Chapter 20
Induced Voltages and Inductance

Quick Quizzes

1. *b*, *c*, *a*. At each instant, the magnitude of the induced emf is proportional to the rate of change of the magnetic field (hence, proportional to the slope of the curve shown on the graph).

2. (a). All charged particles within the metal bar move straight downward with the bar. According to right-hand rule #1, positive changes moving downward through a magnetic field that is directed northward will experience magnetic forces toward the east. This means that the free electrons (negative charges) within the metal will experience westward forces and will drift toward the west end of the bar, leaving the east end with a net positive charge.

3. (b). According to Equation 20.3, because *B* and *v* are constant, the emf depends only on the length of the wire moving in the magnetic field. Thus, you want the long dimension moving through the magnetic field lines so that it is perpendicular to the velocity vector. In this case, the short dimension is parallel to the velocity vector. From a more conceptual point of view, you want the rate of change of area in the magnetic field to be the largest, which you do by thrusting the long dimension into the field.

4. (c). In order to oppose the approach of the north pole, the magnetic field generated by the induced current must be directed upward. An induced current directed counterclockwise around the loop will produce a field with this orientation along the axis of the loop.

5. (b). When the iron rod is inserted into the solenoid, the inductance of the coil increases. As a result, more potential difference appears across the coil than before. Consequently, less potential difference appears across the bulb, and its brightness decreases.

Answers to Conceptual Questions

2. Consider the copper tube to be a large set of rings stacked one on top of the other. As the magnet falls toward or falls away from each ring, a current is induced in the ring. Thus, there is a current in the copper tube around its circumference.

4. The flux is calculated as $\Phi_B = BA\cos\theta = B_\perp A$. The flux is therefore maximum when the magnetic field vector is perpendicular to the plane of the loop. We may also deduce that the flux is zero when there is no component of the magnetic field that is perpendicular to the loop.

6. No. Once the bar is in motion and the charges are separated, no external force is necessary to maintain the motion. An applied force in the x direction will cause the bar to accelerate in that direction.

8. As water falls, it gains velocity and kinetic energy. It then pushes against the blades of a turbine transferring this energy to the rotor or coil of a large alternating current generator. The rotor moves in a strong external magnetic field and a voltage is induced in the coil. This induced emf is the voltage source for the current in our electric power lines.

10. The magnetic field lines around the transmission cable will be circular. If you place your loop around the cable, there will be no field lines passing through the loop, so no emf will be induced. The loop needs to be placed next to the cable, with the plane of the loop containing the cable, to maximize the flux through its area.

12. Let us assume the north pole of the magnet faces the ring. As the bar magnet falls toward the conducting ring, a magnetic field is induced in the ring pointing upward. This upward directed field will oppose the motion of the magnet preventing it from moving as a freely-falling body. Try it for yourself to show that an upward force also acts on the falling magnet if the south end faces the ring.

14. A constant induced emf requires a magnetic field that is changing at a constant rate in one direction — for example, always increasing or always decreasing. It is impossible for a magnetic field to increase forever, both in terms of energy considerations and technological concerns. In the case of a decreasing field, once it reaches zero and then reverses direction, we again face the problem with the field increasing without bounds in the opposite direction.

16. As the magnet moves at high speed past the fixed coil, the magnetic flux through the coil changes very rapidly, increasing as the magnet approaches the coil and decreasing as the magnet moves away. The rapid change in flux through the coil induces a large emf, large enough to cause a spark across the gap in the spark plug.

18. **(a)** Clockwise. As the south end of the magnet approaches the loop from above, the flux through the loop is directed upward and is increasing in magnitude. The induced current will flow clockwise to produce a downward flux through the loop and oppose the increasing flux due to the magnet. **(b)** Counterclockwise. When the north end of the magnet is below the loop and is moving away, the flux through the loop is directed upward and is decreasing in magnitude. The induced current will flow counterclockwise to produce upward flux through the loop and oppose the decreasing flux due to the magnet.

Answers to Even Numbered Problems

2. **(a)** 1.00×10^{-7} T·m² **(b)** 8.66×10^{-8} T·m² **(c)** 0

4. zero

6. 2.96×10^{-5} T·m²

8. 0.10 mV

10. 34 mV

12. 52.0 μT

14. **(a)** 1.88×10^{-7} T·m² **(b)** 6.28×10^{-8} V

16. 8.8 A

18. 1.00 m/s

20. 2.87 mV

22. 2.8 mV

24. **(b)** Larger R makes current smaller, so the loop must travel faster to maintain equality of magnetic force and weight.
    (c) The magnetic force is proportional to the product of field and current, while the current is itself proportional to the field. If B becomes two times smaller, the speed must become four times larger to compensate.

26. left to right

28. **(a)** left to right **(b)** no induced current **(c)** right to left

30. 13 mV

32. **(a)** 8.0 A **(b)** 3.2 A **(c)** 60 V

34. **(a)** 7.5 kV **(b)** when the plane of the coil is parallel to the field

36. 1.36 μH

40. 1.92×10^{-5} T·m²

42. **(a)** 1.00 kΩ **(b)** 3.00 ms

44. **(a)** 0 **(b)** 3.8 V **(c)** 6.0 V **(d)** 2.2 V

46. **(a)** 2.00 ms **(b)** 0.176 A **(c)** 1.50 A **(d)** 3.22 ms

48. **(a)** 4.44×10^{-3} H **(b)** 5.55×10^{-4} J

50. **(a)** $\tau = L/2R$ **(b)** $\tau = 2L/R$

52. 3100 μA, the galvanometer will definitely show the induced current

54. **(a)** Amplitude doubles, period unchanged.
(b) Amplitude doubles, period cut in half.
(c) Amplitude unchanged, period cut in half.

56. **(a)** 6.25×10^{10} J **(b)** 2.00 kN/m

58. 0.121 A, clockwise

60. 6.8 V

62. 1.60 A

64. **(b)** 0.750 mA

66. **(a)** between $t = 0$ and $t = 2.0$ s
(b) between $t = 2.00$ and $t = 4.0$ s
(c) No. When the field is increasing, the direction of the induced current is different than when the field is decreasing.
(d) 0.24 A clockwise, current is zero, 0.12 A counterclockwise

Problem Solutions

20.1   The magnetic flux through the area enclosed by the loop is

$$\Phi_B = BA\cos\theta = B\left(\pi r^2\right)\cos 0° = (0.30\text{ T})\left[\pi(0.25\text{ m})^2\right] = \boxed{5.9\times 10^{-2}\text{ T}\cdot\text{m}^2}$$

20.2   The magnetic flux through the loop is given by $\Phi_B = BA\cos\theta$ where B is the magnitude of the magnetic field, A is the area enclosed by the loop, and θ is the angle the magnetic field makes with the normal to the plane of the loop. Thus,

$$\Phi_B = BA\cos\theta = \left(5.00\times 10^{-5}\text{ T}\right)\left[20.0\text{ cm}^2\left(\frac{10^{-2}\text{ m}}{1\text{ cm}}\right)^2\right]\cos\theta = \left(1.00\times 10^{-7}\text{ T}\cdot\text{m}^2\right)\cos\theta$$

(a)   When $\vec{\textbf{B}}$ is perpendicular to the plane of the loop, $\theta = 0°$ and
$\Phi_B = \boxed{1.00\times 10^{-7}\text{ T}\cdot\text{m}^2}$

(b)   If $\theta = 30.0°$, then $\Phi_B = \left(1.00\times 10^{-7}\text{ T}\cdot\text{m}^2\right)\cos 30.0° = \boxed{8.66\times 10^{-8}\text{ T}\cdot\text{m}^2}$

(c)   If $\theta = 90.0°$, then $\Phi_B = \left(1.00\times 10^{-7}\text{ T}\cdot\text{m}^2\right)\cos 90.0° = \boxed{0}$

20.3   The magnetic flux through the loop is given by $\Phi_B = BA\cos\theta$ where B is the magnitude of the magnetic field, A is the area enclosed by the loop, and θ is the angle the magnetic field makes with the normal to the plane of the loop. Thus,

$$\Phi_B = BA\cos\theta = (0.300\text{ T})(2.00\text{ m})^2\cos 50.0° = \boxed{7.71\times 10^{-1}\text{ T}\cdot\text{m}^2}$$

