Information Loss and Bulk Reconstruction in AdS_3/CFT_2

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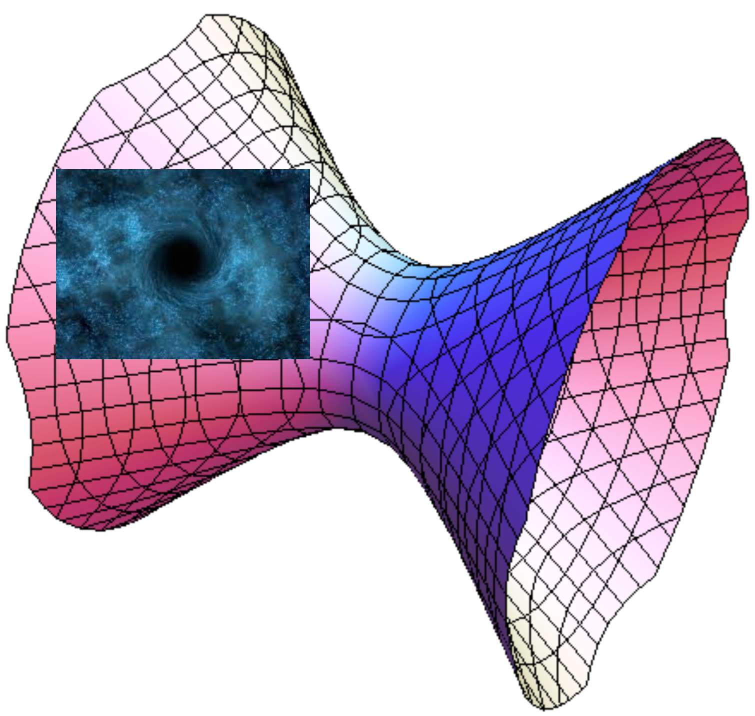
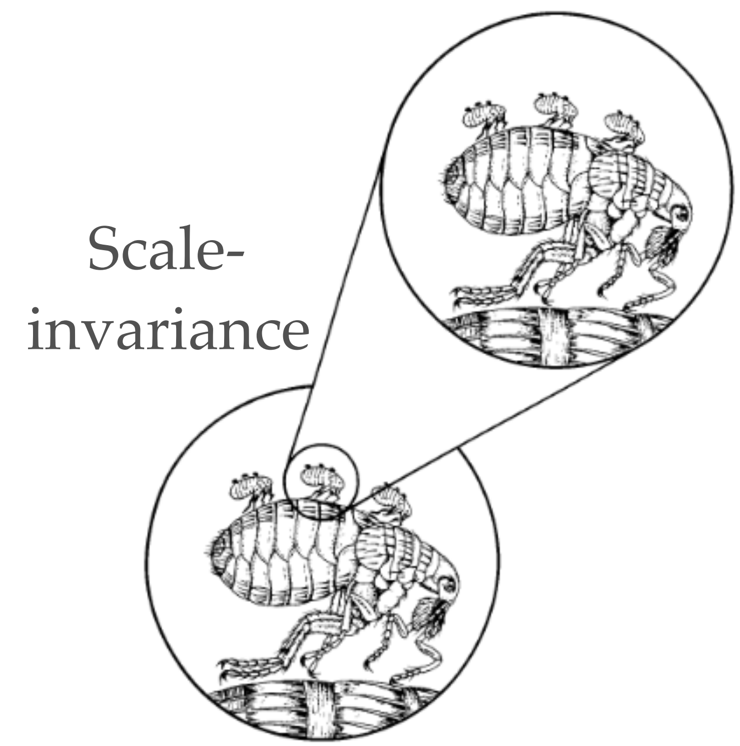
CFTs and Quantum Gravity

Gravity in Anti de Sitter in d+1 dimensions

Conformal Field Theory in d dimensions



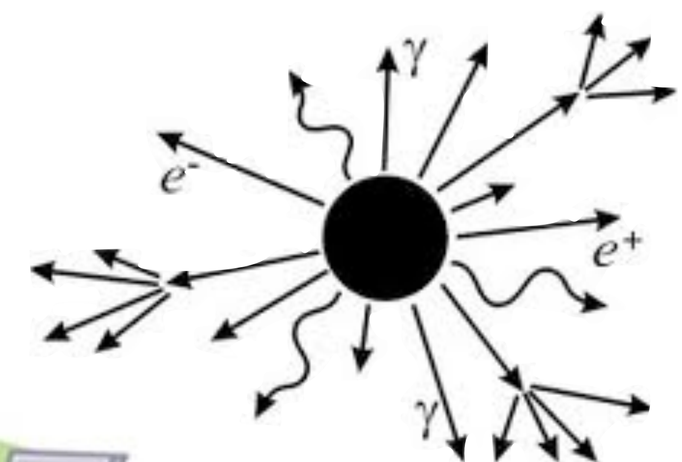
equivalent!



So studying CFTs teaches us about gravity, and vice versa!

CFTs and Quantum Gravity

What can we learn about black hole dynamics?



Hawking radiation: Semi-classical limit says black holes have a temperature.

But if this is exactly true, then information is lost! Not consistent with Quantum Mechanics.

Can we understand "pure" states mimicking a thermal states?

CFTs and Quantum Gravity

Goals:

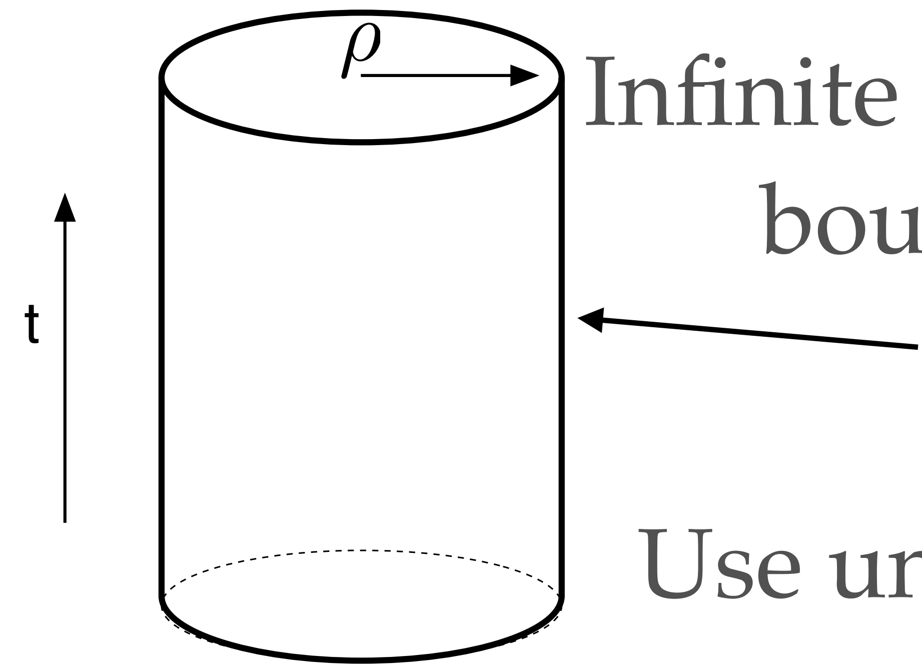
1) Understand how information gets out - need a pure quantum state that looks like a thermal state

2) Harder: understand what black hole looks like just outside horizon, how is this consistent with pure state

Anti-de Sitter

AdS is a very special box.

$$ds^2 = \frac{R_{\rm AdS}^2}{\cos^2\rho}\left(-dt^2 + d\rho^2 + \sin^2\rho d\Omega^2\right)$$



