8

INEQUALITIES IN TRIANGLES

I. INTRODUCTION AND FOCUS QUESTIONS

Have you ever wondered how artists utilize triangles in their artworks? Have you ever asked yourself how contractors, architects, and engineers make use of triangular features in their designs? What mathematical concepts justify all the triangular intricacies of their designs? The answers to these queries are unveiled in this module.

The concepts and skills you will learn from this lesson on the axiomatic development of triangle inequalities will improve your attention to details, shape your deductive thinking, hone your reasoning skills and polish your mathematical communication. In short, this module unleashes that mind power that you never thought you ever had before!

Remember to find out the answers to this essential question: *"How can you justify inequalities in triangles?"*

II. LESSONS AND COVERAGE

In this module, you will examine this question when you take the following lessons:

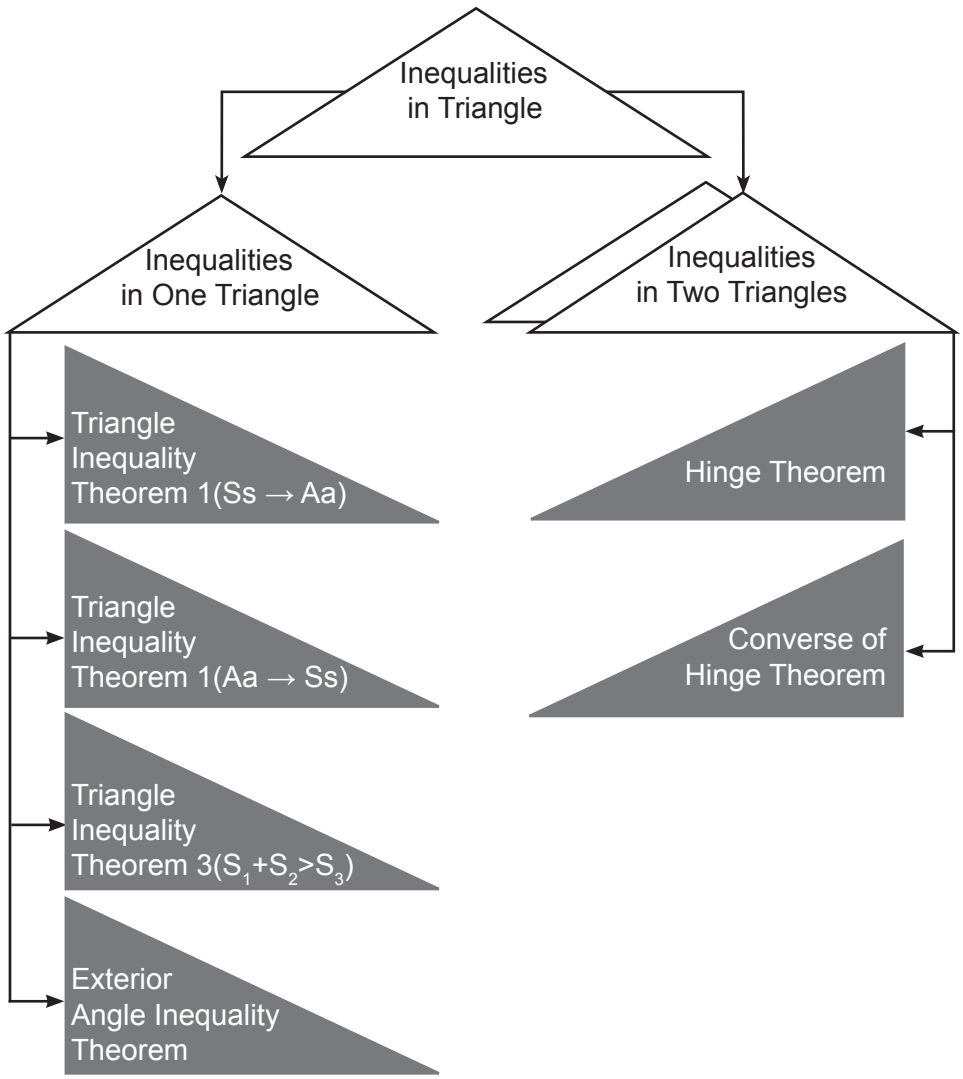
Lesson 1 – Inequalities in Triangles

1.1 Inequalities among Sides and among Angles of a Triangle
1.2 Theorems on Triangle Inequality
1.3 Applications of the Theorems on Triangle Inequality

In these lessons, you will learn to:

Lesson 1	• state and illustrate the theorems on triangle inequalities such as exterior angle inequality theorem, triangle inequality theorem, hinge theorem. • apply theorems on triangle inequalities to: a. determine possible measures for the angles and sides of triangles. b. justify claims about the unequal relationships between side and angle measures; and • use the theorems on triangle inequalities to prove statements involving triangle inequalities.

Module Map



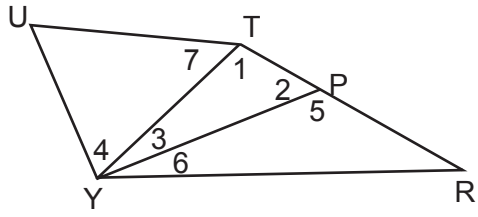
III. PRE - ASSESSMENT

Find out how much you already know about this topic. On a separate sheet, write only the letter of the choice that you think best answers the question. Please answer all items. During the checking, take note of the items that you were not able to answer correctly and find out the right answers as you go through this module.

1. The measure of an exterior angle of a triangle is always _____.

   a. greater than its adjacent interior angle.
   b. less than its adjacent interior angle.
   c. greater than either remote interior angle.
   d. less than either remote interior angle.

2. Which of the following angles is an exterior angle of ΔTYP?

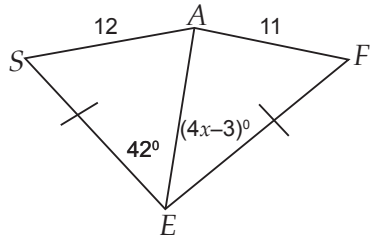


   A. $\angle 4$ B. $\angle 5$ C. $\angle 6$ D. $\angle 7$

3. Each of Xylie, Marie, Angel and Chloe was given an 18-inch piece of stick. They were instructed to create a triangle. Each cut the stick in their own chosen lengths as follows: Xylie—6 in, 6 in, 6 in; Marie—4 in, 5 in, 9 in; Angle—7 in, 5 in, 6 in; and Chloe—3 in, 7 in, 5 in. Who among them was not able to make a triangle?

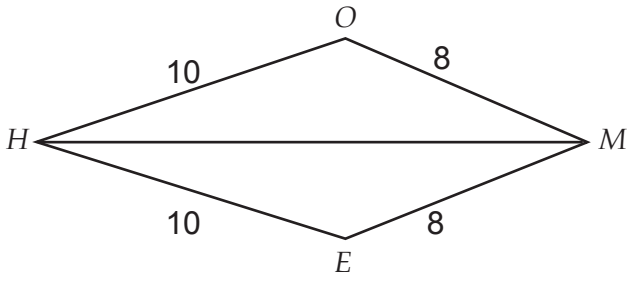
   a. Xylie b. Marie c. Angel d. Chloe

4. What are the possible values for x in the figure?



   a. $x < 11.25$ c. $x \leq 11.25$
   b. $x > 11.25$ d. $x \geq 11.25$

5. From the inequalities in the triangles shown, a conclusion can be reached using the converse of hinge theorem. Which of the following is the last statement?



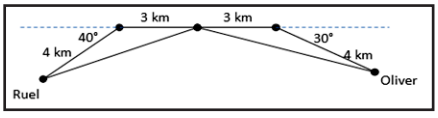
a. $HM \cong HM$
b. $m\angle OHM > m\angle EHM$
c. $HO \cong HE$
d. $m\angle EHM > m\angle OHM$

6. Hikers Oliver and Ruel who have uniform hiking speed walk in opposite directions- Oliver, eastward whereas Ruel, westward. After walking three kilometers each, both of them take left turns at different angles- Oliver at an angle of 30^0 and Ruel at 40^0. Both continue hiking and cover another four kilometers each before taking a rest. Which of the hikers is farther from their point of origin?

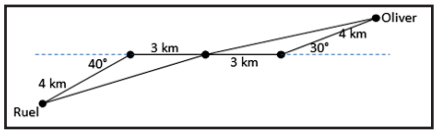
a. Ruel
b. Oliver
c. It cannot be determined.
d. Ruel is as far as Oliver from the rendezvous.

7. Which of the following is the accurate illustration of the problem?

a.



b.



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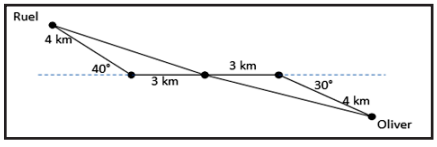
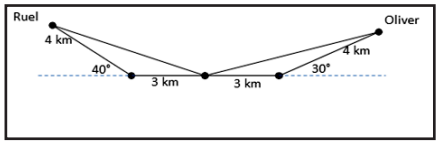
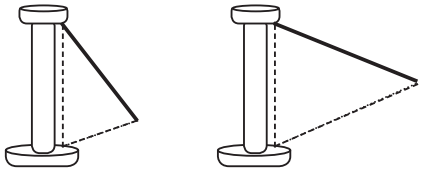


d.



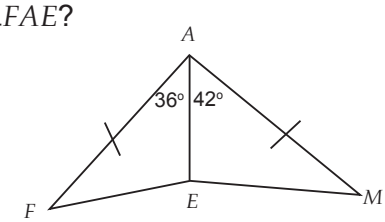
8. The chairs of a swing ride are farthest from the base of the swing tower when the swing ride is at full speed. What conclusion can you make about the angles of the swings at different speeds?



a. The angles of the swings remain constant whether the speed is low or full.
b. The angles of the swings are smaller at full speed than at low speed.
c. The angles of the swings are larger at full speed than at low speed.
d. The angles of the swings are larger at low speed than at full speed.

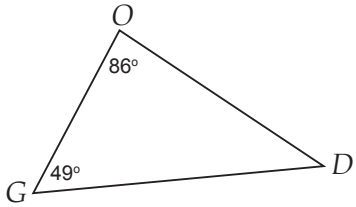
9. Will you be able to conclude that $EM > EF$ if one of the following statements is not established: $\overline{AE} \cong \overline{AE}$, $\overline{AF} \cong \overline{AM}$, $m\angle MAE > m\angle FAE$?

a. Yes, I will.
b. No, I won't.
c. It is impossible to decide.
d. It depends on which statement is left out.



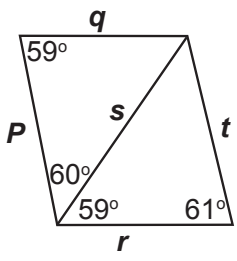
10. Which side of ΔGOD is the shortest?

a. $\overline{GO}$
b. $\overline{DO}$
c. $\overline{DG}$
d. $\overline{GD}$



11. The diagram is not drawn to scale. Which of the following combined inequalities describes p,q,r,s, and t?

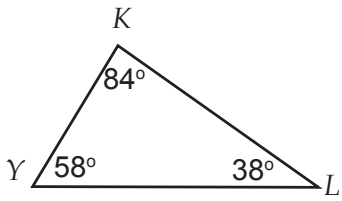
a. $p < q < r < s < t$
b. $s < p < q < r < t$
c. $t < r < s < q < p$
d. $q < p < t < r < s$



12. In ΔTRU, TR = 8 cm, RU = 9 cm, and TU = 10 cm. List the angles in order from least to greatest measure.

a. $m\angle T,\ m\angle R,\ m\angle U$
b. $m\angle U,\ m\angle T,\ m\angle R$
c. $m\angle R,\ m\angle T,\ m\angle U$
d. $m\angle U,\ m\angle R,\ m\angle T$

13. List the sides of ΔLYK in order from least to greatest measure.

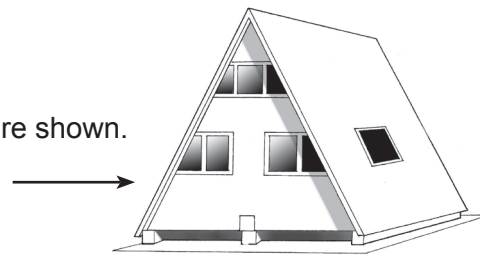


a. $\overline{LY},\ \overline{YK},\ \overline{LK}$
b. $\overline{YK},\ \overline{YL},\ \overline{LK}$
c. $\overline{LY},\ \overline{LK},\ \overline{KL}$
d. $\overline{YK},\ \overline{LK},\ \overline{LY}$

14. What is the range of the values of the diagonal d of a lot shaped like a parallelogram if adjacent sides are 10 inches and 14 inches?

a. $4 \geq d \geq 24$
b. $4 < d < 24$
c. $4 \leq d \leq 24$
d. $4 > d > 24$

For items no. 15-20, use the figure shown.



15. A *balikbayan* chose you to be one of the contractors to design an A-frame house maximizing the size of two square lots with dimensions 18 ft and 24 ft on each side. Which of the following is affected by the dimensions of the lot if the owner would like to spend the same amount of money on the roofs?

I. The width of the base of the house frames
II. Design of the windows
III. The height of the houses
IV. The roof angles

a. I and IV
b. III and IV
c. II, III and IV
d. I, II, III, and IV

16. Which of the following theorems justifies your response in item no. 15?

I. Triangle Inequality Theorem 1
II. Triangle Inequality Theorem 2
III. Triangle Inequality Theorem 3
IV. Hinge Theorem
V. Converse of Hinge Theorem

a. I, II, and III b. IV only c. IV and V d. V only

17. If the owner would like the same height for both houses, which of the following is true?

I. Roof costs for the larger lot is higher than that of the smaller lot.
II. The roof of the smaller house is steeper than the larger house.

a. I only
b. II only
c. neither I nor II
d. I and II

18. What considerations should you emphasize in your design presentation so that the *balikbayan* would award you the contract to build the houses?

I. Kinds of materials to use considering the climate in the area
II. Height of floor-to-ceiling corner rooms and its occupants
III. Extra budget needed for top-of-the-line furnishings
IV. Architectural design that matches the available funds
V. Length of time it takes to finish the project

a. I, II, and IV
b. I, IV, and V
c. I, II, IV, and V
d. I, II, III, IV, V

19. Why is it not practical to design a house using A-Frame style in the Philippines?

I. A roof also serving as wall contributes to more heat in the house.
II. Placement of the windows and doors requires careful thinking.
III. Some rooms of the house would have unsafe low ceiling.
IV. An A-Frame design is an unusually artful design.

a. I and III
b. II and IV
c. I, II, and III
d. I, II, III, IV

20. Why do you think an A-Frame House is practical in countries with four seasons?

A. The design is customary.
B. An artful house is a status symbol.
C. The cost of building is reasonably low.
D. The snow glides easily on steep roofs.

Lesson 1 Inequalities in Triangles

What to Know

Let's start the module by doing three activities that will reveal your background knowledge on triangle inequalities.

