# Input-Output System Models: Leontief versus Ghosh

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## 1. Introduction

The theoretical background of Leontief's Input-Output Analysis is based on Walras' General Equilibrium Theory as was repeatedly stressed by Leontief himself (Leontief, 1941, 1966, and 1986). At the same time it must be pointed out that, there are some differences between them. On the one hand, Walras' approach is purely theoretical and is remote from today's economic life. On the other hand, Leontief's approach is based on the empirical Input-Output tables, but theoretically it also is not suitable for real economics (Davar, 2000a).

The central topic of both approaches is the mutual interdependence between national income, which is determined as the value of used primary factors (quantities multiplied by their prices) and national product, determined as the value of demanded commodities (quantities multiplied by their prices). In Walras' approach this interdependence is directly expressed, i.e., the prices of factors and commodities are changed according to the change in quantities. This is based upon the supply curves of the factors and the demand curves of commodities, assuming that prices are uniform and are measured by the numéraire (money commodity). However, there are two types of price for commodities: the supply (cost of production) and demand, where the state of equilibrium equality between them is required[1].

However, according to Leontief's approach this interdependence is implicitly expressed, because Input-Output is generally described in money terms where prices and physical quantities are amalgamated in one magnitude, which means that Leontief's Input-Output has a uniform measurement, namely money measurement. It must be stressed that in fact today's economics is characterized by the discrimination of prices for both factors and commodities. The uniform money measurement of input-output makes both analysis and forecasting difficult and thus yields some confusions (vide infra).

On the other hand, such uniform money measurement allows us to extend the scope of analysis by the formulation of new models. For example, Ghosh (1964) formulated the allocation model and suggested it be used together with Leontief's model for analysis and planning. Ghosh wrote: 'In a competitive market and with fairly plentiful resources, allocation functions will play a minor role and special conditions can be formulated under which production will determine equilibrium. But in a monopolistic market with scarce resources, allocation functions will determine which of many alternative processes and combinations will be chosen by any particular industry; that is, production functions will play a minor role (Ghosh, 1964, p.111).' Unfortunately, the allocation model was transformed into an "output" (supply, supply-driven) model by Ghosh' followers (Augustinovics, 1970; Dietzenbacher, 1997; Oosterhaven, 1988, & 1996)[2]. Hence, the first question is if Ghosh' model is an output model how can it also complement Leontief's demand model? Perhaps the answer is that in order for it to be an output model, it must be equivalent to Leontief's demand model. Therefore, Dietzenbacher (1997) has attempted to prove that Ghosh's allocation model is equivalent to Leontief's price model, and has suggested that the interpretation of the original Ghosh model is in fact a price model of Leontief, instead of a quantity model. The second question is whether Ghosh's model might be equivalent to Leontief's price model. Furthermore, there are economists who have expressed a doubt regarding the plausibility of Ghosh's model (Oosterhaven, 1988 and 1989).

Therefore, in this paper we will discuss the relationship between the Leontief Input-Output system models and the Ghosh model, namely, whether the results of the latter model might be equivalent to the results of the previous model.

Because of that the primary source of misunderstanding and misinterpretation is based upon fundamental differences in the definition and notation of the terms, hence, in the second section the main terms of both the Leontief and Ghosh model's definition and notation are discussed. The central point, here, is the definition of all type of prices and the connection between them. In the third section Leontief's system models of Input-Output are shortly presented. It will put forward the position that there is a necessity to distinguish between the two types of Input-Output systems: a) in money terms and b) Input-Output where physical quantities and absolute (money) prices are separately presented. Following this, the connection between them will be discussed. Ghosh's original supply quantity model and its according dual price model is described in the fourth section. In the fifth section a problem of equivalence between them is discussed. Finally, the last section will summarize and provide some conclusions.

## 2. Definitions, Notations and Properties (Characteristics) of Input-Output Analysis

2a. Input-Output in Physical Terms

Walras' General equilibrium System consists, as it was mentioned above, of two components: quantities (real, physical) and prices (money); so, let us to start with the structure of Input-Output in physical terms:

| | |
|---|---|
| (I) $\underline{X}=[\underline{x}_{ij}]$ | $\bar{\underline{x}}^d$ |

| | | |
|---|---|---|
| (II) $\underline{Y}=[\underline{y}_{ir}]$ | $\underline{y}^d$ | $\underline{x}^d$ |

| | |
|---|---|
| (III) $\underline{V}=[\underline{v}_{kj}]$ | $\underline{v}^d$ |

Table 1 Input-output in Physical terms, assumed that $(\underline{x}^s)' = \underline{x}^d$

Where

$\underline{X}$ – $[\underline{x}_{ij}]$ – is the square matrix ($n$*$n$) of the quantitative flows of commodities in the production;

$\bar{\underline{x}}^d$ – is the column vector ($n$*1) of the intermediate output quantity of commodities' in the production;

$\underline{Y} = [\underline{y}_{ir}]$ - is the matrix ($n$*$R$) of the quantitative flows of commodities to the categories of final uses;

$\underline{y}^d$ – is the column vector ($n$*1) of commodities' quantities for final uses;

$\underline{x}^d$ – is the column vector ($n$*1) of the total output quantity of commodities;

$\underline{V}$ - $[\underline{v}_{kj}]$ - is the matrix ($m$*$n$) of the quantitative flows of primary factors to the sectors of production;

$\underline{v}^d$ – is the column vector ($m$*1) of the total quantities of primary factors required in the production;

$\underline{A}$ – [$\underline{a}_{ij}$] - is the square matrix ($n*n$) of the direct input coefficients of commodities in real (physical) terms in the production and

$$\underline{A} = \underline{X}\,(\hat{\underline{x}}^d)^{-1}, \text{ i.e., } \underline{a}_{ij} = \underline{x}_{ij}/\underline{x}_j^d; \qquad (2.1)$$

i.e., the input coefficient $\underline{a}_{ij}$ measure quantity of commodity $i$ required for the production of one unit of commodity $j$ in physical terms;

