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The Mathematics of Spiral Wound Body Tubes

By Tim Van Milligan

Mark Bolt writes: “*My students and I are making ‘Egg Rockets’ and making our body tubes from scratch. When I’ve made 1” (OD) rockets in the past, we would cut the gummed paper at a 45-degree angle to align the wrap at the end of the body tube to then start our “candy cane” wrap. Now we are making larger rockets and using 3” gummed tape with 1-1/2”, 2”, and 2-1/2” body tube forms. A 45 degree cut does not work for the candy cane wrap (too sharp an angle). After some trial and error I have found that a 70 degree angle works well on a 2-1/2” body tube form... BUT...is there a formula to figure this angle out for different diameter body tube forms? Also, is there a way to figure out how long to cut the 3” gum tape when making a body tube 12-24” long? Thanks again for all your support and great products we have purchased in the past.*”

This is an excellent question, and I want to answer it because it also makes a great topic for other teachers that want to use rocketry to teach different mathematics concepts. In other words, if you were a teacher, you could use this as a demonstration to teach about geometric shapes, such as a cylinder.

First of all, we do have a technical publication that covers the “how-to” part of making your own spiral wound body tubes, so I will only cover the math concepts here. The how-to-construct-a-tube is in Technical Publication #13 (www.ApogeeRockets.com/technical_publications.asp), which is free to customers that have completed three purchases in our Rocketry Rewards program (www.ApogeeRockets.com/Frequent_flyer.asp).

The math is the harder part, since it isn’t covered in the Technical Publication, so I’ll discuss that part here.

Typically, spiral wound tubes are constructed by taking a roll of paper ribbon (or tape), and wrapping it around a cylindrical form called a “mandrel.” The ends are not squared off like a normal body tube, since the way the paper is wound. They are trimmed to be square later in the process.

There are a lot of advantages to making a tube this way, as it is quicker to set up. But the disadvantage is that

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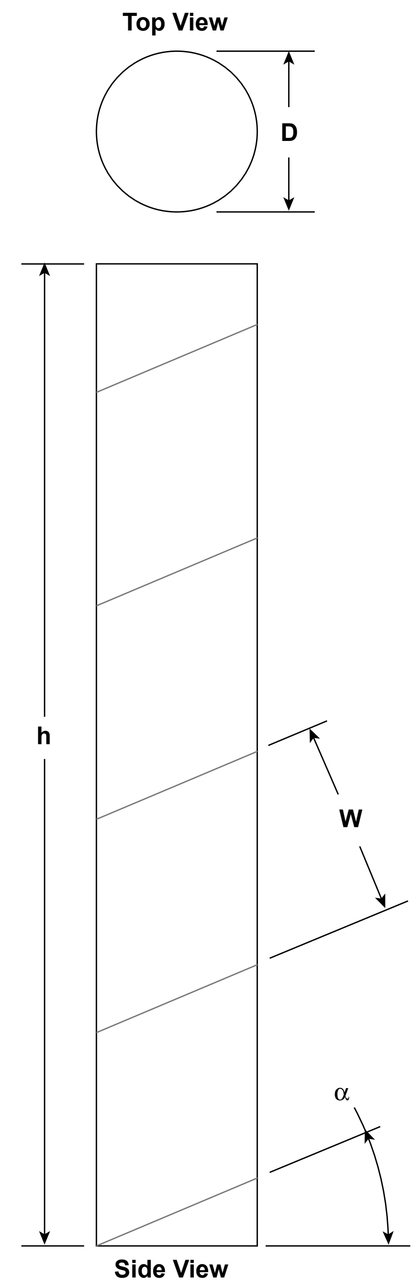

Figure 1 - Basic dimensions of the spiral wound tube.

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you need a mandrel that is much longer than the length of the final tube.

In this article, I'm going to show you how to calculate the wrapping angle, and how long of a ribbon of paper you will need to complete one layer. Obviously, a tube needs multiple layers in order to be strong where the seams are on the tube. So you will need multiple strips of paper, and the length of each strip is calculated the exact same way.

