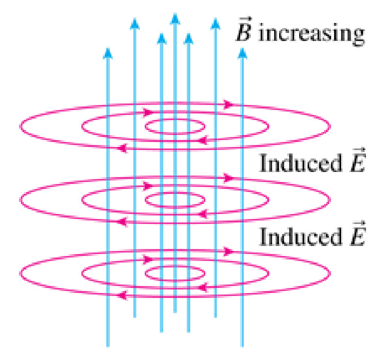
Lecture 18

University of Massachusetts
UMASS Lowell
Physics II

Chapter 33

Induced Electric Field


Michael Faraday
(1791-1867)



Course website:
http://faculty.uml.edu/Andriy_Danylov/Teaching/PhysicsII

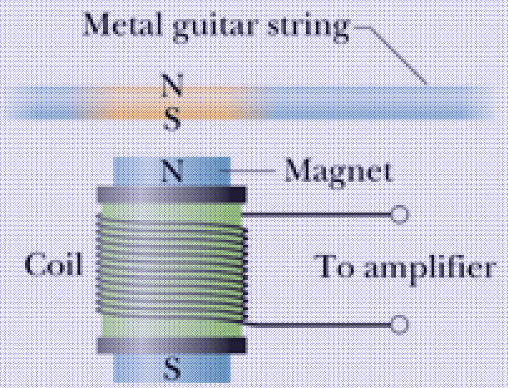
Applications of Faraday's Law

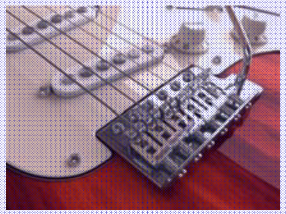
(some leftovers from the previous class)

$$\mathscr{E} = -\frac{d\Phi_B}{dt}.$$

Applications (*Faraday's Law)*

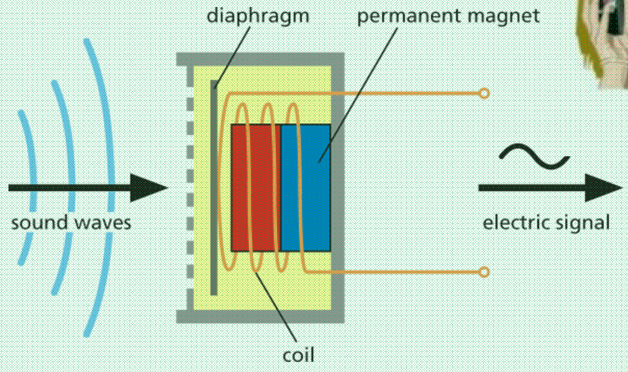
How Electric guitar works?





