INCLUSION IN
MATHEMATICS
IN PRIMARY SCHOOL

- what can it be?

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PROLOGUE

Ever since I started teaching students who struggled with mathematics in primary school I have been interested in finding ways to help them. Help them to understand the mathematics, not just manage it, but to really engage in it and understand it. To see those eyes lighting up, when really understanding something is quite amazing! This is the main reason for conducting this research.

This research process has been like a hard and sweaty spin class. Before beginning it I was enthusiastic, full of energy and expectations. Jumping up on the bike to starting the project, was fun and I managed to make good settings. Shortly into the warm-up I hesitated. Were the settings good enough? Could I do this? Could my body (and mind) cope with the pressure? At the first slope I had to push myself hard. It was difficult to find ways of collecting data and to formulate the research questions. After filling up with water, the question marks became exclamation marks. Yes, I was on the right path. *Keep on going! Believe in yourself!* In the second interval my heart pounded and the sweat was pouring. Is it supposed to be this hard? How on earth will I be able to make sense of all this data? *Keep on going! Believe in yourself!* In the next climb my legs hurt and my mind began to spin. Data and theory was all over my mind, desk, i-pad, laptop and walls… The body (and mind) prepared for defeat. Then, suddenly it happened an adrenalin rush came and my mind was crystal clear and my body was suddenly filled with energy! The data was sorted and analysed in a flash (well). My legs pushed at a furious pace. It all became clear to me what inclusion in mathematics could be. The rest of the cycle-class I just enjoyed. Sure, it was still sweaty and there were climbs and intervals, but I enjoyed every turn of the pedals.

Many people have kept me going and made me believe in myself, when my legs were hurting and my mind span. First of all I would like to thank the

main informant in this study, Barbara. I am so grateful to you! Not only did you allow me to become a part of your important work for two years, you answered all my annoying questions, always with a smile. You always see opportunities where others see difficulties. You are a true inspiration! To all the students, teachers and the principal at Oakdale Primary School – thank you for letting me take part in your everyday struggles. A big hug to all the students in the primary school where I have had the privilege to be a teacher – many of you have been in my thoughts during this research process (even though you are now adults). You are the main reason for my doing this research!

My supervisors. You are the ones who shouted loudest, "Keep on going! Believe in yourself!" Constanta Olteanu, you encouraged me to pursue my quest and believe in my research. You gave me good advice and helped me through the rough parts. Hanna Palmér, my old friend and colleague, you are the best! On our bike trips to and from work we managed to solve many of problems (still do). You challenged me by asking hard questions and you scrutinised my texts. Especially, you always supported me with good advice and took the time to answer and discuss all my questions, high and low. I am very happy to have you by my side! Maria Bjerneby-Häll, thank you for your important contribution at the 50% seminar. Despina Potari, I am very grateful to you for your great response and suggestions at the 90% seminar!

 A big thanks to you, Jörgen Fors! You have made it possible for me to combine teaching with research, organising and supporting me. Anette Bagger, my research friend from the north, thank you for being such a wonderful and wise person. Claes Malmberg, what would this thesis be without your IT support? Thank you! A big thanks to all my supporting colleagues at Linnaeus University, especially my friends at the special education department, Ulla, Elisabeth and Ing-Marie. Thank you, Linnaeus University, Stiftelsen Lars Hiertas minne and Linnéakademien for supporting me financially.

Tobias, Hedda and Alfred, my family and all my good friends in life, thank you for reminding me of what life is really about. Without you I am nothing!

Växjö, December 2014

Helena Roos

1. INTRODUCTION

This study takes of from a special educational point of view regarding mathematics learning and teaching. Special education is a major field within education and is of interest, both in research and practice. The field of special educational needs (SEN) seeks to identify what needs in education have to be met in order to empower all students. Special educational needs *in mathematics* (SEM) is a minor field in this larger context. This field is influenced by theories and research from special education, mathematics education and the psychological research field, among others. Although SEM is often discussed in schools and at the political arena, the research is limited. Further, the limited research stems mainly from the psychological context (Sjöberg, 2006). The research presented in this thesis, has emerged from two of the fields that influence SEM, special education and mathematics education; hence this thesis takes an educational approach. This has been done to provide a counterbalance to the psychological field and to illuminate research in SEM within the educational field.

How to reach all students needs as a teacher is one of the main issues in special education. A dilemma occurs when the identification of a student as of being in need is necessary to receive support. Simultaneously, such an identification risks marginalising and segregating individuals because it identifies them as "not normal".

Research in SEM involves (at least) two different approaches. The pedagogical approach focuses on how to teach mathematics to the SEM-students and the psychological approach focuses on finding a diagnosis (e.g. dyscalculia - even though this term is not generally accepted by educators, it is used in the psychological field). This study is based on the pedagogical side of special educational needs in mathematics; hence, concepts like dyscalculia are not used or discussed in this thesis.

Usually a mathematics teacher and a remedial teacher teach students in special educational needs in mathematics. (In Sweden we have two kinds of remedial teachers, special pedagogues and special teachers and the latter have different specialisations such as reading and writing and mathematics). The remedial teacher in mathematics needs to be able to interpret the students' knowledge to be able to ensure that students' needs are met at all levels. On an overall epistemological level this interpretation can be done from a categorical perspective or a relational perspective (Persson, 2008). Nilholm (2005) has labelled these perspectives as compensatory and critical, which is similar to what Persson (2008) calls categorical and relational. A categorical or compensatory perspective in special education places the problem inside the student and can be described as a deviation from the "normal". Training, compensation and correction of the individual are then necessary. In both the critical and the relational perspective the source of the problem is located in socio-cultural settings. Solutions are then found by adapting the learning environment and relations surrounding the SEM-student. Nilholm (2005, 2007) has furthermore described a third perspective that allows an evaluation of and critique of both the relational and the categorical perspectives used in research: the dilemma perspective. "Dilemma" (Nilholm, 2005, 2007) refers to unsolvable contradictory problems that special pedagogical practice emerge from and has to handle. These can consist of values and motives for supporting the student versus the needs of the society or school system. Problems in the learning situation are then understood as being about for example participation versus exclusion, or equality and democracy.

A relational perspective on mathematics difficulties stresses the need to consider how the teaching and learning activities in question affect the students' learning (Dalvang & Lunde, 2006). The present project adheres to the relational view in striving to reach an understanding of the process of inclusion in mathematics. In this thesis knowledge and mathematical understanding are viewed as cultural and social phenomenon.

Research (e.g. Ballard, 1999; Armstrong, Armstrong & Spandagou, 2009) regarding inclusion in education is a major field of research, which mainly looks at inclusion from a pedagogical point of view. However, little attention has been paid to the meaning of inclusion in mathematics education and to the identification of factors that appear to be important in the students' learning of mathematics.

When Nilholm (2007) discusses inclusion he refers to children with disability; in this concept it is children that have a disorder, a diagnosis of some kind (cognitive or physical). These children are often in special schools or in special teaching groups. Disabled children, special teaching groups and special schools are frequently investigated together with inclusion (e.g. Janhukainen, 2011, Karlsson, 2007, Linikko, 2009). This study does not use disabled children or special education groups in looking at inclusion in mathematics. Instead, I am trying to describe and understand what inclusion in mathematics can be in a regular primary school. The focus is not on a diagnosis or different educational groupings, but on the needs *in school mathematics*.

Many schools use ability grouping in mathematics and the teachers envisions this leading to better goal achievements for students in SEM (Boaler, Wiliam & Brown 2000; Wallby, Carlsson & Nyström, 2001). However, researchers have concluded that ability grouping does not make the positive impact on students' knowledge development that the teachers expect (Boaler, 2008; Slavin, 1990). Educational differentiation and individualisation is a complex issue, which requires more investigation. Moreover, there is still much to learn regarding how different factors work and connect in pursuing inclusive teaching of students in special needs in mathematics. The mathematics focused on in this thesis are those taught in primary schools.

My main interest when I began this research was to find ways for students in SEM to have the opportunity to be engaged in the mathematics education in school. I hope the research will somehow benefit these students.

The aim and the research questions of this study have emerged from the field in that sense that a teacher was eager to focus on inclusion in mathematics in her practice as a remedial teacher. This was an issue she addressed the first time I asked her to participate in this study. It was something she thought was important for the SEM-students. Hence, inclusion in mathematics was in focus when the research questions were formulated. The research questions of the study emerged in the research process through the analysis and data collection.

1.1 Aim, research questions and limitations

The aim of this thesis is to contribute to research and practice in mathematics and special education with more knowledge about, and an understanding of, how all students can be included in the mathematics education in primary school from *a teacher perspective*.

The research questions of the study are:

What can inclusion in mathematics be in primary school and what influences the process of inclusion in mathematics?

What, from an inclusive perspective, appears to be important in the learning and teaching of mathematics?

One might wonder why this study only focuses the teacher perspective of inclusion in mathematics. The simple answer is that the student perspective is far too important to be accommodated within this research. The student perspective needs to be investigated more thoroughly to give the students a voice and justice. If this research project continues that perspective will be investigated.

1.2 Terms and acronyms used

In the thesis there are some notions and acronyms used. The following is a brief definition of these notions.

SEN – special educational needs
SEM - special educational needs in mathematics
Remedial teacher – a teacher working with children in special needs
Special pedagogue – a teacher with further education in special education
Special teacher – a teacher with further education in special education connected to a subject or a disability.
Preschool class – optional preparatory school (age 6)
Primary school – Year 1 to year 6 (students ages 7 to 12)
Lower primary school – Year 1 to 3 (students ages 7 to 9)
Upper primary school - Year 4 to 6 (students ages 10 to 12)
Action plan – Plans that are made when a student is at risk for not, or does not, achieve the curriculum goals in a subject.

2. BACKGROUND

In this chapter research on special education needs in mathematics is presented. Inclusion, exclusion and differentiation are presented from a theoretical point of view. Since this research concerns inclusion in mathematics, mathematics in primary school and representations in mathematics is presented.

2.1 Special education needs in mathematics

Special education is a notion hard to define; yet it is often used in many contexts. The very concept begs the question: Why is it special? What is the difference from ordinary education? According to a Swedish government proposal from the late 1980s, special education can be interpreted as "activities for students that fall outside the natural variability of diversity" (Proposition 1988/89: 4 p. 80). This in itself is hard to interpret and raises the question what is natural variability? Who is defining it and what criteria are used? The variability is strongly connected to knowledge so what does it mean to know something? Knowledge mean different in different contexts but it changes over time and what is knowledge today might not be knowledge tomorrow. "Knowledge is not a static commodity" (Gorard & Smith, 2004 p. 207). Knowledge is thus situated in time and culture. Knowledge is what humans have accumulated over time in order to understand the world and act effectively in it (Wenger, 2004).

When connecting a subject like mathematics to special education needs, the questions are about variability and diversity of knowledge in mathematics. SEM is a relative concept depending on who is defining the natural diversity among students. Because of this, SEM is closely connected with issues of power and democracy. This becomes particularly

clear when it comes to who has the right to make these definitions, establish the criteria and judge who needs special education (The Swedish Research Council, 2007). It is to some degree always an interpretation situated in culture and time. The interpretation and use of the term *special needs* itself "depend ultimately on value judgements about what is important or desirable in human life and not just on empirical fact" (Wilson, 2002, p. 61). Again, it is a question of who or what has the authority and power to make these judgements and state the norm. It is also a question of democracy, of how the people involved influence of these judgements?

The different perspectives in the education of the student in need of support all have used several fields of expertise both in their research and practice (Emanuelsson, Persson, & Rosenqvist, 2001; Heyd-Metzuyanim, 2013; Magne, 2006; Nilholm, 2005). These fields are in some way connected to a psychological, social or pedagogical discourse. SEM is one of these fields. This fields connections to the psychological, social or pedagogical discourse can be seen in the use of the terms and definitions (when there are any). Terms occurring among scholars are for example children with mathematics difficulties (Gifford & Rockliffe, 2012), dyscalculia (Kaufmann, 2008), SEM-student (Magne, 2006), and mathematics anxiety (Hannula, 2012). In this thesis the term *SEM-student* (Magne, 2006) will be used.

SEM and what it means is discussed in practice but unfortunately not as much among scholars. It is also a term that is hard to define and has different definitions depending on from what epistemological field it derives from (Bagger & Roos, 2014). Bagger and Roos (2014) suggest the term *students in special educational needs in mathematics,* which is used in this thesis. The reason for using this term is that the research starts from the relational and pedagogical perspective on mathematics difficulties, which focuses on teaching and learning activities and how they affect students' learning in mathematics. I draw on Silfver, Sjöberg and Bagger (2013) whom writes that the need is something that may occur whether the student is a high or a low-achiever, for a shorter or longer period in time, in a general or in more specific areas in mathematics. Hence, the student is *in* SEM because it signals that it is not a deficiency within the student, it is something the student can get in and out of (Bagger & Roos, 2014).

The notion of special education is closely related to inclusion. This relation can be seen when special education is used to facilitate the learning of students with diverse needs in the classroom and in the urge to solve the problem of individual differences in relation to the curricular goals

(Nilholm, 2005). Stainback and Stainback (1990) argued that the concept of special education should be replaced by inclusive education, in order to highlight that this is about full participation for all. Both concepts (special education and inclusion) have continued to be used and developed, suggesting that they have slightly different meanings.

2.2 Inclusion

The word *inclusion* has been used increasingly over the last decades and has to do with "people and society valuing diversity and overcoming barriers" (Topping, 2012, p. 9). The concept of inclusion is complex and has many interpretations (Brantlinger, 1997; Artiles, Kozleski and Christensen, 2006). Even so, it has come to be used in a wide context covering ethnicity and social issues (Nilholm, 2007, Berhanu, 2011) and has its origin in the civil right movements USA in the 1950s (Persson & Persson, 2012). It is also a well-used term in the educational context, for example, *inclusive education* (e.g. Göransson, Nilholm & Karlsson, 2011). Considerable amounts of research has been done on inclusion and an abundance of definitions and interpretations have been made. Hence, it is not easy to choose a definition. From a broad perspective, however, inclusion is about being able to empower *all* students as well as be able to meet human differences and create meaningful participation in the education (Barton, 1997, Persson & Persson, 2012). In Swedish schools inclusion is often used, even in mathematics education.

Historically, inclusion is a relatively new concept in the school context, and it was first used in this context during the early 1990s; before that, the term "integration" was used (Farrell, 2004). From a school perspective there is a difference between these two concepts, and to sort out what it is, an investigation of the development of the concepts is needed.

The concept of integration was developed towards the end of the 1960s as a critique of the various institutions created for, what were called, "deviant" groups in society. In a school context, this term reflected the use of an assimilation process: children with special needs would be fitted into an existing school context (Nilholm, 2006). The problem with the definition of integration was that it did not address the quality of the education; it only signalled the physical location of the child, that children in SEN was in the same classroom as their peers (Farrell, 2004). At that time, integration was perceived as a development because it was a success to get

the children in SEN in the same classroom as their peers, from a basic values and political point of view. The development and visions of society and the classrooms led to a problem with the definition of integration in the 1990s. The concept did not fully cover the importance of participation, and the term inclusion began to become more common (Rosenqvist, 2003). In the Salamanca Declaration made by UNESCO 1994, there was an international agreement (between 92 governments and 25 international organisations) describing principles and practices regarding SEN (Swedish Unesco Council, 2006). Here the term inclusion was used extensively for the first time and was adopted internationally (Vislie, 2003). Inclusion was used to signal a new way of looking at SEN and dealt with the perceived problems with the concept of integration. Using inclusion in the Salamanca Declaration sought to deal with the problem of seeing SEN inside the child, by instead seeing it in the methods and in the organisation (Swedish Unesco Council, 2006). The intent was on an overall levelling of society to "lay the foundation of a fair and non-discriminating society that encourages people to learn and live together" (Swedish Unesco Council, 2006, p.45, own translation). Hence, inclusive education is a way of trying to change the political cultural view (Slee, 2011) of education for all.

From an inclusive perspective, education is something for all children and should be adjusted according to the specific needs of the children, and the pedagogy should put the learning of the children in the centre. Pedagogy is an important dimension of inclusion (Liasidou, 2012). From a Swedish perspective, education for all children can be seen in the expression "a school for all" (Nilholm, 2006). The idea of "a school for all" is situated on a political and societal level and wishes to have schools where all children have a place. Hence, the concept seeks "that the school (the whole) will be organized based on the fact that children are different (the parts)" (Nilholm, 2006, p.14, own translation). Hence, inclusion is about respecting diversity (Booth, Nes & Strømstad, 2004). This is also highlighted by Ainscow, Booth and Dyson (2006) who is also states "[the] inclusive school is one that is on the move, rather than one that has reached a perfect state" (p.25), implying the need for a continuous process focus on the participation of *all* children in schools. The concept of inclusion refers to a continuous process (Asp-Onsjö, 2006) by which schools attempts to respond to all students as individuals (Vislie, 2003). According to Nilholm (2006) the introduction of the concept inclusion had an intention, a wish to change the perception regarding work with students in SEN, from exclusion and integration to inclusion. It was a symbol of departure from normative thinking (Graham & Jahnukainen, 2011).

Ainscow, Booth and Dyson (2006) recognise a tension trying to define inclusion; "On one side, it was argued that we should keep an open mind about what we meant by inclusion as we engaged in our research. On the other side, it was suggested that without a clear view of what we mean by inclusion we had no way of knowing how to support it" (p. 22-23). Their investigation of inclusion implies that the notion is hard to capture and it is more of a *process* than a static goal to reach.

From an international perspective, inclusion has different meanings. It depends, among other things, on the conditions of the school system, the interpretation of the Salamanca Declaration and the approach to people with disabilities. It is also affected by the political arena, the society and the culture in the countries. From a school system point of view, Sweden uses inclusion much more than, for example, America (Nilholm, 2007).

When looking at successful schools in terms of inclusion, Gregory (2006) has identified a number of factors important for success with an inclusive program. People and relations, an accepting climate, professional development for teachers and the principal, and clearly articulated goals were important factors. In addition, the principal's ability to develop a good climate for learning was a prerequisite. This is also highlighted by Hattie (2003), in discussing the impact of the principal on student achievement through responsiveness to students and creating a "climate of psychological safety to learn" (Hattie, 2003, p. 2). This implies that the principal can influence the climate and students responsiveness through the pedagogical environment and organisation at the school. The principal is thus responsible for organising the pedagogical environment to promote learning. In order to do that, a reorganisation is often done. However, it has been shown that reorganisations in schools do not always improve the practice (Larsson, 1998). Also Cobb, Jackson, Smith, Sorum and Henrick (2013) discuss the role of the organisation in relation to the actual teaching and learning in mathematics. They imply there is a lack of communication between research in mathematics education and research on educational policy and leadership, which limits the impact on the actual mathematics teaching in the classroom.

Inclusion in education
Many different approaches are taken when investigating inclusion in education.

Göransson and Nilholm (2014), who did a conceptual analysis of the concept of inclusion in education, distinguish four categories of definitions

in research: placement definition, specified individualised, general individualised and a community definition. The placement definition refers to student in SEN or with disabilities in general classrooms. The specified individualised definition refers to inclusion as a way of meeting the social and academic needs of student in SEN or with disabilities. The general individualised definition refers to inclusion as a way of meeting social and academic needs of *all* students. The community definition refers to creating special communities. These definitions can be seen in educational research, for example, Karlsson (2007) who investigates social organisation and evaluation of students in a special education group. Karlsson (2007) uses inclusion when discussing marginalisation and exclusion and uses the terms critical, relational and dilemma perspective when discussing the results. If putting this research into the categorisation of Göransson and Nilholm (2014), it would be in the specified individualised category, since it refers to analysing social organisation of individual students. Asp-Onsjö (2006), who highlights inclusion in her investigation of documentation for SEN-students, found that the concept of inclusion was somewhat vague in relation to analysis of the practice. To be able to illustrate inclusion from a practice perspective, she divided it into three parts: spatial inclusion, didactical inclusion and social inclusion. Spatial inclusion basically refers to how much time a student is spending in the same room as his or her classmates. Social inclusion concerns the way in which students interact with his or her peers. Didactical inclusion refers to the student's participation in relation to the subject taught in the classroom. Returning to the categorisation of Göransson and Nilholm (2014), the definition of Asp-Onsjö would be placed in the general individualised definition, since it refers to a way of trying to describe both social and academic needs for all students.

Several studies (e.g. Allan, 2010; Booth, Ainscow & Dyson, 1997; Gregory, 2006) have an overarching view of inclusion, using broad definitions of the concept. This kind of research could be placed in the community definition category. Heimdahl Mattson and Malmgren Hansen (2009) use Booth, Nes and Strømstad (2004) when defining inclusive education as "the idea of supporting all learners within a local community" (p.466). Heimdahl Mattson and Roll-Pettersson (2007) use Skrtic, Sailor & Gee (1996) when discussing an inclusive school. Here inclusive school is defined as one in which the teachers are flexible and open to the problems facing the students and prepared to cooperate with the students. Graham and Jahnukainen (2011), who investigate the development of inclusive education, take disability as their point of departure. Here disability, and disabled students have been categorised into how they receive education, in

regular schools or a special class or a special school. Hence, inclusive education refers to being present in a regular class. Hjörne (2004) investigates the understanding of institutional reasoning and categorising practices in schools when dealing with students in SEN. Hjörne (2004) also defines inclusion as being part of the regular class. Both Graham and Jahnukainen (2011) and Hjörne (2004) can be categorised in the placement definition.

As mentioned, there are few studies in mathematics education with inclusion in focus. DeSimone and Parmar (2006), who investigated teachers' beliefs about inclusion of student with learning disabilities in relation to mathematics, did not define inclusion in their study. The study refers to the No Child Left Behind Act in the USA 2001, which states that all students (with few exceptions) should be able to participate in, and master, the general education curriculum in the United States. DeSimone and Parmar (2006) state that inclusive programs need to be investigated from the view of learning disabilities, but do not define inclusive programs and the questions used in the study to interview the teachers does only refer to inclusion as something known. For example, "How many years have you been teaching mathematics inclusion?" (DeSimone & Parmar, 2006 p. 110). Lindenskov (2006) is writing about an inclusive school with focus on mathematics. Here the terms access and presence are used to describe the inclusive school. This can be compared to a conceptualisation made by Farrell (2004), who introduces a conceptualisation regarding student outcome in relation to inclusion. Here a school needs to fulfil four conditions: presence, acceptance, participation and achievement, in order to be a truly inclusive school. This conceptualisation of inclusion made by both Farrell (2004) and Lindenskov (2006) can be interpreted as a community definition, but also as placement and a specified individualised definition. If comparing Farrell's (2004) conceptualisation to Asp-Onsjö's (2006), presence can be equated to spatial inclusion and acceptance can be equated to social inclusion. Farrell (2004) describes participation as to the extent to which all pupils contribute actively, and achievement refers to learning and developing positive views about themselves. Neither of these two conditions refers to any subject content, which didactical inclusion (Asp-Onsjö, 2006) does, indicating the definitions do not grasp the same issues regarding inclusion. Schmidt (2013) uses the term inclusion when reviewing research dealing with the possibility for the teacher to teach mathematics in a way that include all students in a regular classroom. The result describes how classroom management affects students' opportunity to be included. In this research inclusion is used as an overarching term, describing social and disciplinary aspects of teaching mathematics. This

could be categorised as a placement and/or specified individualised definition. If one compares Schmidt (2013) to the notions of Asp-Onsjö's (2006), social and didactical inclusion, there are specific disciplinary aspects in mathematics. If applying the categorisation of Göransson and Nilholm (2014) to Asp-Onsjö, one can say that spatial inclusion refers to placement and didactical and social inclusion can be both specified individualised definition and the general individualised definition depending on if the appliance is on an individual or on a group.

In the socio-political arena in mathematics education there is research using the term inclusion in connection with terms like equity and diversity, for instance, that of Baldino and Cabral (2006). They use inclusion as an overarching notion when discussing social exclusion and mathematics teaching, but they do not define it. There is abundance of research in mathematics education that uses terms like equity, access, social justice, empowerment and mathematical literacy. In this field of research, inclusion is regarded as an issue of equal access to the mathematics to ensure that the education "allows all students to succeed" (Diversity in Mathematics Education Center for Learning and Teaching [DiME], 2007, p. 406). Inclusion is discussed in terms of access to the mathematics taught for all students. Another issue discussed in the search for what the students need is listening to the students, giving "space for student voices" (Tomlin, 2002, p. 9). There is also talk about the "need to develop meaningful interventions […] to empower marginalized students with mathematics" (Diversity in Mathematics Education Center for Learning and Teaching [DiME], 2007, p. 426). Another issue discussed is the use of tasks, to use tasks embedded in known contexts for the students to achieve access to the mathematics in the tasks for all students (Cahnmann & Remillard, 2002). Also, the context in the tasks influence the way students draw upon high-order thinking regarding mathematical literacy (Meaney, 2007), which strengthens the argument that teachers must consciously choose tasks to be able to challenge the students' mathematical thinking.

Hence, there is talk about inclusion in mathematics on an overall level in the socio-political research in mathematics education, but no definition of the notion, it is used as an overarching notion (e.g. Baldino & Cabral, 2006). However, there is a lot of research that discusses important issues regarding access for all students to the mathematics taught in the classroom (e.g. Meaney, 2007; Cahnmann & Remillard, 2002).

2.3 Exclusion

When talking about inclusion, the opposite, exclusion, may be an important notion to take into consideration. Inclusion and exclusion are interconnected; inclusion involves a fight against exclusion and inclusion is thus a never-ending process (Ainscow et al., 2006). Inclusion and exclusion can be interpreted as students' participation or alienation (Nilholm, 2006). Exclusion can be seen as alienation because it involves not being part of a whole and not to be able to get access to the whole.

