# Integers

2

## Unit Overview

In this unit, you will study negative numbers, and you will learn to add, subtract, multiply and divide them. You will graph positive and negative numbers on number lines and on the coordinate plane.

## Key Terms

As you study this unit, add these and other terms to your math notebook. Include in your notes your prior knowledge of each word, as well as your experiences in using the word in different mathematical examples. If needed, ask for help in pronouncing new words and add information on pronunciation to your math notebook. It is important that you learn new terms and use them correctly in your class discussions and in your problem solutions.

### Academic Vocabulary

- elevation

### Math Terms

- absolute value
- integer
- opposite
- additive inverse
- coordinate plane
- origin
- quadrants
- ordered pair
- reflection

## ESSENTIAL QUESTIONS

- How can integers be represented visually and how can operations with integers be represented with models?
- How are positive and negative numbers used in real-world situations?

## EMBEDDED ASSESSMENTS

These assessments, following activities 8 and 10, will give you an opportunity to demonstrate your understanding of operations with integers to solve mathematical and real-world problems.

**Embedded Assessment 1:**

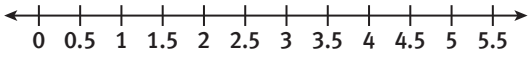
The Number Line and Adding and Subtracting Integers p.107

**Embedded Assessment 2:**

Coordinate Plane and Multiplying and Dividing Integers p.127

# Getting Ready

**1.** Plot the numbers on a number line. Label each number.



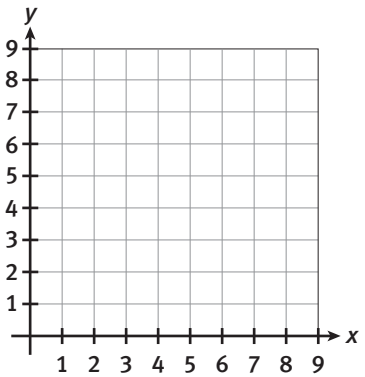

**a.** 4
**b.** $1\frac{1}{2}$
**c.** 2.4

**2.** Create a visual representation to illustrate each of the following.
**a.** the number of pieces of fruit in a basket if there are 3 apples, 2 pears and 4 oranges in the basket
**b.** having 6 pencils and giving your friend 2 of them

**3.** Put the following numbers in order from smallest to largest
**a.** $140 + 38 + 36 - 85$
**b.** $2 \times 21 \times 3$
**c.** $2048 \div 16$
**d.** $150 - 67 + 53 - 9$

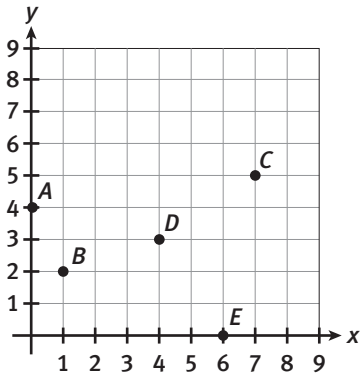
**4.** The numbers 3 and 10 are on a number line.
**a.** Use a model to explain how to find the distance between the numbers.
**b.** Explain in words how to find the distance between 3 and 10 on the number line.

**5.** Plot the following points on the coordinate plane. Label each point.




**a.** $A(1, 4)$
**b.** $B(5, 3)$
**c.** $C(6, 6)$
**d.** $D(0, 2)$
**e.** $E(7, 0)$

**6.** Tell the coordinates of the points on the graph.




**7.** Olivia is getting ready to host a party.
**a.** She bought 13 boxes of cookies and each box has 24 cookies in it. How many cookies does she have?
**b.** She bought 3 packages of invitation and each pack had 15 invitations in it. After she mailed the invitations she had 7 invitations left. How many invitations did she send?

**8.** There are 265 sixth grade students at Rocky River Middle School. Each student participates in exactly one sport. 53 students play tennis, 84 play basketball, 53 play lacrosse, and the rest of the students participate in track.
**a.** Explain in words one way to find the number of students who participate in track.
**b.** Give a visual representation of a different way to find the number of students who participate in track.

# Introduction to Integers

## Get the Point?

## Lesson 7-1 Integers and the Number Line

My Notes

### Lesson Targets:

- Use integers to represent quantities in real-world contexts.
- Position and identify integers on a number line.
- Find the opposite of an integer.
- Find the absolute value of an integer.
- Classify whole numbers, integers, and positive rational numbers.

**SUGGESTED LEARNING STRATEGIES:** Marking the Text, Summarizing, Note Taking, Create Representations, Discussion Groups

Ms. Martinez has a point system in her classroom. Students earn points for participation, homework, teamwork, and so forth. However, students lose points for inappropriate behavior or not completing assignments. At the end of each term, students in the group with the most points receive a book or a DVD. She assigns a letter to each student for tracking his or her total points. One student is letter *A*, the next is *B*, and so on.

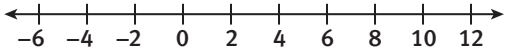
**1.** This table shows each student's total points at the end of the week. Your teacher will assign you a letter.

| *A* | *B* | *C* | *D* | *E* | *F* | *G* | *H* | *I* | *J* | *K* | *L* |
|---|---|---|---|---|---|---|---|---|---|---|---|
| −3 | 3 | 8 | −1 | 0 | −5 | −6 | 10 | 7 | −4 | 1 | 2 |

| *M* | *N* | *O* | *P* | *Q* | *R* | *S* | *T* | *U* | *V* | *W* | *X* |
|---|---|---|---|---|---|---|---|---|---|---|---|
| −3 | −2 | 12 | 1 | −7 | 2 | −1 | 6 | −4 | −1 | 9 | 3 |

**a.** Write the total points and the letter assigned to you on a sticky note and post it on the class number line.

**b.** Copy the letters from the class number line to this one.




**c.** What do the numbers to the right of zero on the number line represent?

**d.** What do the numbers to the left of zero represent?

**e.** Explain how you knew where to place your number on the number line.

My Notes

**f.** Student *E* was in class for only 2 days during the week. On the first day, *E* was awarded points. On the second day, *E* lost points. Explain how *E*'s score can be zero.

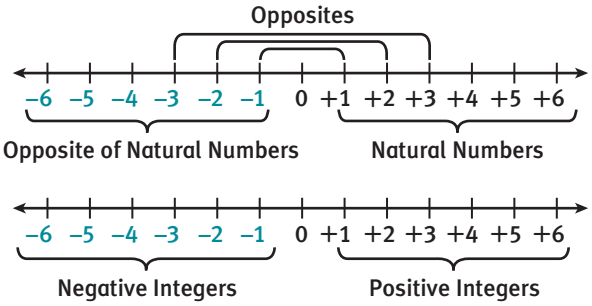
**WRITING MATH**

Positive integers are written with or without a plus sign. For example, +2 and 2 indicate the same number. Negative integers are written with a negative sign. For example, −2.

**WRITING MATH**

To indicate the opposite of a number, place a negative sign in front of the number. The opposite of 4 is −4. The opposite of −4 is −(−4) = 4.

The number lines below give visual representations of integers. Notice that zero is the only integer that is neither positive nor negative. **Integers** are the natural numbers (1, 2, 3, . . . ), their opposites, and zero. The opposite of 0 is 0.




**2.** Ms. Martinez uses negative numbers to represent points lost by students. Work with your group to name at least three other uses for negative numbers in real life. As you discuss the uses, ask your group members for clarification of any terms you do not understand.

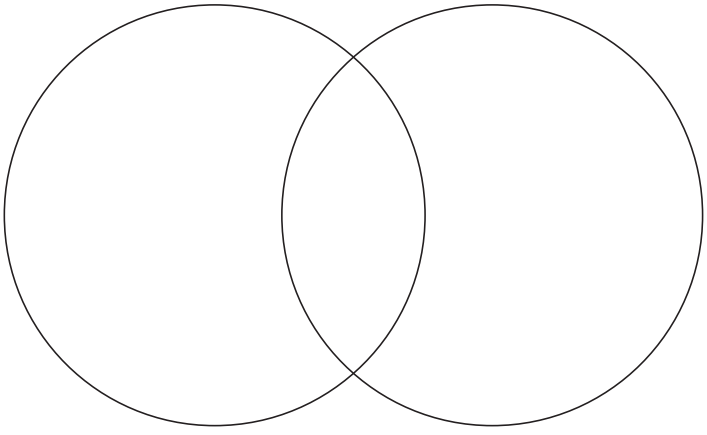
**MATH TERMS**

The symbol ℤ is often used to represent the set of integers. This is because in German, the word *Zahl* means "number."

Numbers may belong to more than one category or set.

- All whole numbers are also integers.
- All whole numbers are also positive rational numbers.
- Some integers are positive rational numbers.
- Some positive rational numbers are integers.

**3. Model with mathematics.** Place the following on the Venn diagram below to show how they are related: Whole Numbers; Integers; Positive Rational Numbers.



My Notes

Numbers that are the same distance from zero but are on different sides of zero on a number line are ***opposites***.

**4.** Recall that the table shows each student's total points at the end of the week.

| *A* | *B* | *C* | *D* | *E* | *F* | *G* | *H* | *I* | *J* | *K* | *L* |
|---|---|---|---|---|---|---|---|---|---|---|---|
| −3 | 3 | 8 | −1 | 0 | −5 | −6 | 10 | 7 | −4 | 1 | 2 |

| *M* | *N* | *O* | *P* | *Q* | *R* | *S* | *T* | *U* | *V* | *W* | *X* |
|---|---|---|---|---|---|---|---|---|---|---|---|
| −3 | −2 | 12 | 1 | −7 | 2 | −1 | 6 | −4 | −1 | 9 | 3 |

Find three pairs of students with scores that are opposites. Explain your reasoning.

**5.** Look at the points for Students *A* and *B*.

**a.** How many points does *A* need to *earn* to have a total of 0? Explain.

**b.** How many points does *B* need to *lose* to have a total of 0? Explain.

**c.** What do you notice about both student *A*'s and student *B*'s point distance from zero?

The ***absolute value*** of a number is the distance of the number from zero on a number line. Absolute value is always positive, because distance is always positive. The symbol for absolute value is a vertical bar on each side of a number. For example, $|3| = 3$ and $|-3| = 3$.

**6.** Why is distance always positive? Use an example in your explanation.

**READING MATH**

Read $|6| = 6$ as the absolute value of 6 is 6. Read $|-6| = 6$ as the absolute value of negative 6 is 6.

**7.** From the ground floor of the school, Ms. Martinez goes down 1 flight of stairs to get to the basement.

**a.** Write an integer to represent the situation.

**b.** What is the opposite of the situation above? What integer represents this situation?

**c.** Explain what 0 represents in this situation.

**d.** Explain what the absolute value | 1 | represents in this situation.

My Notes

## Check Your Understanding

**8.** The football team loses 5 yards on first down.
- **a.** Write an integer to represent this situation.
- **b.** Write an integer to represent the opposite of the situation in part a.
- **c.** What does 0 mean in this situation?

**9.** Write the opposite of each integer.
**a.** 9 **b.** 0 **c.** −12

**10.** Write each absolute value.
**a.** $|10|$ **b.** $|-8|$ **c.** $|0|$

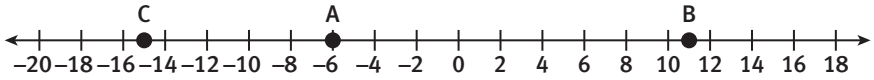
**11. Reason quantitatively.** What values can $x$ have if $|x| = 8$?

