INSIDE LAB 4: Determining the Masses of the Earth and Jupiter

OBJECTIVE: *To determine the mass of the Earth by measuring the acceleration of gravity on the surface of the Earth and to determine the mass of Jupiter using the orbits of its four Galilean moons.*

MEASURING THE EARTH'S MASS:

Newton's law of gravity states that the acceleration of gravity, g, at a distance d from the center of a spherically symmetric sphere of matter whose mass is M, is given by $g = GM/d^2$. Here G is a universal constant equal to 6.673×10^{-11} $m^3/kg/s^2$. To a good approximation, the Earth is a sphere, so near its surface the distance d is just the radius of the Earth, R_E. We can solve the above equation for the mass M_E of the Earth:

$$M_E = gR_E^2 / G$$

Since we know the value of G and we know the radius of the Earth (it was first measured by Eratosthenes), then if we measure g, we can compute the mass of the Earth.

One way to measure g is by measuring the acceleration of a falling object.

MEASURING THE MASS OF THE EARTH:

1. Obtain an iPad, ball and meter stick. After finding a suitable location in front of a blank wall, place the meter stick parallel to the iPad and in the same plane of motion as the falling ball.
2. Open the *Video Physics* app from the home screen. From the *Videos* screen, tap the + icon in the upper left corner of the screen and select *Take a Video*.
3. Have one lab partner stand beside the meter stick and hold the ball up above their head. Be certain that you can see the entire meter stick, the ball and the floor before tapping the red record button.
4. After recording has begun, have your partner release the ball. Be sure to hold the iPad steady and parallel to the falling ball. Do not pan or tilt the iPad during recording.
5. Tap the record button again to stop recording. Once you are satisfied with your video, select *Use*.
6. The video transport controls should be visible at the bottom of the screen. If not, drag up from the bottom to expose the controls. Jog the video (drag the dot indicating the part of the video you are seeing) to find the frame that contains the beginning of the motion of the ball.
7. Select the *Origin & Scale* axis tool near the top and move the crosshairs to your origin and line up the direction of motion with the y axis (this ensures you have motion only in 1 direction.)

8. Next, we will let the software know the scale of the images using the barbell looking tool (a white line with two circles at either end.) Move the circle indicating one of the endpoints of the white line to one end of the meter stick. Now, move the circle indicating the other end of the white line to the other end of the meter stick to set the scale length. Be sure to verify the length and units in the gray box near the top of the screen are correct.
9. Select the *Points* tool and drag the crosshairs over the object. Tap once anywhere in the circle to mark the position of the ball. You don't want to start tracking the position until the ball has started moving. The video will automatically advance one frame and you will again move center the crosshairs on the object and tap.
10. Repeat this process until the ball reaches the floor. You will notice the ball will begin to blur as it accelerates. Be consistent in choosing the leading section of the ball.
11. If you make a mistake, simply go back one frame, adjust the crosshairs and tap again. To delete a point, move to the frame such that the point is highlighted yellow, tap and hold until you see the *Delete* pop-up.
12. Once you have completed the position marking, tap the *Share* menu (a box with an upward pointing arrow) in the upper right-hand corner of the screen and choose *Data File* and then select *Open in.*
13. Choose *Import with Graphical* and the Graphical Analysis app will open with two graphs visible.
14. Tap the *Views* menu icon (the second icon from the right in the upper right-hand corner) and select *1 Graph*.
15. Tap the vertical axis label and check the *Y Velocity* and uncheck everything else. Next, tap the axis again to slide the graph back to full screen.

    a. Why are we using the Y velocity?

    b. Do you expect the X velocity to change and why or why not?

16. The average acceleration for our case is simply the change in velocity divided by the change in time or $a = \frac{v_f - v_i}{t_f - t_i}$. Where v_f is the final velocity and v_i is the initial velocity, and similarly t_f and t_i are the final and initial time respectively. Using the data on your graph find the average acceleration.

