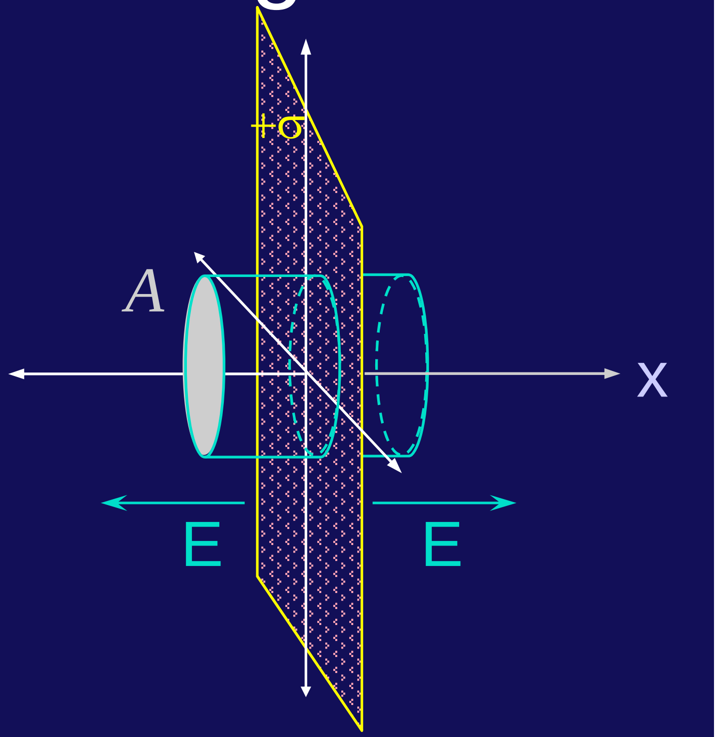
Infinite sheet of charge



Symmetry:

direction of E = x-axis

CHOOSE Gaussian surface to be a cylinder aligned with the x-axis.

Apply Gauss' Law:

Integrate the barrel, $\vec{E} \bullet d\vec{S} = 0$

Now the ends, $\oint \vec{E} \bullet d\vec{S} = 2AE$

The charge enclosed = σA

Therefore, Gauss' Law $\Rightarrow$ $\varepsilon_0 (2EA) = \sigma A$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

Conclusion: An infinite plane sheet of charge creates a CONSTANT electric field .

Two Infinite Sheets

(into screen)

Field outside the sheets must be zero. Two ways to see:

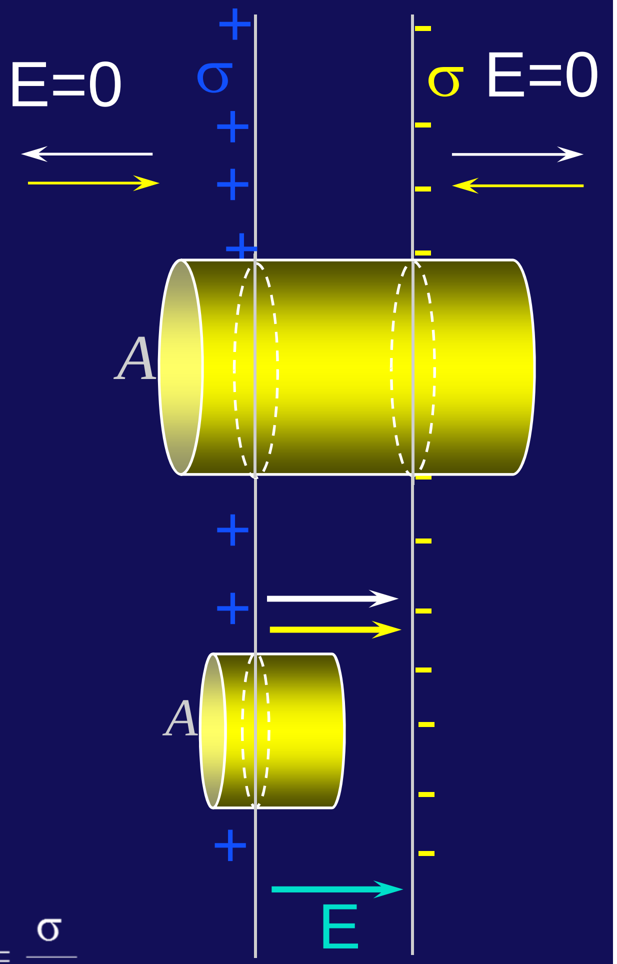
Superposition

Gaussian surface encloses zero charge

Field inside sheets is NOT zero:

Superposition

Gaussian surface encloses non-zero chg



$$Q = \sigma A$$

$$\oint \vec{E} \bullet d\vec{S} = \cancelto{0}{AE}_{outside} + AE_{inside}$$

$$\mathrm{E} = \frac{\sigma}{\varepsilon_0}$$

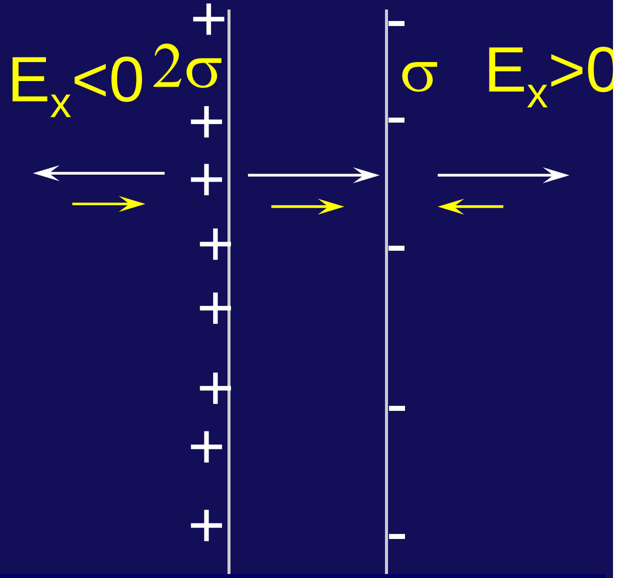
More Sheets

(into screen)

