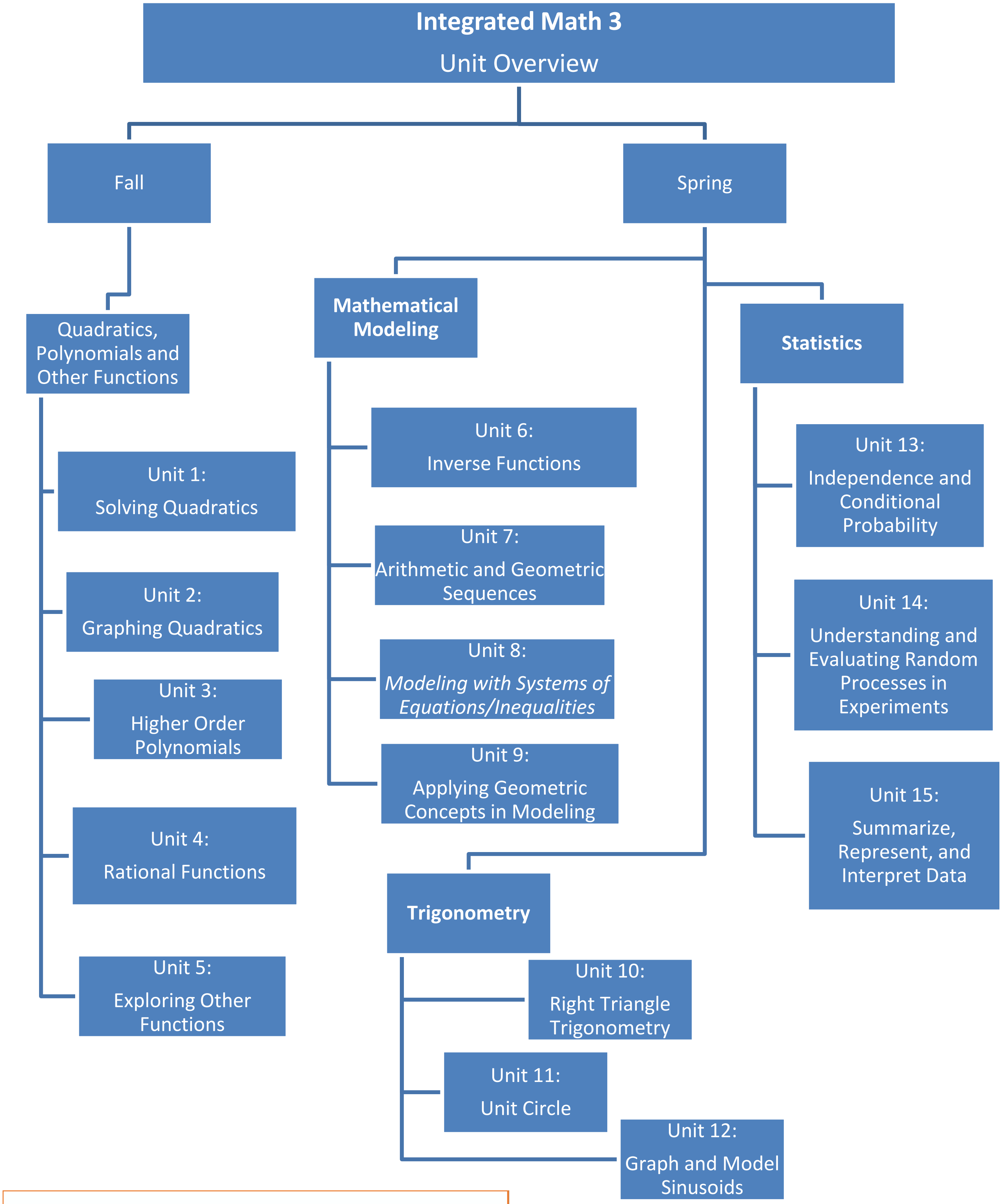
Creating 21st Century Learners

MUHSD

Merced Union High School District

Integrated Math 3

Course Standards
& Resource Guide



Online Learning Center

https://www.**pearsonsuccessnet**.com/

Username: **mytutorIII** Password: **Mathematics3**

<u>Integrated Math 3</u>

Quadratics, Polynomials and Other Functions

Unit 1: Solving Quadratics

Unit 2: Graphing Quadratics

Unit 3: Higher Order Polynomials

Unit 4: Rational Functions

Unit 5: Exploring Other Functions

Mathematical Modeling

Unit 6: Inverse Functions

Unit 7: Arithmetic and Geometric Sequences

Unit 8: *Modeling with Systems of Equations/Inequalities (see note in guide)*

Unit 9: Applying Geometric Concepts in Modeling

Trigonometry

Unit 10: Right Triangle Trigonometry

Unit 11: Unit Circle

Unit 12: Graph and Model Sinusoids

Statistics

Unit 13: Independence and Conditional Probability

Unit 14: Understanding and Evaluating Random Processes in Experiments