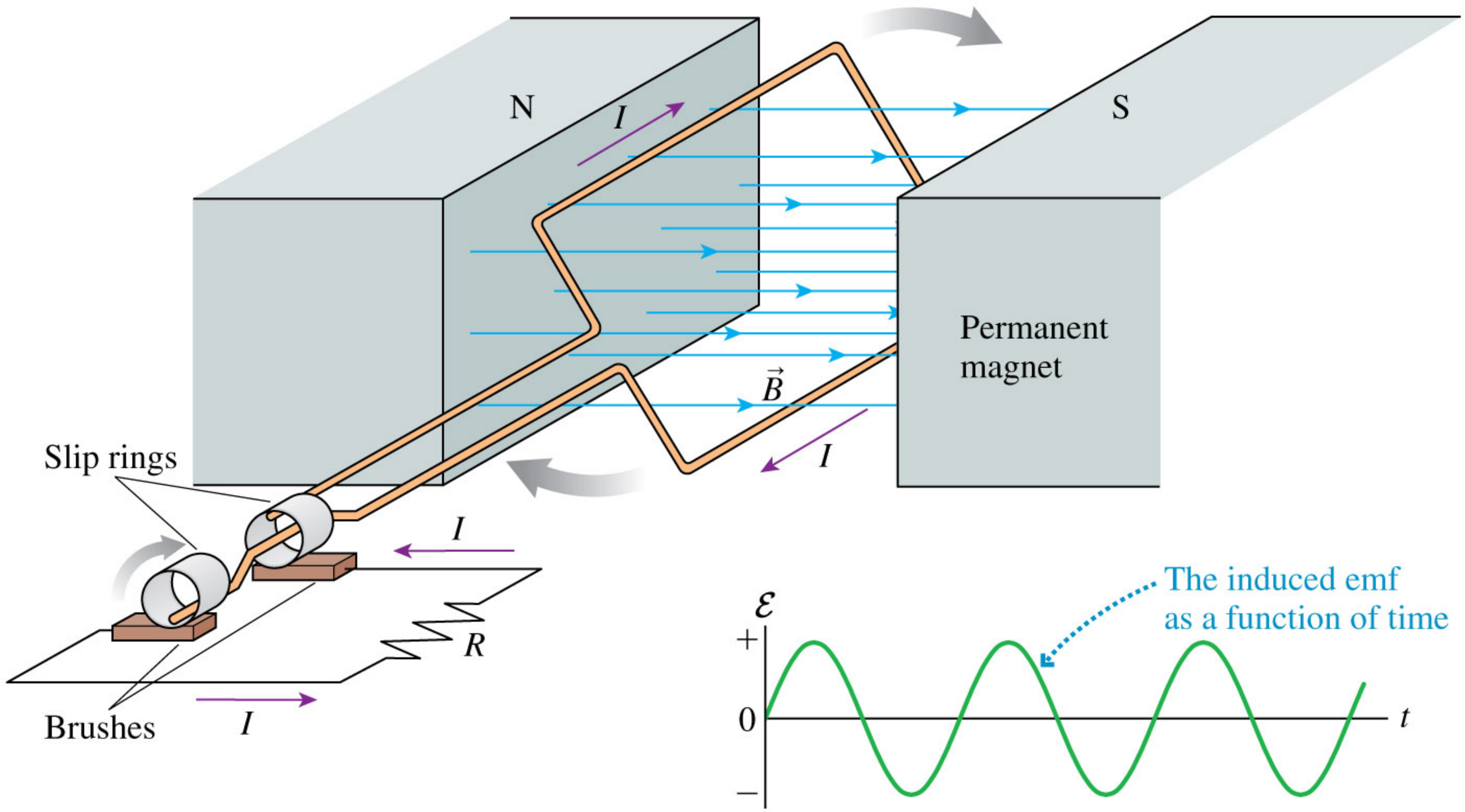
- Presence of a permanent magnet makes the guitar wire a magnet.
- When the magnetic guitar wire vibrates, it changes the magnetic flux in the coil.
- This flux induces a current in the coil which exactly follow the vibrations of the guitar wire.
- This current is fed to the amplifier and we can hear the sound of vibration of the guitar wire.

How Microphone works?



- When a sound wave strikes the diaphragm, a coil fixed to the diaphragm vibrates over a stationary bar magnet, changing the flux in the coil and induced an emf in the coil which is then amplified and sent to speakers.

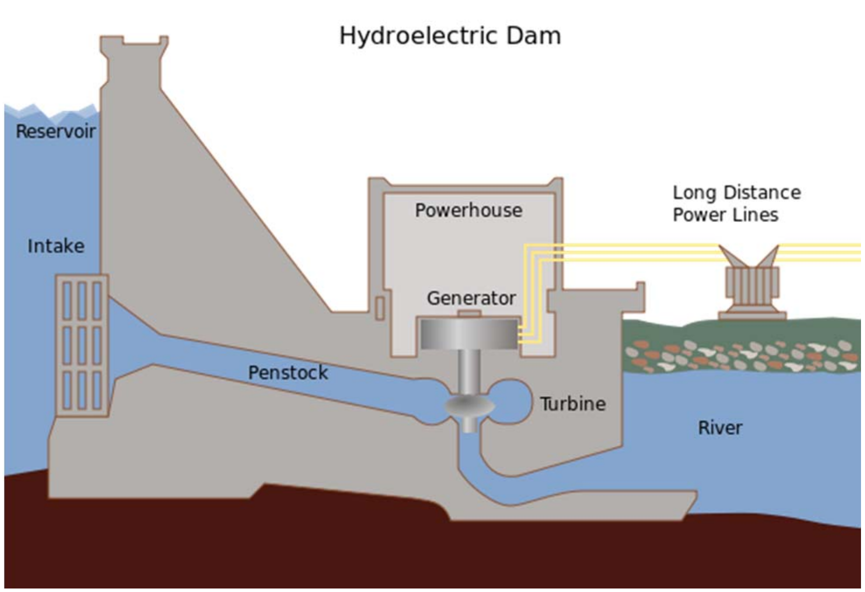
Applications
(An Alternating-Current Generator)