The concept marginalisation can be considered in connection with exclusion. Marginalisation implies a belonging to something, but it is a partial belonging (Svedberg, 1998). Svedberg (1998) talks about marginalisation as a position on a continuum, where at one end an individual has a safe position and the other end an individual is eliminated. In between these two positions marginalisation occurs. Although Svedberg (1998) talks about marginalisation in relation to employment, this can be translated into marginalisation in school. The far ends of the continuum could be inclusion and exclusion and marginalisation as a relational concept between the end points. Because inclusion is seen as a process, the end points are not seen as being static and marginalisation can occur to varying degrees, depending on where on the continuum the individual is.

How marginalisation and exclusion are reflected in practice depends on the interpretation of how to organise the education from the perspective of students' differences (Nilholm, 2006). In this thesis the focus is inclusion in school mathematics. Hence, exclusion in this thesis is when students do not get access to the mathematics in school at all. Marginalisation in this thesis is if the students get access to some parts of the school mathematics.

2.4 Differentiation

Differentiation refers to education shaped differently for different students (Wallby et al., 2001). The goal with differentiation is to fit the education to all students, and since the students are different, the education needs to be different (Wallby et al., 2001; Nyström, 2003). This goal of differentiation relates strongly to inclusion. Hadenius (1990) defines differentiation from a Swedish perspective as grouping of students according to criteria other than age. This definition narrows it down to grouping of students. The National Agency for Education (2010) states that grouping of students can be done differently and over a shorter or longer time, but it is not regulated in policy documents. According to Nyström (2003) grouping of students is particularly prevalent in school

mathematics, and a study done by the National Agency for Education (2010) confirmed this.

Another term used in relation to differentiation in mathematics is ability grouping (i.e. Boaler, William & Brown, 2000; Wallby et al., 2001). When using ability grouping, the students' ability in mathematics is assessed and the students are divided into different groups by ability. This kind of differentiation is also called organisational differentiation (Wallby et al., 2001) or internal differentiation (Nyström, 2003). External differentiation on the other hand means that external factors such as sex or interests affect the grouping (Nyström, 2003).

Pedagogical differentiation is described as differentiation in the classroom by individualisation or occasional small groups (Wallby et al., 2001). This can be compared to a mixed ability mathematics approach, which Boaler (2008) promotes. Mixed ability can be described as "heterogeneous grouping and an associated set of teaching practices allow[ing] students to interact with others from different social classes, cultural groups and ability levels" (Boaler, 2008, p. 21). In both the pedagogical differentiation and mixed ability approaches students are together in the classroom, but their starting points are different. Boaler (2008) start with a focus on interaction between students with differences as a means for learning in mathematics and how to teach to allow this interaction. Pedagogical differentiation, on the other hand, starts with individualisation or how to teach to reach the individual. The key issue here is how to work with the variation in the classroom, regarding both ability and teaching. Variation in student ability and teaching can be seen as a hindrance for learning, which can be reduced by ability grouping. Variation can also be seen as an asset because it provides various opportunities for learning (Stiegler & Hiebert, 1999).

2.5 Mathematics in primary school

In this section mathematics education in primary school from a Swedish perspective is described.

In Sweden students start compulsory school when they are 7 years old. Before that most have gone to a preschool class, a voluntary activity for six year olds, which is usually accommodated in the school (though politicians have suggested that the preschool classes will be mandatory). Lower primary school goes from years 1 to 3 and upper primary school from years 4 to 6. Mathematics is taught from year 1; even though mathematics often is a part of preschool class, there is no curriculum in mathematics for preschool class. The preschool class is governed by the overall formulations

in the Swedish curriculum (National Agency for Education, 2011).

Mathematics in the curriculum
The Swedish mathematics curriculum states that "teaching should aim at helping students develop knowledge of mathematics and its use in everyday life and different subject areas" (National Agency for Education, 2011, p. 59). The *teaching* of mathematics is in focus in the section "aim of mathematics in primary school". Interest, confidence, ability and experience of aesthetic values are emphasised in the description of the teaching of mathematics. The teaching is also connected to interpretations, reflections and development in mathematics. The teaching in mathematics should give students the opportunity to develop five abilities (National Agency for Education, 2011):

> formulate and solve problems using mathematics and also assess selected strategies and methods, use and analyse mathematical concepts and their interrelationships, choose and use appropriate mathematical methods to perform calculations and solve routine tasks, apply and follow mathematical reasoning, and use mathematical forms of expression to discuss, reason and give an account of questions, calculations and conclusions
> (National Agency for Education, 2011, p. 59-60).

These five abilities are meant to be thought of in each area of the core content.

The core content in years 1–3, as well as in years 4–6, is divided into the following main sections: *Understanding and use of numbers, Algebra, Geometry, Probability and statistics, Relationship and change and Problem solving* (National Agency for Education, 2011, p. 60-61). Within the content areas there is a progression between years 1–3 and 4–6, and it is clear that the content in years 4–6 is based on the content in 1–3. To exemplify this, I have chosen to highlight the content area "understanding and use of numbers". The choice was made because this specific content area is in focus in the teaching of SEM-students in this thesis.

For example, this progression can be seen in the area of natural numbers. In years 1–3 one item in understanding and use of numbers is "natural numbers and their properties and how numbers can be divided, and how they can be used to specify quantities and order" (National Agency for Education, 2011, p. 60). In years 4–6 this is developed and focus is rational numbers and their properties. To be able tog grasp rational numbers

natural numbers and their properties is essentially to be able to understand the different constructs of rational numbers (Charalambous & Pitta-Pantazi, 2007).

The positioning system is another topic in the core content that shows progression. In years 1-3, the content is "how the positioning system can be used to describe natural numbers. Symbols for numbers and the historical development of symbols in some different cultures through history" (National Agency for Education, 2011, p. 60). In years 4-6, the positioning system focus is "the positioning system of numbers in decimal form. The binary number system and number systems used in some cultures through history, such as the Babylonian" (National Agency for Education, 2011, p. 61). Here it is clear that natural numbers and how they relate to the positioning system provides the basis for understanding the positioning system of numbers in decimal form. The culture history of number systems is also highlighted in both years 1–3 and 4–6.

Fractions are another topic in the content in both years 1–3 and 4–6: In years 1–3 simple fractions are in focus, and in 4–6 the connection between fractions, percentage and decimals is in focus. "Numbers in fractions and decimals and their use in everyday situations" (National Agency for Education, 2011, p. 61) is an item in the core content for years 4-6. This is preceded by "parts of a whole and parts of a number. How parts are named and expressed as simple fractions, and how simple fractions are related to natural numbers" and " numbers in percentage form and their relation to numbers in fraction and decimal form" and "natural numbers and simple numbers as fractions and their use in everyday situations"(National Agency for Education, 2011, p. 60) in years 1-3. Hence, fractions are content in both years 1-3 and 4-6. In years 1-3 simple fractions is in focus, and in years 4-6 the connection between fractions, percentage and decimals. The progression is obvious and is in line with how fraction can be conceptualised as part-whole, ratio, operator, quotient and measure (Charalambous & Pitta-Pantazi, 2007).

 In years 1–3 the properties of the operations connected to the natural numbers are in focus, while in years 4–6 the connection is to both natural numbers and decimal form. In years 1–3 items in the core content are "properties of the four operations, their relationships and use in different situations." and "main methods of calculating using natural numbers when calculating mental arithmetic and approximate estimates, and calculations

using written methods and calculators. Using the methods in different situations"(National Agency for Education, 2011, p. 60). In years 4–6 the corresponding item is "main methods of calculating using natural numbers and simple numbers in decimal form when calculating approximations, mental arithmetic, and calculations using written methods and calculators. Using the methods in different situations" (National Agency for Education, 2011, p. 61).

Plausibility is the last mentioned item in "understanding and use of numbers" in the Swedish mathematics curriculum. In years 1–3 this is reflected in "assessing plausibility when using simple calculations and estimates" (National Agency for Education, 2011, p. 60) and in years 4–6 in "plausibility assessments when estimating and making calculations in everyday situations" (National Agency for Education, 2011, p. 61). Here the progression lies in where to apply the plausibility.

Assessment
Assessment in the mathematics classroom can be a concept with broad boundaries. This means that it t can be done in many different ways, for example tests, documentation such as written individual action plans or communication between the teacher and the student during day-to-day work. Assessment is an activity that has strong connections with learning and teaching (Björklund Boistrup, 2010). Two notions connected to assessment in school are *formative* and *summative assessment*. Black and Wiliam (1998) define formative assessment as "encompassing all those activities undertaken by teachers, and/or by their students, which provide information to be used as feedback to modify the teaching and learning activities in which they are engaged" (p. 7). Summative assessment is defined as assessment connected with tests on a local or national level and summarised tests of students' performances in relation to stated goals (Björklund Boistrup, 2010). One part of the summative assessment in mathematics is the national tests. Sweden has national tests in mathematics in years 3,6 and 9, which is in line with the knowledge requirements, which are explicitly written for years 3,6 and 9 (National Agency for Education, 2011).

Grades and knowledge requirements
In 2011, the grades in the Swedish school system changed with the implementation of the new curriculum from IG, G, VG and MVG to F, E, D, C, B and A (F is a grade which is used for not achieving the minimum requirement, E). There was also a change in when the grading

started. Before the 2011 curriculum the grading started in year 8; with the new grading system it starts in year 6.

The mathematics knowledge requirements section in the curriculum for the Swedish compulsory school (National Agency for Education, 2011) is divided into three parts: knowledge requirements for acceptable knowledge at the end of year 3, knowledge requirements for grades E, D, C, B and A at the end of year 6 and knowledge requirements for grades E, D, C, B and A at the end of year 9. There are the knowledge requirements for the different grades in year 6 and year 9 and then a matrix of the requirements for years 6 and 9. The years before year 3 and in between 6 and 9 are supposed to be interpreted on the basis of the written knowledge requirements.

According to the commentary on the knowledge requirements in mathematics, the requirements are constructed with value words and based on the two different parts in the curriculum, *abilities and core content* (National Agency for Education, 2012).

Figure 1 is describing how the abilities together with the core content build the knowledge requirements in mathematics.

The abilities in the aim → The knowledge requirements ← The core content
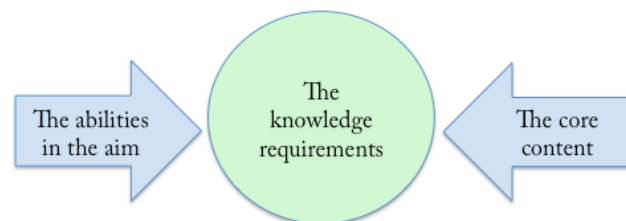
Figure 1. The connection between abilities and core content in the knowledge requirements in the mathematics curriculum. (National Agency for Education, 2012, own translation)

To give an example of the knowledge requirements in the curriculum, I have chosen to highlight the knowledge requirements that are acceptable concerning understanding and use of numbers at the end of year 3 and at the end of year 6, because this is virtually the focus of the teaching in mathematics in this thesis.

Conceptualisation in mathematics is a basis for understanding numbers, which is evident in the curriculum. In year 3 students shall "have basic knowledge about mathematical concepts and show this by using them in commonly recurring contexts in a basically functional way" and be able to "describe the properties of concepts using symbols and concrete materials or diagrams (National Agency for Education, 2011, p. 64). In year 6 (requirements for grade E) students shall "have basic knowledge of mathematical concepts and show this by using them in familiar contexts in a basically functional way" (National Agency for Education, 2011, p. 65). Students need to "describe different concepts using mathematical forms of expression in a basically functional way" and have the ability to switch between different forms of expressions and apply simple reasoning over how the concepts relate to each other (National Agency for Education, 2011, p. 65).

In regard to numbers and operations, for year 3 the focus is both on numbers and the four operations, addition, subtraction, multiplication and division. The students shall have "basic knowledge of natural numbers" and "basic knowledge of numbers as fractions". Even here methods are evident in the requirements, where students shall be able to "choose and use basically functional mathematical methods" and "use mental arithmetic to perform calculation using the four operations when the numbers and the answers are in the range 0-20". When referring to addition and subtraction, the students should also be able to "chose and use written methods [...] when numbers and answers lie within an integer range of 0-200" (National Agency for Education, 2011, p. 64). The equals sign is also mentioned, the students shall be able to use it "in a functional way" (National Agency for Education, 2011, p. 64). For year 6 the acceptable knowledge requirements is focuses on choosing and applying "basically functional mathematical methods with some adaption to the context" and to "carry out simple calculations and solve simple routine tasks in arithmetic" (National Agency for Education, 2011, p. 65).

These requirements contain value words like *basic, basically functional way, familiar and simple*. Because value words are hard to interpret, it is not clear what it mean to be able to describe expressions in a basically functional way. Interpreting a value word depends on the context (National Agency for Education, 2012) and in assessing, it depends on the content area. Hence, it is hard to establish a universal interpretation (National Agency for Education, 2012).

2.6 Representations in mathematics

To be able to think and communicate mathematics, mathematics needs to be represented somehow. A representation is a configuration of signs, images, icons or objects that stands for something else (Goldin, 2000; Duval, 2006). It may include symbols such as 10 or pictures such as two fives on dice. An understanding of these forms of representation and translation between them is crucial for learning mathematics (Ainsworth, 2006; Duval, 2006).

Internal representations are used when thinking mathematics because the person is handling the representations in their mind. These internal representations are harder to describe than the external since they are not observable. Assumptions about how the internal representations are represented in the mind builds on the conclusions of external representations (Hiebert & Carpenter, 1992) and internal representations are the internal language of an individual (Goldin, 2000).

Representations can be divided into different semiotic systems[1]. A register is a semiotic system, containing a certain set of representations. For example, the iconic register contains representations such as sketches, drawings and patterns. According to Duval (2006), learning takes place when a person manages to make a transformation between these registers. There are two different kind of transformation between mathematical registers of semiotic representations, according to Duval (2006): treatments and conversions. Treatments are transformations made within the same register and conversions are more complex transformations between different registers.

Mathematics is done with semiotic representation, hence, mathematical processes automatically involves semiotic representations. Signs in mathematics are not a substitute for other objects, but for other signs, which is one of the difficulties in mathematics. The only way to get access to mathematical representations is to become familiar with them. By combining different representations, the learner is not limited by the strengths and weaknesses of one representation (Ainsworth, 2006). Multiple representations may be assigned different functions in the learning of mathematics, according to Ainsworth (2006). These functions are *complementary*, *constraining* and *constructing*. The complementary

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[1] A semiotic system is a system with signs for meaning making. ”A sign is, basically, an asymmetric relation between a material expression and the content to which it is meant to refer” (Winsløw, 2004, p. 81).

function describes how various representations may support different processes or contain separate information. The constraining function of representations restricts or supports the learner when interpreting another representation. Thus, the constraining function does not have the role of adding new information, but of supporting by providing information to help in interpreting the representations. This can happen, for example when pictures are used with more complex representations such as symbols. The constructing function concerns processes involved in how individuals develop their understanding of content between representations (Ainsworth, 2006). To be able to use the advantages of multiple representations, the learner needs to understand the interrelationship of the representations.

The use of auditory and visual perception uses the short-term memory and facilitates learning (Ainsworth, 2006), thus it is important to take auditory and visual representations into consideration in teaching mathematics. If representations are shown in different registers, individuals with limited understanding of representations have difficulty in seeing the relationship between representation forms.

Given the effectiveness of multiple external representations, the learner must be able to process the information contained in each representation. External representations play an important role in learning by supporting the learning process (Ainsworth, 2006). When students encounters a mathematical task they must be able to encode the content and understand how it can be represented as well as understand the relationships between the represented content and form. The students may need to learn how to choose an appropriate representation or how to construct different representations. One of the advantages of multiple external representations is that information can be distributed over representation boundaries to simplify for learners.

Thus, representation is an important notion in mathematics education. Both research and practice use the term when describing both learning and teaching in mathematics. The students need to be aware of and be able to handle, different representations and the teacher needs to have knowledge of the use of different representations in relation to a mathematical content. Consequently, in this thesis, representations in mathematics need to be considered as a part of the teaching and learning of SEM-students.

In summary

On an overall societal level, changing the perception of work with SEN and moving towards inclusion can be seen as simply a way of saving money. This depends on how inclusion is interpreted. If it is interpreted merely as "put all students in the same class", it results in less money being spent on teachers and teaching. On the other hand, from an ideological perspective, participation, tolerance and humans' differences are cornerstones of our society and needs to be foundations even in the school. If we do not use these cornerstones in our schools, how will we be able to get this way of thinking in our society? Consequently, these concepts can be interpreted as a definition of students' participation or alienation (Nilholm, 2006). Hence, to be included can be seen as a process of participation. The word process signals a continuum. Accordingly, in this thesis inclusion is seen as a process of participation.

When taking an international perspective on inclusion, it is worth to take into consideration that Sweden does have an excluding school system with special schools. However, since there are different meanings in different contexts, it is hard to know whether how much Sweden uses inclusion in comparison with other countries and from what point of view Sweden uses the term.

Despite developments in the research area of inclusion, it still remains a complex issue, and the development of inclusion in schools is not well understood (Ainscow et al., 2006). As Karlsson (2007) highlights, the didactical perspective regarding inclusive education is missing. Hence, inclusion from a didactical perspective in mathematics is what is in focus in this study.

The main findings from research in mathematics education about inclusion in mathematics classrooms are on an overarching level. The concept is used as a tool when investigating for example diversity and equity. However, there is research discussing, for example, the need for being thorough when choosing tasks in order to give all student access to the mathematics (e.g. Cahnmann & Remillard, 2002). Even the need of developing meaningful interventions to include all students in the classroom is highlighted (Diversity in Mathematics Education Center for Learning and Teaching [DiME], 2007). This can be interpreted as interventions towards inclusion.

In an overview of research concerning inclusion in education Göransson and Nilholm (2014) conclude that there are different ways of using (and defining) inclusion in research. In their overview there is lack of studies

defining inclusive education and showing how inclusive practices are to be achieved. Hence, research is lacking on how to operationalise inclusion and how to develop inclusive practices (Göransson & Nilholm, 2014). The research for this thesis is an attempt to operationalise inclusion in mathematics, trying to define inclusion in mathematics and make an attempt to describe how inclusive practices in mathematics can be developed in primary school.

Since it is hard to establish an interpretation of the value words depending on the context, it is difficult to define students' knowledge in mathematics. This can be related to how to define the SEM-student. Magne (2006) draws on the operational education definition of the SEM-student as a low achiever in mathematics. "A SEM-student is an individual at school who has got marks in mathematics below the pass standard according to the valid marking system" (p. 9). Accordingly, depending on the interpretation of the value words in the knowledge requirements, a student could be seen as a SEM-student, or not. Hence SEM is not a fact, but rather an interpretation of what constitutes mathematical knowledge.

Representations in mathematics are used in this thesis, both as internal and external representations. Representations are used from a broad perspective, as both verbal and visual representations within different semiotic systems. As pointed out by Liasdidou (2012), effective teaching requires flexibility regarding the use of methods and material, depending on the student. This can be compared with knowledge of different representations and tasks in mathematics education. Hence, it is important for the teacher to be aware of the students' knowledge, the mathematical content and how to present it.

Mathematics in school is in focus in this study. Whenever mathematics is mentioned in this thesis, it refers to mathematics in primary school.

3. THEORETICAL PERSPECTIVES

In this chapter a presentation of the theoretical perspectives used in this research is made. How these theoretical perspectives are connected is also presented.

Two theoretical perspectives are used in this study, a participatory and an inclusive perspective to capture the research questions of the study: What can inclusion in mathematics be in primary school and what influences the process of inclusion in mathematics? And, what, from an inclusive perspective, appears to be important in the learning and teaching of mathematics? To be able to grasp the process of inclusion an inclusive approach has been used. To be able to identify the process of inclusion and how the participation in the mathematics education looks like, a participatory approach has been used.

3.1 Communities of practice

This investigation of inclusion in mathematics education is grounded in a social theory on learning; learning is considered to be a function of participation (Wenger, 1998). Wengers (1998) social theory on learning is a grand social theory, and thus it is used in many different research areas, such as sociology, education and business, although, initially Wenger developed it for the business area. In the different research areas it is used in different ways. This is not strange, since the theory consists of many detailed parts, which includes both learning and identity and the intimate interplay between. In this study only a part of this social theory is used.

Participation[2] is to be seen as "a process of taking part and also to the

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[2] Participation has been described slightly different prior in the background: Wengers (1998) definition will be used in this thesis.

relations with others that reflect this process" (Wenger, 1998, p. 55). Participation is an active process that involves the whole person and combines the things the person is doing like talking, thinking and feeling (Wenger, 1998). It "goes beyond direct engagement in specific activities with specific people" (Wenger, 1998, p. 57). Participation can involve all different kind of relations, from conflicted relations to competitive or political ones, as well as intimate cooperative relations. Participation in social communities shapes the experience of the members, and the members shape the social community. The process is a continuous process. Participation is broad; it is not restricted to the specific context of the members' engagement and it is a part of who they are and is not something they can turn on and off. Hence, the engagement in the world is social, even when it does not clearly involve direct interaction (Wenger, 1998).

A part of Wengers social theory is about communities of practice. A practice exists because of people's engagement in actions and the negotiation of meaning of those actions between one another. The practices reside in a community of individuals with *mutual engagement,* meaning the members of the community are engaged, but the engagement does not need to be homogeneous, since diversity, disagreements and tensions can create productive relationships. Members of a community of practice are practitioners who develop a *shared repertoire*, such as experiences, tools, artefacts, stories, concepts and so on; this shared repertoire develops over time. The *joint enterprise* is the negotiation that keeps the community of practice together; the members are connected by their negotiation of a joint enterprise, which is linked to a larger social system. The joint enterprise is a process that pushes the community of practice forward, as well as keeps it in check. Hence, it is a collective process of negotiation of the members in the process of pursuing it (Wenger, 1998).

Reification, a notion used by Wenger (1998) with participation shapes the experience of members in concrete ways and can be both a process and its product. There is a fundamental duality of participation and reification where they require and enable each other in interplay. The notion of reification brings how we negotiate meaning among members to the fore. It is the way in which we try to treat an abstraction as an object in order to reach a mutual agreement over something. For example, if we talk about inclusion as a way of dealing with inequalities in society we are using inclusion as reification because we project our meaning of the abstraction of inclusion and perceive it as existing in the world. We use reifications as shortcuts to communication. "Reification occupies much of our collective

energy" (Wenger, 1998, p. 59) through processes of, for example, making, describing and interpreting.

A community can be designed or emergent. If a community is designed the frames comes from the outside, but according to Wenger (1998), it is the members' reaction to the frames that creates the community. In relation to this the sustainability of a community of practice needs to be taken into consideration. In an emergent community, the members of the community form the frames and it is more likely that the needs of the members are met since they are the ones doing the forming.

Communities of practice can be interconnected, according to Wenger (1998), who uses term *constellation* to describe this relationship between communities of practice. This relationship can be that the communities have the same purpose and/or members and/or artefacts etc. Members can also participate in several communities of practice at the same time, which results in connections between the communities of practice. Wenger (1998) describes two types of connections, *boundary objects* and *brokering*. Boundary objects can be artefacts, documents, concepts or other forms of reifications that serve as connections between communities of practice and can be reified. This reification is made to bridge over discontinuous participation. In brokering, a member of multiple communities of practice "transfer[s] elements of on practice into another" (Wenger, 1998, p. 109). The member doing this brokering is called a *broker*; brokering is complex, as it involves making new connections between communities of practice and enabling coordination between them.

3.1.1 Differences in the use of Wengers social theory[3]
Researchers in mathematics education frequently use Wenger's social theory but they have many differences regarding *which* parts of it they use and *how* they use these parts. Hence, Wengers social theory seems to allow for diverse possibilities and use. This is though somewhat problematic since those using it in research may think that they know what using a specific theory implies, but looking behind the surface, researchers using "the same" theory can imply many different things.

One difference in the use of the theory is whether communities of practice

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[3] These differences are also discussed in a forthcoming paper, Roos & Palmér (forthcoming), Communities of Practice, exploring the diverse use of a theory. The paper will be presented at the 9th Congress of European Research in Mathematics Education, February 2015.

are considered as pre-existing or if they are designed within the study. Another difference is that researchers have different focuses, either the individual or groups. The third difference is the use of concepts.

Designed or pre-existing communities of practice
Examples of studies in which communities of practice are designed by the researcher/researchers are those by Bohl and Van Zoest (2003), Cuddapah and Clayton (2011), Goos and Bennison (2008) and Hodges and Cady (2013).

In the study by Goos and Bennison (2008), a web-based community of practice was designed within teacher education. After graduation the interaction in the community of practice continued through the web-based tool developing an "online community" (p. 41). In their article, Goos and Bennison discuss the issue with emergent versus designed communities of practice. Even if Goos and Bennison in their study designed the external frames for the community of practice, their interest was to investigate whether the web-based community would develop into a community of practice. To give the group the best possibilities of developing into a community of practice, the researchers imposed minimal structure on how the members were to communicate with the web-based tool. As such they designed a community of practice but it was its emergence that they investigated.

Hodges and Cady (2013) sought to expand the work of Goos and Bennison (2008) by investigating the development of communities of practice within a professional mathematics teacher's development initiative. In this study, a web-based tool was used to "foster the development of communities of practice" (p.302). In their study, Hodges and Cady designed a virtual space to follow the emergence of communities of practice. However, in the article they do not, like Goos and Bennison (2008), highlight the issue of an emergent or a designed community even though the emergence of potential communities of practice was in focus.

Cuddapah and Clayton (2011) designed a community of practice by arranging physical sessions with a group of novice teachers. The group of their interest was one of several groups of novice teachers that meet every second week within a university-sponsored project. The novice teachers meet 15 times during the study and every session had a theme and the sessions were planned and led by experienced educators. Cuddapah and Clayton write that the group of novice teachers "itself was a community" (p.69) and they used Wenger's theories to analyse the development of the

group and its function as a resource for new teacher support. In their analysis they present how the "community was observed throughout and between the data" (p. 72). As such, the group of novice teachers being a community of practice was both a precondition and a result of their analysis. Hence, these communities were not designed they were pre-existing.

Examples of studies in which communities of practice are treated as pre-existing and developing without the influence of the researcher are some by Corbin, McNamara and Williams (2003), Cwikla (2007), Graven (2004), Pratt and Back (2009), Brown (2009), Palmér (2013) and Siemon (2009). Some studies explain that pre-existing communities of practice were considered developed before a study begins, without the influence of the researcher. In other studies communities of practice were identified in the research process based on concepts from Wenger's theory; other studies do not explained how they are identified as communities of practice.

Bohl and Van Zoest (2003); Graven (2004); Corbin et al. (2003); Cwikla (2007), Brown (2009) as well as Pratt and Back (2009) are examples of studies where communities of practice are considered as pre-existing, in the start of the study, where the researchers do not explain how the communities have been identified as such.

In an article, Bohl and Van Zoest (2003) analysed how the mathematics teaching of novice teachers are influenced by different communities of practices they participate in. In the article, they use an empirical example of one novice teacher and in relation to this example discussed differences in the novice teachers' role in different communities of practice. They do not present how they identified the communities of practice as pre-existing nor how they identified the novice teacher being a member in them.

Palmér (2013) also investigated novice mathematics teachers, analysing identity and identity development with the help of communities of practice. Palmér (2013) investigated professional identity development from the perspective of the teachers themselves. In this study, communities of practice were seen as pre-existing and were identified as such by Wenger's notions, when Palmér identified the different communities of practice the novice teachers participate in.

In an investigation of teacher learning in a mathematics in-service program Graven (2004) considered the program to be a community of practice but did not explain how it has been identified as such. This is also the case in

the study of Corbin et al. (2003), in which the researchers investigated numeracy coordinators in an implementation of a national numeracy strategy. They used communities of practice as a tool to describe the participation of the coordinators in different communities, but did not describe how they define the communities.

Brown (2009) used communities of practice as described by Lave and Wenger (1991) to investigate participation and development of students' agency in mathematics classroom by teaching for social justice. In this research a community of classroom practice is seen as pre-existing but why is not explained.

Pratt and Back (2009), who investigated participation on interactive discussion boards designed for mathematics students, simply stated that "two idealised communities of practice" (p.119) were adopted as a lens to be able to understand the discussion boards. How these communities were created and why they can be seen as such is not explained. They even described the communities of practice as "hypothetical communities" (p.128). Cwikla (2007) used the concept of communities of practice in her study of the evolution of a middle school mathematics faculty to identify bounder encounters but does not present any definition of communities of practice, or which different communities of practice being identified.

Siemon (2009) is an example of a study in which communities of practice were considered as pre-existing, at the beginning of the study, but where the researcher explains how the communities of practice was identified as such. Siemon (2009) investigates indigenous students' numeracy knowledge improvement by working on numeracy key issues in first language. Three pre-existing communities of practice are described and it is explained, by the use of Wenger´s concepts, why they are considered pre-existing. The study investigated the intersection between the acknowledged pre-existing communities of practice but does not describe their members in detail, only as, for example, as "members of the local Indigenous community" (p. 225) or "all those that by virtue of their responsibilities are concerned in some way with school mathematics" (p. 225). The intersection between the communities of practice is not highlighted, though the author states that the edges of the communities of practice took time to emerge.

In this study the communities of practice are considered as pre-existing and identified as such. The communities of practice have emerged in the data in the sense that keywords indicated on the existence of a community and I

was able to identify them as such though these keywords.

Focus of the studies
Wenger's theory makes it possible to foreground groups (communities of practice) or individuals (learning and/or identity) or both. Since Wenger's theory is both broad and detailed, it is not surprising that groups (communities of practice) and/or individuals are being foregrounded in the studies.

In the studies by Cwikla (2007), Cuddapah and Clayton (2011), Goos and Bennison (2008), Hodges and Cady (2013) and Siemon (2009) groups of teachers are in the foreground with the individuals in the background or not mentioned as individuals. Bohl and Van Zoest (2003), Corbin et al. (2003), Graven (2004), Brown (2009) Palmér (2013) as well as Pratt and Back (2009) foreground the individuals in trying to understand how the different communities of practice they participate in influence them.

In this study, the focus is on cases and how those participate regarding inclusion in mathematics in the different communities of practice. Hence, both individual (when the case regards one person) and groups (when the case regards several persons) are in the foreground. When looking at inclusion in mathematics, it will be looked upon from the cases and the communities of practices. Hence, it will be looked upon both from an individual and a community of practice perspective.

The use of the concepts
This study only use a part of Wenger's theory. In order to identify different communities of practice, the concepts of *mutual engagement, joint enterprise and shared repertoire* are used. These "three dimensions" (p.72) are the source of a community of practice, mutual engagement, joint enterprise and shared repertoire, are used in other studies to identify both designed (Goos & Bennison, 2003; Hodges & Cady, 2013) and pre-existing communities of practice (Palmér, 2013; Siemon, 2009). This study also uses the notion of *reification* to discover the participation in the communities of practice. This notion is also used by Palmér (2013) to describe negotiation. Wenger's concept *negotiation of shared meaning* is also used as a tool describing the interplay in and between different communities of practice (Simeon, 2009; Palmér, 2013). *Boundary objects* is a notion also used when referring to items used to negotiate shared meaning (Simeon, 2009: Palmér, 2013; Cwikla, 2007).

Other concepts used in research are the concepts *practice – meaning –*

identity – community. They are used to describe and explain teacher learning (Graven, 2004; Palmér, 2013, Cuddapah and Clayton, 2011). These four concepts are, according to Wenger, "interconnected and mutually defining" (p.5). Although, Graven (2004) wanted to add *confidence* as a supplement to *practice, meaning, identity, community.*

The concepts *co-participation* and *participation* of Lave and Wenger (1991) are mentioned in research (Graven, 2004). The concepts *legitimate peripheral participation/participant* by Lave and Wenger (1991) (Cuddapah and Clayton, 2011; Pratt and Back, 2009) are used to describe a person's participation and change of participation in different communities of practice. Wenger's *modes of belonging* are also used*: engagement*, *alignment* and *imagination* (Pratt and Back, 2009; Corbin et al., 2003)

The concept *broker* is also used in research (Palmér, 2013; Cwikla, 2007) and The verb, *brokering* is also used when describing what the broker does (Corbin et al., 2003).

While many of Wenger's concepts are used in studies, seldom more than three or four concepts are used in the same study. Since the theory is broad and yet detailed, it is not strange that researchers focus on, and use smaller parts.

3.1.2 Use of concepts in this study

In order to capture the teachers' participation I will follow Graven and Lermans (2003) interpretation of Wenger (1998) in regard to the unit of analysis. The reason for this choice is that they argue that the primary unit of analysis in Wenger´s theory is communities of practice, but for teacher learning their interpretation permits the primary unit of analysis to be "the teacher-in-the-learning-community-in-the teacher" (p. 192). This unit of analysis is coherent with Lermans (2000) unit of analysis from the social perspective, "person-in-practice-in-person" (p. 38), where the person has an orientation towards the practice and the practice is in the person. It is also consistent with Wenger's thoughts about this as being two sides of the same coin. This provides access to the individual in the community of practice as well as the community of practice. In addition to the notion of communities of practice, I will also use the terms *reification, constellation, boundary objects* and *brokering*.

3.2 Spatial, social and didactical inclusion

As prior mentioned, the terms *spatial, social* and *didactical* inclusion is used by Asp-Onsjö (2006). Spatial inclusion basically refers to how much time a student is spending in the same room as his or her classmates. The social dimension of inclusion concerns the way in which students are participating in the social, interactive play with the others. Didactical inclusion refers to the ways in which student's participation relates to a teacher's teaching approach and the way in which the students engage with the teaching material, the explanations and the content that the teachers may supply for supporting the student's learning. In this study the content of learning was number sense, because it was covered in the teaching observed.

Although the terms spatial, social and didactical inclusion are used from a student's perspective (Asp-Onsjö, 2006), in this thesis I will use them from a teacher perspective. Hence, I will note how teachers, principal and documents talk and act regarding these three notions. These three analytical categories are used as an overall frame in developing an explanatory framework. This framework seeks to increase our understanding of how students in SEM participate, develop their way of participating or become restricted from participating in the school mathematical practice.

3.3 Connection of theories

When connecting theories it is crucial to know how the connection is made and what it is in the theories that make them work together. According to Wedege (2010) the connection can take place at different levels. In this project, a connection is made between communities of practice (Wenger, 1998) and the theoretical framework of inclusion (Asp-Onsjö, 2006). The connection is at the level of principles, since both theories look at learning as a social phenomenon. Hence, the theories have compatible cores in their view of learning. Although the complexity and size of the theoretical frameworks vary widely, they address different aspects of the research question. Communities of practice is an overarching theory of learning and a connection is made to the three modes of inclusion that Asp-Onsjö (2006) presents.

Different strategies can be used to connect theories, according to Prediger,

Bikner-Ahsbahs, and Arzarello (2008). These strategies include having an understanding of different theories, to combine, coordinate or to integrate them. I coordinate communities of practice with inclusion, since they have consistent assumptions that make it possible. These assumptions are located in their social approach. The coordination creates a conceptual framework with well-fitting elements that help in identifying the teachers' participation and the communities they have access to regarding learning in mathematics. Eisenhart (1991) recognise three types of frameworks (theoretical, practical and conceptual). Since this framework is built from different sources, it is a conceptual framework.

The three modes of inclusion (Asp-Onsjö, 2006), which are deeply interconnected and constantly interacting with each other, put words to and allow for the development of a fine-grained model regarding inclusion in mathematics. This model may inform theory and therefore it has the potential to make new contributions to the field through its explanatory power. The model may also be able to contribute to the solution of the overall research questions: What can inclusion in mathematics be in primary school and what influences the process of inclusion in mathematics? And, what appears to be important in the learning and teaching of mathematics from an inclusive perspective? This is done by identification of factors regarding inclusion in mathematics and their connection in the communities.

In this study the aim is to empirically investigate how all students can be included in the mathematics education in primary school by using necessary theoretical concepts. What inclusion in mathematics can be is an empirical question. In order to identify what inclusion in mathematics can be from the collected data, the conceptual framework will be used as an overall frame.

In summary

As mentioned, I will investigate inclusion from the perspective of teachers. In terms of participation this means that I look at how the teacher and the remedial teacher in mathematics work on including students in special education needs in mathematics to the mathematical practice. A part of a social learning theory that focuses on communities of practice (Wenger, 1998) is used. In this theory learning is seen as a process of social participation. One unit of analysis in this theory is identity and another is the community of practice, which is an informal community where people involved in the same social setting form the practice (Wenger, 1998). In this study the communities of practice are considered to be pre-existing and are identified. The focus is on cases and how they participate regarding inclusion in mathematics in the different communities of practice. Only a part of Wengers (1998) theory is used. This can be limiting, but can also be seen as an advantage, since it can allow for putting a focus on, and delving deeply into the data. Communities of practice offer a way to give structure to the data.

I use Asp-Onsjö's view on inclusion as social, spatial and didactical from a teacher perspective. Here the didactical inclusion strongly can connect to a subject, in this case mathematics in primary school.

The participatory perspective adopted in this study means that inclusion does not just mean being in the classroom physically, it means being included in the mathematical practice of the classroom, which can be anywhere, because this form of inclusion has no physical condition.

4. DESIGN AND ANALYSIS OF THE STUDY

In this chapter the design of the study is presented and motivated. First case studies are discussed and descriptions of the cases in the study are presented. Second, ethnography as a guide to understand the process of inclusion in mathematics is presented. Finally the methods used are presented.

Since this is a study of a process, inclusion in mathematics, a hermeneutical approach was used. This approach offers a perspective to make sense of, and interpret context and meaning for what teachers do to be able to come closer to an understanding of what inclusion in mathematics can be. By using an hermeneutical approach, I have considered my own understanding in the research and am looking at both the whole and the parts in a process. Ethnography has been used as a guide to be able to frame the culture of the group that works with the process.

4.1 Case studies

Case studies can be found inside an interpretative hermeneutic paradigm but also a positivist paradigm. "A paradigm is a worldview, a way of thinking about and making sense of the complexities of the real world" (Patton, 2002, p. 69). The positivist knowledge should be objective, generalisable, and replicable. The methods in the positivist paradigm are often described as quantitative (Bassey, 1999). Ethnographic case studies have a clear hermeneutical approach that makes use of qualitative data and often some kind of interpretation is involved. Hermeneutics offers a perspective to make sense of and interpret context and meaning for what teachers say, express and do. These kinds of studies are thus placed inside the interpretation paradigm. The present case study is in the interpretation

paradigm. In this paradigm, reality does not exist regardless of people (as it does in the positivist paradigm), but concepts of reality varies from one individual to another (Bassey, 1999). Here, the researcher is part of the world in which we observe and also a variable in the study.

According to Patton (2002) a case study tries to seize something in detail and of special interest. This something can be a notion, a process or bounded to objects and it can be theoretical or empirical or both (Ragin, 1992). In this study the object of study is a process, the process of inclusion in mathematics. To be able to grasp this, cases are found at the investigated site. These will be described in the sections below. All names used in this study are fictitious (this will be further described in section ethical considerations).

4.1.1 The starting point of cases

Scholars define a case study differently. Ragin (1992) problematises the notion and stresses the need to recognise the different use of the notion and the possible theories embedded in the notion. We also need to reflect on "what is this a case of?" (Ragin, 1995, p. 6) Ragin (1992) presented a way to cross-tabulate two discectomies in order to understand the starting point for the case (Figure 2). The two dichotomies are key in how cases are perceived, "whether they are seen as involving empirical units or theoretical constructs and whether these, in turn, are understood as general or specific" (Ragin, 1992, p.8).

Case conceptions / Understanding of cases	Specific	General
As empirical units	Cases are found	Cases are objects
As theoretical constructs	Cases are made	Cases are conventions

Figure 2. Ragin's cross-tabulation for stating-points of cases.

In the first field of the figure, cases are specific empirical units. They are empirically real and bounded, that is, the case must be identified and established as cases during the research process. In the second field, cases are general specific units, objects. Here the researcher also views the case as empirically and real but with no need to verify them. The cases here are general and conventionalised with case designations based on existing definitions in research literatures. The third field shows cases as specific theoretical constructs. Here cases are made in the sense that specific theoretical constructs fuse in the course of the research and are gradually imposed on empirical evidence. In the fourth field cases are seen as general theoretical constructs. This means that cases coalesce in the course of research but are seen as conventions.

Consequently, to be able to understand the process of inclusion in mathematics a case study was made. The study object is the process of inclusion in mathematics where several different cases emerge. The different cases have different starting points. These case conceptions helped me to focus my cases at different places in the research process and to understand inclusion in mathematics from the inside.

4.1.2 The case of Barbara

As mentioned, the major interest in the initial phase of this research was to investigate ways to get students in SEM included and engaged in the mathematics in school. In order to do that a practical approach was needed. This implies that the investigation had to be in a school context. Consequently, a remedial teacher in mathematics with great experience of teaching mathematics to SEM-students was contacted, a choice made in order to get "a best case scenario". Patton (2002) describes this kind of choice as an information-rich case for study in depth. From such an in depth study one can learn about issues that are central for the purpose of the study, also called purposeful sampling. Hence, this study is primarily a purposeful case sampling (Patton, 2002). These types of case studies are used to understand a process, in this case, inclusion in mathematics, in depth. The information-rich case (Patton, 2002) in this study was a recognised skilled remedial teacher in mathematics. Using Flyvbjerg (2006) one can also say that this kind of selection is information oriented and the case is an extreme one. An extreme case is a case to "obtain information on unusual cases which can be especially problematic or especially good in a more closely defined sense" (Flyvbjerg, 2006, p. 230). In this investigation of inclusion in mathematics the extreme case is used to obtain information. This case is expected to be an especially good case.

In this study, the information-rich case is Barbara, a 61-year old (at the start of the study) remedial teacher in mathematics primarily working with students in SEM. She has a degree as a lower primary teacher and worked as such for 26 years before becoming a special pedagogue, which she has worked as for 6 years (at the start of the study). She has a special interest in SEM.

The site
It is important to be reflexive about the site and why it fits the study (Walford, 2008b). The choice of site in this study a consequence of choosing the case of Barbara. The site was Oakdale Primary School, a large primary school in the south of Sweden. It has three classes in each year, from preschool class (6 years old students) up to year 6 (12-years old students). Over 40 pedagogues work at the school and are divided into several teams, consisting of preschool teachers, leisure time teachers and primary school teachers. The catchment area has both rural and suburban areas. The school has a principal and a vice principal (who was hired in the last year of the study). The school has a student health team (which every student has access to according to the school law, 25 §). The student health team are supposed to work with medical, psychological and special educational issues in a preventive and health promoting way (SFS 2010:800).

4.1.3 The case of mathematics teachers at Oakdale Primary School

Another case was that of four mathematics teachers at Oakdale Primary School, Anna, Ellie, Amanda and Jonna. These teachers were chosen because of their cooperation with Barbara and the fact that they had SEM-students in their classes. Anna is a 38-year old primary teacher teaching 8-year-old students in lower primary school. She has been working as a primary teacher for 1.5 years and her teacher exam is in Swedish and mathematics. Before that Anna worked as a leisure time pedagogue for 14 years. Ellie is 45 years old and has been working as a primary teacher for seven years. She got her current assignment as a primary teacher three years ago. She has been studying Swedish and mathematics in her teacher exam. Prior Ellie worked as a leisure time pedagogue. Amanda is 40 years old and has been working as a primary teacher for 10 years. She has a teacher exam in Swedish, social studies and English. She had 7.5 credits[4] in mathematics education in her teacher exam. Jonna is a 42-year-old primary teacher with

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[4] 7,5 credits corresponds to five weeks full time studies.

an exam in mathematics and science for grade 1 to 7. She has been working as a primary teacher for 19 years, mostly in upper primary school.

4.1.4 The case of the principal at Oakdale Primary School

Another case at the site and of importance to the aim of the study is the principal. Conrad is 42 years old and became the principal of the school at the same time as the study started. He has been a principal at other schools for 8 years. Before that he was a mathematics and science teacher in a lower secondary school[5].

4.1.5 The starting point of the different cases in the study

Returning to Ragin's (1992) cross-tabulation (Figure 2) the starting point of the cases in this study was a specific empirical unit in the case of Barbara and of the principal at Oakdale Primary School. The case of mathematics teachers was a general specific unit.

4.2 Ethnography as a guide

Ethnography has been used as a guide to understand the process of inclusion in mathematics from the perspective of teachers.

When taking an ethnographic approach, the researcher tries to understand a phenomenon through interpersonal methods. The basis of ethnographic research is social interaction (Aspers, 2007). The ethnographic approach also offers in depth study (Hammersley & Atkinson, 2007), which can be used to follow a process in a particular case, such as inclusion in mathematics. An ethnographic study usually investigates people's actions and accounts in an everyday context. The data are gathered from several sources and the collection of data is relatively unstructured and does not follow a predetermined research plan. The plan is in flux. Categorisation is used for interpretation, which is generated through data analysis (Hammersley & Atkinson, 2007). Sarangi and Roberts (1999) emphasises that institutional workplaces are social and to be able to understand them we need to use "thick descriptions" as a scope to reach from the level of fine-grained analysis to a broader ethnographic description.

However, before doing the fieldwork it is important to think, write and read about the issue you want to investigate (Delamont, 2008). It is also

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[5] Lower secondary school in Sweden is education for students 12–16 years old, year 7 to 9.

important to consider the role as the researcher in the investigation. Lave and Kvale (1995) highlights that researchers are highly influenced by their experience and interests. In this case, I had experience of teaching SEM-students in primary school, which resulted in an urge to investigate SEM issues further. This was made, not only empirically not only in my own teaching practice, but also as a teacher educator in the special teacher program where SEM issues are highlighted. This lead to a great interest in finding ways to get students in SEM included and engaged in the mathematics in school. From a hermeneutic point of view, this makes me a variable in this study.

The research questions emerged from the field because the information-rich case in the study (Barbara) was eager to focus on inclusion in mathematics. It was then important to discuss from the beginning how this research should be done, as Walford (2008b) highlights. This was done through a discussion of how the researcher (I) and the informants were going to collaborate. In the beginning of the study this was in the background but later on, when the teachers wanted me to comment on their teaching after I had observed their lessons, this had to be made clearer. Also it had to be discussed with the remedial teacher, who wanted me to act like a co-teacher; in the beginning it was hard to find my place as a participant observer, not to be too involved in the teaching, but still be a research partner. As a researcher I needed to have deep discussions with the remedial teacher in order to understand the process of inclusion. I needed to place myself, not only beside the remedial teacher, but also in deep interaction. This cooperation with the remedial teacher was worked out over time and in the teaching I was an observing participant and in the discussions I was a participating observer (see further in chapter observations).

It is important to be reflexive in the fieldwork, to see the field in flux (Burawoy, 2003) to find the perspective of the students and teachers. In this study, it was hard to stay alert and be reflexive all the time perhaps because of overfamiliarity (Delamont, 2008). The reflexivity is also visible in the way the researcher thinks about the personal, biographical, and financial and academic reasons for the choices and records these reflections systematically (Delamont, 2008). In this study I had to be reflexive about the academic reasons for making and presenting this study. The academic reason for this study was that there is a gap in the knowledge of inclusion in mathematics.

Ethnographic studies are never finished, the researcher just leaves them and the researcher defines the length of the study (Jeffrey & Troman, 2004). In this particular study I chose to leave the field after two years. This choice is made through the analysis of the constructed data and an awareness that saturation would be reached. During those two years I used a time mode which Jeffrey and Troman (2004) calls "a selective intermittent time mode". I spent a rather long time doing research, but with a flexible approach to the visits at the site. Different foci emerged. In the beginning, a wider scope (Hammersley & Atkinson, 2007) was used to capture different aspects of the research question. During the study a few foci were stable and others either disappeared or were added.

Using ethnography as a guide in this project give me as a researcher tools to be able to explain the data construction and important issues in the data analysis. Ethnography also enabled me to highlight ethical issues, since the project has both external and internal ethical considerations in focus (see 4.6 ethical considerations).

4.3 Construction of data

The data in this research was not given nor discovered; it was created by the questions and answers in the interviews and observations. The data construction was been made during a two-year (autumn 2011 - summer 2013) period. Mathematics lessons have been observed with a focus on the SEM-students. The remedial teacher in mathematics was followed as much as possible, both when she had mathematics lessons with SEM-students in a small group and when she was in the classroom with the whole class. Interviews were conducted with the remedial teacher, the principal and mathematics teachers. In total, 39 interviews were collected and 31 observations made.

Walford (2008a) uses the term construction for data gathering in ethnographical research. In this constructing of data it is essential to recognise the presence of subjectivity (Walford, 2008a) In my case, of how my data is constructed and why I needed these perspectives. The researcher is seen as an instrument and "must aim to have an open mind about what is going on here and what might be the best ways to talk or write about whatever is being studied" (Walford, 2008a p. 10). Hence, the empirical data was constructed in the interaction between the researcher and the respondents. The content knowledge of the researcher influences the quality of the data since the knowledge influences the researchers'

opportunities to ask further questions. In this case I had to be aware of what type of questions were important to ask, what lessons were essential to attend to and who was important to interview. I needed to be aware of what the discussions with the main informant and document such as action plans might bring into the research. I was aware of that I make assumptions implicit in the research. I as a researcher acknowledged my part of the research act so as to be able to get an inside perspective.

In needing to place myself, not only beside the remedial teacher, but also in deep interaction, I was struggling with the problematic issue Sara Delamont (2008) highlights, *how to observe*. I also struggled with the other issues she highlights; what to observe, what to write down, where to record observations and what to do with the field notes. It came down to a use of a notepad and a recording device to capture as much as possible in a situation. In the classroom situation, the video did not capture the whole situation. I tried to use one and an iPad on two occasions in the classroom, but it did not work out. So I wrote field notes when observing in the classroom and recorded audio in the small group situations. When doing the interviews I used a recording device.

Hence, in this study two types of data gathering are used: *interviews and observations*. These two strategies of data collection overlap and there is a dynamic process where they cover different parts and have a different scope. According to Patton (2002), interviewing and observing cannot be separated when using participant observations.

Interviews
Doing interviews allowed me to take an inside perspective. Interviews in ethnographic research range from strictly arranged meetings in bounded settings (formal interviews) to informal conversations arising spontaneously (informal interviews). Hence sometimes it is hard to distinguish what is an informal interview and what is a participant observation. Whatever the degree of formality, all interviews are seen as a social activity where the researcher is a participant (Hammersley & Atkinson, 2007). This participation must be taken into consideration; I made an impact since I had an agenda. In the conversations I sought to understand the perspective of the teachers and I see the discussions with the teachers as informal interviews. I did not use a formal interview guide but I did not come with any suggestions or input.

Both individual and group interviews were done for the study. The individual interviews were with the mathematics teachers and the principal and the remedial teacher. The individual interviews with the mathematics

teachers (interview guide, see appendix 1) as the principal (interview guide, see appendix 2) were all formal. In the formal interviews I stuck to questions and follow up questions, but I do not make any input comments. The interviews with the remedial teacher in mathematics were both formal and informal. These were predominantly informal interviews linked to the lessons hat had just been taught, where I as a researcher asked spontaneous questions and discussed the lessons with the remedial teacher. The informal interviews were conducted before and after the lessons. On three occasions, there were group interviews with mathematics teachers at the school. All of the mathematics teachers were invited to participate and the first time 8 teachers were present, the second time 4 and a student teacher were present and the third time 9 and a student teacher were present. The first group interview was based on questions arising from a lecture on SEM and the second group interview was based on a chapter from the book, *The Elephant in the Classroom* (Boaler, 2011, p. 97-109) on ability grouping. The teachers read the chapter before the interview and we discussed what they had read. The third group interview was based on questions arising from a lecture on problem solving as a tool to reach all students in the classroom, a topic requested by the mathematics teachers' at Oakdale Primary School. The individual interviews with the mathematics teachers have all been formal interviews (interview guide, see appendix 1) as well as the interview with the principal (interview guide, see appendix 2). The interviews with the remedial teacher in mathematics have been both formal and informal. There have been predominantly informal interviews linked to the lessons that just had been taught, where I as a researcher has asked spontaneous questions and discussed the lesson together with the remedial teacher. The informal interviews are before and after the lessons.

Figure 3 provides an overview of the number of interviews and timing, both formal and informal made in the study. The upper part of the figure shows the number and approximately when the interviews were done with the remedial teacher, Barbara. The lower part of the figure shows the number and approximately when the interviews were made with the teachers. The dark square under the timeline shows the interview made with the principal at Oakdale Primary School in the beginning of the semester 2012.

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Interviews and discussions with Barbara

Interviews with Teachers and one with the principal

Year 1 autumn 2011- summer 2012   **Year 2 autumn 2012 – summer 2013**
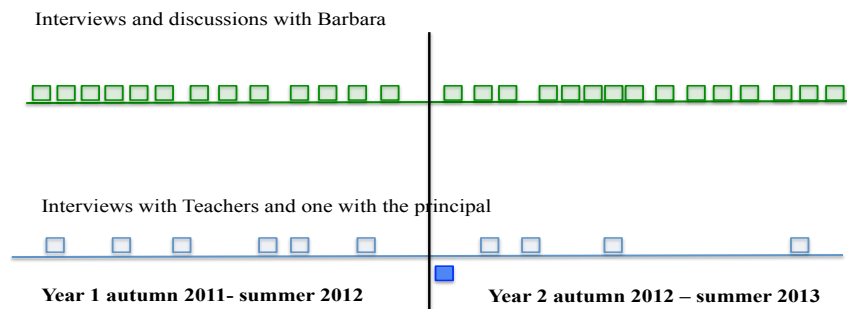
Figure 3. Interviews in the study

Observations

In this study two kinds of observations were made, observing participation and participant observations. In observing participation, the researcher is merely an onlooker. A participant observer interacts with the respondents and is involved to a greater extent in the research. This extent can vary over time; sometimes the researcher is a full participant and sometimes much more of an onlooker as Patton (2002) describes. Taking the role of a participant observer by sharing the activities at the site allows the researcher to develop an insider perspective of what is happening (Patton, 2002).

This type of observation allowed me "to get infinitely closer to the lived experiences of the participants" (Prus, 1996, p. 19). Participant observation implies a more active and interactive role as a researcher in the setting (Prus, 1996). It is important to be reflexive about this role. In the participant observations I did as a researcher, I had a prominent role, since I discussed issues of inclusion in mathematics with Barbara to be able to grasp the process of inclusion from a teacher perspective.

This research project sought to get an insider perspective, to get the teachers' perspective of inclusion in mathematics. I had a rather reserved

role in the lessons I observed, but participated fully in the discussions with Barbara. This implies that I not only asked questions, I raised issues and answered Barbara's questions. This was done to understand fully the complexities of inclusion in mathematics and to generate opportunities to gain insight in the practice at the site.

The remedial teacher in mathematics was followed as much as possible, both when she had mathematics lessons with SEM-students in a small group and when she has been in the classroom with the whole class (See figure 4).

Year 1 autumn 2011- summer 2012   Year 2 autumn 2012 – summer 2013
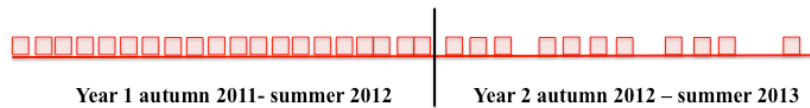
Figure 4. Observations of mathematics lessons

In ethnographic research there is a quest for "intimate familiarity", which means that the researcher uses observations, participant observations and interviews in order to reach and be able to interpret the life-worlds that are studied (Prus, 1996), as I did in this study. Being a participant observer meant I could interact with the teachers to find out what inclusion in mathematics can be and how this phenomenon can be developed. This alternating between being an observer and a participant observer is happening all through the data collection.

As Figure 5 show, interaction occurred between the two forms of observation. After the observing participation the teacher and I reflected over the lesson together and interacted through participation and interaction in form of questions, reflections and answers to the teacher's questions. Here I was a participant observer. Then the teacher has another lesson, which is observed. Then I observed another lesson and had another interaction and so on. The arrows are describing the process, the shift of perspectives in the process.
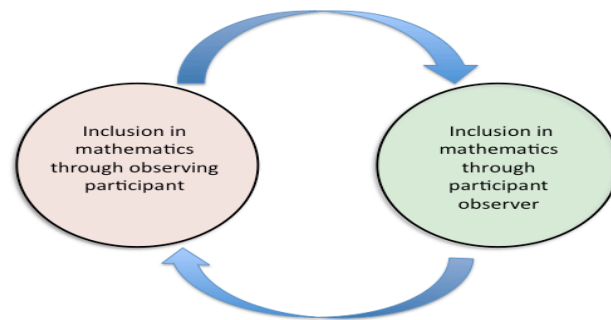
Figure 5. Interaction between the different forms of observation investigation

Other empirical data sources

At the site, there were other sources of data to be taken into consideration in the analysis. As Hammersley and Atkinson (2007) point out, documentary sources and material artefacts can easily be overlooked. These are documentary constructs of reality that are features of the social world the researcher studies. In this study, action plans for the individual SEM-students and a reflection from the remedial teacher written at the end of the project are documentary sources. The school's "local plan for systematic quality" as well as the plan "Results in Focus" and Oakdale Primary School´s student health teams "cycle" were other data sources (See figure 6).

MATERIAL	QUANTITY
Action plans in mathematics	Two
A written reflection from Barbara in the end of the project	One
Oakdale Primary School´s Local Plan for Systematic Quality	One
Oakdale Primary School´s students health teams cycle	One
Results in Focus (at Oakdale Primary School)	One

Figure 6. Overview of documentary sources

4.4 Analysis of data

Lave and Kvale (1995) describes how they work with the constructed data; "you sift, sort, organize develop lines of argument, make descriptions, make connections between events, conversations, insights and so on. You begin to shape a story of what it is" (p. 224). This can be considered as an analysis of the data and is also how the beginning of the analysis in this research was done. Thus analysis is an own part in research; in ethnography the production of data and the analysis are closely related in a process (Delamont, 2008). There might, for example, be a gap in the analysis and the coding of the data, and more data needs to be produced in order to fill the gap. In the present study, analysis was done during the data construction period, which made it easier to find the gaps and produce more relevant data.

Aspers (2007) present several different kinds of analysis techniques used in ethnographic studies, for example, comparative analysis, narrative analysis and static-dynamic analysis. Static-dynamic is a technique in which the researcher first codes the data using a code-scheme developed from the empirical material and theory. Saturation is reached when no further

material is considered likely to change the coding (Aspers, 2007). In this study static-dynamic analysis was used to be able to find key words, make codes and create categories.

The code-scheme was developed and used in three steps. In the first step the empirical data was analysed with static-dynamic analysis, and several communities of practice were identified on the basis of keywords which pointed towards the same practice regarding mathematics at Oakdale Primary School. These code words, for example, "we", "us", "together", indicated mutual engagement, shared repertoire or joint enterprise, which is described by Wenger (1998) as three things that create a community of practice. When this occurred, a community of mathematical practice was considered to exist. This was done in iteration and during the data collection and several communities of practice was considered to exist. In the second step, another analysis of the empirical data was done by applying the theoretical aspects of inclusion of Asp-Onsjö (2006) to the data. The three aspects of inclusion, spatial, didactical and social, were used as lens and several codes regarding inclusion in mathematics were found in the data. These codes were grouped into major codes. As a final step, the major codes regarding inclusion in mathematics were categorised into the constructed communities of mathematical practice at Oakdale Primary School by identifying when and in what community of practice the codes emerged.

Figure 7 illustrates the code-scheme constructed by the two first steps in the analysis. The content was added in the third step.

INCLUSION COMMUNITY OF PRACTICE	SPATIAL	DIDACTICAL	SOCIAL
COP[6] 1			
COP 2			
COP 3			
COP 4			

Figure 7. Code scheme used in the analysis.

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[6] COP stands for community of practice.

The results and analysis in this thesis were based on the identified cases and their participation in the different communities of practice identified at the site. The same code and keywords have been traced in the data, which was gathered over two years. Because of the length of time, the meaning of the code and keywords might have changed, though the code remained the same. The quotations were selected based on the coding and key words of each case and in each community. Numbers, starting with number 1 in each case, will be used to present the quotations in a chronological order.

4.4.1 Generalisation

The results of this qualitative research cannot be generalised in the same sense as results in quantitative research. However, analytical explanations can be made, which subsequently can be challenged or refined, and thus they can be made more general (Hemmi, 2006). This type of generalisation is called fuzzy generalisation and can be defined as a kind of prediction that says that specific things *may* happen, a form of qualified generalisations (Bassey, 1999). In this way, hermeneutic research can be scientific, and iterative processes can refine and develop results, perhaps even make the results generic. The two year long data collection made it possible to use iterative processes in the analysis, which can provide with refined and developed results. These results can maybe be made generic.

There is an on-going debate about generalisability in ethnographic studies (Walford, 2008b). To deal with the dilemma of generalisation, Walford (2008b) highlights the notion of transferability. "It is argued that if the author gives full and detailed descriptions of the particular context studied, readers can make informed decisions about the applicability of the findings to their own or other situations" (p. 17). It is a process in which the researcher provides with thick descriptions to enable the reader to make transfers (Lincoln & Guba, 1985). In this research project it has been important to provide detailed descriptions about the context to be as transparent as possible for the reader.

4.5 The interaction of theories and empirical data

To clarify the relationships between the theoretical framework and the empirical influence and how the construction of data is influenced by the theoretical concepts (Aspers, 2007), I have represented them graphically (Figure 8). In this figure, I have also presented the concepts that were

sensitised in the study – spatial, didactical and social inclusion in mathematics. These concepts give directions where to look. Blumer (1954) describes sensitising concepts as concepts that suggest direction on which way to look. "These concepts are not definitive; they do not have clear definition[s] in terms of attributes or fixed benchmarks" (Blumer, 1954, p. 7). When researchers making use of sensitising concepts, Starrin, Larsson, Dahlgren and Styrborn (1991) sees an opportunity for them "to bring new dimensions to observations and the observations modifies the concepts' theoretical content" (p.20, own translation). In this study the sensitising of concepts is used as a tool for investigating inclusion in mathematics and to find paths in the data that might lead to other sensitised concepts or concepts that can be more definitive.

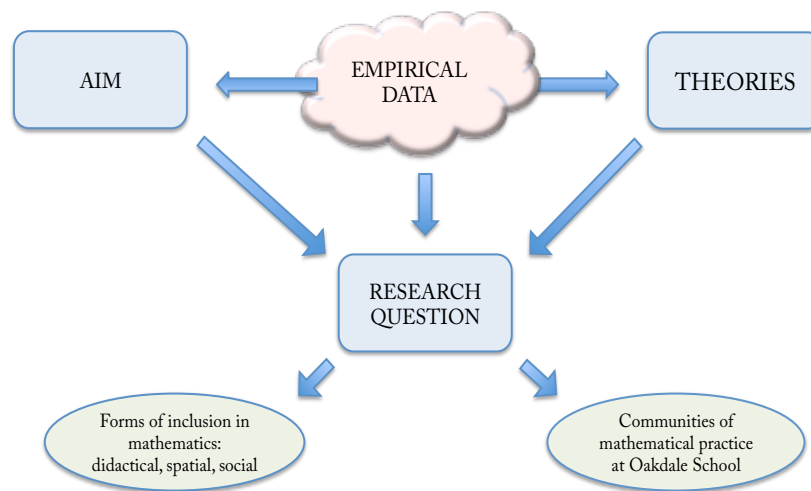*Figure 8. Graphical presentation of the link between theories and empirical data.*

As figure 8 shows, there was an interplay between aim, empirical data and theories to create the research questions. The research questions in turn helped to find the first set results, the communities of mathematical practice at Oakdale Primary School and the different kinds of inclusion in mathematics that emerged.

4.6 Ethical considerations

When conducting educational research it is important to get informed consent from all participants. This means that all participants are to be told, "exactly what the research seeks to investigate" (Walford, 2008b p. 30). This is not unproblematic, since the nature of ethnography is to investigate something over a period of time and this means that the focus of the research often switches and unexpected discoveries are often made (Walford, 2008b). In this investigation the participants were informed and gave their consent about mathematics education, and even though the focus in this study has changed direction several times, mathematics education is still in focus. All students involved were informed and their guardians asked for permission in a letter, which was handed out by the teacher in each class (See appendix 3). The teachers involved in this project have also left a written consent (see appendix 4).

Another aspect of ethics in an investigation is the question of anonymity[7] (Walford, 2005). Walford (2005) challenges the hegemonic norm about anonymity in writing, saying "it is often actually impossible to offer confidentiality and anonymity" (p. 84). It is likely that informants will be more willing to be involved in a research project if they knew that their names would not be mentioned (Walford, 2005). Floyd and Arthur (2012) highlighting the problematic issue of trying to maintain the anonymity of the site. Given the large numbers informed it was hard to keep the anonymity of the present project. Even though it was impossible to offer absolute anonymity, I have chosen to use pseudonyms for the site and to the people involved, not because of the hegemonic norm about anonymity, but because this study involves, student as well as teachers. In addition, these students are in special educational needs in mathematics and I have to do all I can to try to assure them their anonymity.

Yet another aspect of ethics in a research project with an ethnographic approach are the moral dilemmas confronting the researcher. Floyd and Arthur (2012) write of external and internal ethical engagement for insider researchers. Here, external ethical considerations means identifiable ethical issues such as consent from the informants or their guardians. The Swedish Research Council (The Swedish Research Council, 2008) has four main requirements regarding research ethics for an individual's protection: information, approval, confidentiality and appliance. These four requirements are also examples of external ethical considerations. Internal

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[7] Anonymity is the term used by Walford (2005) even though it implies that no one knows, not even I as a researcher, which I obviously do.

ethical considerations deal with ethical issues that are hidden "in the field". These are moral dilemmas facing the researcher below the surface and linked to personal and professional relationships with the informants (Floyd & Arthur, 2012). In this research several of these internal ethical considerations had to be taken into account.

To meet the need for information approval, in the initial phase of this study there were an information meeting with the retiring principal and the new principal. Both the case of Barbara and the other teachers at the site were informed of the research project face to face. When they had given their consent there was information at a staff meeting in order to inform all personal at the school. These information meetings and consents fall under the external ethical considerations.

In regard to the internal ethical considerations in this study, one has to take the relations with the informants into account. It is important to recognise the deep interaction with the informant Barbara affected the research. It is also important to recognise that this long-term research made us friends. When I was observing in the classrooms where the other mathematics teachers were teaching, the teachers wanted me to comment on their teaching afterwards. There was a power relation where they thought of me as a mathematics educator from the university. When they asked questions regarding their teaching I answered that this was research and I was not evaluating their teaching, but looking at issues of inclusion in the teaching.

From this internal ethical perspective, it is also important to consider how I present the informants in the text and was vigilant about the formulations; the informants are our companions in the research and deserve a gentle treatment (Walford, 2005). I took this into consideration when writing up the results. Hence, in this thesis I have chosen to interpret the results in a positive way, meaning I do not focus on failures and deficiencies, but on strengths and development opportunities.

4.7 Summary design and analysis

To be able to grasp the process of inclusion in mathematics a hermeneutical approach was used. It offers a perspective for making sense of and interpreting context and meaning for what teachers do. Based on the aim to understand the process, ethnography was used as a guide to be able to frame the culture of the group that works with the process. To be able to describe the interaction with the informants in this study, the

positions of observing participant and participating observer has been used. Interaction meant that the researcher socially interacted with the teachers in discussions, answering questions and making suggestions. The researcher becomes a part of the research process when becoming a discussion partner. These discussions generated new questions and new ideas in terms of inclusion in mathematics.

This case study has several different cases. Using Ragin's (1992) way of cross tabulating the discectomies of empirical units or theoretical constructs (Figure 2), one can see that the cases have different starting points. Inclusion in mathematics is seen as the study object. Within this study object, three different cases were identified. The principal of Oakdale Primary School and the remedial teacher in mathematics, Barbara, are two of them. These two have their starting points as specific empirical units. Barbara was chosen because of her broad experience teaching mathematics to SEM-students and her recognised skills. The site (in this case Oakdale Primary School) is a consequence of this choice of case. The third case found in this study, that of mathematics teachers had its starting point as a general empirical unit. The empirical data collected at the site were interviews, observations and documentary sources such as action plans and the local work plan of the school. All these different data sources were collected in a selected intermittent way. The analysis was partly been done during the data collection, which enabled the research to move from a wide scope to a narrower focus, by sensitising concepts. This way of looking at the data is a part of the hermeneutics – looking at the whole and the parts. The analysis has been done using a static-dynamic analysis to find key words, make codes and create categories. In this coding and categorisation the theoretical framework has been applied.

5. RESULTS AND ANALYSIS

The results of this study into inclusion in mathematics are presented into four parts. The first part presents the communities of practice identified at the site and what forms and unites them regarding teaching in mathematics. These communities form the basis for the analysis of the cases. The second part presents the cases and their participation in the communities of practice. The third part compares the communities of practice regarding visible forms of inclusion in mathematics. The last part sums up inclusion in mathematics at Oakdale Primary School: the important parts of the process and what influences the process.

5.1 Communities of practice at Oakdale Primary School

In analysing the data, four communities of mathematical practice were identified by using the framework communities of practice by Wenger (1998). These communities[8] are presented in Roos (2013, 2014).

5.1.1 Community of mathematics classrooms

The *community of mathematics classrooms* was created in mathematics classrooms and thus consist of several different visible communities of practice, one in Anna's classroom, and another in Jonna's, another in Amanda's and another in Ellie's. These are the visible communities of mathematics classrooms in the data, since these are the teachers involved in the study. Although there were several small communities of practice, the talk in these communities of practice can be interpreted within one larger community consisting of all the different communities of practice. Ellie, Anna, Jonna and Amanda are members of the community of mathematics

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[8] Community of mathematics classrooms, Community of special education needs in mathematics, Community of mathematics at Oakdale Primary School, Community of student health.

classrooms. There were more mathematics teachers and classrooms at the school, but those are not part of this study.

The mutual engagement of the teachers in these communities of practice was the mathematics learning for all students, that they worked according to the curriculum and all students reach the accepted level of knowledge. Another mutual engagement was the national test in mathematics. Another was talking about cooperation between the different communities of mathematics classrooms and the cooperation with Barbara. The joint enterprise is how to teach mathematics in primary school. The shared repertoire is the mathematics teaching in the classroom, the curriculum and the use of different teaching materials and the mathematics textbook. Even if there are several mathematics classrooms, the actual work with the students regarding mathematics and how to reach them in the classroom(s) is a shared repertoire. Ellie pointed out that it is important to "be involved" in the classroom activities, and Anna argued that it is "valuable that the [SEM] students are present when the teacher presents the content". Jonna pointed out that it is "good for the self-esteem [of the SEM-student] to be in the classroom" and that you as a teacher "need to think about what the students can work with at their own level [in the classroom]".

Barbara, a peripheral member in this community of practice in her role as a remedial teacher, wishes to have more influence; she wanted to be "open about our roles in the class [room] "and "that we discuss together, what I can do". The principal is a peripheral member with an external perspective. He emphasises the importance of all students being in the mathematics classroom. Sometimes there was an extra teacher in the classrooms and they were peripheral members. The students in the classrooms are also members since they are involved in the mathematics education.

5.1.2 Community of special education needs in mathematics

The *community of special education needs in mathematics* is identified by the fact that SEM exists and is dealt with at the school. Barbara is a core member, since she is the only remedial teacher in mathematics at the school: "I serve from the first grade to the sixth grade". She points out "I have been interested [in mathematics] and the others [remedial teachers at the school] are not". She wants to develop the teaching of mathematics for all students at the school, because it is "very easy to see the problem within the student instead of what it is in the teaching that does not benefit all [students]". Anna, Ellie, Jonna and Amanda are members, since they all have SEM-student in their classes. Ellie, Jonna and Anna are core members of the community, since they are interested in developing the teaching of mathematics and eager to cooperate with Barbara regarding the

SEM-students and how to "work with the same things but on different levels" (Jonna). Other mathematics teachers at Oakdale Primary School are also members of this community. Conrad, the principal, is a peripheral member. He is eager to increase goal achievement at the school; thus, how to teach the SEM-students efficiently is of great interest to him. The principal also points out that he wants the staff at the school to "use the resources in the class [room]" and he also wants all pedagogues at the school to take responsibility for all their pupils in the class.

Other remedial teachers at nearby schools are part of this community; Barbara points out that "we need to talk about how we do things …[9] talk about the subject and help each other". In this practice the students in special education needs are peripheral members. They participate and influence the teaching, since "You ask them: How do you want it to be?"(Barbara).

The mutual engagement is the students in special education needs, development of their mathematical knowledge and find ways to get the SEM-students to reach the curriculum goals in mathematics. Another mutual engagement is the development of mathematics teaching for students in SEM. Learning mathematics from an SEM perspective is an overarching mutual engagement.

The shared repertoire consists of the artefacts involved in the teaching of mathematics from an SEM perspective, such as materials, games and tasks. It is also the individual action plans and their content as well as mapping knowledge in mathematics. The shared repertoire also includes the mathematical content *understanding and use of numbers* and how to find different representations to support learning, that is the conversations about how to help the students understand mathematics, and their experiences of the organisation of SEM. The shared repertoire is also the conversations about cooperation. The joint enterprise consists of the desire to develop education in mathematics for SEM-students.

5.1.3 Community of mathematics at Oakdale Primary School

In the *community of mathematics at Oakdale Primary School* Conrad is a core member. All teachers in mathematics at Oakdale Primary School are members and they participate at different levels, depending on how engaged they are in the mathematics teaching. Barbara wants to be a core

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[9] Three dots […] represents a pause in the talk.

member but is struggling: "I haven't got the mandate". Conrad has done a reorganisation in order to increase goal achievement and use competencies at the school in the best possible way. He pointed out that "we shall do the same things […], we shall know what we are doing", referring to the mathematical content of geometry and basic arithmetic. Even Jonna is talking about this; "Ellie and I will cooperate regarding the math, making it the same".

The mutual engagement is the development of mathematics teaching at Oakdale Primary School. The shared repertoire is the conversations about the mathematics teaching overall, the curriculum and "pedagogical concerns" [in mathematics at the school] (Conrad). It is also the discussions about competencies in mathematics education at the school and how to develop cooperation. The subject meetings every third week are the joint enterprise; although they are scheduled for every third week, they do not occur every third week and no one is in charge of them.

5.1.4 Community of student health

The *community of student health* makes an impact on the teaching of the SEM-students, since the members of this community are involved in the decisions about who should receive special education at Oakdale Primary School. The core members are the remedial teachers and the principal. Other members are the school nurse and the school psychologist. The teachers at the school are not members, but they influence and are influenced by it since they refer cases to this group and take part in writing the individual action plans.

The mutual engagement is students in special needs, educational, social and/or physical. The joint enterprise is how to support students in special needs at Oakdale Primary School. A shared repertoire is the "case management" (Conrad), a circle with four steps. It is a "pedagogical mapping resulting in an individual action plan, then an evaluation of the program, actions and follow up." (Conrad). Within this pedagogical mapping, there are thoughts about mapping knowledge in mathematics, which is also a part of the shared repertoire. Another part of the shared repertoire is the *identification form*. When teachers need help they fill in an identification form explaining the problem and the student concerned.

5.1.5 Summary communities of practice at Oakdale Primary School

Based on the data, four communities of practice were identified at the site: community of mathematics classroom, community of special education

needs in mathematics, community of mathematics at Oakdale Primary School, and community of student health (see figure 9). These practices overlap and influence each other; hence, there is a constellation with interconnections. One relationship between the community of mathematics classrooms, community of special education needs in mathematics and community of mathematics at Oakdale Primary School is the shared goal of being to be able to develop mathematics education and enhance learning in mathematics for all students. These three communities of practice also share members: the principal, Barbara and the case of mathematics teachers. Both the community of mathematics classrooms and the community of special education needs in mathematics have the goal of being able to enhance SEM-students to learn mathematics hence it is an interconnection. Another interconnection is also seen between the community of special education needs in mathematics and the community of student health regarding the objective of mapping knowledge to be able to support the SEM-students. The community of special education needs in mathematics and the community of student health also have two members in common, the principal and Barbara.

A boundary object between all the communities of practice is that students are in need in mathematics and mathematics education. The individual action plans can be seen as boundary objects between the four communities of practice at Oakdale Primary School.

Even though there are many similarities between these communities of practice, there are differences in core members, members, mutual engagement and shared repertoires. Figure 9 is showing the four identified communities of practice in the code-scheme.

FORM OF INCLUSION COMMUNITY OF PRACTICE	SPATIAL	DIDACTICAL	SOCIAL
Community of mathematics classrooms			
Community of special education needs in mathematics			
Community of mathematics at Oakdale Primary School			
Community of student health			

Figure 9. Table with the four identified communities of practice

5.2 The cases in the communities of practice and inclusion

This presentation of the results in the communities of practice and inclusion has a descriptive approach, in an attempt to describe the whole picture. Issues in connection to the different kinds of inclusion in the different communities of practice are presented within each case. The communities of practice are presented in order of degree of participation in each case; hence the order might be different in the different cases.

Since Barbara is the remedial teacher in mathematics I have been following, the data from her case is more comprehensive than that in the other cases. When the class teacher is referred to in the quotation, it is the same person as the mathematics teacher. In primary school in Sweden the same teacher teaches most of the subjects, hence is often called a class teacher. As prior mentioned, numbers, starting with number 1 in each case, will present the quotes in a chronological order.

5.2.1 The case of Barbara

Barbara is a member in all of the four identified communities of practice at Oakdale Primary School. She is a core member in one of them, community of special education needs in mathematics. Below her participation in the different communities is presented.

Community of special education needs in mathematics

In this community Barbara is a core member. In this community Barbara talked about student stigmatisation in relation to students being in the classroom versus being alone or with a small group of students or alone with the remedial teacher in mathematics.

[1] This permanent ability grouping, it´s damaging. (2012-08-16)

[2] What if they learn the feeling of being someone who is not able to learn? It overshadows everything. I'm the type that cannot learn. It dominates. (2012-10-18)

[3] It is a risk that they come here all the time and don´t rise in the way that they could [if they were in the classroom]. They don´t! (2012-11-06)

Students being in the classroom versus being alone or in a small group is an issue for her. She referred to the risk of being stigmatised by being in a small group ([1]) or alone with her ([2]). She emphasises this and connected it to the students' mathematical development ([3]). This talk about being stigmatised in a small group fits both within spatial and didactical inclusion (or exclusion), because the students are spatially segregated from their peers and are didactical marginalised, because they do not have the same mathematics as in the classroom.

Barbara emphasised the implications for the SEM-students when using courses.

[4] There are many [students] who really are feeling amazingly good in the small room [alone or in a small group with the remedial teacher]. (2011-09-15)

[5] It is that important, this place value, you have to have ... a little each time so that it stays put. (2012-08-27)

[6] Some children really need continuous [special] education. (2012-08-16)

[7] Sometimes you get small injections [intensive teaching] then you notice that, "I think I need another one", Botox or whatever you call it. Yeah … it's not stupid at all. Because it's as easy as you say, you perpetuating anything without reflecting on whether it is good. (2012-10-18)

[8] Some children can take courses [with Barbara] but not all. (2013-01-10)

[9] She really needs a lot of help with the problem solving. (2013-01-10)

[10] She has been in a large group [the classroom] and I feel that she has not benefited from that. (2013-04-08)

[11] We had a goal that they should be able to do the times tables in multiplication and understand multiplication and division (2013-04-08).

[12] What we will focus on a lot more now is directed courses […] This because we need to cover 63 students at the school who needs [...] help [in mathematics]. […] That they [the students] shall feel…Well simply that they don´t get stigmatised. (2013-06-14)

[13]. What we will focus on a lot more now is courses […] we talk to the teacher when writing the individual action plans, how can we do this in the best way. (2013-06-14)

When talking about courses in relation to stigmatisation ([12]), Barbara also talked about intensive teaching, referring to identifying an issue in mathematics and working intensely with that issue during a period ([7]). But she added a caveat ([8]), meaning there are students needing special education all the time ([6]). She also talked about students who need to be out of the classroom ([10]) Another reflection made by Barbara is that many students feel good being in a small group or alone ([4]). All this refers to spatial inclusion since it concerns the placement, but is connected to didactical inclusion in the sense that it refers to a specific mathematical content. The mathematical content exemplified by Barbara is notions within number sense such as understanding of multiplication and division ([11]), place value ([5]) and basic problem solving ([9]). Time as a mathematical notion is also mentioned. Barbara talked about courses in a future tense and to connecting the courses to the individual action plans, implying thoughts of development ([13]). There is a tension in the

discussion between being in the small group with the remedial teacher, getting special education in mathematics, and the risk of getting stigmatised and marginalised and not being included in the mathematics taught in the classroom. Though, this changes over time. In the beginning of the study Barbara emphasised stigmatisation, but later her focus in SEM was courses, and she did not mention stigmatisation. Hence, this is an indication that the shared repertoire has changed.

Another issue Barbara discussed in this community of practice is whether she can be in the classroom during the mathematics lessons.

[14] His teacher thinks that he must [go to the remedial teacher], he has such large gaps [in mathematical knowledge]. I can see "this I can do" if he is open to it, but he isn´t. […] The mathematics teacher thinks that "no, they cannot be part of the class at all". (Barbara, 2011-09-15)

[15] Well, because then one can also say like this, that you capture the situation […] [for example] - oops, now I saw that these five [students] […] are misusing the equal sign, now you [the students] can follow me into the room [a small group room next to the classroom] so that we can talk a little and use the blackboard there. (2012-10-18)

[16] I can be the one helping the rest of the class while Jonna [the math teacher] is standing there [by the SUM-student]. (2013-01-10)

[17] I´m going to present a little about how to think regarding subtraction in year two. […] She [the mathematics teacher] believes it is good if "somebody else says what I [the mathematics teacher] am saying. (2013-04-13)

Barbara talked about how she and the regular mathematics teacher could change roles, depending on the situation and the students ([16 - 17]). She also spoke about catching the moment in the classroom ([15]) in connection with SEM-students who do not wish to be physically excluded from the classroom ([14]). This thought about being in the classroom is something that developed over time and was discussed by Barbara in the last year of the study. This topic of being in the classroom fits within spatial inclusion. Both Barbara and the SEM-students are physically included in the classroom.

In connection to being with a small group or in the classroom and taking courses, Barbara also talked about listening to the students' voices.

[18] I think that you can listen [to the students] early, early, how do you want it to be? (2011-09-01)

[19] […] feel important and may be involved in determining. […] Really ask [the student], how do you want it? (2011-09-01)

[20] […] ask the student, When do you feel good? When do you feel that you are learning in a good way? (2011-09-01)

[21] You can notice in different ways that some of them do not want to leave the classroom at all. (2011-09-15)

[22] We need to respect all children, we must. (2011-09-15)

 [23] […] ask questions and get quick answers and explanations in many different ways". (2012-10-18)

[24] I said, I have noticed that you do not really need to go here [to the remedial teacher]. You can solve problems; you just need help with automating timetables. How about getting a number of cards a week and practice a little during the day. What do you say about that? Yes [the student said]. I left [cards with multiplication tasks] one day and three days later I checked it. He mastered all of it. I had given him the square numbers [..] I have seen that you have come a bit dejected like this and you do not need special education in that way, you just need a little boost (2013-03-18).

[25] If you just get him to feel extra safe. Then he shows that he can learn. (2013-05-27)

A tension is evident in looking at the feeling of the SEM-student in relation to the special education offered. Does it feel good to be in a small group getting special education, to be able to get quick response and explanations ([25]), or does the student want to be with his or her peers in the classroom ([21],[24])? This can be interpreted within social inclusion – listening to the students' voices when they want to be in the classroom because they want to be socially included in the class. But it can also be interpreted within spatial inclusion – to be in or out of the classroom.

The statements about students' participation in decisions do not fall under any of the three categories of inclusion. Hence the data implies that a new category is needed. Barbara talked a lot about participation ([18-19], [22]). When Barbara talked about the participation, she also mentioned the

importance of feeling safe as a student and of learning to get students to utilise all their competencies ([20], [23]). In the context of participation Barbara spoke of different ways of conducting special education in mathematics in relation to students' wishes and, as a teacher being responsive ([24]).

Barbara discussed tasks.

[26] It´s after all a little bit about how you ask the question: "How many flower pots are there in the window?" Then it's one that can get the chance [to answer]. [...] But if you pose [the question]: I counted something in here and I got it to four, what could I have been counting? (2011-09-01)

[27] There is no one who asks someone to read a book that is a lot harder than they can manage. But when it comes to math tasks however, they [the students] can actually sit and do [tasks] often much harder than they can handle. It's strange in a way. (2011-09-05)

[28] These boys, they rose to the occasion when there was group work and there were some practical tasks involved. Then they were, they were able to do much more in the classroom than when they were able to do with me. […] It was such good tasks with measurement; area was the mathematical content. Find stuff that is between 50 cm and 1.5 m; try to locate it in the room. It was great suggestions and participation. [...] Everybody was doing it and it was important for everybody. [...] The **situation** affected the **achievement**[10]. (2012-08-16)

Barbara highlighted that being able to include all students depends on the formulation of the tasks [26]. The demands on the students regarding the level of mathematics in the tasks are an issue for Barbara on an overall level [27]. How to choose and work with tasks in the classroom and how this stimulates the SEM-students is also discussed [28]. The talk about tasks fits in didactical inclusion, since it concerns students' achievement in relation to the mathematical content.

In this community of practice, Barbara also discussed cooperation regarding the connection of the content between the teaching in the small group and in the classroom.

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[10] Bold text = words are emphasised by the informant

[29] We capture a little planning, what you are doing in the classroom. (2011-12-01)

[30] I have almost all teachers at the school [that she serves], I would like to have it [cooperation] with all. (2011-12-01)

[31] It is important that I as a special pedagogue am informed. It is my obligation to find out to be able to link and prepare here [in the small group with the remedial teacher] to enable them [the SEM-students] to be proficient there [in the classroom] once they attend. (2011-12-01)

[32] [...] we talk about in advance to enable them to be a bit more involved and once Gabriel [a SEM-student] said; "Isn´t it cheating, what we 're doing now ?" No, but there is little that we talk about things before, to be able to understand. There will be no good if you believe it [is cheating], that one gets discriminated that way. (2012-02-06)

[33] Jonna and I help each other to look at what you can work with when they're not in here [in a small group with the remedial teacher] when we are working concretely. (2012-08-27)

[34] […] to get them [the SEM-students] to feel that it is the same things we are working with. (2012-10-18)

[35] Improve our cooperation [...], then they will [the SEM-students] get a feeling it's the same stuff we're working on. (2012-10-18)

[36] [...] to be able to capture and repeat what they do in the group. (2012-11-07)

The connection between the content in the teaching in special education in mathematics and in the classroom is an issue for Barbara ([29], [34-35]). To be able make this connection, Barbara talked about cooperation with the math teacher ([31], [33]). Barbara wants to cooperate with all the math teachers ([30]) and even though she is a member in the communities of mathematics classroom, the data indicates, it is not always possible to have cooperation even though cooperation is a shared repertoire. Barbara also emphasised that she needs to know what is done in the classroom even though she works with the student in a small group ([36]); she needs to be a broker between the communities of mathematics classroom and the community of special educational needs in mathematics. Then it is important that the students know what they do is preparation and what

that means ([32]). The discussions of connection of content fit in didactical inclusion; they concern ways of presenting the content to the SEM-students. These issues are important for Barbara all the time during the study.

Another issue discussed by Barbara in relation to connection of content and cooperation is preparation and immersing.

[37] On Tuesdays they are in the math [classroom], when it's Kangaroo math[11] and it's been great. We've had time; sometimes I have had time to prepare them a little bit so they have little [pre] understanding. They have been active [in the classroom]. (2012-02-13)

[38]
Barbara: Yes, and I know that he has, many times, he has shown the way and he says things like "oh well […], what happens if Jonna [his math teacher] writes the following on the board..." I think …
Researcher: Oh well, he brings questions from the classroom?
Barbara: Yes, I remember very it well; we had division related to multiplication and when it became division with residual, and that he really listened to it. (2013-02-13)

[39] […] it was a request from Ellie, that she [the SEM-student] would become efficient in this [mathematical] field, which they are doing in the class. (2013-04-24)

[40] But it is important nonetheless, we [the teachers] have discussed hot to get our act together, what we are going to work with, so that they [the SEM-students] are more likely to be involved in the discussions, that they feel a secure. (Barbara, 2013-05-27)

In the small group, the SEM-students are encouraged to be active and relate to what they are working on in the classroom ([38]). Here the teaching aims at deepening their understanding of the content worked with ([38-39]) in the classroom and preparing them for upcoming tasks ([37]). The cooperation should also benefit the students' preparation so as to

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[11] The Kangaroo competition is a yearly international problem-solving competition with five levels from preschool class to high school. The tasks from previous years are available and used in the mathematics teaching. http://ncm.gu.se/kangaru, 2014-10-27.

prepare the students before something is taught in the classroom ([40]). Barbara also mentioned that she receive special requests ([39]). This preparation, immersing and special requests requires cooperation regarding the content and how the content is presented, hence it is within didactical inclusion.

Barbara discussed strategies and generalisations in relation to subtraction.

[41]
Barbara: We have kept ourselves in the small range of numbers, but constantly connected to the larger, they have not really understood it yet, but they manage it.

Researcher: They do manage it, but then as you say […] they are stuck in a rather slow and poor strategy.

Barbara: Yes.

Researcher: One must try to help them on with it […] and when he found it out [the student understood generalisation from 5+1 to 50 +10], he began to laugh out loud!

Barbara: Yes, so happy! (2012-02-06)

[42]
Barbara: Like, they have some strategies, but sometimes they only can count from the small to the large, but I'll present some different [strategies]. If the numbers are close it may be useful to compare, if, for example, 101-3. […] To talk to each other and discuss [the strategy].

Researcher: yes, talk, (inaudible) not really count, but the strategies are important now?

Barbara: […] Strategies. 43-21, yes but then everything is covered, […], then it fits quite well with, with, take tens and ones.

Researcher: Yeah, right

Barbara: That they become aware of there is more than take away. (2013-04-29)

Subtraction is often difficult (Fuson, Wearne, Hiebert, Murray, Human, Olivier, Carpenter & Fennema, 1997) and, according to Barbara, the students need to know different ways of thinking about subtraction, both as a difference and as take away [42]. To be able to generalise is also a strategy Barbara is working with to help the SEM-students to understand the mathematics. When they are able to succeed, they can really get excited [41]. When Barbara discusses strategies she only uses representations from one and the same register, abstract numbers. Strategies fit in didactical

inclusion since they involve thoughts about mathematics, to be able to generalise and understand notions.

Barbara talked about recognising similarities.

> [43] You know this thing with subtraction, they [SEM-students] had told Jonna [the math teacher] that they had never worked with … They knew nothing about it … Well ... You know it was 14-6; it was a task that [the students] did not link it to this we've been working on **a lot,** within this very range of numbers. (2012-08-27)

> [44] Then the question arises, is this which we have practiced so extensively here, are they [the SEM-students] able to see that they have a use for this in [the mathematics] class? (2012-08-27)

For the students to be able to recognise that the things they covered in the small group with Barbara are the same things they work with in the regular math classroom is an issue for Barbara ([43]). To be able to recognise similarities is important for the SEM-students according to Barbara ([44]). Recognition of similarities fits within didactical inclusion, since it concerns understanding the mathematical content in different contexts.

Representations in mathematics are discussed.

> [45]
> Barbara: No … what is it that makes …yes …
> Researcher: Thus, they can do it here [in the small group] but when they are in the classroom they don´t take the knowledge with them… it feels like.
> Barbara: No … is it like that? [...]
> Researcher: It is bound to the situation?
> Barbara: Yes. Here they can do it! I thought, I must not forget that, you know, these steps, concrete, it is there, but then sometimes in the representation phase you might draw [a picture] and then you are here
> Researcher: In the abstract
> Barbara: You have to remember that; you have to have that [the concrete representation] because I know I've made the mistake before and thought that it's enough to be there [in the picture]. (2012-08-27)

> [46]
> Barbara: Then she got for example a task, 36+17, maybe it was. 3 and 1 is 4, 7 and 6 is 13, plus 4 then it's 17.

	Consequently, I had not noticed it so clearly as when… She explained exactly [how she understood all the numbers as ones].
Researcher:	Mmm
Barbara:	And then I asked her, because I know that she is quite good when we play games with money; she likes it and gets it correct, but I was not supposed to help them, but .. well, I asked, Does it work? Do you think it feels right? "Yes, I'm finished; I'm finished before you", she said. (2012-10-18)

[47]

Barbara:	And I'm working on this [place value], you know this material and I have worked with those accordions [paper strips that can be folded and unfolded] and I worked and put different [positions].
Researcher:	Mmm
Barbara:	Just to see and follow exactly. And it was fun and it was challenging and it was addition and he [a SEM-Student] managed it […]. But for this to work with intermediaries[12], which are so abstract... It's a pretty big step. I brought… you know it…
Researcher:	The thousand cube[13]
Barbara:	The hundred plate, I took it first. (2012-11-05)

[48]

Barbara:	I´m thinking of this task, they have never seen, 3x2x7, we need of course
Researcher:	Yes, that's right, when there are three factors.
Barbara:	Yes. Yes, exactly. It needs to be really explained; maybe you could build something with blocks or like that, so that it becomes three dimensions. (2012-11-05)

[49] And then it was, for example, in the book: Make a square that has side this and that, and … well ... Make one that is twice as large. Then Gabriel managed this, while with Kevin, you had to say, what does a square look like? And ehh .. What does twice as much mean; how can you think? Really, step by step. (2013-01-10)

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[12] Intermediaries are often used in arithmetic for example 345 + 224 = **500+60+9**= 569. The bold text represents the intermediaries. These can be represented differently.
[13] The thousand cube and the hundred plate is a part of a material used in mathematics with cubes representing a number.

Representations were discussed together with recognising similarities from special education in mathematics to knowledge in the regular math class and using concrete representations to catch the attention of the student ([45]). Recognising similarities from one representation to another can be tricky ([45], [47]) and is known to be complex (DuVal, 2006). The student who misunderstood the addition 36+17 thought that it was just ones and added 3 to 1 and 6 to 7 and then added them all. It seems she knew the procedure but she did not understand place value. However, she understood the addition with money ([46]). Barbara talked about using concrete representations when working with place value and using these representations when working with understanding intermediaries ([47]). The use of representations has a strong connection to the mathematical content ([48]). Different notions and different students need different representations ([49]). My reflection on this is that you might say there are different levels in the teaching that need to be considered: both the content level, which representations are suitable depending on the content, and the student level, in which representations are suitable for this student in this situation. These levels of teaching regarding representations and recognising similarities between representations are within didactical inclusion – how to get the student to reach the mathematics.

Communities of mathematics classroom

In the *communities of mathematics classrooms* Barbara is a peripheral member. In regard to these communities she talked about knowing what the SEM-students work with in the classroom and her participation in the classroom. This is strongly connected to what Barbara said in the community of special education needs in mathematics regarding connection of content.

> [50] I was inside the classroom at the beginning of the week and we talked about why [we do mathematics] and we drew a giant mind map. (2011-10-18)

> [51] In any case, I'm sometimes present [in the classroom] to observe, what's it all about today and like to talk to them [SEM-students]. Yes, here we did this and we did it that and that time as well. (2012-03-05)

To be able to be inside the classroom and participate in the teaching is an issue Barbara discussed ([50]). She mentioned being able to know what mathematics the students do in the classroom and to connect to that content ([51]). She also mentioned this connection of content in community of special education needs in mathematics, when talking about being with SEM-students in the small room. The difference is that in this community of practice the talk is about the SEM-students in the

classroom. Connection of content fits in didactical inclusion since it involves thoughts on how to get the students to recognise the mathematics. Participation in the classroom teaching is both in spatial and didactical inclusion. Here the spatial concerns the remedial teacher, the remedial teacher being spatially included in the classroom.

Approaches in the teaching of mathematics in the classroom were discussed.

> [52] All [of the teachers] have now done a drive with joining and separating numbers; we have had *DIAMANT*[14] and seen the need [..]. Then Anna, she has done this all week: she has had the houses and the squares and the game. (2011-10-18)

> [53] It depends a lot on the teacher and the responsibility you have. [..] It is part of the teacher's knowledge, what variations can you do that is good? (2011-12-01)

From an SEM perspective Barbara discussed approaches and materials in the teaching of mathematics in the classroom. The material she is talking about, the games and tasks, were presented by Barbara to the teachers ([52]). These tasks and games can be seen as boundary objects, especially *DIAMANT*. This talk about approaches and material fits within didactical inclusion. In relation to approaches in the teaching, Barbara spoke about the competence of the teacher, of being able to know what representations to use to vary the teaching and reach the students ([53]). Hence, competence and approaches in the teaching of mathematics are related, according to Barbara.

Barbara discussed the ability to be flexible in the teaching, in and out of the classroom.

> [54]
>
> Researcher: But I think that your resources could be addressed to other students in the class also, not just specifically to Kevin and Gabriel [SEM-students].
>
> Barbara: Yes, sure, then I can also do like this: "This was tricky, you can come along today, or the two of you [can]". Then it doesn't become permanent [who is getting special education in mathematics]. (2012-03-05)

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[14] *DIAMANT* is a material used for mapping knowledge in mathematics, provided by the Swedish National Agency for Education.

[55] […] if they can arrange it with a little flexible groupings in class. (2013-01-10)

To be flexible in the teaching, to be able to utilise recourses in relation to students needs is important for Barbara ([54-55]). This flexibility is a part of spatial inclusion, but also a part of didactical inclusion – to be able to support all children within the classroom.

Another issue highlighted by Barbara was the discussions among mathematics teachers at Oakdale Primary School.

[56] Why was **that** good? Yes, well it was because, as you said, it became a dialogue [talking about a session together with the mathematics teachers discussing mathematics teaching]. (2012-04-25)

[57] This was so much fun, and I needed [it]. I had longed [for talking about mathematics teaching] and sometimes it gets a little […] solitary work for me [here] at the end [referring to her placement at the end of the corridor]. (2012-04-25)

[58]

Barbara:	And then I had a little thought: if I were to gather all grade 3 teachers. We [could] share good teaching tips and […] [what] you can work with if they [the SEM-students] have not managed this. But then it turned out that it is difficult with time. But then I started talking to one [teacher] at a time. [I talked to] Sofia [a teacher], "Look, you can do Digicubes[15], the app". She [Sofia] recommended [it to] a student. So he [the student] has worked with her a lot! It was enough; she caught hold of it right away!
Researcher:	Yea […]
Barbara:	And then I showed it, with the solitaire that tends to be so appreciated. Both of the things he had worked with, you know.
Researcher:	yea […]
Barbara:	It became intensive remedial teaching. She [Sofia the teacher] has it now. **She has it**! (Barbara says this with excitement in her voice). (2013-05-27)

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[15] Digicubes is a free digital application working with basic number sense.

When Barbara has an opportunity to discuss mathematics teaching in the classrooms she is excited ([56,57]). Although the data indicates that it is hard to get opportunities for discussions ([58]) Barbara is persistent though and tries to have the discussions, although it is hard to find time ([58]). When she has discussions with the teachers and is able to inspire them, she gets excited ([58]). Discussions in mathematics education fit within didactical inclusion, since they concern a mathematical content.

Barbara talked about difficulties for the teacher in the teaching of mathematics and knowledge in mathematics education for the teacher.

> [59]
>
> Barbara: Yes, what is it that is that difficult? Finding the things that, you know, can provide this [understanding in mathematics], Are there exercises? And is it even possible to find exercises that everybody can take part in?
>
> Researcher: Mmm … and do you think it is the class teacher who feels that it is difficult or you as a remedial teacher or … ?
>
> Barbara: I think it is the class teacher who finds it difficult. In order to proceed […] you move on in the mathematical content, one base the second... How long can we stay doing this, this basic number sense […]? It's still very much looking at what they [the students] are supposed to know [in mathematics] in year 3. There are so many different parts. Therefore, I believe that you [as a class teacher] are afraid in some way. You're afraid. […] You are afraid that they [the students] have not even heard of these things when it comes to the national tests.
>
> Researcher: Mmm
>
> Barbara: You need to have presented it [to the students] somehow. […] For example, now it's geometry, then you might work a lot with recognising geometric figures, and then you skip step two where you work a lot with describing and telling each other. Instead they concentrate on calculations because they [the teachers] feel that [the students] have to be able to do that.
>
> Researcher: Mmm
>
> Barbara: And then they notice that they [the students] are not able, when it comes to tricky tasks. (2011-12-01)

[60] We must have confidence in our students and we need to have great knowledge when you are a math teacher, that you are able to se how the different eh ... areas are interrelated. That you might work a lot with number sense even if you work with this [meaning other mathematical areas]. (2011-12-01)

[61] It depends a lot on the teacher and the responsibility you have [as a mathematics teacher]. […] Even this falls under teacher's knowledge, what kind of good material is there, and how shall I vary [it]? When you know that you shall make variations, what is there to make the variations with, what is good? (2011-12-01)

[62] It is always difficult for the class teacher if it differs too much, what should I do with the children? (2012-08-27)

In this community of practice, Barbara talked about tasks and representations in relation to teacher knowledge ([59,61]). She also highlighted the difficulties for the mathematics teacher to be able to support all students in the class and to be able to get everyone on the same page ([59, 61]). The complex assignment as a mathematics teacher is to be able to see all children in the class, their knowledge in connection to the mathematical content and the interrelation between the mathematical contents is mentioned by Barbara ([60-62]). This discussion about difficulties for the teacher and knowledge of the teacher in mathematics education fits within didactical inclusion – to be able to reach the students in mathematics through tasks and representations.

Community of mathematics at Oakdale Primary School

Barbara is a member of the community of mathematics at Oakdale Primary School. In this community of practice, she talked about being lonely as a remedial teacher.

[63]
Barbara: Now it has become like this at this school. I've been interested in math and deepened myself [my knowledge] and it has become my niche. And then maybe, that Iris [another remedial teacher], she's amazing there with creativity and language development based on creative activities. […] Math is not her thing. She has math and English, and even Annie [another remedial teacher] has that.
Researcher: Swedish and English, you mean?