20.4   The magnetic field lines are tangent to the surface of the cylinder, so that no magnetic field lines penetrate the cylindrical surface. The total flux through the cylinder is $\boxed{\text{zero}}$

20.5   (a)   Every field line that comes up through the area A on one side of the wire goes back down through area A on the other side of the wire. Thus, the net flux through the coil is $\boxed{\text{zero}}$

(b)   The magnetic field is parallel to the plane of the coil , so $\theta = 90.0°$. Therefore,
$\Phi_B = BA\cos\theta = BA\cos 90.0° = \boxed{0}$

20.6 The magnetic field generated by the current in the solenoid is

$$B = \mu_0 nI = \left(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}\right)\left(\frac{250}{0.200 \text{ m}}\right)(15.0 \text{ A}) = 2.36 \times 10^{-2} \text{ T}$$
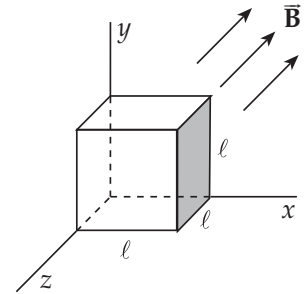
and the flux through each turn on the solenoid is

$$\Phi_B = BA\cos\theta$$

$$= \left(2.36 \times 10^{-2} \text{ T}\right)\left[\frac{\pi\left(4.00 \times 10^{-2} \text{ m}\right)^2}{4}\right]\cos 0° = \boxed{2.96 \times 10^{-5} \text{ T} \cdot \text{m}^2}$$

20.7 (a) The magnetic flux through an area A may be written as

$$\Phi_B = (B\cos\theta)A$$

$$= (\text{component of } B \text{ perpendicular to } A) \cdot A$$

Thus, the flux through the shaded side of the cube is

$$\Phi_B = B_x \cdot A = (5.0 \text{ T}) \cdot \left(2.5 \times 10^{-2} \text{ m}\right)^2 = \boxed{3.1 \times 10^{-3} \text{ T} \cdot \text{m}^2}$$

(b) Unlike electric field lines, magnetic field lines always form closed loops, without beginning or end. Therefore, no magnetic field lines originate or terminate within the cube and any line entering the cube at one point must emerge from the cube at some other point. The net flux through the cube, and indeed through any *closed* surface, is zero .

20.8 $$|\mathcal{E}| = \frac{|\Delta\Phi_B|}{\Delta t} = \frac{(\Delta B)A\cos\theta}{\Delta t} = \frac{(1.5 \text{ T} - 0)\left[\pi\left(1.6 \times 10^{-3} \text{ m}\right)^2\right]\cos 0°}{120 \times 10^{-3} \text{ s}} = 1.0 \times 10^{-4} \text{ V} = \boxed{0.10 \text{ mV}}$$

20.9 From $|\mathcal{E}| = \dfrac{|\Delta\Phi_B|}{\Delta t} = \dfrac{B|\Delta A|\cos\theta}{\Delta t}$, we find that

$$B = \frac{|\mathcal{E}|}{(|\Delta A|/\Delta t)\cos\theta} = \frac{18 \times 10^{-3} \text{ V}}{(0.10 \text{ m}^2/\text{s})\cos 0°} = \boxed{0.18 \text{ T}}$$

20.10 $\left|\mathcal{E}\right| = \dfrac{\Delta\Phi_B}{\Delta t} = \dfrac{B(\Delta A)\cos\theta}{\Delta t}$

$$= \frac{(0.15\ \text{T})\left[\pi(0.12\ \text{m})^2 - 0\right]\cos 0°}{0.20\ \text{s}} = 3.4\times 10^{-2}\ \text{V} = \boxed{34\ \text{mV}}$$

20.11 The magnitude of the induced emf is $\qquad \left|\mathcal{E}\right| = \dfrac{\left|\Delta\Phi_B\right|}{\Delta t} = \dfrac{\left|\Delta(B\cos\theta)\right|A}{\Delta t}$

If the normal to the plane of the loop is considered to point in the original direction of the magnetic field, then $\theta_i = 0°$ and $\theta_f = 180°$. Thus, we find

$$\left|\mathcal{E}\right| = \frac{\left|(0.20\ \text{T})\cos 180° - (0.30\ \text{T})\cos 0°\right|\pi(0.30\ \text{m})^2}{1.5\ \text{s}} = 9.4\times 10^{-2}\ \text{V} = \boxed{94\ \text{mV}}$$

20.12 $\left|\mathcal{E}\right| = \dfrac{\Delta\Phi_B}{\Delta t} = \dfrac{NBA\left[\Delta(\cos\theta)\right]}{\Delta t}$, so $B = \dfrac{\left|\mathcal{E}\right|\cdot\Delta t}{NA\left[\Delta(\cos\theta)\right]}$

or $\qquad B = \dfrac{(0.166\ \text{V})(2.77\times 10^{-3}\ \text{s})}{500\left[\pi(0.150\ \text{m})^2/4\right]\left[\cos 0° - \cos 90°\right]} = 5.20\times 10^{-5}\ \text{T} = \boxed{52.0\ \mu\text{T}}$

20.13 The required induced emf is $\left|\mathcal{E}\right| = IR = (0.10\ \text{A})(8.0\ \Omega) = 0.80\ \text{V}$.

From $\left|\mathcal{E}\right| = \dfrac{\Delta\Phi_B}{\Delta t} = \left(\dfrac{\Delta B}{\Delta t}\right)NA\cos\theta$

$$\frac{\Delta B}{\Delta t} = \frac{\left|\mathcal{E}\right|}{NA\cos\theta} = \frac{0.80\ \text{V}}{(75)\left[(0.050\ \text{m})(0.080\ \text{m})\right]\cos 0°} = \boxed{2.7\ \text{T/s}}$$

20.14 The initial magnetic field inside the solenoid is

$$B = \mu_0 nI = (4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A})\left(\frac{100}{0.200\ \text{m}}\right)(3.00\ \text{A}) = 1.88\times 10^{-3}\ \text{T}$$

(a) $\Phi_B = BA\cos\theta = (1.88\times 10^{-3}\ \text{T})(1.00\times 10^{-2}\ \text{m})^2\cos 0°$

$$= \boxed{1.88\times 10^{-7}\ \text{T}\cdot\text{m}^2}$$

(b)   When the current is zero, the flux through the loop is $\Phi_B = 0$ and the average induced emf has been

$$|\mathcal{E}| = \frac{\Delta\Phi_B}{\Delta t} = \frac{1.88\times10^{-7} \text{ T}\cdot\text{m}^2 - 0}{3.00 \text{ s}} = \boxed{6.28\times10^{-8} \text{ V}}$$

20.15   If the solenoid has current I, the magnetic field inside it is

$$B = \mu_0 nI = \left(4\pi\times10^{-7} \text{ T}\cdot\text{m/A}\right)\left(\frac{300}{0.200 \text{ m}}\right)I = \left(6.00\,\pi\times10^{-4} \text{ T/A}\right)\cdot I$$

(a) $\Delta\Phi_B = (\Delta B)A\cos\theta$

$$= \left[\left(6.00\,\pi\times10^{-4} \text{ T/A}\right)(5.0 \text{ A} - 2.0 \text{ A})\right]\left[\pi\left(1.5\times10^{-2} \text{ m}\right)^2\right]\cos 0°$$

$$= \boxed{4.0\times10^{-6} \text{ T}\cdot\text{m}^2}$$

(b) $|\mathcal{E}| = \dfrac{N(\Delta\Phi_B)}{\Delta t} = \dfrac{4\left(4.0\times10^{-6} \text{ T}\cdot\text{m}^2\right)}{0.90 \text{ s}} = 1.8\times10^{-5} \text{ V} = \boxed{18 \ \mu\text{V}}$

