6.6 Inequalities in Two Triangles

Essential Question

If two sides of one triangle are congruent to two sides of another triangle, what can you say about the third sides of the triangles?

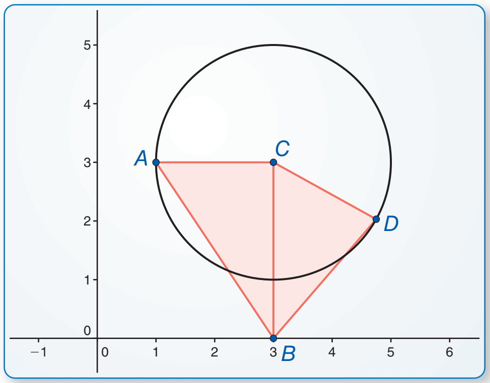
EXPLORATION 1 Comparing Measures in Triangles

Work with a partner. Use dynamic geometry software.

a. Draw $\triangle ABC$, as shown below.

b. Draw the circle with center $C(3, 3)$ through the point $A(1, 3)$.

c. Draw $\triangle DBC$ so that D is a point on the circle.



Sample

Points
$A(1, 3)$
$B(3, 0)$
$C(3, 3)$
$D(4.75, 2.03)$
Segments
$BC = 3$
$AC = 2$
$DC = 2$
$AB = 3.61$
$DB = 2.68$

d. Which two sides of $\triangle ABC$ are congruent to two sides of $\triangle DBC$? Justify your answer.

e. Compare the lengths of $\overline{AB}$ and $\overline{DB}$. Then compare the measures of $\angle ACB$ and $\angle DCB$. Are the results what you expected? Explain.

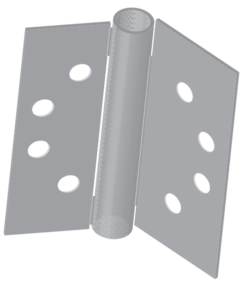
f. Drag point D to several locations on the circle. At each location, repeat part (e). Copy and record your results in the table below.

	D	AC	BC	AB	BD	$m\angle ACB$	$m\angle BCD$
1.	(4.75, 2.03)	2	3				
2.		2	3				
3.		2	3				
4.		2	3				
5.		2	3				

g. Look for a pattern of the measures in your table. Then write a conjecture that summarizes your observations.

CONSTRUCTING VIABLE ARGUMENTS

To be proficient in math, you need to make conjectures and build a logical progression of statements to explore the truth of your conjectures.



Communicate Your Answer

2. If two sides of one triangle are congruent to two sides of another triangle, what can you say about the third sides of the triangles?

3. Explain how you can use the hinge shown at the left to model the concept described in Question 2.

6.6 Lesson

What You Will Learn

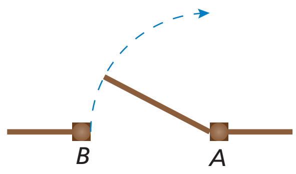
- Compare measures in triangles.
- Solve real-life problems using the Hinge Theorem.

Core Vocabulary

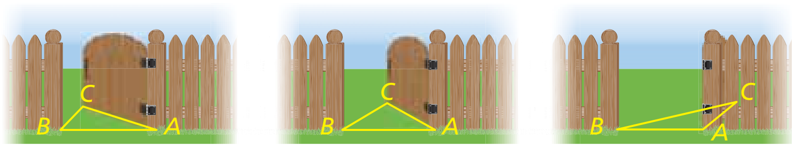
Previous
indirect proof
inequality

Comparing Measures in Triangles

Imagine a gate between fence posts A and B that has hinges at A and swings open at B.



As the gate swings open, you can think of $\triangle ABC$, with side $\overline{AC}$ formed by the gate itself, side $\overline{AB}$ representing the distance between the fence posts, and side $\overline{BC}$ representing the opening between post B and the outer edge of the gate.