Infinite volume near boundary at $\rho = \frac{\pi}{2}$

Use units of $R_{\rm AdS} = 1$

The isometries of AdS are in one-to-one correspondence with the generators of the conformal group

AdS Energy = CFT Scaling Dimension

$$H_{\text{AdS}} = D_{\text{CFT}}$$

AdS Hamiltonian
Generates time evolution

CFT "Dilatation"
Generates scaling

$\xrightarrow{H_{\text{AdS}}}$

$\xrightarrow{D_{\text{CFT}}}$

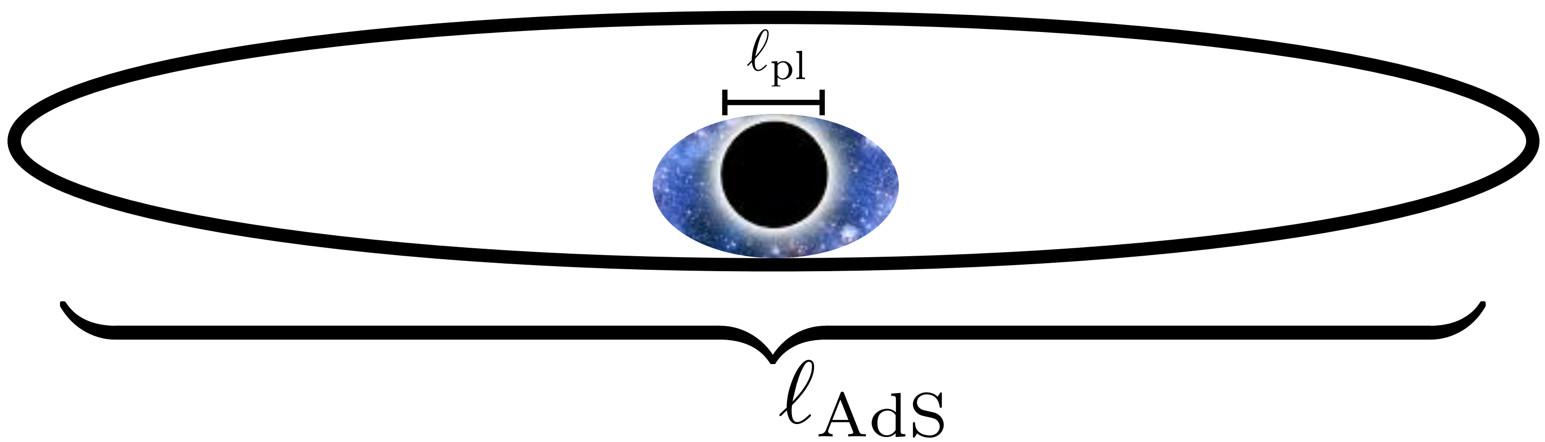
Large C Expansion

Consider large CFT central charge : essentially, large number of degrees of freedom. Like a classical limit.

Brown, Henneaux, '86

$$c = \frac{3\ell_{\rm AdS}}{2G_N}$$

"Semi-classical" gravity limit



"Perturbative" corrections ~ $\frac{1}{c^n}$

"Non-perturbative" corrections ~ e^{-c}

Some Motivation

Want to be able to calculate how information escapes from black hole, hidden in non-perturbative effects

E.g.: - late-time decay of correlators,
- physics near and across horizons.

In AdS_3/CFT_2, many non-perturbative effects are controlled by conformal symmetry; we want to calculate them.

Algebraic Gravity

Power of AdS_3/CFT_2: gravitons are algebraic

AdS		CFT
$h_{\mu\nu}$	$\longleftrightarrow$	$T_{\mu\nu}$
multi-grav		products of T

Virasoro

CFT$_2$ $$T(z) = \sum_n \frac{L_n}{z^{n+2}}$$ generators of Conf. Alg.

Algebra knows about General Relativity!

Focusing on 2d

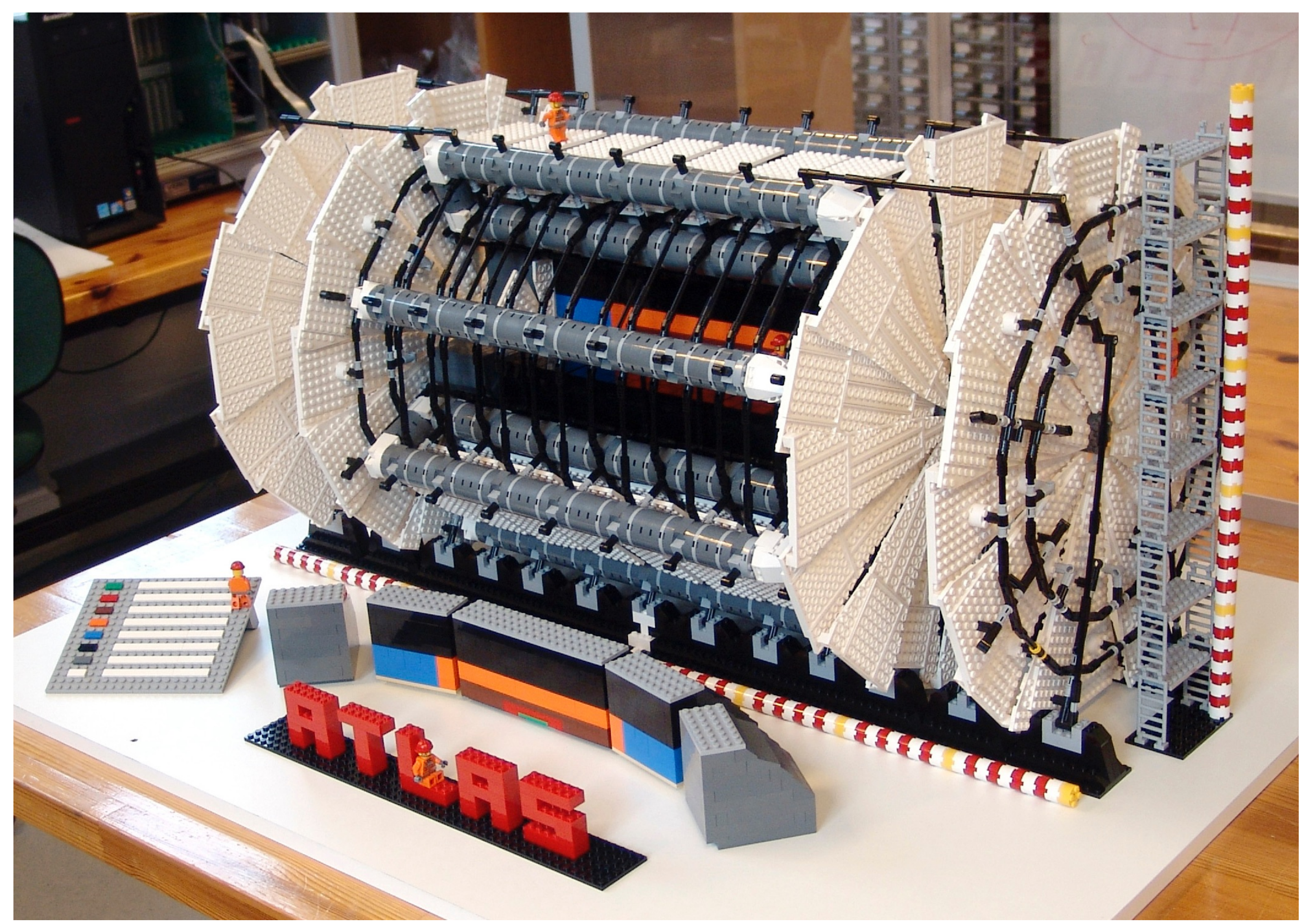
Useful toy model: conformal symmetry is much bigger!

AdS_3: no gravity waves, but there are still black holes.

Some other toy models:

2d QCD at large N: the gluon has no DOFs, and the theory is solvable.

Lego ATLAS



Operators

In conformal theories, a key role is played by “operators”, which can be any local observable

Simple Example: density operator $\rho(x)$

We study correlation functions among operators

$$\langle \rho(x)\rho(y)\rho(z)\rangle$$

$\rho(x)$	$\rho(y)$	$\rho(z)$
0.13	**0.04**	**1.04**
0.22	**0.19**	**0.42**
…	…	…

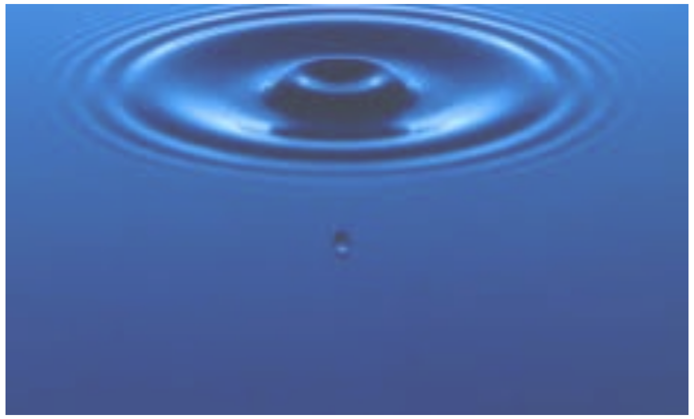
Operators and States

Every operator creates a unique state, and vice versa:

$$\rho(x)|0\rangle \leftrightarrow |\rho\rangle$$

By "measuring" ρ, we perturb the vacuum and put it in a new state.

$$\rho(x)$$



Multiple Operators

Start with insertion of two operators

$Y_{\ell m}$

Decompose into a convenient basis at a fixed radius.

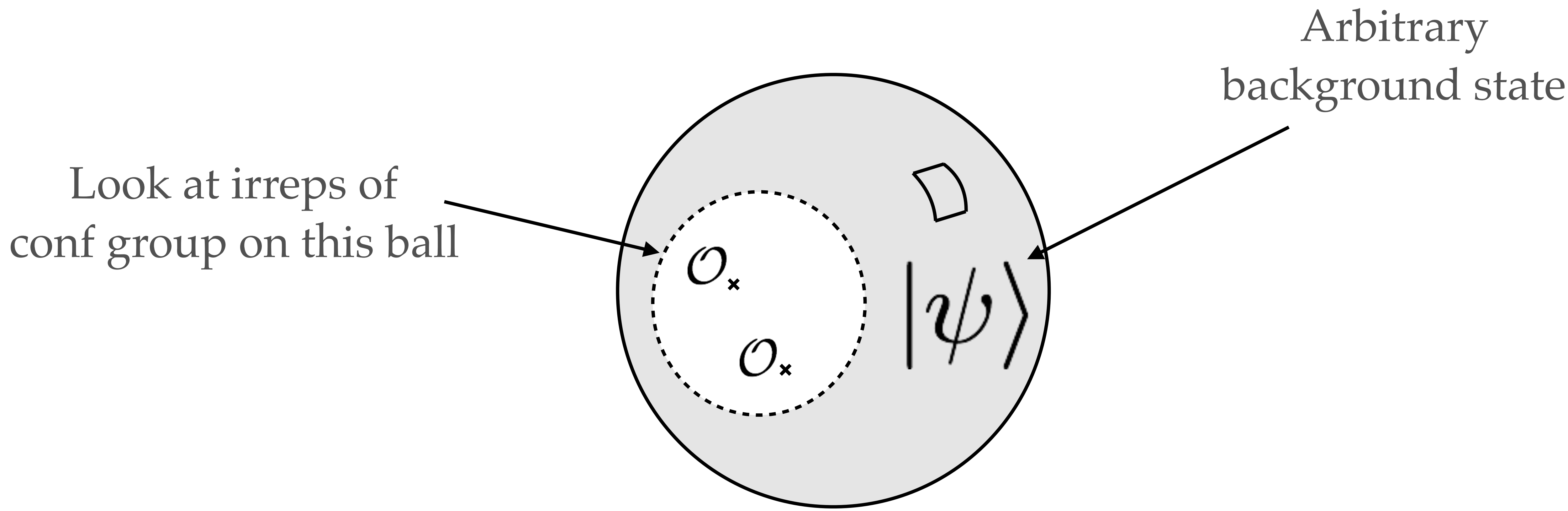
E.g. Spherical harmonics

Quantum: Decompose wavefunction

$$\psi(\theta, \phi) = \sum c_{\ell,m} Y_{\ell,m}(\theta, \phi)$$

Conformal Irreps

"OPE blocks"= contribution to OPE from a single irrep



OPE block is an operator (can be evaluated in any state)

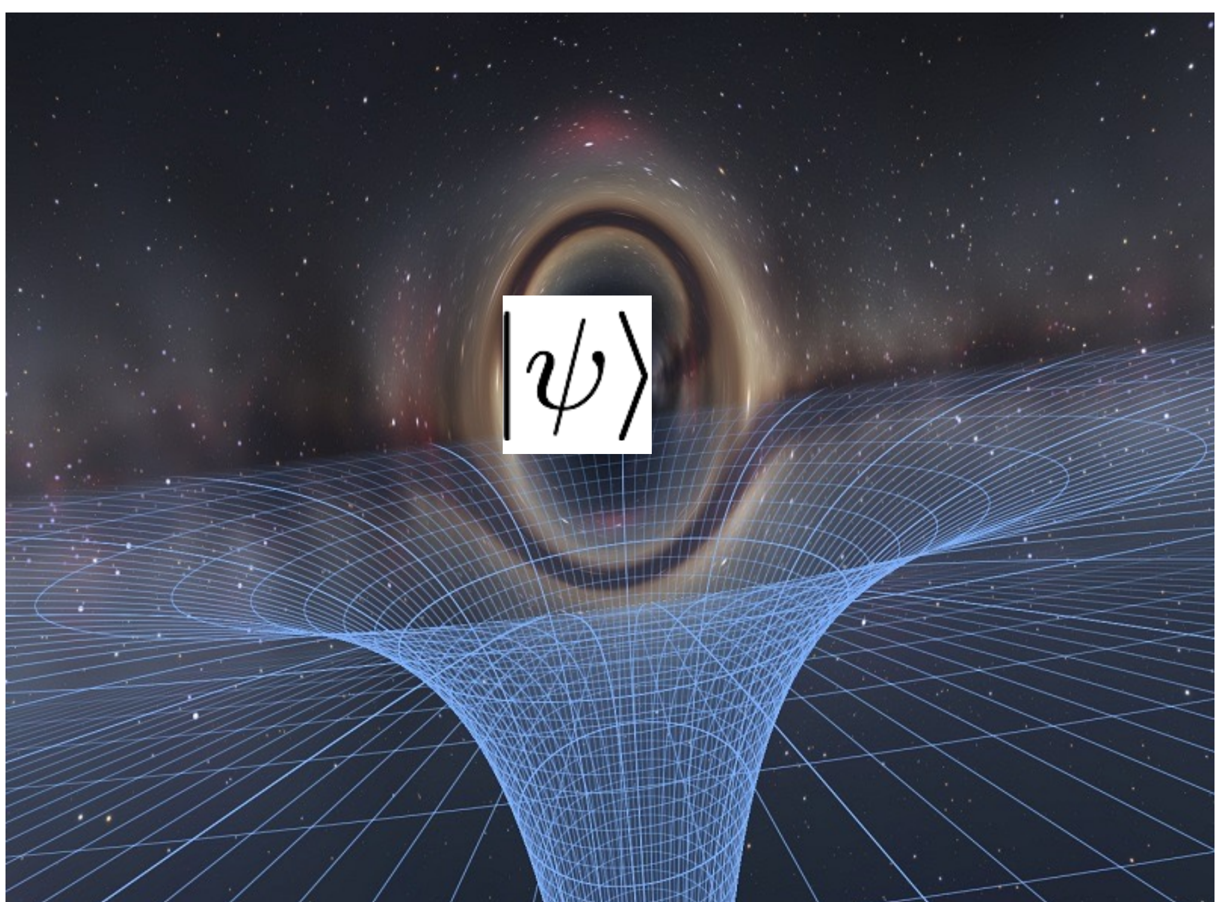
"Vacuum OPE block": $[\mathcal{O}(z_1)\mathcal{O}(z_2)]_{\text{vac}} = \sum_n C_{\mathcal{O}\mathcal{O}T^n}(z_1, z_2) T^n(z_2)$

$\alpha = 1, T, T^2, \ldots$

Large c and "Heavy" states

How do we get interesting effects in gravity at $G_N \to 0$? Keep $G_N M \sim R$ fixed

Heavy state $|\psi\rangle$: $\dfrac{h_\psi}{c}$ fixed, $c \to \infty$



"BH microstate": $G_N \leftrightarrow \dfrac{1}{c}$

$$M_\psi \leftrightarrow h_\psi$$

$$\frac{h_\psi}{c} \leftrightarrow G_N M_\psi \sim R_S$$

Fixed geometry

Large c and "Heavy"

Example: a heavy primary state $|\psi\rangle$

OPE block at large c:

Exactly thermal!

$$\langle\psi|[\mathcal{O}(z_1)\mathcal{O}(z_2)]_{\text{vac}}|\psi\rangle = \left(\frac{1}{\sinh(\pi T_\psi t)}\right)^{h_\mathcal{O}} + \mathcal{O}(\frac{1}{c})$$

~Eigenstate Thermalization

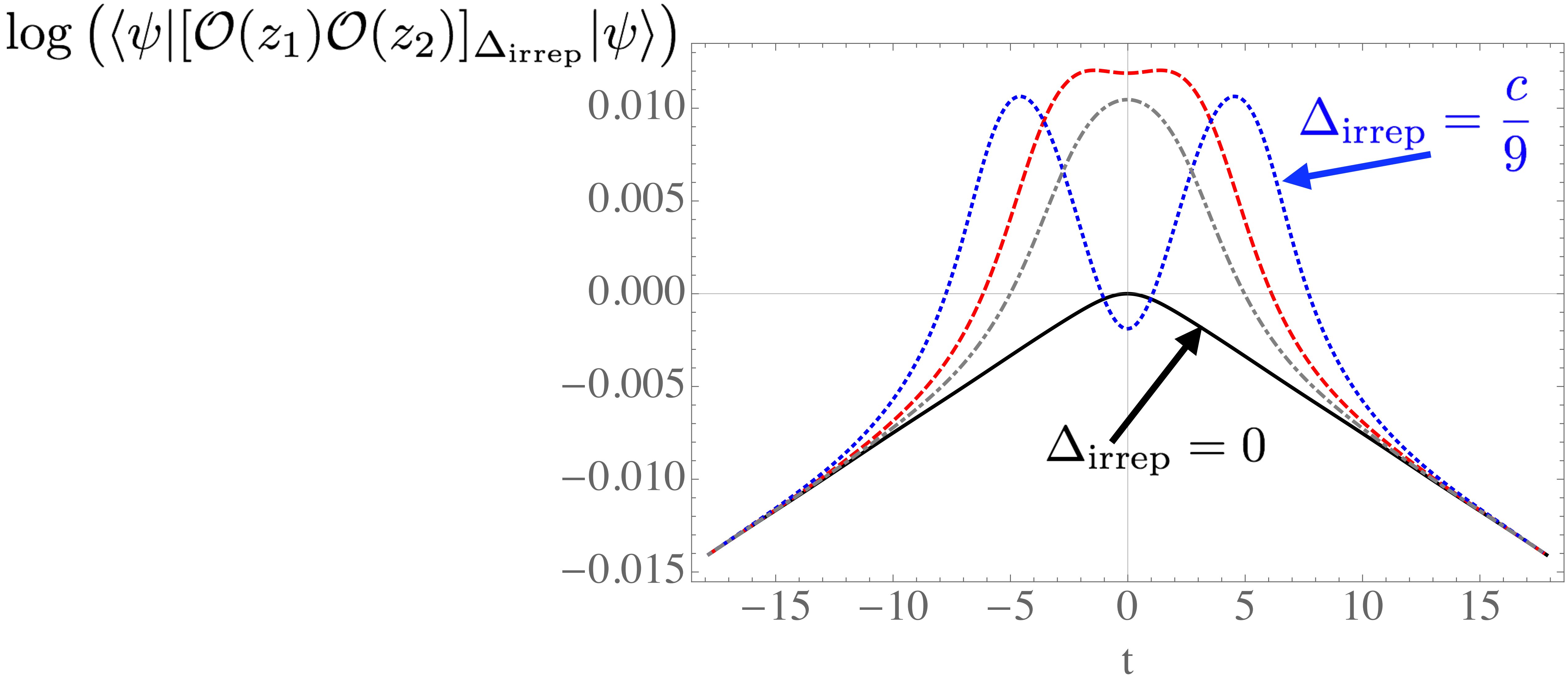
$|\psi\rangle =$ 

$$T_\psi = \frac{1}{2\pi}\sqrt{\frac{24h_\psi}{c} - 1}$$

$$t \to \infty : \left(\frac{1}{\sinh(\pi T_\psi t)}\right)^{h_\mathcal{O}} \sim e^{-\pi h_L T_\psi t}$$

