

APOGEE
COMPONENTS

PEAK OF FLIGHT

N E W S L E T T E R

In This Issue

***Compute How High and Fast
Your Rocket Goes Using The
Basic Equations of Rocket
Motion***

**Cover Photo: U.S. Rockets Micro Sentra SRB Kit. Get one at:
www.ApogeeRockets.com/Rocket_Kits/Skill_Level_4_Kits/Micro-Sentra_SRB**

Apogee Components, Inc. — Your Source For Rocket Supplies That Will Take You To The “Peak-of-Flight”
3355 Fillmore Ridge Heights
Colorado Springs, Colorado 80907-9024 USA
www.ApogeeRockets.com e-mail: orders@apogeerockets.com
Phone: 719-535-9335 Fax: 719-534-9050

ISSUE 320 AUGUST 28, 2012

PEAK OF FLIGHT

How High and Fast? The Equations of Motion

By Steve Hardin

This summer I decided that I would spend some time with my 13-year-old daughters building model rockets. The process of doing this would teach them how to measure and build as well as “engineer” something that actually did something cool such as fly. Building rockets is a good way to study a wide range of physics, engineering, math and science. I built rockets when I was a kid and it was such great fun, I thought I would do it again.

I brushed up on my rocket skills and read G. Harry Stine’s *Handbook of Model Rocketry*. In the book he described rocket equations to estimate the height and velocity that a rocket will go theoretically, ignoring aerodynamic drag. As I started looking at the equations, I was slightly confused as to how they were derived.

I looked around on the Internet and did not see a clear representation of the math in one place that my kids would understand. My daughters have had basic algebra but most of the derivations on the net leave out a lot of the steps. I’m writing about this because along with my daughters, I thought it would help others who are teaching younger students on the basics of the equations of motion.

I’m going to glue together the basics of finding out how fast a model rocket will go and how high it will go theoretically. We will calculate the velocity of the rocket after its initial burn and then after the coasting of the rocket in the air. Remember this only works ignoring aerodynamic drag which is a very big piece of how a rocket flies. Later I may address the more complex math but for now we’ll start with the easy stuff.

{Editor’s note: If you would like to go deeper, see the book *Topics In Advanced Model Rocketry* at: www.ApogeeRockets.com/Rocket_Books_Videos/Books/Topics_In_Advanced_Model_Rocketry}

By definition these equations work with constant acceleration. We use Newton’s laws of motion. All of Newton’s laws work on a rocket and affect how it flies. Remember this is ideal. For much better real world models use RockSim (www.ApogeeRockets.com/RockSim/RockSim_Information).

Given is the graph below with a line in a coordinate

plane where Y is equal to velocity and X is equal to time. In this basic graph of velocity versus time I have defined acceleration as the change in velocity over the change in time. If we add up the area under the line this would be the area of a rectangle plus the area of the triangle. This area is called the displacement. For our equations this will be the altitude of a rocket.

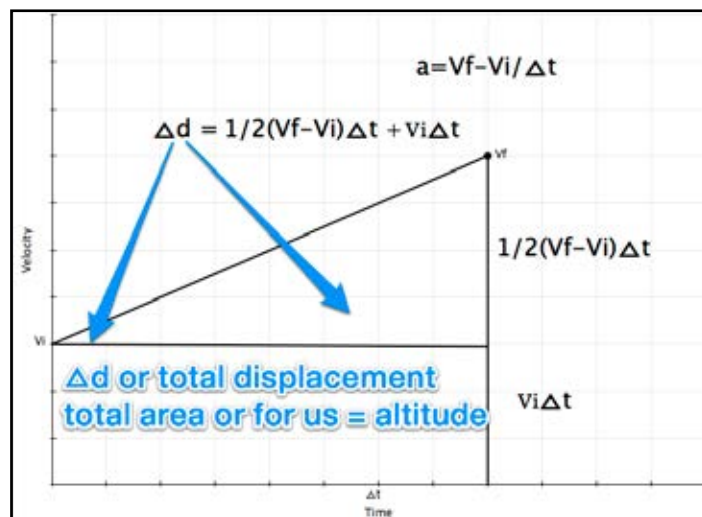


Figure 1: Delta d means total displacement.