Activity 1 MY DECISIONS NOW AND THEN LATER

Directions:

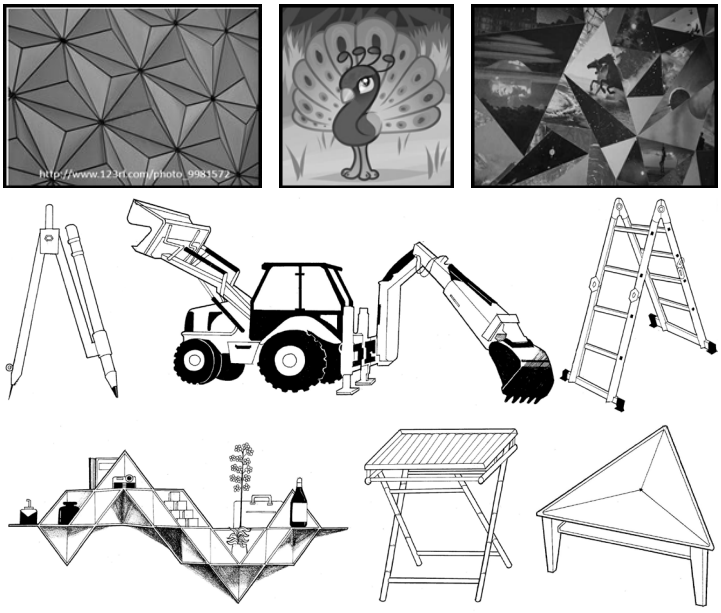
1. Replicate the table below on a piece of paper.
2. Under the my-decision-now column of the first table, write **A** if you agree with the statement and **D** if you don't.
3. After tackling the whole module, you will be responding to the same statements using the second table.

	Statement	My Decision Now
1	To form a triangle, any lengths of the sides can be used.	
2	The measure of the exterior angle of a triangle can be greater than the measure of its two remote interior angles.	
3	Straws with lengths 3 inches, 4 inches and 8 inches can form a triangle.	
4	Three segments can form a triangle if the length of the longest segment is greater than the difference but less than the sum of the two shorter segments.	
5	If you want to find for the longest side of a triangle, look for the side opposite the largest angle.	

	Statement	My Decision Later
1	To form a triangle, any lengths of the sides can be used.	
2	The measure of the exterior angle of a triangle can be greater than the measure of its two remote interior angles.	
3	Straws with lengths 3 inches, 4 inches and 8 inches can form a triangle.	
4	Three segments can form a triangle if the length of the longest segment is greater than the difference but less than the sum of the two shorter segments.	
5	If you want to find for the longest side of a triangle, look for the side opposite the largest angle.	

Activity 2 ARTISTICALLY YOURS!

Direction: Study the artworks below and answer the questions that follow:



QUESTIONS ?

1. What features prevail in the artworks, tools, equipment, and furniture shown?
2. Have you observed inequalities in triangles in the designs? Explain.
3. What is the significance of their triangular designs?
4. How can you justify inequalities in triangles in the designs?

More Triangular Designs and Artworks

1. **Triangular Girl by Caroline Johansson** http://thecaroline-johansson.com/blog/2011/10/triangular-girl-2/
2. **Tile works: Diminishing Triangles** http://sitteninthehills64.blogspot.com/2010/05/tile-house-8.html
3. **Repetitive Graduation by Scott Mihalik** http://mathtourist.blogspot.com/2012/02/flight-of-tetrahedra.html
4. **Maths-the best use for golf balls** http://www.whizz.com/blog/fun/maths-best-use-for-golf-balls/
5. **Luxury sailboat** http://edgeretreats.com/
6. **Triangle Card Stand** http://www.behance.net/gallery/TRIANGLE-CARD-STAND/3883741
7. **Triangular Periodic Table** http://www.metasynthesis.com/webbook/35_pt/pt_database.php?PT_id=40
8. **A triangular approach to fat loss by Stephen Tongue** http://www.flickr.com/photos/32462223@N05/3413593357/in/photostream/
9. **Triangular Petal Card** http://www.flickr.com/photos/32462223@N05/3413593357/in/photostream/

Question:

- Which among these designs and artworks you find most interesting? Explain.
- Which design you would like to pattern from for a personal project?

Activity 3 HELLO, DEAR CONCEPT CONTRACTOR!

The figure on the next page is a concept museum of inequalities in triangles. You will be constructing this concept museum throughout this module.

Each portion of the concept museum, mostly triangular, poses a task for you to perform. All tasks are related to knowledge and skills you should learn about inequalities in triangles.

What is a contractor?

A *contractor* is someone who enters into a binding agreement to build things.

~Wordweb 4.5a by Anthony Lewis~

What is a museum?

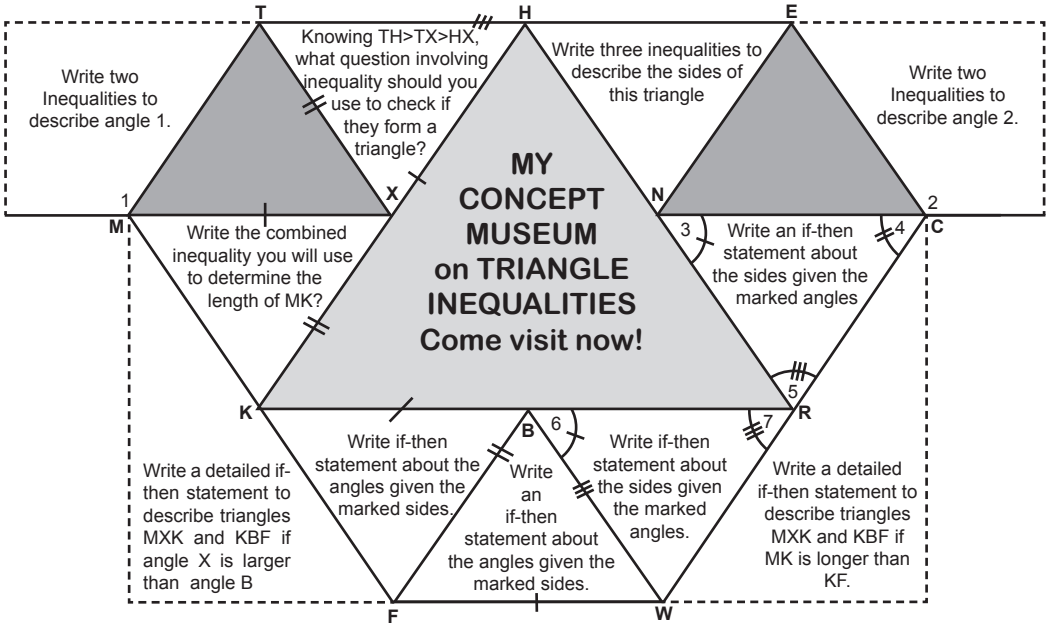
Museum is a depository for collecting and displaying objects having scientific or historical or artistic value.

~Wordweb 4.5a by Anthony Lewis~

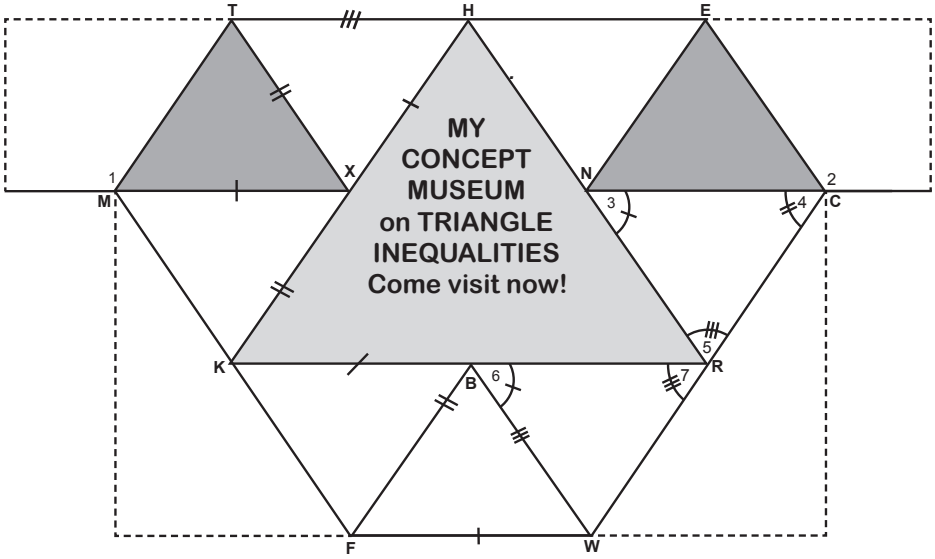
Note that the triangles in this concept museum are not drawn to scale and all sides can be named using their endpoints. Consider using numbers to name the angles of these triangles.

Notice that markings are shown to show which angles are larger and which sides are longer. These markings serve as your hints and clues. Your responses to the tasks must be justified by naming all the theorems that helped you decide what to do.

How many tasks of the concept museum can you tackle now?



Replicate two (2) copies of the unfilled concept museum. Use the first one for your responses to the tasks and the second one for your justifications.



Are you excited to completely build your concept museum, Dear Concept Contractor? The only way to do that is by doing all the succeeding activities in the next section of this module. The next section will also help you answer this essential question raised in the activity ***Artistically Yours: How can you justify inequalities in triangles?***

The next lesson will also enable you to do the final project that is inspired by the artworks shown in ***Artistically Yours****. When you have already learned all the concepts and skills related to inequalities in triangles, you will be required to make a model of a folding ladder and justify the triangular features of its design. Your design and its justification will be rated according to these rubrics: accuracy, creativity, efficiency, and mathematical justification.*

What to Process

Your first goal in this section is to develop and verify the theorems on inequalities in triangles. To succeed, you need to perform all the activities that require investigation.

When you make *mathematical generalizations* from your observations, you are actually making *conjectures* just like what mathematicians do. Hence, consider yourself *little mathematicians* as you perform the activities.

Once you have developed these theorems, your third goal is to prove these theorems. You have to provide statements and/or reasons behind statements used to deductively prove the theorems.

The competence you gain in writing proofs enables you to justify inequalities in triangles and in triangular features evident in the things around us.

Before you go through the process, take a few minutes to review and master again the knowledge and skills learned in previous geometry lessons. The concepts and skills on the following topics will help you succeed in the investigatory and proof-writing activities.

1. Axioms of Equality

1.1 Reflexive Property of Equality
- For all real numbers p, $p = p$.

1.2 Symmetric Property of Equality
- For all real numbers p and q, if $p = q$, then $q = p$.

1.3 Transitive Property of Equality
- For all real numbers p, q, and r, if $p = q$ and $q = r$, then $p = r$.

1.4 Substitution Property of Equality
- For all real numbers p and q, if $p = q$, then q can be substituted for p in any expression.

2. Properties of Equality

2.1 Addition Property of Equality
- For all real numbers p, q, and r, if $p = q$, then $p + r = q + r$.

2.2 Multiplication Property of Equality
- For all real numbers p, q, and r, if $p = q$, then $pr = qr$.

3. Definitions, Postulates and Theorems on Points, Lines, Angles and Angle Pairs

3.1 Definition of a Midpoint
- If points P, Q, and R are collinear (P–Q–R) and Q is the midpoint of $\overline{PR}$,then $\overline{PQ} \cong \overline{QR}$.

3.2 Definition of an Angle Bisector
- If $\overrightarrow{QS}$ bisects $\angle PQR$, then $\angle PQS \cong \angle SQR$.

3.3 Segment Addition Postulate
- If points P, Q, and R are collinear (P–Q–R) and Q is between points P and R, then $\overline{PQ} + \overline{QR} \cong \overline{PR}$.

3.4 Angle Addition Postulate
- If point S lies in the interior of $\angle PQR$, then $\angle PQS + \angle SQR \cong \angle PQR$.

3.5 Definition of Supplementary Angles
- Two angles are supplementary if the sum of their measures is 180º.

3.6 Definition of Complementary Angles
- Two angles are complementary if the sum of their measures is 90º.

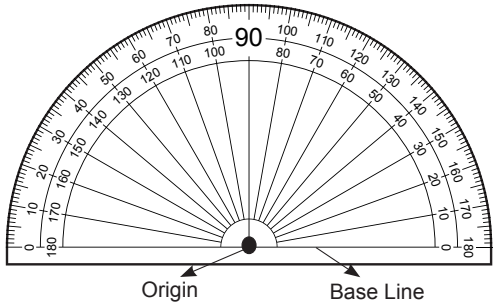
3.7 Definition of Linear Pair
- Linear pair is a pair of adjacent angles formed by two intersecting lines

3.8 Linear Pair Theorem
- If two angles form a linear pair, then they are supplementary.

3.9 Definition of Vertical Angles
- Vertical angles refer to two non-adjacent angles formed by two intersecting lines.

3.10 Vertical Angles Theorem
- Vertical angles are congruent.

4. How to Measure Angles using a Protractor



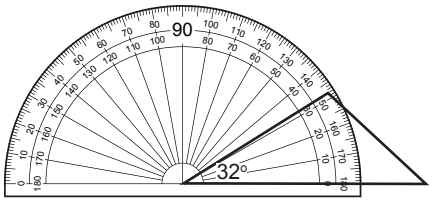
Internet Learning

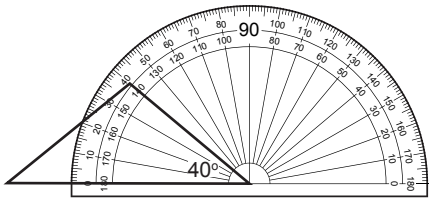
Mastering the Skill in Estimating Measures of Angles

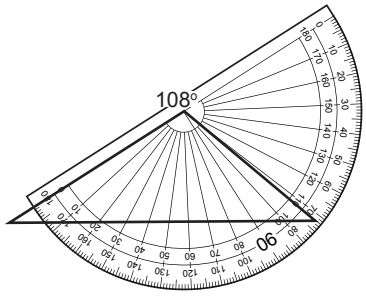
Interactive:
- **http://www.mathplayground.com/measuringangles.html**
- **http://www.teacherled.com/resources/anglemeasure/anglemeasureload.html**

Games:
- **http://www.bbc.co.uk/schools/teachers/ks2_activities/maths/angles.shtml**
- **http://www.innovationslearning.co.uk/subjects/maths/activities/year6/angles/game.asp**
- **http://www.bbc.co.uk/keyskills/flash/kfa/kfa.shtml**
- **http://resources.oswego.org/games/bananahunt/bhunt.html**
- **http://www.fruitpicker.co.uk/activity/**

To measure an angle, the protractor's origin is placed over the vertex of an angle and the base line along the left or right side of the angle. The illustrations below show how the angles of a triangle are measured using a protractor.







Mathematical History
Who invented the first advanced protractor?

Capt. Joseph Huddart (1741-1816) of the United States Navy invented the first advanced protractor in 1801. It was a three-arm protractor and was used for navigating and determining the location of a ship

~Brian Brown of www.ehow.com~

To read more about the history of protractor, visit these website links:

- http://www.counton.org/museum/floor2/gallery5/gal3p8.html
- http://www.ablogabouthistory.com/2011/07/29/the-worlds-first-protractor/

5. Definitions and Theorems on Triangles

5.1 The sum of the measures of the angles of a triangle is 180º.

5.2 Definition of Equilateral Triangle
- An equilateral triangle has three sides congruent.

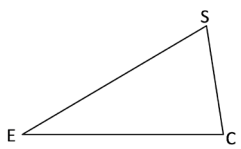
5.3 Definition of Isosceles Triangle
- An isosceles triangle has two congruent sides.
- *Is an equilateral triangle isosceles*? Yes, since it also has two congruent sides.
- Base angles of isosceles triangles are congruent.
- Legs of isosceles triangles are congruent.

5.4 Exterior Angle of a Triangle
- An exterior angle of a triangle is an angle that forms a linear pair with an interior angle of a triangle when a side of the triangle is extended.

5.5 Exterior Angle Theorem
- The measure of an exterior angle of a triangle is equal to the sum of the measures of the two interior angles of the triangle.

5.6 Sides and Angles of a Triangle

- $\angle S$ is opposite $\overline{EC}$ and $\overline{EC}$ is opposite $\angle S$.
- $\angle E$ is opposite $\overline{SC}$ and $\overline{SC}$ is opposite $\angle E$
- $\angle C$ is opposite $\overline{ES}$ and $\overline{ES}$ is opposite $\angle C$.



Internet Learning Mastering the Triangle Congruence Postulates

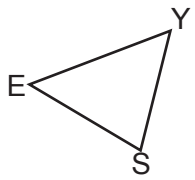
Video
- http://www.onlinemathlearning.com/geometry-congruent-triangles.html

Interactive
- http://www.mrperezonlinemathtutor.com/G/1_5_Proving_Congruent_SSS_SAS_ASA_AAS.html
- http://nlvm.usu.edu/en/nav/frames_asid_165_g_1_t_3.html?open=instructions
- http://www.mangahigh.com/en/maths_games/shape/congruence/congruent_triangles?localeset=en

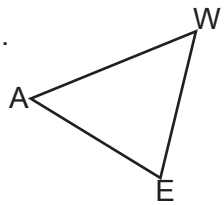
6. Definition and Postulates on Triangle Congruence

6.1 Definition of Congruent Triangles: Corresponding parts of congruent triangles are congruent (CPCTC).

6.2 Included Angle
- Included angle is the angle formed by two distinct sides of a triangle.
  - $\angle YES$ is the included angle of $\overline{EY}$ and $\overline{ES}$
  - $\angle EYS$ is the included angle of $\overline{YE}$ and $\overline{YS}$
  - $\angle S$ is the included angle of $\overline{SE}$ and $\overline{SY}$



6.3 Included Side
- Included side is the side common to two angles of a triangle.
  - $\overline{AW}$ is the included side of $\angle W\boldsymbol{A}E$ and $\angle E\boldsymbol{W}A$
  - $\overline{EW}$ is the included side of $\angle A\boldsymbol{E}W$ and $\angle A\boldsymbol{W}E$
  - $\overline{AE}$ is the included side of $\angle W\boldsymbol{A}E$ and $\angle A\boldsymbol{E}W$



6.4 SSS Triangle Congruence Postulate

6.5 SAS Triangle Congruence Postulate

6.6 ASA Triangle Congruence Postulate

7. Properties of Inequality

7.1 For all real numbers p and q where $p > 0$, $q > 0$:
- If $p > q$, then $q < p$.
- If $p < q$, then $q > p$.

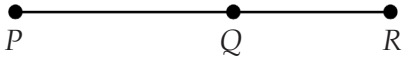
7.2 For all real numbers p, q, r and s, if $p > q$ and $r \geq s$, then $p + r > q + s$.

7.3 For all real numbers p, q and r, if $p > q$ and $r > 0$, then $pr > qr$.

7.4 For all real numbers p, q and r, if $p > q$ and $q > r$, then $p > r$.

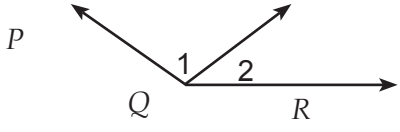
7.5 For all real numbers p, q and r, if $p = q + r$, and $r > 0$, then $p > q$.

The last property of inequality is used in geometry such as follows:



Q is between P and R.

$\overline{PR} \cong \overline{PQ} + \overline{QR}$
Then $PR > PQ$ and $PR > QR$.

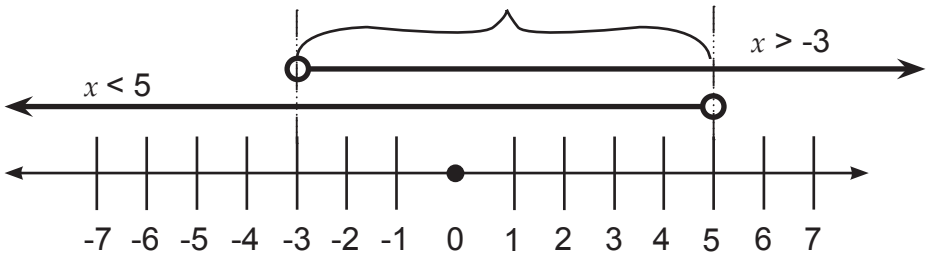


$\angle 1$ and $\angle 2$ are adjacent angles.

$\angle PQR \cong \angle 1 + \angle 2$
Then $m\angle PQR > m\angle 1$ and $m\angle PQR > m\angle 2$

8. How to Combine Inequalities

- *Example: How do you write x < 5 and x > -3 as a combined inequality?*



From the number line, we observe that the value of x must be a value between -3 and 5, that is, x **is greater than -3 but less than 5**. In symbols, -3 < x < 5.

9. Equality and Congruence

Congruent figures (segments and angles) have equal measures such that:

- If $\overline{PR} \cong \overline{PR}$, then $PR = PR$.
- If $\angle PQS \cong \angle PQS$, then $m\angle PQS = m\angle PQS$.

Note that to make proofs brief and concise, we may opt to use $\overline{PR} \cong \overline{PR}$ or $\angle PQS \cong \angle PQS$ instead of $PR = PR$ or $m\angle PQS = m\angle PQS$. Because the relation symbol used is for congruence; instead of writing, say, reflexive property of equality as reason; we just have to write, reflexive property. Note that some other books sometimes call reflexive property as reflexivity.

10. How to Write Proofs

Proofs in geometry can be written in paragraph or two-column form. A proof in paragraph form is only a two-column proof written in sentences. Some steps can be left out when paragraph form is used so that two-column form is more detailed.

A combination of both can also be used in proofs. The first part can be in paragraph form especially when the plan for proof is to add some constructions first in the illustration. Proving theorems sometimes requires constructions to be made.

The first column of a two-column proof is where you write down systematically every step you go through to get to the conclusion in the form of a statement. The corresponding reason behind each step is written on the second column.

Possible reasons are as follows: Given, by construction, axioms of equality, properties of equality, properties of inequality, definitions, postulates or previously proven theorems.

The following steps have to be observed in writing proofs:

- Draw the figure described in the problem. The figure may have already been drawn for you, or you may have to draw it yourself.
- Label your drawn figure with the information from the given by:

    ✓ *marking congruent or unequal angles or sides,*
    ✓ *marking perpendicular, parallel or intersecting lines or*
    ✓ *indicating measures of angles and/or sides*

The markings and the measures guide you on how to proceed with the proof they also direct you whether your plan for proof requires you to make additional constructions in the figure.

- Write down the steps carefully. Some of the first steps are often the given statements (but not always), and the last step is the statement that you set out to prove.

11. How to Write an Indirect Proof

11.1 Assume that the statement to be proven is not true by negating it.
11.2 Reason out logically until you reach a contradiction of a known fact.
11.3 Point out that your assumption must be false; thus, the statement to be proven must be true.

12. Greatest Possible Error and Tolerance Interval in Measurements

You may be surprised why two people measuring the same angle or length may give different measurements. Variations in measurements happen because measurement with a measuring device, according to Donna Roberts (2012), is approximate. This variation is called uncertainty or error in measurement, but not a mistake. She added that there are ways of expressing error of measurement. Two are the following:

Greatest Possible Error (GPE)

One half of the measuring unit used is the greatest possible error. For example, you measure a length to be 5.3 cm. This measurement is to the nearest tenth. Hence, the GPE should be one half of 0.1 which is equal to 0.05. This means that your measurement may have an error of 0.05 cm, that is, it could be 0.05 longer or shorter.

Tolerance Intervals

Tolerance interval (margin of error) may represent error in measurement. This interval is a range of measurements that will be tolerated or accepted before they are considered flawed.

Supposing that a teacher measures a certain angle x as 36 degrees. The measurement is to the nearest degree, that is, 1. The GPE is one half of 1, that is, 0.5. Your answer should be within this range: $36\text{-}0.5 \leq x \leq 36 + 0.5$. Therefore, the tolerance interval or margin of error is $35.5 \leq x \leq 36.5$ or 35.5 to 36.5.

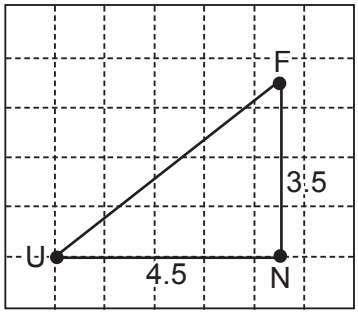
Now that you have already reviewed concepts and skills previously learned that are useful in this module, let us proceed to the main focus of this section—develop, verify, and prove the theorems on inequalities in triangles.

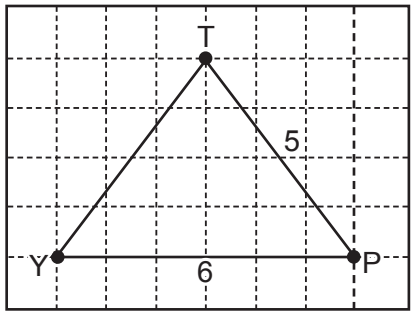
Activity 4 WHAT IF IT'S LONGER?

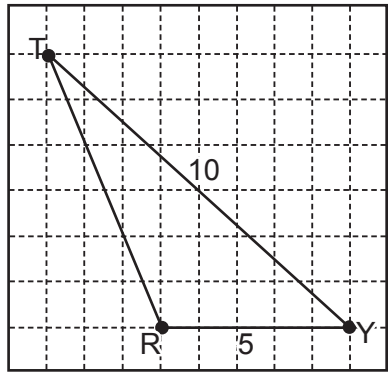
Materials Needed: protractor, manila paper, ruler

Procedures:

1. Replicate the activity table on a piece of manila paper.
2. Measure using a protractor the angles opposite the sides with given lengths. Indicate the measure in your table.
3. Discover the relationship that exists between the lengths of the sides of triangles and the angles opposite them. Write them on manila paper.

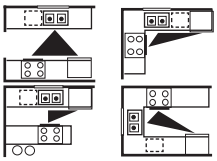






Triangle	Length of Sides		Measures of Angles Opposite the Sides	
ΔFUN	FN	3.5	$m\angle U$	
	NU	4.5	$m\angle F$	
ΔPTY	TP	5	$m\angle Y$	
	PY	6	$m\angle T$	
ΔRYT	RY	5	$m\angle T$	
	TY	10	$m\angle R$	

Mathematics in the Kitchen
The Kitchen Triangle



The Kitchen Triangle was developed in the 1950s as a tool to aid designers in creating an effective kitchen layout. The triangle has a corner at the sink, the refrigerator, and the stove, the three essential locations in the kitchen when cooking. Most kitchen plans still include this today. The idea is to have them close enough that they can easily be moved between, but not too far away to reduce the amount of movement while cooking. The general rule is that the triangles perimeter must be at least 12 ft., but should be larger than 26 ft.

The area inside the triangle should be completely open, making movement between each of these easy. Most modern kitchen plans still include this.

See the whole article at http://bathroomphotogallery.com/kitchen-plans.php~

Questions:

1. Suppose R is for Refrigerator, S is for Sink, and C for cooking stove, describe the relationship of the sides and angles of the kitchen triangle in each of the model.
2. If you were to build a house in the future, which kitchen model do you prefer from the figure? Why?

QUESTIONS ?

1. Is there a relationship between the length of a side of a triangle and the measure of the angle opposite it?
   ☐ Yes, there is. ☐ No, there isn't.

2. *Making Conjecture*: What is the relationship between the sides of a triangle and the angles opposite them?
   - *When one side of a triangle is longer than a second side, the angle opposite the* _____.

3. Your findings in no. 2 describe the Triangle Inequality Theorem 1. Write it in if-then form.
   - *If one side of a triangle is longer than a second side, then* _____.

4. What is the relationship between the longest side of a triangle and the measure of the angle opposite it?

5. What is the relationship between the shortest side of a triangle and the measure of the angle opposite it?

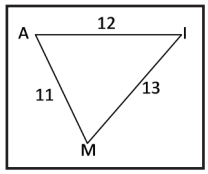
6. Without using a protractor, determine the measure of the third angles of the triangles in this activity.
   (**Hint**: T*he sum of the measures of the angles of a triangle is 180°*.)

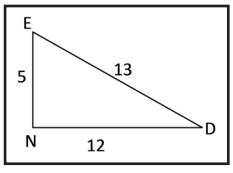
Name of Triangle	Working Equations	Measure of the Third Angle	
ΔFUN		$m\angle N$	
ΔTYP		$m\angle P$	
ΔTRY		$m\angle Y$	

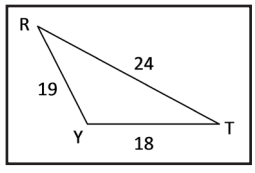
Quiz No. 1

Directions: Write your answer on a separate answer sheet.

A. Name the smallest angle and the largest angle of the following triangles:

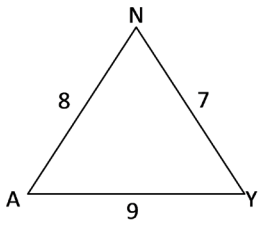


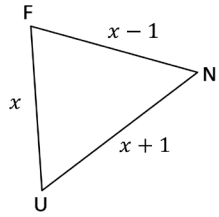


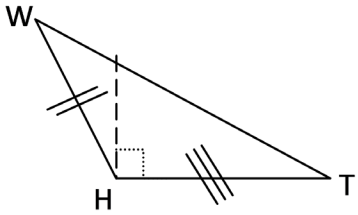


	Triangle	Largest Angle	Smallest Angle
1.	ΔAIM		
2.	ΔEND		
3.	ΔRYT		

B. The diagrams in the exercises are not drawn to scale. If each diagram were drawn to scale, list down the sides and the angles in order from the least to the greatest measure.





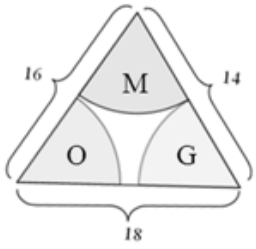


	ΔNAY	ΔFUN	ΔWHT
Sides			
Angle			

C. Your parents support you in your studies. One day, they find out that your topic in Grade 8 Math is on *Inequalities in Triangles*. To assist you, they attach a triangular dart board on the wall with lengths of the sides given.

They say they will grant you three wishes if you can hit with an arrow the corner with the smallest region and two wishes if you can hit the corner with the largest region.

- Which region should you hit so your parents will grant you three wishes?
- Which region should you hit so your parents will grant you two wishes?