Unit 15: Summarize, Represent, and Interpret Data

Integrated Math 3 Course Standard and Resource Guide

Quadratics, Polynomials, and Other Functions

UNIT 1: Solving Quadratics

(Some standards will come from Math 2 to supplement)

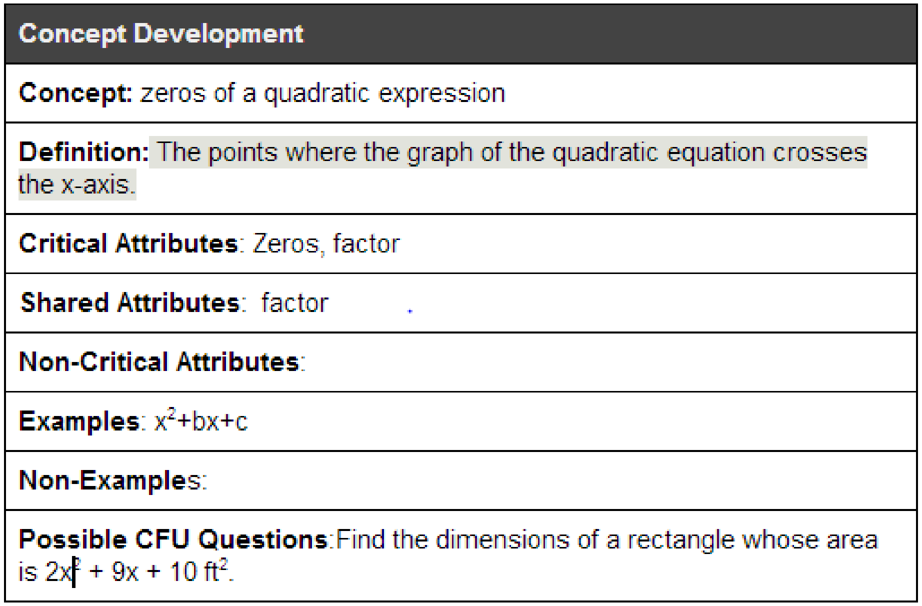
Overview	Solve quadratic functions in various forms

Priority standard

A.APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Supporting standards

A-SSE 3.a Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.)



Concept Development
Concept: zeros of a quadratic expression
Definition: The points where the graph of the quadratic equation crosses the x-axis.
Critical Attributes: Zeros, factor
Shared Attributes: factor
Non-Critical Attributes:
Examples: x^2+bx+c
Non-Examples:
Possible CFU Questions:Find the dimensions of a rectangle whose area is $2x^2 + 9x + 10$ ft^2.

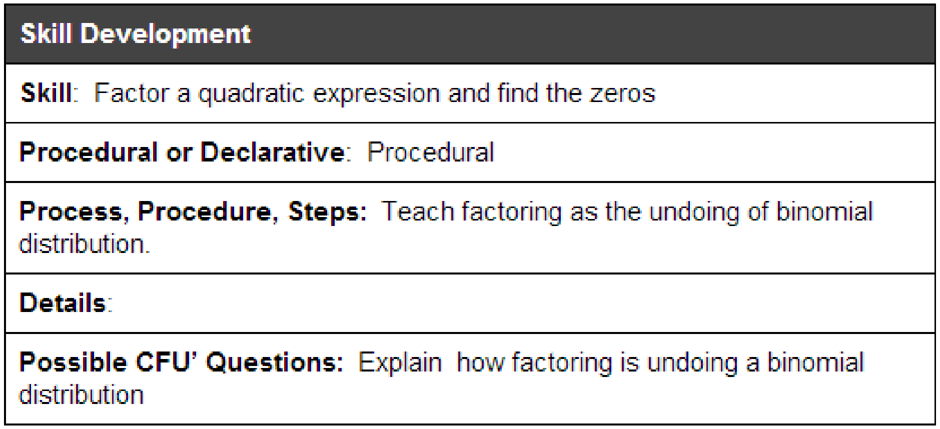
examples:

Three forms of the quadratic function reveal different features of its graph.

