INFORMATION ASYMMETRIES IN COMMON VALUE AUCTIONS WITH DISCRETE SIGNALS

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Microsoft Research, NYC

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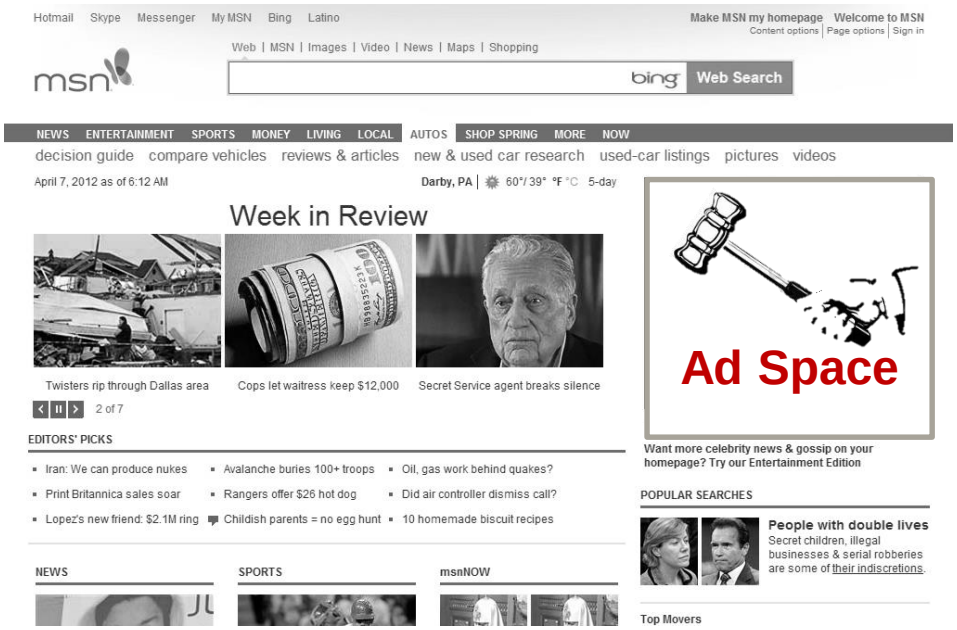
Eva Tardos, Cornell University

Ad Auctions and Information Asymmetries

Web Impression



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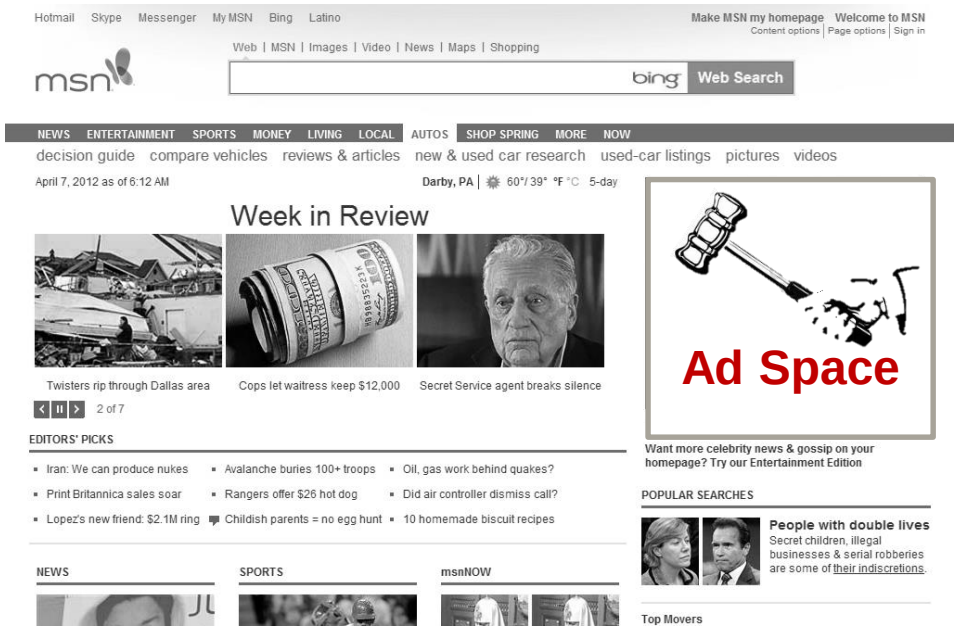
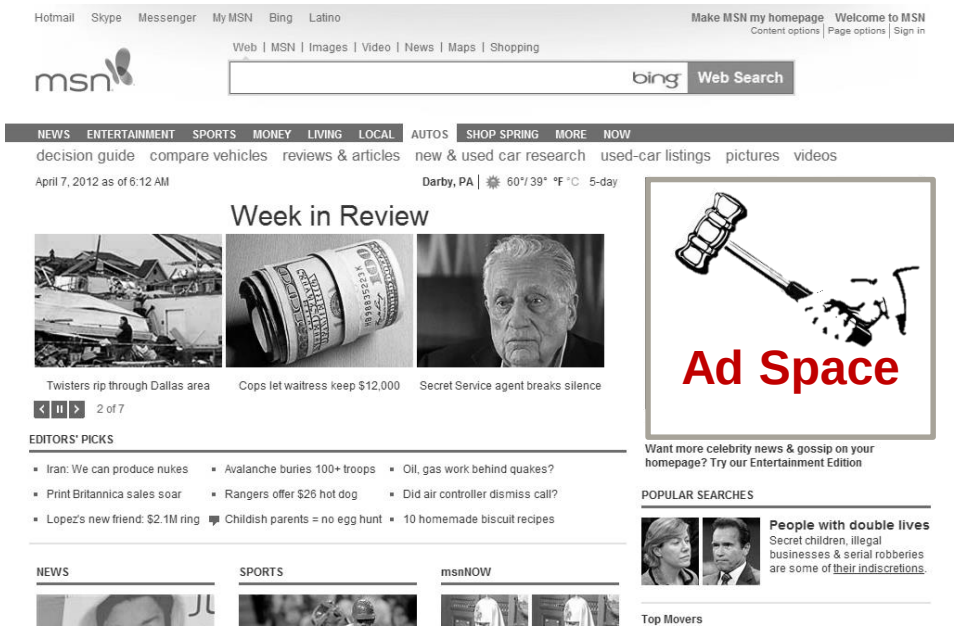


Amazon Cookie x

Kayak Cookie y

Ad Auctions and Information Asymmetries

Web Impression of Unknown Common Value V



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Common Value Single-Item Auction

Item of Unknown Common Value V

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- **Signals Drawn from Arbitrary Asymmetric and Correlated Distribution (with full support)**

Traditional Applications

Oil Lease for land
with unknown
common value V

Main Questions

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- How does extra information affect player utilities and seller's revenue?

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- $\kappa \to 0$: Limit Equilibrium of Second Price Auction (Equilibrium Selection)

Related Work

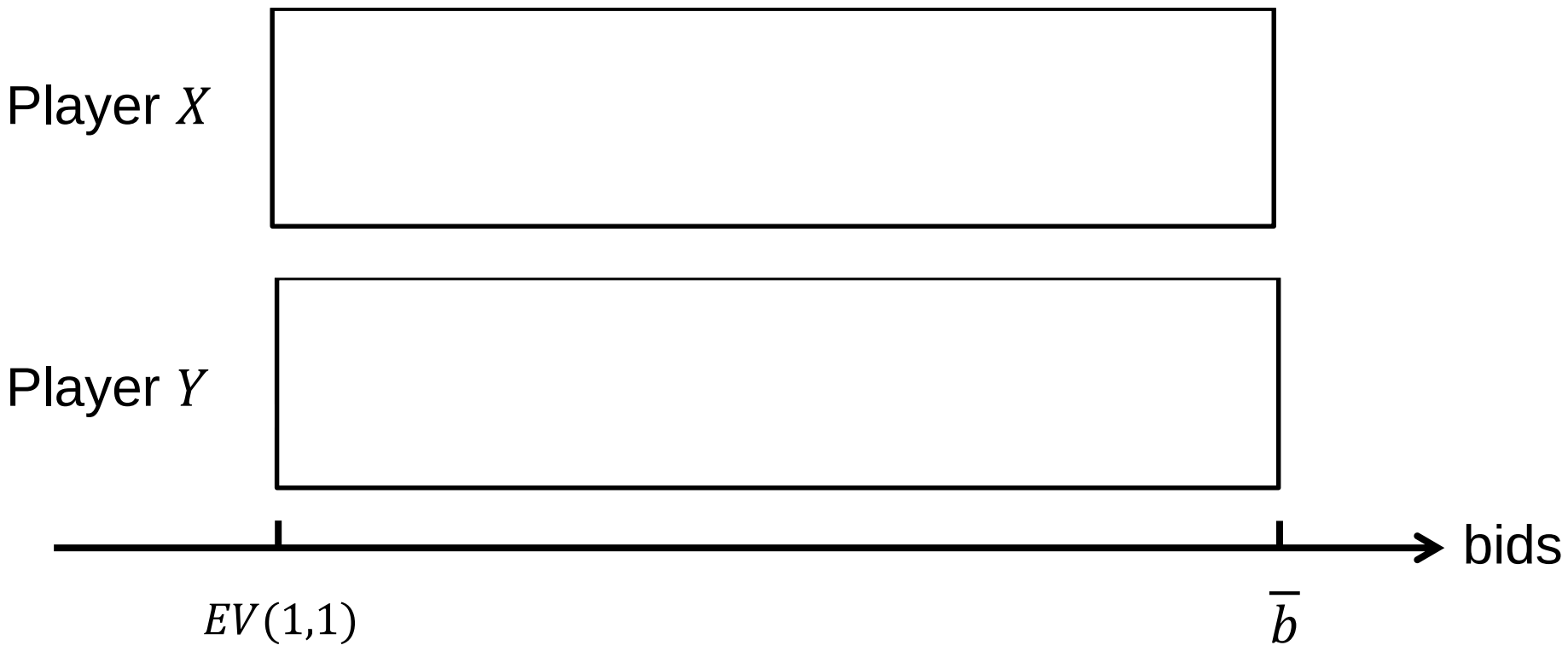
- Value of information in auctions: [Milgrom ’79], [Milgrom/Weber ’82], . . .
- Common-value auctions with binary signals: [Banerjee ’05], [Abraham/Athey/Babaioff/Grubb ’12]
- Continuous values/signals: [Engelbrecht-Wiggans/Milgrom/Weber ’83], . . ., [Parreiras ’06]
- Other common-value models: [Rothkopf ’69], [Reece ’78], [Hausch ’87], [Wang ’91], [Laskowski/Slonim ’99], [Kagel/Levin ’02].
- Value of information: [Lehmann ’88], [Persico ’00], [Athey/Levin ’01], [Compte/Jehiel’07], [Es˝o/Szentes ’07]