	Grant: 3 wishes	**Grant: 2 wishes**
Region to Hit with an Arrow		

Mathematics in Art
Geometric Shapes for Foundation Piecing by Dianna Jesse

Challenge:
1. Which figure is drawn first in the artworks--the smallest polygon or the largest polygon?
2. Make your own design by changing the positions or the lengths of the sides of the triangles involved in constructing the figure.
3. Would you like to try using the hexagon?

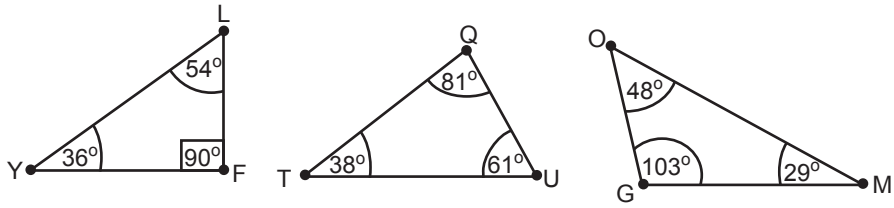
Visit this web link to see the artworks shown: http://dian-najessie.wordpress.com/tag/triangular-design/~

Activity 5 WHAT IF IT'S LARGER?

Materials Needed: ruler, manila paper

Procedures:

1. Replicate the activity table on a piece of Manila paper.
2. Measure using ruler the sides opposite the angles with given sizes. Indicate the lengths (in mm) on your table.
3. Develop the relationship of angles of a triangle and the lengths of the sides opposite them by answering the questions below on a piece of Manila paper.



Triangle	Measure of the Angles		Lengths of Sides Opposite the Angles	
ΔLYF	$m\angle L$		FY	
	$m\angle Y$		LF	
	$m\angle F$		LY	
ΔQUT	$m\angle Q$		TU	
	$m\angle U$		QT	
	$m\angle T$		QU	
ΔOMG	$m\angle O$		MG	
	$m\angle M$		GO	
	$m\angle G$		MO	

QUESTIONS ?

1. Is there a relationship between the size of an angle and the length of the side opposite it?
   ☐ Yes, there is. ☐ No, there isn't.
2. Making Conjecture: What is the relationship between the angles of a triangle and the sides opposite them?
   - When one angle of a triangle is larger than a second angle, the side opposite the _____.
3. Your findings in no. 2 describe Triangle Inequality Theorem 2. Write it in *if-then* form.
4. What is the relationship between the largest angle of a triangle and the side opposite it?
5. What is the relationship between the smallest angle of a triangle and the side opposite it?

6. Arrange in increasing order the angles of the triangles in this activity according to measurement.

Name of Triangle	Smallest Angle	Smaller Angle	Largest Angle
ΔLYF			
ΔQUT			
ΔOMG			

7. Arrange in decreasing order the sides of the triangles in this activity according to their lengths.

Name of Triangle	Shortest Side	Shorter Side	Longest Side
ΔLYF			
ΔQUT			
ΔOMG			

8. Having learned Triangle Inequality 2, answer the question in the table.

Kind of Triangle	How do you know that a certain side is the longest side?
Acute Δ	
Right Δ	
Obtuse Δ	

Mathematics for Eco-Architecture

Triangular Skyscraper with Vegetated Mini-Atriums

The triangular form, which in China is symbolic with balance and stability, also allows the building to shade itself, which lowers the amount of energy required to cool the interiors. The signature feature of the entire design is the atrium, which runs the entire height of the building and also allows each level to be illuminated by natural light.

Questions:

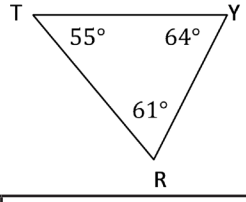
1. Have you seen triangular buildings or structures in your area?
2. When do you think it is best to use a triangular design like the one shown in building a structure?

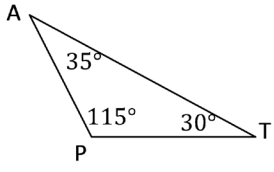
To find out the reasons why the triangular design is eco-friendly, visit this website link: http://www.ecofriend.com/eco-architecture-triangular-skyscraper-designed-with-vegetated-mini-atriums.html

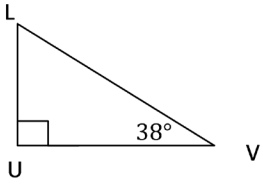
QUIZ No. 2

Directions: Write your answer on a separate answer sheet. Note that the diagrams in the exercises are not drawn to scale**.**

A. Name the shortest side and the longest side of the following triangles:



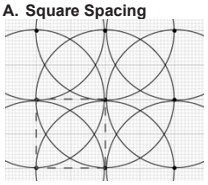




	Triangle	Longest Side	Shortest Side
1.	ΔTRY		
2.	ΔAPT		
3.	ΔLUV		

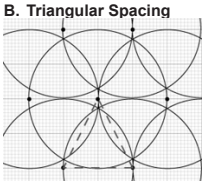
Mathematics in the Garden
How to Space Sprinklers?
~irrigationrepair.com~

A. Square Spacing



Square spacing is the easiest to plot; the downfall is that there will be areas that are going to be covered by all four sprinkler heads causing some over watering. Sprinklers are spaced relatively close when using a square pattern (on average around 50% of the diameter of the throw). This means you will also need more sprinkler heads to cover a given area.

B. Triangular Spacing

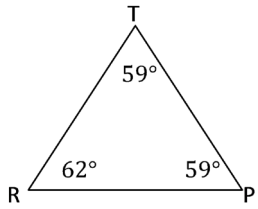


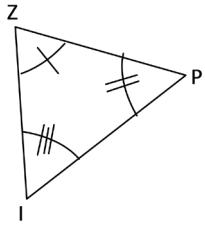
Triangular spacing is plotted using three points which means that more surface area is watered with less overlap .Since you can cover more surface using triangular spacing you will be able to space the sprinkler heads farther apart (usually around 60% of the diameter of the throw). Using a triangular pattern in plotting sprinkler heads can save money because less sprinkler heads are needed to irrigate any given area.

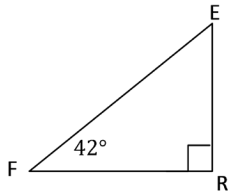
Task:
If you were asked to space sprinklers, which spacing would you use?

To read more about spacing sprinklers, visit this website link: http://www.irrigationrepair.com/how_to_space_rotors_sprays.html

B. List down the sides from longest to shortest.

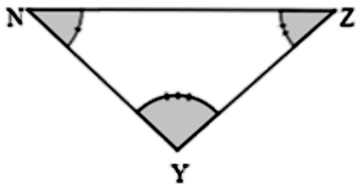






ΔTRP			ΔZIP			ΔFRE		

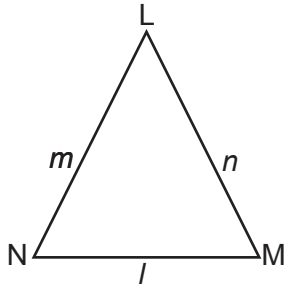
C. Skye buys a triangular scarf with angle measures as illustrated in the figure shown. She wishes to put a lace around the edges. Which edge requires the longest length of lace?



Activity 6 WHEN CAN YOU SAY "ENOUGH!"?

Materials Needed: plastic straws, scissors, manila paper, and ruler
Procedure:

1. Cut pieces of straws with the indicated measures in inches. There are three pieces in each set.
2. Replicate the table in this activity on a piece of Manila paper.
3. With each set of straws, try to form triangle LMN.
4. Write your findings on your table and your responses to the ponder questions on a piece of Manila paper.



Sets of Straw Pieces				Do the straws form a triangle or not?		Compare the sum of the lengths of shorter straws ($l + m$) with that of the longest length c			Compare ($m + n$) and l			Compare ($l + n$) and m		
	l	m	n	YES	NO	$l+m$	<,>,=	n	$m+n$	<,>,=	l	$l+n$	<,>,=	m
1.	3	3	7											
2.	3	3	5											
3.	4	6	10											
4.	4	6	9											
5.	5	5	10											
6.	5	5	8											
7.	6	7	11											
8.	6	7	9											
9.	4	7	12											
10.	4	7	10											

QUESTIONS ?

1. Making Conjectures:

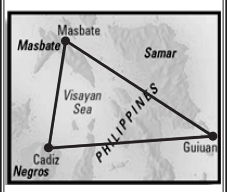
1.1 What pattern did you observe when you compared the sum of the lengths of the two shorter straws with the length of the longest straw? Write your findings by completing the phrases below:

- *If the sum of the lengths of the two shorter straws is equal to the length of the longest straw* _____.
- *If the sum of the lengths of the two shorter straws is less than the length of the longest straw* _____.
- *If the sum of the lengths of the two shorter straws is greater than the length of the longest straw* _____.

1.2 What pattern did you observe with the sets of straws that form and do not form a triangle? Complete the phrases below to explain your findings:

- *When the straws form a triangle, the sum of the lengths of any two straws* _____.

Mathematics in Geography
Feasible Possible Distance
(McDougal Little Geometry, 2001)



Suppose you know the following information about distances between cities in the Philippine Islands:

Cadiz to Masbate ≈ 159 km
Cadiz to Guiuan ≈ 265 km

Considering the angle formed with Cadiz as the vertex, describe the range of possible distances from Guiuan to Masbate.

- *When the straws do not form a triangle, the sum of the lengths of any two straws_____.*

2. Your findings in this activity describe Triangle Inequality Theorem 3. State the theorem by describing the relationship that exists between the lengths of any two sides and the third side of a triangle.
   - *The sum of the lengths of any two sides of a triangle is _____.*

QUIZ No. 3

Directions: Write your answer on a separate answer sheet.

1. Describe sides $\overline{AW}$, $\overline{EW}$ and $\overline{AE}$ of ΔAWE using Triangle Inequality Theorem 3.

2. Check whether it is possible to form a triangle with lengths 8, 10, and 14 by accomplishing the table below. Let the hints guide you.

	Hints	In Symbols	Simplified Form	Is the simplified form true?	Can a triangle be formed? Justify
1	Is the sum of 8 and 10 greater than 14?				
2	Is the sum of 8 and 14 greater than 10?				
3	Is the sum of 10 and 14 greater than 8?				
Which question should be enough to find out if a triangle can be formed?					

3. Is it possible to form a triangle with sides of lengths 5, 8, and 13? Complete the table to find out the answer.

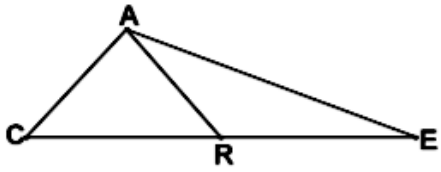
	Find out if:	Simplified Forms	Is the simplified form true?	Can a triangle be formed? Justify
1				
2				
3				
Which question should be enough to find out if a triangle can be formed?				

4. Can you form a triangle from sticks of lengths 7, 9, and 20?

Find out if:	Simplified Forms	Is the simplified form true?	Can a triangle be formed? Justify
1				
2				
3				
Which question should be enough to find out if a triangle can be formed?			

5. Study the figure shown and complete the table of inequalities using Triangle Inequality Theorem 3.

$\overline{CA} + \overline{AR}$	>	
$\overline{ER} + \overline{AR}$	>	
	<	$\overline{RE} + \overline{AE}$
$\overline{AC} + \overline{CE}$	>	
		$\overline{AE} + \overline{CE}$



Mathematics in Architecture

World's Thinnest House: Keret House by Jakub Szczesny

The four-feet-wide house is built on a tiny space between two buildings in Warsaw, Poland.

Questions:
1. Explain why Architect Szczesny used triangular design in building the house?
2. Why is it not advisable to use rectangular design?

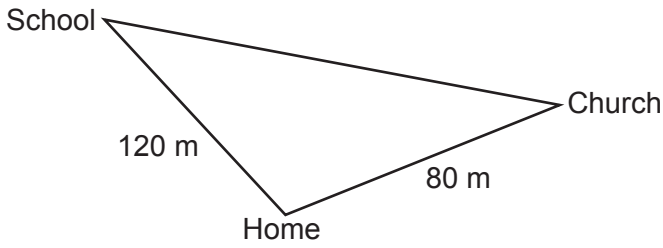
To see more photos of the house, visit this website link: http://design-milk.com/worlds-thinnest-house-keret-house-by-jakub-szczesny/?fb_action_ids=4022332433678&fb_action_types=og.likes&fb_source=aggregation&fb_aggregation_id=288381481237582

6. Using Triangle Inequality Theorem 3, what inequality will you write to check whether segments with lengths s_1, s_2, and s_3 form a triangle if $s_1 < s_2 < s_3$?

7. If two sides of a triangle have lengths 7 feet and 10 feet, what are the possible integral lengths of the third side? Between what two numbers is the third side?

8. The distance Klark walks from home to school is 120 meters and 80 meters when he goes to church from home. Xylie estimates that the distance Klark walks when he goes directly to church, coming from school is 180 meters. Realee's estimation is 210 meters. Which estimation is feasible? Justify your answer.

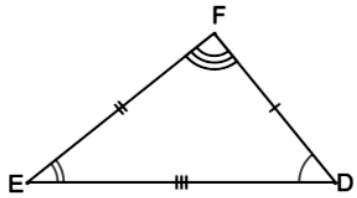
For items no. 8-10, use the figure shown



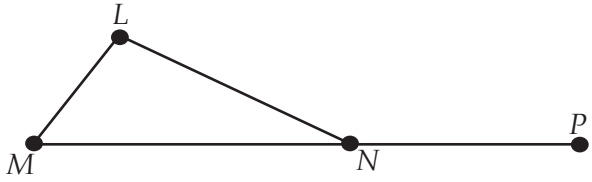
9. Supposing that the shortest distance among the three locations is the school-church distance, what are its possible distances?

10. Which of the following paths to church is the shortest if you are coming from school? Justify your answer.
    - Path No. 1: School to Home then to Church
    - Path No. 2: School to Church

11. Some things are wrong with the measurements on the sides and angles of the triangle shown. What are they? Justify your answer.

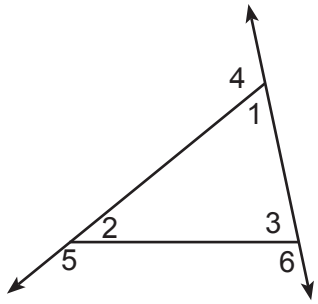


The next activity is about discovering the triangle inequality theorem involving an exterior angle of a triangle. Before doing it, let us first recall the definition of an exterior angle of a triangle.



By extending $\overline{MN}$ of ΔLMN to a point P, $\overline{MP}$ is formed. As a result, $\angle LNP$ forms a linear pair with $\angle LNM$. Because it forms a linear pair with one of the angles of ΔLMN, $\angle LNP$ is referred to as an exterior angle of ΔLMN. The angles non-adjacent to $\angle LNP$, $\angle L$ and $\angle M$, are called remote interior angles of exterior $\angle LNP$.

In the triangle shown, ∠4, ∠5 and ∠6 are exterior angles. The remote interior angles of ∠4 are ∠2 and ∠3; of ∠5, ∠1 and ∠3; of ∠6, ∠1 and ∠2.



Internet Learning
Measures of Interior and Exterior Angles of a Triangle

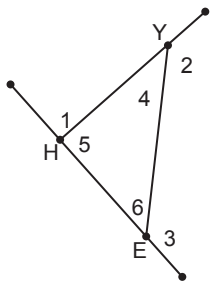
Interactive:
- http://www.mathwarehouse.com/geometry/triangles/angles/remote-exterior-and-interior-angles-of-a-triangle.php

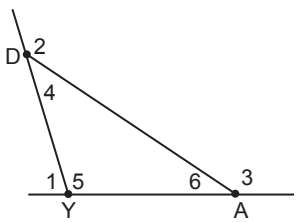
Activity 7 MEASURE MANIA: EXTERIOR OR REMOTE INTERIOR?

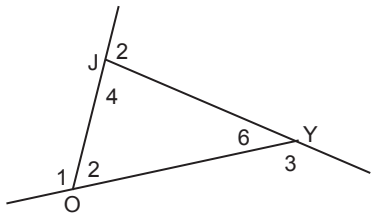
Materials Needed: Protractor, Manila Paper, and Ruler

Procedures:

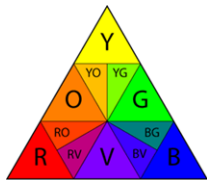
1. Measure the numbered angles of ΔHEY, ΔDAY, and ΔJOY.
2. Replicate the table in this activity on a piece of manila paper.
3. Indicate the measures on your table and write your answers to the questions on a piece of manila paper.







Mathematics in Art
Color Triangle



The Color Triangle makes it easier to determine the resulting color if two colors are combined.

Questions:

1. What is the resulting color with the following combinations?
   - Yellow and Blue
   - Red and Yellow
   - Blue and Red
2. How many possible exterior angles do the following sets of color triangles have?
   - B, R, Y
   - G, O, V
   - YO, YG, RO, RV, BG, BV

To read more about the color triangle, visit this website link: http://www.atpm.com/9.08/design.shtml

Name of Triangle	MEASURES								
	1st Exterior ∠	Remote Interior ∠s		2nd Exterior ∠	Remote Interior ∠s		3rd Exterior ∠	Remote Interior ∠s	
	∠1	∠4	∠6	∠2	∠5	∠6	∠3	∠4	∠5
ΔHEY									
ΔDAY									
ΔJOY									

QUESTIONS ?

1. Compare the measure of exterior ∠1 with either remote interior ∠4 or ∠6 using the relation symbols >, <, or =.
   - In ΔHEY, $m\angle 1$ is _____ $m\angle 4$.
   - In ΔHEY, $m\angle 1$ is _____ $m\angle 6$.
   - In ΔDAY, $m\angle 1$ is _____ $m\angle 4$.
   - In ΔDAY, $m\angle 1$ is _____ $m\angle 6$.
   - In ΔJOY, $m\angle 1$ is _____ $m\angle 4$.
   - In ΔJOY, $m\angle 1$ is _____ $m\angle 6$.

2. Compare the measure of exterior ∠2 with either remote interior ∠5 or ∠6 using the relation symbols >, <, or =.
   - In ΔHEY, $m\angle 2$ is _____ $m\angle 5$.
   - In ΔHEY, $m\angle 2$ is _____ $m\angle 6$.
   - In ΔDAY, $m\angle 2$ is _____ $m\angle 5$.
   - In ΔDAY, $m\angle 2$ is _____ $m\angle 6$.
   - In ΔJOY, $m\angle 2$ is _____ $m\angle 5$.
   - In ΔJOY, $m\angle 2$ is _____ $m\angle 6$.

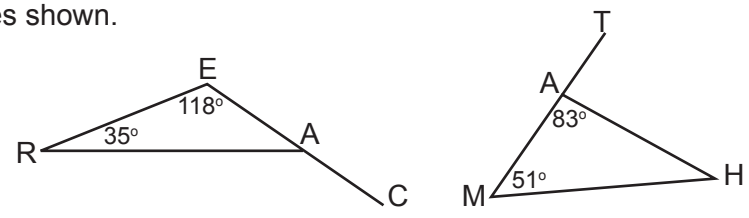
3. Compare the measure of exterior ∠3 with either remote interior ∠4 or ∠5 using the relation symbols >, <, or =.
   - In ΔHEY, $m\angle 3$ is _____ $m\angle 4$.
   - In ΔHEY, $m\angle 3$ is _____ $m\angle 5$.
   - In ΔDAY, $m\angle 3$ is _____ $m\angle 4$.
   - In ΔDAY, $m\angle 3$ is _____ $m\angle 5$.
   - In ΔJOY, $m\angle 3$ is _____ $m\angle 4$.
   - In ΔJOY, $m\angle 3$ is _____ $m\angle 5$.

4. *Making Conjecture:* Your comparison between the measure of an exterior angle of a triangle and either interior angle in this activity describes the Exterior Angle Inequality Theorem. With the pattern that you observed, state the exterior angle inequality theorem.
   - *The measure of an exterior angle of a triangle is*_____*.*

QUIZ No. 4

Directions: Write your answer on a separate answer sheet.

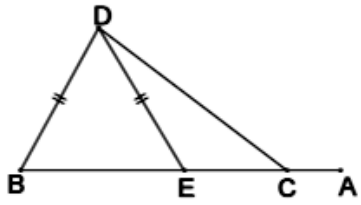
1. Use the Exterior Angle Inequality theorem to write inequalities observable in the figures shown.



Considering ΔREA	Considering ΔHAM

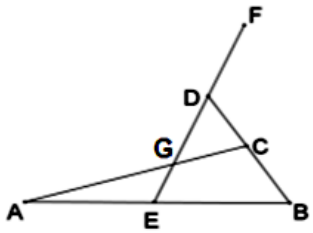
2. Use >, <, or = to compare the measure of angles.

$m\angle AED$		$m\angle CED$
$m\angle DEB$		$m\angle DCE$
$m\angle DEB$		$m\angle DBE$
$m\angle CDE$		$m\angle DEB$
$m\angle DEC$		$m\angle ACD$



3. Name the exterior angle/s of the triangles shown in the figure.

ΔDEB	
ΔCDG	
ΔAGE	
ΔBAC	



You have successfully learned all the theorems on inequalities in one triangle. You can now do Activity No. 8 applying them.

Activity 8 My Grandpa, My Model of Healthy Lifestyle!

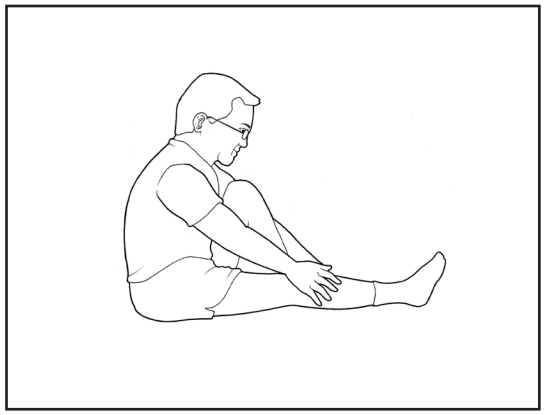
Leruana has a triangular picture frame that her grandpa gave her on her 13th birthday. Like her, his grandpa loves triangular shapes. Since it is going to be his grandpa's 65th birthday soon, her birthday gift idea is to have two triangular frames made so she can place in them photos of his grandpa as health exercise instructor. As her woodworker friend, she asks you to do the triangular frames for her. To determine the shapes of the picture frames, how should the photos be cropped?

Mathematics in Psychology
Robert Sternberg's Triangular Theory of Love

Questions:

1. Study the triangle intently. What do you understand about the triangular theory of love?
2. Which mean more to you—passion, intimacy or commitment?
3. Which love you would like to have in the future—romantic, fatuous, companionate or consummate love?
4. How do you rank romantic love, fatuous love, companionate love and consummate love using combined inequality?

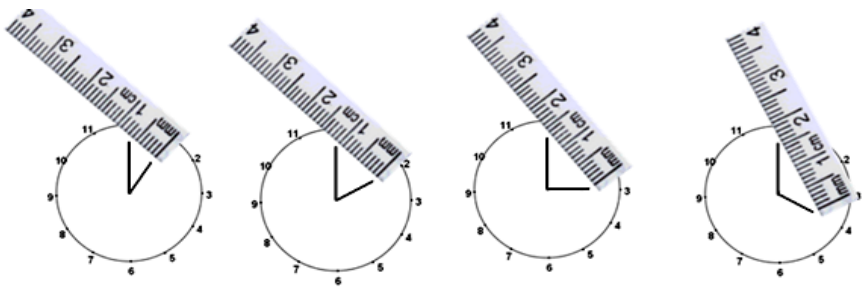
To help you decide, visit http://gentlemencalling.wordpress.com/2012/03/13/for-the-love-of-triangles/



QUESTIONS ?

1. *How do you plan to crop the photographs?*
   - Indicate the vertices of the triangular part of the photos.
   - Mark the sides of the new triangular photos.
2. *What made you decide to have that shape and not something else?*
   - What is your basis for determining the largest corner?
   - What is your basis for determining the longest side?

Activity 9 Clock Wisdom, Pretty One!



A complete revolution around a point is equivalent to 360º. The minute and hour hands of the clock also cover that in a compete revolution.

Materials: Ruler and Manila Paper
Procedure:

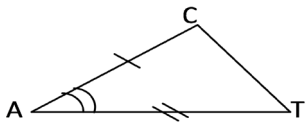
1. Replicate the activity table on a piece of Manila paper.
2. Study the faces of the clock shown at different hours one afternoon and complete your copy of the activity table.

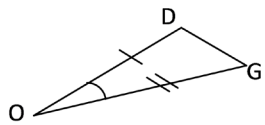
3. Write also your answers to the ponder questions on a piece of manila paper.
4. Compute for the measure of the angle formed by the hands of the clock given that one revolution for each hand is equivalent to 360°.

Clock Face	Time (Exact PM Hours)	Measure of angle formed by the hour hand and minute hand	Distance between the tips of the hour hand and minute hands (in mm)
A			
B			
C			
D			

QUESTIONS ?

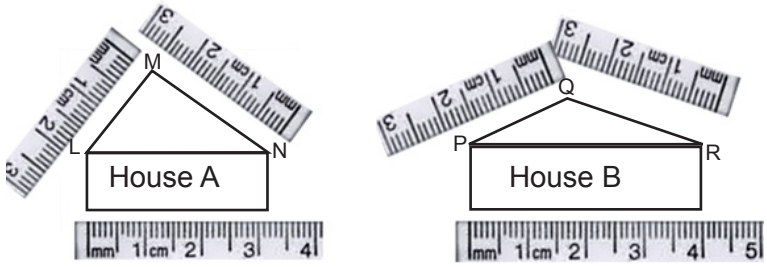
1. How do you describe the lengths of the hour hands of the clock faces using a relation symbol?
2. How do you describe the lengths of the minute hands of the clock faces using a relation symbol?
3. The angles formed by the hands of the clock can be called as_____.
4. In the activity, what do you observe about the measures of the angles formed by the hands of the clock at different hours?
5. What affects the measure of the distance between the tips of the hands of the clock? Explain.
6. *Making a Conjecture:* Your findings describe the Hinge Theorem (This is otherwise known as SAS Triangle Inequality Theorem). How will you state this theorem if you consider the clock hands of two faces (say, Clock Faces A and B) as sides of two triangles and the angles they make as the included angles? State it in if-then form.
   - *If two sides of one triangle are congruent to two sides of another triangle, but the included angle of the first triangle is greater than the included angle of the second, then*_____.
7. Using Hinge Theorem, write an if-then statement about the appropriate sides and angles of ΔCAT and ΔDOG.





8. Is the name Hinge for this theorem suitable? Explain.
9. Hinge theorem characterizes many objects around us. Give examples of these objects.

Activity 10 ROOF-Y FACTS, YEAH!



Materials Needed: protractor, manila paper, and ruler

Procedure: Study the house models and complete your copy of the activity table. For ponder questions, write your answers on a piece of manila paper.

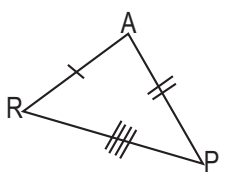
HOUSE								
A								
B								

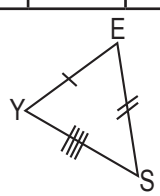
QUESTIONS ?

1. Write your observations on the following:
   - The lengths of the roofs at the left part of both houses __.
   - The Lengths of the roof at the right part of both houses __.
   - The lengths of the roof bases of both houses __.
   - The Roof angles of both houses __.
2. What influences the measures of the roof angles of both houses? Justify.
3. *Making a Conjecture:* Your findings describe the Converse of Hinge Theorem (This is otherwise known as SSS Triangle Inequality Theorem). How will you state this theorem if you consider the two corresponding roof lengths as two sides of two triangles, the roof bases as their third sides, and the roof angles as included angles? State it in *if-then* form.

   If two sides of one triangle are congruent to two sides of another triangle, but the third side of the first triangle is greater than the third side of the second, then _____.
4. Using the Converse of Hinge Theorem, write an if-then statement to describe the appropriate sides and angles of ΔRAP and ΔYES.

HOUSE	Roof Lengths at the Right (in cm)		Roof Lengths at the Left (in cm)		Lengths of Roof Base (in cm)		Roof Angle	
A								
B								



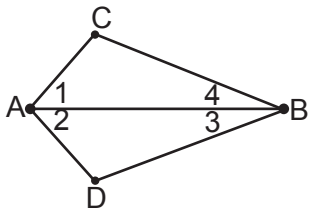


5. With both houses having equal roof lengths, what conclusion can you make about their roof costs?

QUIZ No. 4

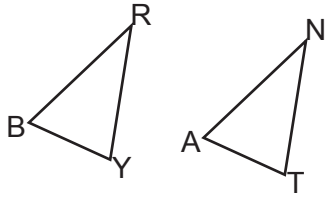
Directions: Write your answer on a separate answer sheet.

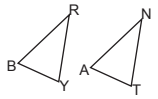
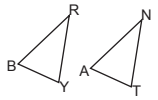
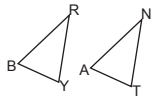
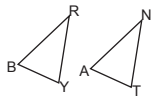
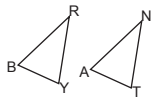
A. Use the symbol <, > or = to complete the statements about the figure shown. Justify your answer.



Statements			Justification
1. If $\overline{AC} \cong \overline{AD}$ and $m\angle 1 = m\angle 2$, then $\overline{BC}$		$\overline{BD}$	
2. If $\overline{BC} \cong \overline{BD}$ and $AC > AD$,then $m\angle 4$		$m\angle 3$	
3. If $\overline{AD} \cong \overline{AC}$ and $m\angle 2 < m\angle 1$,then $\overline{BD}$		$\overline{BC}$	
4. If $\overline{BD} \cong \overline{BC}$ and $AD > AC$,then $m\angle 3$		$m\angle 4$	

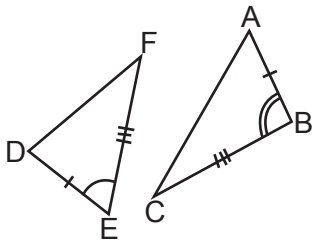
B. Make necessary markings on the illustration based on the given. What conclusion can you make, if there is any, given the facts about the two triangles? Provide justifications to your conclusions.



