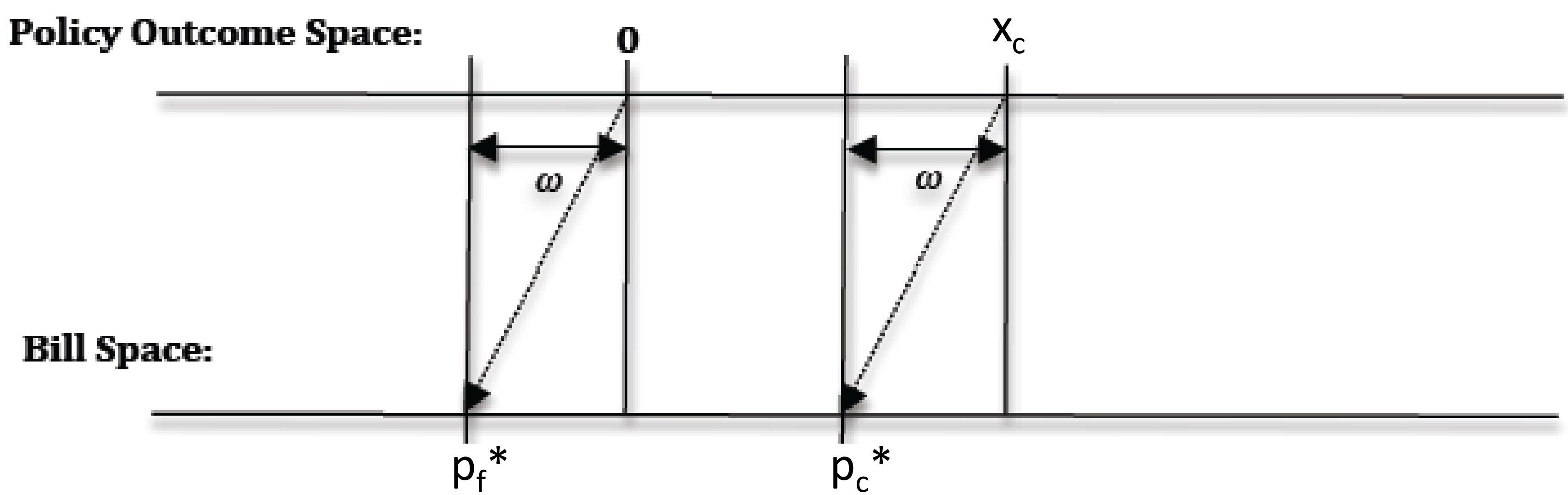
Gilligan and Krehbiel



Gilligan and Krehbiel

Optimal policies p* for each actor

- Here ω is the value drawn.



Gilligan and Krehbiel

- Information Transmission
  - What will the committee reveal about ω in equilibrium?
  - What will the floor do with that information?
  - Can the committee serve as a device that provides valuable policy information to the floor?

Gilligan and Krehbiel

- Perfect Information Transmission
  - Let $r(\omega)$ be the committee's report on ω. The committee's proposal is based on this value.
  - Could we have $r(\omega) = \omega$?
- Separating equilibrium

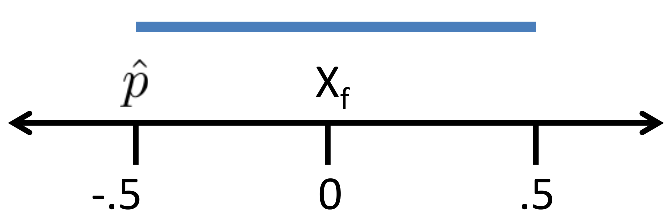
  (i.e. the equilibrium fully reveals previously private information)
  - Consistency requires that the floor median believes that $\omega = r(\omega)$.
  - Optimal policy for floor: $\hat{p} = -\omega$.
  - Optimal committee proposal: $\hat{p} = X_c - \omega$.
  - Hence, committee has incentive to deviate and there is no PBE with full separation.