20.16   The magnitude of the average emf is

$$|\mathcal{E}| = \frac{N(\Delta\Phi_B)}{\Delta t} = \frac{NBA\left[\Delta(\cos\theta)\right]}{\Delta t}$$

$$= \frac{200(1.1 \text{ T})\left(100\times10^{-4} \text{ m}^2\right)(\cos 0° - \cos 180°)}{0.10 \text{ s}} = 44 \text{ V}$$

Therefore, the average induced current is $I = \dfrac{|\mathcal{E}|}{R} = \dfrac{44 \text{ V}}{5.0 \ \Omega} = \boxed{8.8 \text{ A}}$

20.17   If the magnetic field makes an angle of 28.0° with the plane of the coil, the angle it makes with the normal to the plane of the coil is $\theta = 62.0°$. Thus,

$$|\mathcal{E}| = \frac{N(\Delta\Phi_B)}{\Delta t} = \frac{NB(\Delta A)\cos\theta}{\Delta t}$$

$$= \frac{200\left(50.0\times10^{-6} \text{ T}\right)\left[\left(39.0 \text{ cm}^2\right)\left(1 \text{ m}^2/10^4 \text{ cm}^2\right)\right]\cos 62.0°}{1.80 \text{ s}} = 1.02\times10^{-5} \text{ V} = \boxed{10.2 \ \mu\text{V}}$$

20.18   From $\varepsilon = B\ell v$, the required speed is

$$v = \frac{\varepsilon}{B\ell} = \frac{IR}{B\ell} = \frac{(0.500 \text{ A})(6.00 \text{ }\Omega)}{(2.50 \text{ T})(1.20 \text{ m})} = \boxed{1.00 \text{ m/s}}$$

20.19 $\varepsilon = B_\perp \ell v$, where $B_\perp$ is the component of the magnetic field perpendicular to the velocity $\vec{v}$. Thus,

$$\varepsilon = \left[\left(50.0 \times 10^{-6} \text{ T}\right)\sin 58.0°\right](60.0 \text{ m})(300 \text{ m/s}) = \boxed{0.763 \text{ V}}$$

20.20   The speed of the beam after falling freely for 9.00 m, starting from rest $\left(v_{0y} = 0\right)$, is

$$v_y = \sqrt{v_{0y}^2 + 2a_y\left(\Delta y\right)} = \sqrt{0 + 2\left(-9.80 \text{ m/s}^2\right)(-9.00 \text{ m})} = 13.3 \text{ m/s}$$

Since the induced emf is $\varepsilon = B_\perp \ell v$, where $B_\perp$ is the component of the magnetic field perpendicular to the velocity $\vec{v}$, we find

$$\varepsilon = \left(18.0 \times 10^{-6} \text{ T}\right)(12.0 \text{ m})(13.3 \text{ m/s}) = 2.87 \times 10^{-3} \text{ V} = \boxed{2.87 \text{ mV}}$$

20.21   (a)   Observe that only the horizontal component, B_h, of Earth's magnetic field is effective in exerting a vertical force on charged particles in the antenna. For the magnetic force, $F_m = qvB_h \sin\theta$, on positive charges in the antenna to be directed upward and have maximum magnitude (when $\theta = 90°$), the car should move $\boxed{\text{toward the east}}$ through the northward horizontal component of the magnetic field.

(b) $\varepsilon = B_h \ell v$, where B_h is the horizontal component of the magnetic field.

$$\varepsilon = \left[\left(50.0 \times 10^{-6} \text{ T}\right)\cos 65.0°\right](1.20 \text{ m})\left[\left(65.0 \text{ } \frac{\text{km}}{\text{h}}\right)\left(\frac{0.278 \text{ m/s}}{1 \text{ km/h}}\right)\right]$$

$$= \boxed{4.58 \times 10^{-4} \text{ V}}$$

20.22   During each revolution, one of the rotor blades sweeps out a horizontal circular area of radius ℓ, $A = \pi \ell^2$. The number of magnetic field lines cut per revolution is $\Delta \Phi_B = B_\perp A = B_{vertical} A$. The induced emf is then

$$\mathcal{E} = \frac{\Delta \Phi_B}{\Delta t} = \frac{B_{vertical}(\pi \ell^2)}{1/f} = \frac{(5.0 \times 10^{-5} \text{ T})[\pi (3.0 \text{ m})^2]}{0.50 \text{ s}} = 2.8 \times 10^{-3} \text{ V} = \boxed{2.8 \text{ mV}}$$

20.23   (a)   To oppose the motion of the magnet, the magnetic field generated by the induced current should be directed to the right along the axis of the coil. The current must then be $\boxed{\text{left to right}}$ through the resistor.

   (b)   The magnetic field produced by the current should be directed to the left along the axis of the coil, so the current must be $\boxed{\text{right to left}}$ through the resistor.

20.24   (a)   As the bottom conductor of the loop falls, it cuts across the magnetic field lines coming out of the page. This induces an emf of magnitude $\mathcal{E} = Bwv$ in this conductor, with the left end at the higher potential. As a result, an induced current of magnitude

$$I = \frac{\mathcal{E}}{R} = \frac{Bwv}{R}$$

flows clockwise around the loop. The field then exerts an upward force of magnitude

$$F_m = BIw = B\left(\frac{Bwv}{R}\right)w = \frac{B^2 w^2 v}{R}$$

on this current-carrying conductor forming the bottom of the loop. If the loop is falling at terminal speed, the magnitude of this force must equal the downward gravitational force acting on the loop. That is, when $v = v_t$, we must have

$$\frac{B^2 w^2 v_t}{R} = Mg \qquad \text{or} \qquad v_t = \boxed{\frac{MgR}{B^2 w^2}}$$

   (b)   A larger resistance would make the current smaller, so the loop must reach higher speed before the magnitude of the magnetic force will equal the gravitational force.

   (c)   The magnetic force is proportional to the product of the field and the current, while the current itself is proportional to the field. If B is cut in half, the speed must become four times larger to compensate and yield a magnetic force with magnitude equal to the that of the gravitational force.
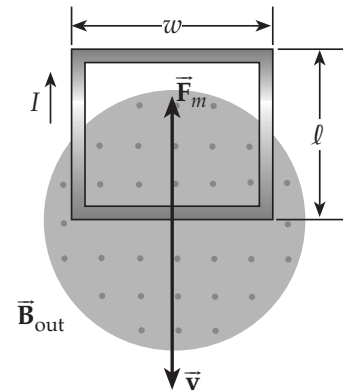
20.25 (a) After the right end of the coil has entered the field, but the left end has not, the flux through the area enclosed by the coil is directed into the page and is increasing in magnitude. This increasing flux induces an emf of magnitude
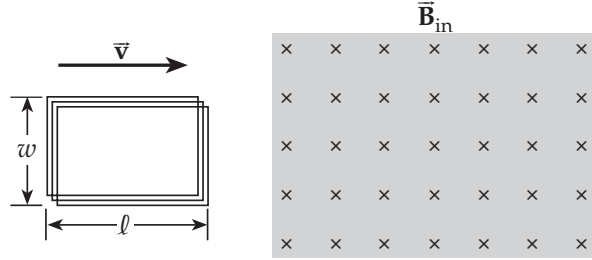
$$|\mathcal{E}| = \frac{\Delta\Phi_B}{\Delta t} = \frac{NB(\Delta A)}{\Delta t} = NBwv$$

in the loop. Note that in the above equation, ΔA is the area enclosed by the coil that enters the field in time Δt. This emf produces a counterclockwise current in the loop to oppose the increasing inward flux. The magnitude of this current is $I = \mathcal{E}/R = NBwv/R$. The right end of the loop is now a conductor, of length Nw, carrying a current toward the top of the page through a field directed into the page. The field exerts a magnetic force of magnitude

$$F = BI(Nw) = B\left(\frac{NBwv}{R}\right)(Nw) = \boxed{\frac{N^2B^2w^2v}{R}} \quad \text{directed} \quad \boxed{\text{toward the left}}$$

on this conductor, and hence, on the loop.

(b) When the loop is entirely within the magnetic field, the flux through the area enclosed by the loop is constant. Hence, there is no induced emf or current in the loop, and the field exerts $\boxed{\text{zero}}$ force on the loop.

