Improving Milling Process Using Modeling

E. Budak Faculty of Engineering and Natural Sciences Sabanci University, Istanbul, Turkey

Abstract

Complex part surfaces such as dies and molds can be generated using CAD/CAM systems and precision machine tools. In many cases, however, part quality and productivity deteriorate during machining due to excessive cutting forces and chatter vibrations. Milling force and stability models can be very effective in optimizing the process by improving part quality and productivity. Process modeling is particularly important for low volume production such as die and mold machining as the opportunity for testing is very limited whereas in mass production process can be somewhat improved over time by trial and error. In this paper, analytical force and stability models developed by the author are presented with applications. An analytical milling force model and its simplified version for feedrate optimization are reviewed with applications. The application of the analytical stability model to spindle speed optimization for chatter suppression is demonstrated. As an alternative approach to chatter suppression, optimal design of variable pitch cutters is also presented. The paper is concluded with some recommendations for effective implementation of the models in production environment which can be used in die and mold machining.

Keywords: Milling, Cutting Force, Chatter

1 INTRODUCTION

Machining time and quality of dies and molds are very critical in many industries as they may be the limiting factors for development of production processes. Reduction of milling time and improvement of surface quality in machining can directly affect lead time and cost of product and process development cycle. Although modern CAD/CAM systems are very efficient in generating tool paths for complex die surfaces for various machine tool configurations and cutter geometries, material removal rate (MRR) and surface quality are usually limited due to the machining process. High cutting forces, chatter vibrations and tool deflections limit depth of cut, spindle speed and feed rate causing reduced productivity, and may also result in poor surface finish and dimensional quality. Machining process modeling can be very effective in improving productivity and quality. Therefore, in this paper, milling force and stability models that can be applied to die and mold machining are briefly discussed with some applications.

Modeling of milling process has been the subject of many studies and an overview is given in [1]. The focus of these studies have been mostly on the modeling of cutting force and stability and prediction of part quality. In several applications, cutting force models have been used in order to increase material removal rate by optimizing feed rate [2,3]. Milling forces have been investigated using different approaches. Koenigsberger and Sabberwal [4] developed equations for milling forces using mechanistic modeling where the cutting force coefficients which relate the chip area to the tangential, radial and axial forces are calibrated through force measurements. The mechanistic approach has been widely used for the force predictions and also extended to predict associated machine component deflections or surface geometrical errors [5-6]. Another approached is

to use mechanics of cutting in determining milling force coefficients as used by Armarego et al. [7]. In this approach, an oblique cutting force model together with an orthogonal cutting database are used to predict milling force coefficients eliminating the need for milling tests as different tool and cutting geometries can be handled by the oblique model [8]. Once the cutting force coefficients are known, the milling forces can be determined by integrating the forces along the cutting edges. Altintas et al. [9] also demonstrated the application of this approach to complex milling cutter geometries.

Tobias [10] and Tlusty [11] identified the most powerful source of self-excitation, regeneration, which is associated with the structural dynamics of the machine tool and the feedback between the subsequent cuts on the same cutting surface. Self-excited chatter vibrations in milling develop due to dynamic interactions between the cutting tool and workpiece which results in regeneration of waviness on the cutting surfaces, and thus modulation in the chip thickness [12]. Under certain conditions the amplitude of vibrations grows and the cutting system becomes unstable. Especially for highly flexible machining systems, such as long slender end mills which are also used in die and mold machining, chatter may develop even at very slow cutting speeds which are commonly used to suppress vibrations in metal cutting. In general, additional operations, mostly manual, are required to clean the chatter marks left on the surface. Thus, chatter vibrations result in reduced productivity, increased cost and inconsistent product quality.

The stability analysis of milling is complicated due to the rotating tool, multiple cutting teeth, periodical cutting forces and chip load directions, and multi-degree-of-freedom structural dynamics, and has been investigated using experimental, numerical and analytical methods. In the early milling stability analysis, Tlusty [12] used his

