Student Explorations in Mathematics

Formerly *Student Math Notes*

January 2010

If you live in a locale that experiences snow, you might recognize the connection among needles, dendrites, and plates. Just like fingerprints, every snowflake has a different signature, depending on the altitude where it forms. With a little mathematics, we just might find the connection. Let it snow, let it snow, let it snow….

Create a Snowflake

Let's start with some basic geometry to draw your own snowflake. Watch for the as you work through this activity. This symbol indicates the steps you are to follow in order to complete the construction.

❄	Use a compass to draw a circle. Draw one radius from the center of the circle to one point on the circle.

1. How many degrees are in the circle?

2. How might we divide the circle into six pieces (arcs) that are all the same size?

3. How many degrees are in each arc?

❄	Using your answer to question 3, draw five additional points on your circle that will divide the circle into six congruent (equal-measure) arcs. Connect each pair of adjacent points with a line segment.

These six points are the **vertices** of a polygon, and the line segments are the **sides** of the polygon.

4. What is the name of the polygon that you created?

When each side is the same length and each of the figure's angles is congruent to each of the others, we have a regular polygon. Let's explore some properties of regular polygons.

❄	Cut out your hexagon. Trace your hexagon onto a second piece of paper.

Label one vertex of the original hexagon with A and the traced hexagon with A´. Align the labeled vertices, and think of this as the starting point. Rotate our original hexagon on top of the traced hexagon in a clockwise manner.

5. What do you notice about the vertices and the sides of the two hexagons as the original hexagon is rotated?

Sometimes the vertices and the sides of the original hexagon line up with the vertices and the sides of the traced hexagon (**rotational symmetry**). A shape has rotational, or turning, symmetry if an outline of the turning figure matches its original shape (Eather 2009). To view examples of rotational symmetry, try using http://www.analyzemath.com/Geometry/rotation_symmetry.html. Each time the hexagons fit on top of one another, count this as one point of rotational symmetry. Rotate the original hexagon on top of the traced hexagon until A is again on top of A´.

6. How many points of rotational symmetry do you have in your hexagon?

❄	Cut out the traced hexagon.

But a snowflake is not just a two-dimensional figure. We will create a three-dimensional shape to represent a snowflake. For this, we must add a few more geometric shapes to connect our two hexagons. We will build what will end up being a **hexagonal prism**, a solid, three-dimensional shape with two identical, parallel bases (hexagons). All other faces are rectangles (Eather 2009). The two cutout hexagons will become the bases for a hexagonal prism (**see fig. 1**).

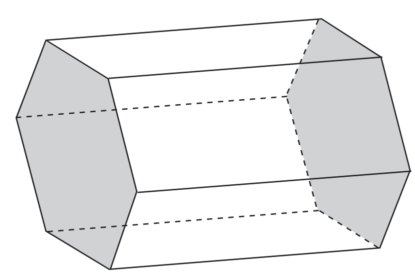

Figure 1

7. What other polygons do we need in order to construct the hexagonal prism?

8. How many of these polygons will we need to form the faces of our hexagonal prism? (Exclude your base hexagons as faces.)

Measure one hexagon's side length. Use this measurement as the length of the rectangles that will be used to finish the prism. Form the rest of the rectangle by choosing a width of your choice (less than 11 inches or 28 centimeters). Record the measures below.

Length = ____________ Width = ____________

Draw and cut out six congruent rectangles of the measurements you determined.

Use the two hexagons and six rectangles to create a hexagonal prism similar to **figure 1**.

Two surfaces of a prism join (**intersect**) at an edge. The points where the surfaces of a prism meet (corner) are **vertices**. The flat surfaces of prisms are **faces** (Eather 2009)

9. How many edges does the hexagonal prism have? How many vertices? How many faces? On a snowflake, these may look a bit different because a snowflake is very small and the three-dimensional aspect may not be obvious to the human eye.

The hexagonal prisms created by your classmates and you all have their own unique surface area and volume. Observe the similarities and differences in these surface areas and volumes.

10. Using mathematical vocabulary, describe how the surface area of your hexagonal prism compares with the surface area of one of your classmate's hexagonal prisms.

11. Next use mathematical vocabulary to compare the volume of your hexagonal prism with the volume of one of your classmate's hexagonal prisms.