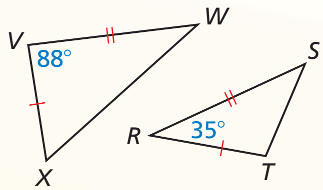


Notice that as the gate opens wider, both the measure of $\angle A$ and the distance BC increase. This suggests the *Hinge Theorem*.

Theorems

Theorem 6.12 Hinge Theorem

If two sides of one triangle are congruent to two sides of another triangle, and the included angle of the first is larger than the included angle of the second, then the third side of the first is longer than the third side of the second.

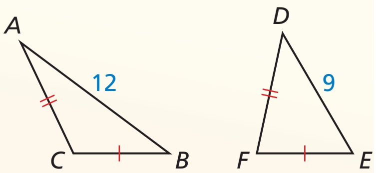


$WX > ST$

Proof BigIdeasMath.com

Theorem 6.13 Converse of the Hinge Theorem

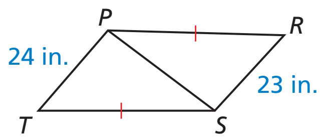
If two sides of one triangle are congruent to two sides of another triangle, and the third side of the first is longer than the third side of the second, then the included angle of the first is larger than the included angle of the second.



$m\angle C > m\angle F$

Proof Example 3, p. 345

EXAMPLE 1 Using the Converse of the Hinge Theorem



Given that $\overline{ST} \cong \overline{PR}$, how does $m\angle PST$ compare to $m\angle SPR$?

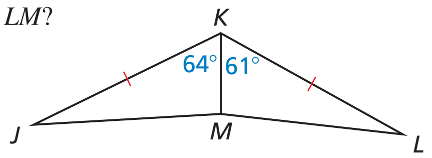
SOLUTION

You are given that $\overline{ST} \cong \overline{PR}$, and you know that $\overline{PS} \cong \overline{PS}$ by the Reflexive Property of Congruence (Theorem 2.1). Because 24 inches > 23 inches, $PT > SR$. So, two sides of $\triangle STP$ are congruent to two sides of $\triangle PRS$ and the third side of $\triangle STP$ is longer.

- By the Converse of the Hinge Theorem, $m\angle PST > m\angle SPR$.

EXAMPLE 2 Using the Hinge Theorem

Given that $\overline{JK} \cong \overline{LK}$, how does JM compare to LM?



SOLUTION

You are given that $\overline{JK} \cong \overline{LK}$, and you know that $\overline{KM} \cong \overline{KM}$ by the Reflexive Property of Congruence (Theorem 2.1). Because $64° > 61°$, $m\angle JKM > m\angle LKM$. So, two sides of $\triangle JKM$ are congruent to two sides of $\triangle LKM$, and the included angle in $\triangle JKM$ is larger.

▶ By the Hinge Theorem, $JM > LM$.

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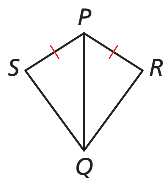
Use the diagram.



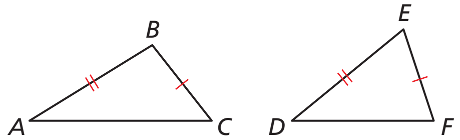
1. If $PR = PS$ and $m\angle QPR > m\angle QPS$, which is longer, $\overline{SQ}$ or $\overline{RQ}$?
2. If $PR = PS$ and $RQ < SQ$, which is larger, $\angle RPQ$ or $\angle SPQ$?

EXAMPLE 3 Proving the Converse of the Hinge Theorem

Write an indirect proof of the Converse of the Hinge Theorem.

Given $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $AC > DF$

Prove $m\angle B > m\angle E$



Indirect Proof

Step 1 Assume temporarily that $m\angle B \not> m\angle E$. Then it follows that either $m\angle B < m\angle E$ or $m\angle B = m\angle E$.

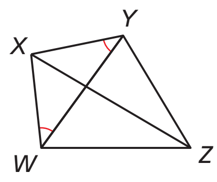
Step 2 If $m\angle B < m\angle E$, then $AC < DF$ by the Hinge Theorem.