orthogonal cutting model considering an average direction for the cut. Later, however, Tlusty et al. [13] showed that the time domain simulations would be required for accurate stability predictions in milling. Sridhar et al. [14-15] performed a comprehensive analysis of milling stability which involved numerical evaluation of the dynamic milling system's state transition matrix. Minis et al. [16-17] used Floquet's theorem and the Fourier series for the formulation of the milling stability, and numerically solved it using the Nyquist criterion. Budak [18] developed a stability method which leads to analytical determination of stability limits. The method was verified by experimental and numerical results, and demonstrated to be very fast for the generation of stability lobe diagrams [19-20]. This method was also applied to the stability of ball-end milling [21]. These methods can be used to generate stability diagrams from which stable cutting conditions, and spindle speeds resulting in much higher stability can be determined. Stability diagrams have been extensively used in high speed milling applications, to utilize the large stability pockets [22]. If the required high spindle speeds are not available on machine tools, then another alternative is to use special geometry cutters to suppress chatter. Milling cutters with irregular tooth spacing, or variable pitch, can be used to improve the stability. The effectiveness of variable pitch cutters in suppressing chatter vibrations in milling was first demonstrated by Slavicek [23]. He assumed a rectilinear tool motion for the cutting teeth, and applied the orthogonal stability theory to irregular tooth pitch. By assuming an alternating pitch variation, he obtained a stability limit expression as a function of the variation in the pitch. Opitz et. al. [24] considered milling tool rotation using average directional factors, however they too considered alternating pitch with only two different pitch angles. Their experimental results and predictions showed significant increase in the stability limit using cutters with alternating pitch. Vanherck [25] considered different pitch variation patterns in the analysis by assuming rectilinear tool motion. His computer simulations showed the effect of pitch variation on stability limit. Tlusty et al. [26] analyzed the stability of milling cutters with special geometries such as irregular pitch or serrated edges, using numerical simulations. These studies mainly concentrated on the effect of pitch variation on the stability limit, however they do not address the cutting tool design, i.e. determination of optimal pitch variation. Altintas et. al. [27] adapted the analytical milling stability model to the case of variable pitch cutters which can be used more practically to analyze the stability with variable pitch cutters. Budak [3,28] recently developed an analytical method for the design of pitch angles for given chatter frequencies and spindle speeds.

Both cutting force and stability models can be used to improve the machining operations on dies and molds. Therefore, cutting force and stability models will be summarized in sections 2 and 3, and applications of the models will be presented in section 4. The paper will be concluded with brief summary of results and suggestions for the implementation of the models.

2 MILLING FORCES

2.1 Analytical Force Model

Milling forces can be modeled for given cutting and cutter geometry, cutting conditions, and work material. Force modeling for cylindrical milling cutters, which are used in roughing of dies and molds, will be given in the following. The model can be extended to ball end mills as presented in [29-32] for the analysis of finishing operations.



Figure 1. Cross sectional view of an end mill showing differential milling force components.

Consider the cross sectional view of an end mill shown in Figure 1. For a helical milling cutter, the radial (*b*) and axial depth of cut (*a*), number of teeth (*N*), cutter radius (*R*) and helix angle (β) determine what portion of a tooth is in contact with the workpiece for a given angular orientation of the cutter (ϕ). For a point on the (*j*th) cutting tooth, differential milling forces in the tangential (*dF*_i) and radial direction (*dF*_r) and can be given as

$$dF_{t_j}(\phi, z) = K_t h_j(\phi, z) dz$$

$$dF_{r_j}(\phi, z) = K_r dF_{t_j}(\phi, z)$$
(1)

where ϕ is the immersion angle measured from the positive *y* axis as shown in Figure 1. The uncut chip thickness can be approximated as

$$h_{i}(\phi, z) = f_{t} \sin \phi_{i}(z) \tag{2}$$

where f_t is the feed per tooth and $\phi_j(z)$ is the immersion angle for the flute (*j*) at axial position *z*. Due to the helical flute, the immersion angle changes along the axial direction as

$$\phi_j(z) = \phi + (j-1)\phi_p - \frac{\tan\beta}{R}z$$
(3)

where the pitch angle is defined as $\phi_p = 2\pi N$. Milling force coefficients K_r and K_r can be expressed as exponential functions of the average chip thickness as follows

$$K_t = K_T h_a^{-p} \quad ; \quad K_r = K_R h_a^{-q} \tag{4}$$

Constants K_{τ} , K_{κ} , p and q are usually determined using mechanistic models where linear regression on measured milling forces is performed for different feedrates. Therefore, they depend on the workpiece material and cutting tool geometry. The average chip thickness is defined as the ratio of the chip volume produced in one revolution of the cutter to the exposed chip area:

$$h_a = f_t \frac{\cos\phi_{st} - \cos\phi_{ex}}{\phi_{ex} - \phi_{st}} = f_t \Phi$$
(5)

Where ϕ_{st} and ϕ_{ex} are the start and exit angles of the tooth to and from the cut, respectively. A more general approach is to use mechanics of milling method as presented in [7]. An oblique cutting force model is used to relate the shear stress in the shear plane, shear angle, friction coefficient on the rake face, rake angle,

chip flow angle and helix angle to the cutting force coefficients. The cutting data are obtained from orthogonal cutting tests as explained in [7-8].