$\underline{C}$ – [$\underline{c}_{kj}$] - is the matrix ($m*n$) of the direct input coefficients of factors in real physical terms in the production and

$$\underline{C} = \underline{V}\,(\hat{\underline{x}}^d)^{-1}, \text{ i.e., } \underline{c}_{kj} = \underline{v}_{kj}/\underline{x}_j^d; \qquad (2.2)$$

i.e., the input coefficient $\underline{c}_{kj}$ measure quantity of factor $k$ required for the production of one unit of commodity $j$ in physical terms;

$\underline{H}$ – [$\underline{h}_{ir}$] - is the square matrix ($n*n$) of the direct allocation coefficients of commodities in real (physical) terms in the production and

$$\underline{H} = (\hat{\underline{x}}^d)^{-1}\,\underline{X}, \text{ i.e., } \underline{h}_{ij} = \underline{x}_{ij}/\underline{x}_i^d; \qquad (2.3)$$

i.e., the allocation coefficient $\underline{h}_{ij}$ measure the share of sector $j$ in the output of one unit of commodity $i$ in physical terms[3];

$\underline{T}$ – [$\underline{t}_{ir}$] - is the matrix ($n*R$) of the direct allocation coefficients of commodities in real (physical) terms to final uses and

$$\underline{T} = (\hat{\underline{x}}^d)^{-1}\,\underline{Y}, \text{ i.e., } \underline{t}_{ir} = \underline{y}_{ir}/\underline{x}_i^d; \qquad (2.4)$$

i.e., the allocation coefficient $\underline{t}_{ir}$ measures the share of category $r$ in the output of one unit of commodity $i$ in physical terms;

From (2.1) and (2.3) the following relations between $\underline{A}$ and $\underline{H}$ is obtained:

$$\underline{X} = \underline{A}\,(\hat{\underline{x}}^d) \text{ and } \underline{X} = (\hat{\underline{x}}^d)\,\underline{H}; \text{ hence } \underline{A} = (\hat{\underline{x}}^d)\,\underline{H}\,(\hat{\underline{x}}^d)^{-1} \text{ and } \underline{H} = (\hat{\underline{x}}^d)^{-1}\,\underline{A}\,(\hat{\underline{x}}^d) \qquad (2.5)$$

From (2.5) this we can conclude that if the output of commodities is changed on the basis of the direct input coefficients ($\underline{A}$) then the magnitudes of the direct allocation coefficients ($\underline{H}$) are changed and vice versa, i.e., if the output is changed on the basis of the direct allocation coefficients then the magnitudes of the direct input coefficients are changed.

It is necessary to point out that for input-output in physical terms the intermediate input of commodities for the sectors and the total inputs of all factors for the sectors do not exist. This is because in this case the sum of inputs for a certain sector is impossible since every input has a different measurement. Yet, for the same reason it is impossible to determine the total input for the sectors. Despite this it is still assumed that $(\underline{x}^s)' = \underline{x}^d$.

2b. Input-Output in Money Terms

Input-Output in money terms, or alternatively, the General Equilibrium of the Input-Output in physical terms and absolute (money) prices, i.e., $(p^s)' = p^d$, and $(x^s)' = x^d$; has the following structure

| (I) $X = (\hat{p}^s) * \underline{X}$ | $(\hat{p}^s)\,\bar{\underline{x}}^d = \bar{x}^d$ |
|---|---|
| $\bar{x}^s$ | |

| (II) $Y = (\hat{p}^d) * \underline{Y}$ | $(\hat{p}^d)\,\underline{y}^d = y^d$ | $(\hat{p}^d)\,\underline{x}^d = x^d$ |
|---|---|---|
| $y^s$ | | |

| (III) $V = (\hat{w}^s) * \underline{V}$ | $v^d = (\hat{w}^s)\underline{v}^d$ |
|---|---|
| $v^s$ | |
| $x^s = \bar{x}^s + v^s$ | |

Table 2 Input-Output in money terms, assumed that $(p^s)' = p^d$, and $(x^s)' = x^d$.

From Table 2 we can see that all the definitions and notations of Input-Output in money terms are similar to Input-Output in physical terms, but here they are measured in money terms rather than in physical terms. In addition

$$A = X\,(\hat{x}^s)^{-1}, \text{ i.e., } a_{ij} = x_{ij}/x_j^s = \underline{x}_{ij}p_i/\underline{x}_j^s p_j = \underline{a}_{ij}\, pi/pj; \tag{2.6}$$

$$C = V\,(\hat{x}^s)^{-1}, \text{ i.e., } c_{kj} = v_{kj}/x_j^s = w_k\, \underline{v}_{kj}/\underline{x}_j^s p_j = \underline{c}_{kj}\, w_k/p_j; \tag{2.7}$$

$$H = (\hat{x}^d)^{-1}\, X, \text{ i.e., } h_{ij} = x_{ij}/x_i^d = p_i\, \underline{x}_{ij}/\underline{x}_i^d p_i = \underline{h}_{ij}; \tag{2.8}$$

$$T = (\hat{x}^d)^{-1}\, Y, \text{ i.e., } t_{ir} = y_{ir}/x_i^d = p_i\, \underline{y}_{ir}/\underline{x}_i^d p_i = \underline{t}_{ir}; \tag{2.9}$$

It is necessary to stress that for I-O in monetary terms (included empirical I-O table) in equilibrium state the sum of all input coefficients (of commodities and factors) for every sector (column sum) is equal to one, i.e., $(i_n)'A + (i_m)'C = (i_n)'$; while for I-O in physical terms the latter could not be calculated (vide supra); and the sum of all allocation coefficients (for commodities and categories of final uses) for every sector (row sum) is also equal to one but for both I-O systems, i.e., $\underline{H}\, i_n + \underline{T}\, i_R = i_n$ and $H\, i_n + T\, i_R = i_n$ (consequently).

From (2.6) and (2.7) we can conclude that input coefficients of commodities and primary factors in money terms are dependant on the prices of commodities and factors, i.e., for different price systems the input coefficients in money terms will be of different magnitudes. While, from (2.8) and (2.9) we can conclude that the allocation coefficients for sectors and categories are not dependent on the price system, i.e., for different price systems the allocation coefficients in money terms will be of the same magnitude Furthermore they will be equal to the allocation coefficients in physical terms. For the following discussion these conclusions have significant meaning.