Again looking at the graph of the line of acceleration as:

$$a = \frac{\Delta v}{\Delta t}$$

Remember that acceleration is the slope of the graph. Rearranging gives us:

$$at = v_f - v_i$$

$$at + v_i = v_f$$

$$v_f = at + v_i$$

This final equation looks a lot like the basic equation for a straight line that everyone is used to.

$$y = mx + b$$

Continued on page 3

About this Newsletter

Newsletter Staff

You can subscribe to receive this e-zine FREE at the Apogee Components web site (www.ApogeeRockets.com), or by sending an e-mail to: ezine@apogeeRockets.com with “SUBSCRIBE” as the subject line of the message.

Writer: Tim Van Milligan
Layout / Cover Artist: Tim Van Milligan
Proofreader: Michelle Mason

PEAK OF FLIGHT

Continued from page 2

The Basic Equations of Rocket Motion

We want to find the area under the line which is our displacement. In order to do this accurately we will use calculus and integrate velocity with respect to time.

$$v = v_i + at \text{ (eq1)}$$

Plugging in equation 1 to the integration and solving gives us:

$$s = \int_0^t v dt = \int_0^t v_i + at dt$$

$$s = v_i t + \frac{1}{2} at^2$$

By integrating we add up all the area under the line, that's all integration really is. "s" is the disposition. Physicist use s for some reason (note s = d in our graph). This equation, we will call equation 2, has acceleration in it. In order for us to use it for our rocket equation we need to get acceleration out by substituting our definition of acceleration. We're going to do some substitution and simplify.

$$s = v_i t + \frac{1}{2} at^2 \text{ (eq2)}$$

$$a = \frac{v - v_i}{t}$$

$$s = v_i t + \frac{1}{2} \left(\frac{v - v_i}{a} \right) t^2$$

$$s = v_i t + \frac{vt}{2} - \frac{v_i t}{2}$$

$$s = \frac{v_i t}{2} + \frac{vt}{2}$$

$$s = \frac{1}{2} (v + v_i) t \text{ (eq3)}$$

The next equation will take equation 2 and eliminate time. The algebra on this is a little messy but I have all the steps and I'm using the binomial theorem (http://en.wikipedia.org/wiki/Binomial_theorem). Substitute t in equation 2 and simplify.

$$t = \frac{v - v_i}{a}$$

$$s = v_i t + \frac{1}{2} at^2$$

$$s = v_i \left(\frac{v - v_i}{a} \right) + \frac{1}{2} a \left(\frac{v - v_i}{a} \right)^2$$

$$t^2 = \frac{v^2 - 2vv_i + v_i^2}{a^2}$$

Continued on page 4

AltimeterOne

"The one altimeter you'll use in every rocket you fly."

AltimeterOne - See how high your rocket flew

- Records peak altitude up to 29,000 feet (ASL). Displays in meters too!
- Easy-to-read LCD display. No need to count beeps or flashes of light.

AltimeterTwo - See how fast and high your rocket went

- Records peak speed and acceleration using 3-axis accelerometer.
- Also tells you how high the rocket flew.

www.ApogeeRockets.com

Penny shown for size comparison

www.ApogeeRockets.com

Your Source For Everything Rocketry

PEAK OF FLIGHT

Continued from page 3

The Basic Equations of Rocket Motion

$$s = v_i \left(\frac{v - v_i}{a} \right) + \frac{1}{2} a \frac{v^2 - 2v v_i + v_i^2}{a^2}$$

$$s = \frac{v_i v}{a} - \frac{v_i^2}{a} + \frac{v^2}{2a} - \frac{2v_i v}{2a} + \frac{v_i^2}{2a}$$

$$s = \frac{v_i v}{a} - \frac{v_i^2}{a} + \frac{v^2}{2a} - \frac{v_i v}{a} + \frac{v_i^2}{2a}$$

$$s = \frac{v^2 - v_i^2}{2a} \text{ (eq4)}$$

After all that work equation 3 is our altitude at burnout and equation 4 is our coasting altitude.

$$s_{\text{altitude@burnout}} = \frac{1}{2} (v + v_i) t \text{ (eq3)}$$

$$s_{\text{coasting altitude}} = \frac{v^2 - v_i^2}{2a} \text{ (eq4)}$$

In order to get the altitude at burn out we will need the velocity at the time our motor runs out of burn. In order to get this we will use Newton's Second Law $F=ma$:

T = thrust or force to put the rocket in the air

t = motor burn time

g = acceleration due to gravity. 9.8 m/s^2

w_{avg} = average weight of rocket

v_m = maximum velocity during motor burn

$$F = ma$$

$$T - w_{\text{avg}} = \frac{w_{\text{avg}}}{g} \left(\frac{v_m}{t} \right)$$

In order to use this law we need to include gravity with mass to figure the force on our rocket and that is the gravity for acceleration. Given our equation above for velocity, we will substitute.