How do bidders behave in equilibrium?

Theorem. There exists a unique equilibrium which is mixed and can be found constructively.

How do bidders behave in equilibrium?

- Mixed Nash Equilibrium must look as follows:



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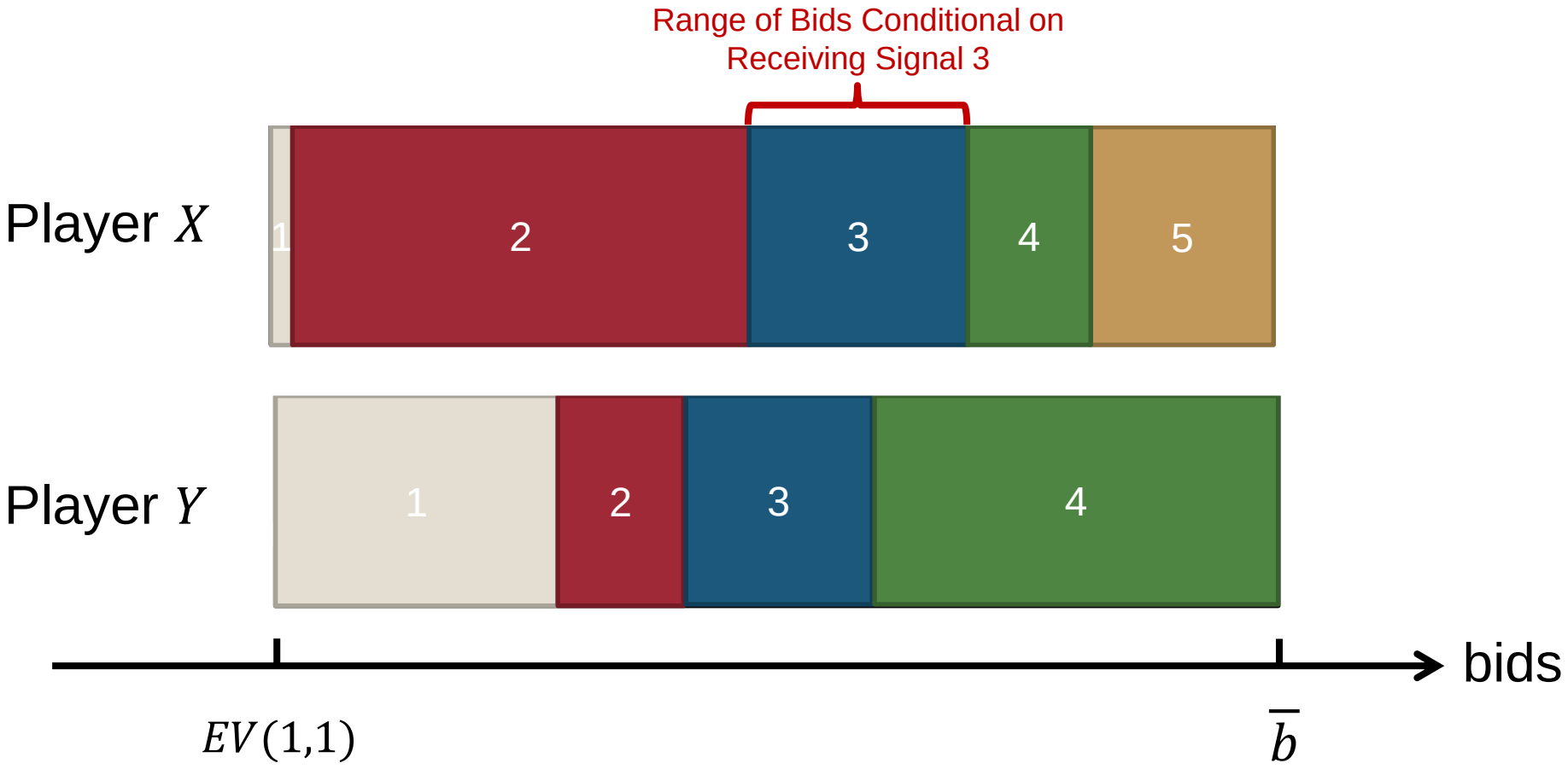

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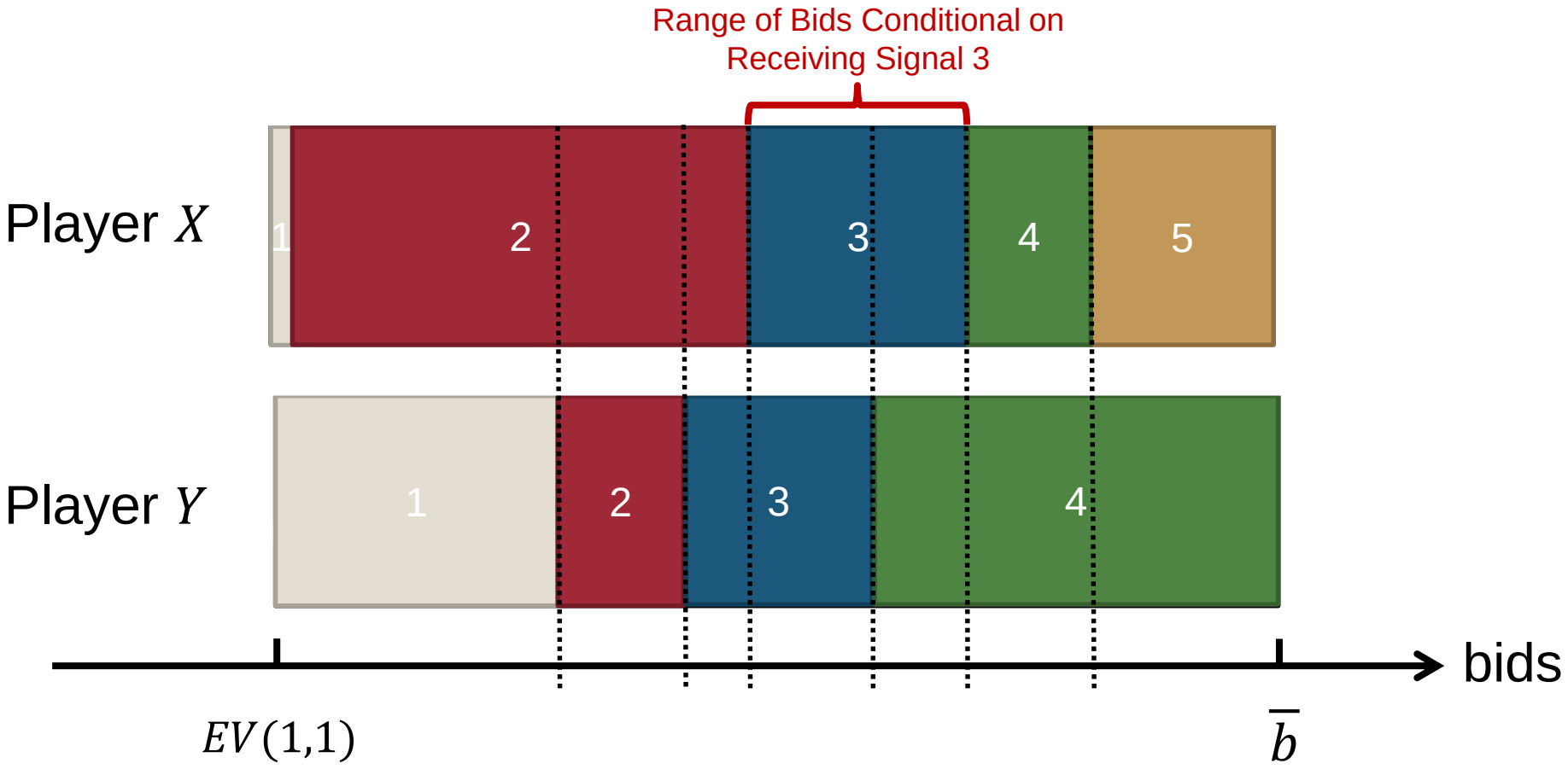