$$\varepsilon = -\frac{d\Phi_m}{dt} = -\frac{d}{dt}(BA\cos\theta) = -\frac{d}{dt}(BA\cos\omega t) = \omega BA\sin\omega t$$

Generators

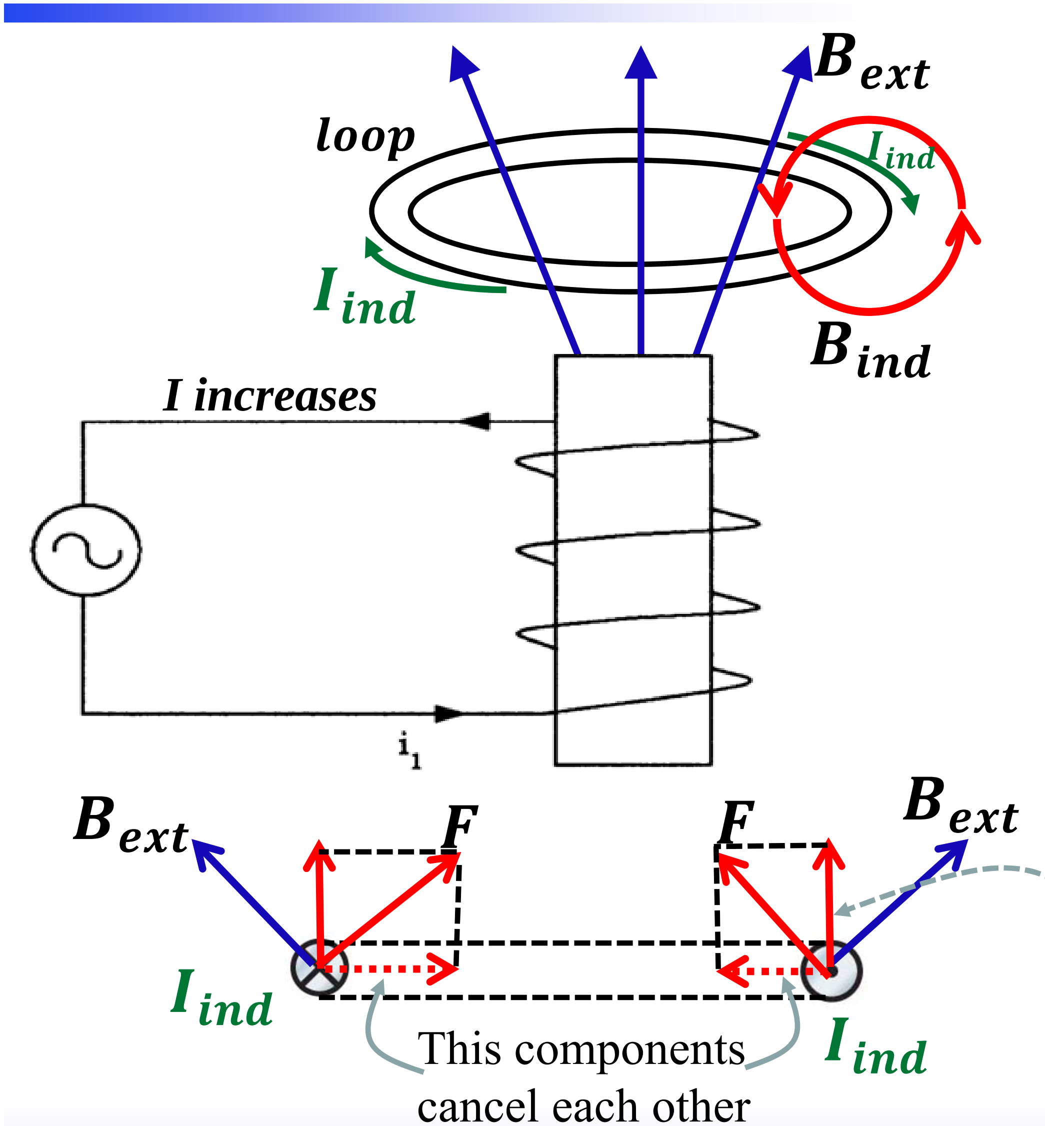
A generator is a device that transforms mechanical energy into electric energy.