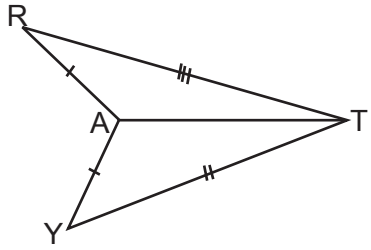
	GIVEN FACTS			FOR MARKINGS	CONCLUSION	JUSTIFICATION
1.	$BY = AT$	$BR = AN$	$m\angle B > m\angle A$			
2.	$BR = AT$	$RY = NT$	$m\angle R > m\angle N$			
3.	$BY = AT$	$BR = AN$	$RY > NT$			
4	$BR = AN$	$RY = NT$	$BY > AT$			
5.	$RY = NT$	$BY = AN$	$\angle N < \angle Y$			

C. Using Hinge Theorem and its converse, write a conclusion about each figure.

1.



3.



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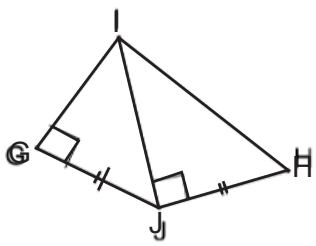

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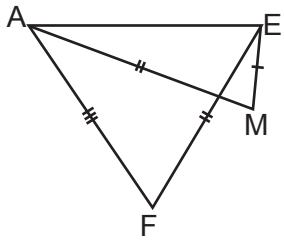
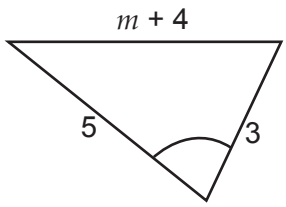
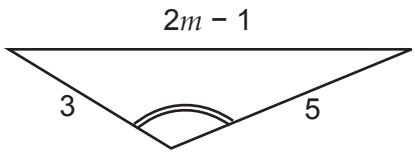


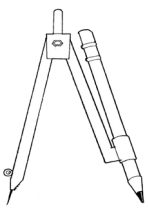
D. Using Hinge Theorem and its converse, solve for the possible values of m.

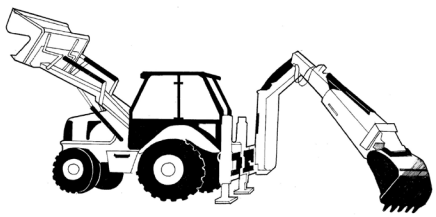


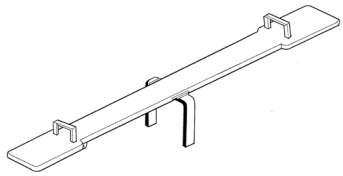


E. Enrichment Activities

1. *Hinges in Tools and Devices*
   Hinges are used to fasten two things together and allow adjustment, rotation, twisting or pivoting. Choose at least one of the following hinged devices and explain how it works.







2. *Mathematics in Fashion: Ladies' Fan*

From the sixteenth century up to the late 1800s throughout the whole of Europe, each fashionable lady had a fan and because of its prominence, it was considered as a "woman's scepter"—tool for communicating her thoughts.

http://www.victoriana.com/Fans/historyofthefan.html

Questions:

1. Do you think that fan is an important fashion item?
2. Describe the concept of inequality in triangles that is evident about a ladies' fan.

From the prior investigations, we have discovered the following theorems on triangle inequalities:

Inequalities in One Triangle:

Triangle Inequality Theorem 1 ($Ss \rightarrow Aa$)
If one side of a triangle is longer than a second side, then the angle opposite the first side is larger than the angle opposite the second side.

Triangle Inequality Theorem 2 ($Aa \rightarrow Ss$)
If one angle of a triangle is larger than a second angle, then the side opposite the first angle is longer than the side opposite the second angle.

Triangle Inequality Theorem 3 ($S_1 + S_2 > S_3$)
The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Exterior Angle Inequality Theorem
The measure of an exterior angle of a triangle is greater than the measure of either remote interior angle

Inequalities in Two Triangles:

Hinge Theorem or SAS Inequality Theorem
If two sides of one triangle are congruent to two sides of another triangle, but the included angle of the first triangle is greater than the included angle of the second, then the third side of the first triangle is longer than the third side of the second.

Converse of Hinge Theorem or SSS Inequality Theorem:
If two sides of one triangle are congruent to two sides of another triangle, but the third side of the first triangle is longer than the third side of the second, then the included angle of the first triangle is larger than the included angle of the second.

How can we prove these theorems?

Writing proofs is an important skill that you will learn in geometry. It will develop your observation skills, deductive thinking, logical reasoning, and mathematical communication. Guide questions are provided to help you succeed in the next activities.

In writing proofs, you have to determine the appropriate statements and give reasons behind these statements. There are cases when you only have to complete a statement or a reason. Make use of hints to aid you in your thinking.

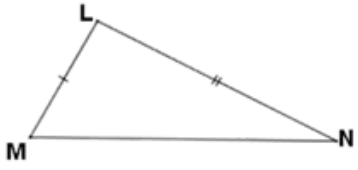
Be reminded that theorems may be proven in different ways. The proofs that follow are some examples of how these theorems are to be proven.

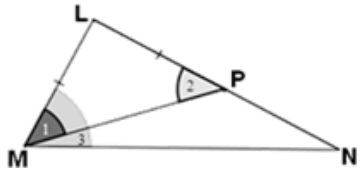
For activity 11-16, you are required to use a piece of manila paper for each proof.

Activity 11 PROVING TRIANGLE INEQUALITY THEOREM 1

Triangle Inequality Theorem 1 ***($Ss \rightarrow Aa$)***
If one side of a triangle is longer than a second side, then the angle opposite the first side is larger than the angle opposite the second side.

Given: ΔLMN; $LN > LM$
Prove: $m\angle LMN > m\angle LNM$





Proof: There is a need to make additional constructions to prove that $m\angle LMN > m\angle LNM$. With compass point on L and with radius $\overline{LM}$, mark a point P on $\overline{LN}$ and connect M and P with a segment to form triangle.

Statements	Reasons
1. How do you describe the relationship between $\overline{LM}$ and $\overline{LP}$?	By construction
2. Based on statement 1, what kind of a triangle is ΔLMP?	Definition of Isosceles Triangle
3. Based on statement 1, how do you describe $\angle 1$ and $\angle 2$?	Converse of Isosceles Triangle Theorem

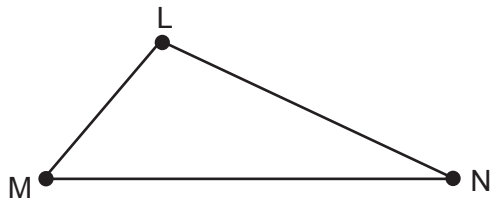
4. Study the illustration and write a statement about $\angle LMN$ if the reason is the one given.	Angle Addition Postulate
5. Basing on statement 4, write an inequality statement focusing on $\angle 1$.	Property of Inequality
6. Using statement 3 in statement 5: $m\angle LMN > m\angle 2$	Substitution Property
7. Study the illustration and write an operation statement involving $\angle MPN$, $\angle N$, and $\angle 3$	The sum of the interior angles of a triangle is 180^o.
8. Study the illustration and write an operation statement involving $\angle 2$ and $\angle MPN$	Linear Pair Theorem
9. $\angle 2 + \angle MPN \cong \angle MPN + \angle N + \angle 3$	What property supports the step where-in we replace the right side of statement 8 with its equivalent in statement 7?
10. What will be the result if $\angle MPN$ is deducted away from both sides of statement 9?	
11. Basing on statement 10, write an inequality statement focusing on $\angle N$.	Property of Inequality
12 Based on statement 6 and 11: If $m\angle LMN > m\angle 2$ and $m\angle 2 > m\angle N$, then	Property of Inequality

Congratulations! You have contributed much in proving Triangle Inequality Theorem 1. In the next activity, you will see that Triangle Inequality Theorem 1 is used in proving Triangle Inequality Theorem 2.

Activity 12 INDIRECT PROOF OF TRIANGLE INEQUALITY THEOREM 2

Triangle Inequality Theorem 2 (Aa→Ss)
If one angle of a triangle is larger than a second angle, then the side opposite the first angle is longer than the side opposite the second angle.

Given: ΔLMN; $\angle L > \angle N$
Prove: $\overline{MN} > \overline{LM}$



Indirect Proof:

Assume: $\overline{MN} \not> \overline{LM}$

Statements	Reasons
1. $\overline{MN} = \overline{LM}$ or $\overline{MN} < \overline{LM}$	1. Assumption that $\overline{MN} \not> \overline{LM}$
2. Considering MN ≅ LM : If MN ≅ LM, then ☐	2. Definition of ☐
Consequently, what can you say about $\angle L$ and $\angle N$? ☐	☐ of isosceles triangles are congruent.
The Assumption that $\overline{MN} \cong \overline{LM}$ is ☐ True ☐ False	The conclusion that $\angle L \cong \angle N$ ☐ the given that $\angle L > \angle N$.
3. Considering $\overline{MN} < \overline{LM}$: If $\overline{MN} < \overline{LM}$, then ☐	3. Base angles of isosceles triangles are congruent.
The Assumption that $\overline{MN} < \overline{LM}$ is ☐ True ☐ False	The conclusion that $\angle L < \angle N$ contradicts the given that ☐
4. Therefore, $\overline{MN} > \overline{LM}$ must be ☐ True ☐ False	4. The ☐ that $\overline{MN} \not> \overline{LM}$ contradicts the known fact that $\angle L > \angle N$.

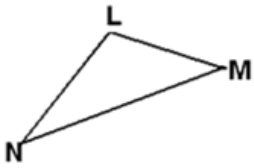
Amazing! You have helped in proving Triangle Inequality Theorem 2. Let us proceed to prove Triangle Inequality Theorem 3 using a combination of paragraph and two-column form. You will notice that Triangle Inequality Theorem 2 is used as reason in proving the next theorem.

Activity 13 PROVING TRIANGLE INEQUALITY THEOREM 3

Triangle Inequality Theorem 3 ($S_1 + S_2 > S_3$)
The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

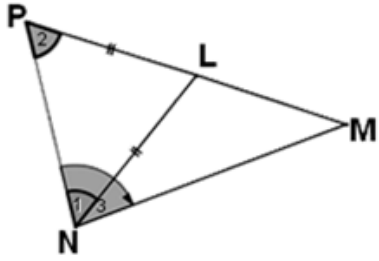
Given: ΔLMN where $\overline{LM} < \overline{LN} < \overline{MN}$

Prove: $\overline{MN} + \overline{LN} > \overline{LM}$
$\overline{MN} + \overline{LM} > \overline{LN}$
$\overline{LM} + \overline{LN} > \overline{MN}$



Proof:

- Notice that since $\overline{MN} > \overline{LN}$ and that $\overline{MN} > \overline{LM}$, then it's obvious that $\overline{MN} + \overline{LM} > \overline{LN}$ and $\overline{MN} + \overline{LN} > \overline{LM}$ are true.
- Hence, what remains to be proved is the third statement: $\overline{LM} + \overline{LN} > \overline{MN}$



Let us construct LP as an extension of $\overline{LM}$ such that L is between M and P, $\overline{LP} \cong \overline{LN}$ and ΔLNP is formed.

Statements	Reasons
1. Write a statement to describe $\overline{LP}$ and $\overline{LN}$. [ ]	1. By construction
2. Describe ΔLNP. [ ]	2. [ ]
3. Describe $\angle LNP$ and $\angle LPN$ [ ]	3. Bases of isosceles triangles are congruent.
4. The illustration shows that $\angle LPN \cong \angle MPN$	4. Reflexive Property of Equality
5. If $\angle LNP \cong \angle LPN$ (statement 3) and $\angle LPN \cong \angle MPN$ (statement 4), then [ ]	5. Transitive Property of Equality
6. From the illustration, $\angle MNP \cong \angle LNM + \angle LNP$	6. [ ]

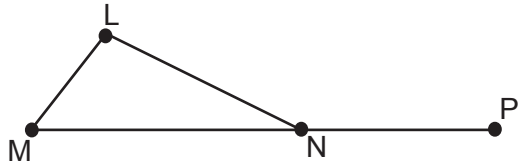
7.	Using statement 5 in statement 6, $\angle MNP \cong \angle LNM + \angle MPN$	7.	
8.	From statement 7, $\angle MNP > \angle MPN$	8.	Property of Inequality
9.	Using statement 8 and the illustration, write a statement with the reason given.	9.	Triangle Inequality Theorem 2
10.	From the illustration, what operation involving $\overline{LM}$ and $\overline{LP}$ can you write?	10.	Segment Addition Postulate
11.	Write a statement using statement 10 in statement 9	11.	Substitution Property of Inequality
12.	Write a statement using statement 1 in statement 11	12.	Substitution Property of Equality

Hurray! Triangle Inequality Theorem 3 is already proven. Let us proceed to writing the proof of Exterior Angle Inequality Theorem.

Activity 14 PROVING THE EXTERIOR ANGLE INEQUALITY THEOREM

Exterior Angle Inequality Theorem
The measure of an exterior angle of a triangle is greater than the measure of either remote interior angle.

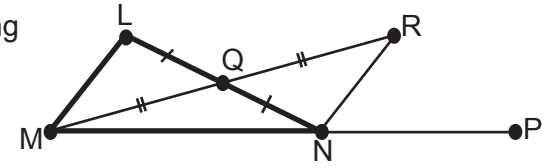
Given: ΔLMN with exterior angle $\angle LNP$
Prove: $\angle LNP > \angle MLN$



Proof:

Let us prove that $\angle LNP > \angle MLN$ by constructing the following:

1. midpoint Q on $\overline{LN}$ such that $\overline{LQ} \cong \overline{NQ}$
2. $\overline{MR}$ through Q such that $\overline{MQ} \cong \overline{QR}$



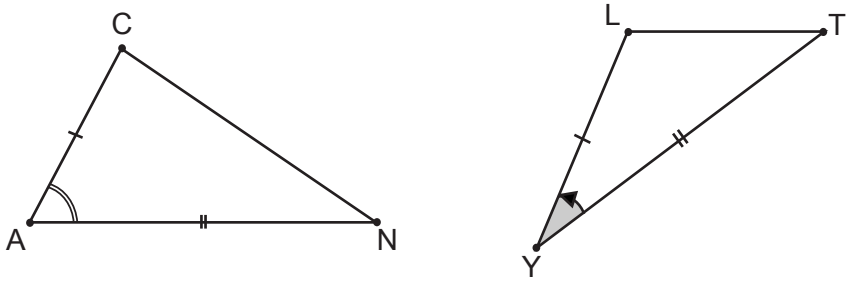
Statements		Reasons	
1.	$\overline{LQ} \cong \overline{NQ}$; $\overline{MQ} \cong \overline{QR}$	1.	
2.	What relationship exists between $\angle 3$ and $\angle 4$?	2.	
3.	Basing on statements 1 and 2, describe two triangles from the illustration:	3.	What triangle congruence postulate supports statement 3?
4.	Basing on statement 3, $\angle MLN \cong$	4.	
5.	Basing on the illustration, $\angle LNP \cong$	5.	Angle Addition Postulate
6.	Basing on statement 5, $\angle LNP > \angle 1$	6.	
7.	Using statement 4 in statement 6,	7.	Substitution Property

Indeed, the measure of an exterior angle of a triangle is greater than the measure of either remote interior angle.