Now, asymmetrically charged

Superposition

Show fields in all locations from each sheet



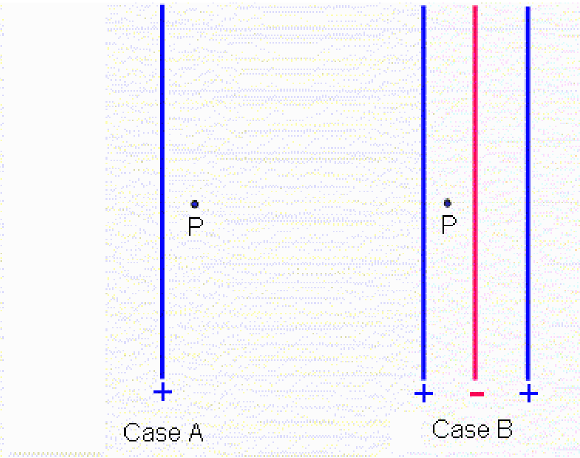
$$E_x = \frac{-2\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}$$

$$E_x = \frac{2\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}$$

$$E_x = \frac{2\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0}$$

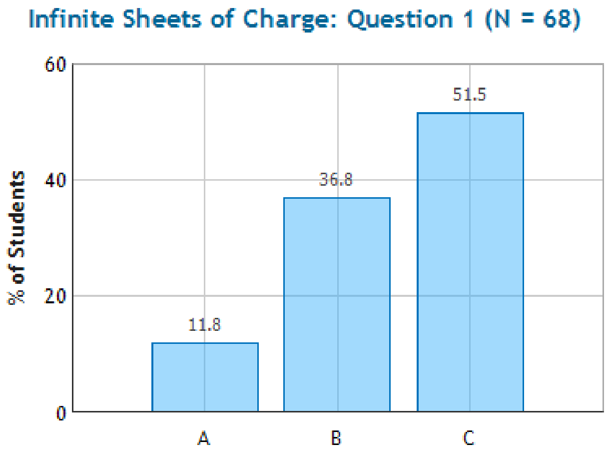
CheckPoint

In both cases shown below, the colored lines represent positive (blue) and negative (red) charged planes.

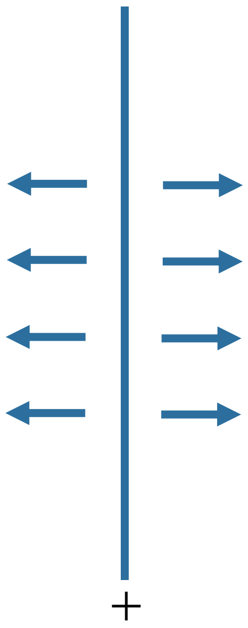


In which case is E at point P the biggest?

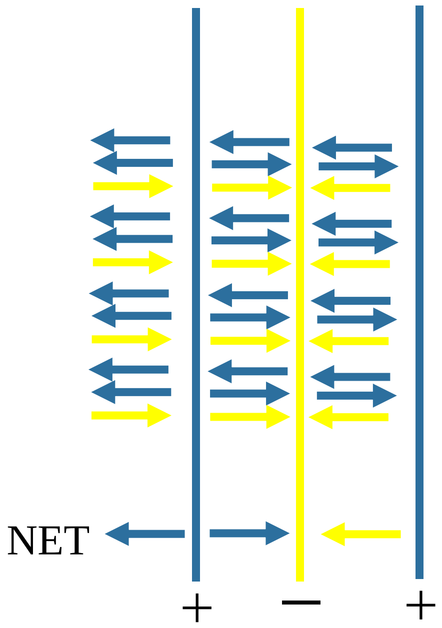
A) A   B) B   [✓ C) the same]



Superposition:



Case A



Case B

This week

- Today:
  - Electric Potential Energy
- Wednesday:
  - Electric Potential
  - Homework #2 is due 9PM
- Thursday:
  - Midterm 1 → Kane Hall; 5 pm sharp
  - See Home Page for content, Practice, Equation sheet …

  - PHYS 122 A → Physics Building Rooms A102 and A118
  - PHYS 122 B → Kane 120
    - No backpacks please
    - Bring a calculator (no fancy stuff allowed of course)
    - We provide the equation sheet
- Friday:
  - No class

Lecture Thoughts

- *I don't recall any of the types of energy we went over in 121 ever being negative. I understand that change in electric energy can be positive or negative, but how can the overall potential energy of a system be negative? What does it mean to have "negative potential energy"?*

- *The concept of the potential energy with more than two bodies and different angles confuses me.*

- *The myriad of different formulas in change in work, change in potential, and the potential energy.*

Big new ideas from Physics 121

- Define Potential Energy Difference (for a Conservative Force)

$$\Delta U = U_2 - U_1 = -W_{1\to 2}$$

Units: Joules

The definition of Work:
The work done on a particle by a force *F* as it moves an object from point 1 to point 2 is

$$W_{1\to 2} = \int_1^2 \vec{F} \cdot d\vec{l}$$

Remember: The work equals the change in kinetic energy

$$W_{1\to 2} = \Delta K$$

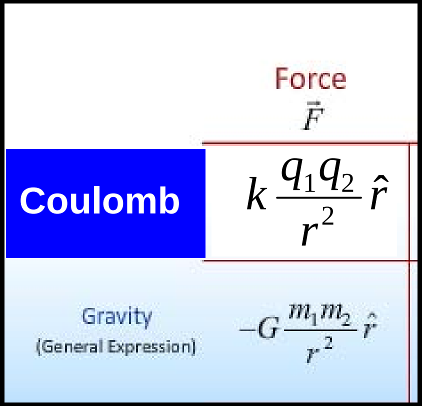
Consequence:

$$\boxed{W_{1\to 2} + \Delta U = 0}$$

Finding the potential energy change:

Use formulas to find the magnitude

Check the sign by understanding the problem…



	Force $\vec{F}$
Coulomb	$k\frac{q_1 q_2}{r^2}\hat{r}$
Gravity (General Expression)	$-G\frac{m_1 m_2}{r^2}\hat{r}$

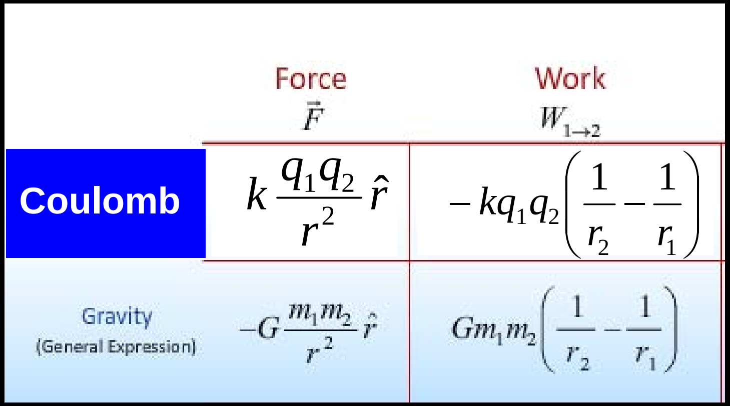
Note: For gravity, m_1 and m_2 are always positive
Thus, it is always: attractive force

But: For Coulomb's Law, q_1 and q_2 can have either sign.
Same sign charges: repulsive force
Opposite signs charges: attractive force

Finding the potential energy change:

Use formulas to find the magnitude

Check the sign by understanding the problem…



	Force $\vec{F}$	Work $W_{1\to 2}$
Coulomb	$k\frac{q_1 q_2}{r^2}\hat{r}$	$-kq_1 q_2\left(\frac{1}{r_2}-\frac{1}{r_1}\right)$
Gravity (General Expression)	$-G\frac{m_1 m_2}{r^2}\hat{r}$	$Gm_1 m_2\left(\frac{1}{r_2}-\frac{1}{r_1}\right)$

$$W_{1\to 2}=\int_1^2 \vec{F}\cdot d\vec{l}$$

Note: For gravity, m_1 and m_2 are always positive
Thus, it is always: attractive force

