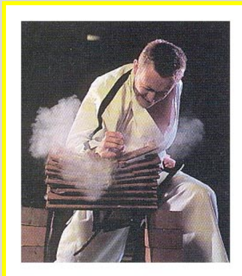
Impulse and Momentum



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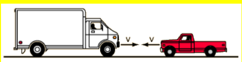

Force	F	=	F
Impulse	F_t	=	F_t
Change in momentum	$m\Delta v$	=	$m\Delta v$
Acceleration	ma	=	ma

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Linear Momentum

"The change of motion is ever proportional to the motive force impressed; and is made in the direction of the right [straight] line in which that force is impressed"

Sir Isaac Newton

What Newton called "motion" translates into "moving inertia".

Today the concept of moving inertia is called momentum which is defined as the product of mass and velocity.

LARGE MASS, SMALL VELOCITY

$$momentum = mass \times velocity$$

momentum is a vector quantity

$$\vec{p} = m\vec{v}$$

SI unit for momentum

$$1\ \text{kilogram} \times 1\ \frac{\text{meter}}{\text{second}} = 1\ \frac{\text{kg}\cdot\text{m}}{\text{s}}$$

SMALL MASS, LARGE VELOCITY

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Impulse and Momentum

Newton's 2nd Law was written in terms of momentum and force

$$F = ma$$

$$F = \frac{m\Delta v}{t}$$

$$F = \frac{\Delta p}{t}$$

IMPULSE MOMENTUM THEOREM

$$Ft = m\Delta v$$

impulse → Ft; momentum change → $m\Delta v$

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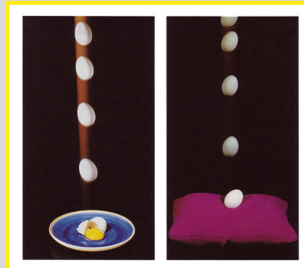

Impulse causes a change of momentum for any object. This is analogous to work, which causes a change of energy for any object.

impulse is a vector quantity

$$\vec{I} = \vec{F}t$$

SI unit of impulse

$$1 \text{ newton} \times 1 \text{ second} = 1 \text{ N} \cdot \text{s}$$

$$1 \text{ N} \cdot \text{s} = 1 \frac{\text{kg} \cdot \text{m}}{\text{s}}$$

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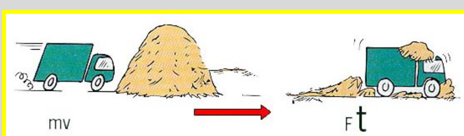
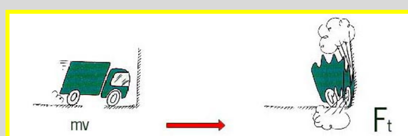
Impulse and Safety



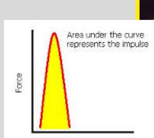
MOMENTUM CHANGED BY A SMALL FORCE OVER A LONG TIME

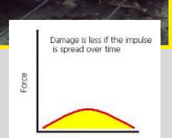


MOMENTUM CHANGED BY A LARGE FORCE OVER A SHORT TIME

Other examples of car safety that involve increased time and decreased force (but result in equal impulse)

- Airbags
- Seatbelts
- Crumple zones
- Bumpers
- Padding





Other examples force/time in impulse

- Air cushioned shoes
- Bending knees when landing
- Natural turf vs. artificial turf
- Gym and playground mats
- Helmets/padding for sports
- Shocks/forks in bicycles

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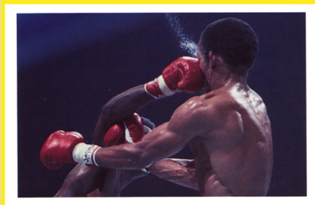
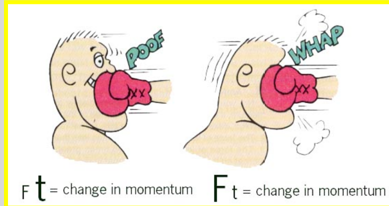
Impulse of Sports





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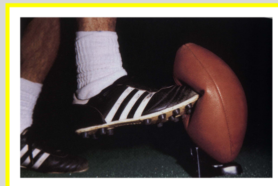
A boxer who "rolls with the punch" will experience less force over more time.