(c) After the right end of the loop emerges from the field, and before the left end emerges, the flux through the loop is directed into the page and decreasing. This decreasing flux induces an emf of magnitude $|\mathcal{E}| = NBwv$ in the loop, which produces an induced current directed clockwise around the loop so as to oppose the decreasing flux. The current has magnitude $I = \mathcal{E}/R = NBwv/R$. This current flowing upward, through conductors of total length Nw, in the left end of the loop, experiences a magnetic force given by

$$F = BI(Nw) = B\left(\frac{NBwv}{R}\right)(Nw) = \boxed{\frac{N^2B^2w^2v}{R}} \quad \text{directed} \quad \boxed{\text{toward the left}}$$

20.26 When the switch is closed, the magnetic field due to the current from the battery will be directed to the left along the axis of the cylinder. To oppose this increasing leftward flux, the induced current in the other loop must produce a field directed to the right through the area it encloses. Thus, the induced current is $\boxed{\text{left to right}}$ through the resistor.

20.27 Since the magnetic force, $F_m = qvB \sin \theta$, on a positive charge is directed toward the top of the bar when the velocity is to the right, the right hand rule says that the magnetic field is directed into the page .

20.28 When the switch is closed, the current from the battery produces a magnetic field directed toward the right along the axis of both coils.

(a) As the battery current is growing in magnitude, the induced current in the rightmost coil opposes the increasing rightward directed field by generating a field toward to the left along the axis. Thus, the induced current must be left to right through the resistor.

(b) Once the battery current, and the field it produces, have stabilized, the flux through the rightmost coil is constant and there is no induced current .

(c) As the switch is opened, the battery current and the field it produces rapidly decrease in magnitude. To oppose this decrease in the rightward directed field, the induced current must produce a field toward the right along the axis, so the induced current is right to left through the resistor.

20.29 When the switch is closed, the current from the battery produces a magnetic field directed toward the left along the axis of both coils.

(a) As the current from the battery, and the leftward field it produces, increase in magnitude, the induced current in the leftmost coil opposes the increased leftward field by flowing right to left through R and producing a field directed toward the right along the axis.

(b) As the variable resistance is decreased, the battery current and the leftward field generated by it increase in magnitude. To oppose this, the induced current is right to left through R, producing a field directed toward the right along the axis.

(c) Moving the circuit containing R to the left decreases the leftward field (due to the battery current) along its axis. To oppose this decrease, the induced current is left to right through R, producing an additional field directed toward the left along the axis.

(d) As the switch is opened, the battery current and the leftward field it produces decrease rapidly in magnitude. To oppose this decrease, the induced current is left to right through R, generating additional magnetic field directed toward the left along the axis.

20.30 $\mathcal{E}_{max} = NB_{horizontal}A\omega = 100\left(2.0\times10^{-5}\text{ T}\right)\left(0.20\text{ m}\right)^2\left[\left(1500\,\dfrac{\text{rev}}{\text{min}}\right)\left(\dfrac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\dfrac{1\text{ min}}{60\text{ s}}\right)\right]$

$= 1.3\times10^{-2}\text{ V} = \boxed{13\text{ mV}}$

20.31 Note the similarity between the situation in this problem and a generator. In a generator, one normally has a loop rotating in a constant magnetic field so the flux through the loop varies sinusoidally in time. In this problem, we have a stationary loop in an oscillating magnetic field, and the flux through the loop varies sinusoidally in time. In both cases, a sinusoidal emf $\mathcal{E} = \mathcal{E}_{max}\sin\omega t$ where $\mathcal{E}_{max} = NBA\omega$ is induced in the loop.

The loop in this case consists of a single band $(N=1)$ around the perimeter of a red blood cell with diameter $d = 8.0\times10^{-6}$ m. The angular frequency of the oscillating flux through the area of this loop is $\omega = 2\pi f = 2\pi(60\text{ Hz}) = 120\pi$ rad/s. The maximum induced emf is then

$$\mathcal{E}_{max} = NBA\omega = B\left(\frac{\pi d^2}{4}\right)\omega = \frac{\left(1.0\times10^{-3}\text{ T}\right)\pi\left(8.0\times10^{-6}\text{ m}\right)^2\left(120\pi\text{ s}^{-1}\right)}{4} = \boxed{1.9\times10^{-11}\text{ V}}$$

20.32 (a) Immediately after the switch is closed, the motor coils are still stationary and the back emf is zero. Thus, $I = \dfrac{\mathcal{E}}{R} = \dfrac{240\text{ V}}{30\ \Omega} = \boxed{8.0\text{ A}}$

(b) At maximum speed, $\mathcal{E}_{back} = 145$ V and

$$I = \frac{\mathcal{E} - \mathcal{E}_{back}}{R} = \frac{240\text{ V} - 145\text{ V}}{30\ \Omega} = \boxed{3.2\text{ A}}$$

(c) $\mathcal{E}_{back} = \mathcal{E} - IR = 240\text{ V} - \left(6.0\text{ A}\right)\left(30\ \Omega\right) = \boxed{60\text{ V}}$

20.33 (a) When a coil having N turns and enclosing area A rotates at angular frequency ω in a constant magnetic field, the emf induced in the coil is

$$\mathcal{E} = \mathcal{E}_{max} \sin \omega t \quad \text{where} \quad \mathcal{E}_{max} = NB_{\perp} A \omega$$

Here, $B_{\perp}$ is the magnitude of the magnetic field perpendicular to the rotation axis of the coil. In the given case, $B_{\perp} = 55.0 \ \mu\text{T}$; $A = \pi ab$ where $a = (10.0 \ \text{cm})/2$ and $b = (4.00 \ \text{cm})/2$; and

$$\omega = 2\pi f = 2\pi \left(100 \ \frac{\text{rev}}{\text{min}} \right)\left(\frac{1 \ \text{min}}{60.0 \ \text{s}} \right) = 10.5 \ \text{rad/s}$$

Thus, $\quad \mathcal{E}_{max} = (10.0)(55.0 \times 10^{-6} \ \text{T})\left[\frac{\pi}{4}(0.100 \ \text{m})(0.040 \ 0 \ \text{m}) \right](10.5 \ \text{rad/s})$

or $\quad \mathcal{E}_{max} = 1.81 \times 10^{-5} \ \text{V} = \boxed{18.1 \ \mu\text{V}}$

(b) When the rotation axis is parallel to the field, then $B_{\perp} = 0$ giving $\quad \mathcal{E}_{max} = \boxed{0}$

It is easily understood that the induced emf is always zero in this case if you recognize that the magnetic field lines are always parallel to the plane of the coil, and the flux through the coil has a constant value of zero.

20.34 (a) Using $\mathcal{E}_{max} = NBA\omega$,

$$\mathcal{E}_{max} = 1\,000(0.20 \ \text{T})(0.10 \ \text{m}^2)\left[\left(60 \ \frac{\text{rev}}{\text{s}} \right)\left(\frac{2\pi \ \text{rad}}{1 \ \text{rev}} \right) \right] = 7.5 \times 10^3 = \boxed{7.5 \ \text{kV}}$$

(b) $\mathcal{E}_{max}$ occurs when the flux through the loop is changing the most rapidly. This is when the plane of the loop is $\boxed{\text{parallel to the magnetic field}}$.