Gilligan and Krehbiel

- No Information Transmission
  - Could we have r(ω) = *c*, where *c* is a constant for all ω?
- Pooling equilibrium

  (i.e. equilibrium does not allow true value of ω to be revealed)

  - Consistency requires that after report *c*, the floor median believes that ω is uniformly distributed on [0,1].
  - Suppose the floor median holds the same prior belief after any report.
  - Optimal policy for floor: $\hat{p}$ = – ½.



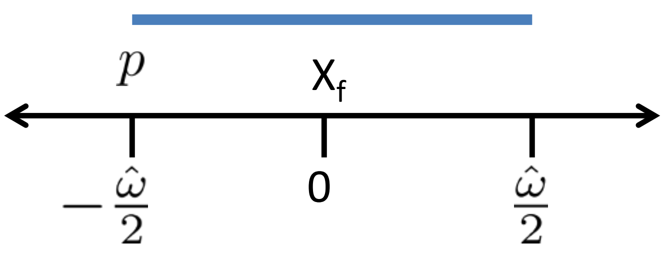
Note that $\hat{p}$= -.5 means that half of the ω values will be above X_f and half will be below, with E[x]=0. The floor is indifferent between open rule and $\hat{p}$= -.5.

Gilligan and Krehbiel

- Optimal policy for floor: = – ½.
- Committee proposal: = – ½ is a best response. …Why?
  - Any deviation to the right would make the floor chose open and set p = -.5
- Hence, there is a pooling PBE where no information is transmitted.

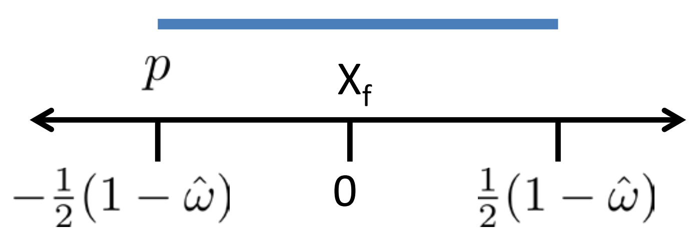
Gilligan and Krehbiel

- Some Information Transmission
  - Suppose the committee sends one of two reports:
    - "ω is low" if $\omega \in [0, \hat{\omega}]$.
    - "ω is high" if $\omega \in [\hat{\omega}, 1]$.
- Equilibria
  - Consistency requires that after report "ω is low", the floor median believes ω is uniformly distributed on $[0, \hat{\omega}]$.
  - Floor's optimal policy for "ω is low": $p = -\frac{\hat{\omega}}{2}$.



Gilligan and Krehbiel

- Some Information Transmission
  - Suppose the committee sends one of two reports:
    - "ω is low" if $\omega \in [0, \hat{\omega}]$.
    - "ω is high" if $\omega \in [\hat{\omega}, 1]$.
- Equilibria
  - Consistency requires that after report "ω is high", the floor median believes ω is uniformly distributed on $(\hat{\omega}, 1]$.
  - Floor's optimal policy for "ω is high": $p = -\frac{1}{2}(1 - \hat{\omega})$.



Gilligan and Krehbiel

- Some Information Transmission
  - **Will the committee send an accurate "low" or "high" report?**

    Consider $X_c = 2, \hat{\omega} = .4, \text{ and } \omega = .8.$ Hence, ω is a high type.

    If committee reports "high," the floor sets

    $$p = -\frac{1}{2}(1 - \hat{\omega}) = -.3.$$

    If committee reports "low," the floor sets

    $$p = -\frac{\omega}{2} = -.2.$$

    The policy outcome for "high" is $x = -.3 + .8 = .5$.

    The policy outcome for "low" is $x = -.2 + .8 = .6$.

  Hence, it is better for the committee to deviate and announce "low" (i.e., not send an accurate report) because it produces an x closer to the committee.

Gilligan and Krehbiel

- Some Information Transmission
  - **Will the committee send an accurate "low" or "high" report?**

Note: the answer depends on the values of X_c and ω.

If $X_c = .5, \hat{\omega} = .4,$ and $\omega = .8,$ then it is better for the committee to truthfully say "high".