A generator inside a hydroelectric dam uses electromagnetic induction to convert the mechanical energy of a spinning turbine into electric energy.

Hydroelectric Power Plant Working Animation (start at 1 min)

Jumping Ring Demonstration



With a metal ring in place, turn on the switch. The solid ring will jump. Why?

$I\nearrow \Rightarrow B_{ext}\nearrow \Rightarrow \Phi_B\nearrow \Rightarrow$

$\Rightarrow B_{ext}\uparrow\downarrow B_{ind}$ $\left\|\begin{matrix} B_{ind}\ opposes\ B_{ext} \\ inside\ the\ loop \\ (Lenz's\ law)\end{matrix}\right\|$

$\Rightarrow$ I_{ind} ***has CW direction***

According to the right – hand rule for a wire

Let's look at a force on the ring:

$$\overrightarrow{dF_{mag}} = I(\overrightarrow{dl} \times \vec{B})$$

Vertical components add each other

Thus, the net force is up and the ring jumps

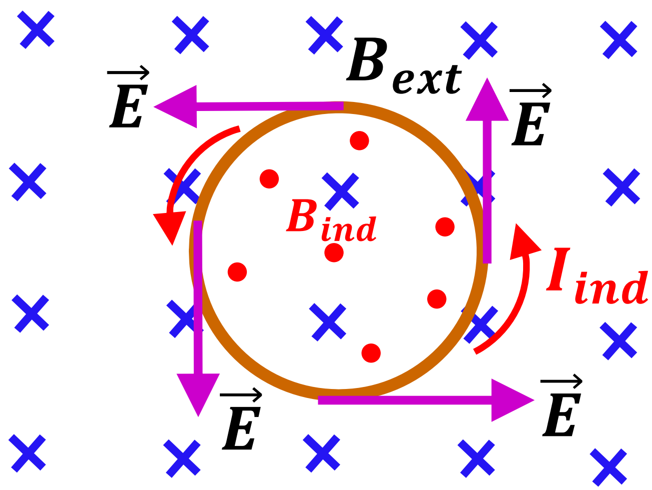
Induced Electric Field

Let's reformulate Faraday's Law in terms of $\vec{E}$

$$\mathcal{E} = -\frac{d\Phi_m}{dt}$$

Induced Electric Field

Consider a conducting loop in an increasing uniform magnetic field.



$B_{ext}\nearrow \;\Rightarrow\; \Phi_B \nearrow \;\Rightarrow\; B_{ext}\uparrow\downarrow B_{ind} \;\Rightarrow\; I_{ind}\ CCW$

So, according to Lenz's law, there is an induced current in the counterclockwise direction.

If there is a current, there must be something that acts on the charge carriers to make them move,

so we infer that there must be an **induced electric field** tangent to the loop at all points

- The conducting loop is not necessary to generate E!

(The electric field arise whether or not circuits are present).

Thus, the space is filled with an induced electric field.