17. A slightly better way to find the acceleration when errors in the measurement are introduced is to use a curve fitting tool. Tap and drag across the data to select the region of interest. Then tap inside this region to bring up the *Selection Options* menu.
18. Scroll through the *Curve Fit* menu and select *Linear*. Next, tap anywhere outside of the *Curve Fit* box.
19. The slope of your velocity vs. time graph will give your average acceleration. Record this value in the blank below. Compare this value to the accepted acceleration due to gravity value (9.8 m/s^2).

    Gravitational acceleration: g = _____________ m/s^2

    Fractional error $= \frac{|g| - 9.8\ \text{m/s}^2}{9.8\ \text{m/s}^2} \times 100\%$_________ %

20. Calculate the mass of the Earth using $M_E = gR_E^2/G$, where $R_E = 6.371\times10^6$ m, and $G = 6.673\times10^{-11}$ m^3/kg/s^2. Compare with the actual value M_{actual} from your text or some other source.

    Earth's Mass $M_E = \frac{gR_E^2}{G} =$__________ kg

    Fractional error $= \frac{M_E - M_{actual}}{M_{actual}} \times 100\% =$_______ %.

21. How well did you expect your value of g to agree with the known value and why?

22. Delete from the iPad the video you have created during this lab.

MEASURING THE MASS OF JUPITER:

Newton's version of Kepler's third law states that

$$(M_1 + M_2)P^2 = \frac{4\pi^2}{G}(a_1 + a_2)^3 .$$

This formula applies to any two objects which are in orbit about their common center of mass. Recall that the center of mass is the point at which you would place the fulcrum of a "seesaw" if the two objects were on either end of it and you wanted to balance them. F

We can use this formula to determine the mass of Jupiter from the orbits of its four Galilean moons. We will apply it to each moon separately.

1. First, we must simplify the formula.

   a) If $M_1 = M_J$ = mass of Jupiter and M_2 is the mass of one of its moons, what do you know to be true about the relationship between these two masses? How can you use this to simplify the above formula?

   b) Given your result about the relationship between the two masses, what do you think will be true about the motion of Jupiter versus the motion of its moon? How can you use that to simplify the above formula?

2. Start up the program *Stellarium*. Remove the atmosphere and ground by pressing 'a' and 'g' respectively or use the bar that appears in the lower left by hovering over the date and time with the mouse pointer. for this portion of the lab.

3. Find Jupiter, 'command' plus 'F' or 'control' plus 'F' on your keyboard, and press enter. Press the ocular view, the left most button in the bar in the upper right of the screen. The ocular view simulates the view you would see through a specific telescope and lens combination. Choose Ocular #1, Telescope #0, and Lens #1 to get a clear view of Jupiter and it's moons(You can use the small arrow keys in the drop down to change these).

4. Pick one of the moons to watch first, say Io. We wish to carefully record exactly how long it takes Io to orbit Jupiter. To determine this distance, note carefully the day and time when Io first passes behind Jupiter (or as close to it as possible). You can use the 'J'(to slow the rate of time) and 'L'(to speed up the rate of time) keys on your keyboard to control the rate of time. You may need to stop the time by pressing '0' on the keyboard. Continue the animation until Io passes behind Jupiter again from the same direction *(we can't distinguish going in front from going behind)*, and again record the day and time. Take the difference to determine the total number of days, hours, and minutes. (Note: Jupiter will appear dark in the software during the daytime.) Record all in the table below.

5. Repeat part 4 for the other three Galilean moons listed.

6. Look up the semi-major axes of these moon's orbits in the table below and convert these distances into m.

7. For each moon, calculate the mass of Jupiter using the formula you found in part 1. Use $G = 6.673 \times 10^{-11}\ m^3/kg/s^2$

Name of Moon	**Semimajor Axis (km)**	**Semimajor Axis (m)**	**Period (d, h m)**	**Period (s)**	**Mass of Jupiter (kg)**
Io	421,800				
Europa	671,100				
Ganymede	1,070,400				
Callisto	1,882,700				

8. What is the average mass that you find for Jupiter? Compare this with the value of Jupiter's mass from your textbook or some other source.

Jupiter's Mass M_J = __________ kg

Fractional error = $\frac{M_J - M_{actual}}{M_{actual}} \times 100\% =$ _______ %.