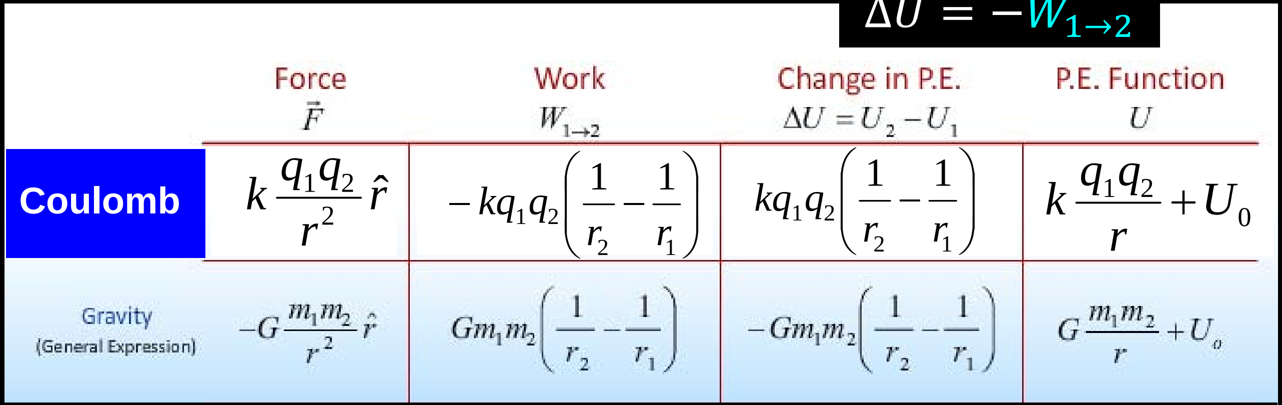
But: For Coulomb's Law, q_1 and q_2 can have either sign.
Same sign charges: repulsive force
Opposite signs charges: attractive force

Finding the potential energy change:

Use formulas to find the magnitude

Check the sign by understanding the problem…

$$\Delta U = -W_{1\to 2}$$



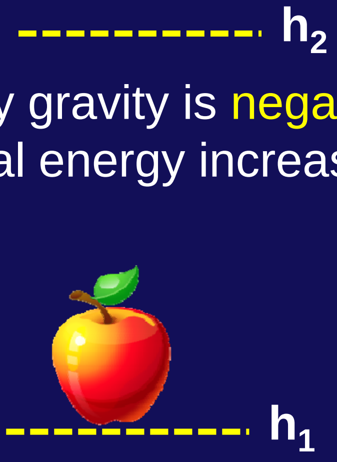
	Force $\vec{F}$	Work $W_{1\to 2}$	Change in P.E. $\Delta U = U_2 - U_1$	P.E. Function U
Coulomb	$k\dfrac{q_1q_2}{r^2}\hat{r}$	$-kq_1q_2\left(\dfrac{1}{r_2}-\dfrac{1}{r_1}\right)$	$kq_1q_2\left(\dfrac{1}{r_2}-\dfrac{1}{r_1}\right)$	$k\dfrac{q_1q_2}{r}+U_0$
Gravity (General Expression)	$-G\dfrac{m_1m_2}{r^2}\hat{r}$	$Gm_1m_2\left(\dfrac{1}{r_2}-\dfrac{1}{r_1}\right)$	$-Gm_1m_2\left(\dfrac{1}{r_2}-\dfrac{1}{r_1}\right)$	$G\dfrac{m_1m_2}{r}+U_o$

Note: For gravity, m_1 and m_2 are always positive
Thus, it is always: attractive force

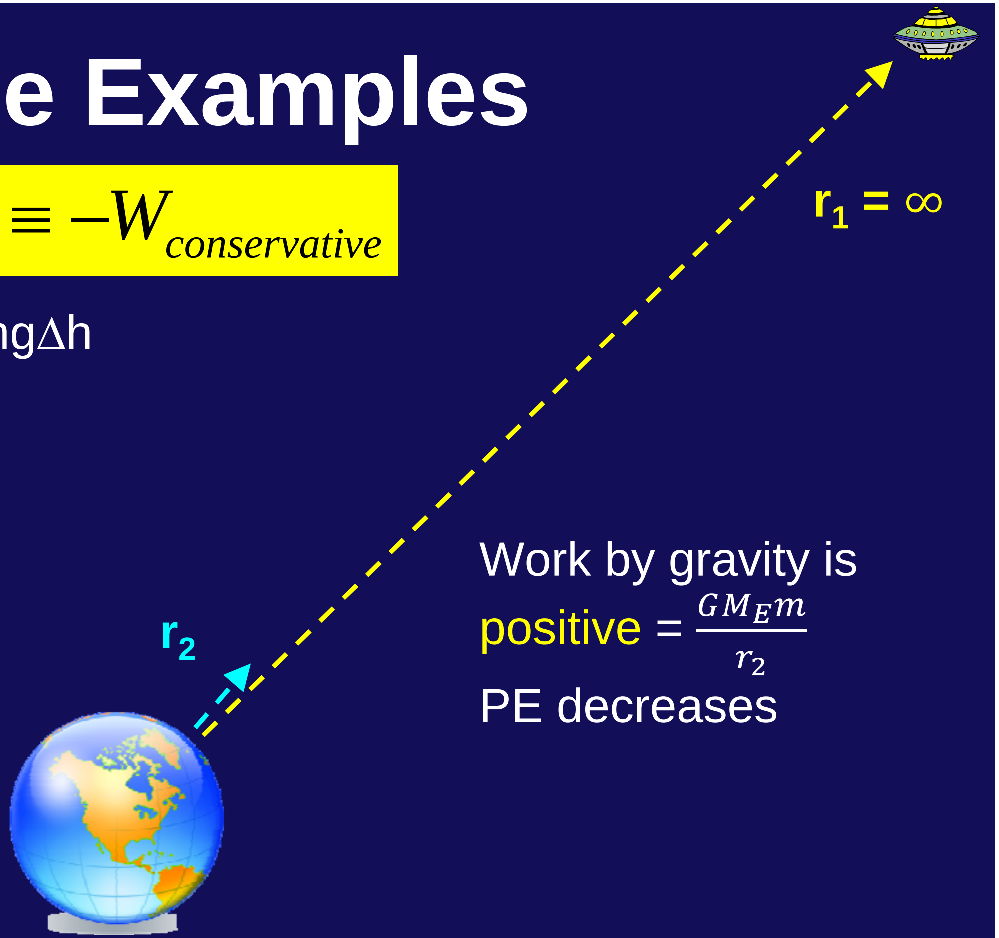
But: For Coulomb's Law, q_1 and q_2 can have either sign.
Same sign charges: repulsive force
Opposite signs charges: attractive force

