INDETERMINACY AND STABILITY

Static Indeterminancy

- To solve a truss using only the equations of equilibrium, the truss must be statically determinate. For a simply supported planar truss, there are three equations of equilibrium available between both supports and two equations of equilibrium available at each joint. If the number of unknowns exceed the available equations, the truss is statically indeterminate
- The difference between the number of unknowns and the number of available equations is called the degree of indeterminacy. The solution of such a truss requires one quantify truss deflection and use those equations to solve for the unkowns
- This technique is beyond the scope of this textbook, but are commonly introduced through a course in strength of materials. Our goal will be to learn how to find out whether or not a truss is statically determinate

Static Indeterminancy

- Overall indeterminacy of a truss may by found as follows:

Let m = number of members
Let r = number of external reaction components
Let j = number of joints including those at the support reactions
Let i = degree of indeterminacy (0 = determinate, greater than 0 is indeterminate)

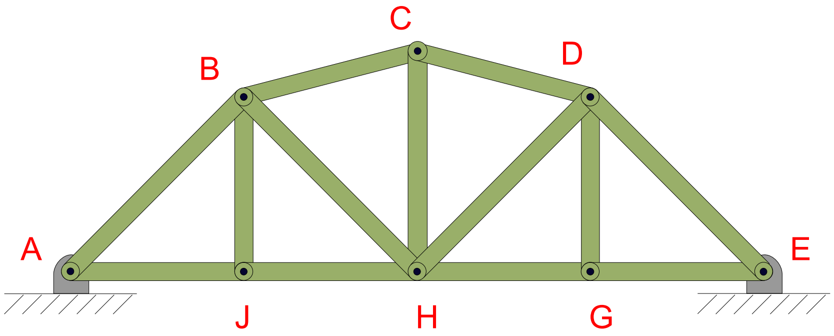
$$i = (m + r) - 2j$$

- Consider the truss below. The degree of static indeterminacy can be found by:

m = 13
r = 4
j = 8

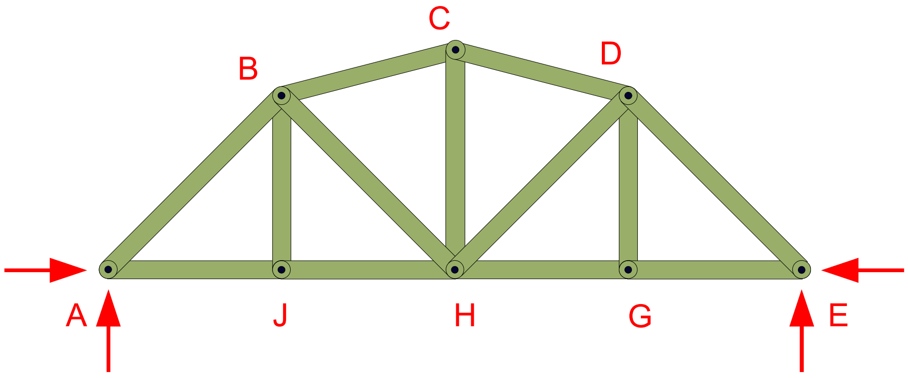
$$i = (13 + 4) - 2(8) = 1$$

This truss is statically indeterminate to the first degree



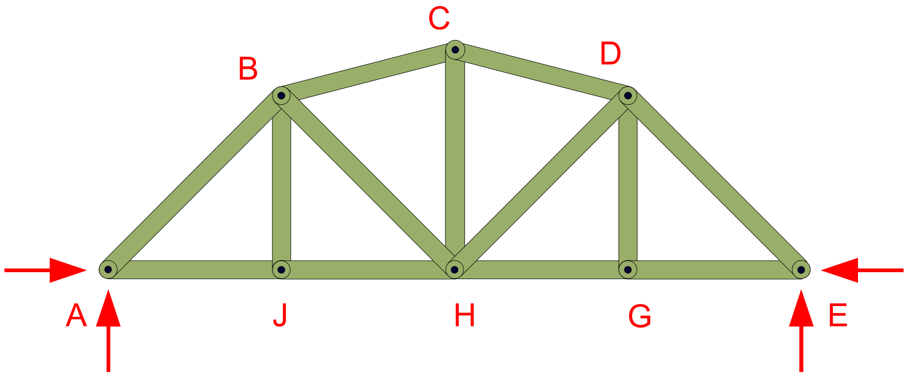
Static Indeterminancy

- A truss may be internally or externally indeterminate
- An externally indeterminate truss is often found easily by inspection. If there are more unknown reaction components than there are equations of equilibrium, then the truss is externally indeterminate
- Consider the following truss used in the previous example. Each end of the truss is hinged resulting in four reaction components. Since there are only three equations of equilibrium available, this truss is externally indeterminate to the 1st degree
- If either hinge were replaced with a roller support, the degree of external indeterminacy would be zero. In other words, truss reactions can be found using only the equations of equilibrium



Static Indeterminancy

- Although it is not generally necessary to determine the degree of internal indeterminacy of a truss, it is easily done. The degree of internal indeterminacy is the difference between overall indeterminacy and external indeterminacy
- In this particular case, since the degree of overall indeterminacy is one and the degree of external indeterminacy is one, then the degree of internal indeterminacy is zero

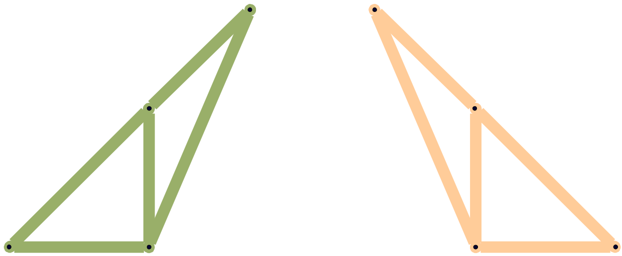


Stability

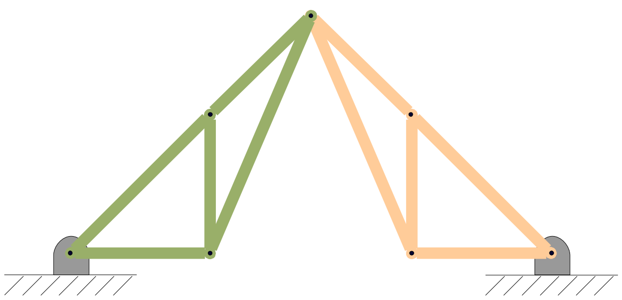
- Truss stability refers to whether or not the truss will collapse under load. A truss may be externally unstable, internally unstable, or both
- A truss is said to be externally unstable if:
  - All reaction components are concurrent
  - All reactions components are parallel
- Internal stability is a function of truss geometry
  - A simple truss is always stable
  - A compound or complex truss is internally stable if the number of members and their arrangement does not allow the truss to collapse or change shape when it is removed from its supporting structure
  - Since a triangle is the only inherently stable polygon, internal stability can often be identified visually if the internal members of the truss form a shape other than a triangle
- Stability can also be determined from the equation used to find degree of indeterminacy. If the degree of indeterminacy is less than zero, the truss is unstable

Example 5
Indeterminacy and Stability

- Consider two simple trusses as shown below. Let us analyze the determinacy and stability as these simple trusses are connected to each other in various manners