Barbara: Yes, Swedish and English. Mmm ... But Annie also
              has a little math, but she feels that she does not want
              [to have] it really, because she feels she does not really
              have the repertoire.
Researcher: Do you feel lonely, one of a kind, at the school?
Barbara: Yes, I probably do. (2011-12-01)

[64] I don´t have anyone to talk to about mathematics at the school, I
have been interested [in mathematics] and the other [remedial
teachers at the school] are not. (2012-06-11)

[65]
Barbara: Yes, Annie [a remedial teacher] works with [..] Annie
              does these overviews [of reading and writing] and we
              work, both me and Iris [another remedial teacher], all
              three of us work with student with reading and
              writing, we do.
Researcher: Mmm […] but who of you work with the math
              students?
Barbara: Yes, that's me.
Researcher: Mmm […]
Barbara: It is [...] Yes, that's the way it is. (2013-04-24)

Although there are other members in this community Barbara feels that
she does not have any one else at the school to discuss mathematics at the
school with and feels lonely ([63-64]). It is also evident that although there
are three remedial teachers at Oakdale Primary School, Barbara is the only
one working with the SEM-students ([65]). It seems that during the study
this does not change. This talk about being lonely does not fall under any
of the three categories of inclusion but it might influence the development
of inclusion in mathematics.

Another issue Barbara is discussed in this community was the development
of mathematics education at Oakdale Primary School.

[66]
Barbara: Something what I think about a little bit about is that
              last year, almost all of the teachers did this course,
              understand and use numbers.
Researcher: Yes, that's right; you had that one.
Barbara: Yes, and then, I think that, I think I'll go to Conrad
              [the principal] and ask why not use this [doing courses
              with teachers] a bit throughout, because you know it

will be like this at a school. You try a little on your own and then […] it becomes a split sometimes. (2011-09-01)

[67] Thus, I feel that we are a huge school ... we are a huge school. It would maybe ... sometimes at smaller schools, you feel; now we focus a bit on mathematics and technology or something. And then you have it on the agenda all the time. Yes ... (2011-12-01)

[68]
Barbara: I noticed when we had these lectures, the teachers became inspired, Anna, she bought problem solving tasks […].
Researcher: Something happened …?
Barbara: Yes, something happened; it became a parallel process. (2012-11-06)

[69] And you can easily fall into the feeling of having no conversation partner who ... Or you get this replenishment with different courses. (2012-12-01)

It seems that Barbara is struggling with developing mathematics education at the school ([66-68]). She feels the need of recurrent meetings or courses together with the other teachers ([66]). This in order to develop the teaching of mathematics at the school and not stagnate ([68-69]). This can be seen as fitting within didactical inclusion – to really discuss and develop mathematics education at the school in order to include all students in the mathematics.

In relation to developing mathematics education, cooperation and didactical discussions is an issue in this community of practice as well as in community of mathematics classrooms. The difference is that in this community of mathematics at Oakdale Primary School the talk is on a more overall level.

[70] We have to talk [to each other]; what do we have at the school? Can we borrow [materials] from each other? (2011-09-15).

[71] Why don't we have time to discuss; it is insane! (2011-12-01).

[72] I am trying, we're talking fast, a little here and a little there; it is not directly planned (2011-12-01).

[73] Then I feel that I need, or we need and long to come together and talk to each other about all the good stuff we are doing, that I believe (2011-12-01).

[74] No, but if the cooperation would be better and more inclusion, I would have fewer [students] to meet, I think. (2011-12-01).

[75] I think we need it [didactical discussions] at the school, with qualified guidance. (2012-04-25)

[76] I feel that when we come together, we talk about, and make our thoughts visible, yes, well, a little like that, and [then] it is easier to connect to it later on. (2012-04-25)

[77]

Barbara:	Now I am a bit … I want to move back a little, because I feel, I do not really know, because I'm not part of the subject planning meetings, I'm not a part of them.
Researcher:	No …?
Barbara:	What I've been involved in terms of subject meetings, it´s when you have been here this semester. Otherwise we usually have meetings with the student health [at the same time], me, Iris and Annie [the remedial teachers].
Researcher:	How do you feel about that, that you are not part of the subject planning?
Barbara:	…
Researcher:	It is positive ... negative ... what is it ...?
Barbara:	Well, I think like this, if, for example, you say subject meetings, then it may be that … well, why am I not doing it?, Math and science, for example? […]. Why am I not a part of that? […]. (2012-06-11)

[78]

Barbara:	Here we have subject meetings again [looking at a schedule].
Researcher:	Are they once a month?
Barbara:	I think it is. It's really, it's all very well intended, but then sometimes it is … in the spring there is much that disappears, actually. (2012-06-11)

[79] […] in that we could have mathematical discussions [at the school]. (2012-08-16)

[80]

Researcher:	Do you have opportunities for collaborative planning or … ?
Barbara:	Yes, we take us [time], we must do. Most often Jonna [a mathematics teacher] arrives [early] in the morning, I usually come at 7:30 or so. Then we'll take a cup of coffee, and then we are able to steal a moment. (2012-11-15)

Barbara is talking about the need for cooperation and discussions regarding the teaching of mathematics ([70,73]). There seems to be very little time to discuss overarching issues and plan mathematics education at the school ([71-80]). The subject meetings is organised to make time for the teachers to plan together, though the planning seem to be dropped because of other priorities ([78]). Barbara is not supposed to be a part of these subject meetings, since she is scheduled to have meetings with the other remedial teachers the same time. This becomes an issue for her during an interview ([77]). Barbara thinks that if they cooperated and included the SEM-students more, there would be fewer SEM-students ([75]). Cooperation does not fall under any of the three forms of inclusion, but it might influence inclusion in mathematics at Oakdale Primary School. Discussions fit in didactical inclusion since they involve how to teach mathematics.

Barbara discussed organisation at the school.

[81] I think that, ehh … on the whole at the school, then it's perhaps more that I pick out [students from the classrooms] because I have to make a schedule. I'm serving all [classes] from grade 1 to 6. (2011-09-01)

[82] My ideal would be, for example, that I got to devote myself to lower primary school then I had fewer [people] to work with. Or what if it was this house, [the school is organisational divided into five houses], me and the house […]. (2011-12-01)

[83]

Barbara:	I feel that I am a little unsure, maybe. Because I know that Conrad [the principal] would like me to be more in the classes and simply be more involved. He expressed that, particularly when he started.
Researcher:	Mmm
Barbara:	And then I'm not sure how it can be done in practice. This I can say.
Researcher:	What are your feeling about this, his wishes?

Barbara: No, but I think that I believe that the basic idea is good; it is. I'm not negative towards it, it is not it, but it requires me to be much more involved in everyday life, the everyday planning. It requires that [from me].

Researcher: Do you see any obstacles in that?

Barbara: That I am more involved?

Researcher: Mmm

Barbara: Ah, that's if I should have, if I'm going to work with everybody like I am now, and then it's an obstacle. Because then it's ... Yes, there will be 18 classes here. (2012-06-11)

[84] I think it is important for the whole school, the principle of how we plan the support [for the SEM-students]. (2012-10-18)

Barbara says it is important that they have a principle for supporting SEM-students at the school ([84]). The data shows that it is overwhelming for her being the only one doing remedial support in mathematics at the school ([81, 83]). She has a suggestion for being able to cope and be more involved in the everyday planning, namely only to serve the lower primary school or be connected to only one of the houses at the school ([82]). This issue of organisation cannot be interpreted in any of the three forms of inclusion, but it can influence the process of inclusion in mathematics at Oakdale Primary School.

Barbara also discussed mandate in this community.

[85] […] I don't really have that mandate to say, "Can I come in [to the classroom] and look at how you are doing things? I think that is on the edge, like, to question [the teaching]. (2011-09-15)

[86] Because I've just recently had a salary discussion with Conrad, we went through the objectives and criteria, and I said at the beginning when I came [to the school] I experienced that, it was Anne who was the principal at that time, then I felt that I had received a mandate from her, just like that. Now, having this study group about this book[16] [Understand and use numbers]. Then it was easy, then I prepared and it became these regular pedagogical discussions. And then they [the teachers] said, We want a continuation […] but now I have not ehh … I'm, well, I do not have more mandate than anyone

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[16] McIntosh, A. (2008). Förstå och använda tal. (1. edt.) Gothenburg: National Centre for mathematics education (NCM), University of Gothenburg.

else ... no ... And then you crawl to the back and become a wallflower suddenly. (2013-04-24)

When Barbara is referring to her self as a wallflower ([86]), she is saying that she wants to have a mandate, but she does not feel that she has one anymore. If she had the mandate, she could pursue pedagogical discussions in mathematics like she had before ([86]). She is also referring to a mandate to be part of mathematics classrooms at the school ([85]). She does not want the teachers to think that she is questioning their work ([85]). Mandate does not fit in any of the three forms of inclusion, but it can affect the development of inclusion.

Barbara mentioned courses.

[87] Then I make a schedule and eh ... we hold a conference ... I just talked to the teacher in year 6… then ... I feel that ... we decided that we should run courses, five week courses. (2011-09-01)

[88] The students are on it, it's five weeks, and it's twice a week; perhaps you really should have even more but … yeah … (2011-09-01)

[89] Let´s say they are working with a mathematical area. They are having geometry in year 4 now, let's say that five there, three there and two and one or like that [referring to student in different classes in year 4]. They need a small turn. (2012-06-11)

On an overall level, Barbara is talking about courses in mathematics for SEM-students in relation to organisation ([87-89]). Courses fit within spatial inclusion since they involve students being out of the classroom ([87-89]).

Another issue Barbara discussed was mapping knowledge in mathematics.

[90]
Barbara: Today I managed [to check] a quarter of all new students, just checked with *DIAMANT*, this with preliminary statistics and quantity and numbers.
Researcher: It is interviews, huh?
Barbara: Yes, it is, and then, it was good, I could already see the concepts fewer half and twice, we need to have a drive with all [students]. Then they were pretty good at counting far.
[…]

Researcher:	Do you hand this information over to the class teachers or how …?
Barbara:	Yes, I will do that ... Yes, that's really important to say that everybody has to work with it much more, with half and twice. It is so amazingly hard. They [the students] do not [understand it]. […] And the same thing goes for the counting chant, it's quite telling actually. (2012-06-11)

[91]

Barbara:	It's not that bad, this little test [*DIAMANT AG 1*], to get, I think it's a pretty good prognostic value […]
Researcher:	Do you think there are many that make them [the tests]? Many schools?
Barbara:	I do not know […] because […] this was previously a different material and it was very comprehensive and it took really long time [to carry out] but it is a matter of **using** the results, otherwise it's like there's no point in spending a lot of time on it, if you do not proceed [according to the results]. (2012-06-11)

[92]

Researcher:	Do you have a screening of math or ...?
Barbara:	Yeah. It is what I do when they [the students] are about to start grade 1. So usually I take preparatory arithmetic and preparatory statistics. How far can you count and like that
Researcher:	Yes that's right.
Barbara:	And some common notions. It ... I do ... and since then we have actually added *DIAMANT*, you know this, composition of numbers 0–9, it is of course, we have added that to our annual cycle. and … em then the class teachers are supposed to do it. (2013-04-24)

[93]

Barbara:	And understand and use numbers, last year's test, we recommend that you do.
Researcher:	Right.
Barbara:	And I haven't got ... more then ... Annie [a remedial teacher] she compiles it, in statistics.
Researcher:	and even that [the mapping of mathematics] then?
Barbara:	No, not that. But it might be good to do. It would be great because then it became a little more ... uh …
Researcher:	[…] transparent?

Barbara:	Transparent and a bit more ... this we will do, oh yes, this we will do.
Researcher:	Mmm …
Barbara:	Yes , and then there is nothing there [looking at the year cycle] and in January it´s screening of reading, which image is correct for year 6 , satisfaction survey in April , screening of reading [in year] 2,4,5 ... and then it is Fonolek[17] [...] it is now [...]
Researcher:	mm […] then you have the arithmetic here [...]
Barbara:	Yes, AG 1 [a preparatory test from *DIAMANT*], it is there [...] it's broadly as we have planed. So Annie, she makes good reminders [for the teachers], […] check at "home" [a place on the intranet] the ones [students in danger to fail] who are marked in red, we have to pay some extra attention to.
[…]	
Barbara:	[...] And it feels like, now when I talk to you, it would probably be good to do it in math too [compile, make statistics and put it on "home"], that […] somehow collect … what is it who is it. […] I'll say it to Annie, praise her for it, **now** check it out at "home", and those who are marked red, the ones we need to keep track on. She has done that. While I have not […] done it that way. Yet. (2013-04-24)

Barbara is doing mapping of knowledge in mathematics in the preschool class ([90, 92]), and she sees what needs to be worked on at an overarching level, especially in year 1. Here she can see that the notions fewer, half and twice needs to be treated in year 1 ([90]). She thinks that it provides a good prognostic measure and emphasises that it is a matter of using the results ([91]). However, she has not done a compilation, like her colleague Annie has done with the mapping of reading ([93]). This became an issue for her during the discussion with the researcher ([93 line 24]). She feels the need to develop a compilation of the mapping of mathematics on an overall level and to be able to inform the teachers formally on the intranet "home" ([93]). Mapping the mathematical knowledge fits within didactical inclusion. *DIAMANT* can be seen as a boundary object between this community of practice and the communities of mathematics classrooms. Tests are a kind of assessment (Björklund Boistrup, 2010), and the *DIAMANT* is a test used at the school for mapping knowledge [93].

Community of student health at Oakdale Primary School

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[17] Fonolek is a group test that assess phonological awareness from 6 years and forward.

In the *community of student health at Oakdale Primary School,* Barbara is a member together with the principal and the other special pedagogues working as remedial teachers, Annie and Iris.

Even in this community of practice Barbara talked about mapping knowledge in mathematics.

> [94] You know, we made a proposal in eehh … in our student health team then that if you have it [understand and use numbers tests] right at the beginning of the semester […] we run it throughout the school, the book [understand and use numbers] now. (2012-08-16)

> [95] The student health team it is maybe about dealing with things that already are troublesome, but it's pretty important in work of the student health team that you talk about things **before** they turns into trouble. And it's that kind of [discussion] as well, what kind of teaching do we have? Do we have a teaching where everyone's questions are important? Or is it a lot of right and wrong? (2013-05-27)

From this community has come a proposal to use a specific material identifying students' knowledge in mathematics ([94]). This fits within didactical inclusion and it concerns summative assessment. To use this type of assessment, understand and use numbers, is also referred to in the community of mathematics at Oakdale Primary School and can be seen as a boundary object. Barbara is talking about what issues need to be discussed in the student health team. She emphasises that the health team need to deal with preventive work and look at the teaching of mathematics, how is it conducted at the school regarding including all students ([95]).

Summary
For a summary of issues influencing inclusion in mathematics that emerged in the case of Barbara, se figure 10. This version of the code-scheme has a new column called other issues, added to be able to understand all the codes emerging in the data.

INCLUSION / COMMUNITY OF PRACTICE	SPATIAL	DIDACTICAL	SOCIAL	OTHER ISSUES
Community of mathematics classrooms	Participation in classroom teaching Flexible solutions	Connection of content Participation in classroom teaching Approaches and material Flexible solutions Discussions Teacher knowledge Representations and tasks		
Community of special education needs in mathematics	Stigmatisation Courses Change roles in the classroom In or out of the classroom	Stigmatisation Courses Intensive teaching Tasks Connection of content Prepare and Immerse Strategies Recognising similarities Representations	In or out of the class- room	Student participation
Community of Mathematics at Oakdale Primary School	Courses	Development of mathematics education Discussions Mapping knowledge		Working alone Cooperation Organisation Mandate
Community of student health		Mapping knowledge		Preventive work

Figure 10. Overview of issues regarding inclusion in mathematics from the case of Barbara

As seen in Figure 10, *in or out of the classroom* in the community of special education needs in mathematics fits in both spatial and social inclusion.

This is the case because the data implied it could be interpreted within both spatial and didactical inclusion. Spatial when the placement only is referred to and didactical when there are issues concerning learning mathematics. *Courses* appear in didactical inclusion and in spatial inclusion. In community of special education needs in mathematics *courses* appears in both and in in spatial inclusion in the community of mathematics classroom. Even here the data implied different forms of inclusion, spatial when only courses were talked about as being out of the classroom and didactical when there was talk about the learning of mathematics through courses. *Stigmatisation* can be an issue of both spatial inclusion and didactical inclusion, didactical when talking about the SEM-students in relation to the mathematical content and the risk of marginalisation, when they don't get the opportunity to do the mathematics taught in the classroom and spatial when the SEM-student are not in the classroom and get the feeling of that they do not belong with their peers. *Flexible solutions* appear in both spatial and didactical inclusion in the community of mathematics classroom, indicating it can both be about placement as well as the content taught.

Other issues emerged in the data that are not in the framework of Asp-Onsjö: *student participation, working alone, organisation, mandate* and *preventive work.*

Barbara can be seen as a broker between the community of special education needs in mathematics and the community of student health, when transferring elements between these communities of practice such as individual education plans. It seems that Barbara also is a broker between the community of special education needs in mathematics and the community of mathematics at Oakdale Primary School but that this is a struggle for her, implying that this translation and coordination addresses conflicting interests. One example of this is that she wants to be able to support the SEM-students by cooperating more with the teachers in practice, but she also wants to develop the discussion among mathematics teachers at an overall level. There is not enough time for her.

5.2.2 The case of mathematics teachers

The case of mathematics teachers are seen as one case, although there were differences between the individual teachers. The mathematics teachers as a case were visible in three of the four communities at Oakdale Primary School. They were not visible in the community of student health and were most visible in the *communities of mathematics classrooms.*

Communities of mathematics classrooms

The teachers are core members of the communities of mathematics classrooms with full participation. They talked about how to get the SEM-students involved in the classroom activities in mathematics.

[1] You [the SEM-student] take part in the lessons and involve them by posing questions that you know they can handle. (Ellie, 2011-11-07)

[2] They do everything […] at their level and you take away some tasks. (Ellie, 2011-11-07)

[3] They [the SEM-students] hadn't learned much by being in here [the classroom]. I could not bring the math level to their level; maybe it´s easier as a special pedagogue. (Amanda, 2012-09-03)

[4] […] you need to think about what some of them can do instead, at their level, in order for them to feel challenged, although at their level. (Jonna, 2013-06-16)

[5] You work with problems on different levels. (Jonna, 2013-06-16)

There is talk about how to reach the SEM-students and challenge them ([1-2, 4]). Both Jonna and Ellie talk about didactical inclusion in terms of challenging the SEM-students in the classroom by adjusting the mathematical level in the tasks and activities given ([4-5]). Amanda disagrees, and thinks that the SEM-students are almost impossible to reach in the classroom ([3]). In her classroom the SEM-students are excluded.

In these communities there are talk about supporting the SEM-students in terms of being able to get time and support in the classroom.

[6] I have the privilege of often being two in the classroom in math. It's Bianca, a leisure time pedagogue, she is a preschool teacher, she joins a lot of math [lessons] and can support and help, and it allows me to take a little extra care of them [the SEM-students]. (Ellie, 2011-11-07)

[7] […] If you are alone then as a teacher in the classroom, then it's hard be sufficient. (Jonna, 2013-06-16)

[8] […] you had the time to sit down in peace and quiet and really help them [the SEM-students]. (Jonna, 2013-06-16)

[9] […] and since I've had Henry [another teacher] joining three lessons a week […]then these students became one in the group. (Jonna, 2013-06-16)

An issue discussed by the teachers is the possibility to be two teachers in the classroom, to be able to support the SEM-students in the classroom ([6, 9]). This possibility seems to provide time for the mathematics teacher to be able to support the SEM-students in the classroom ([6, 8-9]). Jonna is also highlighting the difficulty of being alone with all the students in the classroom ([7]). This talk about time and support is in spatial inclusion, since it is about being in the classroom supporting all students.

There is also talk about self-esteem and self-confidence in this community.

[10] That they [the SEM-students] can grow some confidence […] - Look what I have done today, I managed to do that - it is the far **biggest** thing, to succeed. (Ellie, 2012-11-15)

[11] Get the students to think that it is fun […] and make them to grow. (Jonna, 2013-06-16)

[12] He has received quite a good self-confidence when he has been in the classroom and managed things. (Jonna, 2013-06-16)

How, as a teacher, to develop the students' confidence in connection to mathematics is discussed [11-12]. Jonna talked about being in the classroom as one way to get self-confidence [12]. This talk about self-esteem and self-confidence does not fall under any of the three categories of inclusion. Hence the data indicates a new category of inclusion is needed to describe this.

Community of special education needs in mathematics
In the *community of special education needs in mathematics* there is talk about mathematics teaching in relation to SEM-students.

[13] […] it is difficult to satisfy all the students in the classroom, to give tasks they can work with and understand. It takes time to find. (Anna, 2011-10-03)

[14] When you are alone in a class with over 20 students, 20 to 25, it´s not that easy. (Amanda, 2012-09-03)

[15] You must be aware of that you need to have the math lesson at three different levels and you need to have education at a really basic level. (Ellie, 2011-11-07)

[16] It is a lot to take into consideration. (Jonna, 2013-06-16)

Anna, Amanda, Jonna and Ellie are highlighting the complexity of mathematics education and to be able to give every student support in their learning in the classroom to avoid marginalisation ([13-16]). Although Amanda just refers to being alone with many students, and does not connect it to any content. This talk about the complexity of teaching mathematics, to be able to cope with different levels at the same time has a clear connection to the mathematics; hence it is within didactical inclusion.

Another issue, discussed by the teachers, is if the SEM-students should be in the classroom or with the remedial teacher.

[17] It is valuable if students can join the introductions so they know what we are going to work with. I'd like them to be, even if they go away from the classroom later. (Anna, 2011-10-03)

[18] They should be in the classroom as much as possible, but I think it is good that they can go [to the remedial teacher in mathematics] a session per week to be lifted. It provides synergy effects in the classroom. (Ellie, 2012-11-15)

[19] […] that you [the SEM-student] actually work with the class, when you [the remedial teacher] might normally pick them out [of the classroom] and like… ehh ... And Gabriel [a SEM-student] has been good at saying himself when he thinks it is good and when not good [to be in the classroom]. (Jonna, 2013-06-16)

Here Anna, Jonna and Ellie agreed that the students ought to be in the classroom even though they are talking about support out of the classroom as well ([17–19]). Jonna also mentioned listening to the students, to what do they want ([19])? The talk about being in or out of the classroom fits within spatial inclusion since it refers to the placement. Listening to the students does not fall under any of the three categories of inclusion.

Ellie talked about courses in connection to a mathematical content.

[20] She needs this course […] about counting methods and number sense. (Ellie, 2012-11-15)

The talk about courses here fits both within spatial and didactical inclusion because the students need to be taken out of the classroom to get support regarding specific content, in this case counting methods and number sense ([20]).

Different representations to be able to support the SEM-student were also discussed.

[21] She (a SEM-student) often uses money as a concrete material, manipulatives[18] has been really hard for her, but money she understands and has used. (Ellie, 2011-11-07)

[22] […] some [of the students] had a really hard time to understand [the mathematics], sort of. What I think is difficult is to explain it, and I get frustrated when I cannot explain! For, in that I do not have all these different ways to explain that maybe a math teacher has, in the end [I] become easily frustrated. (Amanda, 2012-09-03)

[23] They need to have concrete material that the others [classmates] have passed a long time ago. […] Then it can feel good that they may do so without receiving any comments. (Jonna, 2013-06-16)

[24]
Ellie:	Money has always been difficult [for a SEM-student].
Researcher:	Aha …
Ellie:	Mmm … She thinks it 's really hard, she is not able to break it down into two tens [referring to 20], or four fives, and that there may be a ten and two fives, and so on. It … It is really hard for her.
Researcher:	Aha
Ellie:	Hence, this classic teaching example related to money that most kids actually understand, it is really hard for her to understand.
Researcher :	Oh well!
Ellie:	[…] then you have to get 20; she still needs a material you can pick with. It [money] becomes abstract for her.
Researcher:	It is not a good representation for her, money?

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[18] Manipulatives is an overarching term for working and visualisation materials and visual charts and diagrams as a means to represent mathematical knowledge (Nührenbörger & Steinbring, 2008).

Ellie:　　　　　　No, it´s not. It is better with blocks or buttons or whatever. Money is abstract for her for some reason. It is … […] You believe that it should be simple and ... it takes a while before you realise that, oh my God, it's better to get some beads for her, and get her to share and put into groups. She's simply on a one-unit-level. She needs 20 beads and put them in two groups of tens, then she sees it. If you put two ten-crowns[19], she cannot see that there are ten one crowns within the ten. (2012-11-15)

Ellie talked about money as a representation ([21,24]), and to her it is better than other materials to elaborate with ([21]). But in [24] Ellie says that money is not a good representation because it is too abstract for the student, and the student cannot understand this representation and transform it into symbols. It is actually the same student she is referring to both in [21] and [24] indicating that she later on understands the problems the student is facing more deeply and what kind of representations the student understands. This talk about representations fits in didactical inclusion – finding good representations supporting the students learning. Amanda is expressing her frustration about not to being able to find different representations in explaining the mathematical content for students who are struggling ([22]). Jonna is discussing different representations together with her thoughts about being exposed in the classroom when needing different support than the classmates. She points out that it is important to be responsive to the student ([23]). This discussion fits in didactical inclusion but the part about being responsive does not fall under any of the three categories of inclusion.

In community of special education needs in mathematics time is an issue.

[25] It's difficult when we do other things in the classroom and they [the SEM-students] are out and work with what they need. No matter how we do, they will never catch up, because when they have done something out there [with the remedial teacher] the others have come further. It is the hard nut to crack. (Anna, 2011-10-03)

[26] Perhaps it is a little easier as special pedagogue to have two or three different levels within the same [classroom], thus you only have three students and it is easier to provide help right away than when you have 20–25 [students]. (Amanda, 2012-09-03)

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[19] A ten-crown is a Swedish coin

[27] It was actually a boy that, little half aloud, said: [when a SEM-student went out] […] "Oh, good, now we can get some more help." So I think they [the other students in the class] also felt that now may Ms [Ellie] also have some time for us, because she took a lot of my time in the classroom. (Ellie, 2012-11-15)

[28] He has so much basic things left, before he can move forward a notch. (Jonna, 2013-06-16)

The teachers' talk about time in relation to SEM-students has differences. Anna and Jonna talked about time in relation to the SEM-students and learning in mathematics ([25, 28]). Jonna talked about consolidation and understanding of basic mathematical notions, procedures and methods ([28]). Anna compared the tempo of learning of the SEM-students with that of the other students ([25]). Amanda and Ellie talked about time to reach everybody in the classroom ([26, 27]). The time issue seem to be stressful for the mathematics teachers. Time does not fit in any of the three different forms of inclusion, but it seems to be a prerequisite to didactical inclusion – to have time to reach the students, and help them to reach understanding for basic mathematical notions.