## LESSON 7-1 PRACTICE

**For Items 12–14, locate each integer on a number line.**

**12.** −4 **13.** 7 **14.** −9

**For Items 15–17, identify the integer for each point.**




**15.** *A* **16.** *B* **17.** *C*

**For Items 18–20, write the opposite of each integer.**

**18.** 23 **19.** −41 **20.** −78

**For Items 21–23, write an integer to represent each situation. Explain the opposite of the situation and write an integer to represent the opposite. Then explain what 0 means in each situation.**

**21.** temperature of 7 degrees below 0

**22.** a gain of 3 pounds

**23.** a withdrawal of $15

**For Items 24–26, write each absolute value.**

**24.** $|9|$ **25.** $|-3|$ **26.** $|-46|$

**27. Model with mathematics.** The lowest elevation of Death Valley is about −282 feet. Find the absolute value of the lowest elevation of Death Valley. Explain what this absolute value means.

**28. Make sense of problems.** The height of an iceberg above the water is 38 meters. The bottom of the iceberg is 21 meters below sea level. Write integers to represent the height and depth of the iceberg. Explain what 0 means in the situation.

**29.** The counting, or natural, numbers are 1, 2, 3,... How could you add counting numbers to the Venn diagram on page 84?

My Notes

## Lesson Targets:

- Compare and order integers.
- Interpret statements of inequality of integers in terms of a number line and of real-world contexts.
- Distinguish comparisons of absolute value from statements about the order of integers.

**SUGGESTED LEARNING STRATEGIES:** Marking the Text, Summarizing, Create Representations, Sharing and Responding, Use Manipulatives

Ms. Martinez sometimes assigns students to cooperative learning groups. She assigns each group member a role based on his or her total points. The roles are reporter (lowest total), recorder (next to lowest total), facilitator (next to highest total), and timekeeper (highest total).

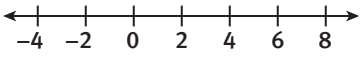
**MATH TIP**

Numbers increase from left to right on a number line.

**1.** Use the number lines to plot each student's total for Groups 1 through 4. For Groups 5 and 6, create your own number lines. Then order the points for the members in each group from lowest to highest and determine who will have each role in the group.

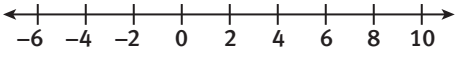
**a.** Group 1:

| *A* | *B* | *C* | *D* |
|---|---|---|---|
| −3 | 3 | 8 | −1 |



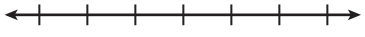

**b.** Group 2:

| *E* | *F* | *G* | *H* |
|---|---|---|---|
| 0 | −5 | −6 | 10 |




**c.** Group 3:

| *I* | *J* | *K* | *L* |
|---|---|---|---|
| 7 | −4 | 1 | 2 |



My Notes

**d.** Group 4:

| *M* | *N* | *O* | *P* |
|---|---|---|---|
| −3 | −2 | 12 | 1 |

**e.** Group 5:

| *Q* | *R* | *S* | *T* |
|---|---|---|---|
| −7 | 2 | −1 | 6 |

**f.** Group 6:

| *U* | *V* | *W* | *X* |
|---|---|---|---|
| −4 | −1 | 9 | 3 |

**2. a.** Student *T* has 4 points and Student *X* has 3 points. Who has more points? Explain.

**b.** Student *S* has −1 point and Student *P* has 1 point. Who has more points? Explain.

**c.** Student *U* has −4 points and Student *N* has −2 points. Who has more points? Explain.

**d.** Complete using $>$ or $<$.

$4 \bigcirc 3$ $\quad -1 \bigcirc 1$ $\quad -4 \bigcirc -2$

My Notes

**3.** Student $F$ has $-5$ points and Student $R$ has 2 points. Explain how to determine which student has the greater number of points.

You used a number line to compare and order integers. You can also use absolute value to compare and order integers.

## Example

Compare $-98$ and $-90$.

**Step 1:** Find the absolute value of each integer.

$|-98| = 98$ $\quad$ $|-90| = 90$

**Step 2:** Compare the distances from 0.

$-98$ is farther left from 0 than $-90$, so $-98$ is less than $-90$.

**Solution:** $-98 < -90$

## Try These

Use absolute value to compare. Write $>$ or $<$.

**a.** $-58 \bigcirc -63$

**b.** $-21 \bigcirc -19$

**c.** $-100 \bigcirc -110$

**4.** The first week a student in Ms. Martinez's class had a total of $-6$ points. The second week the student had a total of $-2$ points. In which week did the student have the greater number of points? Explain your thinking.

**5.** Student $M$ has a total of $-3$ points, Student $Q$ has $-7$ points, Student $S$ has $-1$ point, and Student $U$ has $-4$ points. Explain how to use absolute value to order each student's points from least to greatest.

My Notes

## Check Your Understanding

**6.** Explain how to use a number line to order the integers −5, 6, and −2 from least to greatest.

**7.** Explain how to use absolute value to determine which integer is greater, −43 or −39.

## LESSON 7-2 PRACTICE

**For Items 8–16, compare each pair of integers. Write > or <.**

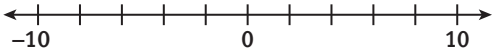
**8.** 17 ◯ 20 **9.** 0 ◯ −5 **10.** 23 ◯ −46

**11.** −9 ◯ 3 **12.** −1 ◯ −16 **13.** −82 ◯ 0

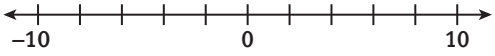
**14.** −39 ◯ −90 **15.** −4 ◯ 4 **16.** −71 ◯ −28

**For Items 17 and 18, use the number line to write the integers in order from least to greatest.**

**17.** 4, −7, 3, −2, 0




**18.** −5, 8, 2, −6, −9, 1




**For Items 19–22, write the integers in order from least to greatest.**

**19.** −8, 1, −11, 10 **20.** 3, −26, 0, −17, 15

**21.** 7, −8, 9, −16, −13 **22.** 48, −53, 32, −41, −35, 20

**23. Critique the reasoning of others.** Lexie says that −68 feet is a greater depth than −75 feet, since −68 > −75. Is Lexie correct? Explain why or why not.

**24. Make sense of problems.** In golf, the person with the lowest score wins the round. At the end of a golf round, Henry's score was 2 under par, Maria's score was 1 above par, Jorge's score was 3 under par, and Setsuko's score was 4 above par. Write each player's score as an integer. Then order the scores from least to greatest. Who won the round? Explain your reasoning.

**25. Reason quantitatively.** On Monday, the prices of four of Kyle's stocks fell. At market close, Stock A was −8 points from the opening price, Stock B was −3 points, Stock C was −12 points, and Stock D was −7 points. Which stock lost the greatest amount on Monday? Which stock lost the least amount? Explain your reasoning.

**26.** Consider the inequality −4 > −6. What does the inequality tell you about the relative positions of −4 and −6 on a horizontal number line?

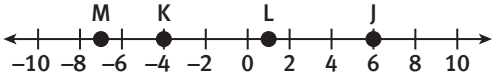
## ACTIVITY 7 PRACTICE

**Write your answers on a separate piece of paper. Show your work.**

### Lesson 7-1

1. Locate each integer on a number line.
   - **a.** 3
   - **b.** −1
   - **c.** −7
   - **d.** 8
   - **e.** 12
   - **f.** −15

2. Identify the integer for each point.




   - **a.** *J*
   - **b.** *K*
   - **c.** *L*
   - **d.** *M*

**For Items 3–6, write an integer to represent each situation.**

3. loss of 8 yards
4. change in score after an inning with no runs
5. depth of 15 meters below sea level
6. go up 4 flights of stairs

**For Items 7–11, write an integer to represent each situation. Then tell what 0 means for the situation.**

7. loss of 6 pounds
8. a withdrawal of $25
9. an elevation of 1,200 feet
10. a score of 2 under par
11. an elevator going down 7 floors
12. Write the opposite integer.
    - **a.** 2
    - **b.** −13
    - **c.** −68
    - **d.** 94
    - **e.** 187
    - **f.** −275
13. What is the value of −(−8)? Explain your answer.
14. Write the answer for items a–b.
    - **a.** Write an integer that represents the opposite of 15.
    - **b.** Write an integer that represents the opposite of the opposite of 15.
    - **c.** What is the result of finding the opposite of the opposite of a number?
15. Which integer is its own opposite? Explain your thinking.

**For Items 16–19, explain the opposite of each situation. Then write an integer to represent the opposite.**

16. a deposit of $20
17. 9 degrees below zero
18. a loss of 12 points
19. 35 feet above sea level
20. Write each absolute value.
    - **a.** $|-14|$
    - **b.** $|0|$
    - **c.** $|123|$
    - **d.** $|-80|$
    - **e.** $|-204|$
    - **f.** $\left|1\frac{1}{2}\right|$
21. Which integer is neither positive nor negative?
22. Classify each number as a whole number, an integer, or a positive rational number. Use as many ways as possible.
    - **a.** 8
    - **b.** −15
    - **c.** 0
    - **d.** $\frac{3}{4}$
23. On first down, a football team moved −15 yards. Did the team gain or lose yards on the play? How many yards? Explain.
24. The base of a volcano is on the ocean floor 20,000 feet below sea level. Write an integer to represent the depth of the volcano in the ocean. Find the absolute value of the integer and explain what it means.
25. At dawn, the temperature on the mountain peak was 3 degrees below 0. Which integer below represents this temperature?
    - **A.** −3°
    - **B.** 3°
    - **C.** −15°
    - **D.** 15°
26. Sketch a Venn diagram showing the relationship between the sets of numbers.
    - **a.** the positive integers and positive rational numbers
    - **b.** the integers and the whole numbers

## Lesson 7-2

**For Items 27–37, Write $>$ or $<$ to compare the integers.**

**27.** $6 \bigcirc -3$

**28.** $-1 \bigcirc 0$

**29.** $9 \bigcirc 7$

**30.** $-14 \bigcirc -2$

**31.** $-7 \bigcirc -5$

**32.** $12 \bigcirc 0$

**33.** $-4 \bigcirc -19$

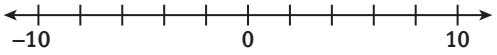
**34.** $-28 \bigcirc 1$

**35.** $0 \bigcirc -8$

**36.** $-21 \bigcirc -17$

**37.** $-49 \bigcirc -50$

**For Items 38–40, use the number line to write the integers in order from least to greatest.**




**38.** 6, −2, 4, −5

**39.** −4, −8, 3, 0, 7

**40.** 10, −1, 5, −9, −6

**For Items 41–45, write the integers in order from least to greatest.**

**41.** −2, 8, −12, 3

**42.** 10, −9, 14, −5, −1

**43.** −30, −25, 20, −50, 35

**44.** 7, 14, −21, 35, −42

**45.** −40, −32, 8, −48, 24

**46.** Which choice below shows the integers in order from least to greatest?
**A.** 3, −3, −1, 1
**B.** −1, −3, 1, 3
**C.** −3, −1, 1, 3
**D.** 1, 3, −3, −1

**The table shows the high and low temperatures for 5 consecutive days in February at North Pole, Alaska. Use the table for Items 47–49.**

| | Mon | Tues | Wed | Thurs | Fri |
|---|---|---|---|---|---|
| **High** | 1 | −29 | −27 | 5 | 7 |
| **Low** | −13 | −45 | −54 | −2 | 1 |

**47.** Order the high temperatures from warmest to coldest over this five-day period. Which day had the warmest high temperature?