20.35 $\omega = \left(120 \ \frac{\text{rev}}{\text{min}} \right)\left(\frac{1 \ \text{min}}{60 \ \text{s}} \right)\left(\frac{2\pi \ \text{rad}}{1 \ \text{rev}} \right) = 4\pi \ \frac{\text{rad}}{\text{s}}$

and the period is $\quad T = \frac{2\pi}{\omega} = 0.50 \ \text{s}$

(a) $\mathcal{E}_{max} = NBA\omega = 500(0.60 \ \text{T})\left[(0.080 \ \text{m})(0.20 \ \text{m}) \right](4\pi \ \text{rad/s}) = \boxed{60 \ \text{V}}$

(b) $\mathcal{E} = \mathcal{E}_{max} \sin(\omega t) = (60 \ \text{V})\sin\left[(4\pi \ \text{rad/s})\left(\frac{\pi}{32} \ \text{s} \right) \right] = \boxed{57 \ \text{V}}$

(c)   The emf is first maximum at $t = \dfrac{T}{4} = \dfrac{0.50 \text{ s}}{4} = \boxed{0.13 \text{ s}}$

20.36   Treating the coiled telephone cord as a solenoid,

$$L = \frac{\mu_0 N^2 A}{\ell} = \frac{\left(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}\right)(70.0)^2 \left[\frac{\pi}{4}\left(1.30 \times 10^{-2}\right)^2\right]}{0.600 \text{ m}}$$

$$= 1.36 \times 10^{-6} \text{ H} = \boxed{1.36 \ \mu\text{H}}$$

20.37 $\left|\mathcal{E}_{\text{av}}\right| = L\left|\dfrac{\Delta I}{\Delta t}\right| = \left(3.00 \times 10^{-3} \text{ H}\right)\left(\dfrac{1.50 \text{ A} - 0.20 \text{ A}}{0.20 \text{ s}}\right) = 2.0 \times 10^{-2} \text{ V} = \boxed{20 \text{ mV}}$

20.38   The units of $\dfrac{N\Phi_B}{I}$ are $\dfrac{\text{T} \cdot \text{m}^2}{\text{A}}$

From the force on a moving charged particle, $F = qvB$, the magnetic field is $B = \dfrac{F}{qv}$ and we find that

$$1 \text{ T} = 1 \frac{\text{N}}{\text{C} \cdot (\text{m/s})} = 1 \frac{\text{N} \cdot \text{s}}{\text{C} \cdot \text{m}}$$

Thus, $\text{T} \cdot \text{m}^2 = \left(\dfrac{\text{N} \cdot \text{s}}{\text{C} \cdot \text{m}}\right) \cdot \text{m}^2 = \dfrac{(\text{N} \cdot \text{m}) \cdot \text{s}}{\text{C}} = \left(\dfrac{\text{J}}{\text{C}}\right) \cdot \text{s} = \text{V} \cdot \text{s}$

and $\dfrac{\text{T} \cdot \text{m}^2}{\text{A}} = \dfrac{\text{V} \cdot \text{s}}{\text{A}}$ which is the same as the units of $\dfrac{\mathcal{E}}{\Delta I / \Delta t}$

20.39   (a) $L = \dfrac{\mu_0 N^2 A}{\ell} = \dfrac{\left(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}\right)(400)^2 \left[\pi\left(2.5 \times 10^{-2} \text{ m}\right)^2\right]}{0.20 \text{ m}}$

$$= 2.0 \times 10^{-3} \text{ H} = \boxed{2.0 \text{ mH}}$$

(b)   From $\left|\mathcal{E}\right| = L\left(\Delta I / \Delta t\right)$, $\dfrac{\Delta I}{\Delta t} = \dfrac{\left|\mathcal{E}\right|}{L} = \dfrac{75 \times 10^{-3} \text{ V}}{2.0 \times 10^{-3} \text{ H}} = \boxed{38 \text{ A/s}}$

20.40 From $|\mathcal{E}| = L(\Delta I/\Delta t)$, the self-inductance is

$$L = \frac{|\mathcal{E}|}{\Delta I/\Delta t} = \frac{24.0 \times 10^{-3} \text{ V}}{10.0 \text{ A/s}} = 2.40 \times 10^{-3} \text{ H}$$

Then, from $L = N\Phi_B/I$, the magnetic flux through each turn is

$$\Phi_B = \frac{L \cdot I}{N} = \frac{(2.40 \times 10^{-3} \text{ H})(4.00 \text{ A})}{500} = \boxed{1.92 \times 10^{-5} \text{ T} \cdot \text{m}^2}$$

20.41 The inductive time constant is $\tau = L/R$. From $|\mathcal{E}| = L(\Delta I/\Delta t)$, the self-inductance is

$L = \dfrac{|\mathcal{E}|}{\Delta I/\Delta t}$ with units of $\dfrac{\text{V}}{\text{A/s}} = \left(\dfrac{\text{V}}{\text{A}}\right) \cdot \text{s} = \Omega \cdot \text{s}$. Thus, the units of the time constant are

$$\frac{\Omega \cdot \text{s}}{\Omega} = \text{s}.$$

20.42 (a) The time constant of the RL circuit is $\tau = L/R$, and that of the RC circuit is $\tau = RC$. If the two time constants have the same value, then

$$RC = \frac{L}{R}, \text{ or } R = \sqrt{\frac{L}{C}} = \sqrt{\frac{3.00 \text{ H}}{3.00 \times 10^{-6} \text{ F}}} = 1.00 \times 10^3 \text{ } \Omega = \boxed{1.00 \text{ k}\Omega}$$

(b) The common value of the two time constants is

$$\tau = \frac{L}{R} = \frac{3.00 \text{ H}}{1.00 \times 10^3 \text{ } \Omega} = 3.00 \times 10^{-3} \text{ s} = \boxed{3.00 \text{ ms}}$$

20.43 The maximum current in a RL circuit $I_{max} = \mathcal{E}/R$, so the resistance is

$$R = \frac{\mathcal{E}}{I_{max}} = \frac{6.0 \text{ V}}{0.300 \text{ A}} = 20 \text{ } \Omega$$

The inductive time constant is $\tau = L/R$, so

$$L = \tau \cdot R = (600 \times 10^{-6} \text{ s})(20 \text{ } \Omega) = 1.2 \times 10^{-2} \text{ H} = \boxed{12 \text{ mH}}$$

20.44 The current in the *RL* circuit at time *t* is $I = \dfrac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right)$. The potential difference across

the resistor is $\Delta V_R = RI = \mathcal{E}\left(1 - e^{-t/\tau}\right)$, and from Kirchhoff's loop rule, the potential

difference across the inductor is

$$\Delta V_L = \mathcal{E} - \Delta V_R = \mathcal{E}\left[1 - \left(1 - e^{-t/\tau}\right)\right] = \mathcal{E}e^{-t/\tau}$$

(a) At $t = 0$, $\Delta V_R = \mathcal{E}\left(1 - e^{-0}\right) = \mathcal{E}(1 - 1) = \boxed{0}$

(b) At $t = \tau$, $\Delta V_R = \mathcal{E}\left(1 - e^{-1}\right) = (6.0\text{ V})(1 - 0.368) = \boxed{3.8\text{ V}}$

(c) At $t = 0$, $\Delta V_L = \mathcal{E}e^{-0} = \mathcal{E} = \boxed{6.0\text{ V}}$

(d) At $t = \tau$, $\Delta V_L = \mathcal{E}e^{-1} = (6.0\text{ V})(0.368) = \boxed{2.2\text{ V}}$

20.45 From $I = I_{max}\left(1 - e^{-t/\tau}\right)$, $e^{-t/\tau} = 1 - \dfrac{I}{I_{max}}$

If $\dfrac{I}{I_{max}} = 0.900$ at $t = 3.00$ s , then

$$e^{-3.00\text{ s}/\tau} = 0.100 \text{ or } \tau = \frac{-3.00\text{ s}}{\ln(0.100)} = 1.30\text{ s}$$

Since the time constant of an *RL* circuit is $\tau = L/R$, the resistance is

$$R = \frac{L}{\tau} = \frac{2.50\text{ H}}{1.30\text{ s}} = \boxed{1.92\ \Omega}$$

20.46 (a) $\tau = \dfrac{L}{R} = \dfrac{8.00\text{ mH}}{4.00\ \Omega} = \boxed{2.00\text{ ms}}$

(b) $I = \dfrac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right) = \left(\dfrac{6.00\text{ V}}{4.00\ \Omega}\right)\left(1 - e^{-250\times 10^{-6}\text{ s}/2.00\times 10^{-3}\text{ s}}\right) = \boxed{0.176\text{ A}}$

(c) $I_{max} = \dfrac{\mathcal{E}}{R} = \dfrac{6.00\text{ V}}{4.00\ \Omega} = \boxed{1.50\text{ A}}$

(d) $I = I_{max}\left(1 - e^{-t/\tau}\right)$ yields $e^{-t/\tau} = 1 - I/I_{max}$,

and $t = -\tau\ln\left(1 - I/I_{max}\right) = -(2.00\text{ ms})\ln(1 - 0.800) = \boxed{3.22\text{ ms}}$