Info loss at large c

All blocks decay semiclassically



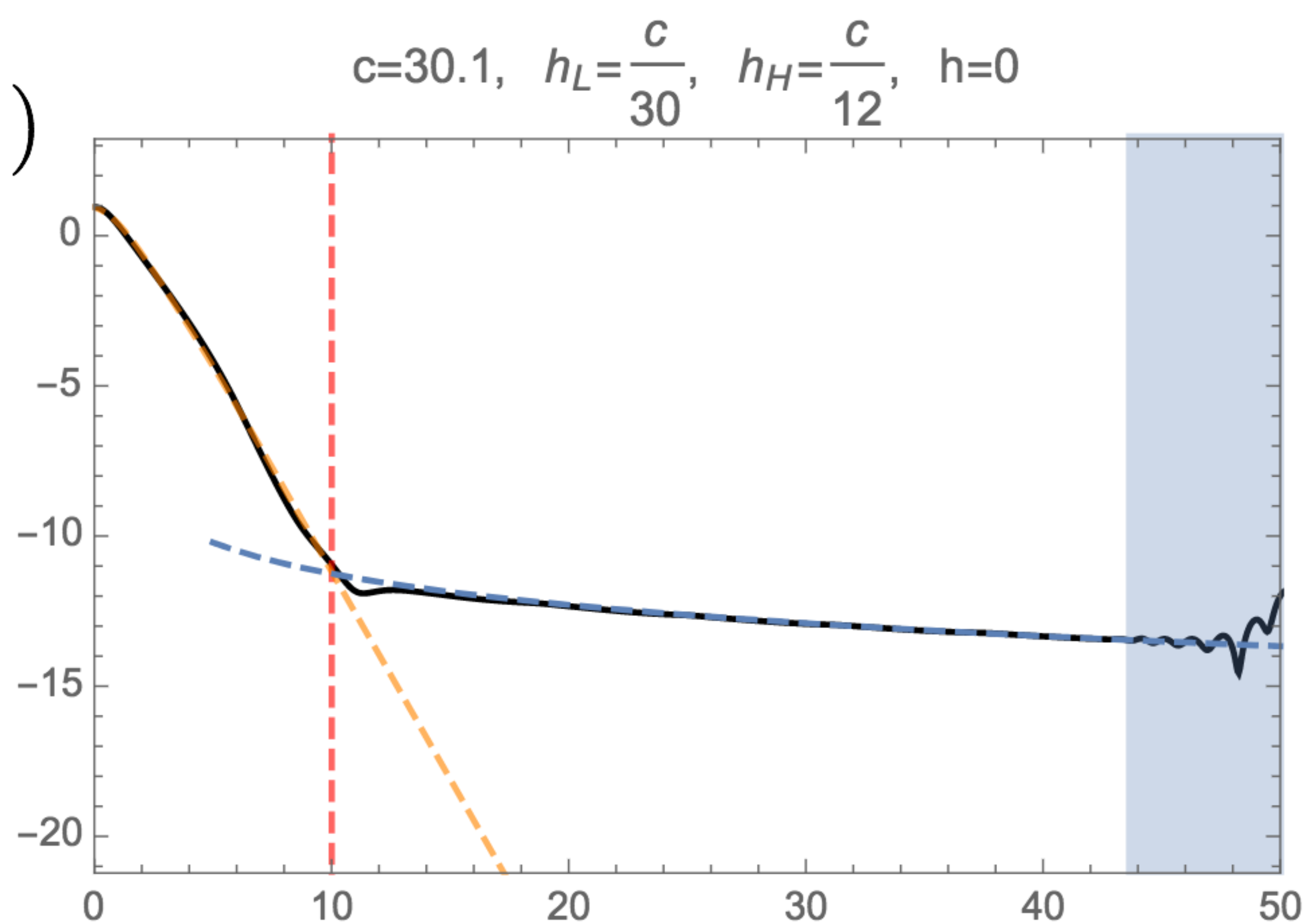
All blocks decay at same rate at late time in semiclassical limit

Can't resolve info loss by including just a few heavy states semiclassically

Exact Numeric Behavior

In the exact block, late-time exponential decay becomes power-law $t^{-3/2}$ at t ≳ c

$\log\left(\langle\psi|\mathcal{O}\mathcal{O}|\psi\rangle\right)$



$t_D = \frac{\pi c}{6 h_L}$

t

Chen, Hussong, Kaplan, Li, '17

Euclidean time periodicity and forbidden singularities

Periodic in Euclidean time (KMS condition):

If a singular event occurs…

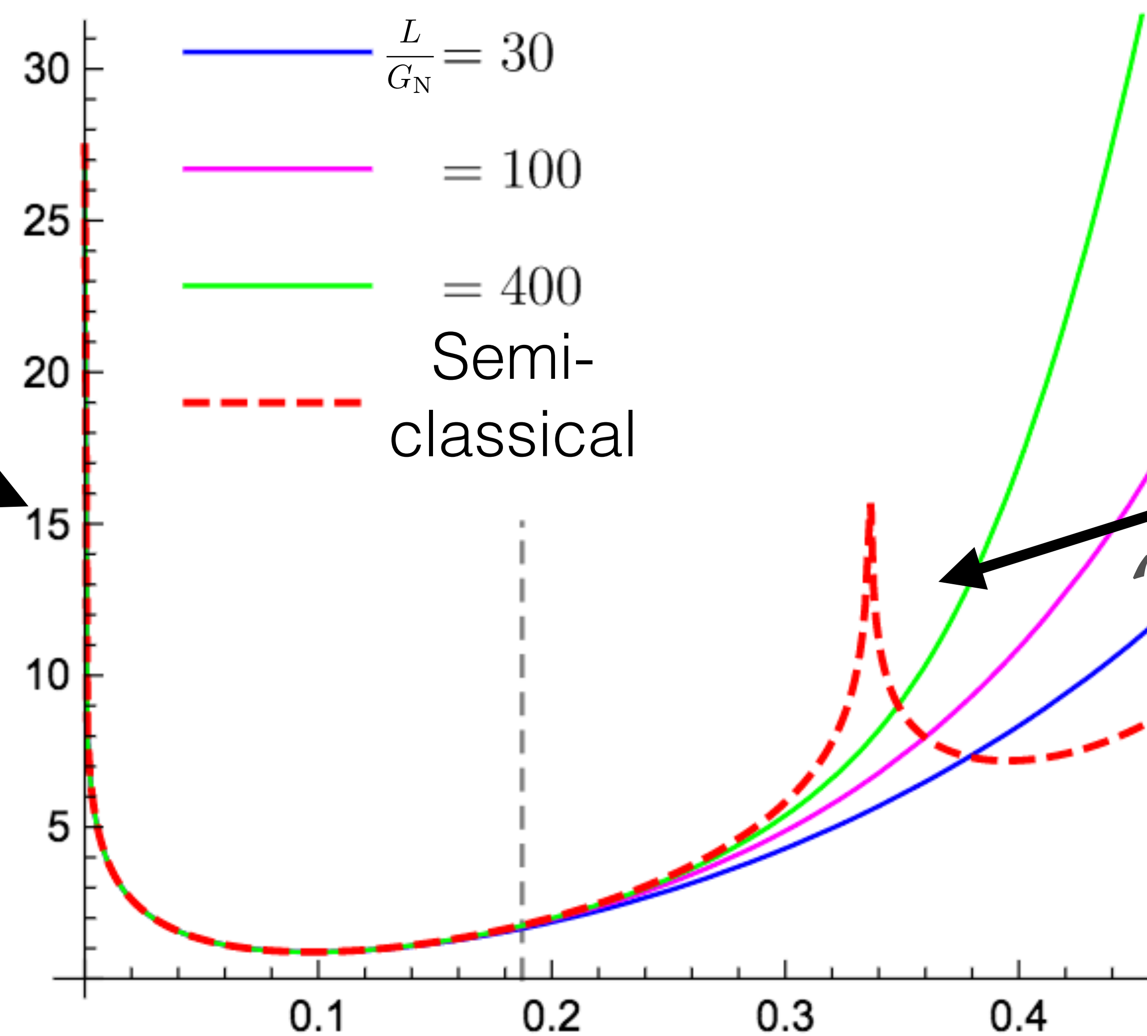
it gets repeated again and again for a thermal background

But the black hole is really a *pure* state not a thermal state, so this can't be true exactly

Going beyond the Semi-classical Limit

Unitarity restoration can be seen in the exact quantum theory!