In practice, unfortunately, it is not always possible to separate quantities and prices with objective and subjective reasons (Davar, 2000b). For example, the problems of the aggregation of commodities and factors, the high cost of gathering of satisfactory data, and the fact that individuals are not always interested in making the separation between quantities and prices (black markets, taxes, and so on). Hence, the results of economic activities are usually presented in money form and all existing empirical I-O are compiled in monetary terms except for the former USSR and China which together with an I-O in money terms compile an I-O table in real (physical) terms too.

Therefore, empirical (Marxian-Leontievian) I-O is characterized by "quantity" in the monetary terms (Dorfman, et al, 1958). This means that in these cases prices and quantities are not separated and they are amalgamated into one magnitude (element). Each element in Table 2 is included as quantity and prices. Therefore, empirical I-O has a uniform measurement for all parts: commodities, factors and categories of final uses, namely, money measure. On the one hand, this creates some problems in the analysis its use for planning and analysis. On the other hand this allows to extend a scope of analysis by the formulation of additional models (vide infra).

Since each element of matrices $X$, $Y$, and $V$ of monetary I-O, in general, include two components, quantity and price, then a change of its magnitude might be the result of changing either each component separately or both simultaneously. It is necessary to stress that some elements of $Y$ relate to export and there are those of $V$ which relate to imports consisting of three components including the exchange rate. Hence, in the case of monetary I-O, it is necessary to make clear beforehand what kind of assumptions should be taken into account regarding quantity and price. With regards to quantity there is a minor problem, because "quantity" in money terms is understood in the same way as in other terms. By the way, I-O in physical terms includes some sectors whose quantities are measured in money terms, for example some sectors of industry, financial and services sectors and so on (Davar, 2000b).

The serious problem arises when one desires to analyse monetary I-O from the point view of prices. Because of that prices (money) are already included (presented). So, a new price system for monetary I-O should be take into account this the existence of money prices, and have to be considered as supplementary to them, but not replacing them, as sometimes they are interpreted. What this meant is that these two prices systems coexist. So, new prices for monetary I-O must be related to money prices as quantity in money terms is related to quantity in real (physical) terms.

At the same time, monetary I-O has a certain advantage. The dual character of each element of monetary I-O and the uniformity of their measurement allow us to consider a special (additional) system of prices not only for prices of commodities for production and final use (according to rows) on the factor side (supply prices), but also the prices of commodities and factors (according to columns) on the categories side of final uses (demand prices). In addition, the demand of a certain category for the different commodities in monetary I-O becomes homogenous. This allows us to determine a price which is in relation to the unit expenditure of a certain category, when the structure of demand of this category does not change. The same is true for commodities and factors too.

In the literature of I-O analysis there is no agreement on the character and interpretation of a new additional price system. Firstly, many authors consider only prices for commodities (Rose and Miernyk, 1989) and the prices for factors are considered without distinguishing (difference) between prices from the supply side and prices from the demand side in one system models (Dietzenbacher, 1997; Seton, 1985; Oosterhaven, 1994, describes such a difference, but only for prices of commodities and assumes that they are equal and uses the same notation for both). Secondly, these prices are interpreted in various forms: 1) as unit prices (Folloni and Miglierina, 1994); 2) as relative prices (Leontief, 1986, Tokutsu, 1994); 3) as index prices (Rose and Miernyk, 1989; Dietzenbacher, 1997; Oosterhaven, 1994); and 4) as eigenprices (Seton, 1985). Finally, it seems that these prices substitute absolute (money) prices because the connection between them is not discussed (except Dietzenbacher, 1997 (vide infra)).

The common failure of these interpretations is that there are prices considered without any measurement. However, since monetary I-O is characterized by the quantities in money terms, prices should be determined by the relation of unit flows in money terms and therefore, the measure of such prices should be a *measure of money per* 1 *unit of measure of money*, for example $ per 1$. Moreover it is uniform for commodities, factors and categories. Since, in I-O monetary terms there is no direct relation to the real (physical) quantities and their prices I have suggested previously to call them a latent price (Davar, 1989 and 1994). They may be determined by the following form:

$\lambda$ - $[\lambda_k]$ – is the row vector (1*$m$) of the latent prices of primary factors, and it has two meanings: 1) it is given and is determined from the supply side of factors which are identical to their supply money prices; and are used for the determination of the latent supply prices of commodities and consequently for the determination of the latent supply prices of categories In this case it is called the latent supply price of primary factors and notated as $\lambda^s$ In the case of a general equilibrium state (included the base year I-O table) $\lambda^s$ is a unit vector (vide infra); 2) it is unknown, and it is obtained by means of the latent demand prices of commodities determined on the base of the latent demand prices of categories; in this case it is called the latent demand prices of primary factors and is notated as $\lambda^d$;

$\mu$ - $[\mu_r]$ – is the column vector ($R$*1) of the latent prices of categories, and it also has two meanings: 1) it is given and is determined from the demand side of categories; and is used for the determination of the latent demand prices of commodities and consequently for the determination of the demand supply prices of primary factors; in this case it is called the latent demand price of categories and notated as $\mu^d$. In the case of a general equilibrium state (included the base year I-O table) $\mu^d$ is a unit vector (vide infra); 2) it is unknown, and it is obtained by means of the latent supply prices of commodities determined on the basis of the latent supply prices of primary factors; in this case it is called the latent supply prices of categories and notated as $\mu^s$;

$\pi$ – $[\pi_{i,j}]$ – is either the column vector ($n$*1) or the row vector (1*$n$) of the latent prices of commodities and they are obtained in two ways: 1) by means of the latent supply prices of primary factors and it is called the latent supply prices of commodities and notated as $\pi^s$ and it is row vector; 2) by means of the latent demand prices of categories and notated as $\pi^d$ which is column vector.