Activity 15
PROVING THE HINGE THEOREM

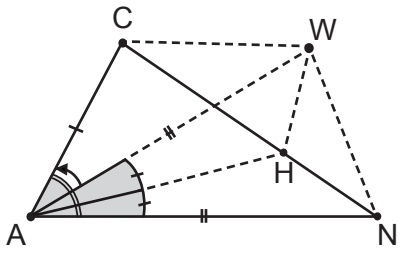
Hinge Theorem or SAS Triangle Inequality Theorem
If two sides of one triangle are congruent to two sides of another triangle, but the included angle of the first triangle is greater than the included angle of the second, then the third side of the first triangle is longer than the third side of the second.

Given: ΔCAN and ΔLYT; $\overline{CA} \cong \overline{LY}$, $\overline{AN} \cong \overline{YT}$, $\angle A > \angle Y$
Prove: $\overline{CN} > \overline{LT}$



Proof:

1. Construct $\overline{AW}$ such that :
   - $\overline{AW} \cong \overline{AN} \cong \overline{YT}$
   - $\overline{AW}$ *is between* $\overline{AC}$ *and* $\overline{AN}$, *and*
   - $\angle CAW \cong \angle LYT$.



Consequently, $\Delta CAW \cong \Delta LYT$ *by SAS Triangle Congruence Postulate. So,* $\overline{CW} \cong \overline{LT}$ *because corresponding parts of congruent triangles are congruent.*

2. Construct the bisector $\overline{AH}$ of $\angle NAW$ such that:
   - H *is on* $\overline{CN}$
   - $\angle NAH \cong \angle WAH$

Consequently, $\Delta NAH \cong \Delta WAH$ by SAS Triangle Congruence Postulate because $\overline{AH} \cong \overline{AH}$ by reflexive property of equality and $\overline{AW} \cong \overline{AN}$ from construction no. 1. So, $\overline{WH} \cong \overline{HN}$ *because corresponding parts of congruent triangles are congruent.*

Statements	Reasons
1. From the illustration: $\overline{CN} \cong \overline{CH} + \overline{HN}$	1.
2. $\overline{CN} \cong \overline{CH} + \overline{WH}$	2.
3. In ΔCHW, $\overline{CH} + \overline{WH} > \overline{CW}$	3.
4. Using statement 2 in 3: $\overline{CN} > \overline{CW}$	4.
5. Using statement in construction 1 in statement 4: $\overline{CN} > \overline{LT}$	5.

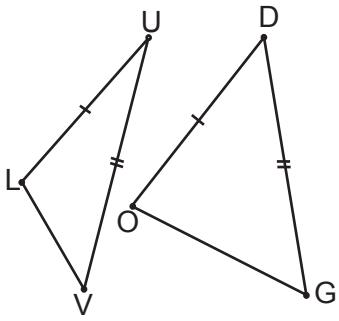
Bravo! The Hinge Theorem is already proven. Notice that the use of paragraph form on the first part of the proof of the Hinge Theorem shortens the proof process.

Activity 16 INDIRECT PROOF OF THE CONVERSE OF HINGE THEOREM

Converse of Hinge Theorem or SSS Triangle Inequality Theorem
If two sides of one triangle are congruent to two sides of another triangle, but the third side of the first triangle is longer than the third side of the second, then the included angle of the first triangle is larger than the included angle of the second.

Given: ΔODG and ΔLUV;
$\overline{OD} \cong \overline{LU}$, $\overline{DG} \cong \overline{UV}$,
$\overline{OG} > \overline{LV}$

Prove: $\angle D > \angle U$



Indirect Proof:

Assume: $\angle D \not> \angle U$

Statements	Reasons
1. $\angle D \cong \angle U$ and $\angle D < \angle U$	1. Assumption that [ ]
2. Considering $\angle D \cong \angle U$: It's given that $\overline{OD} \cong \overline{LU}$, $\overline{DG} \cong \overline{UV}$. If ∠D ≅ ∠U, then $\Delta ODG \cong \Delta LUV$.	2. [ ] Triangle Congruence Postulate
$\overline{OG} \cong \overline{LV}$	[ ]
The Assumption that $\angle D \cong \angle U$ is false.	[ ]
3. Considering $\angle D < \angle U$: If $\angle D < \angle U$, then [ ]	3. Hinge Theorem
[ ]	$\overline{OG} < \overline{LV}$ contradicts the given that $\overline{OG} > \overline{LV}$
4. [ ]	4. Assumption that ∠D ≯ ∠U is proven to be false.

After proving the theorems on inequalities in triangles, you are now highly equipped with skills in writing both direct and indirect proofs. Moreover, you now have a good grasp on how to write proofs in paragraph and/or two-column form.

You will be undergoing more complex application problems involving inequalities in triangles in the next section.

Dear Concept Contractor, your task is to revisit your concept museum. How many more tasks can you tackle? Which concepts you have built previously need revision? Check also your decisions in Activity No.1. Would you like to change any decision?

How can you justify inequalities in triangles? *Do you have a new insight on how to address this essential question raised in the activity* ***Artistically Yours?***

Now that you know the important ideas about this topic, let's go deeper by moving on to the next section.

What to Understand

Having developed, verified, and proved all the theorems on triangle inequalities in the previous section, your goal now in this section is to take a closer look at some aspects of the topic. This entails you to tackle more applications of the theorems on triangle inequalities.

Your goal in this section is to use the theorems in identifying unknown inequalities in triangles and in justifying them.

The first set of activities showcases model examples that will equip you with ideas and hints on how to conquer problems of the same kind but already have twists. When it is your turn to answer, **you have to provide justifications** to every step you take as you solve the problem. The model examples provide questions for you to answer. Your answers are the justifications.

The second set of activities requires you to use the theorems on inequalities in triangles in solving problems that require you to write proofs.

There are no limits to what the human imagination can fathom and marvel. Fun and thrill characterize this section. It is also where you will wrap up all the concepts you learned on Triangle Inequalities.

Activity 17 SHOW ME THE ANGLES!!!

Watch this!

For extra fun, groups of students in a class are tasked to create algebraic expressions to satisfy the measures of the angles of their triangular picture frame project. If the measure of the angles are as follows: $m\angle A = 5x - 3$, $m\angle C = 2x + 5$, $m\angle E = 3x - 2$, arrange the sides of the frame in increasing order.

Solution:

To solve for x:	**Solving for $m\angle A$**	**Solving for $m\angle C$**	**Solving for $m\angle E$**
$(5x - 3) + (2x + 5) + (3x - 2) = 180$ $5x + 2x + 3x - 3 + 5 - 2 = 180$ $10x - 5 + 5 = 180$ $10x = 180$ $x = 18$	$m\angle A = 5x - 3$ $= 5(18) - 3$ $= 90 - 3$ $= \mathbf{87}$	$m\angle C = 2x + 5$ $= 2(18) + 5$ $= 36 + 5$ $= \mathbf{41}$	$m\angle E = 3x - 2$ $= 3(18) - 2$ $= 54 - 2$ $= \mathbf{52}$

Therefore, listing the sides in increasing order should follow this order: Sides opposite $\angle C$, $\angle E$, and $\angle A$. That is, $\overline{AE}$, $\overline{AC}$, and $\overline{CE}$.

QUESTIONS ?

1. Why is the value x being solved first?
2. Why is the sum of the angles being equated to 180°?
3. What theorem justifies the conclusion that the increasing order of the sides is $\overline{AE}$, $\overline{AC}$, and $\overline{CE}$?
4. What makes us sure that our answer is correct considering that we have not exactly seen the actual triangle and have not used tools to measure the lengths of its sides and the measures of its angles?

It's Your Turn!

Angle S of the triangular picture frame of another group is 58°. The rest of the angles have the following measures: $m\angle E = 2x - 1$, $m\angle A = 4x - 3$. Determine the longest and the shortest side. Give justifications.

Activity 18 BELIEVE ME, THERE ARE LOTS OF POSSIBILITIES!

Watch this!

Problem:

You are tasked to draw a triangle wherein the lengths of two sides are specified. What are the possible lengths for the third side of the triangle you will draw if two sides should be 11 and 17, respectively? How many possible integer lengths has the third side?

Solution:

Since the third side is unknown, let's represent its length by t.

Inequality 1	Inequality 2	Inequality 3
$11 + 17 > t$ $28 > t$ $t < 28$ t must be less than **28**	$11 + t > 17$ $t > 17 - 11$ $t > 6$ t must be greater than **6**	$17 + t > 11$ $t > 11 - 17$ $t > -6$ Values of t to be disregarded

The resulting inequalities show that t must be between 6 and 28, that is, 6 as the lower boundary and 28 as the higher boundary. Using combined inequality, the order by which they will be written should be 6, t, then 28.

Therefore,

- the possible length for the third side is $\mathbf{6 < t < 28}$.
- the set of possible integer lengths for the third side of the triangle is described as follows: {7, 8, 9, …,27}. Hence, there are 27 – 6 = **21** possible integer lengths for the third side.

QUESTIONS ?

1. *What theorem justifies the three inequalities being written about the sides?*
2. *Are you convinced that $6 < t < 28$ is accurate even if you have not tried drawing all the possible lengths of the third side to form a triangle with 11 and 17? Why?*
3. *Do you observe a relationship existing between 6 in **$6 < t < 28$** and the two known lengths 11 and 17? Describe the relationship.*
4. *Do you observe a relationship existing between 28 in **$6 < t < 28$** and the two known lengths 11 and 17? Describe the relationship.*
5. *If the known lengths are l and s, where l is longer and s is shorter, what should be the formula in solving for the unknown third side t?*
6. *There are 21 possible integer lengths for the third side when two respective sides of a triangle have lengths 11 and 17. Can you count all the possible lengths other than the integer lengths? Explain.*

It's Your Turn!

Problem:

The lengths of the sides of a triangle are $16 - k$, 16, and $16 + k$. What is the range of the possible values of k? Create a table of the possible integer lengths of the sides of the triangle. Is $16\text{-}k$ always the shortest length? Develop a general formula for lengths with this description. Provide justifications.

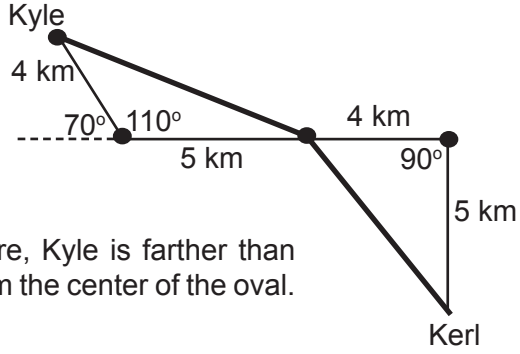
Activity 19 AND YOU THOUGHT ONLY SURVEYORS TRACE, HUH!

Watch this!

Problem:

Kerl and Kyle play with their roller skates at the town oval. From the centre of the oval, Kerl skates 4 meters east and then 5 meters south. Kyle skates 5 meters west. He then takes a right turn of 70° and skates 4 meters. Who is farther from the centre of the oval?

Solution:



Therefore, Kyle is farther than Kerl from the center of the oval.

QUESTIONS ?

1. How are 110° and 90° produced?
2. What theorem justifies the conclusion that Kyle is farther than Kerl from the center of the oval?
3. Would this problem be answered without a detailed illustration of the problem situation? Explain.
4. Had the illustration of the problem not drawn, what would have been your initial answer to what is asked? Explain.
5. We have not actually known Kerl and Kyle's distances from the center of the oval but it is concluded that Kyle is farther than Kerl. Are you convinced that the conclusion is true? Explain.

It's Your Turn!

1. *Problem:*
   From a boulevard rotunda, bikers Shielou and Chloe who have uniform biking speed, bike 85 meters each in opposite directions—Shielou, to the north and Chloe, to the south. Shielou took a right turn at an angle of 50^{o} and Chloe, a left turn at 35^{o}. Both continue biking and cover another 60 meters each before taking a rest. Which biker is farther from the rotunda? Provide justifications.

2. *Enrichment Activity*
   Career in Mathematics: Air Traffic Controller

   Air traffic controllers coordinate the movement of air traffic to make certain that planes stay a safe distance apart. Their immediate concern is safety, but controllers also must direct planes efficiently to minimize delays.

   They must be able to do mental math quickly and accurately. Part of their job is directing aircraft at what altitude and speed to fly.

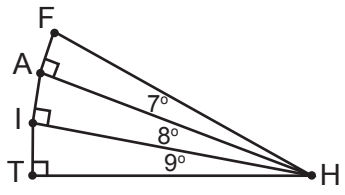
   Task:

   Make a research of problems related to the work of air traffic controllers. Solve it and present it in class

Activity 20 TRUST YOURSELF, YOU'RE A GEOMETRICIAN!

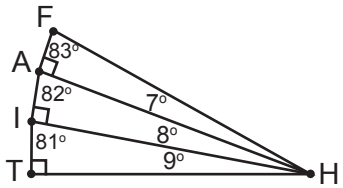
Watch this!

Problem:



The diagram is not drawn to scale. Which of the lengths $\overline{HF}$, $\overline{HA}$, $\overline{HI}$, and $\overline{HT}$ of polygon FAITH is the longest? Which is the shortest?

Solution:



Considering ΔHIT:
$\overline{HT} < \overline{HI}$

Considering ΔHAI:
$\overline{HI} < \overline{HA}$

Considering ΔHFA:
$\overline{HA} < \overline{HF}$

$\overline{HT} < \overline{HI} < \overline{HA} < \overline{HF}$

Therefore, the longest side is $\overline{HF}$ and the shortest side is $\overline{HT}$.

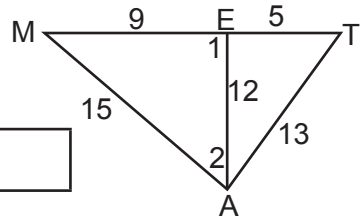
QUESTIONS ?

1. By just looking at the original figure, which side do you think is the longest? There is a misconception to explain why HT would have been the initial choice as having the longest side. Explain.
2. Why is it necessary to consider each right triangle in the figure individually?
3. What theorem justifies the choice of the longer side in each triangle?
4. Notice that the diagram is not drawn to scale. However, we are still able to tell which side is the longest and which side is the shortest. Are you convinced that your answer is true? Explain.

It's Your Turn!