20.47 $PE_L = \frac{1}{2}LI^2 = \frac{1}{2}(70.0 \times 10^{-3} \text{ H})(2.00 \text{ A})^2 = \boxed{0.140 \text{ J}}$

20.48   (a)   The inductance of a solenoid is given by $L = \mu_0 N^2 A/\ell$, where N is the number of turns on the solenoid, A is its cross-sectional area, and ℓ is its length. For the given solenoid,

$$L = \frac{\mu_0 N^2 (\pi r^2)}{\ell} = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(300)^2 \pi (5.00 \times 10^{-2} \text{ m})^2}{0.200 \text{ m}} = \boxed{4.44 \times 10^{-3} \text{ H}}$$

(b)   When the solenoid described above carries a current of $I = 0.500 \text{ A}$, the stored energy is

$$PE_L = \frac{1}{2}LI^2 = \frac{1}{2}(4.44 \times 10^{-3} \text{ H})(0.500 \text{ A})^2 = \boxed{5.55 \times 10^{-4} \text{ J}}$$

20.49   The current in the circuit at time t is $I = \frac{\mathcal{E}}{R}(1 - e^{-t/\tau})$, and the energy stored in the inductor is $PE_L = \frac{1}{2}LI^2$

(a)   As $t \to \infty$, $I \to I_{max} = \frac{\mathcal{E}}{R} = \frac{24 \text{ V}}{8.0 \text{ }\Omega} = 3.0 \text{ A}$, and

$$PE_L \to \frac{1}{2}LI_{max}^2 = \frac{1}{2}(4.0 \text{ H})(3.0 \text{ A})^2 = \boxed{18 \text{ J}}$$

(b)   At $t = \tau$, $I = I_{max}(1 - e^{-1}) = (3.0 \text{ A})(1 - 0.368) = 1.9 \text{ A}$

and $PE_L = \frac{1}{2}(4.0 \text{ H})(1.9 \text{ A})^2 = \boxed{7.2 \text{ J}}$

20.50   (a)   When the two resistors are in series, the total resistance is

$$R_{eq} = R + R = 2R \text{ , and the time constant of the circuit is } \tau = \frac{L}{R_{eq}} = \boxed{\frac{L}{2R}}$$

(b)   With the resistors now connected in parallel, the total resistance is

$$R_{eq} = \frac{(R)(R)}{R + R} = \frac{R}{2} \text{ , and the time constant is } \tau = \frac{L}{R_{eq}} = \boxed{\frac{2L}{R}}$$

20.51   According to Lenz's law, a current will be induced in the coil to oppose the change in magnetic flux due to the magnet. Therefore, current must be directed from b to a through the resistor, and $V_a - V_b$ will be $\boxed{\text{negative}}$.

20.52   While the coil is between the poles of the magnet, the component of the field perpendicular to the plane of the coil is $B_i = 0.10 \text{ T}$. After the coil is pulled out of the field, $B_f \approx 0$.

The magnitude of the average induced emf as the coil is moved is

$$|\mathcal{E}| = N\frac{\Delta\Phi_B}{\Delta t} = N\frac{\Delta(BA)}{\Delta t} = N\frac{(\Delta B)A}{\Delta t}$$

and the average induced current in the galvanometer is

$$I = \frac{|\mathcal{E}|}{R} = \frac{N(\Delta B)A}{R(\Delta t)} = \frac{10(0.10 \text{ T} - 0)\left[\pi(0.020 \text{ m})^2\right]}{(2.0 \ \Omega)(0.20 \text{ s})}$$

$$= 3.1 \times 10^{-3} \text{ A} = \boxed{3100 \ \mu\text{A}}$$

This means the galvanometer will $\boxed{\text{definitely show the induced current}}$ and even be overloaded.

20.53   (a)   The current in the solenoid reaches $I = 0.632 I_{max}$ in a time of $t = \tau = L/R$, where

$$L = \frac{\mu_0 N^2 A}{\ell} = \frac{\left(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}\right)(12\,500)^2\left(1.00 \times 10^{-4} \text{ m}^2\right)}{7.00 \times 10^{-2} \text{ m}} = 0.280 \text{ H}$$

Thus, $t = \dfrac{0.280 \text{ H}}{14.0 \ \Omega} = 2.00 \times 10^{-2} \text{ s} = \boxed{20.0 \text{ ms}}$

(b)   The change in the solenoid current during this time is

$$\Delta I = 0.632 I_{max} - 0 = 0.632\left(\frac{\Delta V}{R}\right) = 0.632\left(\frac{60.0 \text{ V}}{14.0 \ \Omega}\right) = 2.71 \text{ A}$$

so the average back emf is

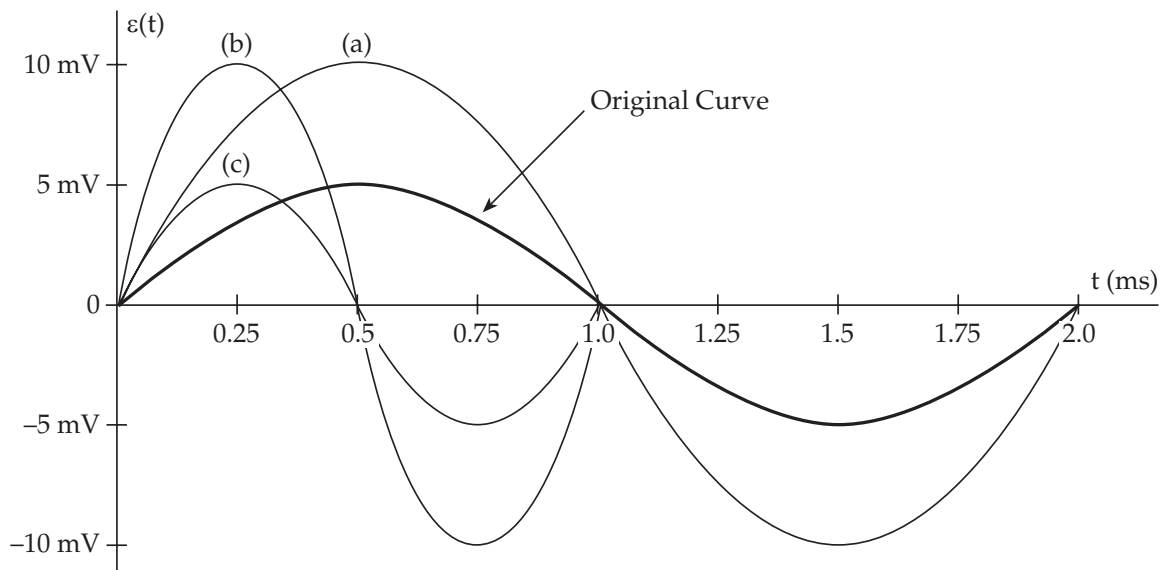$$\mathcal{E}_{back} = L\left(\frac{\Delta I}{\Delta t}\right) = (0.280 \text{ H})\left(\frac{2.71 \text{ A}}{2.00 \times 10^{-2} \text{ s}}\right) = \boxed{37.9 \text{ V}}$$

(c) $\dfrac{\Delta\Phi_B}{\Delta t} = \dfrac{(\Delta B)A}{\Delta t} = \dfrac{\frac{1}{2}\left[\mu_0 n(\Delta I)\right]A}{\Delta t} = \dfrac{\mu_0 N(\Delta I)A}{2\ell\cdot(\Delta t)}$

$$= \dfrac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(12\,500)(2.71\ \text{A})\left(1.00\times10^{-4}\ \text{m}^2\right)}{2\left(7.00\times10^{-2}\ \text{m}\right)\left(2.00\times10^{-2}\ \text{s}\right)} = \boxed{1.52\times10^{-3}\ \text{V}}$$

(d) $I = \dfrac{\left|\mathcal{E}_{coil}\right|}{R_{coil}} = \dfrac{N_{coil}\left(\Delta\Phi_B/\Delta t\right)}{R_{coil}} = \dfrac{(820)\left(1.52\times10^{-3}\ \text{V}\right)}{24.0\ \Omega} = 0.0519\ \text{A} = \boxed{51.9\ \text{mA}}$

20.54

(a) Doubling the number of turns doubles the amplitude but does not alter the period.

(b) Doubling the angular velocity doubles the amplitude and also cuts the period in half.

(c) Doubling the angular velocity while reducing the number of turns to one half the original value leaves the amplitude unchanged but does cut the period in half.

20.55 $Q = I_{av}\left(\Delta t\right) = \dfrac{\left|\mathcal{E}_{av}\right|}{R}\left(\Delta t\right) = \dfrac{1}{R}\left(\dfrac{\Delta\Phi_B}{\Delta t}\right)\left(\Delta t\right) = \dfrac{B(\Delta A)}{R}$

or $Q = \dfrac{\left(15.0\times10^{-6}\ \text{T}\right)\left[(0.200\ \text{m})^2 - 0\right]}{0.500\ \Omega} = 1.20\times10^{-6}\ \text{C} = \boxed{1.20\ \mu\text{C}}$

20.56   (a) $PE_L = \dfrac{1}{2}LI^2 = \dfrac{1}{2}(50.0 \text{ H})(50.0 \times 10^3 \text{ A})^2 = \boxed{6.25 \times 10^{10} \text{ J}}$

(b) $\dfrac{F}{\ell} = \dfrac{\mu_0 I_1 I_2}{2\pi d} = \dfrac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(50.0 \times 10^3 \text{ A})^2}{2\pi (0.250 \text{ m})}$

$= 2.00 \times 10^3 \ \dfrac{\text{N}}{\text{m}} = \boxed{2.00 \ \dfrac{\text{kN}}{\text{m}}}$

20.57   (a)   To move the bar at uniform speed, the magnitude of the applied force must equal that of the magnetic force retarding the motion of the bar. Therefore, $F_{app} = BI\ell$. The magnitude of the induced current is

$$I = \dfrac{|\varepsilon|}{R} = \dfrac{(\Delta \Phi_B / \Delta t)}{R} = \dfrac{B(\Delta A / \Delta t)}{R} = \dfrac{B\ell v}{R}$$

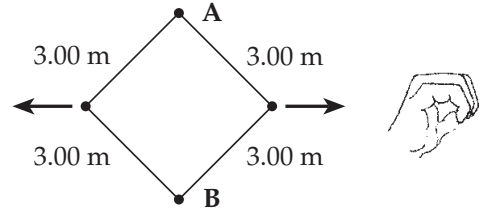so the field strength is $B = \dfrac{IR}{\ell v}$, giving $F_{app} = I^2 R / v$

Thus, the current is

$$I = \sqrt{\dfrac{F_{app} \cdot v}{R}} = \sqrt{\dfrac{(1.00 \text{ N})(2.00 \text{ m/s})}{8.00 \ \Omega}} = \boxed{0.500 \text{ A}}$$

(b) $\mathcal{P} = I^2 R = (0.500 \text{ A})^2 (8.00 \ \Omega) = \boxed{2.00 \text{ W}}$

(c) $\mathcal{P}_{input} = F_{app} \cdot v = (1.00 \text{ N})(2.00 \text{ m/s}) = \boxed{2.00 \text{ W}}$

20.58 When A and B are 3.00 m apart, the area enclosed by the loop consists of four triangular sections, each having hypotenuse of 3.00 m, altitude of 1.50 m, and base of $\sqrt{(3.00 \text{ m})^2 - (1.50 \text{ m})^2} = 2.60$ m The decrease in the enclosed area has been

$$\Delta A = A_i - A_f = (3.00 \text{ m})^2 - 4\left[\frac{1}{2}(1.50 \text{ m})(2.60 \text{ m})\right] = 1.21 \text{ m}^2$$

The average induced current has been

$$I_{av} = \frac{|\mathcal{E}_{av}|}{R} = \frac{(\Delta\Phi_B/\Delta t)}{R} = \frac{B(\Delta A/\Delta t)}{R} = \frac{(0.100 \text{ T})(1.21 \text{ m}^2/0.100 \text{ s})}{10.0 \text{ }\Omega} = \boxed{0.121 \text{ A}}$$

As the enclosed area decreases, the flux (directed into the page) through this area also decreases. Thus, the induced current will be directed $\boxed{\text{clockwise}}$ around the loop to create additional flux directed into the page through the enclosed area.

20.59 If d is the distance from the lightning bolt to the center of the coil, then

$$|\mathcal{E}_{av}| = \frac{N(\Delta\Phi_B)}{\Delta t} = \frac{N(\Delta B)A}{\Delta t} = \frac{N\left[\mu_0(\Delta I)/2\pi d\right]A}{\Delta t} = \frac{N\mu_0(\Delta I)A}{2\pi d(\Delta t)}$$

$$= \frac{100\left(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}\right)\left(6.02 \times 10^6 \text{ A} - 0\right)\left[\pi(0.800 \text{ m})^2\right]}{2\pi(200 \text{ m})\left(10.5 \times 10^{-6} \text{ s}\right)}$$

$$= 1.15 \times 10^5 \text{ V} = \boxed{115 \text{ kV}}$$

20.60 The flux through the surface area of the tent is the same as that through the tent base. Thus, as the tent is flattened, the change is flux is

$$\Delta\Phi_B = B(\Delta A_{base}) = B\left[L(2L) - L(2 \cdot L\cos\theta)\right] = 2L^2B(1 - \cos\theta)$$

The magnitude of the average induced emf is then

$$|\mathcal{E}_{av}| = \frac{\Delta\Phi_B}{\Delta t} = \frac{2L^2B(1 - \cos\theta)}{\Delta t} = \frac{2(1.5 \text{ m})^2(0.30 \text{ T})(1 - \cos 60°)}{0.10 \text{ s}} = \boxed{6.8 \text{ V}}$$

20.61 (a) $\left|\mathcal{E}_{av}\right| = \dfrac{\Delta \Phi_B}{\Delta t} = \dfrac{B(\Delta A)}{\Delta t} = \dfrac{B\left[\left(\pi d^2/4\right) - 0\right]}{\Delta t}$

$$= \dfrac{(25.0 \text{ mT})\pi \left(2.00 \times 10^{-2} \text{ m}\right)^2}{4\left(50.0 \times 10^{-3} \text{ s}\right)} = \boxed{0.157 \text{ mV}}$$

As the inward directed flux through the loop decreases, the induced current goes clockwise around the loop in an attempt to create additional inward flux through the enclosed area. With positive charges accumulating at *B*,

$\boxed{\text{point } B \text{ is at a higher potential than } A}$

(b) $\left|\mathcal{E}_{av}\right| = \dfrac{\Delta \Phi_B}{\Delta t} = \dfrac{(\Delta B)A}{\Delta t} = \dfrac{\left[(100 - 25.0) \text{ mT}\right]\pi \left(2.00 \times 10^{-2} \text{ m}\right)^2}{4\left(4.00 \times 10^{-3} \text{ s}\right)} = \boxed{5.89 \text{ mV}}$

As the inward directed flux through the enclosed area increases, the induced current goes counterclockwise around the loop in an attempt to create flux directed outward through the enclosed area.