For the following discussion there are some issues which have to be noted:

1) The latent demand prices of commodities, $\pi^d$, differs from the latent prices of commodities, $\pi^s$, not only in that the first is a column vector, while the second is a row vector, but also in its meaning, because the first relates to output (demand), while the second – to input (supply). Therefore, it is incorrect to substitute $\pi^d$ with $\pi^s$ and vice versa. Only in a general equilibrium state are they equal to each other. The same is true also for $\mu^d$ and $\mu^s$, and for $\lambda^s$ and $\lambda^d$. Of course, it is also incorrect to substitute the latent prices of commodities with the latent prices of factors and categories, and vice versa.
2) $\mu^d$ and $\mu^s$ – are the latent prices of categories of final uses, but not commodities. This does not mean that they might not identify with each other. For example, when a structure of demand of a certain category includes only one commodity.

3) It is necessary to keep the rule of homogeneity, which means that the latent prices of commodities, factors and categories must have an identical character. For example, if the demand side is discussed the latent prices must have the same character, namely, $\mu^d$, $\pi^d$ and $\lambda^d$; or, if the supply side is discussed then - $\lambda^s$, $\pi^s$ and $\mu^s$; and any other combination will be incorrect, for example, $\mu^d$, $\pi^s$ and $\lambda^d$, or $\mu^s$, $\pi^d$ and $\lambda^s$, and so on.
4) Due to the fact that the base year empirical I-O table is conveniently considered as being in a general equilibrium state (not necessary Walras' type) the magnitudes of components of all latent prices – commodities, factors and categories – are equal to one (vide infra). But this does not mean that their money (absolute) prices are also equal to one, except for the prices of commodities of sectors where output is measured in money terms.

## 3. Input-Output System Models a là Leontief

According to Walras' approach the establishment of an equilibrium state is divided into two stages: firstly, equilibrium from the side of quantities, either real (physical) or money; and secondly, equilibrium from the side of prices, either absolute (money) and latent. In addition, Walras also assumed that there are total demand curves, (decreasing) for goods and the total supply curves (increasing) of primary factors. It is necessary to point out that the total supply functions are also bound, which means that either the supplied or required quantities of factors must be less or equal to their available quantities. Such an approach allows us to determine which factors are unemployed, what the character (voluntary or involuntary) of it is .This approach also allows us to determine the magnitude of unemployment and which factors are scarce.

3a Quantitative Equilibrium for I-O in Physical Terms from the Demand Side

The quantitative equilibrium for I-O in physical terms consists into two systems according to Table 1:

$$\underline{x}^d = \underline{A}\,(\underline{x}^d) + \underline{\hat{y}}^d, \qquad \text{or} \quad \underline{x}^d = (I\text{-}\underline{A})^{-1}\,\underline{y}^d, \qquad \text{or} \quad \underline{x}^d = \underline{B}\,\underline{y}^d, \tag{3.1}$$

$$\underline{v}^d = \underline{V}\,i_n = (\underline{C}\,\underline{x}^d)\,i_n = \underline{C}\,\underline{x}^d \leq \underline{v}_0, \qquad \text{or} \qquad \underline{v}^d = \underline{C}\,\underline{x}^d = \underline{C}\,\underline{B}\,\underline{y}^d \leq \underline{v}_0, \tag{3.2}$$

where in addition to the above notation $\underline{B}$ – is Leontief's inverse matrix, and $\underline{b}_{ij}$ – is the complete required quantities (direct and indirect inputs) of good $i$ to a satisfied one unit of demand of the good $j$;

$\underline{v}_0$ – is the vector of the available quantities of primary factors;

$i_n$ – is a unit column vector ($n$*1);

So, by means of system (3.1) the total required outputs of commodities are obtained for given quantities for final uses for the certain conditions for the direct input coefficients $\underline{A}$ Consequently, the substitution of the obtained required output quantities in the system (3.2) the required quantity of primary factors are defined as $\underline{v}^d$. It is necessary to stress that the system (3.2) is generally ignored, but it is necessary not only in order to determine money prices according to required quantities of factors according to their supply curves, but also in order to observe whether the available quantities of factors might satisfy their required quantities. Therefore, if the required quantities of factors are within the limits given by their supply curves, i.e., if the required quantities are less or equal to the available quantities ($\underline{v}^d \leq \underline{v}_0$).If this is so a quantitative equilibrium and a price equilibrium might be considered. In the opposite case, namely, when if at least the required quantities for one factor is larger then its available quantity then the process must be carried out for the new different quantities for final uses, until the above conditions are satisfied.

It is necessary to stress that for every new I-O, which is obtained according to the new magnitude of final uses, the direct input coefficients ($\underline{A}$ and $\underline{C}$) are not changed, while the direct allocation coefficients ($\underline{H}$ and $\underline{T}$) are changed (see (2.5)).

3b Price Equilibrium and Consequently General Equilibrium for I-O in Physical Terms from the Supply Side

The establishment of quantitative equilibrium for any given quantity of commodities for final uses when according required quantities of factors are determined, allows us to consider the second stage of general equilibrium, namely, price equilibrium. The results of the first stage, simultaneously with quantitative equilibrium, allow us to determine two types of prices: prices of the commodities from the demand side according to their demand curves (the prices of the commodities where the categories of final uses are paid to the producer for equilibrium quantities) and prices for primary factors from the supply side according to their supply curves (the prices of factors which the owners of factors wish (request) to receive from the producers). Therefore, in order to verify whether there is general equilibrium, a comparison between the supply prices (cost of production) of commodities, obtained by means of the prices of factors and the demand prices of commodities is needed.