Problem:

The diagram is not drawn to scale. Using $\angle 1$, $\angle 2$, $\angle T$, $\angle M$, and $\angle MAT$, complete the combined inequalities below:



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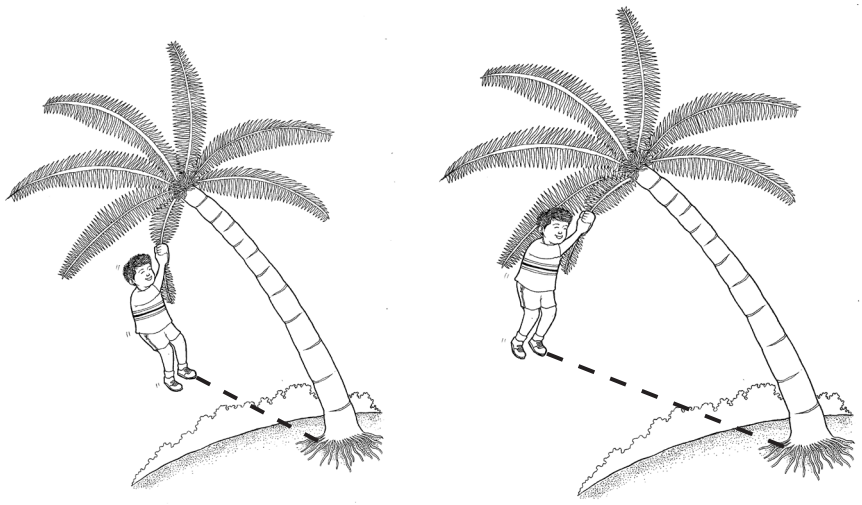
Activity 21 I BELIEVE I CAN FLY



The figure shows two pictures of a kid swinging away from the coco trunk while holding on a stalk of coco leaf. Compare the distances of the kid from the bottom of the coco trunk in these pictures. Note that the kid's distance from the bottom of the coco trunk is farthest when he swings at full speed.

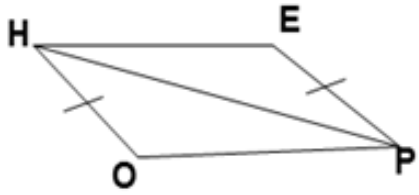
QUESTIONS ?

1. Name the sides of the triangle formed as the kid swings away holding on to the stalk of coco leaf.
2. An inequality exists in the two triangles shown. Describe it.
3. Compare the angles formed by the coco leaf stalk and the coco trunk at the kid's full speed and low speed.
4. How can you justify the inequality that exists between these triangles?
5. Many boys and girls in the province have great fun using coco leaf stalks as swing rides. Have you tried a coco leaf swing ride?
6. Aside from coco leaf swing rides, what other swing rides do you know in your area or from your knowledge or experience?
7. If you were asked to improvise a swing ride in your community, how would you design the swing ride? Explain.
8. Concepts on inequalities in triangles are useful in improvising a swing ride. What are the disadvantages if a designer of a swing ride does not apply these concepts?
9. What are the qualities of a good improvised swing ride?
10. What are the things you should do to attain these qualities?
11. Should all designers of tools and equipment comply with standards and guidelines in designing them? Why?

Activity 22 YOU ARE NOW PROMOTED AS PROOFESSOR!

1. Write the statements supported by the reasons on the right side of the two-column proof.

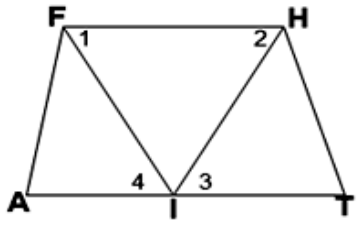
Given: $\overline{HO} \cong \overline{EP}$, $\angle OHP > \angle EPH$
Prove: $\overline{OP} > \overline{EH}$



	Statements	Reasons
1		Given
2		Reflexive Property of Equality
3		Given
4		Hinge Theorem

2. Make necessary markings to the congruent angles and sides as you analyze the given and the meanings behind them. Write the reasons for the statements in the two-column proof.

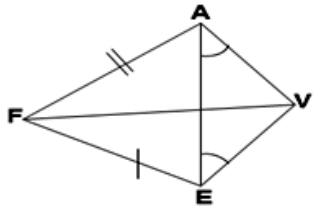
Given: I is the midpoint of $\overline{AT}$, $\angle 1 \cong \angle 2$, $\angle 3 > \angle 4$
Prove: $\overline{HT} > \overline{FA}$



	Statements	Reasons
1	$\angle 1 \cong \angle 2$	
2	ΔFIH is isosceles	
3	$\overline{FI} \cong \overline{HI}$	
4	I is the midpoint of AT	
5	$\overline{AI} \cong \overline{TI}$	
6	$\angle 3 > \angle 4$	
7	$\overline{HT} > \overline{FA}$	

3. Write the statement or reason in the two-column proof.

Given: ∠VAE ≅ ∠VEA, $\overline{AF} > \overline{EF}$
Prove: ∠AVF ≅ ∠EVF



	Statements	Reasons
1	$\angle VAE \cong \angle VEA$	
2	ΔAVE is an isosceles triangle.	
3		Legs of isosceles triangles are congruent.
4	$\overline{FV} \cong \overline{FV}$	
5		Given
6	$\angle AVF \cong \angle EVF$	

In this section, the discussion focuses mainly on using the triangle inequality theorems in solving both real-life problems and problems that require writing proofs.

Considering the application and proof-writing problems found in this module, share your insights on the following questions:

- Can you solve these problems without accurate illustrations and markings on the triangles?
- Can you solve these problems without prior knowledge related to triangles and writing proofs?
- Has your knowledge in algebra helped you in solving the problems?
- Have the theorems on triangle inequalities helped you in writing proofs of theorems?

Having tackled all concepts and skills to be learned on inequalities in triangles, revisit your decisions in Activity No.1 and write your responses to the statements under ***My Decisions Later****. Are there changes to your responses? Explain.*

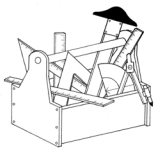
*What would be your reply to the essential question "****how can you justify inequalities in triangles"****?*

Now that you have a deeper understanding of the topic, it is time for you to put your knowledge and skills to practice before you do the tasks in the next section.

What to Transfer

Your goal in this section is to apply your learning to real life situations. You will be given a practical task which will enable you to demonstrate your understanding of inequalities in triangles.

Activity 23 DISASTER PREPAREDNESS: MAKING IT THROUGH THE RAIN



Goal: To design and create a miniature model of a folding ladder
Role: A design engineer
Audience: Company head

Situation: The lessons learned from the widespread flooding in many parts of the country during typhoons and monsoon season include securing tools and gadgets needed for safety. More and more people are buying ladders that could reach as high as 10 feet, long enough for people to gain access to their ceiling or their roof. There is a high demand for folding ladders for they can be stored conveniently. Being the design engineer of your company, your boss asks you to submit a miniature model of that ladder and justify the design.

Product: design of a folding ladder that can reach up to 10-feet height and its miniature

Standards: accurate, creative, efficient, and well-explained/well-justified

RUBRIC

CRITERIA	Outstanding 4	Satisfactory 3	Developing 2	Beginning 1	RATING
Accuracy	The computations are accurate and show a wise use of the geometric concepts specifically on triangle inequalities.	The computations are accurate and show the use of geometric concepts specifically on triangle inequalities.	The computations are erroneous and show some use of the concepts on triangle inequalities.	The computations are erroneous and do not show the use of the concepts on triangle inequalities.	
Creativity	The overall impact of the presentation of highly impressive and the use of technology is highly commendable.	The overall impact of the presentation is impressive and the use of technology is commendable.	The overall impact of the presentation is fair and the use of technology is evident.	The overall impact of the presentation is poor and the use of technology is not evident.	
Efficiency	The miniature is very effective and flawlessly done. It is also attractive.	The miniature is effective and flawless.	The miniature has some defects.	The miniature has many defects.	
Mathematical Justification	Justification is logically clear, convincing, and professionally delivered. The concepts learned on triangle inequalities are applied and previously learned concepts are connected to the new ones.	Justification is clear and convincingly delivered. Appropriate concepts learned on triangle inequalities are applied.	Justification is not so clear. Some ideas are not connected to each other. Not all concepts on triangle inequalities are applied.	Justification is ambiguous. Only few concepts on triangles inequalities are applied.	

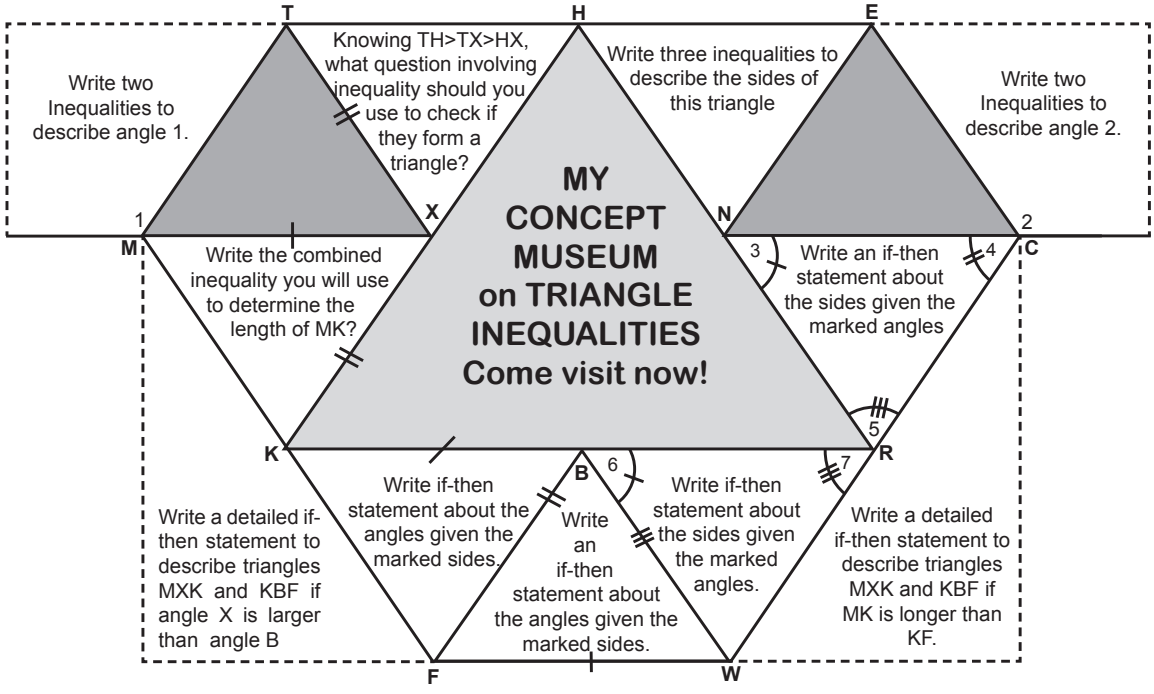
QUESTIONS ?

1. How do you find the experience of designing?
2. What insights can you share from the experience?
3. Has the activity helped you justify inequalities in triangles? How?
4. How did the task help you see the real world use of the concepts on inequalities in triangles?
5. Aside from designing a folding ladder, list down the real-life applications of concepts learned in Inequalities in Triangles from this module.
6. Can you think of other real-life applications of this topic?

SUMMARY

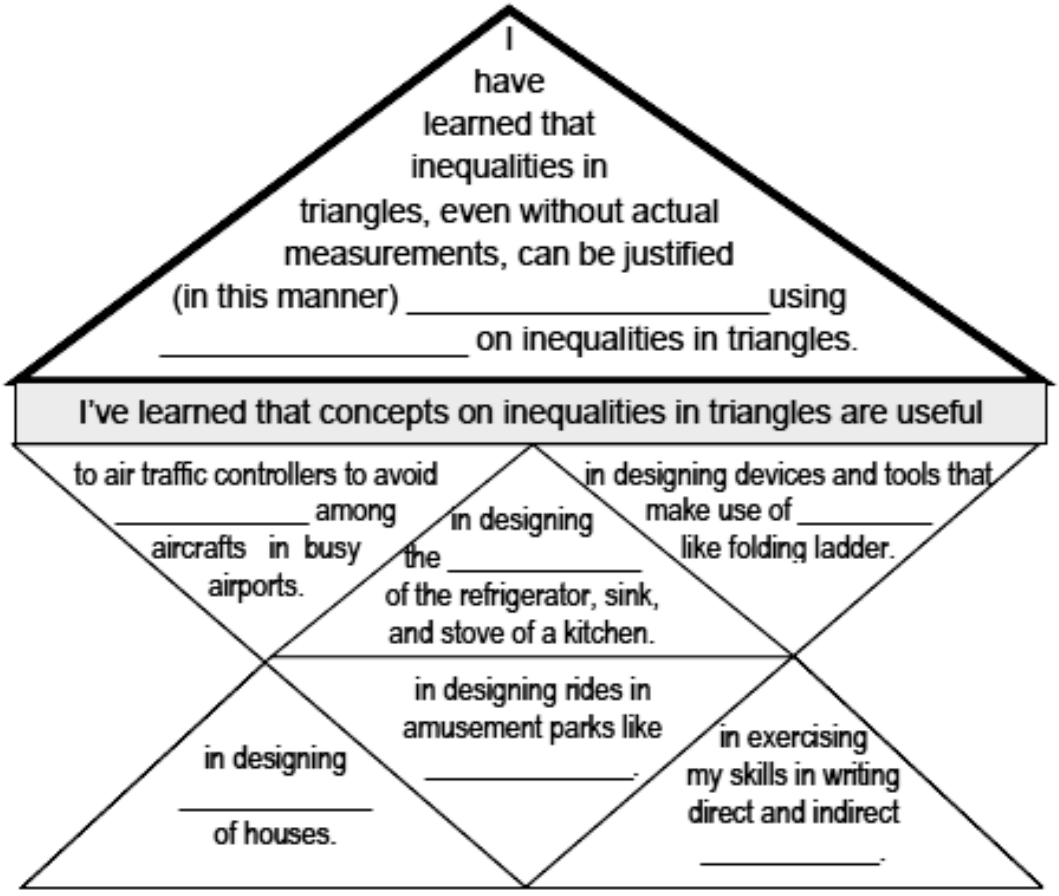
Activity 24 FINAL CONSTRUCTION OF CONCEPT MUSEUM

Directions: After learning all the concepts and skills on Inequalities in Triangles, take a final visit to your responses in Activity No.3— *Hello, Dear Concept Contractor*—of this module and make some modifications or corrections to your responses and their corresponding justifications.



Activity 25 CONCEPTS I'VE LEARNED THAT LAST FOREVER

Direction: Fill in the blanks with the right words to make the statements complete.



*You have completed the lesson on **Inequalities in Triangles**. Before you go to the next geometry lesson on **Parallelism and Perpendicularity**, you have to answer a post-assessment and a summative test.*

GLOSSARY OF TERMS USED IN THIS LESSON:

Inequalities in One Triangle:

Triangle Inequality Theorem 1 ($Ss \rightarrow Aa$)
If one side of a triangle is longer than a second side, then the angle opposite the first side is larger than the angle opposite the second side.

Triangle Inequality Theorem 2 ($Aa \rightarrow Ss$)
If one angle of a triangle is larger than a second angle, then the side opposite the first angle is longer than the side opposite the second angle.

Triangle Inequality Theorem 3 ($S_1 + S_2 > S_3$)
The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Exterior Angle Inequality Theorem
The measure of an exterior angle of a triangle is greater than the measure of either remote interior angle

Inequalities in Two Triangles:

Hinge Theorem
If two sides of one triangle are congruent to two sides of another triangle, but the included angle of the first triangle is greater than the included angle of the second, then the third side of the first triangle is longer than the third side of the second.

Converse of Hinge Theorem
If two sides of one triangle are congruent to two sides of another triangle, but the third side of the first triangle is longer than the third side of the second, then the included angle of the first triangle is larger than the included angle of the second.

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