The loop was used as a probe system to convince ourselves that there was the E field

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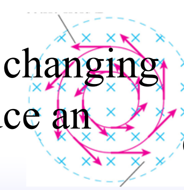
From Eq.29.3: $\Delta V = V_f - V_i = -\int_i^f \vec{E}\bullet d\vec{s}$ Let's calculate ΔV or Ɛ (emf) over the closed loop: $\Delta V = \varepsilon = \oint \vec{E}\bullet d\vec{s}$

From the other side, Faraday's law: $\boldsymbol{\varepsilon = -\dfrac{d\Phi_m}{dt}}$

Combining them we can get

$$\oint \vec{E}\bullet d\vec{s} = -\frac{d\Phi_m}{dt}$$

This law implies that a changing magnetic flux will induce an induced electric field



General form of Faraday's law

Two types of the electric field

Let's look at the special case, when A=const and θ=0.

$$\Phi_m = \vec{B}\cdot\vec{A} = BA \quad \Rightarrow \quad \oint \vec{E}\cdot d\vec{s} = A\left|\frac{dB}{dt}\right|$$

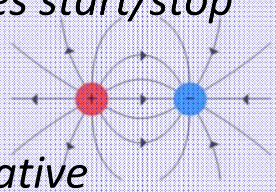
Amazing!

A changing magnetic field produces an electric field!!!

Thus, now we know two types of electric field:

Coulomb electric field :

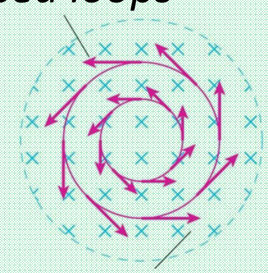
1) *E is produced by charges*
2) *Coulomb el. field lines start/stop on charges*
3) *Induced E is conservative*



$$\varepsilon = \oint \vec{E} \bullet d\vec{s} = 0$$

Induced electric field:

1) *E is produced by changing B, not by charges*
2) *Induced el. field lines form closed loops*
3) *Induced E is nonconservative*



$$\varepsilon = \oint \vec{E} \bullet d\vec{s} = -\frac{d\Phi_m}{dt}$$

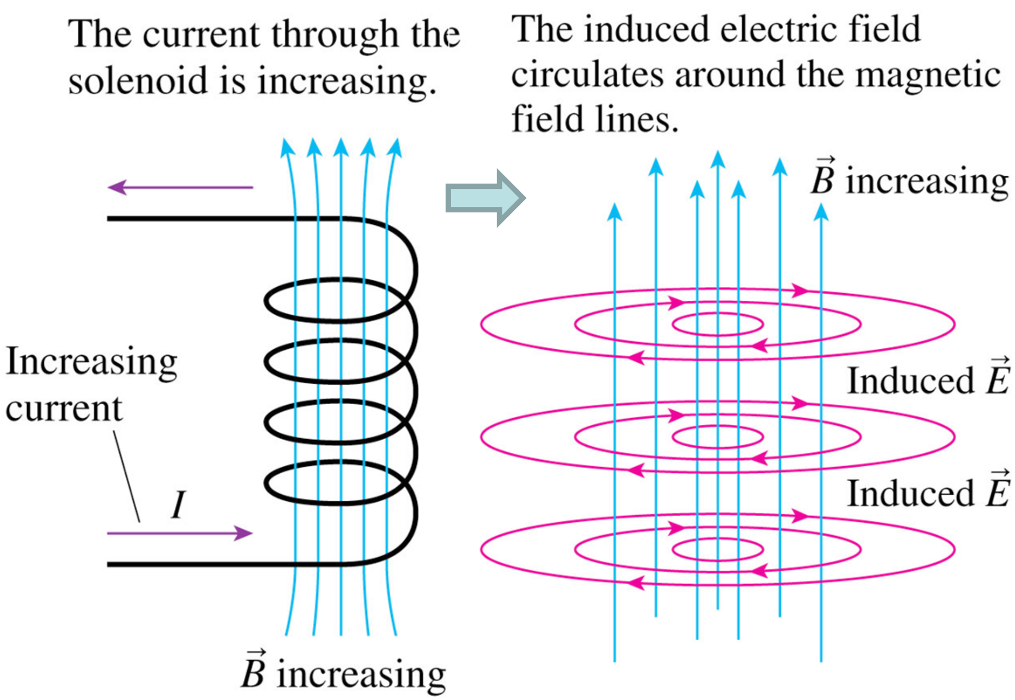
Faraday's law

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_{\mathrm{m}}}{dt}$$

Michael Faraday
(1791-1867)