The tangential and radial forces can be resolved in the feed, x, and normal, y, directions as follows

$$dF_{x_j} = -dF_{t_j} \cos\phi_j - dF_{r_j} \sin\phi_j$$

$$dF_{y_j} = dF_{t_j} \sin\phi_j - dF_{r_j} \cos\phi_j$$
(6)

By integrating the differential forces given in equation (6) within the portion of the cutting flute that is in contact with the workpiece, the forces contributed by one tooth of the cutter can be determined as [6]

$$F_{x_{j}}(\phi) = \frac{K_{t}f_{t}R}{4\tan\beta} \left[-\cos 2\phi_{j} + K_{r} \left(2\phi_{j}(z) - \sin 2\phi_{j}(z) \right) \right]_{z_{jl}(\phi)}^{z_{ju}(\phi)}$$

$$F_{y_{j}}(\phi) = -\frac{K_{t}f_{t}R}{4\tan\beta} \left[\left(2\phi_{j}(z) - \sin 2\phi_{j}(z) \right) + K_{r}\cos 2\phi_{j}(z) \right]_{z_{jl}(\phi)}^{z_{ju}(\phi)}$$
(7)

where $z_{jl}(\phi)$ and $z_{jul}(\phi)$ are the lower and higher limits of the contact for the tooth (*j*). The total milling forces on the cutter in both directions can then be determined as

$$F_{x}(\phi) = \sum_{j=1}^{N} F_{x_{j}}(\phi) \quad ; \qquad F_{y}(\phi) = \sum_{j=1}^{N} F_{y_{j}}(\phi)$$
(8)

Milling forces for a more complicated tool geometry such as ball end mills can be determined similarly [29-31]. As it can be seen from the above equations, prediction of milling forces require that the tool geometry, cutting coefficients, cutting conditions and tool-workpiece intersection geometry, i.e. axial and radial depth of cuts, be known accurately. If the required information is available, then the model can be used to optimize or schedule feedrate in a cutting cycle for a target cutting force as shown in [2] or maximum allowed tool deflection as demonstrated in [6,32-33]. However, for many cases there is a need for a more practical model as all of the required data may not be available for production implementation. This simplified model is presented next.

2.2 Simplified Force Model For Feedrate Optimization

As shown by equation (7), milling forces vary periodically as the cutter rotates. In order to maximize the feedrates, the peak resultant force in the (x,y) plane, F, which cause bending stresses in the tool, has to be kept under control. The resultant force can be expressed as

$$F_{R}(\phi) = \sqrt{F_{x}^{2}(\phi) + F_{y}^{2}(\phi)}$$
(9)

and the peak resultant force as

$$F = \max\{F_R(\phi) ; \phi : [0 - 2\pi]\}$$
(10)

From equations (1-10), F can also be described as

$$F = K_t f_t C(\beta, N, R, a, b, K_r)$$
(11)

where *C* is function of tool and cutting geometry, and K_r . Substituting equations (4-5) into (11):

$$F = K_T f_t^{1-p} \Phi^{-p} C(\beta, N, R, a, b, K_R, q)$$
(12)

If equation (12) is written for two different feedrates, f_{t1} and f_{t2} , corresponding to two peak forces, F_1 and F_2 , and they are divided side by side (neglecting the effect of feed on K_r for simplicity), the following is obtained:

$$\frac{F_1}{F_2} \approx \left(\frac{f_{t1}}{f_{t2}}\right)^{1-p} \tag{13}$$

Equation (13) can be used to optimize feedrates to maintain the peak force constant in a cycle. For this, the peak resultant milling force, F_{M} , using the original feedrate, f_t , need to be measured. Then, the optimal feedrate, f_o , for a reference target force, F_{ref} , can be determined from equation (13) as

$$f_o = f_t \left(\frac{F_M}{F_{ref}}\right)^{\frac{1}{1-p}}$$
(14)

The feed rate override FOR can also be determined as

$$FOR = \frac{f_o}{f_t} = \left(\frac{F_M}{F_{ref}}\right)^{\frac{1}{1-p}}$$
(15)

Equation (14) can be used to optimize feedrates in a milling cycle provided that the milling forces are known. A force dynamometer can be used to measure the milling forces. Therefore, this method is applicable to the cases where more than one die need to be machined.

3 MILLING STABILITY

3.1 Analytical Model for Stability Lobes

Another very important limitation in milling is the self excited chatter vibrations which cause poor surface finish and tool life resulting in reduced productivity. In this section, the stability model developed by Budak [18-20] will be reviewed. In this analysis, both milling cutter and workpiece are considered to have two orthogonal modal directions as shown in Figure 2. Milling forces excite both cutter and workpiece causing vibrations which are imprinted on the cutting surface. Each vibrating cutting



Figure 2. Chatter model for milling.