A Blizzard of Numbers

According to snow scientist Charles Knight at the National Center for Atmospheric Research in Boulder, Colorado, there are about 10,000,000,000,000,000,000 water molecules in a typical snow crystal.

12. Write this number in scientific notation.

David Phillips, the senior climatologist with Environment Canada, has estimated that the number of snowflakes that have fallen on Earth over the course of time is 10 followed by 34 zeros.

13. Write this number both in standard form and in scientific notation.

14. Use scientific notation (and the properties of exponents) to find the total number of water molecules that have fallen in snow crystals over the course of time.

Temperature Formulas

15. The table below shows the temperatures at which different types of snowflake crystals form. Scientists often report temperatures in both Celsius and Fahrenheit. To change the Celsius temperatures to Fahrenheit and complete the table, use this formula:

$$C = (F - 32)\,\frac{5}{9}$$

Snowflake Temperatures

Fahrenheit (in degrees)	Celsius (in degrees)	Crystal Form
3–10		Dendrites (lacy hexagonal shapes)
10–14		Sector plates (hexagons with indentations)
14–21		Hollow columns
21–25		Needles
25–32		Thin hexagonal plates

16. Create two number lines to identify the type of crystal formed at the temperature ranges—one for temperatures in Fahrenheit degrees and one for Celsius degrees.

View "The Curious Growth of a Snow Crystal" applet from the Why Files at http://whyfiles.org/interactives/index.php?g=4.txt. Be sure to read all the information with the applet. Use the applet to "grow" a snowflake.

17. As you can see, actual snowflakes are more complex than the prism we constructed. Describe your snowflake using such mathematical vocabulary as *vertices*, *faces*, and *edges*. You may wish to also include the temperature at which your snowflake was created.

Which Clouds Produce Snow?

The troposphere, which is the lower level of the atmosphere, is divided into three layers. You would find upper-level clouds (cirrocumulus, cirrostratus, and cirrus) between 5 and 13 kilometers, mid-level clouds (altostratus and altocumulus) from 2 km to 7 km, and the lowest-level clouds up to 2 km (stratus, stratocumulus, nimbostratus). One place that you might find some more information is www.windows.ucar.edu/tour/link=/earth/Atmosphere/clouds/cloud_types.html.

18. Create a scale drawing of the troposphere. Include the various levels of clouds.
19. How did you determine the scale used in your drawing?

Using NASA Data

The National Aeronautics and Space Administration (NASA) has a number of satellites currently collecting data about many aspects of the earth, including cloud coverage, precipitation, and temperature, all of which certainly play a part in snowflake formation. One place students and teachers can use some of these data is MY NASA DATA at http://mynasadata.larc.nasa.gov/data.html. Follow these directions:

- Choose +*Live Access Server (Basic Edition)*.
- Click on *Clouds*, check *Monthly Low Cloud Coverage*, and click on *Next*.
- Select
  - view: *Map*;
  - output: *Graph output*;
  - region: *World*. As default, *88.75 N Latitude, 180 W Longitude, 180 E Longitude, and 88.75 S Latitude* should appear. If not, type them in.
  - time: *January 15, 2005.*
- Click on *Next*. Now you are ready to answer the following questions.

20. What percent of low monthly cloud coverage was found in most of Australia?
21. What might the white areas represent?
22. Which region(s) in the contiguous United States has/have over 45 percent low cloud coverage?
23. What was the low monthly cloud coverage for your region?

Go back to the previous page on the NASA site.

- Select
  - view: *Graph vs time*;
  - output: *Graph output*;
  - region: *World*;
  - Find the latitude and longitude of your school's location by using http://www.infoplease.com/atlas/latitude-longitude.html and typing in the name of your school. If your school does not appear, you may have to use your town, city, state (province), or nearest landmark. Type your latitude into the top box (the bottom box should automatically change when clicked outside the box) and your longitude into the left box.
  - Select the date range of *January 15, 2000 to January 15, 2005*.
- Click on *Next*.

24. Describe the patterns observed in the graph of Cloud Coverage versus Time. Include which months have the most and least cloud coverage percentage each year.
25. Use 0° N Latitude and 0° W Longitude with the same date range. Compare and contrast the new graph with the graph for your location.