IN SPORTS THE IMPACT TIME IS SHORT, BUT EVERY BIT COUNTS!

In many sports you are taught to "follow through". Why?

As you "follow through" the time of contact with the ball is increased, so the amount of momentum change is also increased. You get more momentum and more speed and more distance!

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Impulse of Sports

Ball	**Mass** (kg)	**speed imparted** (m/s)	**impact time** (ms)
Baseball	0.149	39	1.25
Football (punt)	0.415	28	8
Golf ball (drive)	0.047	69	1
Handball (serve)	0.061	23	12.5
Soccer ball (kick)	0.425	26	8
Tennis ball (server)	0.058	51	4

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Third Law and Impulses

Every action has an equal and opposite reaction:

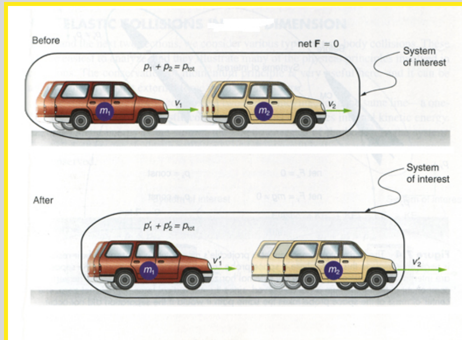
$$F_1 = -F_2$$

Every action takes just as long as the reaction so:

$$t_1 = t_2$$

Every impulse has an equal and opposite impulse:

$$F_1 t_1 = -F_2 t_2$$



The momentum changes are equal and opposite:

$$m_1 \Delta v_1 = -m_2 \Delta v_2$$

The momentum changes are equal and opposite:

$$\Delta p_1 = -\Delta p_2 \quad \text{or} \quad \Delta p_1 + \Delta p_2 = 0$$

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Conservation of Momentum

If two isolated objects interact (collide or separate), then the total momentum of the system is conserved (constant).

$$p_i = p_f \quad \text{or} \quad m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

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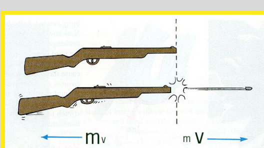
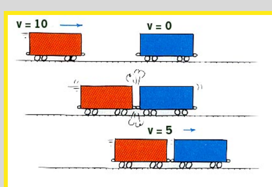
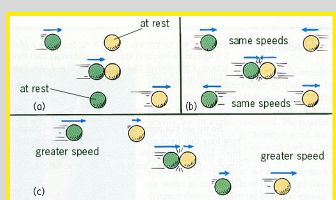


SEPARATION OR EXPLOSION



INELASTIC COLLISION



ELASTIC COLLISION

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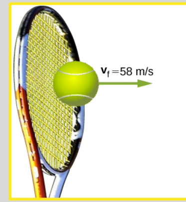
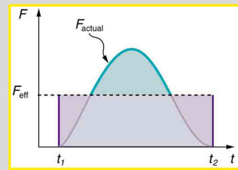
Examples of Impulse and Momentum Problems

In 2007, Venus Williams hit the fastest recorded serve in a women's match, at a speed of 58 m/s. What is the average force exerted on the 0.057-kg tennis ball by her racquet, assuming she hit the ball from rest, and the ball remained in contact with the racquet for 5.0 milliseconds.



$$Ft = m\Delta v \;\Rightarrow\; F(0.005\text{ s}) = (0.057\text{ kg})(58 - 0\text{ m/s}) \;\Rightarrow\; \boxed{F = 661\text{ N}}$$