Simple Examples

$$\Delta U \equiv -W_{conservative}$$

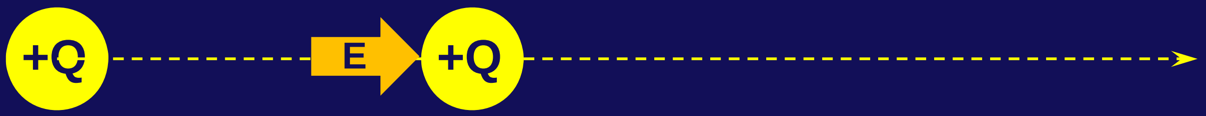


Work by gravity is negative = -mgΔh
Potential energy increases



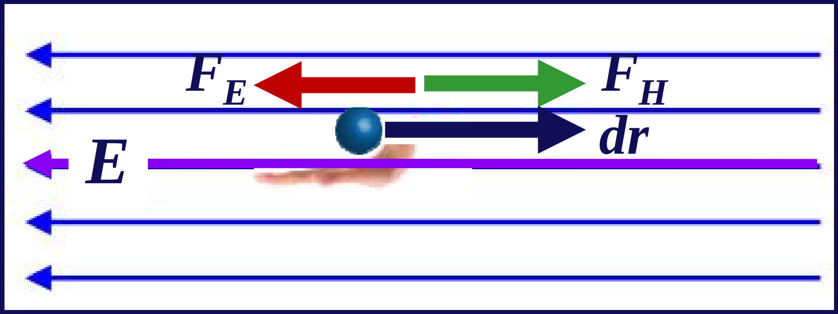
Work by gravity is
positive = $\frac{GM_Em}{r_2}$
PE decreases

Same idea for Coulomb force… if Coulomb force does positive work, potential energy decreases.



Clicker

You hold a positively charged ball and walk to the right in a region that contains an electric field directed to the left.



W_H is the work done by the hand on the ball
W_E is the work done by the electric field on the ball

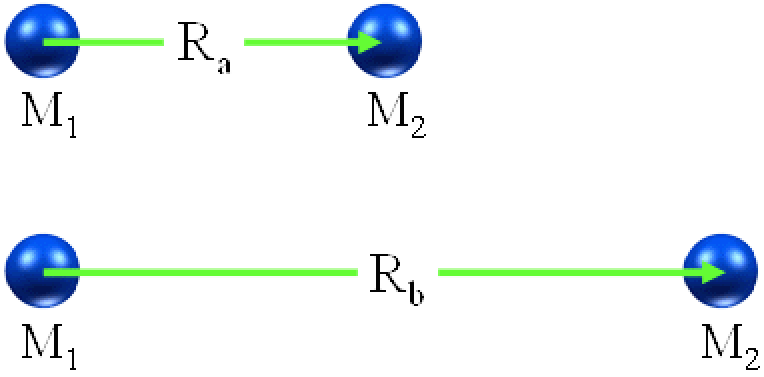
Which of the following statements is true:

A) $W_H > 0$ and $W_E > 0$
[✓ B) $W_H > 0$ and $W_E < 0$]
C) $W_H < 0$ and $W_E < 0$
D) $W_H < 0$ and $W_E > 0$

Prelecture: Electric Potential Energy

Masses M_1 and M_2 are initially separated by a distance R_a. Mass M_2 is now moved further away from mass M_1 such that their final separation is R_b.

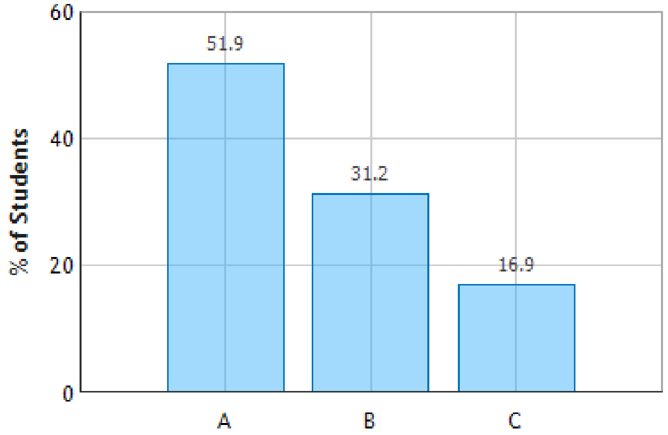
Which of the following statements best describes the work W_{ab} done by the force of gravity on M_2 as it moves from R_a to R_b?



○ $W_{ab} > 0$

● $W_{ab} = 0$

○ $W_{ab} < 0$

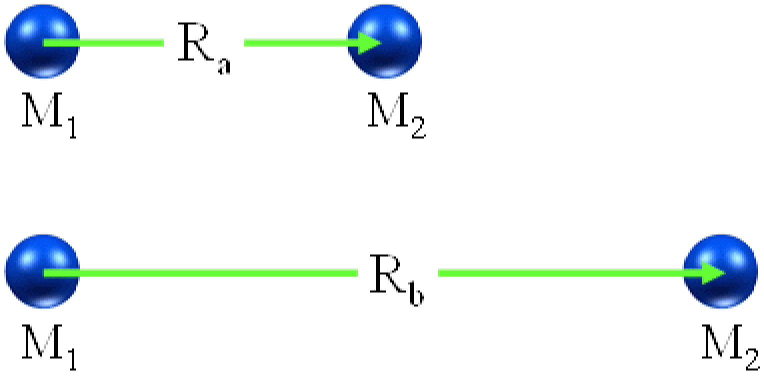


First Answer Choice Distribution (N = 77)

Prelecture: Electric Potential Energy

Masses M_1 and M_2 are initially separated by a distance R_a. Mass M_2 is now moved further away from mass M_1 such that their final separation is R_b.

Which of the following statements best describes the work W_{ab} done by the force of gravity on M_2 as it moves from R_a to R_b?



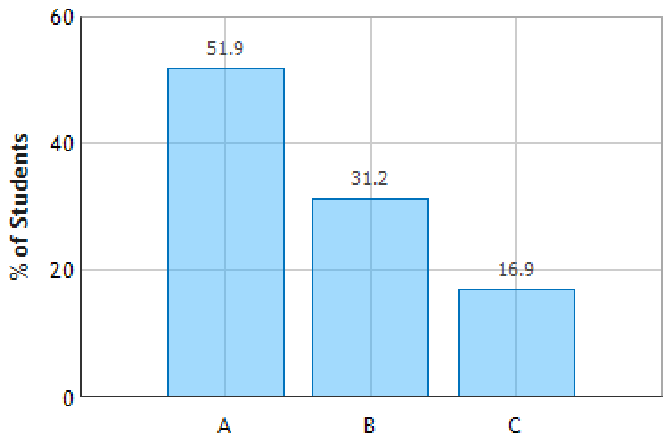
○ $W_{ab} > 0$

○ $W_{ab} = 0$

● $W_{ab} < 0$

Force and displacement in opposite directions → Work is *negative*.