“True” singular event

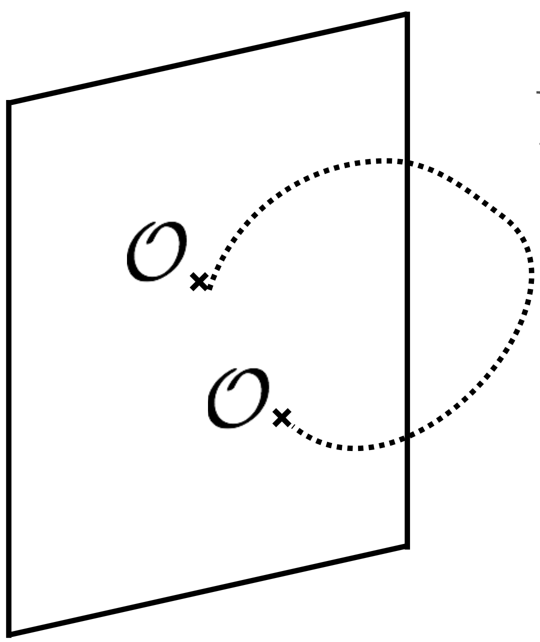


Approximate “thermal image”

Chen, Hussong, Kaplan, Li, ‘17

Blocks from Wilson Lines

AdS$_3$ gravity Chern-Simons description: $e_\mu^a \, , \, \omega_\mu^{ab} \longrightarrow A_\mu$



Blocks from Wilson Lines:

$$[\mathcal{O}(z_1)\mathcal{O}(z_2)] = Pe^{\int_{z_1}^{z_2} dz A_z} \frac{1}{x^{2h}}$$

Verlinde '89

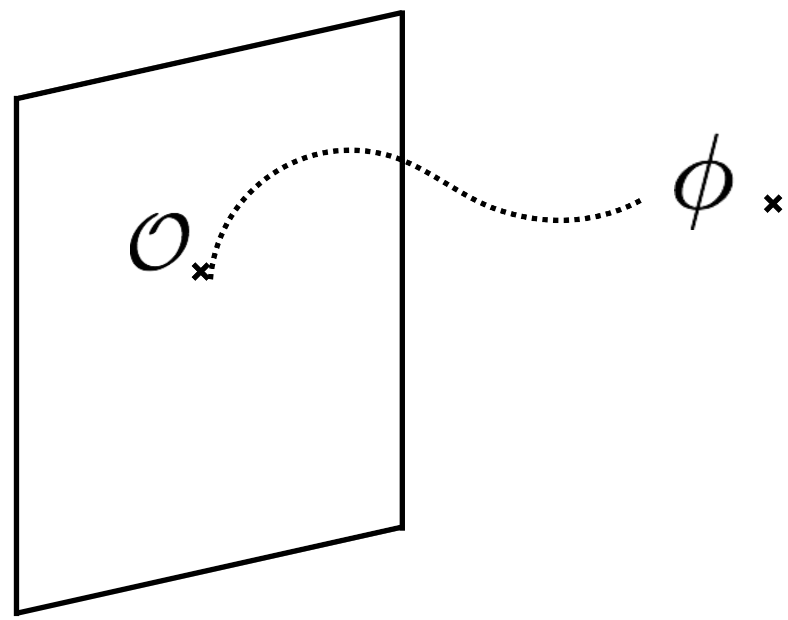
ALF, Kaplan, Li, Wang '16

+many others

Bulk Reconstruction

Bulk Reconstruction

Take one end into bulk



Physically: like ϕ attached to boundary with WL

$$\phi\mathcal{O} \sim P e^{\int_{z_1}^{(z_2, y_2)} dz A_z}$$

We want to construct an exact definition of ϕ

Basic strategy: 1) reconstruct ϕ from $\mathcal{O}$ in fixed background metric

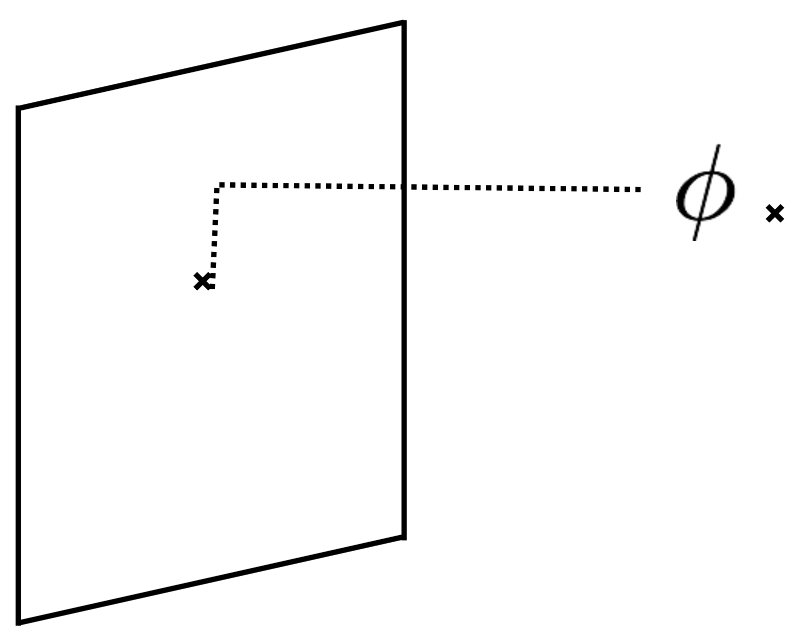
2) Then, promote T to operator

Bulk Reconstruction

We will use Fefferman-Graham gauge for vacuum metric:

$$ds^2 = \frac{dy^2 + dzd\bar{z}}{y^2} - \frac{6T(z)}{c}dz^2 - \frac{6\bar{T}(\bar{z})}{c}d\bar{z}^2 + y^2\frac{36T(z)\bar{T}(\bar{z})}{c^2}dzd\bar{z}$$

In terms of Wilson line: line goes straight toward boundary along y direction, then along boundary to z=0



For practical purposes, we will develop an algebraic definition of ϕ

Algebraic Definition of ϕ

Let's do a warm-up:
reconstruction of ϕ in the bulk in a free AdS theory.

Metric: $$ds^2 = \frac{dy^2 + dz d\bar{z}}{y^2}$$

$$\langle \phi \mathcal{O} \rangle_{\text{vac}} = \left(\frac{y}{y^2 + z\bar{z}} \right)^{\Delta}$$

is an exact relation for the bulk to boundary propagator

This fixes the contribution to ϕ from all "global" descendants of $\mathcal{O}$

$$\phi(y, 0) = \sum_n \lambda_n y^{\Delta + 2n} (L_{-1} \bar{L}_{-1})^n \mathcal{O}(0)$$

translation generators,
the simplest elements of the conformal algebra

Algebraic Definition of ϕ

$$\phi(y,0) = \sum_n \lambda_n y^{\Delta+2n}(L_{-1}\bar{L}_{-1})^n \mathcal{O}(0)$$

translation generators

Substituting into the LHS of $\langle \phi \mathcal{O} \rangle_{\text{vac}} = \left(\frac{y}{y^2 + z\bar{z}} \right)^\Delta$

and demanding that we reproduce the RHS fixes

$$\lambda_N = \frac{(-1)^N}{N!(\Delta)_N}$$

Algebraic Definition of ϕ

Same basic idea let's us fix contributions from all Virasoro descendants of $\mathcal{O}$:

We know $\langle \phi \mathcal{O} \rangle_T = \left(\frac{y'}{y'^2 + z'\bar{z}'} \right)^\Delta$ from the ***T-dependent*** coord transformation between Fff-Graham metric and pure AdS

This fixes the contribution to ϕ from the entire Virasoro irrep of $\mathcal{O}$

$$\phi(y,0) = \sum_n \lambda_n y^{\Delta+2n} (\mathcal{L}_{-n} \bar{\mathcal{L}}_{-n}) \mathcal{O}(0)$$

some specific combination of Virasoro generators

for example: $\mathcal{L}_{-2} = \frac{(2h+1)(c+8h)}{(2h+1)c + 2h(8h-5)} \left(L_{-1}^2 - \frac{12h}{c+8h} L_{-2} \right)$