Therefore, price equilibrium for I-O in physical terms also consists of two systems according to Table 2:

$$(p^s)\,\underline{X} + (w^s)\,\underline{V} = (\underline{x}^d)'\,(\hat{p}^s),\ \text{ or } p^s = p^s\,\underline{A} + w^s\,\underline{C},\ \text{ or } (p^s)' = \underline{A}'\,(p^s)' + \underline{C}'\,(w^s)',$$

$$\text{or }\ (p^s)' = (I-\underline{A}')^{-1}\,\underline{C}'\,(w^s)', \qquad \text{or } (p^s)' = \underline{B}'\,\underline{C}'\,(w^s)' \qquad (3.3)$$

$$(\hat{p}^s\,\underline{X})\,i_n + (\hat{p}^d\,\underline{Y})\,i_R <=> \hat{p}^s\,\underline{x}^d, \text{or } \hat{p}^s\,\underline{y}^d <=> \hat{p}^d\,\underline{y}^d, \text{ or } (p^s)' <=> p^d, \qquad (3.4)$$

Thus by means of the system (3.3) the cost of productions (supply prices) for commodities are obtained for the given required quantities of primary factors. The system (3.4) also allows us to establish whether there is price equilibrium. If the cost of production for all commodities is equal to its relevant demand price, then there is price equilibrium and consequently a general equilibrium. Even for one commodity there is an inequality between these prices, then there is disequilibrium and the process of equilibrium establishment must be started from the beginning, i.e., from the quantitative side. The problems of proving the existence of a solution are not discussed here (Davar, 1994), since this complicates the discussing subject and has a minor influence. It must be stressed that the system (3.4) is also generally ignored, and automatically assumed that the cost of productions (supply prices) replace demand prices.

It is necessary to stress that for every new I-O, which is compiled according to new supply prices (cost of production) of commodities obtained according to new prices of primary factors, here in opposite to the quantitative models, the direct input coefficients ($\underline{A}$ and $\underline{C}$) are changed, while the direct allocation coefficients ($\underline{H}$ and $\underline{T}$)

are not changed (see (2.6)-(2.9)), despite of that these new prices were obtained on basis of current (old, previous) direct input coefficients.

### 3c Quantitative Equilibrium for I-O in Monetary Terms from the Demand Side

The quantitative equilibrium for I-O in monetary terms is identical to quantitative equilibrium for I-O in physical terms and also consists of two systems according to Table 2:

$$x^d = A\,(x^d) + y^d, \qquad \text{or} \quad x^d = (I\text{-}A)^{-1}\, y^d, \qquad \text{or} \quad x^d = B\, y^d, \tag{3.5}$$

$$v^d = V\, i_n = (C\, x^d)\, i_n = C\, x^d \leq v_0, \qquad \text{or} \qquad v^d = C\, x^d = C\, B\, y^d \leq v_0, \tag{3.6}$$

All notations, determinations and indexes here are identical to systems (3.1) and (3.2), except that they are in monetary terms.

Here as well as for I-O in physical terms, by means of system (3.5) the total required outputs of commodities are obtained for the given quantities of final uses in the certain conditions for the matrix of the direct input coefficients ($A$); and consequently, by the substitution of the obtained required output quantities in the system (3.6) the required quantity of primary factors are defined as $v^d$. It is necessary to stress that the system (3.6) is also generally ignored, but it is required in order to observe whether the available quantities of factors might satisfy their required quantities, that is, which factors are unemployed and which factors are scarce. Therefore, if required quantities are less or equal to the available quantities ($v^d \leq v_0$), then there is a quantitative equilibrium and the price equilibrium might be considered. In the opposite case, namely, when at least the required quantity for one factor is larger than its available quantity then the process must be carried out for the new different quantities for final uses, until the above condition will be satisfied.

Here also, it is necessary to stress that for every new I-O, which is obtained according to the new magnitude of final uses, the input coefficients ($A$ and $C$) are not changed, while the allocation coefficients ($H$ and $T$) are changed (see (2.5)).

### 3d Prices' equilibrium and consequently General Equilibrium for I-O in Monetary Terms from the Supply Side

Prices' equilibrium for I-O in monetary terms also consists of two systems according to Table 3:

| | |
|---|---|
| $\widehat{(\pi^s)} * X$ | $\bar{x}^d$ |
| $\bar{x}^s$ | |

| | | |
|---|---|---|
| $Y * \widehat{(\mu^s)}$ | $y^d$ | $\widehat{(\pi^s)}\, x^d$ |
| $y^s$ | | |

| | |
|---|---|
| $\widehat{(\lambda^s)} * V$ | $v^d$ |
| $v^s$ | |
| $x^s$ | |

Table 3 The General Equilibrium of the Input-Output in monetary terms and latent prices, i.e., $\lambda^s$, $\pi^s$, and $\mu^s$ are unit vectors; and $(x^s)' = x^d$.

$$\pi^s = \lambda^s C B, \quad \text{or} \qquad (\pi^s)' = B' C' (\lambda^s)' \tag{3.7}$$

$$\hat{\pi}^s y^d = Y \mu^s, \tag{3.8}$$

Therefore, if the latent supply prices of factors are given by means of system (3.3) then the latent supply prices of commodities are obtained under certain conditions (Davar, 1994). For Leontief's original I-O system, in this stage it is assumed that these latent supply prices of commodities are also used for the commodities for final uses. This is also is the same for the prices in the I-O physical terms, where it is assumed that the demand prices of commodities are equal to their supply prices (cost of productions).

However, when the latent prices for the categories of final uses are considered, the system (3.4) allows us to obtain the latent supply prices of categories, under certain conditions too. It is important to point out that for an equilibrium state (included basis year empirical I-O table) all latent supply prices of factors, commodities and categories are equal to one (Davar, 1994). It must be stressed that for the I-O in physical terms and absolute (money) prices the prices for categories have no economic interpretation. Because of this a structure of a certain category is heterogenic, i.e., it includes commodities characterized by different measurement. But for I-O in money terms a structure is homogeneous, as it has been shown above, and therefore, this is a legitimate consideration regarding the latent price of categories. At the same time, a certain *caution* must be taken when using them for practical purposes.

Here also, it is necessary to stress that every new I-O, which is compiled according to new latent supply prices of commodities obtained according to new latent supply prices of primary factors, unlike where there are quantitative models, the direct input coefficients ($A$ and $C$) are changed, while the direct allocation coefficients ($H$ and $T$) are not changed (see (2.6)-(2.9)). Despite this these new latent prices were obtained on the basis of current (previous) direct input coefficients.

## 4. Input-Output system models a là Ghosh

Before describing Ghosh's I-O system models, it is important to stress that it is impossible to formulate these models for the I-O in physical terms and money prices. This is due to the heterogeneous character of both the structure of the way that the use of factors is structured for the production of certain products and the structure of commodities for a certain category of final uses (vide supra).