**48.** Order the low temperatures from coldest to warmest over this five-day period. Which day had the coldest low temperature?

**49.** Is the order of days from warmest to coldest daily low temperatures the same as for the daily high temperatures? Explain.

**50.** Use the absolute value of the given integer to explain what each statement represents.
**a.** The elevation of the cave is below −45 feet.
**b.** The temperature is below −10 degrees.

**51.** A diver made two dives. The first dive was to a sunken ship located at a depth of −45 feet. The second dive was to a coral reef at a depth of −29 feet. Which was the deeper dive, to the sunken ship or to the coral reef?

**52.** Randy played four rounds of golf on his vacation. Randy's score for his first round was 6 above par, for his second round 1 above par, for his third round 2 under par, and for his last round 1 under par. Order his scores from least to greatest. Which round was Randy's best round?

**53.** Which integer is not between 2 and −18?
**A.** −9 **B.** 0
**C.** 3 **D.** −10

**54.** Consider the inequality $-8 < -2$. What does the inequality tell you about the relative positions of -8 and -2 on a horizontal number line?

### MATHEMATICAL PRACTICES
**Reason Abstractly and Quantitatively**

**55.** What is the opposite of $|-6|$? Explain your reasoning.

# Adding and Subtracting Integers

## What's the Temperature?

## Lesson 8-1 Using Models to Add Integers

**Learning Targets:**

- Using models, create several representations of a given integer.
- Using models, add any two integers with absolute value less than 10.

**SUGGESTED LEARNING STRATEGIES:** Summarizing, Create Representations, Use Manipulatives

Students are tracking winter temperature changes throughout their state for a class science project. They will determine temperatures at different times as the temperature rises or falls. To find the temperatures, they will have to add integers. Models can be used to add integers, but first the students must create models to represent the integers.

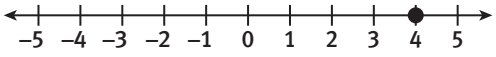
Two ways to represent integers with models are on a number line or with counters.

### Example A

Represent the number 4 using counters and on a number line.

**Solution:** Line up 4 positive counters to represent 4. ⊕⊕⊕⊕

Place a dot at 4 on a number line.




### Try These A

Represent each integer below using both a number line and counters. You may draw the counters or use counters from your teacher.

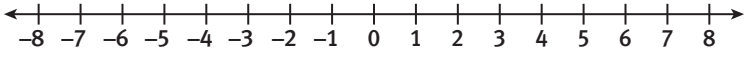
**a.** 7 **b.** −3

**1.** This morning, the temperature was 2°F at 6 A.M. The temperature rose 3°F by 7 A.M. What was the temperature at 7 A.M.?

**a.** Use counters to represent the addends of the problem.

**b.** Explain how these counters model $2 + 3 = 5$.

**c.** Explain how the number line models $2 + 3 = 5$.




My Notes

**MATH TIP**

A positive counter, ⊕, represents 1. A negative counter, ⊖, represents −1.

**MATH TIP**

To add using a number line, start at 0. Move the number of units represented by the first addend. From that point, move the number of units represented by the second addend. The final location is the sum.

My Notes

**MATH TIP**

Use a negative sign to represent a drop in temperature.

**WRITING MATH**

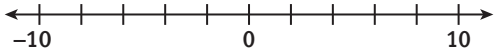
To avoid confusion, use parentheses around a negative number that follows an operation symbol: $2 + (-5)$.

**2.** At a ski resort, the temperature is $-3°$F at 9 P.M. The temperature drops $4°$F by midnight. What is the temperature at midnight?

**a.** Use counters to represent:the problem.

**b.** Explain how the counters model $-3 + (-4)$.

**c.** Show and explain how to use a number line to model $-3 + (-4)$.




**d.** Explain how both models can be used to find the temperature at midnight.

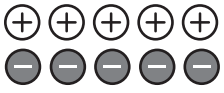
You used models to add integers with like signs. You can also use models to add integers with unlike signs.

**3.** What happens when you add an integer and its opposite? For example, what is $3 + (-3)$? Model this on the number line.

**4.** A number and its opposite are called **additive inverses.** Another name for a number and its opposite is a **zero pair**.

**a.** Why do you think they are also called a **zero pair**?

**b.** Which zero pair is modeled below using counters?



You can also use counters to add two integers with unlike signs.

My Notes

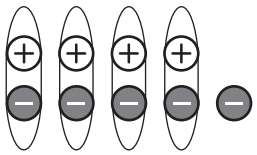
## Example B

Use counters to add $4 + (-5)$.

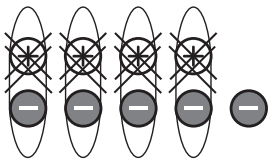
**Step 1:** Use counters to represent each integer.

4: ⊕ ⊕ ⊕ ⊕ $\qquad$ $-5$: ⊖ ⊖ ⊖ ⊖ ⊖

**Step 2:** Combine the counters to make zero pairs.



**Step 3:** Eliminate the zero pairs. Find the number of counters that remain.



One negative counter remains.

**Solution:** $4 + (-5) = -1$

## Try These B

Use counters to find each sum.

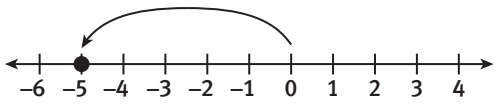
**a.** $-3 + 7$ $\qquad$ **b.** $5 + (-2)$ $\qquad$ **c.** $-6 + 1$

Number lines can be used to add two integers with unlike signs as well.

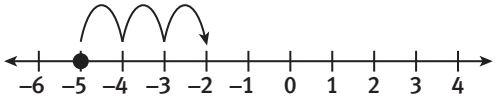
## Example C

Use a number line to add $-5 + 3$.

**Step 1:** Start at 0 and move 5 units to the left to $-5$.




**Step 2:** To add 3, move 3 units to the right of $-5$.




**Step 3:** The sum is the point where you land. The point is $-2$.

**Solution:** $-5 + 3 = -2$

### MATH TIP

Remember, zero pairs have a sum of zero ($-1 + 1 = 0$), so they are eliminated.

### MATH TIP

Remember, to add a positive integer move to the right on the number line. To add a negative integer, move to the left.

My Notes

## Try These C

Use a number line to find each sum.

**a.** $3 + (-8)$ **b.** $-4 + 6$ **c.** $(-5) + 1$

## Check Your Understanding

**5.** Show how to use counters to find the sum of $3 + (-7)$. What is the sum?

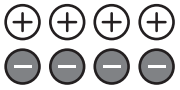
**6.** Show how to add $-5 + 4$ using a number line. What is the sum?

## LESSON 8-1 PRACTICE

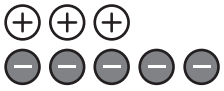
**Use counters or a number line to find each sum.**

**7.** $2 + 7$ **8.** $-3 + (-1)$

**Write an addition number sentence for the models shown.**

**9.** 

**10.** 

**11.** 

**12.** 

**Use counters to find each sum.**

**13.** $1 + (-8)$ **14.** $-6 + 2$

**15.** $-3 + 9$ **16.** $7 + (-6)$

**Use a number line to find each sum.**

**17.** $8 + (-5)$ **18.** $4 + (-9)$

**19.** $-7 + 1$ **20.** $-2 + 8$

**Use a model to add.**

**21.** $-6 + 6$ **22.** $-5 + (-4)$ **23.** $-3 + 7$

**24.** $2 + (-2)$ **25.** $9 + (-5)$ **26.** $-8 + 4$

**27. Construct viable arguments.** Is adding two integers with the same sign on a number line the same as or different from adding two whole numbers on a number line? Support your answer.

**28. Critique the reasoning of others.** Matt used counters to add $-4 + 6$. He said that the sum is 10 since there are 10 counters in the model. Is he correct? Explain why or why not.

My Notes

## Learning Targets:

- Add two or more integers.
- Solve real-world problems by adding integers.

**SUGGESTED LEARNING STRATEGIES:** Summarizing, Sharing and Responding, Look for a Pattern

You have used models to add integers. You can also use absolute value to add integers.

**Rule for adding integers with the same sign:**

- Find the sum of the absolute values of the addends.
- Use the same sign as the integers in the sum.

### Example A

Add $-23 + (-9)$.

**Step 1:** Find the absolute value of each addend.
$|-23| = 23 \qquad |-9| = 9$

**Step 2:** Add the absolute values.
$23 + 9 = 32$

**Step 3:** Use the same sign as the integers in the sum.
$-32$

**Solution:** $-23 + (-9) = -32$

### Try These A

Find each sum.

**a.** $-8 + (-7)$
**b.** $35 + 48$
**c.** $-42 + (-26)$

**1.** At the ski resort where the students are tracking temperatures, the low temperature on Monday is $-12°$F. Tuesday's low temperature is 15°F lower than on Monday.

**a.** Write an addition expression to find the low temperature on Tuesday.

**b.** What is the low temperature at the ski resort on Tuesday? Explain your reasoning.

My Notes

You can also use absolute value to add integers with unlike signs.

**Rules for Adding Integers with Unlike Signs:**

- When adding integers with different signs, find the difference of the absolute values.
- Use the sign of the integer with the greater absolute value for the sum.

## Example B

Add $31 + (-45)$.

**Step 1:** Find the absolute value of each addend.
$|31| = 31$ $|-45| = 45$

**Step 2:** Subtract the lesser absolute value from the greater absolute value.
$45 - 31 = 14$

**Step 3:** Use the sign of the integer with the greater absolute value.
$|-45| > |31|$, so the sum is negative.

**Solution:** $31 + (-45) = -14$

## Try These B

Determine if the sum is positive or negative. Then find each sum.

**a.** $-16 + 4$
**b.** $23 + (-15)$
**c.** $-50 + 72$

**2.** At the ski resort, the high temperature on Thursday is $-7°$F. Friday's high temperature is $14°$F higher than on Thursday.

**a.** Write an addition expression to find the high temperature on Friday.

**b.** What is the high temperature at the ski resort on Friday? Explain.

**3.** At 8 A.M., the temperature at the peak on the mountain is $-24°$F. The temperature at mid-mountain is 8 degrees warmer than at the peak. The temperature at the base of the mountain is 11 degrees warmer than it is at mid-mountain. Explain how to find the temperature at the base of the mountain.