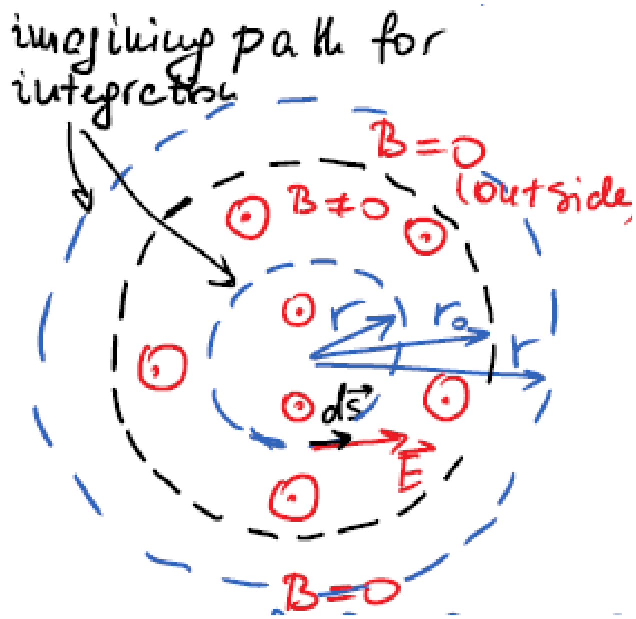
A changing magnetic field creates an induced electric field.

Induced Electric Field in a Solenoid



- The current through the solenoid creates an upward pointing magnetic field.
- As the current is increasing, *B* is increasing, so it must induce an electric field.
- We could *use Lenz's law to determine that if there were a conducting loop in the solenoid*, the induced current would be clockwise.
- The induced electric field must therefore be clockwise around the magnetic field lines.

Induced Electric Field in a Solenoid



- $r < r_0$, $B \neq 0$: $\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$; let's loop at the magnitude (drop "−")

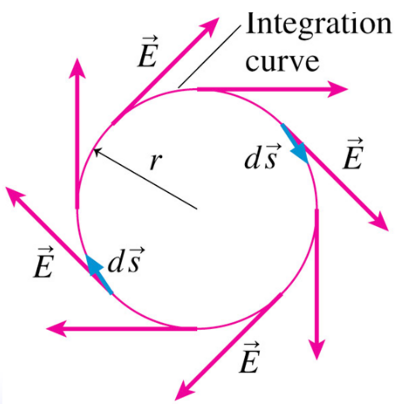
circle r

Due to asymmetry, $\vec{E}$ is tangent to the loop

$$\oint \vec{E}_1 \cdot d\vec{s} = \left\| \vec{E} \uparrow\uparrow d\vec{s},\ E = const \right\| = E_1 \oint ds = E_1 2\pi r = \left| \frac{d(\vec{B} \cdot \vec{A})}{dt} \right| ; \vec{B} \uparrow\uparrow \vec{A}$$

$$E_1 \cdot 2\pi r = \left\| A = \pi r^2 = const \right\| = \pi r^2 \left| \frac{dB}{dt} \right|$$

$$\left\| E_1 = \frac{r}{2} \left| \frac{dB}{dt} \right| \right\|$$ El. field inside



- $r > r_0$ (outside the magn. field)

$$E_2 \cdot 2\pi r = \left| \frac{d}{dt} \Phi_{tot} \right| = \frac{d}{dt} \left| B \cdot \pi r_0^2 \right|$$