With positive charges now accumulating at *A*,

$\boxed{\text{point } A \text{ is at a higher potential than } B}$

20.62 The induced emf in the ring is

$$\left|\mathcal{E}_{av}\right| = \dfrac{\Delta \Phi_B}{\Delta t} = \dfrac{(\Delta B)A_{solenoid}}{\Delta t} = \dfrac{\left(\Delta B_{solenoid}/2\right)A_{solenoid}}{\Delta t} = \dfrac{1}{2}\left[\mu_0 n\left(\dfrac{\Delta I_{solenoid}}{\Delta t}\right)\right]A_{solenoid}$$

$$= \dfrac{1}{2}\left[\left(4\pi \times 10^{-7} \text{ T·m/A}\right)(1\,000)(270 \text{ A/s})\left(\pi \left[3.00 \times 10^{-2} \text{ m}\right]^2\right)\right] = 4.80 \times 10^{-4} \text{ V}$$

Thus, the induced current in the ring is

$$I_{ring} = \dfrac{\left|\mathcal{E}_{av}\right|}{R} = \dfrac{4.80 \times 10^{-4} \text{ V}}{3.00 \times 10^{-4} \text{ }\Omega} = \boxed{1.60 \text{ A}}$$

20.63 (a) As the rolling axle (of length $\ell = 1.50$ m) moves perpendicularly to the uniform magnetic field, an induced emf of magnitude $\left|\mathcal{E}\right| = B\ell v$ will exist between its ends. The current produced in the closed-loop circuit by this induced emf has magnitude

$$I = \dfrac{\left|\mathcal{E}_{av}\right|}{R} = \dfrac{\left(\Delta \Phi_B/\Delta t\right)}{R} = \dfrac{B(\Delta A/\Delta t)}{R} = \dfrac{B\ell v}{R} = \dfrac{(0.800 \text{ T})(1.50 \text{ m})(3.00 \text{ m/s})}{0.400 \text{ }\Omega} = \boxed{9.00 \text{ A}}$$

(b) The induced current through the axle will cause the magnetic field to exert a retarding force of magnitude $F_r = BI\ell$ on the axle. The direction of this force will be opposite to that of the velocity $\vec{v}$ so as to oppose the motion of the axle. If the axle is to continue moving at constant speed, an applied force in the direction of $\vec{v}$ and having magnitude $F_{app} = F_r$ must be exerted on the axle.

$$F_{app} = BI\ell = (0.800\text{ T})(9.00\text{ A})(1.50\text{ m}) = \boxed{10.8\text{ N}}$$

(c) Using the right-hand rule, observe that positive charges within the moving axle experience a magnetic force toward the rail containing point b, and negative charges experience a force directed toward the rail containing point a. Thus, the rail containing b will be positive relative to the other rail. Point $\boxed{b\text{ is then at a higher potential than }a}$, and the current goes from b to a through the resistor R.

(d) $\boxed{\text{No}}$. Both the velocity $\vec{v}$ of the rolling axle and the magnetic field $\vec{B}$ are unchanged. Thus, the polarity of the induced emf in the moving axle is unchanged, and the current continues to be directed from b to a through the resistor R.

20.64 (a) The flowing water is a conductor moving through Earth's magnetic field. A motional emf given by $|\mathcal{E}| = B(w)v$ will exist in the water between the plates and the induced current in the load resistor is

$$I = |\mathcal{E}|/R_{total} = B(w)v/R_{total}$$

where $\quad R_{total} = R_{water} + R = \rho\dfrac{\ell}{A} + R = \rho\dfrac{w}{ab} + R$

Thus, $\quad I = \dfrac{B(w)v}{\rho(w/ab) + R} = \boxed{\dfrac{abvB}{\rho + abR/w}}$

(b) If $R = 0$,

$$I = \frac{(100\text{ m})(5.00\text{ m})(3.00\text{ m/s})(50.0\times 10^{-6}\text{ T})}{100\ \Omega\cdot\text{m} + 0}$$

$$= 7.50\times 10^{-4}\text{ A} = \boxed{0.750\text{ mA}}$$

20.65 Consider the closed conducting path made up by the horizontal wire, the vertical rails and the path containing the resistance R at the bottom of the figure. As the wire slides down the rails, the outward directed flux through the area enclosed by this path is decreasing as the area decreases. This decreasing flux produces an induced current which flows counterclockwise around the conducting path (and hence, right to left through the horizontal wire) to oppose the decrease in flux.
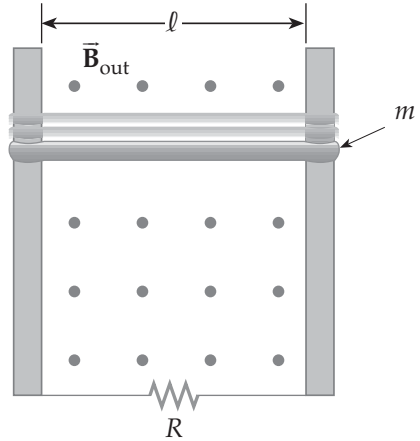
The wire is now carrying a current toward the left through a magnetic field directed out of the page. The field then exerts an upward magnetic force on the wire of magnitude

$$F = BI\ell\sin 90° = B\left(\frac{|\mathcal{E}|}{R}\right)\ell = \frac{B\left(|\Delta\Phi_B|/\Delta t\right)\ell}{R} = \frac{B\left(B|\Delta A|/\Delta t\right)\ell}{R} = \frac{B^2\left(\ell v\right)\ell}{R} = \frac{B^2\ell^2 v}{R}$$

Observe that this upward directed magnetic force opposes the weight of the wire, and its magnitude is proportional to the speed of the falling wire. The wire will be at its terminal speed $(v = v_t)$ when the magnitude of the magnetic force equals the weight of the wire. That is, when

$$\frac{B^2\ell^2 v_t}{R} = mg \qquad \text{which gives} \qquad \boxed{v_t = \frac{mgR}{B^2\ell^2}}$$

20.66 The flux through the area enclosed by the coil is given by $\Phi_B = BA$, where B is the magnetic field perpendicular to the plane of the coil, and A is the enclosed area. Compare Figures (a) and (b) at the right and observe that when $B > 0$, Φ_B is directed into the page through the interior of the coil. The induced current in the coil will have a magnitude of

(a)

$$I = \frac{|\mathcal{E}|}{R} = \frac{|\Delta\Phi_B/\Delta t|}{R} = \left(\frac{|\Delta B|}{\Delta t}\right)\frac{A}{R}$$

where R is the resistance of the coil.
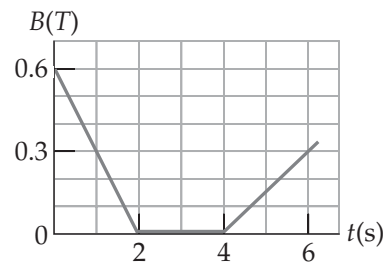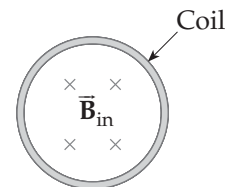
(b)

(a) From the above result, note that the induced current has greatest magnitude when $|\Delta B|/\Delta t$ is the greatest (that is, when the graph of B vs. t has the steepest slope).

From Figure (a) above, this is seen to be in the interval $\boxed{\text{between } t = 0 \text{ and } t = 2.0 \text{ s}}$.

(b) $I = 0$ when $|\Delta B|/\Delta t = 0$ (that is, when B is constant). This is true in the interval

$\boxed{\text{between } t = 2.0 \text{ s and } t = 4.0 \text{ s}}$.

(c) No. The direction of the induced current will always be such as to oppose the change that is occurring in the flux through the coil. When B is decreasing the change in the flux (and hence the induced current) is directed opposite to what it is when B is increasing.

(d) Between $t = 0$ and $t = 2.0$ s , the flux through the coil is directed into the page and *decreasing* in magnitude. The induced current must flow $\boxed{\text{clockwise}}$ around the coil so that the flux it generates through the interior of the coil is into the page opposing the change that is occurring in the primary flux. The magnitude is

$$I = \left(\frac{|\Delta B|}{\Delta t} \right) \frac{A}{R} = \left(\frac{0.60 \text{ T}}{2.0 \text{ s}} \right) \frac{0.20 \text{ m}^2}{0.25 \ \Omega} = \boxed{0.24 \text{ A}}$$

Between $t = 2.0$ s and $t = 4.0$ s , the induced $\boxed{\text{current is zero}}$ because the flux is constant.

Between $t = 4.0$ and $t = 6.0$ s , the flux through the coil is directed into the page and *increasing* in magnitude. The induced current must flow $\boxed{\text{counterclockwise}}$ around the coil so that the flux it generates through the interior of the coil is out of the page opposing the change that is occurring in the primary flux. The current magnitude is

$$I = \left(\frac{|\Delta B|}{\Delta t} \right) \frac{A}{R} = \left(\frac{0.30 \text{ T}}{2.0 \text{ s}} \right) \frac{0.20 \text{ m}^2}{0.25 \ \Omega} = \boxed{0.12 \text{ A}}$$