$$v = v_i + at$$

$$v_i = 0 \text{ at liftoff}$$

$$v = at$$

$$\frac{v_m}{t} = a$$

$$T - w_{\text{avg}} = \frac{w_{\text{avg}}}{g} \left(\frac{v_m}{t} \right)$$

$$v_m = \left(\frac{T - w_{\text{avg}}}{w_{\text{avg}}} \right) gt$$

Finally we can calculate some ideal values for our rocket. Given the picture of our rocket in Figure 2, lets figure out our altitude at burnout and our maximum altitude theoretically.

Using my kids' Alpha rocket with an A8-3 motor installed, it weighs 38.8 grams at lift off. The weight of the propellant is 3.12 grams giving an average weight of 37.2 grams during the thrust of the flight. The motor thrusts for .32 seconds and has an impulse of 2.50 Newton seconds. In our equations you will want to convert grams to kilograms as the standard unit of measurement.

Continued on page 5





North Coast Rocketry

Mid & High Power Rocket Kits!

- Big Kits with Classic Styling and Bold Graphics
- All Rockets Feature Laser-Cut Plywood Fins and Rings
- Easy-to-Build. Durable. Exciting, and a Real Joy to Fly!

Sold Exclusively at ApogeeRockets.com

www.ApogeeRockets.com

Everything Rocketry

PEAK OF FLIGHT

Continued from page 4

The Basic Equations of Rocket Motion

$$v_m = \left(\frac{T}{w_{avg}} - 1 \right) gt$$

$$T = \frac{\text{total impulse in Newton - seconds}}{\text{burn time in seconds}}$$

$$T = \frac{2.50 \text{ Nsec}}{.32 \text{ sec}} = 2.50 \text{ N}$$

$$37.2g = .0372kg$$

$$w_{avg} = .0372kg * 9.8 \frac{m}{sec^2} = .365 \frac{kgm}{sec^2} = N$$

$$v_m = \left(\frac{7.81 \text{ N}}{.365 \text{ N}} - 1 \right) 9.8 \frac{m}{sec^2} (.32 \text{ sec}) = 64.03 \frac{m}{sec}$$

$$v_m = 64.03 \frac{m}{sec}$$

The maximum velocity of our rocket ideally will be 64.03 m/s or 143 mph. Now let's figure our altitude after the rocket motor burns.

Using equation 3:

$$s = \frac{1}{2} (v + v_i) t \text{ (eq3)}$$

$$v_m = 64.03 \frac{m}{sec}$$

$$v_i = 0 \text{ initial velocity}$$

$$t = \text{burn time} = .32 \text{ sec}$$

$$s = \frac{1}{2} \left(64.03 \frac{m}{sec} + 0 \right) .32 \text{ sec} = 10.24 \text{ meters}$$

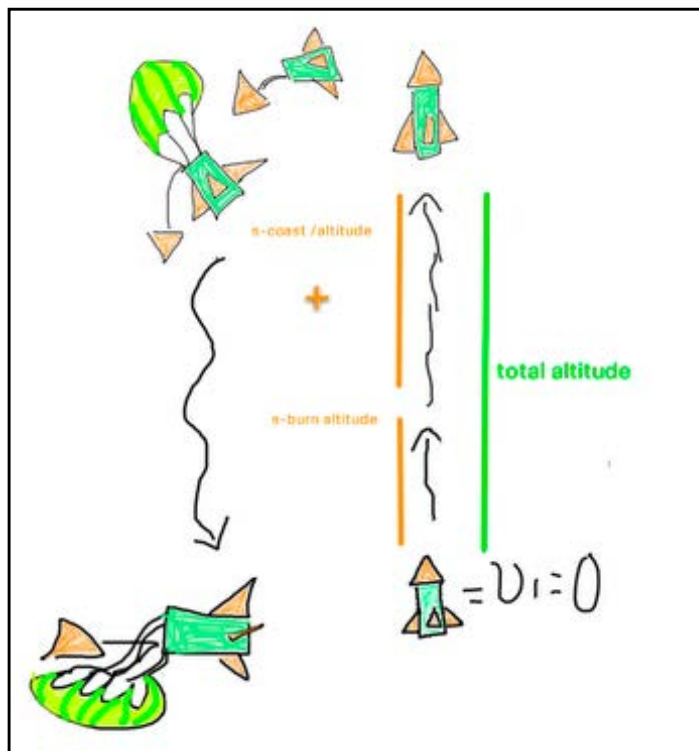
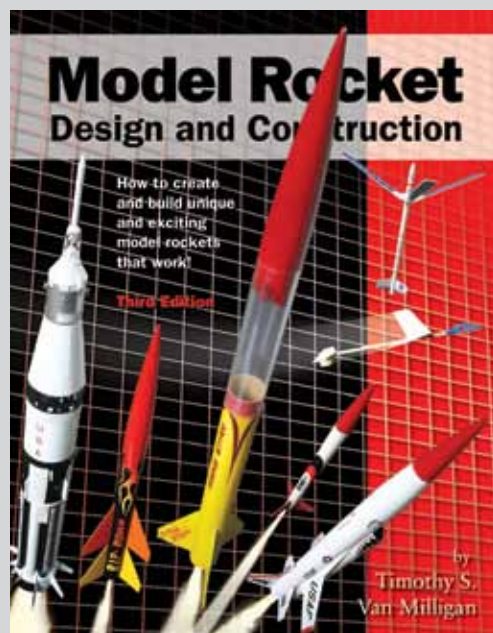