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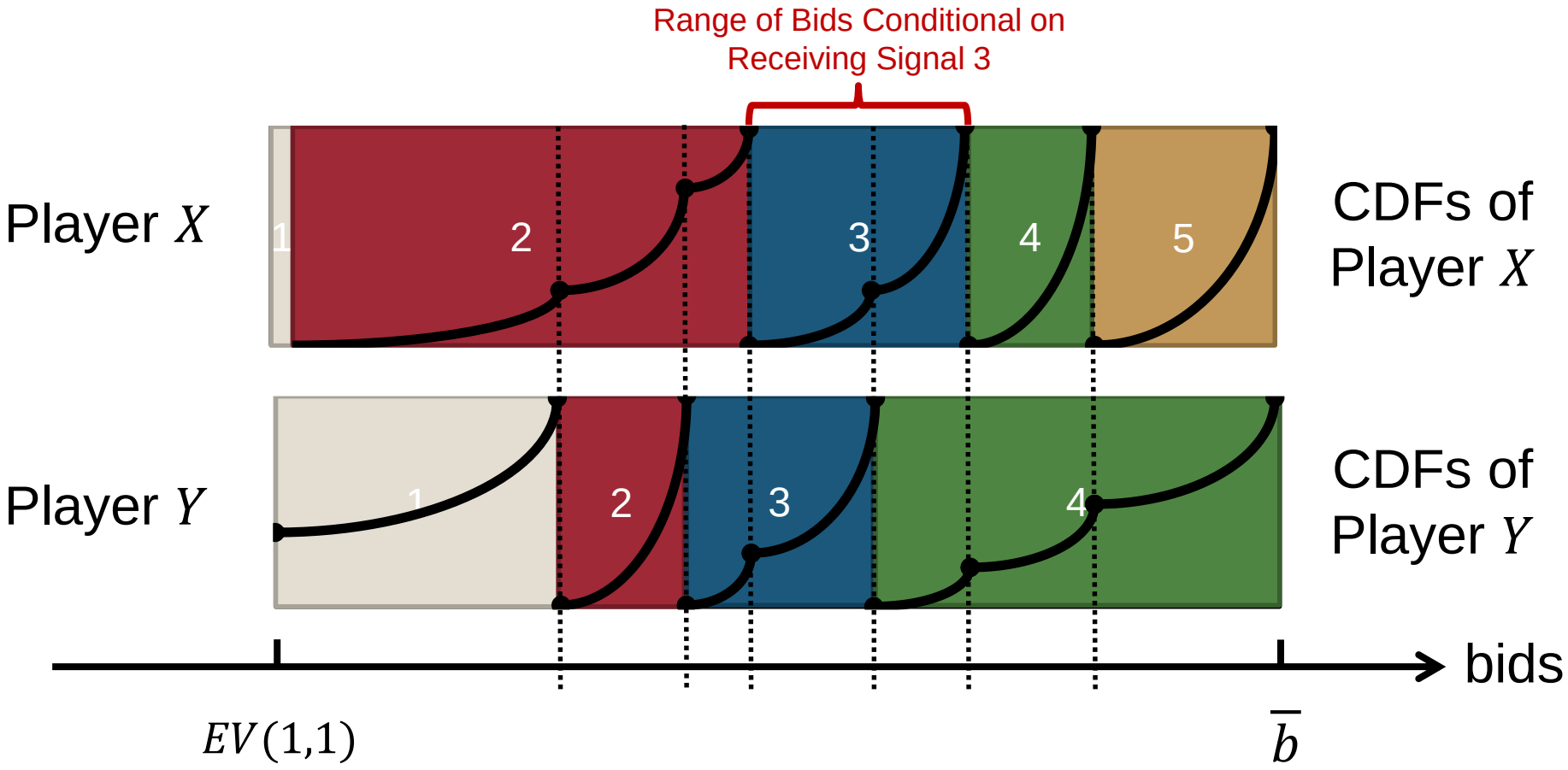

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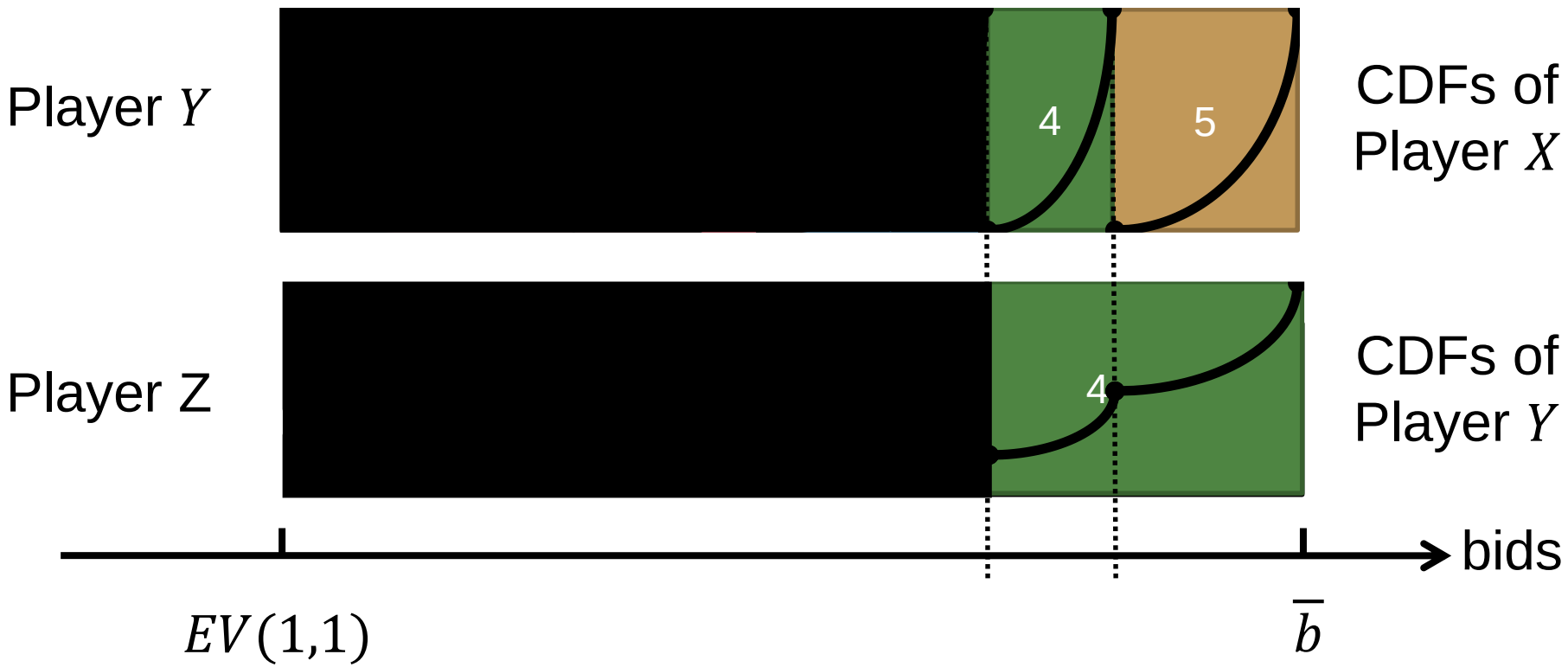

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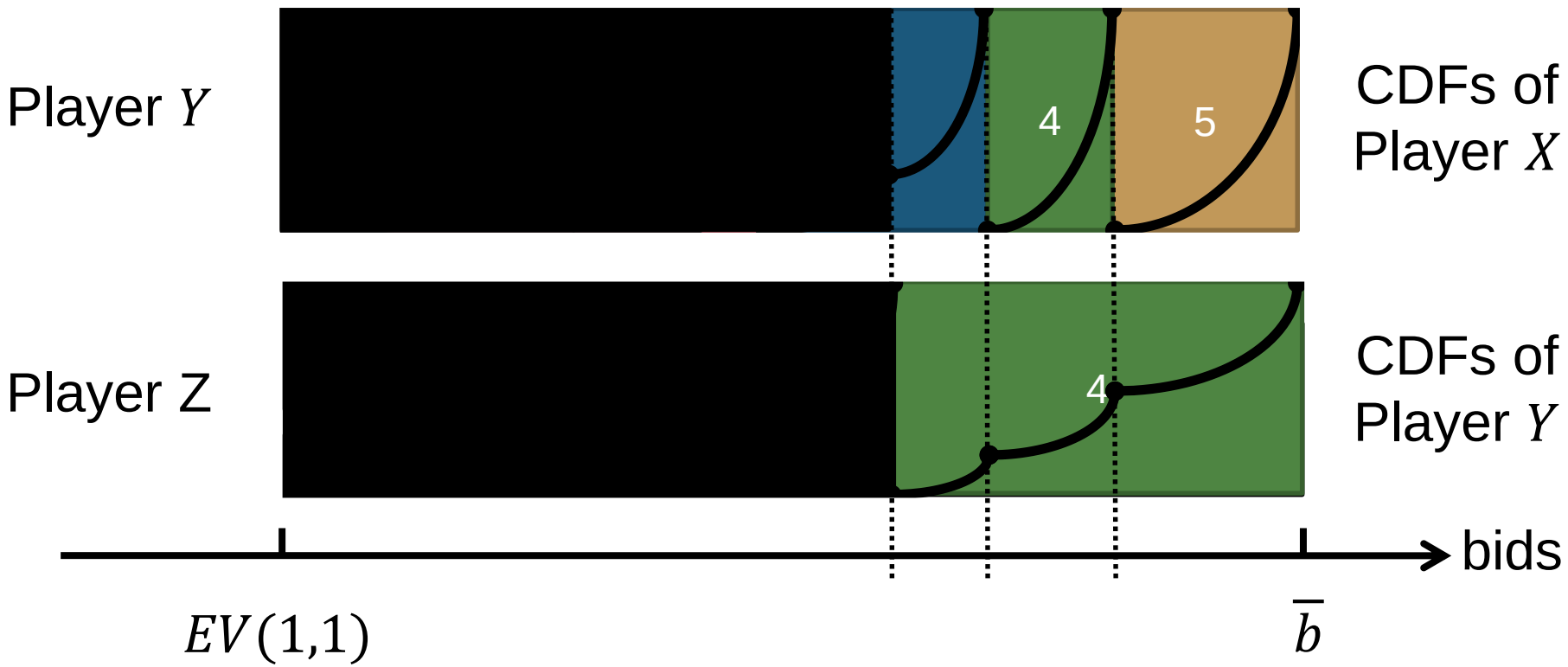