My Notes

## Check Your Understanding

4. Explain how to use absolute value to add $-17 + (-4)$.
5. Explain how to use absolute value to add $-8 + 12$.

## LESSON 8-2 PRACTICE

**For Items 6–9, determine if each sum is positive or negative.**

6. $-10 + (-4)$
7. $-51 + 25$
8. $16 + (-11)$
9. $-32 + 39$

**For Items 10–19, find the sum.**

10. $-3 + 9$
11. $-5 + (-7)$
12. $-14 + 6$
13. $-32 + 39$
14. $16 + (-11)$
15. $-16 + 24$
16. $-3 + (-29)$
17. $48 + (-27)$
18. $58 + (-65)$
19. $-72 + 41$
20. Is $-9 + 14$ the same as $-14 + 9$? Explain.
21. Is $-8 + 6$ the same as $6 + (-8)$? Explain.
22. One winter morning, the temperature fell below $-6°$F. What does this temperature mean?
23. At dawn the temperature was $-3°$C. By noon the temperature increased by $5°$C. What was the temperature at noon?
24. A submarine at a depth of $-26$ meters rises 8 meters. What is the elevation of the submarine after it rises?
25. A liquid solution at $53°$C is cooled $60°$C. If the solution freezes at $0°$C, is the solution frozen? Explain your reasoning.
26. **Make sense of problems.** A football team gained 5 yards, lost 8 yards, lost another 2 yards, and then gained 45 yards. What were the total yards gained or lost? Explain your reasoning.
27. **Construct viable arguments.** When is the sum of two integers negative? When is the sum positive? Write a short note explaining your thinking.

My Notes

**Learning Targets:**

- Use models to subtract one integer with absolute value less than 10 from another.
- Subtract integers.
- Solve real-world problems by subtracting integers.

**SUGGESTED LEARNING STRATEGIES:** Summarizing, Sharing and Respondiing, Create Representations, Use Manipulatives

To track the winter temperatures, the students also have to subtract integers to find temperature differences. One way to subtract integers is to use models.

## Example A

Use counters to subtract $-5 - (-3)$.

**Step 1:** Use counters to represent $-5$.

$-5$: 

**Step 2:** Subtract 3 negative counters.

**Step 3:** The difference is the number of counters that remain.

**Solution:** $-5 - (-3) = -2$

## Try These A

Use counters to find each difference. You may draw the counters or use counters from your teacher.

**a.** $-6 - (-1)$
**b.** $-4 - (-2)$
**c.** $-5 - (-5)$

1. **Make Use of Structure.** Consider the subtraction method used in Example A.
   **a.** You can use the method to subtract positive integers when the number being subtracted is less than the beginning number. Explain why.

   **b.** To subtract negative integers using the method in Example A, the number being subtracted must be greater than the beginning number. Explain why.

My Notes

Zero pairs can be used to subtract other combinations of integers.

## Example B

Use counters to subtract $6 - (-2)$.

**Step 1:** Start with 6 positive counters.

**Step 2:** You need to subtract 2 negative counters. There are no negative counters in Step 1. So insert 2 zero pairs.

**Step 3:** Subtract the 2 negative counters.

**Step 4:** The difference is the number of counters that remain.

**Solution:** $6 - (-2) = 8$

## Try These B

Use counters to find each difference. You may draw the counters or use counters from your teacher. Check students' models.

**a.** $-2 - (-3)$ **b.** $7 - (-1)$ **c.** $-6 - (-4)$

**2.** Look at each subtraction sentence and its corresponding addition sentence.

| Subtraction Sentence | Addition Sentence |
|---|---|
| $-5 - (-3) = -2$ | $-5 + (+3) = -2$ |
| $6 - (-2) = 8$ | $6 - (+2) = 8$ |
| $-8 - (+5) = -13$ | $-8 + (-5) = -13$ |
| $7 - (+4) = 3$ | $7 + (-4) = 3$ |

**a.** How did the operation symbol change?

**b.** How did the sign of the integer being subtracted change?

**c.** What do you notice about the answer to each subtraction sentence and its corresponding addition sentence?

### MATH TIP

Remember, zero pairs have a sum of zero ($-1 + 1 = 0$), so adding a zero pair is like adding zero to an expression. It does not change the value of the expression.

### CONNECT TO ENGLISH

Subtracting a negative number is somewhat similar to stating a double negative in English.

In English, the two negatives result in a positive to form an affirmative, or nonnegative, statement. Example: *I do not disagree.*

My Notes

**3.** Write a corresponding addition sentence for the following subtraction sentence.

$-4 - (+6) = -10$

You have used models to subtract integers. You have used addition to subtract integers.

**Rule for Subtracting Integers:** Change the subtraction sign to an addition sign and change the integer being subtracted to its opposite. Then use the rule for adding integers.

**4.** Use the rule for subtracting integers. Show your work.

**a.** $-5 - 7$

**b.** $4 - (-9)$

**c.** $-2 - (-4)$

**d.** $-8 - (-6)$

**e.** $3 - (+8)$

**f.** $6 - (-2)$

**g.** $-6 - (-6)$

**h.** $-3 - (-3)$

**i.** $4 - (+9)$

**j.** $-5 - (+8)$

**k.** $-3 - (+5)$

**l.** $7 - 9$

**m.** $7 - (-2)$

**n.** $-6 - (+3)$

My Notes

**5.** At the ski resort, the outside temperature is −7°F. The wind chill factor makes it feel like it is −12°F.

**a. Model with mathematics.** Write an expression to model the difference between the actual temperature and what the temperature feels like with the wind chill.

**b.** What is the difference between the actual temperature and what the temperature feels like with the wind chill?

**6.** At the state capital, the outside temperature is 18°F. The wind chill factor makes it feel like it is −3°F. What is the difference between the actual temperature and what the temperature feels like with the wind chill? Explain your reasoning.

**7.** The temperature at 6 A.M. was −15°F. At 6 P.M. the temperature was 20°F. What was the change in temperature?

**8. a.** Evaluate 8 − 3 and 3 − 8.

**b.** Does reversing the order of the numbers in subtraction change the difference? Explain your thinking.

My Notes

## Check Your Understanding

**9.** Show how to use counters to subtract $5 - (-7)$.

**10.** How is the expression $-8 - 2$ related to the expression $-8 + (-2)$?

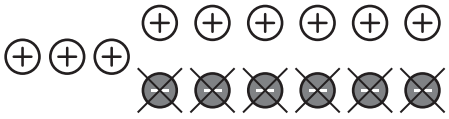
**11.** Use the rules for subtracting integers to find each difference.

**a.** $-2 - (-3)$ **b.** $1 - 6$

## LESSON 8-3 PRACTICE

**Write a subtraction number sentence for the models shown.**

**12.** 

**13.** 

**Use counters to find each difference.**

**14.** $1 - (-8)$

**15.** $-6 - (-2)$

**16.** $-3 - 9$

**17.** $7 - (-6)$

**Subtract.**

**18.** $-4 - (-4)$

**19.** $5 - (-12)$

**20.** $-8 - (-17)$

**21.** $-10 - (-8)$

**22.** $34 - (-3)$

**23.** $9 - (-9)$

**24.** $-20 - (-4)$

**25.** $7 - 16$

**26. Reason quantitatively.** Is $5 - (-5)$ the same as $-5 - 5$? Explain your reasoning.

**27. Make sense of problems.** A diver jumps from a platform 16 feet above the water to a depth of 6 feet below the surface of the water. What is the distance he dives?

**28.** A skyscraper is 1,150 feet above ground level. It extends 35 feet below ground level. What is the total length of the skyscraper from top to bottom?

**29.** A submarine at an ***elevation*** of $-48$ meters descends 12 meters. What is its elevation after it descends?

**30.** From Monday to Tuesday, the temperature fell from 23°F to $-6$°F. How many degrees is this temperature change?

### ACADEMIC VOCABULARY

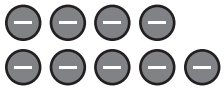
An ***elevation*** is a distance above or below a point of reference, such as ground level or sea level.

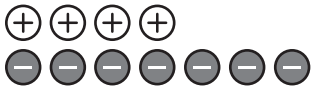
## ACTIVITY 8 PRACTICE

**Write your answers on a separate piece of paper. Show your work.**

### Lesson 8-1

**1.** Write an addition number sentence for the model shown.

**a.** 

**b.** 

**2.** Use counters to find each sum.
- **a.** $-8 + 2$
- **b.** $9 + (-7)$
- **c.** $-4 + (-6)$

**3.** Use a number line to find each sum.
- **a.** $6 + (-1)$
- **b.** $3 + (-7)$
- **c.** $-9 + 6$

**4.** Use models to add.
- **a.** $2 + (-5)$
- **b.** $-3 + (-4)$
- **c.** $-7 + 1$
- **d.** $-5 + 8$
- **e.** $9 + (-4)$

**5.** What is the sum of a number and its opposite? Use a model in your explanation.

### Lesson 8-2

**6.** Determine if each sum will be positive or negative.
- **a.** $9 + (-1)$
- **b.** $-6 + 3$
- **c.** $5 + (-8)$
- **d.** $-3 + 7$

**For Items 7–16, find each sum.**

**7.** $-6 + 14$

**8.** $-3 + (-10)$

**9.** $12 + (-5)$

**10.** $-8 + (-4)$

**11.** $9 + (-11)$

**12.** $-24 + (-15)$

**13.** $-19 + 16$

**14.** $50 + (-21)$

**15.** $-46 + 32$

**16.** $6 + (-12)$
- **a.** $-18$
- **b.** $-6$
- **c.** $6$
- **d.** $18$

**17.** The Commutative Property of Addition says that the order of the addends does not change the sum. Is the Commutative Property true for integers? Explain your reasoning.

**18.** A scuba diver is swimming at an elevation of 52 feet below sea level. She sees a school of fish and rises 16 feet to watch the fish. What is her elevation now?

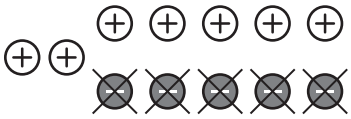
**19.** The temperature of some frozen chicken was $-18°C$. After thawing in the refrigerator overnight, the temperature of the chicken rose by $22°C$. What was the temperature of the chicken after thawing overnight?

**20.** At 6 A.M., the temperature was $-9°F$. By 11 A.M., the temperature had increased $34°C$. What was the temperature at 11 A.M.?

## Lesson 8-3

**21.** Write a subtraction number sentence for the model shown.

**a.** 

**b.** 

**For Items 22–25, use counters to find each difference.**

**22.** $3 - (-1)$

**23.** $-6 - (-4)$

**24.** $2 - 7$

**25.** $-4 - 3$

**For Items 26–37, find each difference.**

**26.** $9 - (-9)$

**27.** $-17 - 6$

**28.** $1 - 14$

**29.** $-24 - 11$

**30.** $-13 - (-8)$

**31.** $31 - (-10)$

**32.** $-8 - (-4)$

**33.** $-14 - (-20)$

**34.** $-10 - (-1)$

**35.** $-17 - (-6)$

**36.** $5 - 21$

**37.** $16 - (-16)$

**38.** $9 - 14 =$

**A.** $-23$
**B.** $-5$
**C.** $5$
**D.** $23$

**39.** The highest elevation in North America is Denali, in Alaska, with an elevation of 20,320 feet above sea level. The lowest elevation in North America is Death Valley, in California, with an elevation of 282 feet below sea level. What is the difference in the highest and lowest elevations in North America? Explain your reasoning.

**40.** The temperature was 12°C before the storm. During the storm, the temperature dropped 19°C. By the end of the storm, the temperature dropped another 7°C. What was the temperature at the end of the storm? Explain your reasoning.