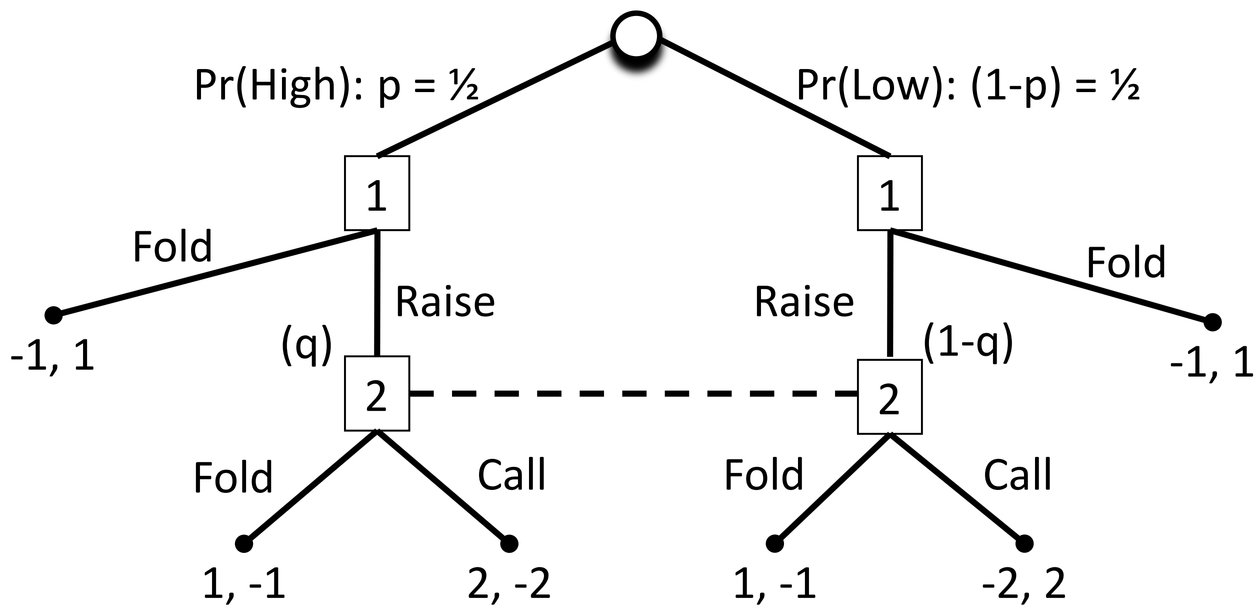
The difference, between this case and the last, is that the committee median is now very close to the floor median, making the "low" deviation overshoot the committee.

Gilligan and Krehbiel

- Conclusions
  - The committee's policy expertise can NEVER be fully revealed to the floor in a PBE if there is any policy divergence between the committee and the floor.
  - The committee's policy expertise may be completely ignored in a PBE.
  - SOME information transmission is possible but only if the preferences of the committee converge with those of the floor.
  - As a result, the floor has reason to assure that committee members are not preference outliers.
- Discussion
  - Any questions or thoughts about what's going on in the paper?