In real-life collisions, the forces acting on an object are not constant. For example, when a bat strikes a baseball, the force increases with time, and then decreases, much like the figure to the right. However, if an effective force (or average) is know, the impulse-momentum theorem works! Suppose that a 42 m/s (94 mph) fastball is hit with an effective force of 760 N for 0.017 s. How fast does a 145 gm ball leave the bat if hit directly back at the pitcher?



$$Ft = m\Delta v \;\Rightarrow\; (760\text{ N})(0.017\text{ s}) = (0.145\text{ kg})(v_f - 42\text{ m/s}) \;\Rightarrow\; \boxed{v_f = 49.1\text{ m/s}}\text{ or 110 mph!}$$

An 8,000 kg railroad car is moving along a low-friction track at 3 m/s when a load of gravel is dropped into the railroad car. After the load is gravel is dropped, the railroad car moves at 2.5 m/s. How much is the mass of the dropped gravel?

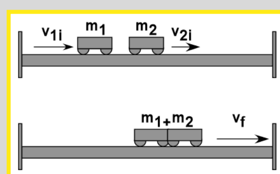
$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$(8000\text{ kg})(3\text{ m/s}) + m_2(0) = (8000\text{ kg} + m_2)(2.5\text{ m/s}) \;\Rightarrow\; m_2 = \boxed{1600\text{ kg}}$$

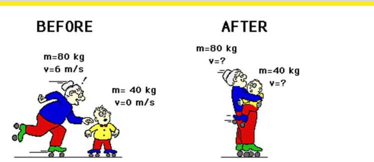
Perfectly Inelastic Collisions

Most collision are somewhat inelastic, with some kinetic energy converted into thermal energy

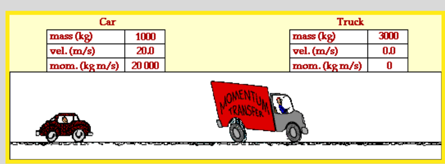
When two objects collide they may combine into one object in a perfectly inelastic collision

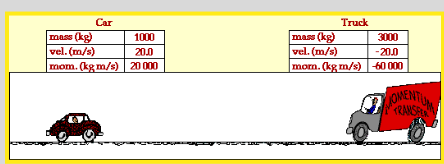


$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$



An 80 kg roller skating grandma collides inelastically with a 40 kg kid as shown. What is their velocity after the collision?

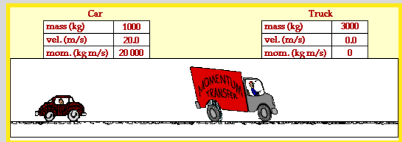


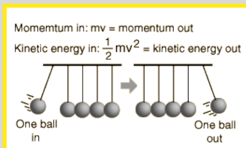


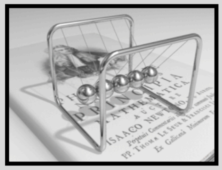
Elastic Collisions

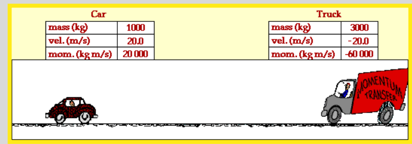
Ideal collisions are elastic, with zero kinetic energy converted into thermal energy (heat)

Springs and magnets can create elastic collisions. Other examples: air molecules, billiard balls, spacecraft gravitational boost.

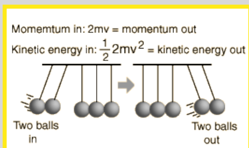


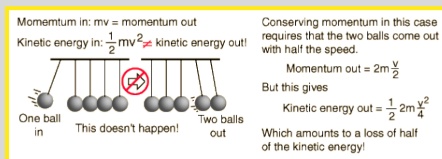






NEWTON'S CRADLE



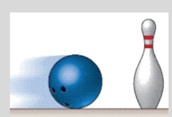


Examples of Inelastic and Elastic Collisions

Train cars are coupled together by being bumped into one another. Suppose two loaded train cars are moving toward one another, the first having a mass of 1.50×10^5 kg and a velocity of 0.30 m/s to the right, and the second having a mass of 1.10×10^5 kg and a velocity of 0.15 m/s to the left, as shown in the figure. What is their final velocity?