**41.** Explain how to evaluate the following expression:

$6 + (-3) - 9.$

**42.** Delia evaluated the expression $2 - (-7)$ and found a difference of $-5$. Is she correct? Explain why or why not.

**43.** Compare and contrast subtracting integers and subtracting whole numbers.

**44.** The rule for subtracting integers can be used to subtract whole numbers. Explain why.

### MATHEMATICAL PRACTICES
**Express Regularity in Repeated Reasoning**

**45.** Explain how to use a number line to add and subtract integers. Use examples in your explanation.

**Write your answers on notebook paper. Show your work.**

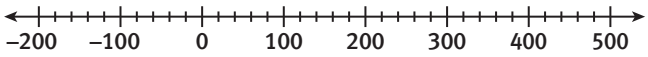
Antonia's hobby is astronomy. She is learning about the surface temperatures of the planets in our solar system. She knows that the temperatures on a given planet can vary widely, just as Earth's temperatures do. She has compiled a table that gives average temperatures on each of the eight planets.

| Planet | Temperature (°C) |
|---|---|
| Earth | 20 |
| Jupiter | −120 |
| Mars | −20 |
| Mercury | 440 |
| Neptune | −200 |
| Saturn | −140 |
| Uranus | −180 |
| Venus | 460 |

**1.** Which two temperatures are opposites? Explain your choice.

**2. a.** Order the temperatures from least to greatest. Explain how you determined the order.

**b.** Draw a number line like the one below and place a dot at the location of each of the eight temperatures. Write the name of each planet above its temperature.




**3.** Find the difference between the temperatures of each pair of planets.

**a.** Venus and Earth

**b.** Mercury and Saturn

**c.** the warmest planet and the coldest planet

**d.** Mars and Uranus

**4. a.** The sum of the temperatures of Earth and Neptune is the same as the temperature of which planet?

**b.** The sum of the temperatures of Jupiter and Mars is the same as the temperature of which planet?

**c.** The sum of the temperatures of Jupiter, Saturn, and Neptune has the same absolute value as the temperature of which planet?

# Integer Sums and Differences
## HOT AND COLD

| Scoring Guide | Exemplary | Proficient | Emerging | Incomplete |
|---|---|---|---|---|
| | The solution demonstrates these characteristics: | | | |
| **Mathematics Knowledge and Thinking** (Items 1, 2a, 3a-d, 4) | • A clear understanding of ordering integers, finding opposites, and absolute value.<br>• An effective understanding of and accuracy in adding and subtracting integers. | • A functional understanding of ordering integers, finding opposites, and absolute value.<br>• Addition and subtraction of integers that is usually correct. | • Partial understanding of integers and opposites; difficulty using absolute value.<br>• Difficulty with addition and subtraction of integers. | • An inaccurate understanding of integers and opposites; no understanding of absolute value.<br>• Inaccurate addition and subtraction of integers. |
| **Problem Solving** (Items 4a-c) | • An appropriate and efficient strategy that results in a correct answer. | • A strategy that may include unnecessary steps but results in a correct answer. | • A strategy that results in some incorrect answers. | • No clear strategy when solving problems. |
| **Mathematical Modeling / Representations** (Items 2a-b, 3a-d, 4a-c) | • Clear and accurate ordering and graphing of integers on a number line.<br>• Clear and accurate rewriting of problems as addition and subtraction expressions that can be solved. | • Graphing of integers on a number line with little difficulty.<br>• Some difficulty in rewriting addition and subtraction of integer problems as expressions, but can get the correct answer. | • Partially accurate graphing of integers on a number line.<br>• Difficulty in writing expressions leading to errors in solving addition and subtraction of integer problems. | • Inaccurate graphing of integers on a number line.<br>• No understanding of addition and subtraction of integers. |
| **Reasoning and Communication** (Items 1a, 4a-c) | • Precise use of appropriate math terms and language to explain opposites of integers and ordering of integers.<br>• Relating a mathematical result to a table of data accurately and easily. | • An adequate explanation of opposites of integers and ordering of integers.<br>• Relating a mathematical result to a table of data with little difficulty. | • A misleading or confusing explanation of opposites of integers and ordering of integers.<br>• Poor understanding of relating a mathematical result to a table of data. | • An incomplete or inaccurate description of opposites of integers and ordering of integers.<br>• No understanding of how a mathematical result might relate to a table of data. |

# The Coordinate Plane

## Map It Out!

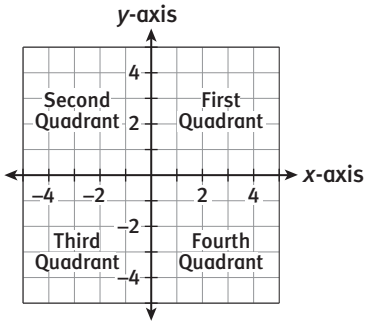
## Lesson 9-1 Integers in the Coordinate Plane

### Learning Targets:

- Graph and identify ordered pairs of rational numbers.
- Understand and use terms such as *origin, quadrant, x-axis, y-axis, first coordinate,* and *second coordinate* associated with graphing on the coordinate plane.

**SUGGESTED LEARNING STRATEGIES:** Summarizing, Note Taking, Sharing and Responding, Create Representations

Student volunteers at the zoo are making a new map of the zoo grounds with additional information about the exhibits. They decide to use a coordinate plane to design the map since it is a quick way to locate points.




A ***coordinate plane***, or **grid,** is formed by a horizontal number line, called the ***x*-axis,** and a vertical number line, called the ***y*-axis.** The two axes intersect at right angles at a point called the ***origin***. The axes divide the coordinate plane into four sections called ***quadrants***.

You can locate any point in the coordinate plane using a pair of numbers called an ***ordered pair***.

The *first coordinate* in an ordered pair is the ***x*-coordinate**. It tells how far to move on the *x*-axis from the origin.

The *second coordinate* is the ***y*-coordinate**. It tells how far to move on the *y*-axis from the origin.

To graph a point in the coordinate plane, start at the origin and go right (+) or left (−) along the *x*-axis by the number of units given by the first coordinate. From that point, go up (+) or down (−) by the number of units given by the second coordinate.

My Notes

**CONNECT TO HISTORY**

The coordinate plane is also called a Cartesian coordinate plane. It is named after Rene Descartes, a 17th century French philosopher and mathematician who first used a coordinate plane.

**MATH TIP**

You can identify the quadrant in which a point is located by looking at the signs of the coordinates.

Quadrant I: (+,+)
Quadrant II: (−,+)
Quadrant III: (−,−)
Quadrant IV: (+,−)

**MATH TIP**

Remember that the second coordinate in an ordered pair is the *y*-coordinate by remembering that *y* comes after *x* in the alphabet.

My Notes

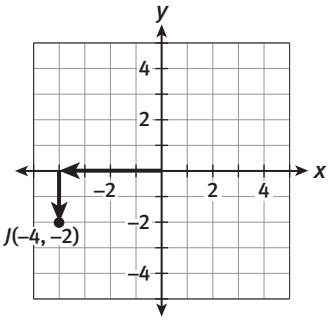
## Example

Graph and label the point $J(-4, -2)$ on the coordinate plane.

**Step 1:** Start at the origin and go left 4 units.

**Step 2:** Move down 2 units. Draw and label the point.

**Solution:** The grid shows point $J$ graphed at $(-4, -2)$.




## Try These

Graph and label each point on the grid above.

**a.** $K(3, 4)$ **b.** $L(-1, 3)$ **c.** $M(2, -4)$

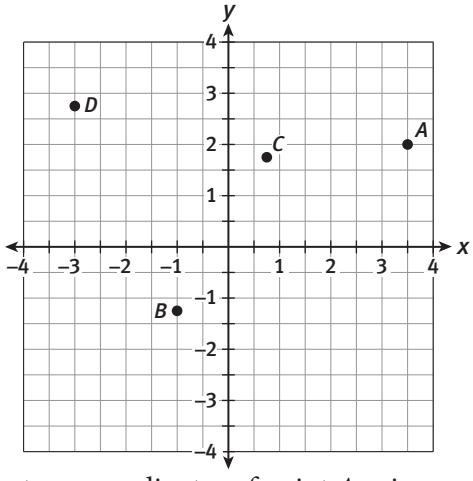
### READING MATH

An ordered pair is always named in order (first $x$, then $y$).

Write an ordered pair $(x, y)$.

Read "point $x$ $y$."

The student volunteers have started a map. The origin represents the education center at the zoo. Each point on the grid shows the location of a different exhibit.




Sometimes, a decimal, a fraction, or a mixed number is used to name a point on a grid.

**1.** Point $A$ shows the location of the African Plains exhibit. The $x$-coordinate of point $A$ is halfway between 3 and 4. What are coordinates of point $A$ using decimals? How do you know?

**2.** Point $B$ shows the location of the Bat Exhibit. The $y$-coordinate of point $B$ is one quarter of the way from $-1$ to $-2$. What are the coordinates of point $B$ using mixed numbers? How do you know?

**3.** The coordinates of the petting zoo are $(-3, 2.75)$. Which letter names the location of the petting zoo?

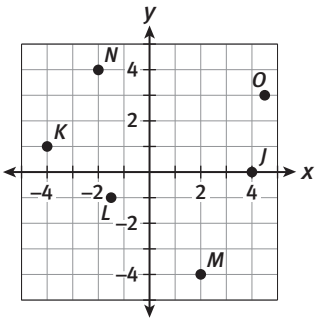
My Notes

## Check Your Understanding

**4.** Explain how to use the first coordinate and the second coordinate of the ordered pair $P\,(7, -2)$ to graph the point in the coordinate plane. Then plot the point on a coordinate plane.

**5.** Write the ordered pair for each point.

**a.** $J$ **b.** $K$ **c.** $L$
**d.** $M$ **e.** $N$ **f.** $O$



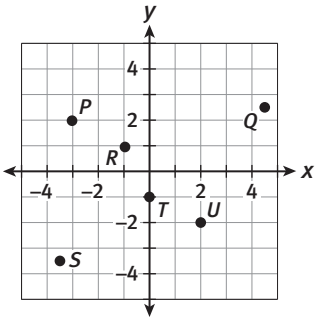

## LESSON 9-1 PRACTICE

**6.** Graph and label each point on a coordinate plane.

**a.** $A\,(6, 3)$ **b.** $B\,(2, -7)$ **c.** $C\,(8.5, -4)$
**d.** $D\,(-5, -3)$ **e.** $E\,(-1\frac{1}{2}, 6)$ **f.** $F\,(-3, 4)$

**7. Attend to precision.** Write the ordered pair for each point graphed on the coordinate grid below.

**a.** $P$ **b.** $Q$ **c.** $R$
**d.** $S$ **e.** $T$ **f.** $U$




**8.** Graph the following points on a coordinate grid: $X\,(0, 4)$, $Y\,(0, 0)$, and $Z\,(4, 0)$. Then draw line segments $XY$, $YZ$, and $XZ$. Name the type of figure that you have drawn.

**9.** Graph the following points on a coordinate grid: $A\,(0, 0)$, $B\,(0, 3)$, $C\,(4.5)$, and $D\,(4.5, 0)$. Then draw line segments $AB$, $BC$, $CD$, and $DA$. Name the type of figure that you have drawn.

**10.** A point is located in the second quadrant of the coordinate plane. What can you conclude about the signs of the coordinates of the point?

**11.** A point is located in the third quadrant of the coordinate plane. What can you conclude about the signs of the coordinates of the point?

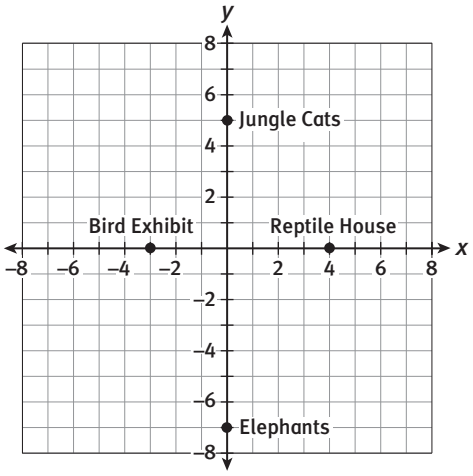
My Notes

**Learning Targets:**

- Find the distance between points in the coordinate plane with the same first coordinate or the same second coordinate.
- Solve real-world and mathematical problems by graphing points in the coordinate plane and finding the distances between them.
- Find the reflection of a point over one or both axes.

**SUGGESTED LEARNING STRATEGIES:** Summarizing, Note Taking, Sharing and Responding, Create Representations, Discussion Groups

Ryan has a map of the zoo. He has just gone through the Bird exhibit and wants to go to the Reptile house next. Each unit on the map represents 10 yards.




## Example A

How far must Ryan go to get from the Bird exhibit to the Reptile house?

**Step 1:** Count the number of units between 0 and the Bird exhibit.

There are 3 units.

**Step 2:** Count the number of units between 0 and Reptile house.

There are 4 units.

**Step 3:** Find the sum of these two distances.

$3 + 4 = 7$

**Solution:** Ryan must go 7 units. Since 1 unit = 10 yards, he must go $7 \times 10$ yards, or 70 yards.

My Notes

## Example B

How far apart are the Jungle cats and the Elephants?

**Step 1:** Write the coordinates for each exhibit.

Jungle cats: (0, 5) Elephants: (0, −7)

**Step 2:** Since both exhibits are on the *y*-axis. Find the difference of the *y*-coordinates.

$5 - (-7) = 5 + (7) = 12$

**Solution:** The distance between the Jungle cats and the Elephants is 12 units, or 120 yd.

## Try These A–B

Find the distance between each pair of points.

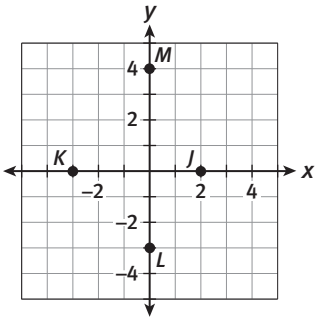
**a.** (0, 3) and (0,12)
**b.** (−1, 0) and (6, 0)
**c.** (−8, 0) and (−2, 0)

**MATH TIP**

If Step 2 in Example B showed the subtraction of the *y*-coordinates as $-7 - 5 = -7 + (-5)$, the result would be −12. However, distance cannot be negative. To avoid negative results when finding distances, find the absolute value of the difference of the coordinates:

$|-7 - 5| = |-7 + (-5)|$
$= |-12|$
$= 12$

**1.** Students plan to list the distances between some of the exhibits on the back of the map. Each unit on the map shown represents 1 yard.



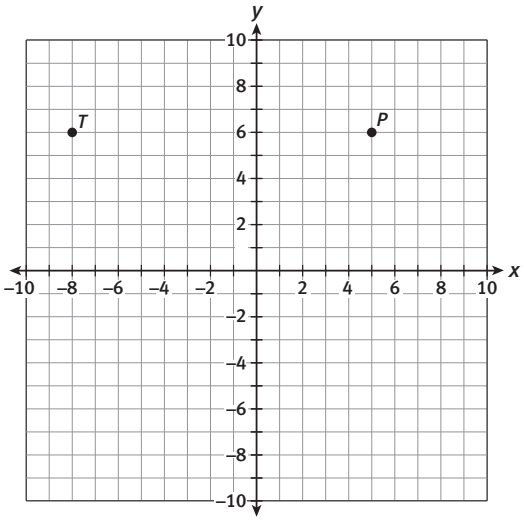

**a.** How far apart are Exhibits *J* and *K*? Explain.

**b.** How far apart are Exhibits *L* and *M*? Explain.

You can use distance on a coordinate grid to find the **reflection** of a point. The reflection of a given point across the *x*-axis or *y*-axis is another point that is the same distance from the axis as the given point.

My Notes

Designers are using a coordinate grid, as shown, to draw the new Native American exhibit. The exhibit will be in the shape of a rectangle. Each corner will have a totem pole. The designers have drawn points $T$ and $P$ where two of the totem poles will be located.




**2. a.** What is the distance from point $P$ to the $x$-axis directly below point $P$? Explain.

**b.** Point $Q$ will be the reflection of point $P$ across the $x$-axis. What will be the coordinates of point $Q$? Explain.

**c.** Point $S$ will be the reflection of point $T$ across the $x$-axis. What are the coordinates of point $S$?

**3.** Work with your group to create a simple exhibit on a coordinate grid. Prepare a brief report to share with your class describing the location of objects on your map making sure to include terms such as point, distance, and reflection. As needed, refer to the Glossary to review translations of key terms.

## GROUP DISCUSSION TIPS

Be sure to use appropriate vocabulary, both real-world and mathematical, to describe your exhibit. Refer to the Word Wall as needed to help you to choose words for your description. Assign each group member a role in creating the exhibit and make sure to set a reasonable time frame for completing the exhibit.

## Check Your Understanding

**4.** Explain how to find the distance between the points −2 and −5 on a number line.

**5.** Point $C$ has coordinates (1, 3) and Point $D$ has coordinates (1, −4).
**a.** Graph the points on a coordinate plane.
**b.** Explain how to find the distance between the points using their ordered pairs.

My Notes

## LESSON 9-2 PRACTICE

**6.** Find the distance between each pair of points on a number line.

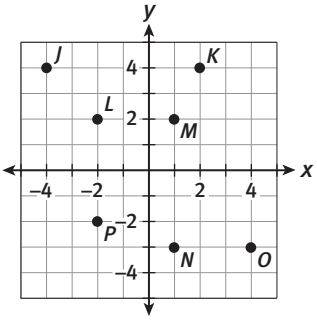
**a.** 4 and 7

**b.** −5 and 1

**c.** −6 and 8

**d.** −3 and −11

**Use the coordinate plane at the right for items 7–10. Find the distance between each pair of points.**



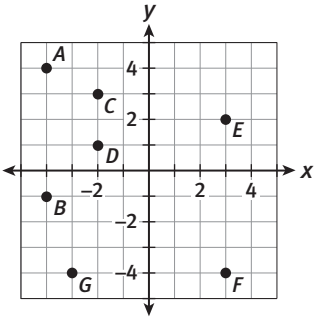

**7.** *J* and *K*

**8.** *L* and *M*

**9.** *N* and *O*

**10.** Which point is the reflection of point *L* across the *x*-axis?

**Use the coordinate plane at the right for Items 11–14. Find the distance between each pair of points.**



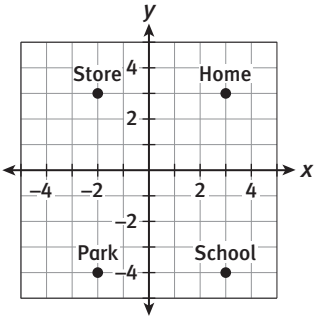

**11.** *A* and *B*

**12.** *C* and *D*

**13.** *E* and *F*

**14.** Which point is the reflection of point *F* across the *y*-axis?

**15. Make sense of problems.** Kurt used a coordinate plane to make a map of his neighborhood. Each unit on the map is 1 block.




**a.** How many blocks is Kurt's school from home?

**b.** How far is the school from the park?

**c.** If Kurt begins at his house, goes to the store and then to the park, how far will he travel? Explain your reasoning.

**16. Reason abstractly.** The point (−3, 8) is reflected over the *x*-axis. What is the distance between the point and its reflection? Explain your thinking.

## ACTIVITY 9 PRACTICE

**Write your answers on a separate piece of paper. Show your work.**

### Lesson 9-1

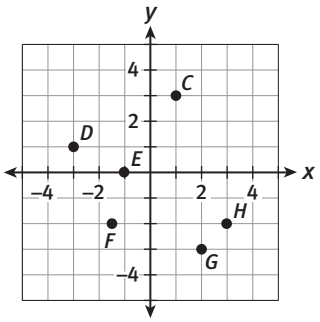
**1.** Graph and label each point on a coordinate plane.

**a.** $A(1, 7)$ **b.** $B(0, -5)$

**c.** $C(-4, 6)$ **d.** $D(-8\frac{1}{4}, -3)$

**e.** $E(9, 0)$ **f.** $F(6.5, -2)$

**2.** Write the ordered pair for each point.




**a.** $C$ **b.** $D$ **c.** $E$

**d.** $F$ **e.** $G$ **f.** $H$

**3.** If the first coordinate in an ordered pair is 0, where does the point lie? If the second coordinate in an ordered pair is 0, where does the point lie? How do you know?

**4.** The coordinates of point $H$ are $(12, -15)$. In which quadrant is point $H$ located?

**A.** First **B.** Second

**C.** Third **D.** Fourth

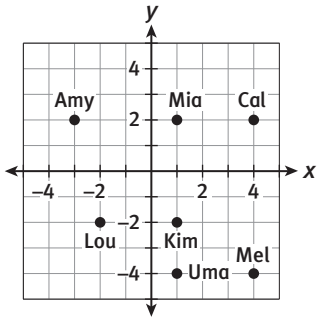
**5.** David graphed the point $(6\frac{1}{2}, -9\frac{3}{4})$ on a coordinate grid. In which quadrant is the point? He then decided to graph a point that had coordinates opposite those of the first point. In which quadrant did he graph this point?

**6.** Do the points $(-4, 6)$ and $(6, -4)$ describe the same location? Explain your thinking.

**7.** Which point is common to both axes?

### Lesson 9-2

**Use the coordinate plane for Items 8–16. The coordinate plane shows the locations of some classmates' homes. Each unit on the grid represents 1 block.**




**8.** How far does Cal live from Mel?

**9.** How far does Mia live from Amy?

**10.** How far apart do Amy and Cal live?

**11.** How far apart do Mia and Uma live?

**12.** Does Kim live closer to Lou or to Uma? Explain.

**13.** Does Mia live closer to Amy or to Cal? Explain.

**14.** Whose house is a reflection of Mia's house across the $x$-axis? Explain.

**15.** Is Cal's house a reflection of Amy's house across the $y$-axis? Explain.

**16.** Starting at her house, Lou walks to see each of her classmates in the following order: Kim, Uma, Mel, Cal, Mia, and Amy. How far does Lou walk to reach Amy's house? Explain your thinking.