The process starts by creating a drawing of the tube, and defining the dimensions that we will be needing. Figure 1 shows the top and side views of a spiral-wound paper tube.

D = Diameter of the mandrel that you're wrapping around.

h = Height of the finished tube.

W = Width of the ribbon of paper that is being wrapped around the tube.

α = Angle that the paper is wrapped around the tube.

The main thing to comprehend is that if you unwound the strip of paper shown in Figure 1, it doesn't make a long rectangle. It makes a parallelagram. If you have an old toilet paper roll, you can unwind it, and you'll see that it looks like Figure 3 on page 4.

In Figure 2, you can see a picture of the start of the parallelagram.

Figure 2 - Spiral-wound strips of paper can be used to create your own body tubes.

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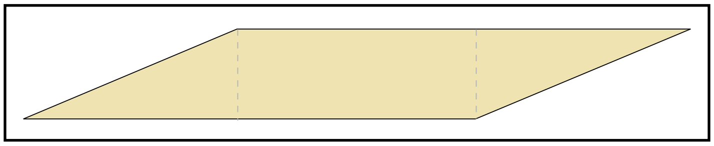

Figure 3: If you unwind the tube, you find the shape is a long parallelagram.

Start by looking at the ends of the parallelagram. They can be thought of as two identical right triangles. Let's use that concept to figure out the dimensions of this triangle.

In Figure 4, I've taken the triangle off the tube, and flipped it over and then rotated it around so that it looks just like the end of the parallelagram.

The hypotenuse of the triangle is exactly equal to the circumference of the tube. It has to be this way if you want to wrap the tube with no overlap of paper.

From this point, we can start to figure out the angle α, which is the critical angle you'd wrap the paper around the tube. Notice that α changes based on the width of the paper strips, and the diameter of the tube, because the circumference is dependant on the diameter of the tube.

$$\text{Sine } \alpha = \frac{W}{\text{Circumference}} \qquad \text{Eq. 1}$$

$$\text{Sine } \alpha = \frac{W}{\pi D} \qquad \text{Eq. 2}$$

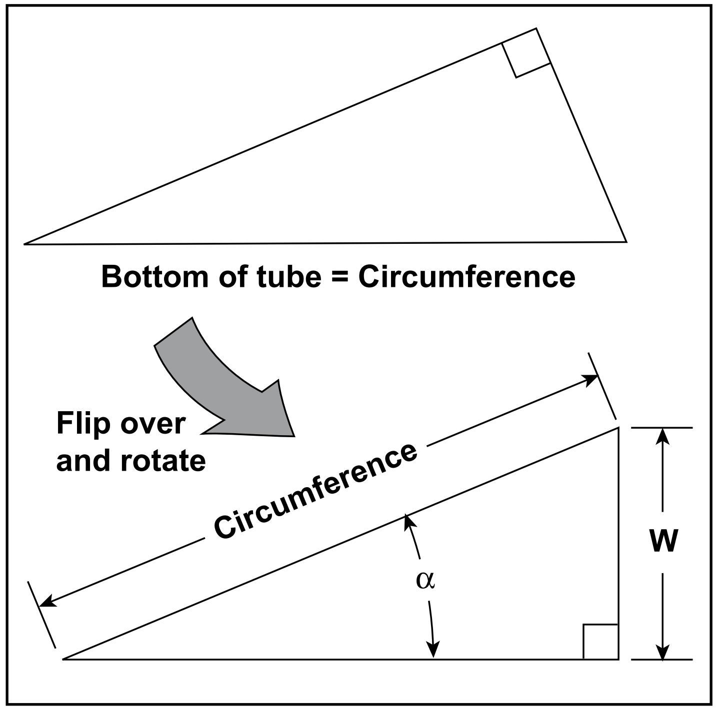

Figure 4: If you cut off the ends of the parallelagram, you would create a triangle.