tooth removes the wavy surface left from the previous tooth resulting in modulated chip thickness which can be expressed as follows:

$$h_{j}(\phi) = s_{t} \sin \phi_{j} + (v_{j_{c}}^{o} - v_{j_{w}}^{o}) - (v_{j_{c}} - v_{j_{w}})$$
(16)

where the feed per tooth s_t represents the static part of the chip thickness, and $\phi_{j=(j-1)}\phi_{p}+\phi}$ is the angular immersion of tooth (j) for a cutter with constant pitch angle $\phi_p=2\pi/N$ as shown in Figure 2. $\phi=\Omega_t$ is the angular position of the cutter measured with respect to the first tooth and corresponding to the rotational speed Ω (rad/sec). v_j and v_j^o are the dynamic displacements due to tool and workpiece vibrations for the current and previous tooth passes, for the angular position ϕ_{j_t} and can be expressed in terms of the fixed coordinate system as follows:

$$v_{j_p} = -x_p \sin\phi_j - y_p \cos\phi_j \qquad (p = c, w)$$
(17)

where *c* and *w* indicate cutter and workpiece, respectively. The static part in equation (1), $(s_i \sin \phi_j)$, is neglected in the stability analysis. If equation (17) is substituted in equation (16), the following expression is obtained for the dynamic chip thickness in milling:

$$h_j(\phi) = [\Delta x \sin \phi_j + \Delta y \cos \phi_j]$$
(18)

where

$$\Delta x = (x_c - x_c^o) - (x_w - x_w^o)$$

$$\Delta y = (y_c - y_c^o) - (y_w - y_w^o)$$
(19)

where (x_c, y_c) and (x_w, y_w) are the dynamic displacements of the cutter and workpiece in x and y directions, respectively. Similar to the static force analysis, dynamic cutting forces can be obtained using the dynamic chip thickness as

$$\begin{cases} F_x \\ F_y \end{cases} = \frac{1}{2} a K_t \begin{bmatrix} a_{xx} & a_{xy} \\ a_{yx} & a_{yy} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$
(20)

where the directional coefficients are given as:

$$a_{xx} = -\sum_{j=1}^{N} \sin 2\phi_j + K_r (1 - \cos 2\phi_j)$$

$$a_{xy} = -\sum_{j=1}^{N} (1 + \cos 2\phi_j) + K_r \sin 2\phi_j$$

$$a_{yx} = -\sum_{j=1}^{N} -(1 - \cos 2\phi_j) + K_r \sin 2\phi_j$$

$$a_{xx} = -\sum_{j=1}^{N} -\sin 2\phi_j + K_r (1 + \cos 2\phi_j)$$
(21)

The directional coefficients depend on the angular position of the cutter which makes equation (20) time-varying :

$$\{F(t)\} = \frac{1}{2} a K_t [A(t)] [\Delta(t)]$$
(21)

[A(t)] is periodic with the tooth passing frequency of $\omega = N\Omega$ and with the corresponding period of $T = 2\pi/\omega$. In general, the Fourier series expansion of the periodic term is used for the solution of the periodic systems [34]. Budak and Altintas [18-20] have shown that the higher harmonics do not affect the accuracy of the predictions, and it is sufficient to include only the average term in the Fourier series expansion of [A(t)]:

$$[A_0] = \frac{1}{T} \int_0^T [A(t)] dt$$
 (22)

As all the terms in [A(t)] are valid within the cutting zone between start and exit immersion angles (ϕ_{st} , ϕ_{ex}), equation (22) reduces to the following form in the angular domain:

$$[A_0] = \frac{1}{\phi_p} \int_{\phi_{xx}}^{\phi_{ex}} [A(\phi)] d\phi = \frac{N}{2\pi} \begin{bmatrix} \alpha_{xx} & \alpha_{xy} \\ \alpha_{yx} & \alpha_{yy} \end{bmatrix}$$
(23)

where the integrated, or average, directional coefficients are given as:

$$\alpha_{xx} = \frac{1}{2} [\cos 2\phi - 2K_r \phi + K_r \sin 2\phi]_{\phi_{xt}}^{\phi_{ex}}$$

$$\alpha_{xy} = \frac{1}{2} [-\sin 2\phi - 2\phi + K_r \cos 2\phi]_{\phi_{xt}}^{\phi_{ex}}$$

$$\alpha_{yx} = \frac{1}{2} [-\sin 2\phi + 2\phi + K_r \cos 2\phi]_{\phi_{xt}}^{\phi_{ex}}$$

$$\alpha_{yy} = \frac{1}{2} [-\cos 2\phi - 2K_r \phi - K_r \sin 2\phi]_{\phi_{xt}}^{\phi_{ex}}$$
(24)

Substituting equation (23), equation (21) reduces to the following form:

$$\{F(t)\} = \frac{1}{2} a K_t [A_0] [\Delta(t)]$$
(25)