### MATHEMATICAL PRACTICES
**Make Use of Structure**

**17.** Explain how to find the distance between the points $(28, -17)$ and $(-15, -17)$ on a coordinate plane.

# Multiplying and Dividing Integers

## Temperature Ups and Downs

## Lesson 10-1 Multiplying Integers

### Learning Targets:

- Multiply integers.
- Solve real-world problems by multiplying integers.

**SUGGESTED LEARNING STRATEGIES:** Marking the Text, Visualization, Predict and Confirm, Create Representations, Look for a Pattern

In science class, Mariah is learning about temperature. Mariah has decided to use the Celsius scale to investigate the three states of water.

- When the temperature of water is below 0°C, it is a solid called ice.
- From 0°C to 100°C, it is a liquid called *water*.
- Above 100°C, it is a gas called *steam*.

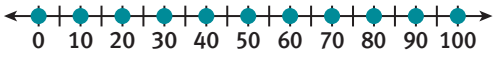
Mariah starts with a container of water with a temperature of 0°. That is the temperature at which ice changes to water. To study changes in the water's state, she increases the temperature at a constant rate for 10 minutes until it begins to steam.

1. What is the increase in temperature?

2. What is the rate at which the temperature increased? Write the answer in degrees per minute.

3. Would it be more appropriate to represent this rate of increase as a positive integer or as a negative integer? Explain your reasoning.

Mariah drew this number line to represent the minute-by-minute changes in temperature:




4. Let ⊕ represent the rate you found in Item 2. Use ⊕ to represent the total change in temperature.

My Notes

**CONNECT TO SCIENCE**

Substances can exist in three different forms:

- solid,
- liquid, and
- gas.

Each form is called a *state*. Heating and cooling causes a substance to change from one state to another. Heating causes ice, the solid form of water, to become water, the liquid form, and finally to become steam, the gas form.

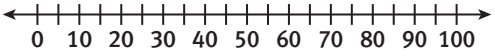
My Notes

Next, Mariah places the container in the laboratory freezer. Over the next 20 minutes, she lowers the temperature at a steady rate until the water begins to freeze.

**5.** What is the rate of decrease in temperature? Write the answer in degrees per minute.

**6.** Would it be more appropriate to represent this rate of decrease as a positive integer or a negative integer? Explain your reasoning.

**7.** Mark the number line to represent the minute-by-minute changes in temperature:




**8.** Let $\triangledown^{-}$ represent the rate that you found in Item 5. Use $\triangledown^{-}$ to represent the total change in temperature.

In Items 4 and 8, you used triangles to represent multiplication.

$\triangle^{+}$ represents a temperature increase of 10 degrees.

$\triangledown^{-}$ represents a temperature decrease of 5 degrees.

So, $10 \triangle^{+} = +100$ and $20 \triangledown^{-} = -100$.

**9. a.** If $\boxed{-}$ represents $-8$, what does $\boxed{-}\boxed{-}\boxed{-}\boxed{-}\boxed{-}$ represent?

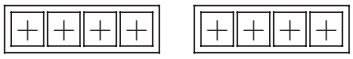
**b.** If $\boxed{-}$ represents $-5$, what does $\boxed{-}\boxed{-}\boxed{-}$ represent?

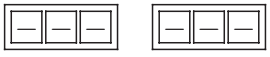
**c.** If $\boxed{+}$ represents $+3$, what does $\boxed{+}\boxed{+}\boxed{+}\boxed{+}$ represent?

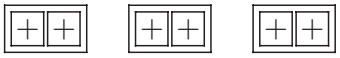
**d.** If $\boxed{+}$ represents $+9$, what does $\boxed{+}\boxed{+}\boxed{+}\boxed{+}\boxed{+}\boxed{+}$ represent?

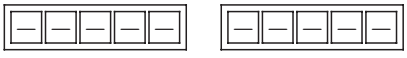
My Notes

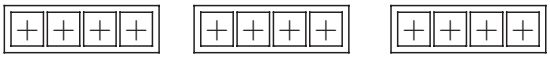
**10.** Write the number sentence represented by each diagram. Each counter stands for 10 or −10. The first one has been done for you.

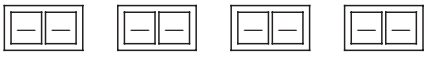
**a.**  $2 \times 40 = 80$

**b.** 

**c.** 

**d.** 

**e.** 

**f.** 

**11.** Use  and  to represent each number sentence.

**a.** $2 \times 50 = 100$

**b.** $3 \times (-30) = -90$

**12.** Look at your results for Items 10 and 11. Answer the following questions:

**a.** What was the sign of the product when you multiplied two positive integers?

**b.** What was the sign of the product when you multiplied a positive integer and a negative integer?

**c.** Use these observations to write rules to find the sign of the product of integers.

My Notes

**13.** You can use patterns to find the sign of the product of two negative integers.

**a.** Fill in ONLY the first four squares in the table below.

| × | 3 | 2 | 1 | 0 | −1 | −2 | −3 |
|---|---|---|---|---|---|---|---|
| −3 | | | | | | | |

**b. Make use of structure.** Describe the pattern in the four squares you filled in.

**c.** Fill in the last three squares by continuing this pattern.

**d.** Use the same procedure you used in parts a–c to complete the multiplication tables below.

| × | 3 | 2 | 1 | 0 | −1 | −2 | −3 |
|---|---|---|---|---|---|---|---|
| −5 | | | | | | | |

| × | 3 | 2 | 1 | 0 | −1 | −2 | −3 |
|---|---|---|---|---|---|---|---|
| −8 | | | | | | | |

| × | 3 | 2 | 1 | 0 | −1 | −2 | −3 |
|---|---|---|---|---|---|---|---|
| −11 | | | | | | | |

**e.** Use your results in parts a and d to write a rule to find the sign of the product of two negative integers.

**14.** Complete the table at the right showing the sign of the product of integers. Write a positive sign or negative sign in each box.

| × | positive | negative |
|---|---|---|
| positive | | |
| negative | | |

**15. Make use of structure.** In parts a–c, state whether the product is positive or negative.

**a.** the product of two negative integers

**b.** the product of three negative integers

**c.** the product of four negative integers

**d.** State a rule for finding the sign of the product of an even number of negative integers.

**e.** State a rule for finding the sign of the product of an odd number of negative integers.

My Notes

## Check Your Understanding

**16.** Find each product.

**a.** $3(-9)$ **b.** $7 \cdot 6$

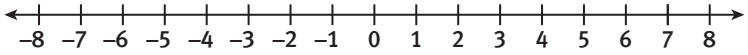
**c.** $-10(-2)$ **d.** $5(-8)$

**17.** Complete.

**a.** $3 \times (-4) = -3 \times$ _____ **b.** $2 \times 5 = -2 \times$ _____

**c.** $-6 \times 7 = 6 \times$ _____ **d.** $-9 \times (-9) = 9 \times$ _____

**18.** Copy the number line below and use it to show the product of 2 and $(-4)$.




$2(-4) =$ _____

**19.** Which property justifies this equation? $-5 \times 12 = 12 \times -5$?

## LESSON 10-1 PRACTICE

**20.** At 4 P.M. the temperature in Clarksville was 0°C. Over the next 8 hours the temperature fell at a rate of 3 degrees per hour.

**a.** Write the rate that the temperature fell as an integer.

**b.** Determine the temperature at midnight.

**21.** **a.** Blake borrowed $25 from his brother. Write an integer to express the $25 that Blake owes his brother.

**b.** Blake borrowed $25 from his brother a total of 7 times. Write an integer that represents the amount that Blake owes his brother.

**22.** Find the 11th number in the pattern below:

0, −7, −14, −21,…

**23.** A submarine on the surface of the ocean descended at a rate of 7 feet per second for 2 minutes. Then it ascended at a rate of 4 feet per second for 3 minutes, Finally, it descended at a rate of 9 feet per second for 5 minutes. What was the final elevation of the submarine? (The elevation of the ocean surface is 0 feet.)

My Notes

**Learning Targets:**

- Divide integers.
- Solve real-world problems by dividing integers.

**SUGGESTED LEARNING STRATEGIES:** Marking the Text, Visualization, Predict and Confirm, Look for a Pattern

The low temperatures in Bismarck, North Dakota, during five days in January are given in the table. The local TV weather reporter wants to announce the mean low temperature for the week. To do this, the reporter will need to divide negative integers.

| Day | Temperature (°F) |
|---|---|
| Monday | −7 |
| Tuesday | −9 |
| Wednesday | 3 |
| Thursday | 5 |
| Friday | −2 |

**1.** Use what you know about multiplying integers to draw a diagram using [+] and [−] for each problem. Give the answer. The first one has been done for you.

**a.** $12 \div 2 = 6$ [+][+][+][+][+][+] [+][+][+][+][+][+]

**b.** $20 \div 4 =$ _____

**c.** $-12 \div 4 =$ _____

**d.** $-14 \div 7 =$ _____

**2. Make use of structure.** The equation $4 \cdot 7 = 28$ shows that the numbers 4, 7, and 28 are related by multiplication. Write two equations to show that 4, 7, and 28 are related by division.

**3. Make use of structure.** Use the fact $(-5)(-9) = 45$ to write two equations showing that −5, −9, and 45 are related by division.

**4. Make use of structure.** Use the fact $6(-3) = -18$ to write two equations showing that 6, −3, and −18 are related by division.

My Notes

**5.** Use your results from Items 3 and 4 to complete these statements:

**a.** When a positive integer is divided by a negative integer, the quotient is _____.

**b.** When a negative integer is divided by a positive integer, the quotient is _____.

**c.** When a negative integer is divided by a negative integer, the quotient is _____.

**d.** The quotient of two integers with the same sign is _____.

**e.** The quotient of two integers with different signs is _____.

**6.** Complete the table at the right to show the sign of the quotient of two integers. Write a positive sign or negative sign in each box.

| ÷ | positive | negative |
|---|---|---|
| positive | | |
| negative | | |

**7.** Compare the rules you determined for finding the sign of the product of two integers with the rules you determined for finding the sign of the quotient of two integers.

**8. a.** Use the table on the preceding page to find the sum of the low temperatures in Bismarck for the five given days.

**b.** What was the mean low temperature in Bismarck for the five days?

**c.** Draw a diagram showing how you can use $\boxed{+}$ and $\boxed{-}$ to find the mean temperature.

**9.** Use the fact that multiplication and division are inverse operations to explain why the expression $6 \div 0$ has no answer.

**MATH TIP**

To find the mean, find the average of the data items by taking the sum of the data values and dividing by the total number of data items.

My Notes

## Check Your Understanding

**10.** Find each quotient.

**a.** $-15 \div 3$ **b.** $24 \div (-6)$ **c.** $30 \div 15$

**d.** $-25 \div (-5)$ **e.** $36 \div (-9)$ **f.** $-50 \div (-25)$

**g.** $\frac{-42}{6}$ **h.** $\frac{-56}{-7}$ **i.** $\frac{40}{-8}$

**11.** Multiplication and division are inverse operations. Use inverse operations to determine each missing number.

**a.** $15 \times \_\_\_\_\_ = -75$ **b.** $-12 \times \_\_\_\_\_ = 156$

**c.** $20 \times \_\_\_\_\_ = 260$ **d.** $-18 \times \_\_\_\_\_ = -126$

**e.** $-23 \times \_\_\_\_\_ = 207$ **f.** $-16 \times \_\_\_\_\_ = -256$

## LESSON 10-2 PRACTICE

**12.** The low temperatures in the town of Rigby for 5 consecutive days were −12°F, −17°F, −11°F, −12°F, and −18°F. What was the mean low temperature for the 5 days?