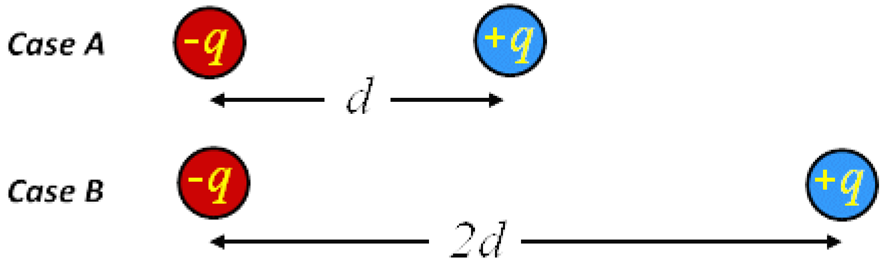
First Answer Choice Distribution (N = 77)



Prelecture: Electric Potential Energy

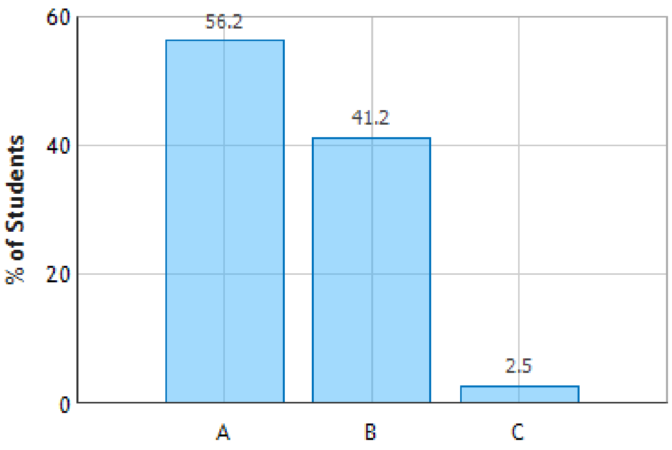
In Case A two charges which are equal in magnitude but opposite in charge are separated by a distance *d*. In Case B the same charges are separated by a distance 2*d*.

Which configuration has the highest potential energy *U*?



- ○ Case A has the highest potential energy
- ○ Case B has the highest potential energy
- ○ Both cases have the same potential energy

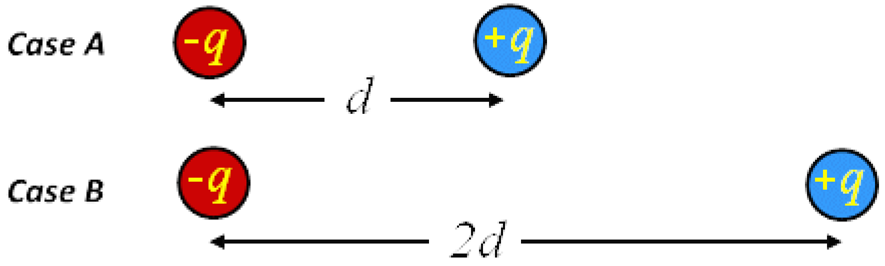
First Answer Choice Distribution (N = 80)



Prelecture: Electric Potential Energy

In Case A two charges which are equal in magnitude but opposite in charge are separated by a distance d. In Case B the same charges are separated by a distance $2d$.

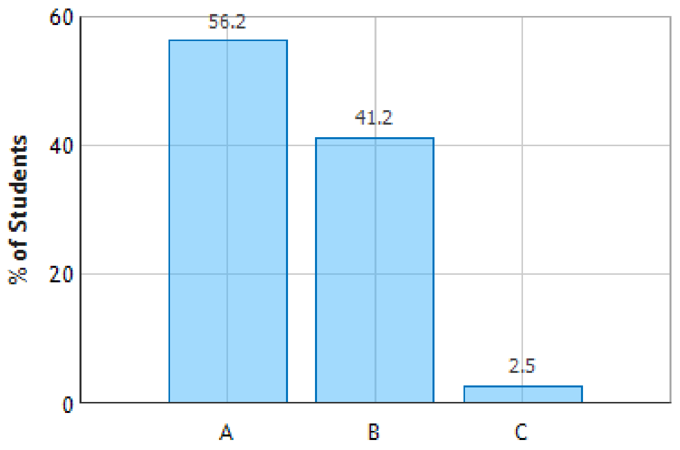
Which configuration has the highest potential energy U?



- ○ Case A has the highest potential energy
- ● Case B has the highest potential energy
- ○ Both cases have the same potential energy

Think what the charges would tend to do in the absence of other forces. They would "fall" from case B to case A. → Case B has higher U.

First Answer Choice Distribution (N = 80)



Example: Two Point Charges

Calculate the change in potential energy for two point charges originally very far apart moved to a separation of "*d*"

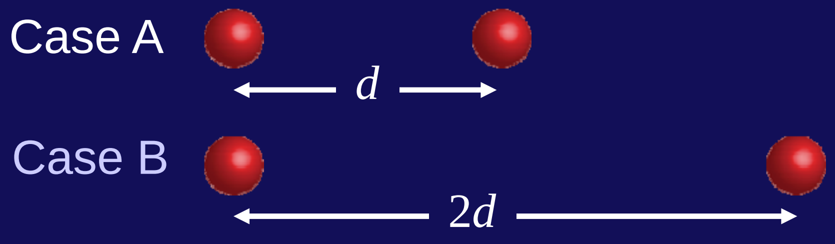
$$\Delta U = -\int_{\infty}^{d} k \frac{q_1 q_2}{r_{12}^2} dr$$

$$\Delta U = k \frac{q_1 q_2}{d} \equiv \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{d}$$

Charged particles with the same sign have an increase in potential energy when brought closer together.

For point charges often choose $r =$ infinity as "zero" potential energy.

Clicker



In Case A two negative charges which are equal in magnitude are separated by a distance d. In Case B the same charges are separated by a distance $2d$. Which configuration has the highest potential energy?

- [✓ A) Case A]
- B) Case B

Followup

As usual, choose U = 0 at infinity:

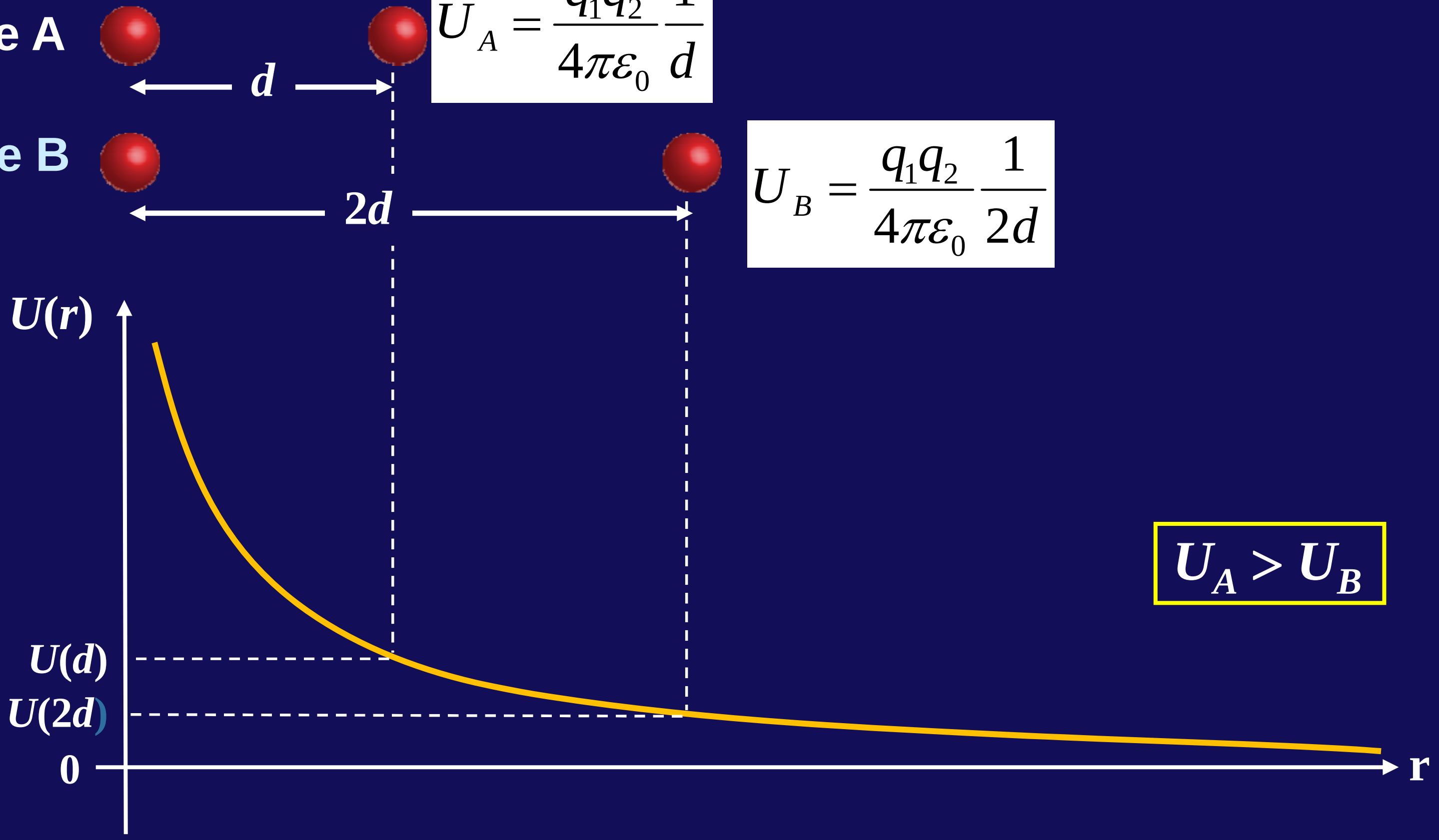
$$U(r) = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{1}{r}$$

Case A

$$U_A = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{1}{d}$$

Case B

$$U_B = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{1}{2d}$$



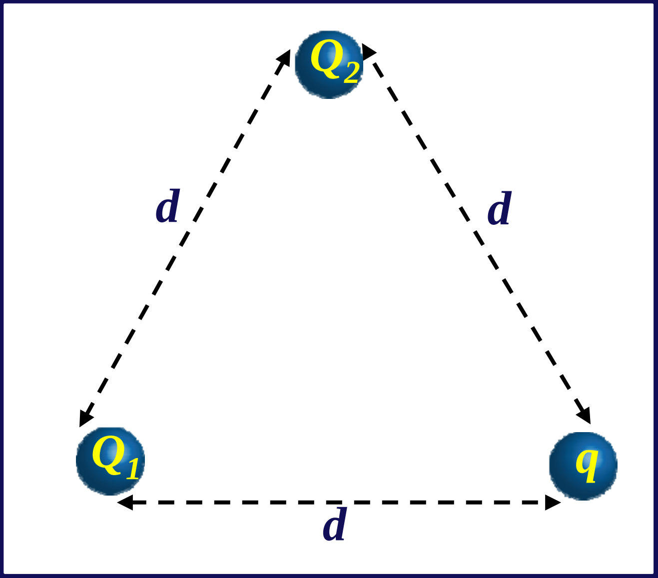
$$\boxed{U_A > U_B}$$

Potential Energy of Many Charges

Two charges are separated by a distance d. What is the change in potential energy when a third charge q is brought from far away to a distance d from the original two charges?

$$\Delta U = \frac{qQ_1}{4\pi\varepsilon_0}\frac{1}{d} + \frac{qQ_2}{4\pi\varepsilon_0}\frac{1}{d}$$

(consider each pair separately and add)



Clicker

What is the total energy* required to bring in three identical charges, from infinitely far away to the points on an equilateral triangle shown.

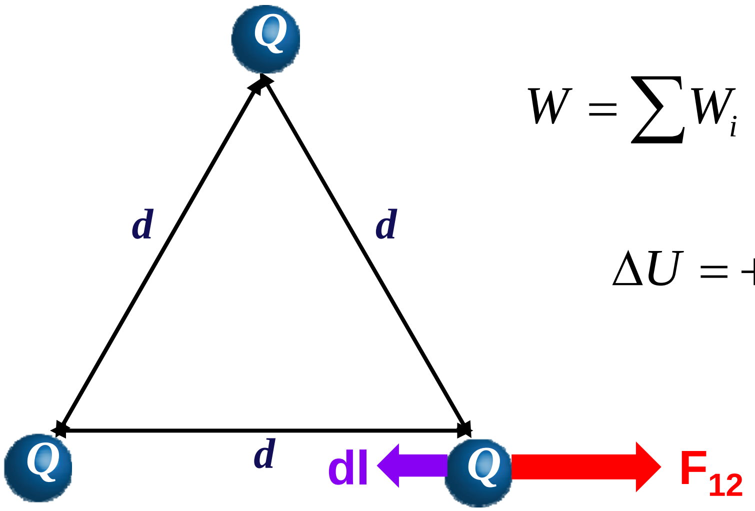
A) 0

B) $\Delta U = \frac{Q^2}{4\pi\varepsilon_0}\frac{1}{d}$

C) $\Delta U = 2\frac{Q^2}{4\pi\varepsilon_0}\frac{1}{d}$

[✓ **D)** $\Delta U = 3\frac{Q^2}{4\pi\varepsilon_0}\frac{1}{d}$]

E) $\Delta U = 6\frac{Q^2}{4\pi\varepsilon_0}\frac{1}{d}$



$$W = \sum W_i = -\frac{3}{4\pi\varepsilon_0}\frac{Q^2}{d}$$

$$\Delta U = +\frac{3}{4\pi\varepsilon_0}\frac{Q^2}{d}$$

Work to bring in first charge: $W_1 = 0$

Work to bring in second charge : $W_2 = -\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{d}$

Work to bring in third charge : $W_3 = -\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{d} - \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{d} = -\frac{2}{4\pi\varepsilon_0}\frac{Q^2}{d}$

***total energy is equivalent to the change in Potential Energy**

Clicker follower

Suppose one of the charges is negative. Now what is the total energy required to bring the three charges in infinitely far away?