Substituting harmonic functions for dynamic forces and vibrations, the characteristics equation is obtained as

$$\det[I] + \Lambda[G_0(i\omega_c)]] = 0$$
(26)

where [1] is the unit matrix, and the oriented transfer function matrix is defined as:

$$[G_0] = [A_0][G]$$

$$[G(i\omega_c)] = [G_c(i\omega_c)] + [G_w(i\omega_c)]$$

$$[G_p] = \begin{bmatrix} G_{p_{xx}} & G_{p_{xy}} \\ G_{p_{yx}} & G_{p_{yy}} \end{bmatrix} (p = c, w)$$
(27)

and the eigenvalue (Λ) in equation (26) is given as

$$\Lambda = -\frac{N}{4\pi} K_t a \left(1 - e^{-i\omega_c T} \right) \tag{28}$$

If the eigenvalue Λ is known, the stability limit can be determined from equation (28). Λ can easily be computed from equation (26) numerically. However, an analytical solution is possible if the cross transfer functions, G_{xy} and G_{yx} , are neglected in equation (26):

$$\Lambda = -\frac{1}{2a_0} \left(a_1 \pm \sqrt{a_1^2 - 4a_0} \right)$$
(29)

where

 Λ_R

 $1 - \cos \omega_c T$

$$a_{0} = G_{xx}(i\omega_{c})G_{yy}(i\omega_{c})(\alpha_{xx}\alpha_{yy} - \alpha_{xy}\alpha_{yx})$$

$$a_{1} = \alpha_{xx}G_{xx}(i\omega_{c}) + \alpha_{yy}G_{yy}(i\omega_{c})$$

$$\Lambda_{I} \qquad \sin \omega_{c}T$$
(30)

The above can be solved to obtain a relation between the chatter frequency and the spindle speed [19-20]

$$\omega_{c}T = \varepsilon + 2k\pi$$

$$\varepsilon = \pi - 2\psi; \ \psi = \tan^{-1}\kappa \qquad (31)$$

$$n = \frac{60}{NT}$$

where ε is the phase difference between the inner and outer modulations, k is an integer corresponding to the number of vibration waves within a tooth period, and n is the spindle speed (rpm). After the imaginary part in equation (28) is vanished, the following is obtained for the stability limit [19-20]:

$$a_{\rm lim} = -\frac{2\pi\Lambda_R}{NK_t} \left(1 + \kappa^2 \right) \tag{32}$$

Equations (31-32) can be used to determine the stability limit and corresponding spindle speed. When this procedure is repeated for a range of chatter frequencies and number of vibration waves, k, the stability lobe diagram for a milling system is obtained.

3.2 Stability of Variable Pitch Cutters

In general, stability diagrams can be used to increase the chatter free material removal rate particularly at high cutting speeds where the stability lobes are larger. However, this approach may not be very effective if a high speed spindle is not available. Variable pitch cutters can be very effective in suppressing chatter even at slow speeds. The stability limit in terms of axial depth of cut (*a*) with equal pitch cutters can be put in the following form

$$a_{\lim} = -\frac{4\pi}{K_t} \frac{\Lambda_I}{N \sin \omega_e T}$$
(33)

The analytical stability model was used in [28] for the stability of variable pitch cutters and the stability limit was obtained as

$$a_{\lim}^{vp} = -\frac{4\pi}{K_t} \frac{\Lambda_I}{S}$$
(34)

where

$$S = \sum_{j=1}^{N} \sin \omega_c T_j \tag{35}$$

Note that *S* in equation (34) replaces $Nsin\omega_c T$ in equation (33) as the tooth period *T*, and thus the phase between

subsequent vibration waves, ε_i is different for each tooth due to nonequal pitch angles:

$$\varepsilon_j = \omega_c T_j \qquad (j = 1,..,N)$$
 (36)

The fundamental idea behind the variable pitch cutters is to alter the phase difference ε which is the main source of regenerative chatter. It is possible to increase the stability by selecting the pitch angles in such a way that the cumulative effect of the phase delay is minimized, i.e. S=0 in equation (34). The phase difference in equation (36) can be re-written as:

$$\varepsilon_{j} = \varepsilon_{1} + \Delta \varepsilon_{j} \quad (j = 2, ..., N)$$
 (37)

where $\Delta \varepsilon_j$ is the phase difference between tooth *j* and tooth (1) corresponding to the difference in the pitch angles for these teeth. For an introduced variation in pitch, ΔP , the corresponding change in the phase difference can be determined from the geometry as [28]

$$\Delta \varepsilon = \frac{\omega_c}{\omega_s} \Delta P \tag{38}$$

where $\omega_s = 2\pi n/60$ and *n* is the spindle speed in (rpm). Although different pitch variation patterns can be used, Budak [28] shows that the linear variation gives the widest range of high stability area where

Pitch:
$$P_0, P_0 + \Delta P, P_0 + 2\Delta P, P_0 + 3\Delta P,...$$
 (39)

Then, S takes the following form

$$S = \sin \varepsilon_1 (1 + \cos \Delta \varepsilon + \cos 2\Delta \varepsilon + ...) + \cos \varepsilon_1 (\sin \Delta \varepsilon + \sin 2\Delta \varepsilon + ...)$$
(40)

S=0 can be satisfied for

$$\Delta \varepsilon = k \frac{2\pi}{N} \qquad (k = 1, 2, ..., N - 1)$$
(41)

This is the amount of additional phase that has to be introduced to satisfy S=0. The corresponding optimal pitch variation can be determined from equation (38). As the sum of all pitch angles must be equal to 2π (rad), P_o can be determined as

$$P_0 = \frac{2\pi}{N} - \frac{(N-1)\Delta P}{2}$$
(42)



Figure 3. Variation in the stability limit with the phase difference $\Delta \varepsilon$ introduced by a variable pitch cutter with linear pitch variation.