Algebraic Definition of ϕ

Equivalent algebraic definition of ϕ from thinking about how it transforms under Virasoro

$$L_m \phi = ((\delta_m y)\partial_y + (\delta_m z)\partial_z + (\delta_m \bar{z})\partial_{\bar{z}})\phi$$

There is a unique extension of boundary conf txn into the bulk that preserves Fefferman-Graham gauge

Easy to check that $\delta_m y = 0, \delta_m z = 0$ for all m ≥ 2

$$L_m \phi = 0 \qquad m \geq 2$$

This plus normalization condition fixes ϕ

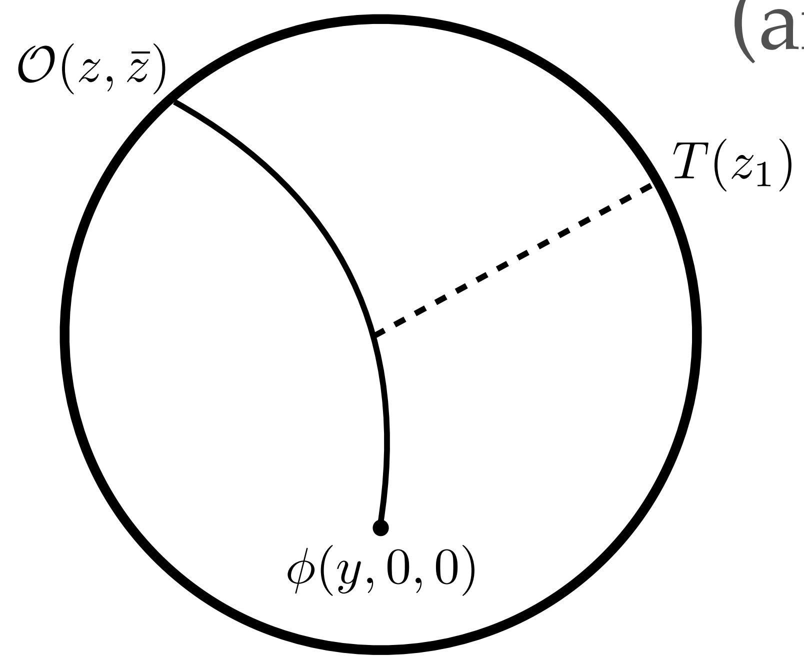
“Vacuum sector” Correlators

This definition of ϕ correctly reproduces all bulk correlators of the form

$$\langle \mathcal{O}\phi T \dots T \bar{T} \dots \bar{T} \rangle$$

(any number of T, $\bar{T}$’s)

E.g. $\langle \phi \mathcal{O} T \rangle$



$$\frac{\langle \phi(y,0)\mathcal{O}(z_2)T(z_1)\rangle}{\langle \phi(y,0)\mathcal{O}(z_2)\rangle} = \frac{\Delta z_2^2}{2z_1^3 z_{12}^2}\left(z_1 + \frac{2y^2 z_{12}}{y^2 + z_2\bar{z}_2}\right)$$

matches Witten diagram computation

Let's Compute Stuff

There are several available techniques for computing correlators of ϕ

"projectors" aka "Brute force"	}	Exact
Recursion relations	}	Exact
Monodromy method	}	Large c
Degenerate Operators	}	Large c
Uniformizing coordinates	}	Large c

For example: $\langle \phi\phi \rangle$ and $\langle \psi | \phi \mathcal{O} | \psi \rangle$

"Two bulk fields approach each other" (bulk locality?) → $\langle \phi\phi \rangle$

"Bulk field near a horizon" → $\langle \psi | \phi \mathcal{O} | \psi \rangle$

Exact $\langle \phi \, \phi \rangle$

We want to compute $\langle \phi \, \phi \rangle$

To get our bearings: recall tree-level result in AdS_3

$$\langle \phi(X_1)\phi(X_2)\rangle = \frac{1}{\ell_{\text{AdS}}}\frac{\rho^{\frac{\Delta}{2}}}{1-\rho} \qquad \rho = e^{-\frac{2\sigma(X_1,X_2)}{\ell_{\text{AdS}}}}$$

geodesic distance

Flat-space limit:

$$\Delta \to m\ell_{\text{AdS}}$$

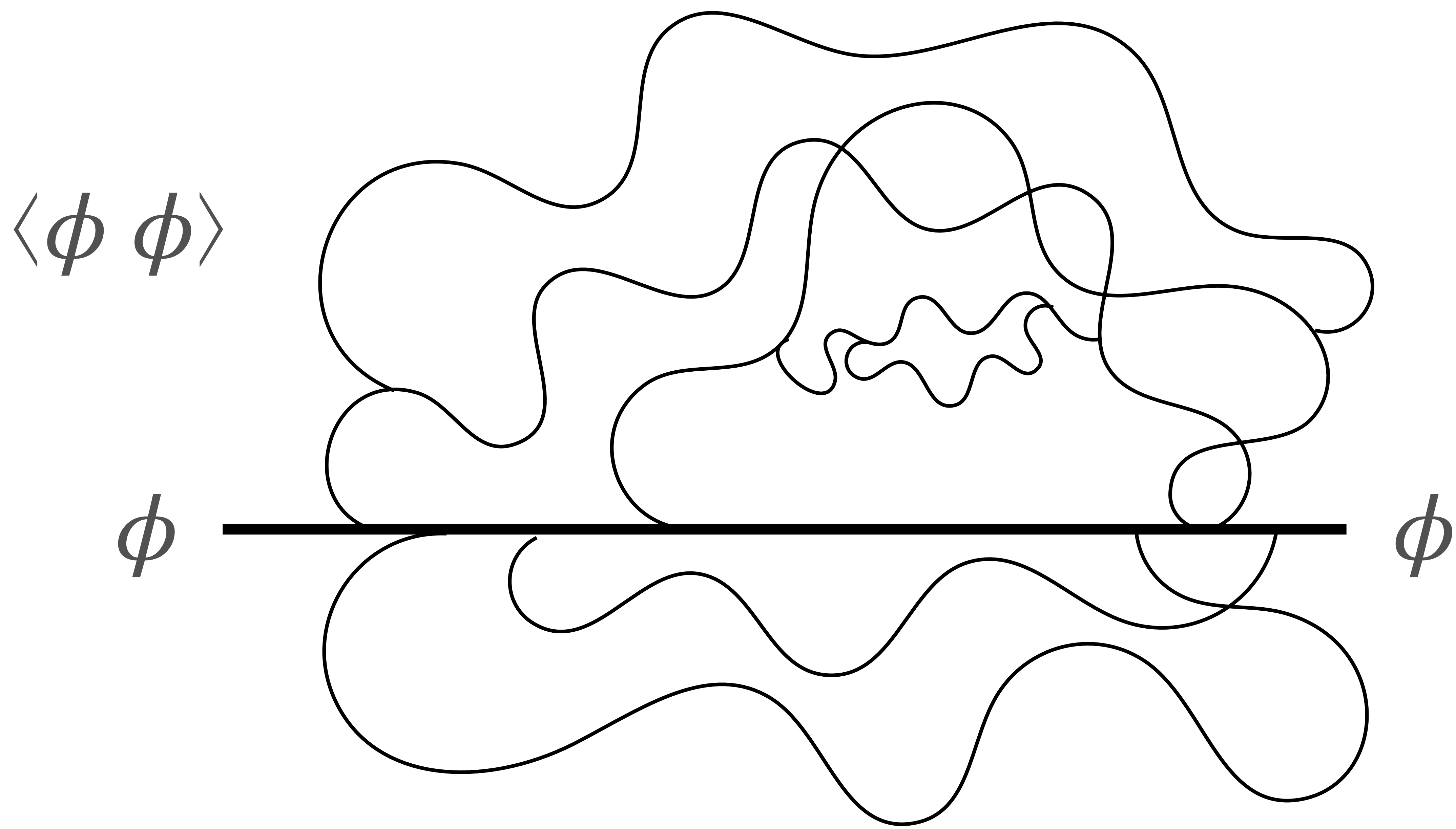
$$\sigma \to r$$

$$1-\rho \to 2\frac{r}{\ell_{\text{AdS}}}$$

$$\langle \phi\phi \rangle \approx \frac{e^{-mr}}{r}$$

Exact $\langle \phi \, \phi \rangle$

The exact $\langle \phi \, \phi \rangle$ is the propagator dressed by gravitons



But does not include ϕ loops

Will consider various limits

1) $\frac{\Delta^2}{c}$ fixed, large c - like taking G_N to zero with fixed Newtonian force $\frac{G_N m_1 m_2}{r}$

Simplest limit to see exponentiation in action

2) large Δ - the limit of very massive fields.

Also a necessary input to a recursion relation

3) small Δ - the limit of massless phi

We will see the breakdown of bulk locality in the exact answer

Brute Force Computation

Most straightforward in principle, also the most work

$$\langle \phi(X_1)\phi(X_2)\rangle = \sum_{n,m} \lambda_n \lambda_m y_1^{\Delta+2n} y_2^{\Delta+2m} \langle (\mathcal{L}_{-n}\bar{\mathcal{L}}_{-n}\mathcal{O}(z_1))(\mathcal{L}_{-m}\bar{\mathcal{L}}_{-m}\mathcal{O}(z_2))\rangle$$

Sum can be done to any order in y

Holomorphic Case

In the following slides, I'll actually be computing a "holomorphic" version $\langle\phi\phi\rangle_{\text{holo}}$ where drop all anti-holomorphic Ts in ϕ

Why?

1) It's easier to do analytically - results are more transparent and under better control

2) It is possible to extract the full result from just the holomorphic parts, so in a sense it's the "hard" part of the numeric computation

3) From numeric exploration, it doesn't appear to be very different from the full two-point function

“Semiclassical” pieces

At large c with Δ/ c fixed, $\langle \phi(X)\phi(Y)\rangle \sim e^{cf(X,Y)}$

$$\langle \psi|[\phi(X)\mathcal{O}(z)]|\psi\rangle \sim e^{cf(X,z)}$$

f is like a “semiclassical action” piece

(imagine a gravity action)

$$\sim e^{\frac{1}{G_N}\int d^dx\sqrt{g}R} \qquad \frac{1}{G_N} \sim c$$

f can be computed with Zamolodchikov “monodromy method”

Semiclassical h^2/c piece

At large c with Δ/c fixed, $\langle \phi\phi \rangle \sim e^{cf(\rho;\frac{\Delta}{c})}$

$\rho \equiv e^{-2\sigma}$ ← geodesic distance

Example of semi-classical piece — can compute order-by-order in Δ/c:

$$cf(\rho) = \Delta \log \rho + \frac{3\Delta^2}{c}\left(\frac{\rho}{(1-\rho)^2} + \log(1-\rho)\right) + \mathcal{O}(\frac{\Delta^3}{c^2})$$

singular at $\rho=1$, ie at $\sigma=0$

large Δ limit

At large Δ/c we can go farther and get the exact result:

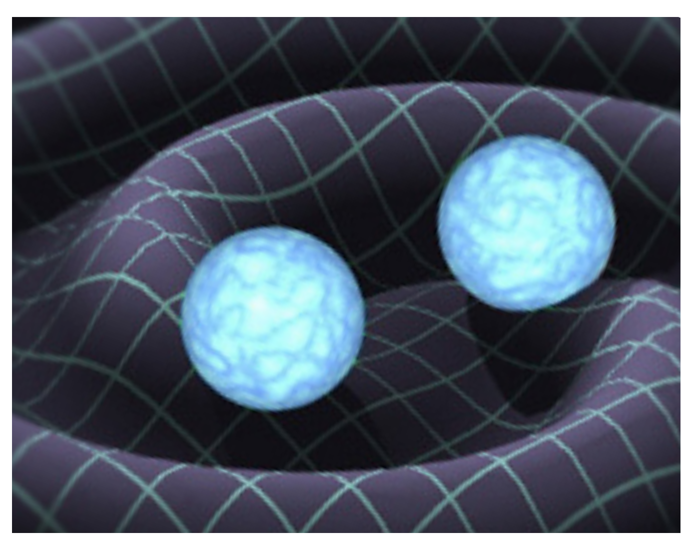
$$\langle \phi\phi \rangle = q^{\frac{\Delta}{2} - \frac{c-1}{24}} \left(\frac{s}{8}\right)^{\frac{c-1}{12}} (1-s)^{\frac{c-13}{144}} \left(\frac{2E(s)}{\pi}\right)^{\frac{19-7c}{36}}$$

$$q = 4e^{2\pi \frac{E(1-s)-K(1-s)}{E(s)} - 4}$$

$$\frac{s}{2(2-s)} = \frac{2\sqrt{\rho}}{1+\rho}$$

Branch cut at s=1

$$\sigma(X, Y) = 1.3\ell_{\mathrm{AdS}}$$



Δ~0 limit

$\langle \phi\ \phi \rangle$ also simplifies somewhat in massless case

$$\langle \phi\phi \rangle \overset{\sigma}{\simeq}{}^{0}\ \frac{1}{2\sigma}\left(\sum_{n=0}^{\infty}\frac{(4n-1)!!}{n!}\left(\frac{3}{4\ c\ \sigma^4}\right)^n\right)$$

Looks like an expansion in $c\ \sigma^4$

This is an asymptotic series

⟶ non-perturbative ambiguity ~ $e^{-c\sigma^4}$

A fundamental scale in gravity at $c^{-1/4}$??

$c^{1/4}$ and AdS_3 string compactifications

The scale $c^{1/4}$ also shows up as the smallest string length in known stable AdS_3 compactifications

E.g. $AdS_3 \times S^3 \times T^4$

Smallest one can make the radius of T is ~ ℓ_s

$$\longrightarrow \ell_{\rm pl,3d}\ell^3_{\rm AdS}\ell_s^4 = \ell^8_{\rm pl,10d} \lesssim \ell_s^8$$

$$\longrightarrow \frac{\ell_s}{\ell_{\rm AdS}} \gtrsim \left(\frac{\ell_{\rm pl,3d}}{\ell_{\rm AdS}}\right)^{1/4} \sim c^{-1/4}$$

$c^{1/4}$ and strings

The scale $c^{1/4}$ also shows up as the smallest string length in known stable AdS_3 compactifications

$$\ell_s \gtrsim c^{-1/4}$$

Possible interpretations:

— Coincidence? Could be
After all, ϕ isn't completely local (due to gauge-fixing)

— Fundamental breakdown of spacetime locality at this scale, prevents string length from being smaller?

Summary

Huge amount of information about gravity is contained in CFT_2 irreps

This includes BH thermodynamics, information paradox, many non-perturbative $e^{-\frac{1}{G_N}}$ corrections

These corrections are computable and in some cases ameliorate or even resolve unitarity issues at infinite c

These techniques can be applied to bulk fields

In progress: what do they tell us about bulk physics near horizon?

The End