Community of mathematics at Oakdale Primary School
The *community of mathematics at Oakdale Primary School* is not that visible in the case of mathematics teachers. Hence, the mathematics teachers are peripheral members of this community.

There was some talk about teacher knowledge in mathematics education.

[29] […] it should be a requirement that you have education to get to work with subjects [mathematics] in school. It is … not that easy to step in and be a teacher. Many believe it until they have got an education. I was among those ... (Ellie, 2012-11-15)

Ellie expressed the need for teachers to have knowledge in mathematics education, because it is a complex assignment ([29]). Since teacher knowledge in this context is strongly connected to mathematics it is a factor in didactical inclusion.

There was also discussion about cooperation in the planning of lessons in mathematics at the school.

[30] It would have been great if we [she and Barbara] had more time to plan together; now we only wave and speaks quickly. (Anna, 2011-10-03)

[31] You are fairly alone in the planning [of mathematics lessons], Amanda and I try to talk, but we have very different opinions about how to work with the math. (Ellie, 2012-11-15)

[32] I have collaborated with [Barbara], and been talking and been having a dialogue all the time... […] Things that I do in the classroom, she can capture in her [lessons]. (Jonna, 2013-06-16)

The teachers talked about cooperation in different ways. Anna talked about the lack of cooperation with Barbara, the remedial teacher in mathematics ([30]). Ellie spoke about cooperation when planning mathematics lessons together with other mathematics teachers at the school ([31]). Jonna is speaking about an efficient cooperation with Barbara ([32]). This could indicate that there has been a development over time (Anna talked about this in 2011 and Jonna talked about it in 2013) in cooperation with the remedial teacher in mathematics. Cooperation does not fall into any of the three categories of inclusion.

Summary
For a summary of issues influencing inclusion in mathematics that emerged in the case of Mathematics teachers, see figure 11.

INCLUSION ⟍ COMMUNITY OF PRACTICE	SPATIAL	DIDACTICAL	SOCIAL	OTHER ISSUES
Community of mathematics classrooms	Time and support	Challenge the SEM-students		Self-esteem and self confidence
Community of special education needs in mathematics	In or out of the classroom Courses	Teaching mathematics Courses Representations		Listening to the students Being responsive Time
Community of Mathematics at Oakdale Primary School		Teacher knowledge		Cooperation
Community of student health				

Figure 11. Overview of issues regarding inclusion in mathematics from the case of Mathematics Teachers

As seen in Figure 11, courses appear in both spatial inclusion and didactical inclusion in community of special education needs in mathematics. In this case, the same code, *courses*, can hence mean different things within the same community. In inclusion (exclusion), courses only refers to the placement of the student. In didactical inclusion, courses refers to the students receiving access to the mathematics taught. Even *time* is a code that appears in both community of mathematics classrooms as spatial inclusion and in community of special education needs in mathematics as other issues. When it is referred to as spatial, it refers to receiving *time and support* in the classroom. When time is a code in other issues, it is a prerequisite for being able to support the SEM-students.

Other issues that the framework of Asp-Onsjö does not include are *self-esteem* and *self-confidence*, *listening to the students*, *being responsive* and *cooperation*.

5.2.3 The case of the principal

The principle, Conrad, is visible in all four of the communities at Oakdale Primary School, although with different emphasis.

Community of special education needs in mathematics

In the community of special education needs in mathematics Conrad talked about teacher knowledge.

> [1] You have [at the school] a special pedagogue or another teacher with expertise [in mathematics education]. (2012-08-15)

Conrad highlights the competence of the remedial teacher or mathematics teacher ([1]). This can be seen as teacher knowledge. Here he is referring to expertise in mathematics education, which fits within didactical inclusion.

Conrad discussed courses in the community of special education needs in mathematics.

> [2] We solve it [special education in mathematics] in a good way because we have courses (2012-08-15).

> [3] […] in a course, maybe you go away somewhere else and do it this, drilling, particular hard (2012-08-15).

> [4] […] the student health team staff will do accrual work with courses in mathematics (from the documents; Oakdale Primary School´s local plan for systematic quality and Results in Focus).

Conrad talked about courses ([2]) and "drillings" in connection to courses ([3]), which means that the students get a lot of education within a special field of mathematics to be able to grasp it. Conrad also spoke about creating courses for the SEM-students in order to get them more included in the mathematics, hence in this context courses is in didactical inclusion. This is reified in the documents Oakdale Primary School´s Local Plan for Systematic Quality and Results in Focus ([4]).

In this community of special education needs Conrad also talked about SEM-students being in the classroom.

> [5] […] being as much as ever possible in the group. (2012-08-15)

[6] Culturally [at the school] students who are struggling have been picked out [of the classroom]. Occasionally you need to do that, though I want to turn things around. My remedial teachers and special pedagogues have got this. (2012-08-15)

[7] [...] it is not okay to hand over the responsibility to someone else. (2012-08-15)

Conrad refers to spatial inclusion being physically in the mathematics classroom ([5]). He is determined to change the prior school culture of excluding SEM-students from the classrooms ([6]). Conrad is implying that the intention of spatial inclusion has not really been achieved in the actual teaching at the school ([6]). Conrad emphasises that the person responsible for the SEM-students is not the remedial teacher; it is the responsibility of the regular mathematics teacher ([7]).

Community of mathematics at Oakdale Primary School

The teaching of mathematics on an overall level was discussed in the community of mathematics at Oakdale Primary School.

[8] […] we should do the same things [in mathematics]. (2012-08-15)

[9] Everything for increased goal achievement […] if you're going to have an ulterior motive, this [goal achievement] is it, because I believe in increased goal achievement if you actually have a discussion between the teachers who work in these grades and then you take a wider perspective, the F-3 and 4-6. (2012-08-15)

[10] Then you get to this collaboration and use the competences that you have in the **little** unit [meaning the teachers working with, for example, year 3]. (2012-08-15)

[11] […] attended courses and further developed themselves. (2012-08-15)

[12] […] it is organised as follows. Wednesday one, then the teacher teams[20] meet. Then they have the opportunity to meet, they can sit for several hours. Then [team] F-3, or at least 1-3, […] and [team] 4-6. Then if they split into smaller units, it is up to them. They solve it

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[20] In Swedish schools there are teams of teachers that cooperates regarding the students in different issues (lärarlag). Mostly it is organisational and pedagogical issues in focus in the team discussions.

themselves. Wednesday 2 – subject meetings. […] Wednesday 3, then I meet team leaders and I have short information […] And then it start again. (2012-08-15)

[13] I thought that when you have subject meetings … you have some who are leaders for math and science, they are in charge of the dialogue, picks out issues, sews it together, spread the word, keep order, so you don't sit and talk about something else here. Instead, now it is a subject meeting and that is what we should have. (2012-08-15)

[14] […] we will have a discussion leader, and that you actually base the discussions on pedagogical issues. (2012-08-15)

One of the aims of the reorganisation at the school is to reach consensus about the mathematics teaching ([8]); In fact, the main reason for the reorganisation is to have increased achievement of goals ([9]). Conrad also underlined cooperation and utilisation of competences in this community of practice as well as in community of special education needs in mathematics ([10]). Conrad mentioned that the teachers at Oakdale Primary School are competent in mathematics education ([11]). To develop the mathematics teaching and science there are subject meetings every third week ([12]). Conrad mentioned the subject meetings as an opportunity for mathematical discussions ([13]). He also mentioned the need to have guidance in the discussions ([13-14]). He spoke in the future tense since this guidance did not exist at the time ([14]). These three issues discussed by Conrad within community of mathematics at Oakdale Primary School, development of mathematics teaching, teacher knowledge and mathematical discussions, fit within didactical inclusion. It refers to the content and the teaching of the mathematical content on an overall level and developing mathematics education at Oakdale Primary School.

Community of student health at Oakdale Primary School
Official plans and procedures were discussed in the community of student health at Oakdale Primary School.

[15] Then you step into the circle I have made, with four steps. It is an educational mapping rendering in an individual action plan; then it is the evaluation of the individual action plan, actions and follow ups. (2012-08-15)

[16] The pedagogical plans also form the basis for one semester or for goal achievement in an academic year, for the individual student and

the class. You do it once; you do it when you do the pedagogical planning. It is also the base when you write your actions; you pick your goals […] for the individual action plans from the pedagogical plan. (2012-08-15)

More specifically, what and how to write the individual action plans and how to execute them were important ([15-16]). The pedagogical plans made by the teachers were, according to Conrad, connected to the individual action plans made for the SEM-students ([16]).

Communities of mathematics classrooms
In the communities of mathematics classrooms Conrad is least visible. He is a peripheral member of these communities.

Here Conrad spoke about responsibility and knowledge of the mathematics teacher.

> [17] You as a pedagogue actually own your class with all your students. You can't leave the responsibility of any student to someone else. (2012-08-15)

> [18] You think about math in a different way [at Oakdale Primary School]; one need not to be traditional, stand at the blackboard or count in the book […], but they are using laboratory material too and even [go] outside. (2012-08-15)

> [19]
> Conrad: I have tried to do this as far as it ever goes. I have tried to spread the competence in a grade. But I don't switch the teachers now, in grade 6, for example.
> Researcher: No, no.
> Conrad: It will surely come later on. [In grade] 5 I have also made a change. And then I have not fully but almost, in math and science. I have Swedish and English and social science ehh ... I have in some more languages. Since it may be that there are two math and science [teachers] or two Swedish and English [teachers] in one team, but I have […]. I have competence in all, if I remember correctly, from year 2 up to year 6. (2012-08-15)

Conrad talked about didactical inclusion from an overall perspective ([17]). This is consistent with the talk of responsibility of the mathematics teacher

in the community of special education needs in mathematics, indicating that it is the mathematics teacher who is responsible for the goal achievement in mathematics. In these communities of practice, as well as in communities of special education needs in mathematics and mathematics at Oakdale Primary School, the competence in mathematics is an issue ([18]). Conrad seemed to equate mathematics within a teaching degree with competence ([19]). Conrad is struggling to achieve as much mathematical competence in every classroom as possible ([19]) and doing that is in line with the teacher certificate that was introduced in Sweden 2012.

Summary

The summary in Figure 12 is illustrating issues influencing inclusion in mathematics emerging from data form the case of the principal.

INCLUSION ⟍ COMMUNITY OF PRACTICE	SPATIAL	DIDACTICAL	SOCIAL	OTHER ISSUES
Community of mathematics classrooms		Teacher knowledge		
Community of special education needs in mathematics	In or out of the classroom	Teacher knowledge Courses		
Community of Mathematics at Oakdale Primary School		Development of mathematics education Teacher knowledge Mathematical discussions		
Community of student health				Individual action plans

Figure 12. Overview of issues regarding inclusion in mathematics from the case of the principal.

In Figure 12, it can be seen that *teacher knowledge* occurs in three of the four practices and seems to be of great importance for the principal. An issue that the framework of Asp-Onsjö doesn't grasp is individual action plans, which in a way can be seen as an overall notion because it is supposed to contain an overarching view of the situation of the student.

5.3 Comparison of the communities and the cases

In the first part of the results (5.1) the similarities in the form of *mutual engagement*, *shared repertoire* and *joint enterprise* regarding teaching in mathematics at Oakdale Primary School resulted in four identified communities of practice. In that part of the results *inclusion in mathematics* was not focused on. In the second part of the results (5.2) *inclusion in mathematics* was the focus in three cases, that of Barbara, the mathematics teachers and the principal. Here the four communities of practice were in the background. In this part of the results (5.3) similarities and differences found in the cases regarding *inclusion in mathematics* will be connected to the four communities of practice regarding teaching in mathematics at Oakdale Primary School (see Figure 13, 14, 15 and 16).

Community of special education needs in mathematics
In Figure 13, all the issues regarding inclusion in the community of special education needs in mathematics is summoned.

INCLUSION / CASE	SPATIAL	DIDACTICAL	SOCIAL	OTHER ISSUES
Barbara	Stigmatisation Courses Change roles in the classroom In or out of the classroom	Stigmatisation Courses Intensive teaching Tasks Connection of content Prepare and Immerse Strategies Recognising similarities Representations	In or out of the classroom	Student participation
Mathematics teachers	In or out of the classroom Courses	Teaching mathematics Courses Representations		Listening to the students Being responsive Time
Principal	In or out of the classroom	Courses Teacher knowledge		

Figure 13. Overview of issues regarding inclusion in mathematics in community of special education needs in mathematics.

In looking at similarities in community of special education needs in mathematics, *courses* are mentioned within didactical inclusion in all three cases. Also the issue of being *in or out of the classroom* is mentioned by all three cases in community of special education needs in mathematics. In this community both Barbara and the case of mathematics teachers mention *representations*. Both mathematics teachers and the principal mention *teacher knowledge* in the community of special education needs in mathematics.

In looking at differences in the community of special education needs in mathematics, Barbara highlighted *stigmatisation, intensive teaching* and *recognising similarities* as well as *prepare* and *immerse*. She is also mentioned *tasks* and *strategies*. Even *didactical discussions* and being able to *change roles in the classroom* was highlighted by Barbara in this community. These were not mentioned in the other cases; hence this is not a shared repertoire in this community of special education needs in mathematics, but is something that is important for Barbara in the process of inclusion in mathematics. This can affect the process of inclusion in mathematics at Oakdale Primary School. Barbara highlighted issues of *student participation* and *student voices* in this community. The case of mathematics teachers highlighted *listening to students* and *being responsive*. These issues does not fit into any of the forms of inclusion of Asp-Onsjö, but seem important for the process of inclusion. Hence, there might be a need for a category containing these issues. In the case of mathematics teachers *time* was an issue of inclusion in mathematics. Time does not belong in any of the three forms of inclusion, but it seems to be a prerequisite for being able to develop didactical inclusion.

Community of mathematics classrooms

Figure 14 shows issues regarding inclusion in the community of mathematics classrooms.

INCLUSION / CASE	SPATIAL	DIDACTICAL	SOCIAL	OTHER ISSUES
Barbara	Participation in classroom teaching Flexible solutions	Connection of content Participation in classroom teaching Approaches and material Flexible solutions Discussions Teacher knowledge Representations and tasks		
Mathematics teachers	Time and support	Challenge the SEM-students		Self-esteem and self confidence
Principal		Teacher knowledge		

Figure 14. Overview of issues regarding inclusion in mathematics in community of mathematics classrooms.

Looking at similarities between the cases shows that both Barbara and the principal talked about *teacher knowledge*.

Looking at differences within this community of practice shows that Barbara talked about *connection of content* and *participation in classroom teaching* as well as *approaches and material*, *flexible solutions* and *didactical discussions*. These were not mentioned by the other cases. The case of Barbara also talked about *representations and tasks*. In this community, the case of mathematics teachers spoke of *challenge the SEM-students*, which was not mentioned by the other cases. The case of mathematics teachers also highlighted *self-esteem and self-confidence*, issues which do not fit into any of the three forms of inclusion and signals a need for a new category.

Community of mathematics at Oakdale Primary School

Figure 15 displays all the issues regarding inclusion in the community of mathematics at Oakdale Primary School are shown.

INCLUSION ⟍ CASE	SPATIAL	DIDACTICAL	SOCIAL	OTHER ISSUES
Barbara	Courses	Development of mathematics education Discussions Mapping knowledge		Working alone Cooperation Organisation Mandate
Mathematics teachers		Teacher knowledge		Cooperation
Principal		Development of mathematics education Teacher knowledge Mathematical discussions		

Figure 15. Overview of issues regarding inclusion in mathematics in community of mathematics at Oakdale Primary School.

Reflecting upon similarities in this community of practice shows that both Barbara and the principal talked about *development of mathematics education*. In connection to this, the case of mathematics teachers talked about *teacher knowledge*. The case of mathematics teachers talked about *cooperation* in this community of practice, which was also mentioned by Barbara. This does not fit into any of the forms of inclusion, but it seems to influence the process of inclusion.

Looking at differences reveals that Barbara, who in this community of practice talked about *working alone* and having a *mandate*, which is not visible in the other cases. The case of Barbara also mentioned *organisation* in this community, which is not visible in any other case or community of practice. *Organisation* does not fit into any of the forms of inclusion, but seems to influence the process of inclusion at Oakdale Primary School. Barbara is also talked about *courses,* which in this context fit within spatial inclusion because she refers to being out of the classroom. A last difference in this community of practice is *mapping knowledge* in mathematics, which was spoken of in the case of Barbara, but not visible in any other case.

Community of student health
In Figure 16, all the issues regarding inclusion in the community of student health are shown.

INCLUSION \ CASE	SPATIAL	DIDACTICAL	SOCIAL	OTHER ISSUES
Barbara		Mapping knowledge		Preventive work
Mathematics teachers				
Principal				Individual action plans

Figure 16. Overview of issues regarding inclusion in mathematics in community of student health.

In this community of practice there were no similarities between the cases regarding inclusion in mathematics. In this community of practice Barbara talked about *mapping knowledge* and *preventive work*, which not is seen in any other case. The case of the principal talked about *individual action plans* in this community of practice, which was not visible in any other case. As mentioned, it is notable that the teachers in the case of mathematics teachers are not members of the community of student health.

5.4 Inclusion in mathematics

Even though there are differences between the communities of practice regarding inclusion in mathematics, there are also similarities. In both the community of mathematics classroom and the community of special education need in mathematics Barbara talked about *in or out of the classroom* and about both students and herself as a remedial teacher being in or out of the classroom. When talking about students, *in or out of the classroom* can be an issue of social inclusion for Barbara. In the community of special education needs in mathematics both the principal and the case of mathematics teachers also talked about *in or out of the classroom*. Hence, *in and out of the classroom* is a reification of inclusion in mathematics in both the community of mathematics classroom and the community of special education need in mathematics, but the data imply this reification is twofold; it concerns both the students and/or the remedial teacher.

Teacher knowledge is visible in the community of mathematics classroom, community of mathematics at Oakdale Primary School and in the community of special education needs in mathematics. Barbara talked about *teacher knowledge* in the community of mathematics classroom and the case of mathematics teachers mentioned it in the community of mathematics at Oakdale Primary School. The principal spoke about *teacher knowledge* in the community of special education needs in mathematics, the community of mathematics at Oakdale Primary School and the community of mathematics classroom. In relation to teacher knowledge, *development of mathematics education* is highlighted. This is seen in the community of mathematics at Oakdale Primary School in the cases of both Barbara and the principal. This can be related to the case of the mathematics teachers' talk about *teaching mathematics* in the community of special education needs in mathematics. *Teacher knowledge* is reification in the communities of mathematics classrooms, the community of mathematics at Oakdale Primary School and in the community of special education needs in mathematics. The members of these communities of practice negotiate in a process what kind of teacher knowledge is necessary to be able to support the SEM-students. The notion *development of mathematics education* is a reification in form of a process in the community of mathematics at Oakdale Primary School.

Representations are visible in the community of special education needs in mathematics in the cases of both Barbara and mathematics teachers. Barbara also mentioned *representations* in the community of mathematics classroom. Hence, *representations* are reifications in the community of special education need in mathematics.

Time and *time and support* are mentioned by the case of mathematics teachers both in the community of special education needs in mathematics and the community of mathematics classroom, although there were differences in how they spoke about it. In the community of special education needs in mathematics they talked about to give time for the SEM-students to learn mathematics and to have time for them, which seems to be a prerequisite for didactical inclusion. In the community of mathematics classroom they talked about having time and extra support in the classroom when teaching mathematics, which is about being able to have all students in the classroom – spatial inclusion.

Courses were mentioned by all three cases in the community of special education needs in mathematics. *Courses* are talked about as a mean for including all students in the mathematics taught – didactical inclusion. But it is also talked about as spatially be out of the classroom. *Courses* were also mentioned in the community of mathematics at Oakdale Primary School,

as spatially excluding students from the classroom. At the beginning of the study *courses* were not mentioned much, but they were mentioned more over time by Barbara, implying that courses were a notion Barbara used as a boundary object. Hence, Barbara was a broker between the communities of special education needs in mathematics and mathematics at Oakdale Primary School.

Mapping knowledge was highlighted by the case of Barbara in the community of student health as well as in that of mathematics at Oakdale Primary School. It seems that mapping knowledge in mathematics is a part of the didactical inclusion. To be able to support the SEM-students, the teachers and the remedial teacher need to know the students' knowledge in mathematics. The materials used to map knowledge in mathematics *DIAMANT* and "Förstå och använda tal" (McIntosh, 2008) (Case of Barbara, [95]) can be seen as boundary objects between the communities of student health and mathematics at Oakdale Primary School.

5.5 Summary Results and Analysis

In this research, four communities of mathematical practice were identified at Oakdale Primary School: Community of mathematics classrooms, Community of special education needs in mathematics, Community of mathematics at Oakdale Primary School and Community of student health. In these communities the process of inclusion is visible in both similarities and differences and hence, *inclusion in mathematics* came to the fore.

In the community of special education needs in mathematics *courses* and *in and out of the classroom* were visible in all three cases. *Representations* were visible in two of the three cases (Those of Barbara and the mathematics teachers). In the case of Barbara *student participation, stigmatisation, intensive teaching, change roles in the classroom, tasks and strategies, recognising similarities, connection of content* and *prepare and immerse* were visible in this community of special education needs in mathematics. In the case of mathematics teachers, *listening to* students, *being responsive, time* and *teaching mathematics* was visible. In the case of the principal *teacher knowledge* was visible.

In the community of mathematics classrooms there were no similarities between the cases regarding inclusion in mathematics. In the case of Barbara *participation in classroom teaching, flexible solutions, connection of content, approaches and material, didactical discussions, teacher knowledge, representations* and *flexible solutions* were visible. In the case of mathematics

teachers, *time and support* in the classroom, being able to reach and *challenge the SEM-students* and *self-esteem and self-confidence* were visible. In the case of the principal *teacher knowledge* was visible.

In the community of mathematics at Oakdale Primary School *development of mathematics education* was visible in two of the cases (those of Barbara and the Principal) *Cooperation* was also visible in two of the cases (the case of Barbara and the case of mathematics teachers). In the case of Barbara *working alone*, *cooperation and discussions, mandate*, *courses* and *mapping knowledge* were visible. In the case of mathematics teachers and the case of the principal, *teacher knowledge* was visible. *Mathematical discussions* were seen in the case of the principal. The case of Barbara and of the principal highlighted discussions, but there was a difference. The principal highlighted mathematical discussions, while Barbara highlighted discussions together with cooperation.

In the community of student health there were no similarities between the cases regarding inclusion in mathematics. *Mapping knowledge* and *preventive work* was visible in the case of Barbara. *Individual action plans* were visible in the case of the principal. Notable is that no members from the case of mathematics teachers participated in this community.

6. DISCUSSION

6.1 Inclusion in mathematics at Oakdale Primary School

As seen in the result, inclusion in mathematics at Oakdale Primary School consists of several integrated components that emerged from the communities of practice and the different cases. Using the terms in the theoretical framework with the three different components spatial, didactical and social inclusion, the focus is on didactical inclusion in all communities and in all cases. Here several categories have been identified, for example, *representations* and *teacher knowledge* in mathematics education. This is visible in the case of Barbara and in the case of mathematics teachers. In spatial inclusion the focus is on *courses* and where the SEM-students are, in the classroom or with the remedial teacher alone or in a small group. This appears in all cases. Social inclusion does not occur too often in the data. Though, one thing mentioned is a student's wish to be a part of the community of the classroom, which can be interpreted as social inclusion. This is visible in the case of Barbara.

The different communities of practice at Oakdale Primary School all have an overarching purpose, to enhance all students learning in mathematics. But, as seen in the results, the communities of practice at the school also have different purposes and a comparison of the identified communities of practice shows there are differences regarding inclusion in mathematics. The discrepancy between purposes can create problems in the development of the process of inclusion in mathematics but it may also create opportunities for discussions and benefit the process. Perhaps there have to be some differences since the communities of practice have somewhat different agendas; for example, in the community of mathematics classroom all students learning is in focus, while in the community of

special education needs in mathematics SEM-students learning is in focus. The problem that may occur is lack of communication between the members of communities of practice. This could increase the differences and there would be no consensus regarding inclusion in mathematics. More interconnections between the communities of practice at Oakdale Primary School are needed to enhance the process of inclusion in mathematics and enhance the constellation. Boundary objects and brokers can enhance these interconnections: for example, Barbara's brokering between the community of special education needs in mathematics and the community of mathematics classrooms. Like Meaney and Lange (2013) I see a problem with, what they call transition between contexts, which in this research this is understood as transition between the communities of practice. Problems with the transition could be one of the reasons why Barbara struggles to be a full member of the community of mathematics classrooms (Barbara, [57-58]). The transition can also concern the students. This is most obvious in the transition between communities of mathematics classroom and of special education needs in mathematics. It seems that, according to the teachers, the students do not always recognise the mathematical content that is worked with in the different communities as being the same; for them it is not the same (case of Barbara, [43]).