In order to determine the stability increase with the variable pitch cutters, the ratio of the stability limit with variable pitch cutters, as given in equation (34), to the minimum, or absolute, stability limit for equal pitch cutters, i.e. equation (33) with (sin $\omega_r T = I$), is considered

$$r = \frac{a_{\lim}^{vp}}{a_{cr}} = \frac{N}{S}$$
(43)

Figure 3 shows the variation of the stability gain (r) for a 4-tooth milling cutter. As ε_1 in general depends on the cutting conditions and the dynamic characteristics of the milling system, r is plotted for 3 different values of ε_1 . As it can be seen from figure 2 the stability increase is maximized for the specific values of $\Delta \varepsilon$ as also predicted by equation (9). It should be noted that variations in the chatter frequency and speed may cause $\Delta \varepsilon$ and thus r to vary. However, at least 4 times stability improvement is guaranteed for a cutter with 4 teeth for $\pi/2 < \Delta \varepsilon < 3\pi/2$. Therefore ΔP should be selected such $\pi/2 < \Delta \varepsilon < 3\pi/2$ must be satisfied always that considering possible variations in chatter frequency.

4 APPLICATIONS

The applications of the models presented in the previous sections will be demonstrated through several examples in the following.

4.1 Feedrate Optimization in Roughing

The optimization of the feedrates in a 5-axis slotting operation is considered. The work material is titanium alloy (Ti6Al4V) and a taper-ball-end-mill is used in this heavy roughing operation where the productivity is limited due to cutter breakage. Therefore, feedrates can be optimized to keep the cutting forces at a safe level. The variation of the peak cutting force determined from the measured milling forces is shown in Figure 4. As it can be seen from the figure, the peak force in this cycle increases 3 times due to changes in the axial depth of cut and the tool orientation with respect to the feed direction. The target force of 900 N for feed optimization is selected from a database which contains safe force levels for different cutter sizes. Finite element analysis can also be used to determine the cutting load capacity of end mills as presented in [35]. The parameter p has been identified as 0.58 from the cutting force data. The FOR determined using equation (15) is shown in Figure 5. FOR is limited to maximum of 3 as it corresponds to the maximum chip thickness for this cutter. The modified FOR is also shown in the figure. The forces in this slotting operation were measured again, this time using the new feedrates. The simulated forces from equation (15) and the measured forces before and after the feed rate optimization are shown in Figure 6. Although the measured forces are close to the target force of 900 N for most of the cycle, there are some discrepancies especially in the beginning of the cycle. This is mainly due to the neglected effect of feedrate on K_r in the formulation which is amplified in the beginning of the cycle due to very high FOR values. Dynamic effects and the changes in the orientation of the cutter also contribute to the differences. The feedrates can be updated again by using the new forces with the new feedrates to get closer to the target force. However, this was not pursued in this case since the difference is only about 100 N as it can be seen from the figure. The important point to note here is that the cycle time has been reduced by about 35% with the new feedrates. Depending on the force variation in a cutting cycle higher reductions can also be obtained.



Figure 4. Variation of the peak forces in a 5-axis milling operation



Figure 5. Variation of the calculated and modified feed override values in the example slotting operation.



Figure 6. Peak force variations with the original and optimized feedrates. Simulated forces are also shown.

4.2 Stability Lobes

Demonstration of the method will be done on a 2-DOF end milling example given in [20]. The dynamic properties of the 3-flute end mill in two orthogonal directions were identified using modal analysis as [36] follows

	ω_n (Hz)	<i>k</i> (kN/m)	ζ
X	603	5600	0.039
Y	666	5700	0.035



Figure 7. Analytical and numerical stability diagrams for the example case considered.

The aluminum workpiece is considered to be rigid compared to the cutter. The stability diagram for this system has been generated using the analytical model and is shown in Figure 7. As it can be seen from the figure, the stability limit predictions using the analytical method and the numerical time-domain solutions are very close. It should be noted that the analytical stability diagram can be generated in a few seconds whereas time domain simulations usually take several hours (up to a full day) depending on the precision required. In time domain simulations, dynamic system equations have to be simulated over several tool rotations using very small time steps. As it can be seen from the figure, the chatterfree depth-of-cut could be increased about 8 times by using about 12500 rpm compared to, for example, 8000 rpm. With the increased spindle speed, this would result in more than 10 times increase in chatter-free material removal rate.