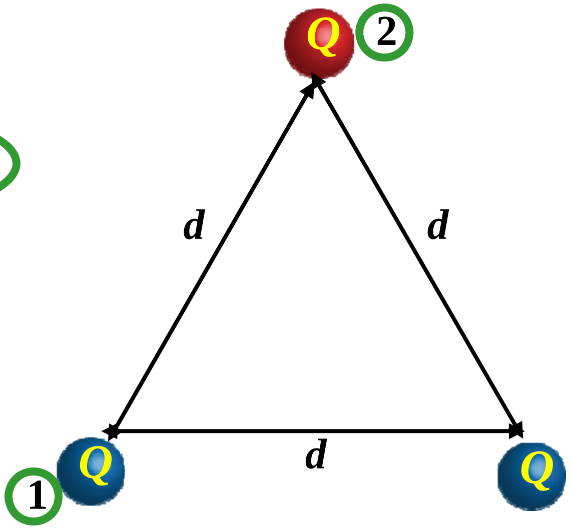
A) 0

B) $\Delta U = +1\frac{Q^2}{4\pi\varepsilon_0}\frac{1}{d}$

[✓ C) $\Delta U = -1\frac{Q^2}{4\pi\varepsilon_0}\frac{1}{d}$]

D) $\Delta U = +2\frac{Q^2}{4\pi\varepsilon_0}\frac{1}{d}$

E) $\Delta U = -2\frac{Q^2}{4\pi\varepsilon_0}\frac{1}{d}$



$$W = \sum W_i = +\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{d}$$

$$\boxed{\Delta U = -\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{d}}$$

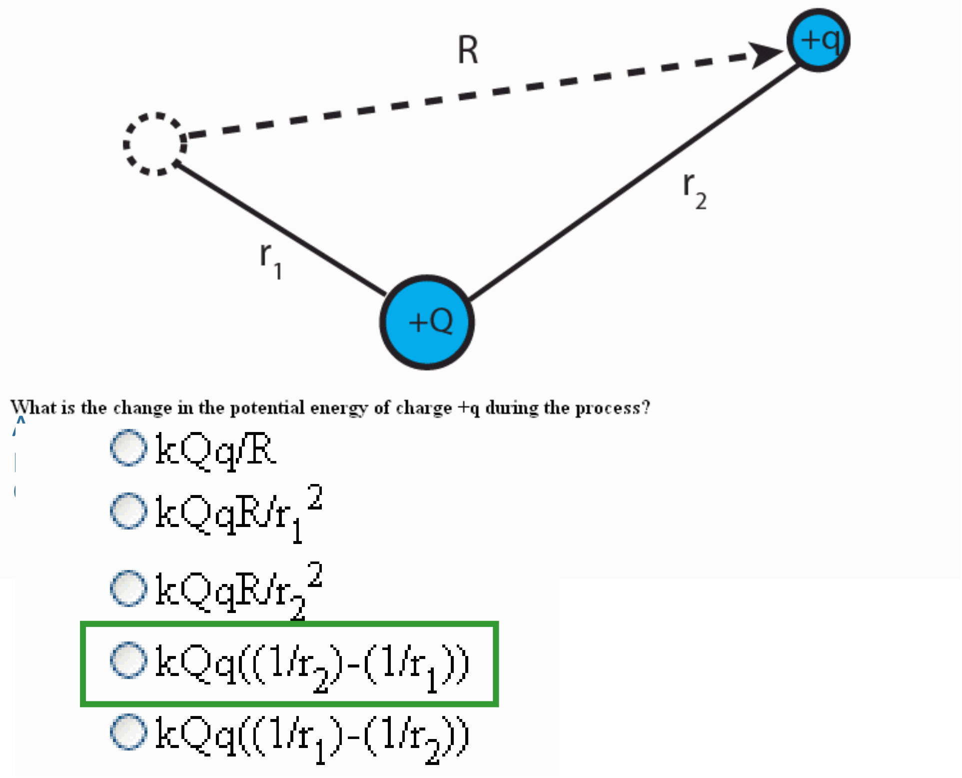
Work to bring in first charge: $W_1 = 0$

Work to bring in second charge : $W_2 = +\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{d}$

Work to bring in third charge : $W_3 = +\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{d} - \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{d} = 0$

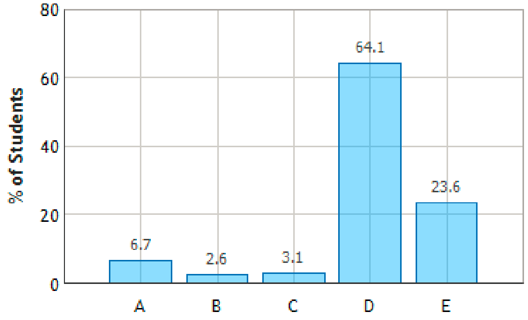
Checkpoint review

A charge of +Q is fixed in space. A second charge of +q was first placed at a distance r_1 away from +Q. Then it was moved along a straight line to a new position at a distance R away from its starting position. The final location of +q is at a distance r_2 from +Q.



What is the change in the potential energy of charge +q during the process?

- ○ kQq/R
- ○ kQqR/r_1^2
- ○ kQqR/r_2^2
- ○ kQq((1/r_2)-(1/r_1))
- ○ kQq((1/r_1)-(1/r_2))



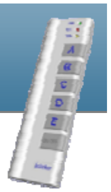
$$U_1 = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r_1} \qquad U_2 = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r_2}$$

$$\Delta U \equiv U_2 - U_1 = \frac{Qq}{4\pi\varepsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

The change in potential energy is final minus initial.

Note: $+q$ moves AWAY from $+Q$.
Its Potential energy MUST DECREASE
$\Delta U < 0$

CheckPoint Review

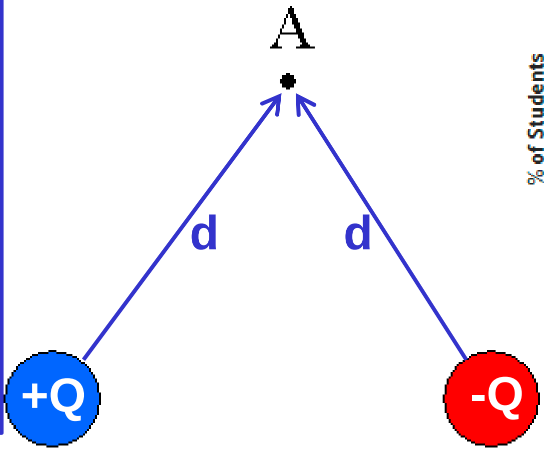


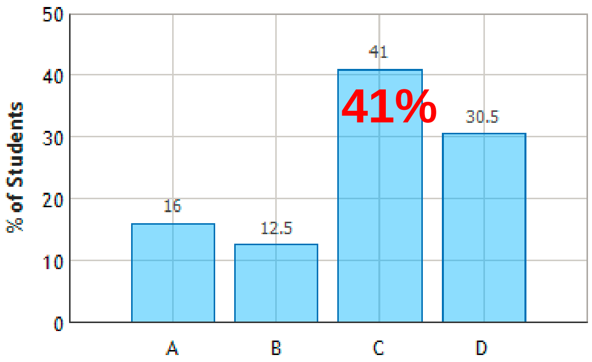
4) Two charges which are equal in magnitude, but opposite in sign are placed at equal distances from point A.