Solving for the angle α yields:

$$\alpha = \arcsin \frac{W}{\pi D} \qquad \text{Eq. 3}$$

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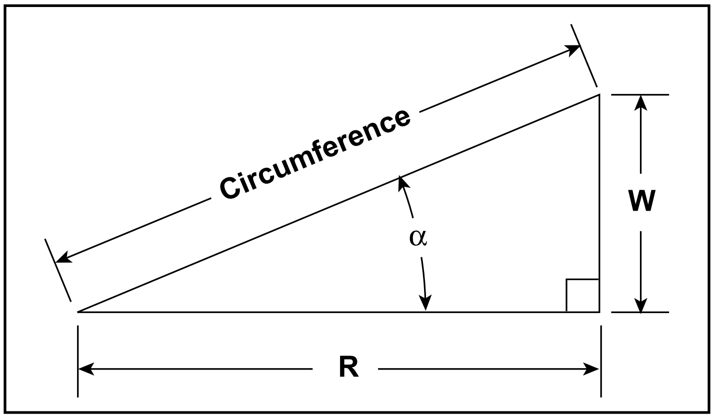

Figure 5: You also want to determine the base length of the triangle, which we'll call "R".

Next, we can calculate R, which is just the base leg of the right triangle. We'll need this distance later when we calculate the overall length of the paper strip.

Distance R can be calculated in one of two ways. It doesn't matter which one you pick. The first way is:

$$\mathbf{R} = \sqrt{(\pi\ \mathbf{D})^2 - \mathbf{W}^2} \qquad \text{Eq. 4}$$

The second way is to use some trigonometry:

$$\textbf{tangent}\ \alpha = \frac{\mathbf{W}}{\mathbf{R}} \qquad \text{Eq. 5}$$

$$\mathbf{R} = \frac{\mathbf{W}}{\textbf{tangent}\ \alpha} \qquad \text{Eq. 6}$$

Now, we'll lay out the parallelagram and label all the dimensions we need to find. This is shown in Figure 6.

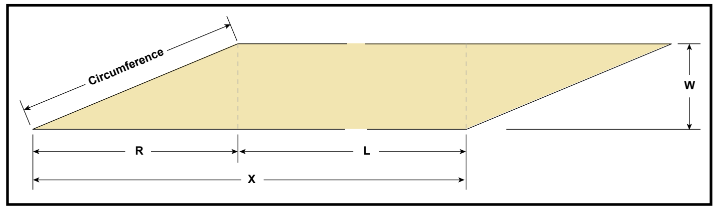

Figure 6: The basic dimensions of the parallelagram.

The area of this parallelagram, which I call the area of the tape, is given by the formula:

Tape Area = Parallelagram = W X Eq. 7

The unique thing is that this is exactly the same as the surface area of the cylinder that we're making.

Tape Area = Cylinder Area Eq. 8

The area of the cylinder is given by the formula: π **D h**. Setting the two area equal yields:

$$\mathbf{W\,X} = \pi\ \mathbf{D\ h} \qquad \text{Eq. 9}$$

Solving for the length X gives us:

$$\mathbf{X} = \frac{\pi\ \mathbf{D\ h}}{\mathrm{W}} \qquad \text{Eq. 10}$$

Since you can see from Figure 5 that distance X is simply L+R, we can substitute that into equation 10, and then solve for distance L:

$$\mathbf{L} = \frac{\pi\ \mathbf{D\ h}}{\mathrm{W}} - \mathbf{R} \qquad \text{Eq. 11}$$

From here it is pretty simple to find the overall length of the tape that we'll need to make the tube. You take the length L and add two times the length of R:

Total Length = L + 2R Eq. 12

This is shown in Figure 7 on page 6.