4.3 Chatter Suppression with Variable Pitch Cutters

The variable pitch cutter design is demonstrated through a finishing operation on a very flexible workpiece (compressor blade). The original process with equal pitch cutter was highly unstable, and the sound spectrum shown in Figure 8 was used to determine the chatter frequency. The depth of cut varies throughout the cutting cycle, maximum being more than 100 mm. A taper-ballend mill made out of carbide with length-to-average diameter ratio of over 10 and 6 flutes is used to flank mill the blade. The chatter in this operation was so severe that a very slow spindle speed, 300 rpm, was being used to reduce the intensity. However, the surface finish was still very poor, and additional finishing cuts and manual finishing were necessary to remove the chatter marks. The optimal variable pitch cutter with linear pitch variation can be designed using equations (38-42). For the chatter frequency of 420 Hz (as shown in Figure 8), spindle speed of 300 rpm and $\Delta \varepsilon = \pi$, the pitch variation is determined as $\Delta P \approx 2^{\circ}$ and $P_{o} = 55$. Note that although equation (41) gives multiple solutions, $\Delta \varepsilon$ should be chosen as π when N is even, and $(N \pm 1)\pi/N$ when N is odd, since the high stability region is the widest close to the center, as it can also be seen from Figure 3. The variable pitch cutter with pitch angles (55, 57, 59, 61, 63, 65) suppressed the chatter completely in this operation, at 300 rpm. The improvement in surface finish is shown in Figure 9. The cutting tool life was also improved significantly, by about a factor of 2.



Figure 8. Sound spectrum of the blade finish-milling operation using a 6-fluted equal pitch end mill.



Figure 9. Improvement of surface finish using the variable pitch cutter.

5 SUMMARY

In this paper, milling process models which can be used in the optimization of die and mold machining are reviewed. The cutting force models can be used in feedrate optimization of roughing operations. For complete analytical modeling of the forces, cutting geometry and the material dependent milling force coefficients have to be known. If multiple parts are to be machined, feedrates can be optimized using the simplified force model based on the force measurements performed on the first part. For chatter suppression, stability lobes can be used in order to determine preferred spindle speeds with high chatter-free material removal rate when a high speed machine is available. Stability lobes can be generated using the analytical model which requires the modal data for the cutting system. Variable pitch cutters, on the other hand, can be used to improve stability at slow speeds as well. Chatter frequency must be known for the design of these cutters using the analytical model presented in this paper. Therefore, force and stability models can be used to improve productivity and surface finish quality in machining of dies and molds.

6 REFERENCES

- Smith, S. and Tlusty, J., 1991, "An Overview of Modeling and Simulation of the Milling Process", Trans. ASME Journal of Engineering for Industry, Vol.113, pp.169-175.
- [2] Yazar, Z., Koch, K.F., Merrick, T. and Altan, T., 1994, "Feed Rate Optimization Based on Cutting Force Calculations in 3-Axis Milling of Dies and Molds with Sculptured Surfaces", Int. Journal of Machine Tools and Manufact., Vol.34, No.3, pp.365-377.

- [3] Budak, E. (Kops, L.), 2000, "Improving Part Quality in Milling of Titanium Based Impellers by Chatter Suppression and Force Control", Annals of the CIRP, Vol.49/1, pp.31-38.
- [4] Koenigsberger, F. and Sabberwal, A.J.P., 1961, "An Investigation into the Cutting Force Pulsations During Milling Operations", International Journal of Machine Tool Design and Research, Vol. 1, pp. 15-33.
- [5] Kline, W.A., DeVor, R.E., and Shareef, I.A., 1982, "The Prediction of Surface Accuracy in End Milling", Trans ASME Journal of Engineering for Industry, Vol. 104, pp. 272-278.
- [6] Budak, E. and Altintas, Y., 1995, "Modeling and Avoidance of Static Deformations in Peripheral Milling of Plates", International Journal of Machine Tools and Manufacture, Vol. 34, No. 3, pp.459-476.
- [7] Armarego, E.J.A. and Whitfield, R.C., 1985, "Computer Based Modeling of Popular Machining Operations for Force and Power Predictions", Annals of the CIRP, Vol. 34, pp.65-69.
- [8] Budak, E., Altintas, Y. and Armarego, E.J.A., 1996, "Prediction of Milling Force Coefficients From Orthogonal Cutting Data", Trans. ASME Journal of Manufacturing Science and Engineering, Vol. 118, pp. 216-224.
- [9] Altintas, Y. and Lee, P., 1996, "A General Mechanics and Dynamics Model for Helical End Mills", Annals of the CIRP, Vol. 45, pp. 59-64.
- [10] Tobias, S.A., 1965, Machine Tool Vibration, Blackie and Sons Ltd.
- [11] Tlusty, J. and Polacek, 1963, "The Stability of Machine Tools Against Self Excited Vibrations in Machining", International Research in Production Engineering, ASME, pp. 465-474.
- [12] Koenigsberger, F. and Tlusty, J., 1967, Machine Tool Structures-Vol I: Stability Against Chatter, Pergamon Press.
- [13] Tlusty, J. and Ismail, F., 1981, "Basic Nonlinearity in Machining Chatter", Annals of the CIRP, 30:21-25.
- [14] Sridhar, R., Hohn, R.E. and Long, G.W., 1968a, "General Formulation of the Milling Process Equation", ASME Journal of Engineering for Industry, pp. 317-324.
- [15] Sridhar, R., Hohn, R.E. and Long, G.W., 1968b, "A Stability Algorithm for the General Milling Process", ASME Journal of Engineering for Industry, pp.330-334.
- [16] Minis, I, Yanushevsky, T., Tembo, R. and Hocken, R., 1990, "Analysis of Linear and Nonlinear Chatter in Milling", Annals of the CIRP, 39:459-462.
- [17] Minis, I. and Yanushevsky, T., 1993, "A New Theoretical Approach for the Prediction of Machine Tool Chatter in Milling", ASME Journal of Engineering for Industry, Vol. 115, pp. 1-8.
- [18] Budak, E., 1994, "The Mechanics and Dynamics of Milling Thin-Walled Structures", Ph.D. Dissertation, University of British Columbia.
- [19] Altintas, Y. and Budak, E., 1995, "Analytical Prediction of Stability Lobes in Milling", Annals of the CIRP, 44/1:357-362.
- [20] Budak,E. and Altintas, Y., 1998, "Analytical Prediction of Chatter Stability in Milling-Part I: General Formulation; Part II: Application to