Figure 2: Burnout altitude plus coast distance will give us the maximum altitude of the rocket.

Our rocket altitude at motor burnout is 10.24 meters or 32 feet above the ground theoretically.

Using equation 4 we can find out the total theoretical altitude that our rocket will go.

Continued on page 6



Model Rocket Design and Construction

By Timothy S. Van Milligan

The Expanded 3rd Edition

This massive, 328 page guidebook for serious rocket designers contains the most up-to-date information on creating unique and exciting models that really work. With 566 illustrations and 175 photos, it is the ultimate resource if you want to make rockets that will push the edge of the performance envelope. Because of the number of pictures, it is also a great gift to give to beginners to start them on their rocketry future.

For more information, and to order this hefty book, visit the Apogee web site at: www.ApogeeRockets.com

Apogee Components
3355 Fillmore Ridge Heights
Colorado Springs, Colorado 80907

telephone: 719-535-9335
website: www.ApogeeRockets.com

APOGEE
COMPONENTS

PEAK OF FLIGHT

Continued from page 5

The Basic Equations of Rocket Motion

$$s_{\text{coasting altitude}} = \frac{v^2 - v_i^2}{2a} \text{ (eq4)}$$

$$\text{total altitude} = \text{coasting} + \text{burnout}$$

$$v = 0 \text{ at the peak altitude}$$

$$s_{\text{coasting altitude}} = \frac{0 - (64.03 \frac{m}{sec})^2}{2(-9.8 \frac{m}{sec^2})} = 208.9m$$

$$\text{total altitude} = 208.9m + 10.24m = 219.22m$$

Our rocket theoretically will go 219.22 m or 719 feet in the air ignoring aerodynamic drag with constant acceleration.

A few notes on this last equation to solve for the coasting altitude. Many of the units I've used here are called vectors. They have a magnitude and direction. Gravity has a downward direction. This is why the 9.8 is in the negative direction. It works out because negative divided by a negative is a positive and altitude needs to be positive. I didn't mention vectors throughout because it may have been more confusing. Be aware however that a rocket doesn't go just straight up, it has an X, Y and Z coordinate. We know when we shoot off a rocket it tends to point into the wind which would decrease its overall altitude.

About The Author

This article was originally published on Steve's web blog.

Steve Hardin has worked in business and technology his entire career, and is currently a business executive in charge of an e-commerce company. On occasion, however, he drifts back to his days as a classically trained chemist from Indiana University and has to dig into some science and math.

Steve is working his way up the rocketry high-power certification ladder with his 13-year-old twin daughters. In his idle time he also enjoys reading, writing sports and science. You can follow his blog at <http://stevehardin.blogspot.com/> and follow him on twitter #stevehardin

Steve resides and flourishes in Southern Indiana with his wonderful daughters and his remarkable wife Kim.



We're Paying Cash For Great Articles for This Newsletter

Are you a writer looking for some serious pocket change? We're paying up to \$350 for good how-to articles for this newsletter. If you're interested, see our submission guidelines on the Apogee web site.

www.ApogeeRockets.com/Newsletter/Newsletter_Guidelines