$$m_1v_{1i} + m_2v_{2i} = (m_1 + m_2)v_f$$

$$(1.5\times10^5)(0.3) + (1.1\times10^5)(-0.15) = (1.5\times10^5 + 1.1\times10^5)v_f \quad \Rightarrow \quad v_f = \boxed{0.11 \text{ m/s}}$$

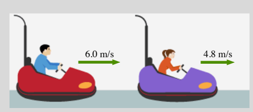


A 6.3-kg bowling ball moving at 9.0 m/s collides directly with a 1.5-kg bowling pin, which is sent forward with a speed of 13.0 m/s. Calculate the final velocity of the bowling ball. Is the collision elastic?

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f} \quad \Rightarrow (6.3)(9) + (1.5)(0) = (6.3)v_{1f} + (1.5)(13) \Rightarrow v_{1f} = \boxed{5.90 \text{ m/s}}$$

$$KE_i \neq KE_f \text{ ??} \Rightarrow \tfrac{1}{2}m_1v_{1i}^2 \neq \tfrac{1}{2}m_1v_{1f}^2 + \tfrac{1}{2}m_2v_{2f}^2 \Rightarrow \tfrac{1}{2}(6.3)(9^2) \neq \tfrac{1}{2}(6.3)(5.9^2) + \tfrac{1}{2}(1.5)(13^2) \Rightarrow 255 \text{ J} > 236 \text{ J}$$

HONORS: In an elastic collision, a 450-kg bumper car collides directly from behind with a second, 400-kg car that is traveling in the same direction. The initial speed of the leading bumper car is 4.8 m/s and that of the trailing car is 6.0 m/s. What are their final speeds? (Assume the masses include the drivers.)



$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$ and $\tfrac{1}{2}m_1v_{1i}^2 + \tfrac{1}{2}m_2v_{2i}^2 = \tfrac{1}{2}m_1v_{1f}^2 + \tfrac{1}{2}m_2v_{2f}^2$ combine to give:

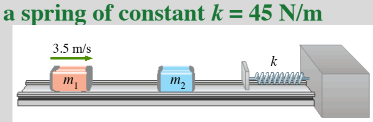
$v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$ which is a "relative velocity" equation. In other words, $v_{rel,approach} = -v_{rel,recede}$

$$450(6) + 400(4.8) = 450v_{1f} + 400v_{2f} \quad \textit{and} \quad 6 - 4.8 = -(v_{1f} - v_{2f}) \Rightarrow v_{2f} = v_{1f} + 1.2$$

$$4620 = 450v_{1f} + 400(1.2 + v_{1f}) \quad \Rightarrow \quad v_{1f} = (4620 - 400(1.2))/850 = \boxed{4.87 \tfrac{\text{m}}{\text{s}}}, \text{ and } v_{2f} = \boxed{6.07 \tfrac{\text{m}}{\text{s}}}$$

Examples of Momentum and Energy Problems

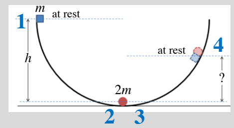
Two carts are placed on a frictionless track that has a spring of constant k = 45 N/m attached to one end, as shown. m_1 = 0.75 kg has an initial velocity 3.5 m/s to the right and m_2 = 0.50 kg is initially at rest. (a) If the carts collide and stick, find the velocity of the carts just after the collision. (b) Find the maximum compression in the spring.



$$m_1v_{1i}+m_2v_{2i}=(m_1+m_2)v_f \Rightarrow (0.75)(3.5)+(0.50)(0)=(0.75+0.50)v_f \Rightarrow v_f=\boxed{2.10\text{ m/s}}$$

$$KE_i=EPE_f \Rightarrow \tfrac{1}{2}mv^2=\tfrac{1}{2}kx^2 \Rightarrow \tfrac{1}{2}(1.25)(2.1^2)=\tfrac{1}{2}(45)(x^2) \Rightarrow x=\boxed{0.35\text{ m}}$$