Common Milling Systems", Trans. ASME, Journal of Dynamic Systems, Measurement, and Control, Vol. 120, pp. 22-36.

- [21] Altintas, Y., Shamoto, E., Lee, P. and Budak, E., 1999, "Analytical Prediction of Stability Lobes in Ball-End-Milling", Trans. ASME Journal of Manufacturing Science and Engineering, Vol. 121, No. 4, pp. 586-592, Nov. 1999
- [22] Smith, S. and Tlusty, J., 1993, "Efficient Simulation Programs for Chatter in Milling", Annals of the CIRP, 42/1:463-466.
- [23] Slavicek, J., 1965, "The Effect of Irregular Tooth Pitch on Stability of Milling", Proceedings of the 6th MTDR Conference, Pergamon Press, London, pp.15-22.
- [24] Opitz, H., Dregger, E.U. and Roese, H., 1966, "Improvement of the Dynamic Stability of the Milling Process by Irregular Tooth Pitch", *Proceedings of the Adv. MTDR Conference*, No.7, p.213.
- [25] Vanherck, P., 1967, "Increasing Milling Machine Productivity by Use of Cutters with Non-Constant Cutting Edge Pitch", 8th MTDR Conference, Manchester, pp. 947-960.
- [26] Tlusty, J., Ismail, F. and Zaton, W., 1983, "Use of Special Milling Cutters Against Chatter", NAMRC 11, SME, pp.408-415.
- [27] Altintas, Y., Engin, S. and Budak, E., 1999, "Analytical Stability Prediction and Design of Variable Pitch Cutters", Trans. ASME Journal of Manufacturing Science and Engineering, 121:173-178.
- [28] Budak, E, 2000, "An Analytical Design Method For Milling Cutters with Non Constant Pitch to Increase Stability", Proceedings of ASME-IMECE, Orlando, USA.
- [29] Altintas, Y. and Lee, P., 1998, "Mechanics and Dynamics of Ball End Milling", Trans. ASME Journal of Manufactruring Science and Engineering, Vol.120, pp.684-692.
- [30] Yucesan, G. and Altintas, Y., 1996, "Prediction of Ball End Milling Forces", Trans. ASME Journal of Manufactruring Science and Engineering, Vol.118, pp.95-103.
- [31] Yang, M. and Park, H., 1991, "The Prediction of Cutting Force in Ball End Milling", International Journal of Machine Tools and Manufact., Vol.31, No.1,pp.45-54.
- [32] Yang, M.Y. and Sim, C.G., 1993, "Reduction of Machining Errors by Adjustment of Feedrates in the Ball-End Milling Process", International Journal of Production Research, Vol.31,No.3, pp.665-689.
- [33] Budak, E., Altintas, Y., "Flexible Milling Force Model for Improved Surface Error Predictions", ASME 1992 European Joint Conference on Engineering Systems Design and Analysis, Istanbul, Turkey, ASME PD-Vol. 47-1, 1992.
- [34] Magnus, W. and Winkler, S., 1966, Hill's Equation, Wiley, N.Y.
- [35] Nemes, J.A., Asamoah-Attia, S. and Budak, E., 2001, "Cutting Load Capacity of End Miils with Complex Geometry", to appear in Annals of the CIRP.
- [36] Weck, M., Altintas, Y. and Beer, C., 1994, "CAD Assisted Chatter Free NC Tool Path Generation in Milling ", International Journal of Machine Tools and Manufact., Vol.34, p.879-891.