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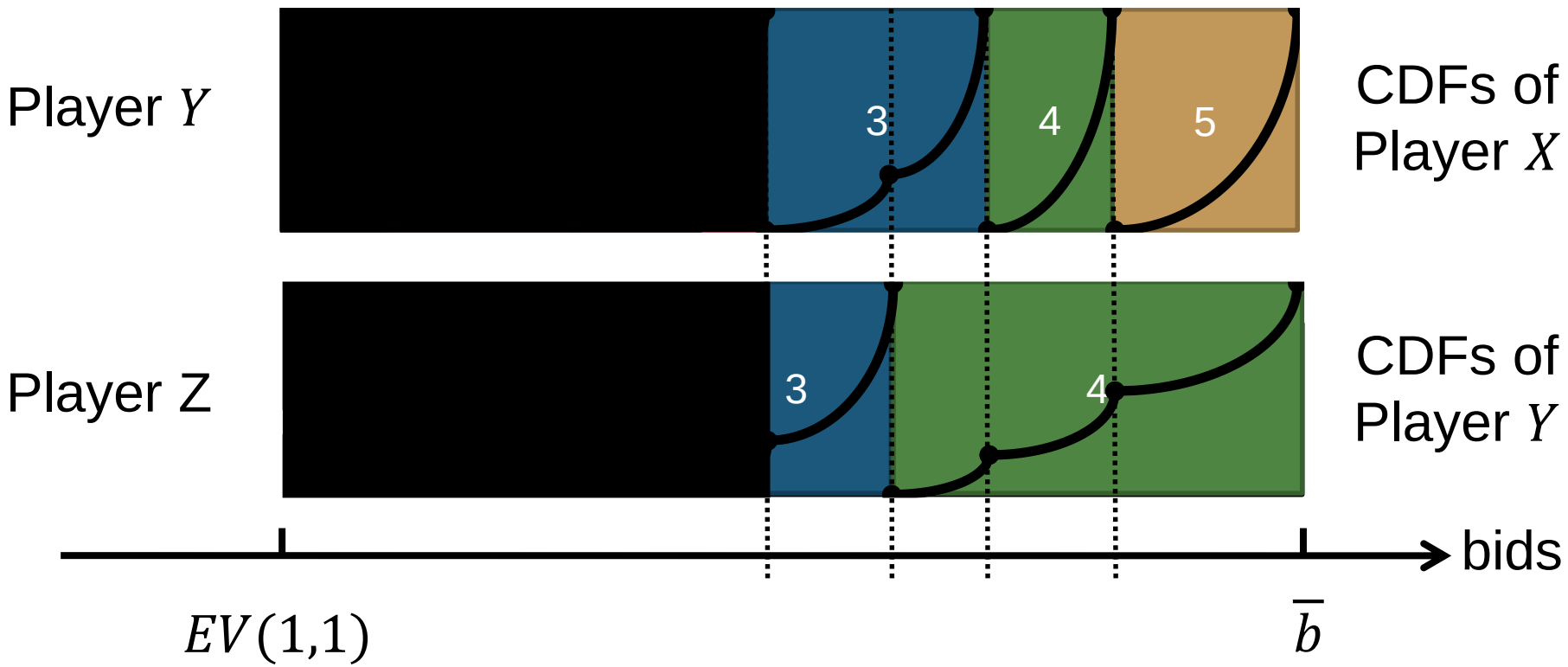

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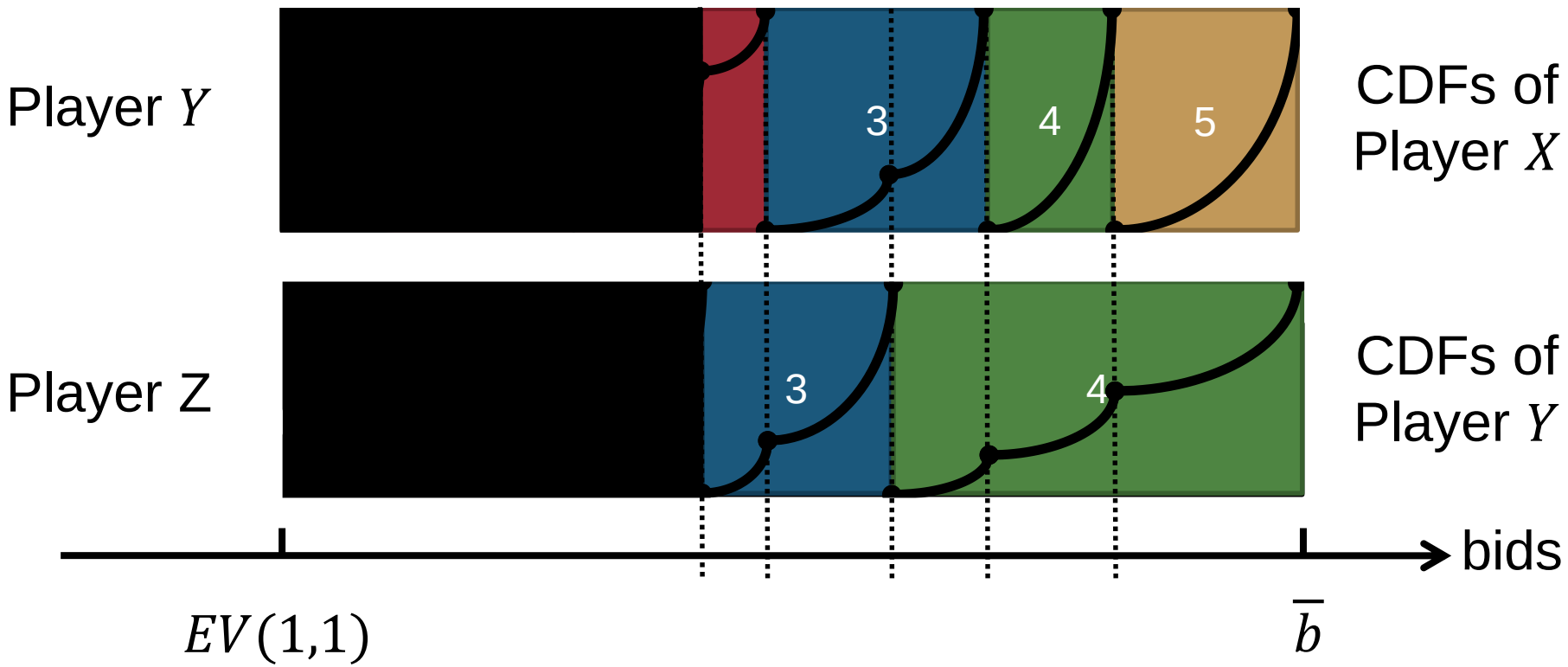

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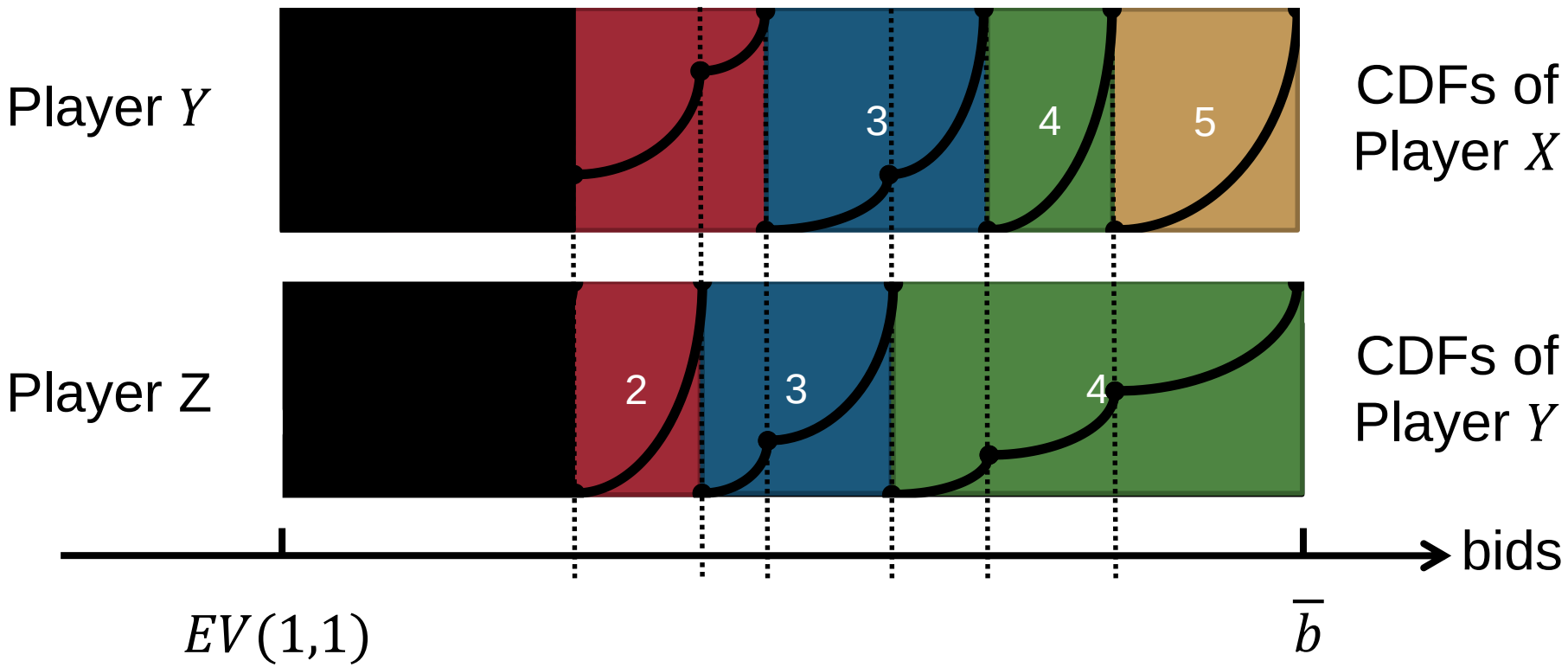

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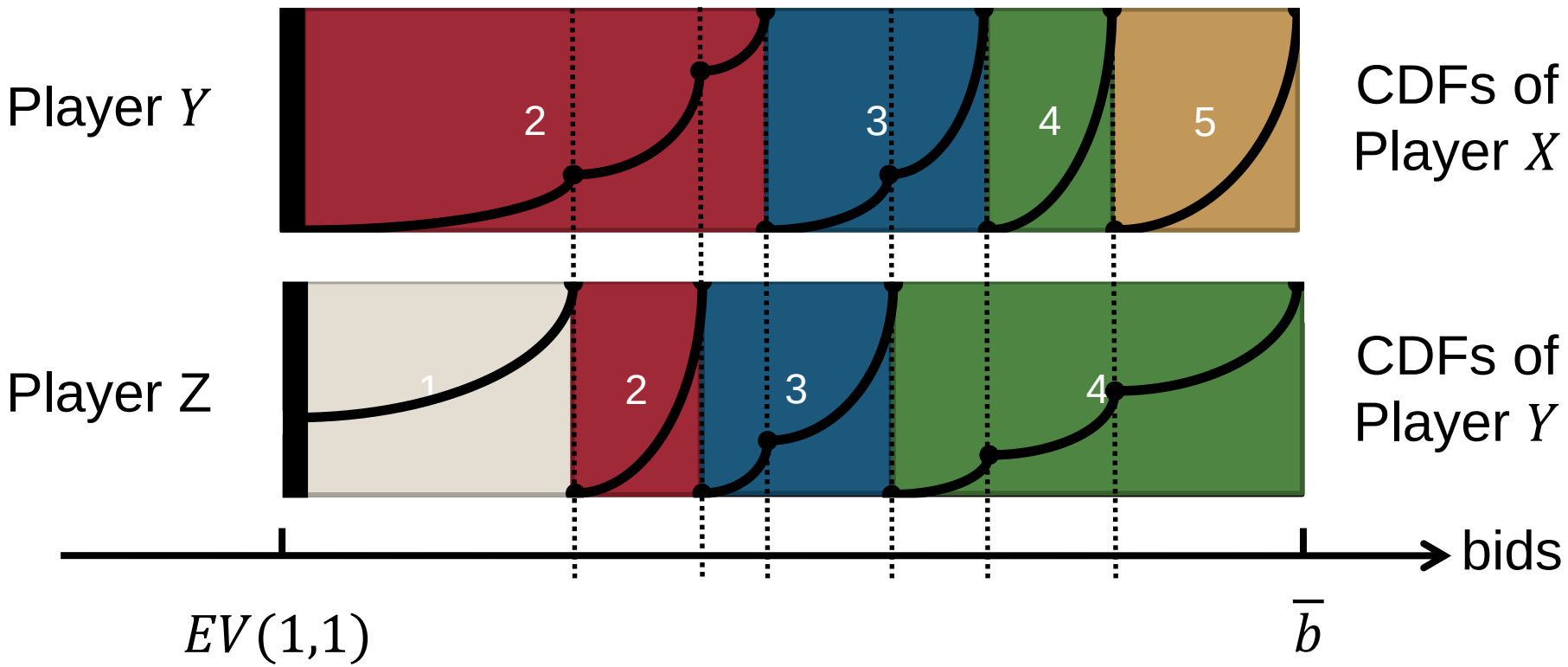

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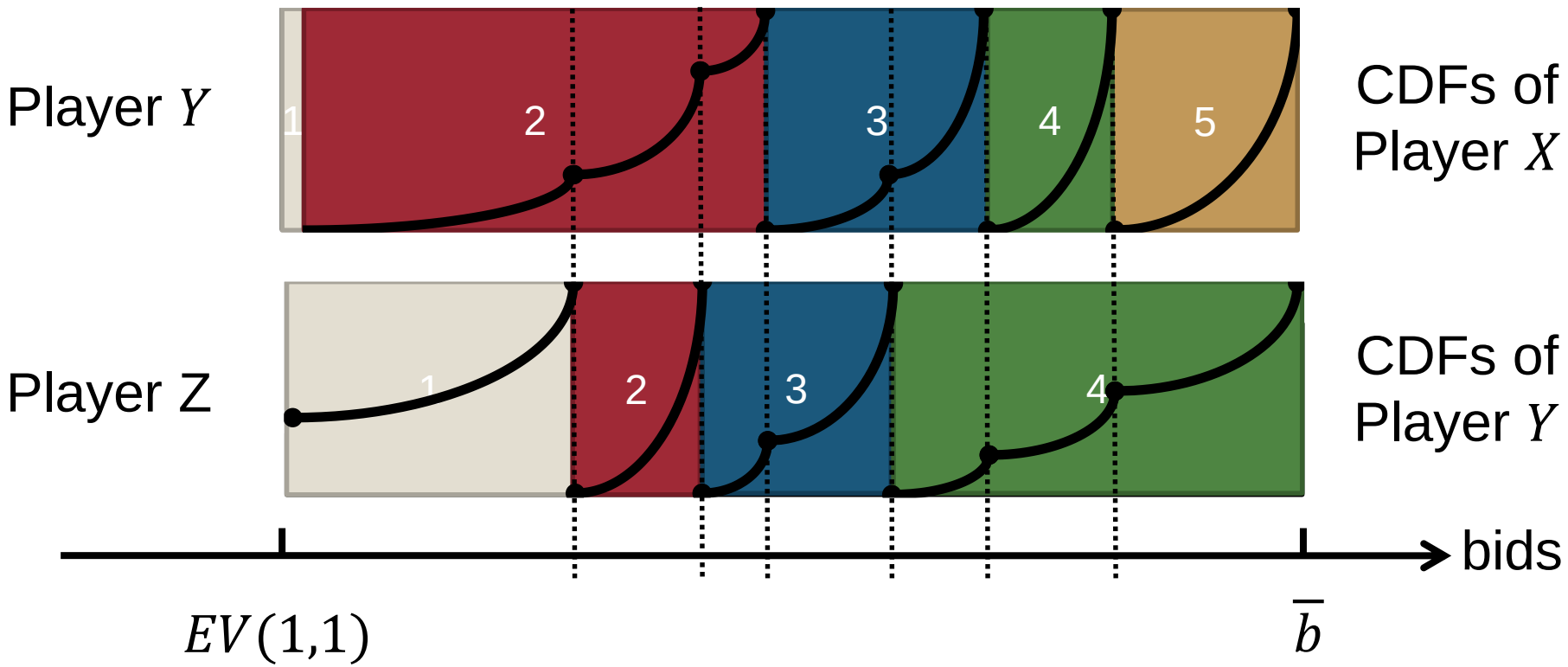
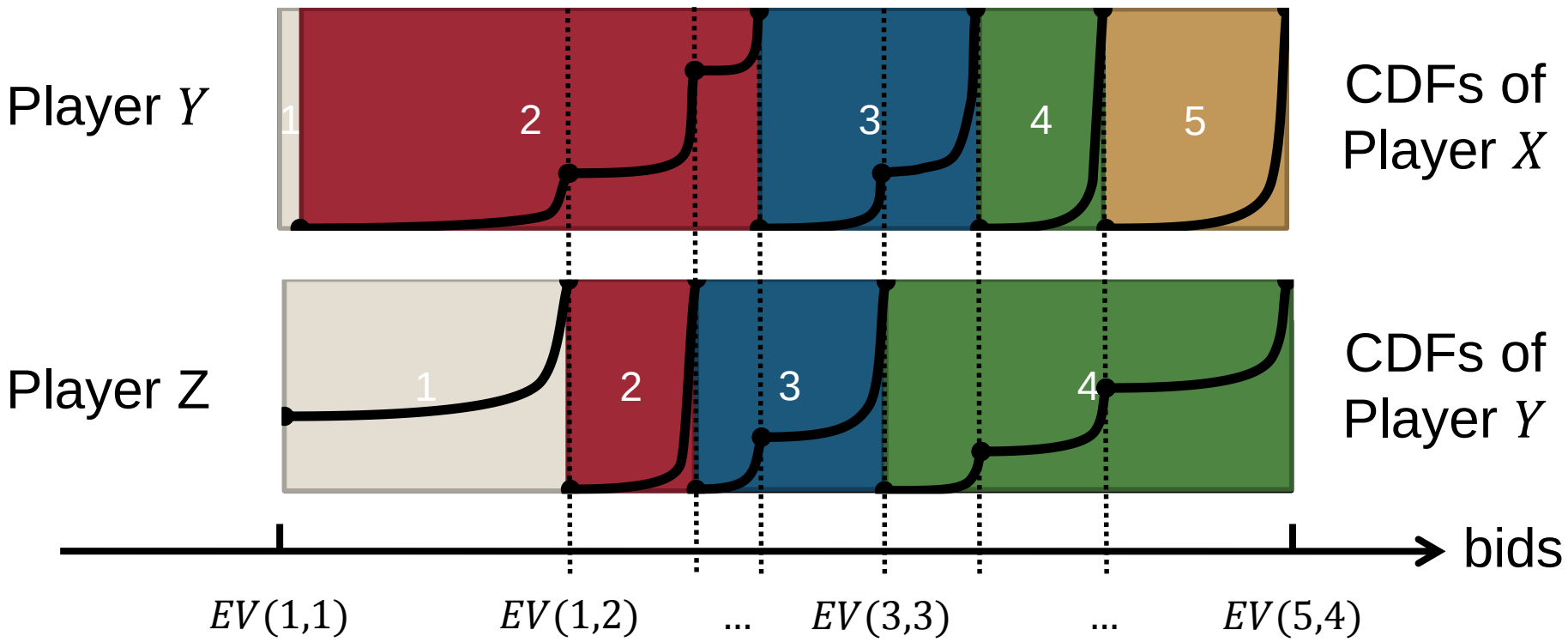


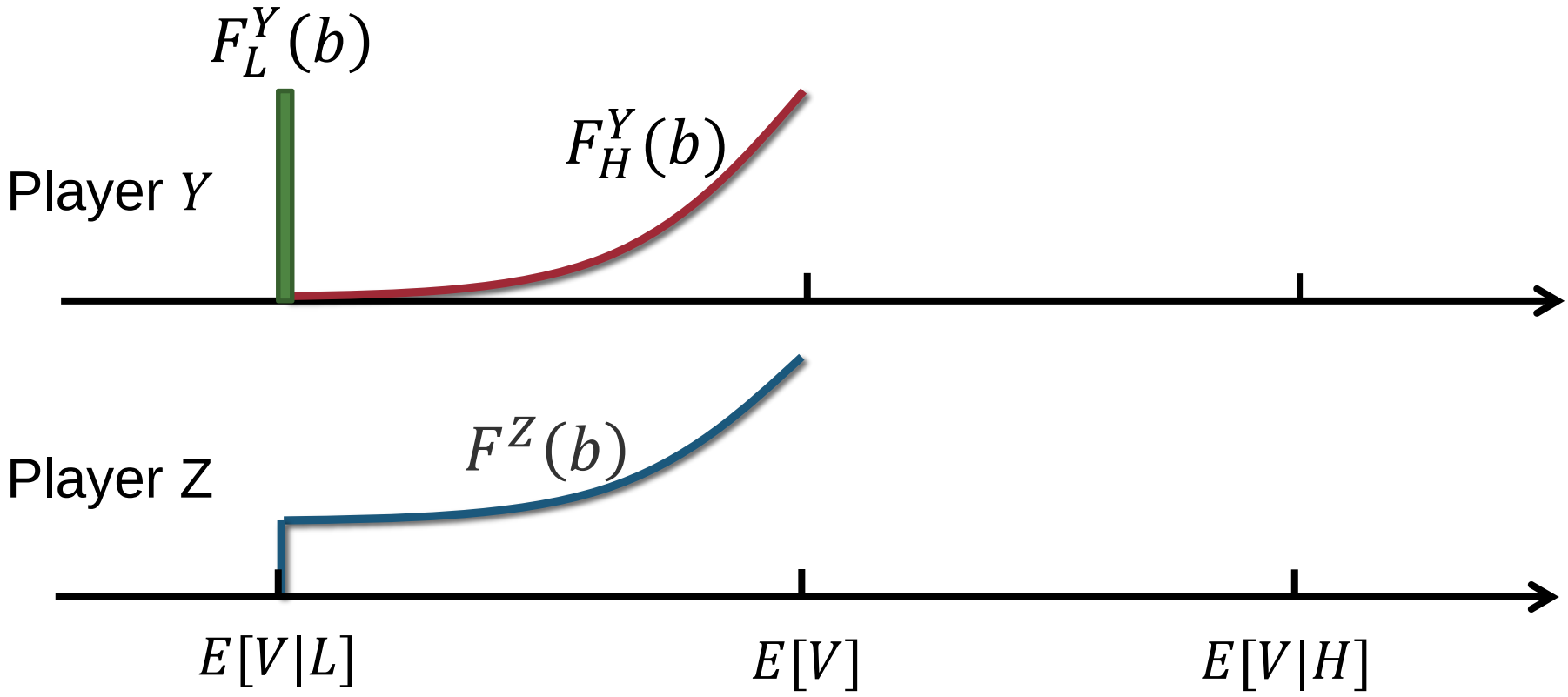
Unique Equilibrium

- As κ approaches 0 (Second Price Auction)



A Simple Example: First Price – Binary Signal

- One player receives a binary signal and the other is uninformed



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$$F^{Z}(b)(E[V|H] - b) = \mathrm{E}[\mathrm{V}|\mathrm{H}] - \mathrm{E}[\mathrm{V}]$$

$$\Pr[H]\, F_{H}^{Y}(b)(E[V|H] - b) + \Pr[L]\,(E[V|L] - b) = 0$$

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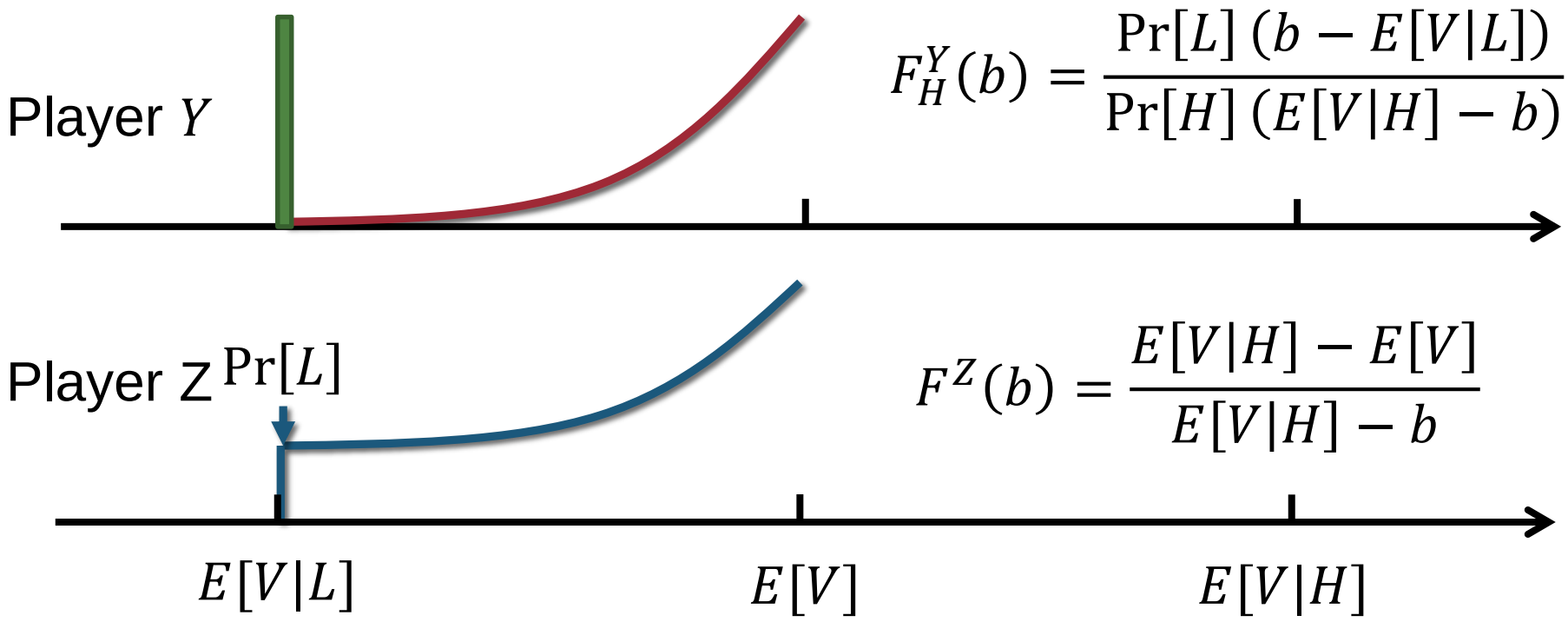

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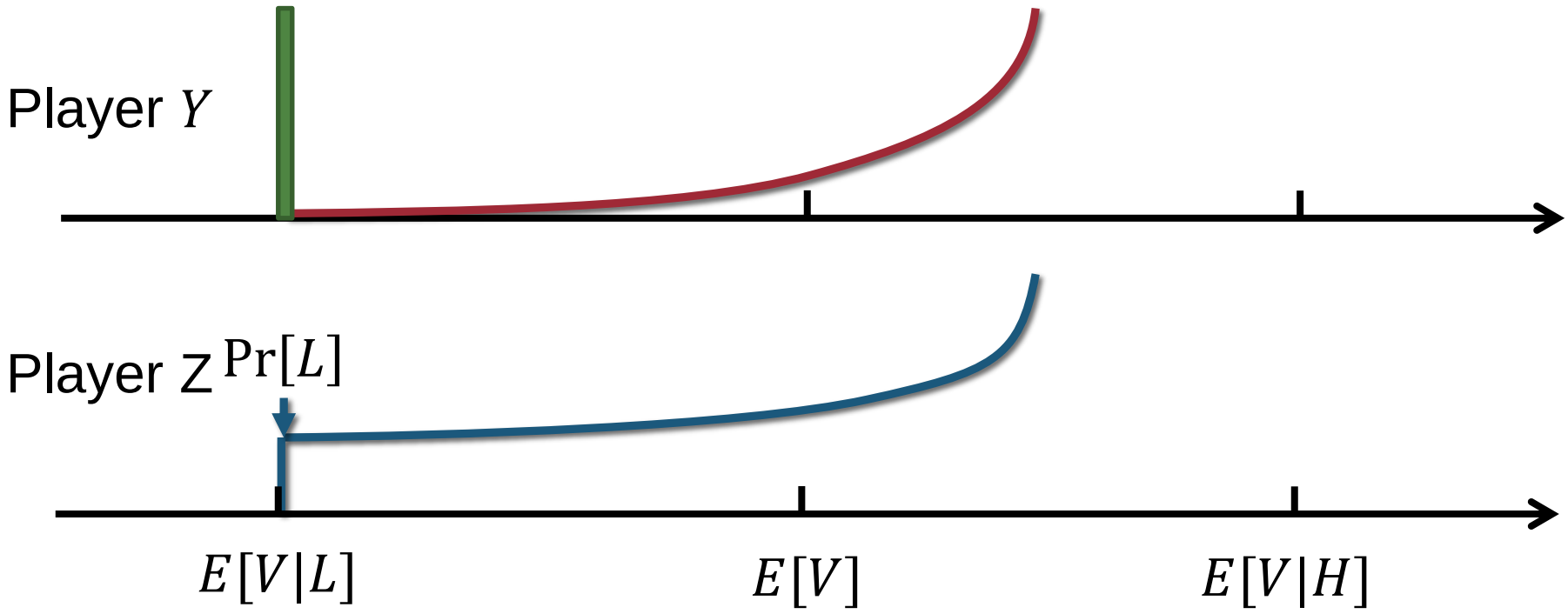

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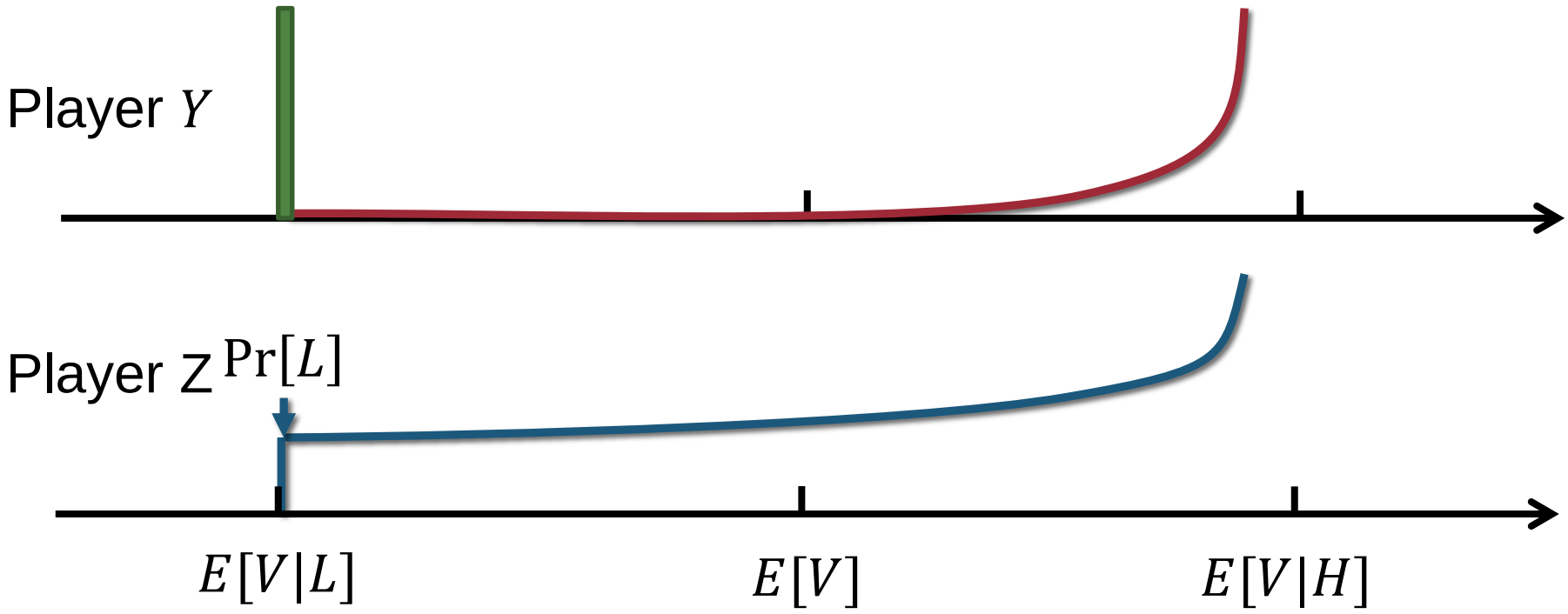
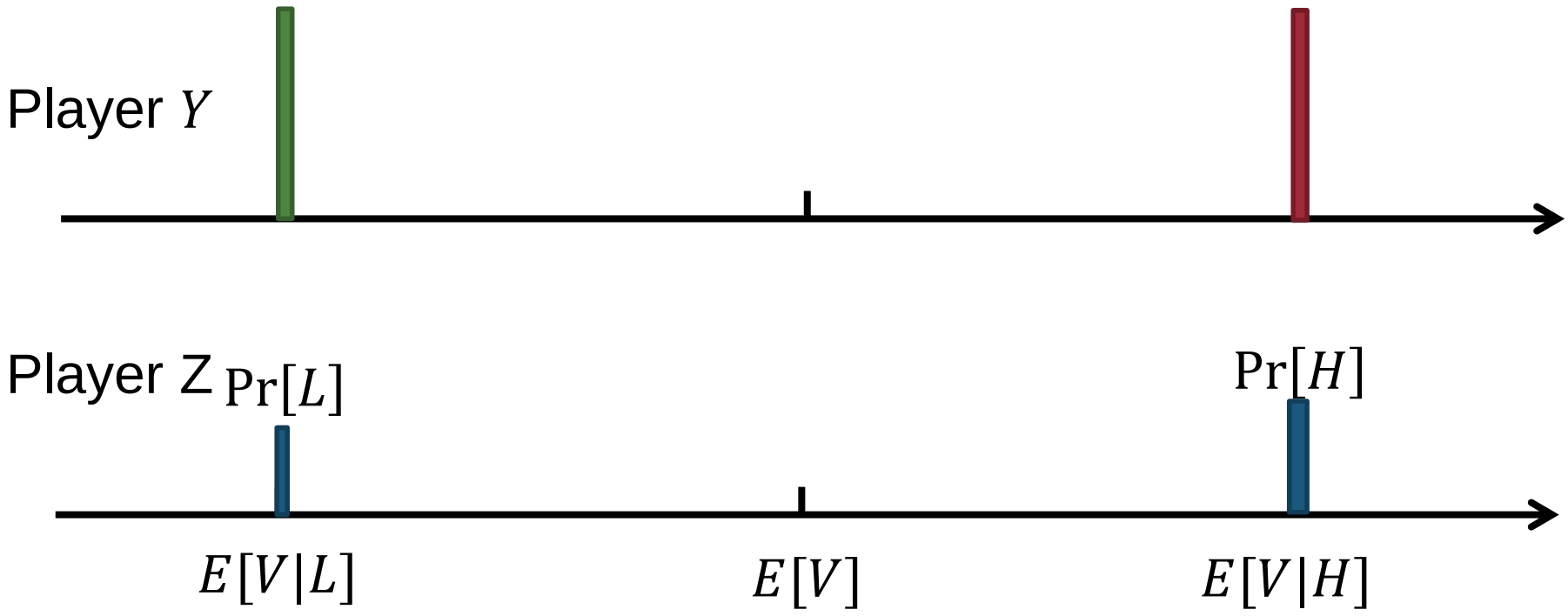


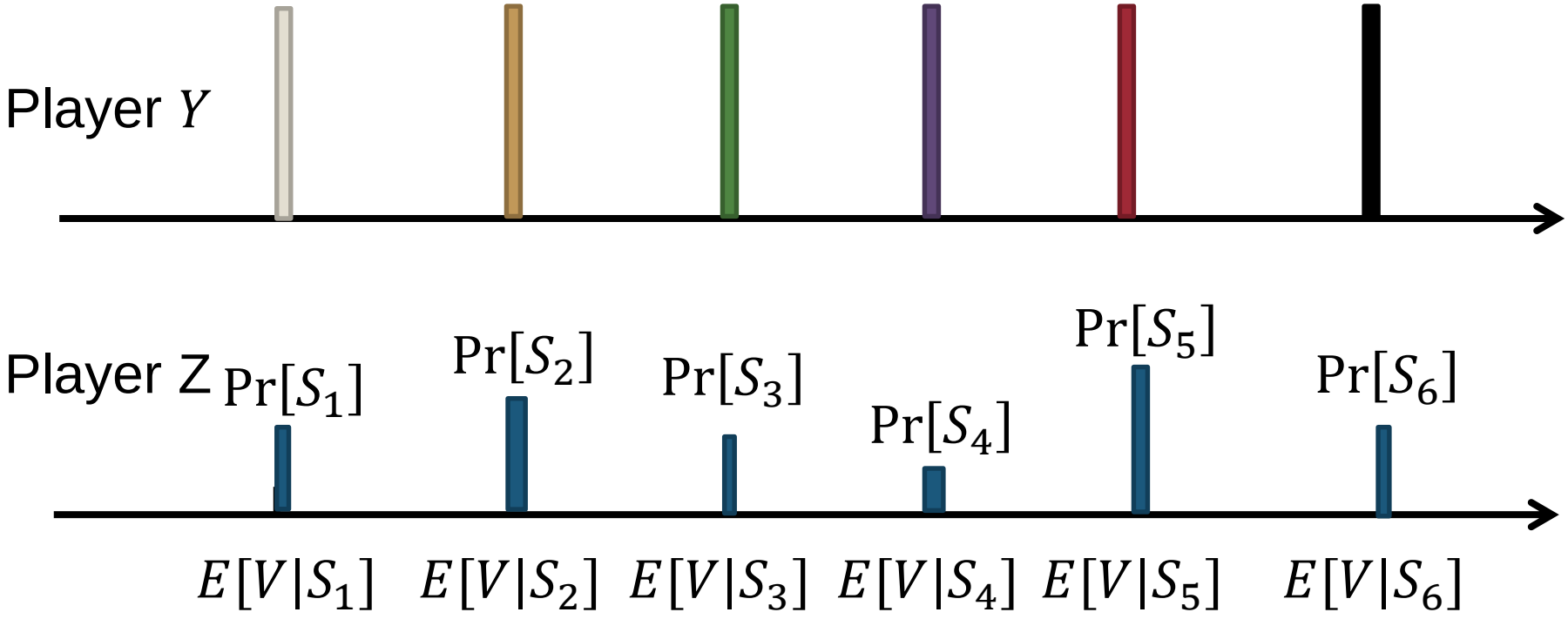
Second Price Selection – No Revenue Collapse



- Different prediction than the collapsed revenue equilibrium predicted by tremble-robust equilibrium selection of Abraham et al.

Only one informed bidder

- Informed Bidder bids “truthfully”
- Uninformed Bidder simulates informed bidder’s bid
- First and Second Price: Revenue Equivalent



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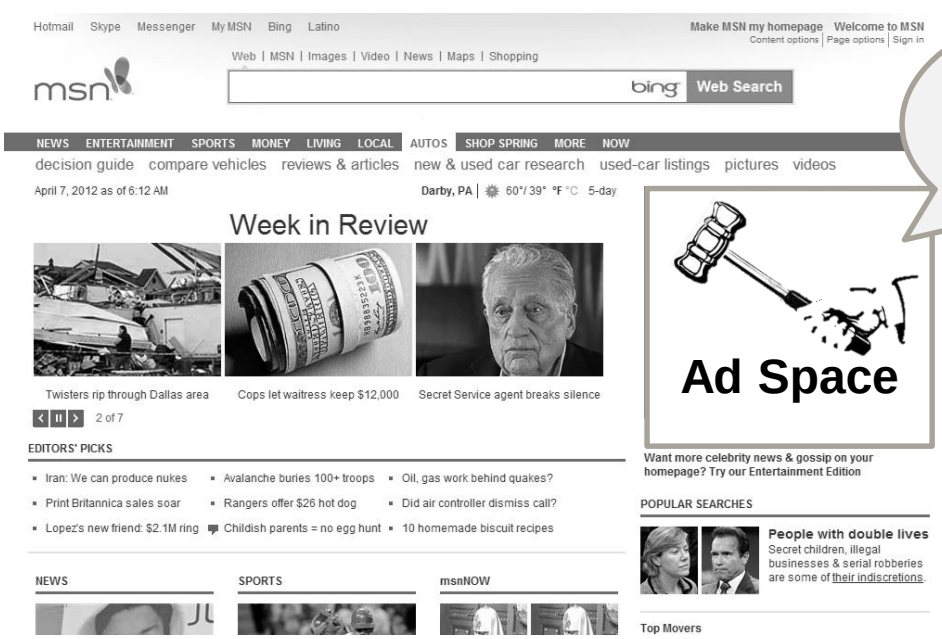
  - Limit Equilibrium of Second Price Selected, has highest revenue among hybrid auctions

Should seller reveal his private signals?

Web Site Visitor of Unknown Common Value V



Amazon Cookie x

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Failure of the Linkage Principle

- **Linkage Principle [Milgrom-Weber'82]:** In common value settings, the more information you link to the price of the winning bidder the higher the revenue.
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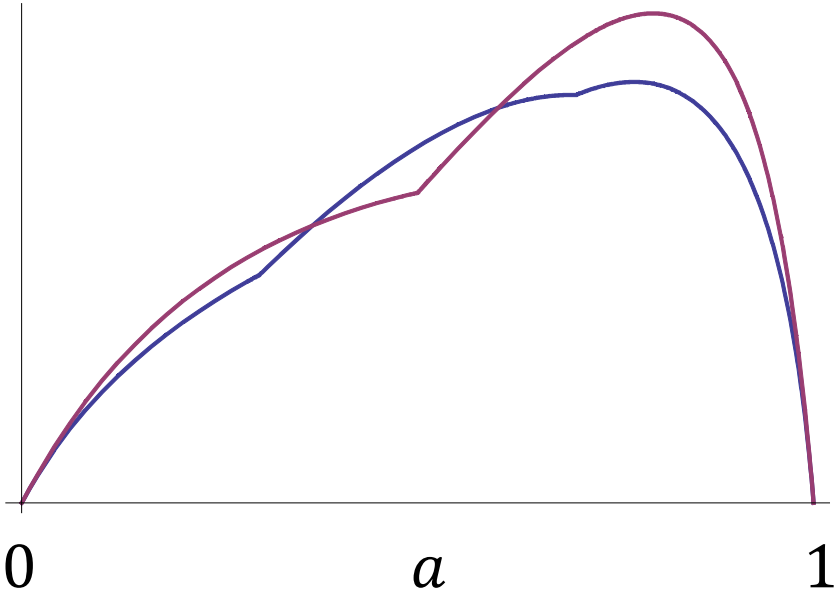
- **Our Result:** Fails when bidders have asymmetric information!
- **Implication:** Revealing policy not necessarily optimal in a market with information asymmetry!
  - Breaks even in first price auction when each bidder and the auctioneer have binary signals of different accuracy

Failure of the Linkage Principle

- First Price Auction
- Value either 0 or 1, a prior is 1 with prob. a
- Player Y gets a binary signal that is correct with p_Y
- Player Z gets a binary signal that is correct with p_Z
- Seller has a signal that is correct with q



$U_Y + U_Z$: without revelation

$U_Y + U_Z$: with revelation

$p_Y = 0.9, p_Z = 0.75, q = 0.7$

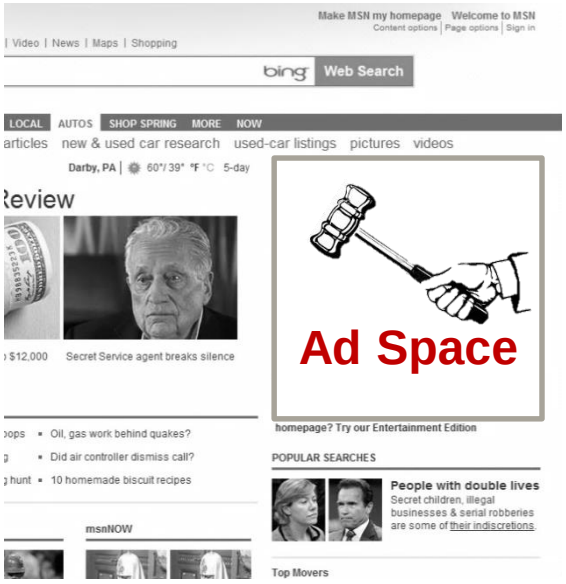
How does extra information affect player utilities?

Third Party Information Sellers

bluekai

exelate

Buy Access to Extra Cookies



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  - E.g. both bidders might strictly prefer that a specific bidder receives the extra signal

Recap

- Information Asymmetries in Common Value Auctions
- Unique Equilibrium if winner pays his bid with positive probability
- Failure of the Linkage Principle – Not always optimal for seller to reveal information even in pure common value
- Complete Revenue Ranking

  Limit Equilibrium of Second Price $\geq$ Hybrid $\geq$ First Price
- Extra Information can have positive externalities