$$U_{Left} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{d}$$

$$U_{Right} = \frac{1}{4\pi\varepsilon_0}\frac{-Qq}{d}$$

They add to 0, no matter what q is



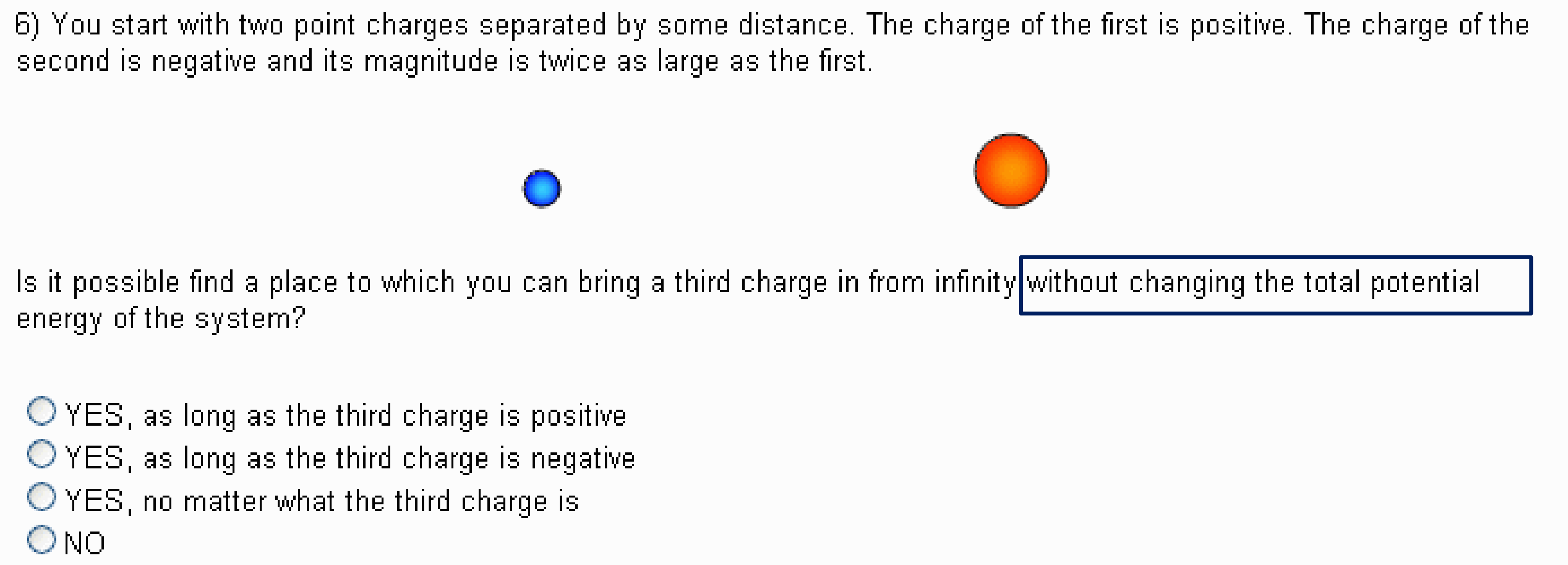


If a third charge is added to the system and placed at point A, how does the electric potential energy of the charge collection change?

○ increases ○ decreases ○ doesn't change ○ the answer depends on the sign of the third charge

A B C D **(and, it doesn't depend on the sign)**

CheckPoint Review



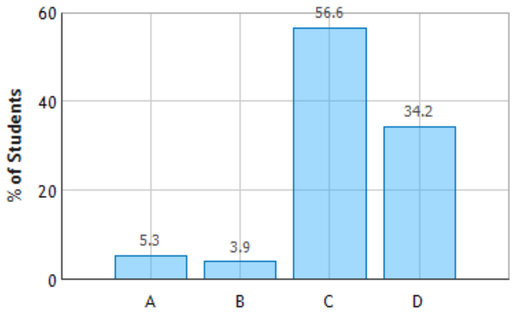
6) You start with two point charges separated by some distance. The charge of the first is positive. The charge of the second is negative and its magnitude is twice as large as the first.

Is it possible find a place to which you can bring a third charge in from infinity without changing the total potential energy of the system?

○ YES, as long as the third charge is positive
○ YES, as long as the third charge is negative
○ YES, no matter what the third charge is
○ NO

50% success. LET'S DO THE CALCULATION!

Electric Potential Energy of a System of Point Charges, II: Question 1 (N = 76)

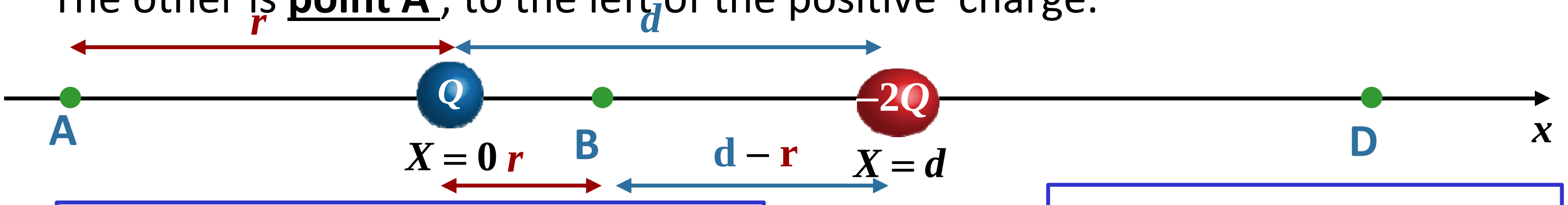


Calculations

There are **two solutions along line that connects the charges**

One is the **<u>point B</u>** located between the charges.

The other is **<u>point A</u>** , to the left of the positive charge.



$$\Delta U = +\frac{1}{4\pi\varepsilon_0}\frac{Qq}{r} - \frac{1}{4\pi\varepsilon_0}\frac{2Qq}{r+d}$$

Set $\Delta U = 0$

$$\downarrow$$

$$\frac{1}{r} = \frac{2}{r+d}$$

$$\boxed{r = d}$$

Set $\Delta U = 0$

$$\frac{1}{r} = \frac{2}{d-r}$$

$$\downarrow$$

$$2r = d - r$$

$$\boxed{r = \frac{d}{3}}$$

Makes Sense! Q **is twice as far from** $-2q$ **as it is from** $+q$

Reminder

- **Thursday at 5 pm. Exam 1**
- **Covers material through Gauss' Law**