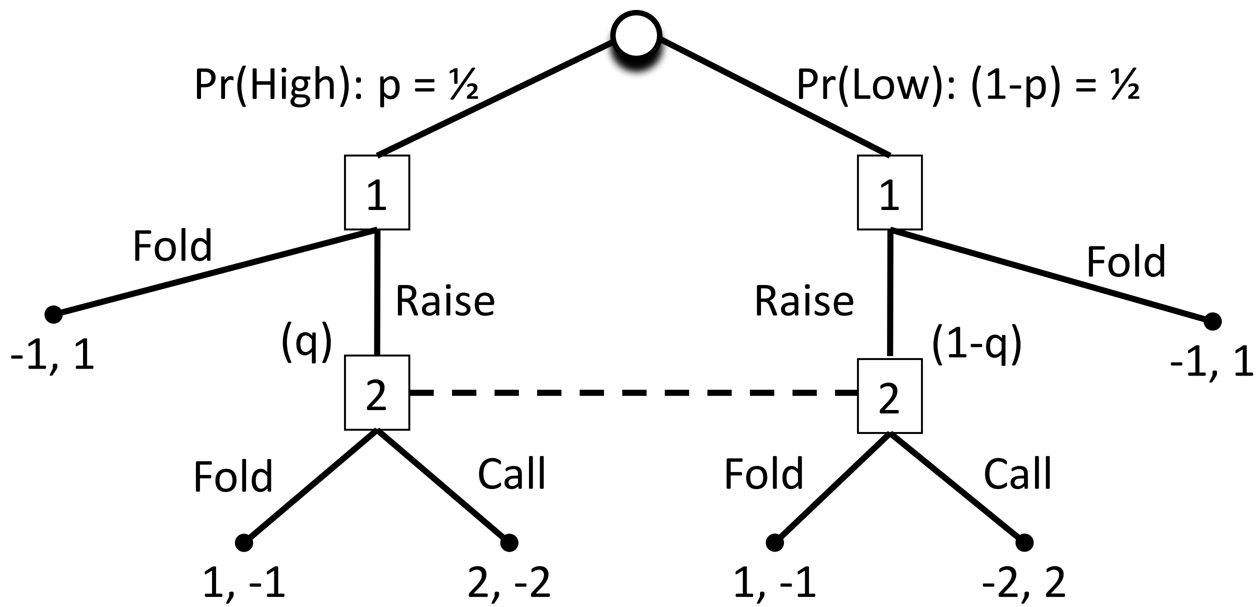
Simplified Poker
(practice 2)

- Nature chooses a High or Low card for player 1
- Player 1 observes the card and chooses to Fold or Raise
- If player 1 raises, player 2 can Fold or Call (but does not observe player 1's card)



Find the PBE.

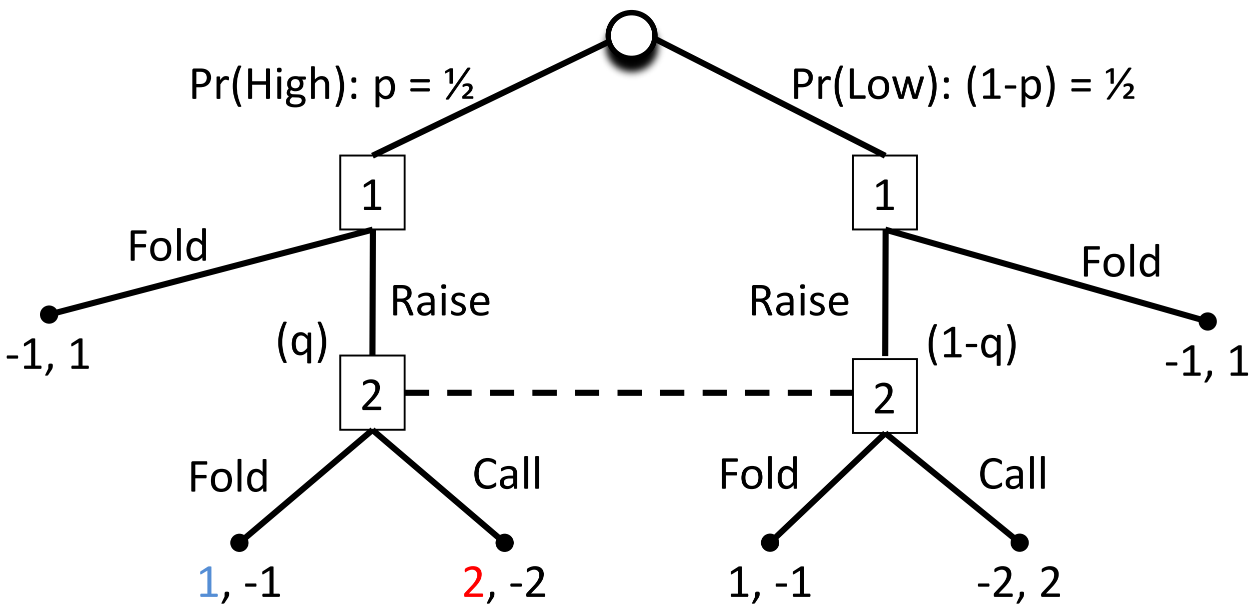
Simplified Poker
(practice 2)



Separating with $F^H R^L$: Given this strategy for player 1, it must be that q = 0. Thus, player 2's optimal strategy is to call. But then the low type of player 1 strictly prefers not to play R^L. Therefore, there is no PBE in which $F^H R^L$ is played.