If $m\angle B = m\angle E$, then $\angle B \cong \angle E$. So, $\triangle ABC \cong \triangle DEF$ by the SAS Congruence Theorem (Theorem 5.5) and $AC = DF$.

Step 3 Both conclusions contradict the given statement that $AC > DF$. So, the temporary assumption that $m\angle B \not> m\angle E$ cannot be true. This proves that $m\angle B > m\angle E$.

EXAMPLE 4 Proving Triangle Relationships

Write a paragraph proof.



Given $\angle XWY \cong \angle XYW$, $WZ > YZ$

Prove $m\angle WXZ > m\angle YXZ$

Paragraph Proof Because $\angle XWY \cong \angle XYW$, $\overline{XY} \cong \overline{XW}$ by the Converse of the Base Angles Theorem (Theorem 5.7). By the Reflexive Property of Congruence (Theorem 2.1), $\overline{XZ} \cong \overline{XZ}$. Because $WZ > YZ$, $m\angle WXZ > m\angle YXZ$ by the Converse of the Hinge Theorem.

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3. Write a temporary assumption you can make to prove the Hinge Theorem indirectly. What two cases does that assumption lead to?

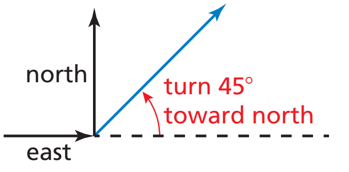
Solving Real-Life Problems

EXAMPLE 5 **Solving a Real-Life Problem**

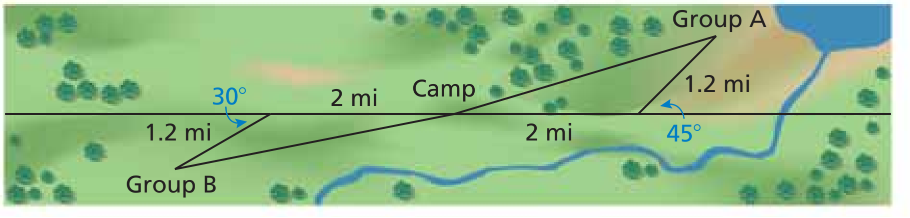
Two groups of bikers leave the same camp heading in opposite directions. Each group travels 2 miles, then changes direction and travels 1.2 miles. Group A starts due east and then turns 45° toward north. Group B starts due west and then turns 30° toward south. Which group is farther from camp? Explain your reasoning.

SOLUTION

1. **Understand the Problem** You know the distances and directions that the groups of bikers travel. You need to determine which group is farther from camp. You can interpret a turn of 45° toward north, as shown.



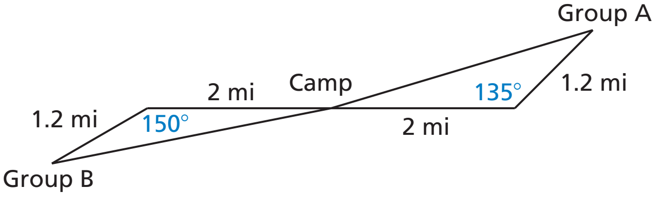
2. **Make a Plan** Draw a diagram that represents the situation and mark the given measures. The distances that the groups bike and the distances back to camp form two triangles. The triangles have two congruent side lengths of 2 miles and 1.2 miles. Include the third side of each triangle in the diagram.



3. **Solve the Problem** Use linear pairs to find the included angles for the paths that the groups take.

   Group A: $180° - 45° = 135°$ **Group B:** $180° - 30° = 150°$

   The included angles are 135° and 150°.



   Because $150° > 135°$, the distance Group B is from camp is greater than the distance Group A is from camp by the Hinge Theorem.

   ▶ So, Group B is farther from camp.

4. **Look Back** Because the included angle for Group A is 15° less than the included angle for Group B, you can reason that Group A would be closer to camp than Group B. So, Group B is farther from camp.

Monitoring Progress

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4. **WHAT IF?** In Example 5, Group C leaves camp and travels 2 miles due north, then turns 40° toward east and travels 1.2 miles. Compare the distances from camp for all three groups.

6.6 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

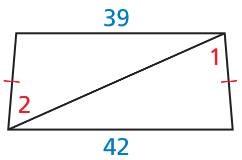
1. **WRITING** Explain why Theorem 6.12 is named the “Hinge Theorem.”

2. **COMPLETE THE SENTENCE** In $\triangle ABC$ and $\triangle DEF$, $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $AC < DF$. So $m\angle$_____ $>$ $m\angle$_____ by the Converse of the Hinge Theorem (Theorem 6.13).

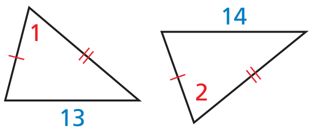
Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, copy and complete the statement with <, >, or =. Explain your reasoning. *(See Example 1.)*

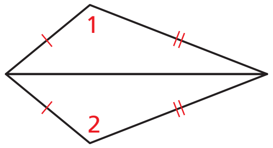
3. $m\angle 1$ _____ $m\angle 2$



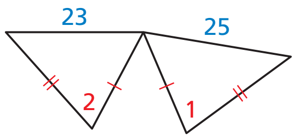
4. $m\angle 1$ _____ $m\angle 2$



5. $m\angle 1$ _____ $m\angle 2$

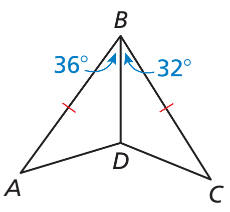


6. $m\angle 1$ _____ $m\angle 2$

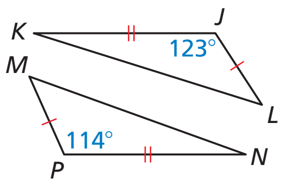


In Exercises 7–10, copy and complete the statement with <, >, or =. Explain your reasoning. *(See Example 2.)*

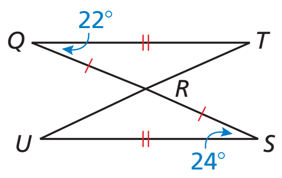
7. AD _____ CD



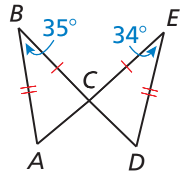
8. MN _____ LK



9. TR _____ UR



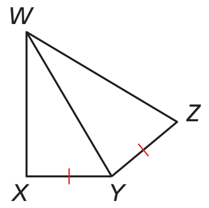
10. AC _____ DC



PROOF **In Exercises 11 and 12, write a proof.** *(See Example 4.)*

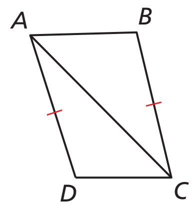
11. **Given** $\overline{XY} \cong \overline{YZ}$, $m\angle WYZ > m\angle WYX$

    Prove $WZ > WX$



12. **Given** $\overline{BC} \cong \overline{DA}$, $DC < AB$

    Prove $m\angle BCA > m\angle DAC$



In Exercises 13 and 14, you and your friend leave on different flights from the same airport. Determine which flight is farther from the airport. Explain your reasoning. *(See Example 5.)*

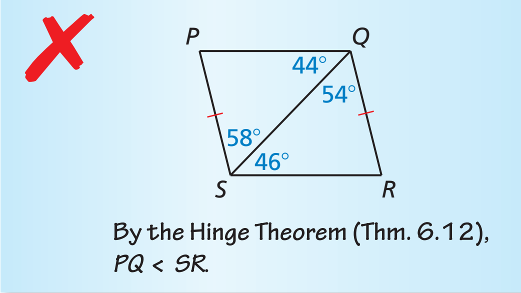
13. **Your flight:** Flies 100 miles due west, then turns 20° toward north and flies 50 miles.

    Friend’s flight: Flies 100 miles due north, then turns 30° toward east and flies 50 miles.

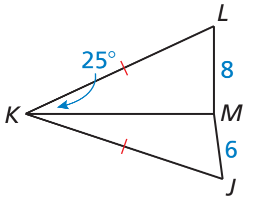
14. **Your flight:** Flies 210 miles due south, then turns 70° toward west and flies 80 miles.

    Friend’s flight: Flies 80 miles due north, then turns 50° toward east and flies 210 miles.

15. ERROR ANALYSIS Describe and correct the error in using the Hinge Theorem (Theorem 6.12).

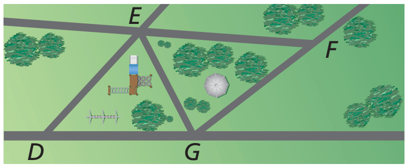


16. REPEATED REASONING Which is a possible measure for $\angle JKM$? Select all that apply.



(A) 15° (B) 22° (C) 25° (D) 35°

17. DRAWING CONCLUSIONS The path from E to F is longer than the path from E to D. The path from G to D is the same length as the path from G to F. What can you conclude about the angles of the paths? Explain your reasoning.

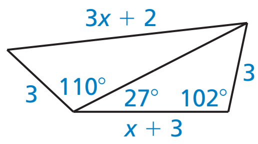


18. ABSTRACT REASONING In $\triangle EFG$, the bisector of $\angle F$ intersects the bisector of $\angle G$ at point H. Explain why $\overline{FG}$ must be longer than $\overline{FH}$ or $\overline{HG}$.

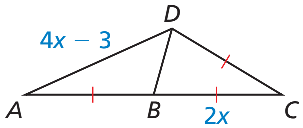
19. ABSTRACT REASONING $\overline{NR}$ is a median of $\triangle NPQ$, and $NQ > NP$. Explain why $\angle NRQ$ is obtuse.

MATHEMATICAL CONNECTIONS **In Exercises 20 and 21, write and solve an inequality for the possible values of *x*.**

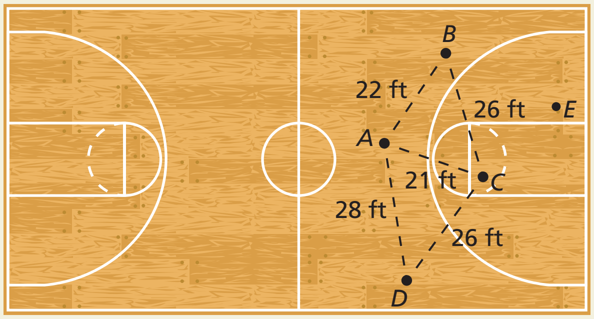
20.



21. Given B is the midpoint of $\overline{AC}$.



22. HOW DO YOU SEE IT? In the diagram, triangles are formed by the locations of the players on the basketball court. The dashed lines represent the possible paths of the basketball as the players pass. How does $m\angle ACB$ compare with $m\angle ACD$?



23. CRITICAL THINKING In $\triangle ABC$, the altitudes from B and C meet at point D, and $m\angle BAC > m\angle BDC$. What is true about $\triangle ABC$? Justify your answer.

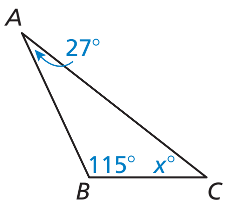
24. THOUGHT PROVOKING The postulates and theorems in this book represent Euclidean geometry. In spherical geometry, all points are on the surface of a sphere. A line is a circle on the sphere whose diameter is equal to the diameter of the sphere. In spherical geometry, state an inequality involving the sum of the angles of a triangle. Find a formula for the area of a triangle in spherical geometry.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the value of *x*. *(Section 5.1 and Section 5.4)*

25.



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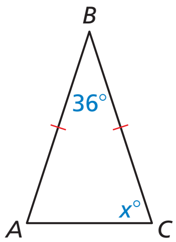

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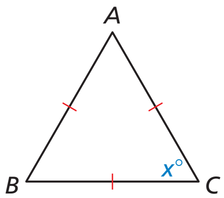

28.