Standard form: $f(x) = ax^2 + bx + c$ reveals the y intercept, (0, c).

Vertex form: $f(x) = a(x-h)^2 + k$ reveals the vertex (h, k) and thus the maximum or minimum value of the function.

Factored form: $f(x) = a(x-x_1)(x-x_2)$ reveals the x-intercepts $(x_1,0)$ and $(x_2,0)$.



Skill Development
Skill: Factor a quadratic expression and find the zeros
Procedural or Declarative: Procedural
Process, Procedure, Steps: Teach factoring as the undoing of binomial distribution.
Details:
Possible CFU' Questions: Explain how factoring is undoing a binomial distribution

A.SSE.1 Interpret expressions that represent a quantity in terms of its context.
a. Interpret parts of an expression, such as terms, factors, and coefficients.
b. Interpret complicated expressions by viewing one or more of their parts as a single entity

Concept Development	
Concept:	Polynomial Expressions
Definition:	A monomial or the sum of monomials
Critical Attributes:	exponent of each monomial must be a whole number
Shared Attributes:	a leading coefficient, a constant term, the degree of the polynomial
Non-Critical Attributes:	the number of monomials
Examples:	$4x^2 + 2x - 8,\ 10x^5,\ -3$
Non-Examples:	$x^{\frac{2}{3}},\ 2^x,\ \frac{1}{2}^{-1},\ 0x$
Resources:	http://ccssmath.org/?page_id=2085 purplemath, mathisfun, algebralab.org, regentsprep.org, ccss.org, illustrativemathematics.org

Skill Development	
Skill:	explain (declarative): coefficient, variable, constant, exponent, degree of polynomial, polynomial type
What do I teach?:	Declarative
How do I teach?:	Given various polynomials, have students identify and/or find terms and factors
CFU Questions:	Given that the volume of a box is $x^3 + 4x^2 + 5x + 2$ with at height x+1, what are the other dimensions.

A.APR.3 (part 1)

Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Concept Development	
Concept:	Identifying the real zeros of a (quadratic) polynomial function (note: sketching rough graph will be done in Unit 2, Part 2)
Definition:	Values of x that make a polynomial function equal to zero.
Critical Attributes:	if f(x)=0, then x is a real zero for the polynomial
Shared Attributes:	one or more zeros may exist
Non-Critical Attributes:	solutions that are imaginary may exist but are not used at this point
Examples:	$x^2 - 8x + 12 = 0,\ x = 2,\ x = 6;$ $n^2 - 6n = 0,\ n = 0,\ n = 6;$ $24x^2 + 8x + 2 = 5 - 6x,\ x = \frac{1}{6}, x = -\frac{3}{4}$
Non-Examples:	anything that finds zeros incorrectly; for example **(x+2)(x-1)=4, x=2, x=5**

Skill Development	
Skill:	Identifying the zeros of a polynomial.
What do I teach?:	Procedural
How do I teach?:	In factored form, we set each factor equal to zero and solve.
CFU Questions	Sketch a graph of f(x)=(x-2)(x+3)(x+1) and identify roots. Identify the factors of a graphed polynomial.

N.CN.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials (quadratics only)

Concept Development	
Concept:	Fundamental Theorem of Algebra (FTA) and its Corollary
Definition:	If f(x) is a polynomial of degree n (n>0) then f(x)=0 has at least one solution in the set of complex numbers. Corollary: number of solutions equals the degree, n.
Critical Attributes:	The degree of polynomial will match the number of linear factors.
Shared Attributes:	Find all zeros of polynomials from linear factorization
Non-Critical Attributes:	Some solutions could be real or imaginary, or a combination of both.
Examples:	How many solutions does the equation $x^3 + 5x^2 + 4x + 20 = 0$ have? Justify your answer.
Non-Examples:	You can't use FTA to find solutions to non-polynomials, like $x^{\frac{1}{2}} - 2^x = 0$.
Resources:	http://ccssmath.org/?page_id=2046 Alg 2 text section 5.7 p 379-386

Examples:

- How many zeros does -2x^2 + 3x – 8 have? