



Implied Volatility using Python's Pandas Library

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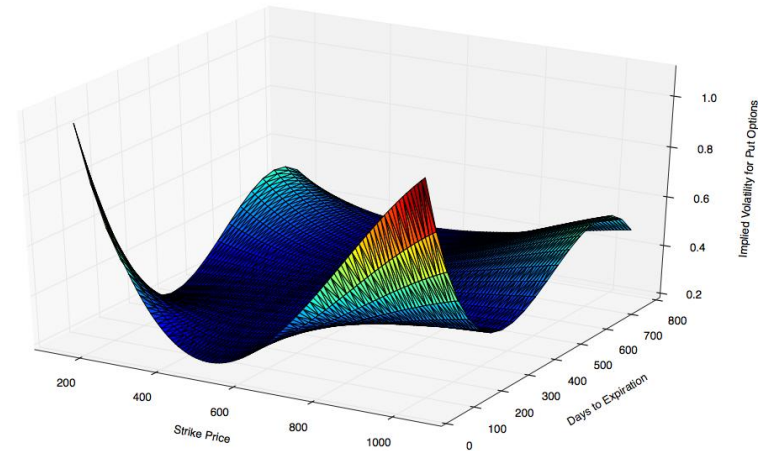
*New York Quantitative Python Users Group
March 6th 2014*



Experts in numerical algorithms
and HPC services

Overview

- Introduction
- Motivation
- Python
- Pandas
- Implied Volatility
 - Timings in python
 - Different Volatility Curves
 - Fitting data points



Numerical Algorithms Group

- Not-for-profit organization committed to research & development
- NAG provides mathematical and statistical algorithm libraries and services widely used in industry and academia
- Library code written and contributed by some of the world's most renowned mathematicians and computer scientists
- NAG Libraries available in C, MATLAB, .NET, Fortran, Java, SMP/Multicore, Excel, Python

NAG Library Contents

- Root Finding
- Summation of Series
- Quadrature
- Ordinary Differential Equations
- Partial Differential Equations
- Numerical Differentiation
- Integral Equations
- Mesh Generation
- Interpolation
- Curve and Surface Fitting
- Optimization
- Approximations of Special Functions
- Dense Linear Algebra
- Sparse Linear Algebra
- Correlation & Regression Analysis
- Multivariate Methods
- Analysis of Variance
- Random Number Generators
- Univariate Estimation
- Nonparametric Statistics
- Smoothing in Statistics
- Contingency Table Analysis
- Survival Analysis
- Time Series Analysis
- Operations Research

Motivation

- Data available from CBOE:
 - <https://www.cboe.com/delayedquote/QuoteTableDownload.aspx>

Motivation

- Data available from CBOE:

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Mar 04 2014 @ 12:18 ET,Bid,531.03,Ask,531.21,Size,5x1,Vol,3803030,  
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```

Python

- Why use python?
 - Cheap
 - Easy to learn
 - Powerful

Python

- Why use python?
 - Cheap
 - Easy to learn
 - Powerful
- Why use python over R?
 - “I would rather do math in a programming language than programming in a math language.”

Python

- What python has:
 - Many built-in powerful packages
 - OO programming
 - Classes
 - Base + Derived Classes
 - Plotting
- What python does not have:
 - Multiple constructors
 - Pointers
 - ???

numpy

- Has made numerical computing much easier in recent years.
- numpy matrices / arrays
- numpy.linalg
- Behind many of these functions are LAPACK + BLAS!

scipy

- Special functions (`scipy.special`)
- Integration (`scipy.integrate`)
- Optimization (`scipy.optimize`)
- Interpolation (`scipy.interpolate`)
- Fourier Transforms (`scipy.fftpack`)
- Signal Processing (`scipy.signal`)
- Linear Algebra (`scipy.linalg`)
- Sparse Eigenvalue Problems with ARPACK
- Compressed Sparse Graph Routines `scipy.sparse.csgraph`
- Spatial data structures and algorithms (`scipy.spatial`)
- Statistics (`scipy.stats`)
- Multidimensional image processing (`scipy.ndimage`)

nag4py

- nag4py (The NAG Library for Python)
- Built on top of NAG C Library + Documentation
- 1600 NAG functions easily accessible from python
- 15 examples programs to help users call NAG functions

```
from nag4py.c05 import c05ayc
```

```
from nag4py.util import NagError,Nag_Comm
```

pandas

- Data Analysis Package
- Many nice built in functions
- Common tools:
 - Series / DataFrame
 - Reading + Writing CSVs
 - Indexing, missing data, reshaping
 - Common time series functionality

(Examples)

Implied Volatility

- Black Scholes Formula for pricing a call/put option is a function of 6 variables:

$$- C(S_0, K, T, \sigma, r, d) = S_0 N(d_1) - K e^{-rT} N(d_2)$$

- Where

$$- d_{1,2} = \frac{1}{\sigma\sqrt{T}} \left[\ln\left(\frac{S}{K}\right) + T\left(r \pm \frac{\sigma^2}{2}\right) \right]$$

$$- N(x) = \text{Standard Normal CDF}$$

Implied Volatility

- We can observe the following in the market:
- $C(S_0, K, T, \sigma, r, d) = C$
- But what is σ ?
- $\sigma_{imp} \rightarrow C_{BS}(S_0, K, T, \sigma_{imp}, r, d) = \text{Market Price}$

Implied Volatility

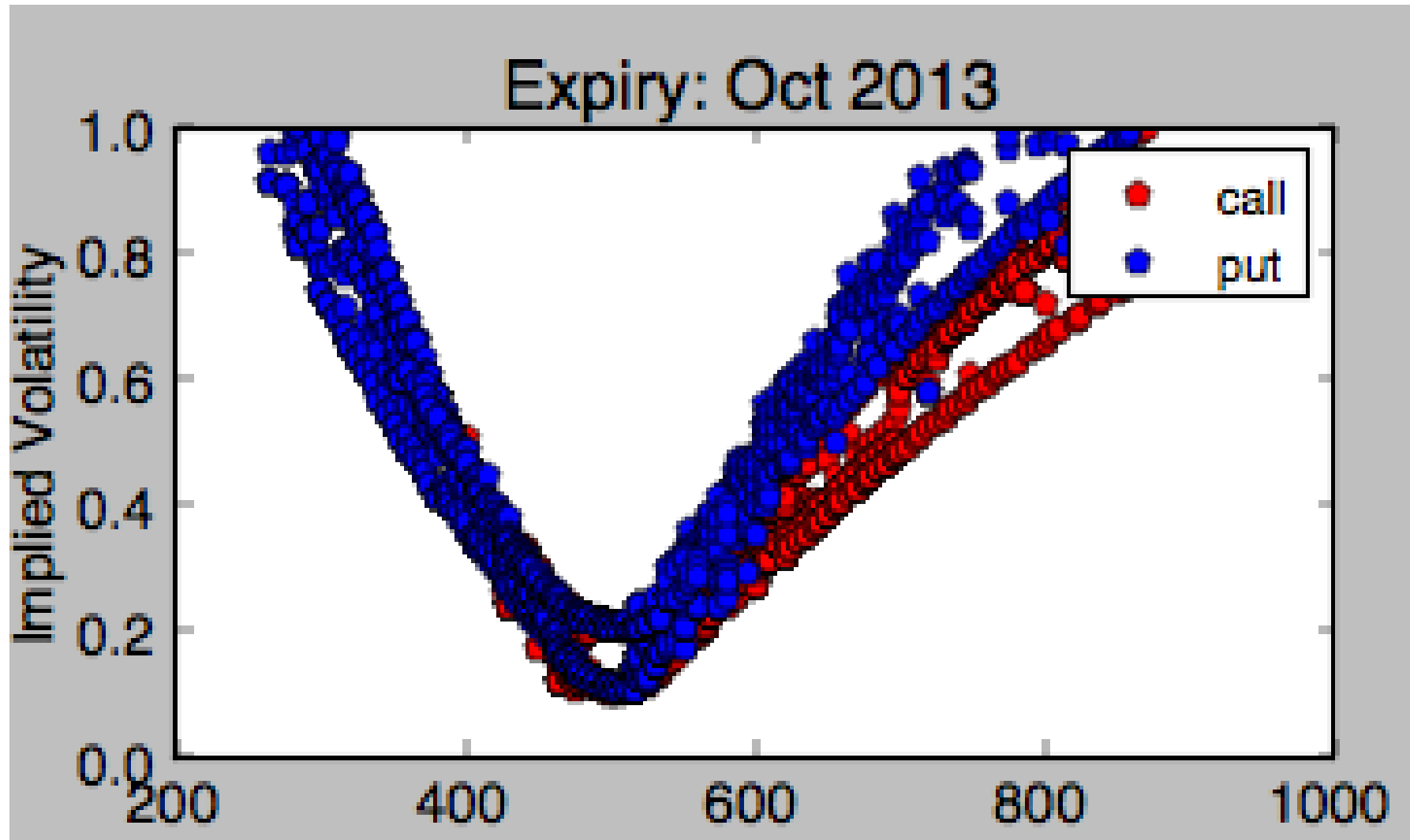
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- Does σ_{imp} exist?
 - Yes