View a short video of how NASA scientists analyze the process of snow and its effects at the following link: http://www.nasa.gov/audience/foreducators/nasaeclips/realworld/earth.html.

Can you …

- examine the snowflakes with hand lenses to determine what their structure is? On a snowy day, take a cold piece of black paper. Collect snowflakes on this paper. You can Use NOAA or National Weather Service sites to find out what the cloud temperature was as well as the amount of water vapor.

Did you know that …

- a huge cumulonimbus cloud can form in just 30 minutes and can tower as high as 15 kilometers, lifting millions of tons of water from the ground into the sky?
- the United States Postal Service featured 39-cent stamps as Holiday Snowflakes? These showed multi-branched stellar dendrites with randomly placed side branches and sectored plates—just one of seven main types of snowflake patterns.
- the most snow ever to fall in one winter was at Mount Baker in Washington State? In the winter of 1998–1999, 1140 inches fell, almost the height of the Statue of Liberty from head to toe.
- in 1885, W. A. (Snowflake) Bentley was the first person to photograph a single snow crystal? He once said, "I have found no exact duplicate. In this storehouse of crystal treasures, what a delight is in store for all future lovers of snowflakes and of the beautiful in nature." In Bentley's lifetime, he captured photographs of more than 5380 snowflakes and did not find any two alike.

- One cloud physicist, Jon Nelson at Ritsumeikan University in Kyoto, Japan, believed that larger snowflakes do not have duplicates, but that smaller crystals (as small as one-tenth of a millimeter across) that fall before they have a chance to develop may have duplicates.

Mathematical Content

Two-dimensional and three-dimensional geometry; symmetry; algebra; data analysis; number: scientific notation and conversions

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Thanks to Mary Stander, Princeton, Illinois, for the topic and many of the ideas in this *Student Explorations in Mathematics*.

Answers

Create a Snowflake

1. 360 degrees
2. One possible method is to divide 360 by 6 and use a protractor.
3. 60 degrees
4. You have created a hexagon. (Note: Be sure that all students know this is a hexagon before moving on.)
5. With each 60 degrees of rotation, the sides and vertices match the original.
6. There will be six points of rotational symmetry.
7. Rectangles
8. You need six rectangular faces.
9. The hexagonal prism has eighteen edges, twelve vertices, and eight faces.
10. Answers may vary but should use mathematical vocabulary, including face, edge, area, volume, and so on.
11. Answers may vary but should use mathematical vocabulary, including face, edge, area, volume, and so on.

A Blizzard of Numbers

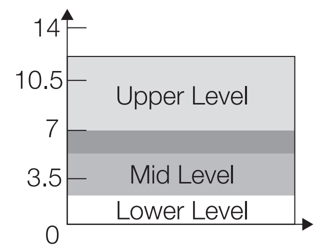
12. 1.0×10^{19}
13. 100,000,000,000,000,000,000,000,000,000,000,000; 1.0×10^{35}

Temperature Formulas

14. 1.0×10^{54}
15. Missing Celsius ranges are the following:
    –16.1 ⇒ –12.2
    –12.2 ⇒ –10.0
    –10.0 ⇒ –6.1
    –6.1 ⇒ –3.9
    –3.9 ⇒ 0.0
16. See number lines below.
17. Answers may vary.

Which Clouds Produce Snow?

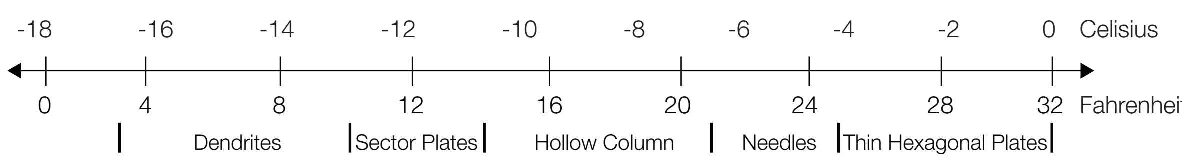
18.



Using NASA Data

19. Answers will vary.
20. About 20 percent (This may vary slightly.)
21. No data (Others answers are possible.)
22. None
23. These answers will vary depending on the location.
24. These answers will vary depending on the location and interpretation.
25. These answers will vary.

Number lines for #16.



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