There are also differences between the cases regarding inclusion in mathematics. One difference is in the talk about representations in the community of special education needs in mathematics. The teachers and Barbara both talk about representations but in a slightly different way. Barbara talked about the need to use representations in different registers and as a teacher to be aware of the difficulty in moving between registers (Barbara, [45-46]). Barbara is hence talking about the function of the representations, where the constructing function (Ainsworth, 2006) is in focus. The case of mathematics teachers only talk about concrete material used to support learning (mathematics teachers, [21-24]). Hence, the talk about representations differs and the students might not be able to see the interrelationship of the representations since the mathematics teachers might not be aware of the importance of interrelationships between representations. Problems that may occur when Barbara and the mathematics teachers think they talk about the same things are misunderstandings and failure to develop a shared repertoire. This can increase the differences and there will be no consensus regarding inclusion in mathematics at Oakdale Primary School. Benefits of the discrepancy between cases can be that the different needs of the SEM-students are met depending on the situation and the discrepancy can create space for discussions in the communities of practice at the school and the process of inclusion can be driven forward. This would require time for discussions, something that the data showed there is a lack of.

6.1.1 Conceptual framework – Inclusion in mathematics

A conceptual framework has emerged trying to explain the process of inclusion in mathematics at Oakdale Primary School and answer the research question "What can inclusion in mathematics be in primary school?" This framework further develops the framework of Asp-Onsjö (2006) regarding inclusion and focuses on mathematics. It seeks to increase our understanding of how SEM-students can participate and how to enhance their participation in the school mathematical practice, from a teacher perspective. Inclusion is a set of principles "embodied in different ways in different contexts" (Dyson, 2014, p. 282). This research has been an attempt to operationalise inclusion in mathematics by providing a set of principles in a conceptual framework.

Courses, intensive teaching, in and out of the classroom (both the remedial teacher and the SEM-students), listening to the students and *change roles* signals the need for flexible solutions in the school. Courses in mathematics, depending on the demands in the classes, are one way to contribute to flexible solutions, which can be interpreted as a flexible kind of internal differentiation. Working intensely with some students during a period of time is also a part of a flexible solution. Even this can be interpreted as internal differentiation over a shorter period of time. To be able to teach SEM-students both in the regular math classroom and sometimes alone is another way to be flexible. All this put demands on the remedial teacher in mathematics to be able to be both in and out of the classroom and to be able to change roles with the regular mathematics teacher. Sometimes the remedial teacher is the mathematics teacher and vice versa. This also puts demands on the mathematics teacher in communicating with the remedial teacher about these roles. This requires the teachers to be able to make transitions between the different communities of practices smoothly.

To be able to fulfil these demands, the organisation and the members of the organisation needs to be dynamic or practice what I call *Dynamic inclusion.*

Representations, *tasks*, *strategies and generalisations*, *didactical discussions*, *teacher knowledge, recognising similarities, connection of content* and to be able to *reach and challenge the SEM-students* are all very much connected to the mathematical content. To be able to give the SEM-students access to the mathematical content and to reach and challenge them, the teachers in

mathematics need to have knowledge of different representations and tasks that invites the students to be part of the mathematics. If the tasks are inviting the students their achievement in the classroom can be enhanced ([28], the case of Barbara). The teachers need to be able to provide different representations from both the same and different registers and be able to help the students connect these representations. The teachers also need to be aware of the different functions (Ainsworth, 2006) the representations have in order to adjust to the particular student. As Liasdidou (2012) highlights, flexibility in teaching is required to have an effective teaching. This flexibility in teaching of mathematics requires knowledge of representations within different registers and the functions of the representations. There is a need to connect between the mathematical content taught in the classroom and the mathematical content that the remedial teacher works with. There is also a need to help the SEM-students recognise similarities in mathematics in other situations, to help them make the mathematics generic and able to recognise it in different situations. Here representations and strategies used in a specific situation needs to be used in another situation in order to help the student to be able to recognise similarities.

I call this approach *content inclusion[21]*.

Asp-Onsjö (2006) writes about didactical inclusion and, as mentioned, the results of this study appeared mostly in didactical inclusion. Some of the results also appeared in spatial inclusion. The categories *dynamic inclusion* and *content inclusion* are influenced by spatial and didactical inclusion, but there are differences. *Dynamic inclusion* refers to where the SEM-students are at some level, but also to how to work with the organisation at the school, working with flexible solutions, depending on the need of the SEM-students at the moment. *Content inclusion,* like didactical inclusion, concerns the content of the subject, in this case mathematics. Since it is mathematics, this inclusion is specific because of the nature of mathematics. A specific feature of mathematics that needs to be considered is that it is an abstraction that needs to be represented. To be able to reach all students the mathematics teacher needs to be able to represent mathematics in different ways and to choose tasks that helps the student recognise similarities and form strategies. To reach and challenge the SEM-students, the teachers in mathematics need to be able to have didactical discussions with both students and colleagues to make connections of content between special education in mathematics and the mathematics taught in the classrooms.

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[21] In this research the content is mathematics.

The framework of Asp-Onsjö (2006) does not cover the issues that appeared in the data concerning participation of the student. Aspelin (2013) calls involving the students in the education, a pedagogical approach[22]. In this research *being responsive*, *listening to students*, *students' participation, self-esteem* and *self-confidence* signal the need for such a category. A teacher in mathematics (both remedial and mathematics teacher) needs to be responsive to the SEM-student, listen to what they want and how they feel regarding mathematics. The teacher needs to let the students take part in the mathematics and decisions regarding the teaching. Also, the teacher needs to be responsive to enhance the students' self-esteem and self-confidence in mathematics.

I call this approach *participating inclusion*.

6.1.2 Two dimensions of dynamic inclusion

In the data, *dynamic inclusion* has two dimensions, a student dimension and a teacher dimension. The student, to be mathematically didactically included, sometimes needs to be physically excluded from the classroom and sometimes physically included. This implies that the remedial teacher also needs to be both in and out of the classroom. Hence, this is a dynamic process and to be able to pursue this process the remedial teacher in mathematics needs to be a member in the community of mathematics classroom, the community of mathematics at Oakdale Primary School, and the community of special education needs in mathematics. Accordingly, working as a remedial teacher in mathematics is a complex assignment that involves cooperation with many teachers and the other remedial teachers, as well as the principal and the student health team. Even though the cooperation with the mathematics teachers is important for Barbara it is almost impossible for her to be able to discuss and plan lessons with all the 18 mathematics teachers at school every week (Case of Barbara, [83]). Consequently, the remedial teacher in mathematics cannot interact with too many classes.

6.1.3 Important aspects

Several categories appearing in the data seem to be important in the process of inclusion in mathematics at Oakdale Primary School. The fear of *stigmatisation* seemed to affect Barbara in the beginning of the study. She worried that students who are physically excluded from the classroom and with her all the time during mathematics lessons become stigmatised.

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[22] In Swedish: pedagogiskt tillvägagångssätt inom relationell pedagogik (Aspelin, 2013)

Later in the study she spoke more about courses and the fear of stigmatisation is not visible anymore. This indicates that courses can be one way to counteract stigmatisation.

Being lonely as a remedial teacher in mathematics seems to influence the process of inclusion in mathematics negatively. Barbara does not have anyone to discuss SEM issues with at the school and she wants to in order to develop, for instance, preventive work and the use of mapping knowledge in mathematics. This is in line with the thoughts Barbara has regarding mandate. She expressed a need for getting a *mandate* from the principal to develop mathematics education at Oakdale Primary School. She does not want to be a "wallflower" (the case of Barbara, [86]); she wants to influence the mathematics teaching at the school to help all students. Hence, mandate is important, and a question that arises is whether a mandate is something to be given or to take.

The *development of mathematics education* at Oakdale Primary School is something both Barbara and the principal talk about and it seem to influence the process of inclusion. The subject meetings are supposed to be a place where this development takes place in form of discussions and didactical issues in mathematics and how to cooperate between teachers and years. Although the subject meetings are scheduled every third week, they do not occur every third week (the case of Barbara, [78]) and there is no one in charge of them, even though the principal plan to have someone in charge of them. The data also shows that *time for discussions and planning* in the daily work is hard to get (the case of Barbara, [71-74]). Hence, the data indicates that the community of mathematics at Oakdale Primary School is a weak community and it is uncertain who or whom are the core member/s. Consequently, this community needs to be strengthened and engage the mathematics teachers at the school in order to develop the mathematics at Oakdale Primary School. The community needs to have core members who are focusing on the development in mathematics overall at the school. Fulfilling Barbara's wish of getting a mandate (the case of Barbara, [86]) might be one way to strengthen the community. The principal has recently made a reorganisation at the school. The main reason for the reorganisation is to get increased goal achievement. Other reasons for the reorganisation are cooperation and utilisation of competences. The reorganisation may be one way of enhancing the development of mathematics education and strengthen community of mathematics at Oakdale Primary School. If the scheduled subject meetings did take place every time with focus on didactical issues in mathematics and the

mathematics teachers were able to prepare and plan together this community would most certainly benefit from that.

To have *time and support* for the SEM-students in the classroom is something the mathematics teachers talk about (The case of mathematics teachers, [6-9, 26-27]) and it can be seen a prerequisite in order to be able to reach the SEM-students. To be able to have two teachers in the classroom is one way to get more time for the SEM-students in the classroom.

6.1.4 Content flow to support recognition of similarities

The data shows three different ways to make *the connection of content* in the teaching in order to help the students *recognise similarities* in mathematics in the community of special education in mathematics and of mathematics classrooms. This can be seen as a way of supporting the transition between these two communities in order to highlight the mathematical content and to enhance recognition of similarities in the content for the SEM-students.

The first way is *preparing* the SEM-students for upcoming mathematical tasks and content in the classroom by working with it with the remedial teacher in advance (case of Barbara, [32] and [37]). The second way is working on the same mathematical issues as in the classroom at the same time, using more concrete representations and basic tasks (case of Barbara, [33-35]) in order to *immerse* the knowledge in mathematics. The third way is working with mathematical content that the students have not grasped after they have worked with it in the classroom, *repeating* it (case of Barbara, [36]). These three aspects, *prepare, immerse and repeat* I call *content flow*. All three can be applied, but depending on the student(s), the mathematical content and the situation, only one or two aspect(s) could be applied. Hence, the *content flow* is used in the teaching of mathematics between the community of special education needs in mathematics and of mathematics classrooms. This is a way of getting the SEM-student included in the mathematics (content) taught.

To strengthen the content flow, the mathematics teachers and remedial teachers need to be aware of these different ways of supporting the SEM-students, but even more important they need to be aware of *how* the mathematical content is worked with in each situation. For instance, they need to know which representations and which tasks are used and whether the same representations could be used in the different situations to enhance the students' recognition of similarities. They need to know whether the support of the recognition of representations in different

semiotic registers could be enhanced. If this were done at Oakdale Primary School, there might be a closer interconnection between the community of special education needs in mathematics and community of mathematics classroom and the representations could serve as boundary objects. The SEM-students can contribute themselves to the *content flow* by suggesting content, tasks and asking questions based on the content in the mathematics taught in the classroom (case of Barbara, [38]). This is also a way of encouraging the students' participation and a way to help students *recognise similarities* of content in different situations. However, it would be important to take into consideration that the attitude of the teachers involved (both mathematics teacher and remedial teacher) affects the work with the *content flow*. If the teachers do not think that the SEM-student should be in the classroom and that the mathematics in the classroom does not concern the SEM-student, it is difficult to discuss and use *content flow*. An example of this occurs in the case of mathematics teachers, where one teacher says, "they [the SEM-students] hadn't learned much by being in here [the classroom]. I could not put the math level on their level".

6.1.5 Summary inclusion in mathematics at Oakdale Primary School

Summing up the discussion of the process of inclusion in mathematics at Oakdale Primary School, three notions act as a set of principles demonstrating inclusion in mathematics at Oakdale Primary School: dynamic inclusion, content inclusion and participating inclusion. These three forms of inclusion naturally interact with each other; there are no clean borders between them and they influence each other. To achieve learning in mathematics we need to let the SEM-student participate in the teaching, using participating inclusion. To be able to do that, the SEM-student needs to be aware of the different ways of receiving support, in or out of the classroom, from the mathematics teacher or the remedial teacher or in a course; this is dynamic inclusion. This also influences the way the content is presented and worked with, depending on needs and wishes of the SEM-student and the mathematical content, which is content inclusion. The three forms of inclusion imply that a teacher and remedial teacher in mathematics need to be aware of all forms of inclusion, because they interact. The notions content flow and recognition of similarities are used to describe ways of supporting SEM-students in mathematics from an inclusive perspective.

In comparing the three forms of inclusion described above to Farrell's (2004) conceptualisation of inclusion (with the notions presence,

acceptance participation and achievement) presence can be seen in the dynamic inclusion, when looking at the students' need and ways to reach the student in the classroom. Acceptance can be interpreted within participating inclusion as responding to diversity in the teaching of mathematics. Participation can be seen in both participating inclusion and content inclusion, since Farrell (2004) defines it as active contribution by the students. Achievement is about students' positive views about themselves and can thus be interpreted both in content inclusion and in participating inclusion.

The case of Barbara is seen as an especially good case. Even though, some things at Oakdale Primary School need to be developed in order to enhance inclusion in mathematics, data indicates that this would not be an easy task in practice. From an inclusive perspective several aspects in the mathematics education in this study are important. One aspect is how the organisation makes room for the development of mathematics education in terms of time for cooperation and discussions. Both Gregory (2006) and Cobb et al. (2013) highlight the importance of a connection between the organisation and the practice, and the results of this thesis point in the same direction. Another important aspect is the need to have a well functioning team working with SEM at the school in order to develop the teaching of SEM and work with prevention. Yet another aspect is the importance of knowledge of mathematics and learning of mathematics of the remedial and the mathematics teachers. And finally, but certainly the most important aspect is listening to the student's voice.

6.2 Reflections of the Design of the study

Looking back, the research conducted in this study has not been easy. If I had done a design study with an intervention my interaction had been easier to justify. It also would been easier to know what to look for. Yet, it would have been difficult to investigate inclusion in mathematics with an intervention, because inclusion in mathematics is not defined in the research, which would have made it difficult to find methods to support, as Ainscow et al. (2006) highlighted when investigating inclusion from an overall perspective. "On one side, it was argued that we should keep an open mind about what we meant by inclusion as we engaged in our research. On the other side, it was suggested that without a clear view of

what we mean by inclusion we had no way of knowing how to support it"
(p. 22–23). Göransson and Nilholm (2014) also highlight the lack of these
studies regarding inclusion. They are hard to find, indicating they are hard
to do. Since this is a case study, these results are specific. However, because
it is an ethnographic study with thick descriptions, the readers can do
transfers, discuss these transfers and the results in relation to their practice
and the results may be made generic.

Using ethnography as a guide allowed me to get hold of the process of
inclusion, as it develops. It allowed me as a researcher to become intimately
close to the practice and go beyond the obvious in the data collection.
Though, with this approach it is hard to make any generalisations.

Ragin's (1992) cross-tabulation offered a way of looking at different cases
as specific or general, as empirical units or theoretical constructs. This
helped me to identify the three cases in the study, the case of Barbara, the
case of the principal and the case of mathematics teachers.

The case of Barbara, which is an extreme case, is viewed as an especially
good and information-rich case in order to get a best-case scenario. If I
hadn't come in contact with Barbara, it would have been difficult to
investigate inclusion in mathematics as a case study. A larger interview
study with remedial teachers in mathematics might have been able to
illuminate the notion of inclusion in mathematics. But even though I
would have collected several remedial teachers' voices, it would have been
impossible to discern a process and it would have been hard to go to the
depth of what the process of inclusion in mathematics can be.

6.3 Implications

...for special education needs in mathematics

One of the major interests when I began this research was to find ways to
have students in SEM included and engaged in the mathematics education
in school. I hope the results of this project will somehow benefit these
students. This research shows that the competence of the mathematics
teacher and even more, the competence of the remedial teacher regarding
learning in mathematics are very important. Another striking fact is that
there is a shortage of educated remedial teachers in mathematics in
Sweden. Often primary schools have special teachers and/or special

pedagogues who do not have any mathematics education. A question that needs to be raised is Do the SEM-students get *the right* support?

The remedial teacher in mathematics needs to have someone to discuss SEM issues with and to have a reasonable number of teachers to cooperate with. There needs to be room for mathematical didactical discussions at the school. Here the organisation is important. It seems that if the schools manage to have a well-functioning organisation for mathematics education it will benefit the SEM-students, as well as all students in mathematics.

From a society perspective the government in Sweden has noticed students declining knowledge in mathematics in the latest PISA-study (National Agency for Education, 2013). This has resulted in a loud debate about mathematics education and SEM in Sweden. Has the efforts made during the latest years not been effective? Is the support the SEM-students good enough? In 2009-2011 large investments were made in mathematics education. The government has funded different project to a total of 352 million Swedish crowns. The latest investment in "matematiklyftet" (in English, "raise the mathematics") is due in 2016 at the cost of approximately 649 million Swedish crowns (Department for Education, 2012). A major issue related to this is whether PISA 2015 will show the efforts have paid off. Other questions that need to be asked are Do the PISA tests test the same knowledge in mathematics as the Swedish mathematics curriculum requires? Are the efforts made in the matematiklyftet in line with PISA? Are the teachers able to translate the issues discussed in the matematiklyftet to the mathematics taught in the classrooms? As a school development researcher told one of the responsible persons of the matematiklyftet at NCM[23]:

> From my experience, teachers really like a "smorgasbord" of ideas. The problem I have found is that it doesn´t change practice sufficiently to make a difference to learning outcomes for students unless they understand why it might be better than what they already do and how they can apply it to the issues their students are experiencing (Jahnke, 2014 p. 68).

Another question is When the time of the matematiklyftet is over, how does the development continue at the schools? Who will ensure the progress of the mathematics education at the schools? What will ensure that this initiative becomes more than a mere flash in the pan? As this

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[23] NCM is the National Centre for Mathematics education in Sweden

study shows it is important to have someone responsible for the process. Hopefully someone has taken the initiative and mandate to proceed with the development at the schools. Oakdale Primary School will have the matematiklyftet in 2016. Hopefully the staff at the school has been able to create an organisation taking care of the knowledge and the process.

The results of this study in connection with PISA suggests that the teachers and remedial teachers in mathematics need to have time to discuss didactical issues and time to use this knowledge in the classrooms in order to reach the SEM-students and optimise learning. The National Agency for Education (2013) suggests that the reasons for the declining results need to be investigated primarily at a system level. If translating the system level into the schools, it may be at the organisation level. As prior mentioned, the organisation at the school is important. If the organisation provides time for the mathematics teachers, and the teachers take the time, to reflect on the mathematics and the students' development of mathematical knowledge, I think that the education will be able to meet the needs of the SEM-students.

From a teacher perspective, including all students in mathematics is about being responsive, competent and able to express and explain the mathematics in many different ways.

...for further research

As mentioned, one limitation is that only the teacher perspective was investigated. Hence, the student perspective needs to be investigated. The students' voices are missing in research regarding inclusion in mathematics. Brolin and Petersson (2013) did their master's thesis investigating this in a small case study and found interesting results pointing to didactical inclusion (Asp-Onsjö, 2006). This needs to be further investigated in a larger study.

One result in this study is that intensive teaching in mathematics might be a way of supporting the SEM-students in the context of inclusion. Intensive teaching in mathematics is almost a blind spot in mathematics education research. There is research concerning intensive teaching in other areas such as reading (e.g. Torgesen, Alexander, Wagner, Rashotte, Voeller & Conway, 2001) but intensive teaching in mathematics needs to be investigated.

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Another result is the importance of the content flow and recognition of similarities in making the knowledge in mathematics generic regardless of the situation. This interesting issue that needs to be further investigated.

Summary in Swedish

Syftet med denna studie är att förstå hur alla elever kan bli inkluderade i matematikundervisningen från ett lärarperspektiv. Forskningsfrågorna, som har gjorts med hjälp av empiri, var: Vad kan inkludering i matematik vara i grundskolans tidigare år och vad påverkar inkluderingsprocessen i matematik? samt, vad förefaller vara viktigt i lärande och undervisning av matematik från ett inkluderande perspektiv? Inkludering är ett komplext begrepp som är svårt att definiera (Brantlinger, 1997; Artiles, Kozleski and Christensen, 2006). Trots det är inkludering ett begrepp som används både inom forskning och i praktik. I utbildningssammanhang är ett välanvänt begrepp och det finns många olika definitioner. Inkludering i skolan är dock fortfarande ett område som är svårförståeligt (Ainscow et al., 2006) och det didaktiska perspektivet gällande inkludering saknas inom forskningen (Karlsson, 2007). I denna studie är det didaktiska perspektivet i fokus, att förstå vad inkludering i matematik kan vara i grundskolan.

Ett deltagande perspektiv används i denna studie, vilket innebär att lärande ses som deltagande. En del av en lärande teori som fokuserar på praktikgemenskaper (Wenger, 1998) har använts tillsammans med ett begreppsligt ramverk som behandlar inkludering ur tre aspekter, spatial, didaktisk och social inkludering (Asp-Onsjö, 2006) för att analysera hur lärare talar om inkludering i matematik. I studien användes etnografi som guide när en stor låg och mellanstadieskola följdes under två år i en fallstudie. Tre fall på identifierades på skolan. Utifrån dessa fall identifierades fyra praktikgemenskaper. Fallens deltagande i dessa praktikgemenskaper i form av inkludering i matematik analyserades. Resultatet visar att processen inkludering i matematik på den undersökta skolan kan beskrivas med tre begrepp, *dynamisk inkludering, innehålls inkludering* samt *deltagande inkludering*. Dessa tre olika former av inkludering interagerar med varandra i en process. Begreppen *innehållsflöde* och *igenkännande av likheter* används för att beskriva sätt att stödja elever i särskilda utbildningsbehov i matematik. Viktiga aspekter för inkludering i matematik är *organisatoriska åtgärder* för att stödja *samarbete och diskussioner*; att ha *väl fungerande team* som arbetar med förebyggande åtgärder i matematikundervisningen; *lärarkunskap* om och i matematikdidaktiska frågor och sist, men inte minst; att lyssna på *elevernas röst*.

EPILOGUE

It has been a hard task conducting this research. As a senior scholar I met at a conference said: "Oh, how interesting, inclusion in mathematics! But I would never have given the task to one of my doctorial students, it is to hard!" I then considered giving it up, but as the stubborn woman I am, I chose not to listen. I chose to pursue my quest of finding a way to describe inclusion in mathematics from a SEM perspective.

In the prologue I compared my research process with a cycle class. Now this particular cycle-class has come to its end. But I promise you I will be right back in the saddle again, searching for that adrenalin rush. Nothing beats it!

And, fortunately, this research of inclusion in mathematics has not come to its end; I am only half way, so there will be more cycle-classes. I will continue my quest and develop the thoughts about inclusion in mathematics realising that the really hard part is yet to come. But before that, I will take a break, reflect on the results and which path I am going to take when continuing the research. *And live a little…*

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Linnéuniversitetet
Institutionen för datavetenskap, fysik och matematik

Interview guide for interviewing mathematics teachers

• Background: Education, number of years in the profession, age

• How do you perceive teaching mathematics? (planning and teaching)

• Do you have any students in mathematical difficulties in your class?

• What kind of problems does these students encounter when doing math?

• How do these problems look like? What do you think are the reasons for them?

• What actions have been taken for these students?

• Can the students be finished with special support and move back to the regular teaching? If this is the case, what assessments are made and who or whom assesses, what criteria should be fulfilled?

• How are students and parents informed about the special support?

• Are students and parents involved in the decision about the special support? Is the teaching "discussable" or mandatory?

• Are "the other" students in the class (and their parents) informed that some have special support and how would you describe this in that case?

• What are the needs as "satisfied" in the special support?

• What is inclusion for you?

• What are critical factors for inclusion in mathematics?

• If the students are working with the following types of tasks, are there any that you think is easier to do in the small group with the remedial teacher? Why are these easier? Are there any tasks the students more easily can

master in the classroom? Why? (The tasks are collected from the students' ordinary book in mathematics).

a. 1085+198   4025-2025   3x2x7 $\frac{28}{7}$

b. At a birthday party there were five girls and twice as many boys. How many were there at the party?

c. In how many ways can you change a 20-crowns banknote? Find at least five solutions.

d. How much do a micro stereo and a speaker package cost together?

Picture from Tänk och Räkna 4a, Gleerups Utbildning. Illustratör Ralph Branders.

e. First, make estimation and then calculate with an algorithm 543-426.

Interview guide for interview with the Headmaster

• Background, number of years as a principal, number of years at school and age.

• How do you perceive mathematics education at school?

• How do you perceive the support for students in special educational needs in mathematics?

• How is the support organised?

• What is inclusion for you?

• What are the critical factors for inclusion in Mathematics?

• How is the Student Health organised? Who is included? What is the mission of the student health?

Consent

Dear Sir/madam.

I am a researcher in mathematics education at Linnaeus University. Over the next three years I will conduct a research project at Oakdale Primary School regarding the teaching of mathematics. In my investigation I will make audio and video recordings of teachers, students and groups of students during mathematics lessons.

All data collected will be treated confidentially and all participants will be anonymous. The material will only be used for this investigation. The results will be compiled in a thesis that will eventually be published.

The surveys will take place from autumn 2011 to spring 2014.

I hope that your child will be allowed to participate in the research. Please contact me if you have any questions.

Best wishes,

Helena Roos, Lecturer, Linnaeus University Växjö, Helena.Roos@lnu.se 0470-708834

☐   I/ we accept our child's participation in the research project at Oakdale Primary School 2011-2014.

☐   We / I do not want our children involved in the research project

Name of the child:
_____

Guardian's Signature

Name in block letters

Linnéuniversitetet
Institutionen för datavetenskap, fysik och matematik

Consent

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I hope that you are willing to participate in the research project.
Best wishes,

Helena Roos, Lecturer, Linnaeus University Växjö, Helena.Roos@lnu.se
0470-708834

I would like to participate in the research project at Oakdale Primary School 2011-2014

Signature

Name in block letters