A block of mass m starts from rest and slides down a frictionless track from a height h = 0.9 m as shown. When it reaches the lowest point of the track, it collides with a stationary piece of putty of mass $2m$. The block and the putty stick together and continue to slide. What is the maximum height that the block-putty can reach?



$$GPE_i=KE_f \Rightarrow mgh_1=\tfrac{1}{2}mv_2^2 \Rightarrow v_2=\sqrt{2gh_1}=\sqrt{2(9.8)(0.9)}=4.2\ \tfrac{\text{m}}{\text{s}}$$

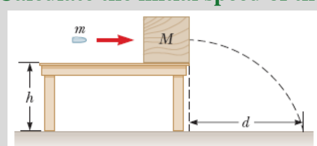
$$mv_2+0=(m+2m)v_3 \Rightarrow v_3=\frac{m}{m+2m}(4.2)=1.4\ \tfrac{\text{m}}{\text{s}} \Rightarrow \tfrac{1}{2}3mv_3^2=3mgh_4 \Rightarrow h_4=\frac{v_3^2}{2g}=\frac{1.4^2}{2(9.8)}=\boxed{0.1\text{ m}}$$

HONORS: A pellet of mass m is fired into a block of mass M initially at rest at the edge of a frictionless table of height h as shown. The pellet remains in the block, and after impact the block lands a distance d from the bottom of the table. Calculate the initial speed of the pellet in terms of m, M, h, d and g.

$$m_1v_{1i}+m_2v_{2i}=m_1v_{1f}+m_2v_{2f} \Rightarrow mv_{1i}+M(0)=(m+M)v_f$$

$$\Delta x=v_xt \Rightarrow d=v_ft \quad \text{and} \quad \Delta y=\tfrac{1}{2}gt^2 \Rightarrow h=0t+\tfrac{1}{2}gt^2 \Rightarrow t=\sqrt{2h/g}$$

$$v_f=d/t=d/\sqrt{2h/g} \quad \text{and} \quad v_{1i}=\frac{(m+M)}{m}v_f=\frac{(m+M)d}{m}\sqrt{\frac{g}{2h}}$$



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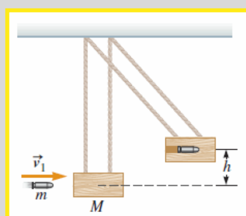
Ballistic Pendulum Example

A useful device for measuring the speed of a ballistic (in the 18th century) combines the use of conservation of energy and conservation of momentum.

A bullet is fired into a heavy block of wood, which swings upward with the bullet embedded inside. The height of the swinging pendulum is measured and the bullet's speed is then calculated.

Which conservation law is useful for the bullet/block collision? Which is best for the pendulum swing? Why?



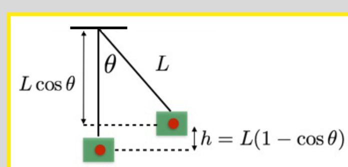
A bullet of mass 45.0 g is fired into a wooden block of mass 5.3 kg. The block–bullet pendulum rises to a maximum height of 13.0 cm above the block's initial height.

$$\tfrac{1}{2}\left(m_{\text{bullet}}+m_{\text{block}}\right)v^2=\left(m_{\text{bullet}}+m_{\text{block}}\right)gh$$

$$v=\sqrt{2gh}=\sqrt{2(9.8\text{ N/kg})(0.13\text{ m})}=1.60\text{ m/s}$$

$$m_{\text{bullet}}v_{1i}+m_{\text{block}}v_{2i}=(m_{\text{bullet}}+m_{\text{block}})v$$

$$(0.045)(v_{1i})=(0.045+5.3)(1.60) \Rightarrow v_{1i}=190\text{ m/s}$$



HONORS: what angle does the pendulum swing to if L = 40 cm?

$$\theta=\cos^{-1}(1-\tfrac{h}{L})=\cos^{-1}(1-\tfrac{13}{40})=47.5°$$

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