(Examples)

Implied Volatility – Different Curves?



Implied Volatility – Different Curves?

- **No hyphen or letter present** = Composite
 - A** = AMEX American Stock Exchange
 - B** = BOX Boston Stock Exchange - Options
 - E** = CBOE Chicago Board Options Exchange
 - I** = BATS
 - J** = NASDAQ OMX BX
 - O** = NASDAQ OMX
 - P** = NYSE Arca
 - X** = PHLX Philadelphia Stock Exchange
 - Y** = C2 Exchange
 - 4** = Miami Options Exchange
 - 8** = ISE International Securities Exchange

Implied Volatility

- Reasons for skews/smiles?
 - Risk Preferences
 - Fat tailed distributions

Implied Volatility Timings

Method	Timing
fsolve + python BSM	
fsolve + NAG BSM	
nag4py	
NAG C	

Implied Volatility Timings

Method	Timing
fsolve + python BSM	~60 seconds
fsolve + NAG BSM	
nag4py	
NAG C	

Implied Volatility Timings

Method	Timing
fsolve + python BSM	~60 seconds
fsolve + NAG BSM	~10 seconds
nag4py	
NAG C	

Implied Volatility Timings

Method	Timing
fsolve + python BSM	~60 seconds
fsolve + NAG BSM	~10 seconds
nag4py	~3 seconds
NAG C	

Implied Volatility Timings

Method	Timing
fsolve + python BSM	~60 seconds
fsolve + NAG BSM	~10 seconds
nag4py	~3 seconds
NAG C	~.15 seconds

Implied Volatility Timings

Method	Timing
fsolve + python BSM	~60 seconds
fsolve + NAG BSM	~10 seconds
nag4py	~3 seconds
NAG C	~.15 seconds

- **Derivatives?**
- **We have the derivative, vega**
 - $\frac{\partial C}{\partial \sigma} = S * T * N'(d_1)$

Fitting Data Points

- In our script we had $k = l = 3$...
 - What if we try different values?

Fitting Data Points

- In our script we had $k = l = 3$...
 - What if we try different values?
 - Poor results, can we do better?
 - Two dimensional spline

Thank You

Questions?

- Further reading see:
- <http://pandas.pydata.org/>
- <http://www.nag.co.uk/python.asp>
- <http://blog.nag.com/2013/10/implied-volatility-using-pythons-pandas.html>