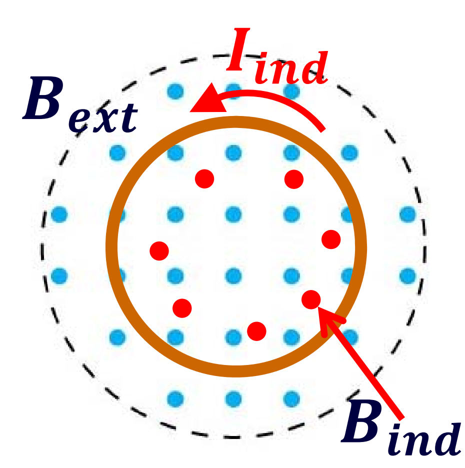
$$\Phi_{tot} = \Phi_{r_0} + \Phi_{r_0 \leftrightarrow r} = B \cdot \pi r_0^2 + B_{out} \cdot A_{r_0 \leftrightarrow r} = B \cdot \pi r_0^2$$

(B_{out} → 0 (outside))

$$E_2 = \frac{\pi r_0^2}{2\pi r} \left| \frac{dB}{dt} \right| = \frac{r_0^2}{2 r} \cdot \left| \frac{dB}{dt} \right|$$

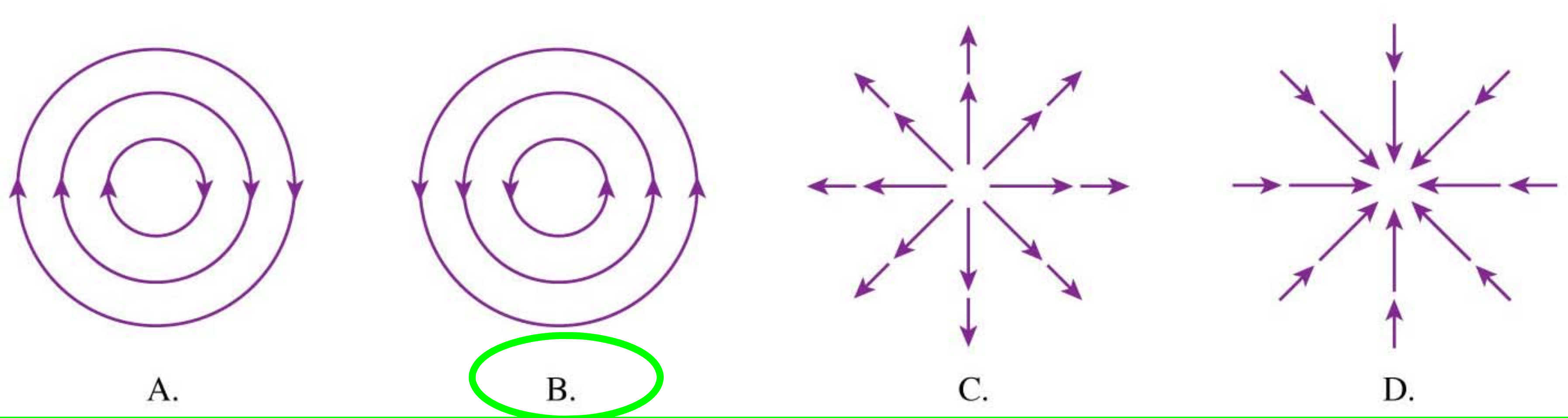
ConcepTest Faraday's Law

- The magnetic field is **decreasing**. Which is the induced electric field?



Imagine a loop ⇨ $B_{ext}\downarrow$ ⇨ $\Phi_B\downarrow$ ⇨ $B_{ext}\uparrow\uparrow B_{ind}$ ⇨ I_{ind} *CCW* ⇩

The field is the same direction as induced current would flow if there were a loop in the field. ⇦ E_{ind} *CCW*



E. There's no induced field in this case.

What you should read

Chapter 33 (Knight)

Sections

- ***33.6***
- ***33.7 (skip "Transformers")***
- ***33.5***

Thank you

See you on Friday