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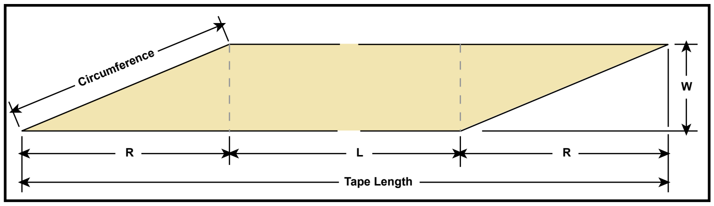

Figure 7: The overall tape length is L plus 2R.

You don’t necessarily have to find the α angle (in equation 3), but it doesn’t hurt to have it handy. However, you will need it if you are not cutting the triangles onto the end of your tape ribbon. One instance might be if you’re fiberglassing the outside of your airframe.

Other than that, the key formulas you want to use are equations 4, 11, and 12.

Multiple Layers of Paper

Don’t forget that with each layer of paper you put on the tube, you will have to recalculate those three equations. The reason is that the thickness of the paper is increasing the diameter of the cylinder. You might not think it matters much, but it makes a big difference. I don’t know what each layer of paper adds, and it is something you’ll have to measure. Typically a sheet of copy-paper is 0.006 inches thick. But you have to remember to account for the glue on the paper too. It is NOT an insignificant amount. It does add up when you get multiple layers.

The nice thing about this method is that you get to find the minimum length of ribbon to cut. You can always cut it longer if you wish.

Conclusion

This is a nice little geometry problem that you can have your students go through, as it shows how math is used in rocketry. You might give them a couple different diameter tubes, and a few widths of paper strips to play with.

About The Author:

Tim Van Milligan (a.k.a. “Mr. Rocket”) is a real rocket scientist who likes helping out other rocketeers. Before he started writing articles and books about rocketry, he worked on the Delta II rocket that launched satellites into orbit. He has a B.S. in Aeronautical Engineering from Embry-Riddle Aeronautical University in Daytona Beach, Florida, and has worked toward a M.S. in Space Technology from the Florida Institute of Technology in Melbourne, Florida. Currently, he is the owner of Apogee Components (http://www.apogeerockets.com) and the curator of the rocketry education web site: http://www.apogeerockets.com/education/. He is also the author of the books: *“Model Rocket Design and Construction,” “69 Simple Science Fair Projects with Model Rockets: Aeronautics”* and publisher of a FREE e-zine newsletter about model rockets. You can subscribe to the e-zine at the Apogee Components web site or by sending an e-mail to: ezine@apogeerockets.com with “SUBSCRIBE” as the subject line of the message.

A New Use For Clothespins

By Bernard J. Herman

Picture 1: Not a very safe way to store hobby knives.

Having a multitude of hobby knives and no container to keep them in has lead to a few problems for me as I am sure for other hobbyists. Trying to keep them together on my work table was almost impossible as they seemed to roll off of their own volition and sometimes even with evil intent as they fell off the table towards my bare feet. A pencil cup was a less than ideal answer as putting the knives in point first would lead to more frequent broken points and putting them in point up was a very dangerous idea.

Another issue was remembering which blade had recently been replaced, which was on its last leg and which needed to be replaced. I prefer a certain handle type and therefore just trying to use my aging brain was not really an option. Marking the handle (i.e. knife 1, knife 2) would again not be suitable for the same reason (less than stellar memory), especially if it was more than a week before I was again in my workshop.

Then one day while bending over to pick up another knife which had rolled off the table (in what seemed like an endless cycle), my eye fell upon the answer. Sitting on the tabletop was the ubiquitous clothespin. Another job for that multi-talented ever useful hobbyist's answer to the question, I need something to….

Picking up my sharpie, I quickly printed "new, good, so-so and replace" on the handles of some clothespins. Now, my knives stay in one place when I set them down, and I immediately know the condition of the blade in the knife. When the blade's condition degrades, I simply replace the clothespin with one of the other conditions marked on the handle.

Simple, Easy and Cheap. I like that.

Picture 2: Keeping knifes from rolling and letting you know the blade condition.