Separating with $R^H F^L$: Given this strategy for player 1, it must be that q = 1. Thus, player 2's optimal strategy is to call. But then the low type of player 1 strictly prefers not to play R^L. Therefore, there is no PBE in which $F^H R^L$ is played.

Simplified Poker
(practice 2)

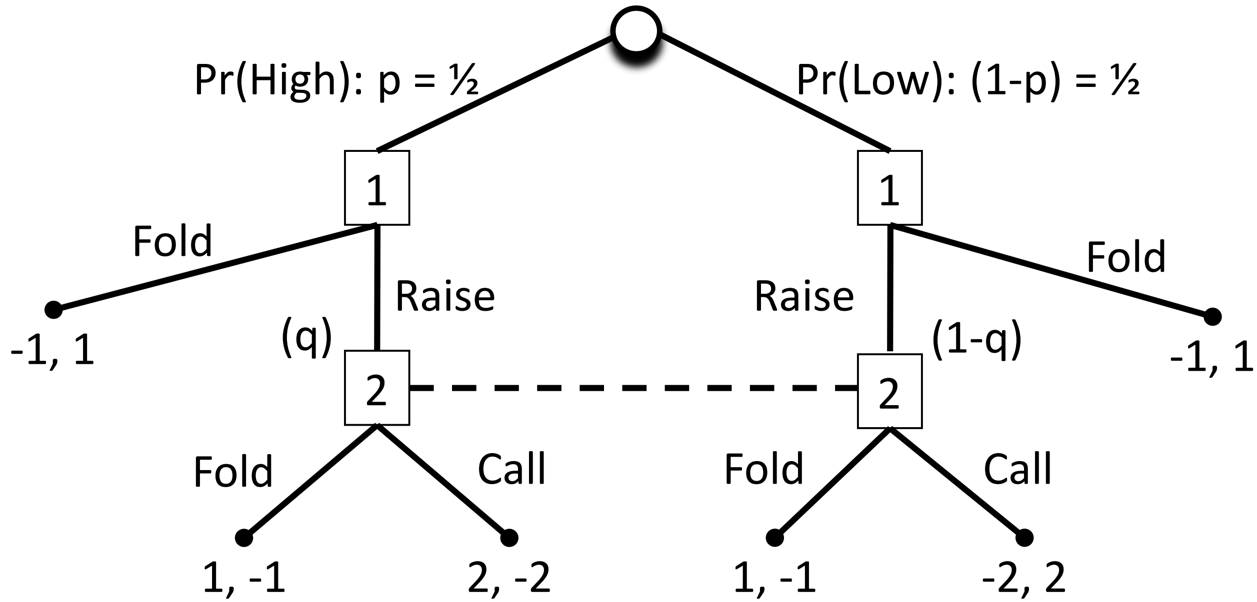


Pooling with F^HF^L: In this case, player 1 has incentive to deviate to R^HF^L regardless of q (which is not determined by Bayes' rule). If player 2 folds, then $EU_1(R^HF^L) > EU_1(F^HF^L)$, because 1 > -1 when raising as the high type. If player 2 calls, then $EU_1(R^HF^L) > EU_1(F^HF^L)$ because 2 > -1 when raising as the high type.

Hence, F^HF^L is not a PBE.

Simplified Poker

(practice 2)



Pooling with $R^H R^L$: In this case, Bayes' rule requires q = ½. Player 2 optimally selects fold iff:

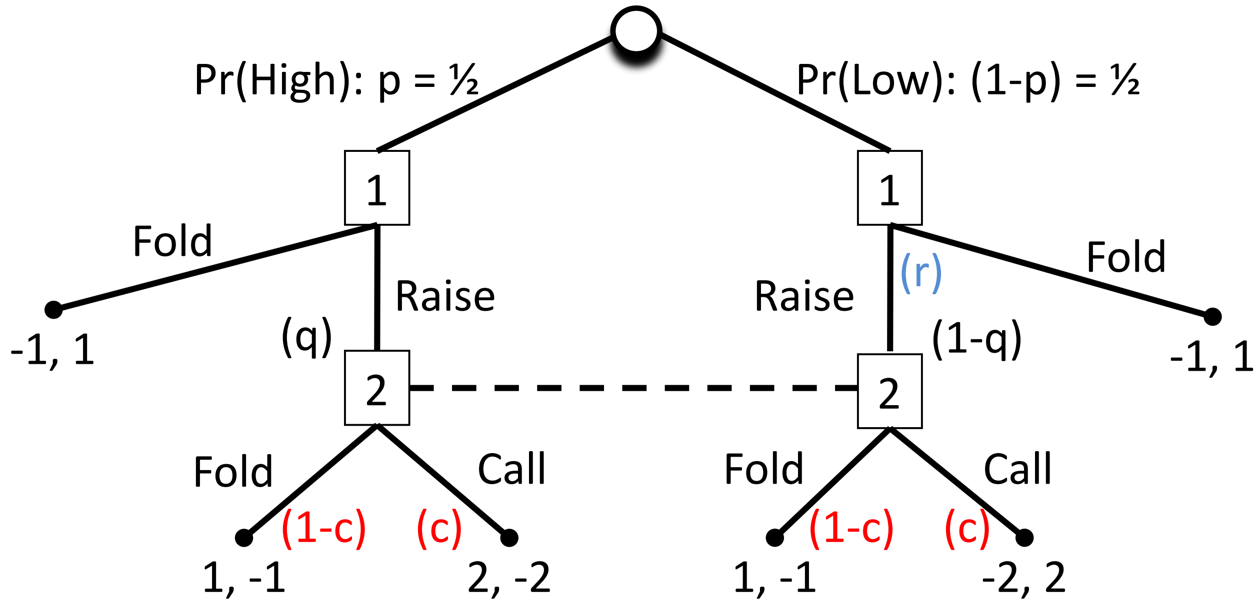
$(q)(-1) + (1-q)(-1) > (q)(-2) + (1-q)(2)$

$-1 > 2 - 4q$

$q > 3/4$.

Since q = ½, by Bayes rule, player 2 cannot optimally chose F. However, player 2 can optimally chose to call. But if player 2 calls, player 1 expects (1/2)-2 + (1/2)2 = 0 from $R^H R^L$ and would prefer to deviate to $R^H F^L$ because $R^H F^L$ gives player 1 (1/2)(2)+(1/2)(-1) = 1/2. Hence, $R^H R^L$ is not a PBE.

Simplified Poker
(practice 2)



Mixed strategies: In a mixed strategy equilibrium we need to make player 2 indifferent between folding and calling and player 1 indifferent between raising and folding. Note that player 1 will always raise if the high type, so player 1's mix will be over raise and fold only if she is the low type.

Let r be the probability 1 raises if she is the low type (r = Prob[R|L]).

Let c be the probability that player 2 calls.

Player 2 is indifferent between folding and calling iff q = 3/4 (see previous slide).

Simplified Poker
(practice 2)

Mixed strategies:

Notes: Prob[R&H] = ½, because player 1 will always raise if she is the high type.
Prob[R&L] = (1/2)r.
Prob[R] = Prob[R&H] + Prob[R&L] = 1/2(1+r).

$$Prob[H|R] = \frac{1/2}{1/2(1+r)} = \frac{1}{1+r}$$

Now set q = Prob[H|R]
3/4 = 1 / (1+r)
…
r = 1/3.

To make player 1 indifferent,

$EU_1(R|L) = EU_1(F|L)$
1(1-c)-2c = -1
…
c = 2/3.

Hence, player 1 raises if type high;
player 1 mixes with r = 1/3 if type low;
player 2 calls with c = 2/3.

Summary of simplified poker

- No pure strategy equilibrium
- Mixed strategy involves bluffing on the part of player 1 and a randomized response on the part of player 2 (in order to ensure that player 1 is indifferent)
- Player 2's beliefs are on the equilibrium path and determined by Player 1's behavioral strategy, Nature's probabilities, and Bayes' Rule