**13.** During the 7 minutes previous to landing, an airplane decreased in elevation by −3,192 feet. Determine the mean elevation change per minute.

**14.** Biff's Bakery had a slow year. Biff recorded the store's loss as −$10,308 for the year. How would Biff record the store's mean monthly loss?

**15.** Evaluate each expression.

**a.** $-2 \times 9 \div (-6)$ **b.** $40 \div (-10) \div 2$

**c.** $-54 \div (-6) \div (-3)$ **d.** $\frac{4 \times (-15)}{-2 \times (-6)}$

**16.** Use $<$, $=$, or $>$ to complete each statement.

**a.** $3 \times (-9) \bigcirc 50 \div (-2)$

**b.** $-46 \div 2 \bigcirc 13 \times (-2)$

**c.** $-5 \times (-6) \bigcirc -96 \div (-3)$

**d.** $-2 \times (-3) \times (-4) \bigcirc -96 \div (-2) \div (-2)$

# ACTIVITY 10 PRACTICE

**Write your answers on notebook paper.**
**Show your work.**

## Lesson 10-1

**1.** Find: $-20(-4)$

**A.** $-80$ **B.** $-5$
**C.** $5$ **D.** $80$

**2.** Which is the most accurate statement about the sign of the product of a positive integer and a negative integer?

**A.** It is positive.
**B.** It is negative.
**C.** It depends on the sign of the first of the two numbers.
**D.** It cannot be predicted without knowing the integers.

**3.** Find the numerator of the fraction: $\frac{?}{5} = -10$

**A.** $-50$ **B.** $-2$
**C.** $2$ **D.** $50$

**For Items 4–13, find each product.**

**4.** $2 \times (-9)$

**5.** $-3 \times (-3)$

**6.** $8 \times 6$

**7.** $-12 \times 6$

**8.** $-10 \times (-11)$

**9.** $7 \times (-7)$

**10.** $-1 \times (-1)$

**11.** $-1 \times (-1) \times (-1) \times (-1)$

**12.** $(-1)^7$

**13.** $-15(0)$

**For Items 14–17, use $<$ or $>$ to complete each statement.**

**14.** $-2(-5)$ ◯ $3 \times 3$

**15.** $-8(3)$ ◯ $5(-5)$

**16.** $10(-6)$ ◯ $-10(-6)$

**17.** $-3(12)$ ◯ $7(-5)$

**18.** A dolphin swimming in the ocean dove deeper, at a rate of $-3$ feet per second. Find the change in the dolphin's elevation one minute later.

**19.** **a.** The lowest temperature ever recorded in Michigan was 3 times the lowest temperature ever recorded in Georgia, which was $-17°$F. What was Michigan's lowest temperature?

**b.** The highest temperature ever recorded in Oregon was $-7$ times the lowest temperature ever recorded in Georgia. What was Oregon's highest temperature?

**20.** Name the number property illustrated by this equation:

$$-2(-3 + 7) = -2(-3) + (-2)(-7)$$

**21.** Complete:

**a.** $7 \times (-8) = -7 \times$ _____
**b.** $-5 \times (3) = 5 \times$ _____
**c.** $9 \times 2 = -9 \times$ _____

**22.** In golf, *par* is is the expected number of strokes needed to complete a course. Numbers of strokes greater than par (over par) are indicated with positive integers. Numbers of strokes less than par (under par) are indicated with negative integers. Clyde scored 6 under par each day of a 4-day tournament. Find his final score for the entire tournament.

## Lesson 10-2

**23.** Find: $12 \div (-2)$

**A.** $-24$ **B.** $-6$
**C.** 6 **D.** 24

**24.** Which answer best explains why the following numerical relationship is true?

$2 \times (-6) = -12$, so $\frac{-12}{-6} = 2$

**A.** the Distributive Property
**B.** the Associative Property
**C.** the Commutative Property of Multiplication
**D.** the inverse relationship between multiplication and division

**25.** Complete: $-6 \times \_\_\_\_ = 12$

**A.** $-72$ **B.** $-2$
**C.** 2 **D.** 72

**For Items 26–34, find each quotient.**

**26.** $8 \div (-4)$

**27.** $14 \div 2$

**28.** $-28 \div (-7)$

**29.** $-35 \div 5$

**30.** $\frac{-1}{-1}$

**31.** $\frac{0}{-12}$

**32.** $\frac{1}{-1}$

**33.** $60 \div (-4) \div 3$

**34.** $-80 \div (-2) \div 5 \div (-4)$

**35.** Which of the following expressions is not equivalent to the others?

**A.** $\frac{-24}{3}$
**B.** $\frac{24}{-3}$
**C.** $-\frac{24}{3}$
**D.** $-\frac{24}{-3}$

**36.** The temperature in Allenville fell from 25°F to −23°F in 8 minutes.

**a.** Write a numerical expression you can evaluate to find the average change in temperature per minute.
**b.** Find the mean change in temperature per minute.

**37.** The numbers below record the number of feet a hot-air balloon rose and fell each minute during a 6-minute period.

+410, +350, −570, +190, −470, −600

**a.** Find the balloon's final position in relation to its original position.
**b.** Find the balloon's mean change in position per minute.

**38.** Casey's bank statement records money taken from his checking account as a negative transaction and money added to his account as a positive transaction.

| Date | Amount ($) |
|---|---|
| 3/12 | +60.00 |
| 3/15 | −95.00 |
| 3/16 | −34.00 |
| 3/20 | −128.00 |
| 3/26 | +75.00 |
| 3/29 | −16.00 |

**a.** Find the net change in the amount Casey had in his account during the period shown.
**b.** Find the mean change per transaction for the same period.

## MATHEMATICAL PRACTICES
### Look For and Make Use of Structure

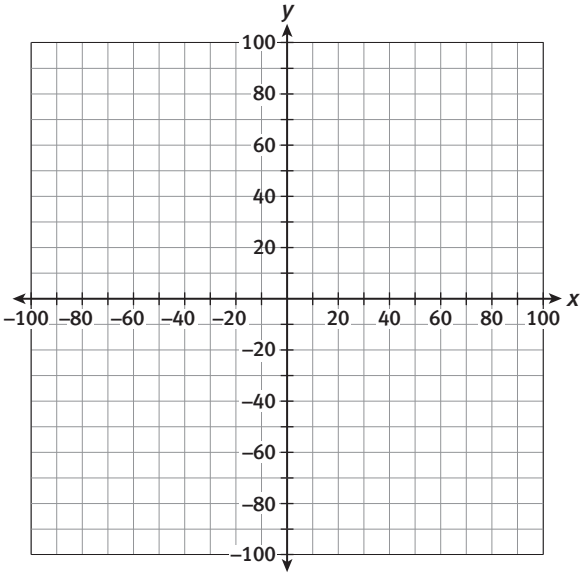
**39.** The product of two integers is positive. Is the quotient of those same integers positive, negative, or impossible to predict? Explain your reasoning

**Write your answers on notebook paper. Show your work.**

Pioneer School is holding a Scavenger Hunt. Teams are given clues that will lead them to places where prizes have been buried. You are the leader of your team.

The tract of farmland for the scavenger hunt measures 200 yards in each direction and has been marked with coordinate grid lines every 10 yards.

- The *x*-axis runs east-west.
- The *y*-axis runs north-south.




**1.** CLUE 1: Start at the origin. Go to (−10, 30). Find Prize 1 at the point that is a reflection of (−10, 30) over the *x*-axis.

**a.** Give the coordinates of Prize 1 and explain how you found these coordinates.

**b.** In which quadrant is Prize 1 located? Explain how you could determine this using only the coordinates of the point.

**2.** CLUE 2: Start at (−60, 70). Go to (−10, 70), keeping track of the distance walked between the two points. Continue by walking this same distance in the same direction.
There you will find Prize 2.

**a.** In which quadrant and what are the coordinates of Prize 2?

**b.** Explain how you determined which quadrant Prize 2 was in using only the coordinate grid.

**c.** What is the distance from the starting point to Prize 2?

**3.** CLUE 3: Start at the origin. Walk 20 yards south and 60 yards east. Divide the first coordinate of your present location by −3 to determine the first coordinate of Prize 3. Multiply the second coordinate of your present location by −3 to find the second coordinate of Prize 3.

What are the coordinates of Prize 3 and in which quadrant is it located?

**4.** CLUE 4: Find Prize 4 at the point that is a reflection of Prize 3's location over the *y*-axis. Give the coordinates of Prize 4.

# Coordinate Plane and Multiplying and Dividing Integers

## SCAVENGER HUNT

5. When you reach the spot for Prize #4, you have to dig 4 inches down 7 times. Write a number sentence to represent this situation and determine how far down you have to dig to find the prize.

6. The last clue tells you to run up the hill. You run at 3 feet per second and reach the top of the hill in 32 seconds. Write a number sentence to represent this situation and determine the distance you ran.

7. Explain how the signs of your answers to Items 5 and 6 make sense for the situation that they describe.

| Scoring Guide | Exemplary | Proficient | Emerging | Incomplete |
|---|---|---|---|---|
| | **The solution demonstrates:** | | | |
| **Mathematics Knowledge and Thinking** (Items 1a, 2a, c, 3, 4, 5, 6) | • A clear understanding of ordered pairs of integers, and reflections and distance in the coordinate plane.<br>• Effective understanding of and accuracy in multiplying and dividing integers. | • A functional understanding of ordered pairs of integers, and reflections and distance in the coordinate plane.<br>• Multiplication and division of integers that is usually correct. | • Partial understanding of ordered pairs of integers, and reflections and distance in the coordinate plane.<br>• Difficulty with multiplication and division of integers. | • An inaccurate understanding of ordered pairs of integers, and reflections and distance in the coordinate plane.<br>• Inaccurate multiplication and division of integers. |
| **Problem Solving** (Items 2c, 5, 6) | • An appropriate and efficient strategy that results in a correct answer. | • A strategy that may include unnecessary steps but results in a correct answer. | • A strategy that results in some incorrect answers. | • No clear strategy when solving problems. |
| **Mathematical Modeling/ Representations** (Items 1a, 2a, c, 3, 4, 5, 6) | • Clear and accurate graphing and reflecting of points in the coordinate plane.<br>• Clear and accurate writing and solving of problems with integer multiplication and division. | • Graphing and reflecting points on a coordinate plane with little difficulty.<br>• Some difficulty in writing integer multiplication and division problems but with correct answers. | • Partially accurate graphing and inaccurate reflection of points on a coordinate plane.<br>• Difficulty in writing expressions leading to errors in solving problems. | • Inaccurate graphing of ordered pairs of integers on a number line.<br>• No understanding of multiplication and division of integers. |
| **Reasoning and Communication** (Items 1a-b, 2a-b, 5-7) | • Precise use of appropriate math terms and language to explain signs of integers and quadrants. | • An adequate explanation of signs of integers and quadrants in the coordinate plane. | • A misleading or confusing explanation of signs of integers and quadrants. | • An incomplete or inaccurate description of signs of integers and quadrants in the coordinate plane. |