Example 5a – Hinged supports and pinned at apex



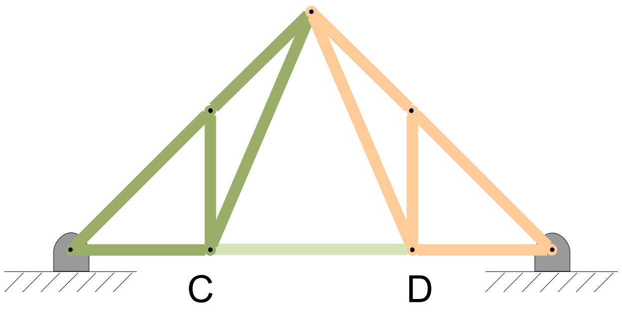
m = 10
r = 4
j = 7

$$i = (10 + 4) - 2(7) = 0$$

This truss is statically determinate and stable

Example 5
Indeterminacy and Stability

Example 5b – Hinged supports, pinned at apex, tied at B & C

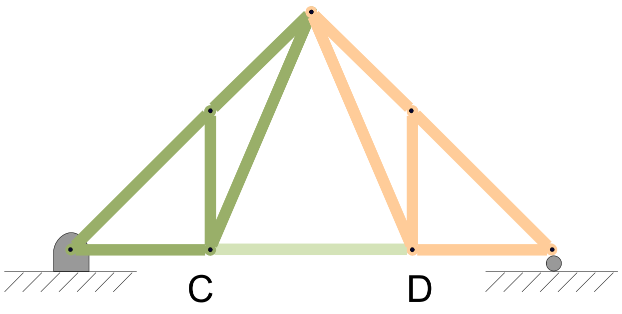


m = 11
r = 4
j = 7

$i = (11 + 4) - 2(7) = 1$

This truss is statically indeterminate and stable

Example 5c – Simply supported, pinned at apex, tied at B & C



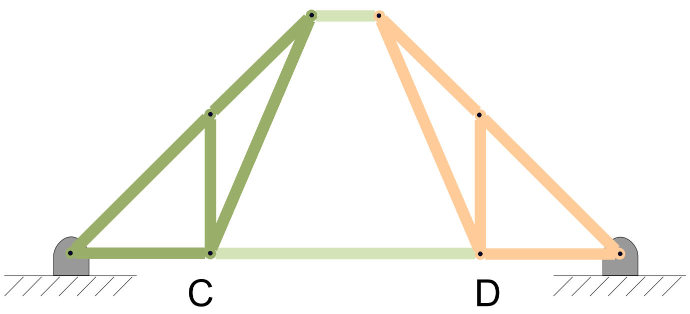
m = 11
r = 3
j = 7

$i = (11 + 3) - 2(7) = 1$

This truss is statically determinate and stable

Example 5
Indeterminacy and Stability

Example 5d – Hinged supports with a two-bar connection

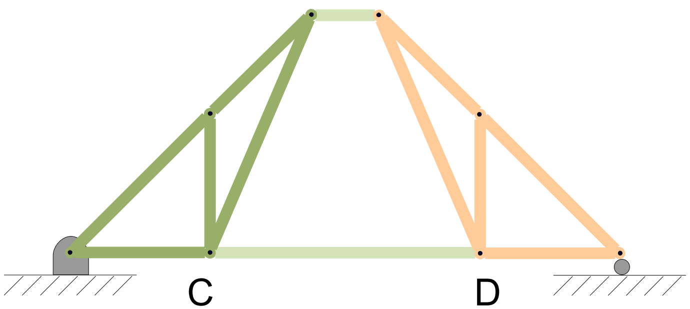


m = 12
r = 4
j = 8

$$i = (12 + 4) - 2(8) = 0$$

This truss is statically determinate but geometrically unstable

Example 5e – Simply supported with two-bar connection



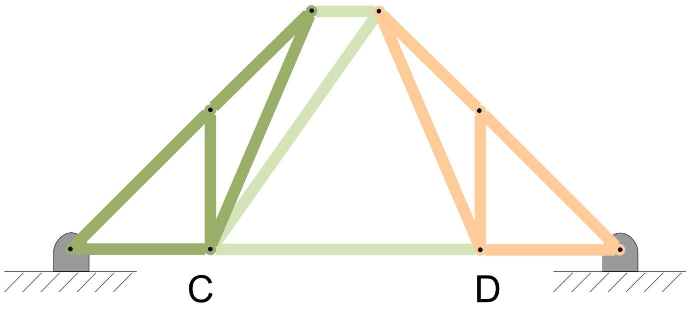
m = 12
r = 3
j = 8

$$i = (12 + 3) - 2(8) = -1$$

This truss is unstable

Example 5
Indeterminacy and Stability

Example 5f – Hinged support with a three-bar connection

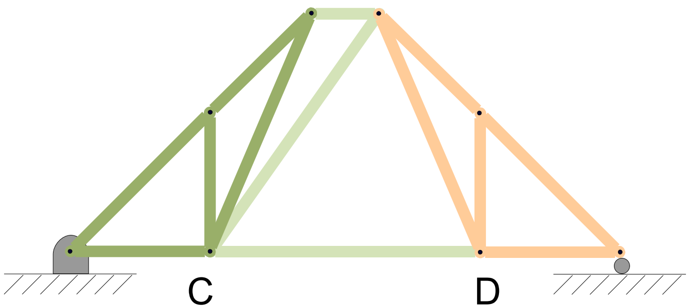


m = 13
r = 4
j = 8

$$i = (13 + 4) - 2(8) = 1$$

This truss is statically indeterminate and stable