4a Quantitative Equilibrium for I-O in Monetary Terms from the Supply Side

Quantitative equilibrium for I-O in monetary terms from the supply side also consists of two systems according to Table 2:

$$(x^s)' = H' (x^s)' + (v^s)', \text{ or } (x^s)' = (I - H')^{-1} (v^s)', \text{ or } (x^s)' = D' (v^s)' \tag{4.1}$$

$$y^d = Y i_R = (\hat{x}^s T) i_R , \tag{4.2}$$

Here as well as for I-O monetary terms from the demand side, by the means of system (4.1) the total quantities (outputs) of commodities may be obtained for the given quantities for value added in the certain conditions for the direct allocation coefficients *H*; and consequently, by the substitution of the obtained total quantities in the system (4.2) the quantity of commodities for final uses can be defined as $y^d$. It is necessary to stress that the system (4.2) is also generally ignored, but it is necessary in order to judge whether there is equilibrium from the categories of final uses, in the case when the magnitude of demand is limited An example of this would be where there is equilibrium the obtained quantity of commodities for final uses must be less or equal to their maximal quantity.

Here also, it is necessary to stress that contrary to the quantitative models from the demand side, for every new I-O, which is obtained according to a new magnitude of primary factors, the input coefficients (*A* and *C*) are changed, while the allocation coefficients (*H* and *T*) are not changed (see (2.5)).

4b Price Equilibrium and Consequently General Equilibrium for I-O in Monetary Terms from the Demand Side

Before describing these types of models, it is necessary to point out that Ghosh did not consider them; they have however appeared in our works (Davar, 1989 & 1994). Price equilibrium for I-O in monetary terms from the demand side also consists of two systems according to Table 4:

| | |
|---|---|
| $X * \widehat{(\pi^d)}$ | $\bar{x}^d$ |
| $\bar{x}^s$ | |

| | | |
|---|---|---|
| $Y * \widehat{(\mu^d)}$ | $y^d$ | $x^d$ |
| $y^s$ | | |

| | |
|---|---|
| $\widehat{(\lambda^d)} * V$ | $v^d$ |
| $v^s$ | |
| $x^s \widehat{(\pi^d)}$ | |

Table 4 The General Equilibrium of the Input-Output in money terms and latent prices, i.e., $(x^s)' = x^d$.

$$\pi^d = H\pi^d + T\mu^d \text{, or } (I-H)\pi^d = T\mu^d \text{, or } \pi^d = DT\mu^d, \tag{4.3}$$

$$\hat{\pi}^d (v^s)' = V' (\lambda^d)', \tag{4.4}$$

Thus, if the latent demand prices for categories are given, then using the system (4.3) the latent demand prices of commodities are obtained, under certain conditions Consequently, by substituting the latter prices in the system (4.4) the latent demand prices of factors are obtained. Thus the two systems establishes a connection between categories' latent demand prices and factors' latent demand prices due to the latent demand prices of commodities. For the equilibrium state (included basis year I-O table) to exist, all latent demand prices of categories, commodities and factors must equal one.

It is necessary to stress that for every new I-O, is compiled according to the new latent demand prices of commodities obtained by using the new latent demand prices of categories of final uses, the direct input coefficients (*A* and *C*) are not changed, while the direct allocation coefficients (*H* and *T*) are changed; which is contrary to the supply side prices models. Because of this the latent demand prices are related to categories and for commodities (sectors) via columns. This is to be distinguished from (2.6)-(2.9) where the latent demand prices are related to them via rows despite this these new latent prices were obtained on basis of current direct allocation coefficients.

## 5 The Relationship between Leontief's and Ghosh I-O System Models

Since the appearance of Ghosh's allocation model in the I-O literature the following problems have been discussed. First, the relationship between them, i.e., between Leontief's and Ghosh's models and second, consequently, whether they might be equivalent; finally, the plausibility of Ghosh's quantitative model and the demand price model. The first two problems must be separated from the problem of plausibility. These models might not be equivalent but Ghosh's model might be plausible in practice.

It is necessary to point out that Ghosh discussed these problems from the very beginning. Firstly; he claimed that there is a difference between Leontief's I-O model and his allocation model. He wrote: 'The allocation model is formally similar to the input-output model, but a certain amount of care is needed to avoid confusion (Ghosh, 1964, p.113)' Secondly, he discussed the conditions and where his allocation model should be used. He wrote: 'It is possible to build up a similar model with allocation functions in an economy where different sectors are under monopoly control and all except one resource is scarce. We can consider a planned economy under centralized control with scarce material resources and productive capacity with ample supply of available labour (Ghosh, 1958, p.59)'. So, from these statements we can conclude that, Ghosh not only understood the essential difference between these I-O systems, but also formulated clearly the conditions and goals when the allocation model was to be used.

In this paper, as has been mentioned above, we shall concentrate our attention only on the two first problems As to the plausibility of Ghosh's I-O system models, the current stage of knowledge (research), in our opinion, is not sufficiently advanced so as to decide whether it is plausible or not, i.e., there additional research is needed.

One of the main reasons for preferring, Leontief or Ghosh's model, is the problem of the stability of direct input coefficients and direct allocation coefficients. Ghosh writes: 'Actually, allocation coefficients may prove to be more stable over a short period than technical coefficients, because rationing authorities, ones the relative shares of each industry have been settled, dislike changing them since they are the outcome of a delicate balance of claims and counterclaims (Ghosh, 1964, p.112)'. But as it has been shown above none of these coefficients are generally stable. Therefore, the problem was transformed (transmuted) to whether these models are equivalent.

For example, Dietzenbacher (1997) tried to prove that Ghosh' quantitative model is equivalent to Leontief's supply price model. At the same time he claimed that his demand price model according to Ghosh's model is also equivalent to the Leontief's quantity model. Dietzenbacher writes: 'In this paper it was shown that the supply-driven input-output model (Ghosh' model – Ezra D.) yields exactly the same results as the Leontief price model. … Therefore, this paper suggested using the term Ghosh price model instead of supply-driven model. … The results for a demand-pull are the same for both models. The Leontief quantity model drives the output values from exogenous final demands, whereas the Ghosh model obtains quantity ratios for the outputs from quantity indexes for the final demand. Since prices are fixed, there is a one-to-one correspondence between output values and quantity ratios (Dietzenbacher, 1997, p.645)'. In the following it will be shown that these conclusions are generally incorrect.

Before discussing these problems, let us clarify the issue of whether Ghosh's model complements Leontief's model? Since, the basis of both models is an Input-Output system many authors have changed Ghosh's original title of it being an "allocation model" to a "supply-driven (supply)" model parallel to the "demand-driven (demand)" model of Leontief. This would be correct only if the results of those models are the same (equivalence) for identical assumptions. Fortunately, it is easy to show that these results are generally different (vide infra; see also Oosterhaven, 1988). Thus, Ghosh's statement that these models must not be confused is valid.

Careful analysis of the latent supply price model and the supply quantitative model (Ghosh's model) allows us to conclude that the results of both models would be equivalent only in the case when the total inputs of all factors ($v^s$) after change are equivalent for every sector in both models. This means that when there is a change in the latent prices of factors or when there is a change in the input quantities of factors for sectors, the structure of the factors' inputs is not changed, i.e., all components of every sector are changed by the same rate for both of these changes, thus results of both models are equivalent. However, this condition might be satisfied, in the general model, that is, for all types of value-added matrix, i.e. a rectangular matrix, when the latent supply prices of all factors are changed by the same rate and the total supply inputs of factors ($v^s$) for all sectors are also changed by the same rate. In a partial model, when the value-added matrix $V$ is a diagonal matrix, as was originally discussed by Dietzenbacher, then the results would also be equivalent. This occurs only when the latent prices of different factors and quantities of factor inputs for different sectors change their various rates, and when the ordinal number of the factor must be identical with the ordinal number of the sector, since in this case the number of factors is equal to the number of sectors. For example, when there is a change in the latent price of the first factor there must consequently be a change in the first sector's quantity of factor input and so on. Because of this, in this partial model every row and column consists of only one element, and therefore the structure of every sector is not connected to the structure of other sectors by the means of factors and their prices. However, even in these two unrealistic and exceptional models, it would be incorrect to say that these two system models are equivalent (vide infra) in an economic sense. This is because in the first model, change is related to the latent price of factors, and in the latter model, change is related to the quantities of factors for sectors.

Therefore, a careful analysis both, of the theoretical proof and numerical illustrations of equivalence between Ghosh's supply quantitative model and Leontief's supply prices model, we can see that they are both in a broad sense a starting point for the I-O albeit in a simplified form. Furthermore it has been assumed that 'The row vector $\mathbf{v'_0}$ gives the value added in each sector, containing for example the payments for primary factors labor and capital (Dietzenbacher, p.432)'. But on the next page he writes 'For the simplicity, assume that the value-added vector consists only of wages (have to be: the payments for labor (quantity of labor × wage). Suppose that in each sector the workers require a raise (which may differ across the sectors) in their wages. Let the increases be such that the new wage bill is given by the vector $v'_1$. The sector-specific percentage increases may be obtained from the sector-specific wage ratios, which can be expressed as $v' = v'_1 v_0^{-1}$. The direct requirements (per dollar of output) of labour (or, in general, value added) are give by the diagonal matrix $L_0 = v_0 x_0^{-1}$ (ibid., p. 633)'. This means that the row vector of value added becomes the diagonal matrix (vide infra). This is not just for simplicity's sake as the author noted, but is in order to demolish the linkage between sectors by means of value added and make a possibility changing in the different rate of the factors input for various sectors and of the latent prices for various factors. It is necessary to stress that, as has been shown above, this is a partial model. However, the results obtained for the partial model is not always correct for the whole (general) model. Unfortunately, the results of the partial model were interpreted for a general model (see the Summary and Conclusions of the paper, where it is not mentioned that this result is correct only in the case when the value-added matrix is considered as a diagonal). In other words, the results of a partial model were used for the general model. This is incorrect from the point of scientific methodology. It is necessary to point out that the stability and joint stability theorems of input and allocation coefficients are considered either for a diagonal matrix or when all final demands (or alternatively all value added) are changed by the same percentage (Dietzenbacher, pp. 642-643). It must be stressed that the stability and joint stability of coefficients do not guarantee that the results will be equivalent for Ghosh's supply quantitative model and Leontief's supply price model (vide infra).

It is incumbent upon us now to show that when the value-added is presented as a matrix which is not diagonal, though as a square matrix, the result is different. As an illustration of this the hypothetical I-O table from the appendix of Dietzenbacher's paper is used:

(a) the basic version (ibid. p.648)

| | | | |
|---|---|---|---|
| 800 | 800 | 400 | 2000 |
| 500 | 2000 | 1500 | 4000 |
| 700 | 1200 | - | 1900 |
| 2000 | 4000 | 1900 | |

(b) the using version (ibid. p.649)

| | | | |
|---|---|---|---|
| 800 | 800 | 400 | 2000 |
| 500 | 2000 | 1500 | 4000 |
| 700 | - | - | 700 |
| - | 1200 | - | 1200 |
| 2000 | 4000 | 1900 | |

Let us now divide the value added row and present it as two rows, so

| | | | |
|---|---|---|---|
| 800 | 800 | 400 | 2000 |
| 500 | 2000 | 1500 | 4000 |
| 400 | 200 | - | 600 |
| 300 | 1000 | - | 1300 |
| 2000 | 4000 | 1900 | |

Here the value added matrix $V$ is presented as:

$$V = \begin{matrix} 400 & 200 \\ 300 & 1000 \end{matrix}$$

And the sum of columns $v_0^s$ is the same (700 1200) as in the original example.

Firstly, we will show that Ghosh's model and Leontief's demand model are not complimentary models. For the first example it was assumed that the value-added vector is increased by 20 percent for the first sector and 5 percent for the second sector. Thus the according results were: $x_1^s$ = (2310; 4368) and $y_1^d$ = (462; 1638)'; (Dietzenbacher, P.648). Now, let us determine the total outputs for the new obtained final uses $y_1^d$ by means of Leontief's demand model – $(x_1^d)' = [(I-A_0)^{-1}\, y_1^d]'$= (2234.4; 4393.2), **which is not equal to $x_1^s$**.

Secondly, let us study the original case when value added is the row vector, i.e., there is only one factor. In this case if the rate of increase of the latent price of factor is 20 percent then of course the latent prices of all commodities will also increase by 20 percent and will be different from the prices of the text.

Thirdly, let us assume as Dietzenbacher did that the first factor's new price is 1.2, i.e., it is increased by 20%, and the new price of the second factor is 1.05, i.e., it is increased by 5 %. In such a case the new prices of products would be:

$$\pi_1^s = \lambda_1^s L_0 (I-A_0)^{-1} = (1.2\ \ 1.05) \begin{matrix} 0.2 & 0.05 \\ 0.15 & 0.25 \end{matrix} \begin{matrix} 2 & 0.8 \\ 1 & 2.4 \end{matrix} = (1.2\ \ 1.05) \begin{matrix} 0.45 & 0.28 \\ 0.55 & 0.72 \end{matrix} = (1.118\ \ 1.112)$$

Therefore these latent prices of commodities $\pi_1^s$ (1.118; 1.112) differ from the prices of Dietzenbacher, where ($\pi_1$ = 1.155; and $\pi_2$ = 1.092); and therefore, the new outputs x'= (2000 * 1.118; 4000 * 1.112) = (2236; 4448), also differ from the original result where the total outputs are (2310; and 4368).

Moreover let us assume that the latent prices of all factors and the quantities of factors input for all sectors are increased by 20 percent. In this case it is clear that the latent prices of all commodities and consequently the total output of all sectors will increase by 20 percent, i.e., $\pi_2^s$ (1.2 1.2) and $x_2^s$ (2400 4800). However, if the rate of change of the latent supply prices of all factors differs from the rate of change of factor inputs for all sectors then the results will also different. This is despite the fact that, both direct coefficients (inputs *A* and allocation *H*) are stable, as they were in the previous case. This means that the stability of both coefficients does not guarantee that the results of these two models will be equivalent.

Furthermore, the same is also true for the results of the Example 2 (ibid. pp.649-650) where I-O in physical terms is considered by Dietzenbacher. If instead of the diagonal matrix

$$\underline{L}_0 = \begin{matrix} 100 & - \\ - & 1200 \end{matrix} \text{ (ibid. pp.649-650) we used matrix } \underline{V}_0 = \begin{matrix} 30 & 200 \\ 70 & 1000 \end{matrix}$$

then new prices would be $p^s$ = (150; 75) $\underline{C}_0\, (I-\underline{A}_0)^{-1}$ = (125; 65). This differs which from the results Of Dietzenbacher (120; 57). Consequently, the value of outputs also differs and all the following computations and conclusions are irrelevant.

Finally, in Example 3 (ibid. pp.650-651), where it 'illustrates the equivalence between the standard Leontief quantity model and the Ghosh quantity model' there are two points worth noting. Firstly, the discussed model is not Ghosh's quantity model, but the latent demand price model according to Ghosh's I-O system models (vide supra); and secondly, if instead of diagonal matrix $D_0$ (ibid. p.651) we used any rectangular matrix (not diagonal) the results would also differ:

2.0 1.6 0.05 0.15 1.25 1.229

$$\pi_1^d = (I\text{-}H_0)^{-1}\, T_0\, \mu_1^d = \; 0.5 \;\; 2.4 \quad 0.3 \;\; 0.075 \quad 1.2 = \; 1.237$$

Therefore, the results, here, also differ from the results of the texts, where (1.22; 1.205)'.

On the basis of the results of the above illustrations we can conclude that the results of Leontief's supply price model and Ghosh's model are equivalent only in two unrealistic and unusual cases. Therefore the suggestion to replace the title of Ghosh's quantitative model by the Ghosh price model, as was suggested by Dietzenbacher, is both erroneous and unworthy of consideration.

## Conclusions

The paper shows that Leontief's Input-Output system model differs from Ghosh's system, therefore they cannot be equivalent. Even in two unrealistic and unusual cases, namely:

(1) When the value added matrix (or the final uses matrix) is a diagonal matrix, and the rate of change is different for the various factor's latent prices (or category's latent prices) and for the sector's input of factors (or sector's output for categories). In this case the ordinal number of the factor (or category) must be identical to the ordinal number of the sector.

(2) When these matrixes are rectangular and all prices and all quantities, in both cases, are changed by the same rate. Even though the results of these models are formally equivalent, it would be inaccurate to say that these models are equivalent, from the point of view of methodology. This is because in the first model, change is related to the latent price of factors, and in the latter model change is related to the quantities of factor inputs for sectors.

Therefore, to replace Ghosh's quantitative model with the Ghosh price model, as was suggested by Dietzenbacher, is both erroneous and unworthy of consideration.

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[1] Therefore, we use the label 'supply price model' and 'demand model', for both types of input-output to stress their organic connection to the general equilibrium model of Walras, because such a connection is sometimes doubted (denied) (see Kurz & Salvadori).

[2] I also, unfortunately, finally called Ghosh's model an output model, despite that in my first paper (Davar, 1989) the distribution and output coefficients are equivalently used, but in my book (Davar, 1994) only the ones I mention are distribution coefficients and after that 'output coefficients' and 'output models' were used.

[3] There is certain confusion in the determination and interpretation of allocation coefficients. Even Ghosh firstly called them "supply coefficients" (Ghosh, 1958, p.61). Almost all post-Ghosh' economists (Augustinovics, 1970; Dietzenbacher, 1997; Oosterhaven, 1988, & 1996; Davar, 1989 &1994) used the alternative title "output coefficients" or alternatively "distribution coefficients". Yet, our interpretation of allocation coefficients is similar to other interpretations (Ghosh, 1964, p.113; Dietzenbacher, p.632), but clearer and easier to understand.

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