Example 5g – Simply supported with three-bar connection



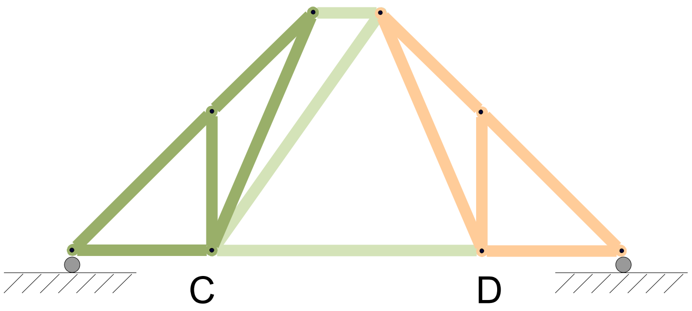
m = 13
r = 3
j = 8

$$i = (13 + 3) - 2(8) = 0$$

This truss is determinate and stable

Example 5
Indeterminacy and Stability

Example 5h – Roller support at both ends with a three-bar connection

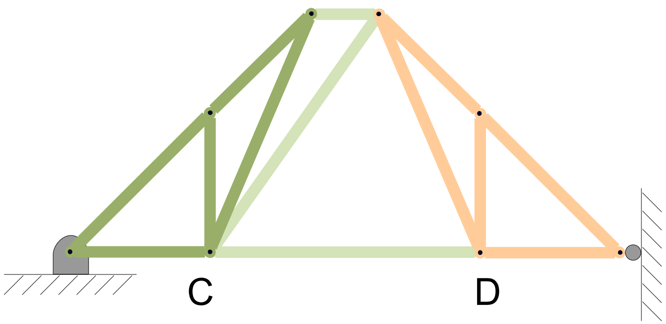


m = 13
r = 2
j = 8

$$i = (13 + 2) - 2(8) = -1$$

This truss is statically determinate but externally unstable since reactions are parallel

Example 5i – Hinged at one end, roller support on vertical surface, with three-bar connection



m = 13
r = 3
j = 8

$$i = (13 + 3) - 2(8) = 0$$

This truss is determinate but externally unstable due to concurrency of all reaction components

Summary

Degree of indeterminacy (i) is found using $i = (m + r) - 2j$

m = number of members
r = number of external reaction components
j = number of joints including those at the support reactions

- If i=0
  - Truss is determinate
  - Truss is stable if
    - reactions are non-parallel and non-concurrent
    - There are no collapsable mechanics
- If i>0
  - Truss is indeterminate
  - Truss is stable if
    - reactions are non-parallel and non-concurrent
    - There are no collapsable mechanics
- If i<0
  - Truss is determinate but unstable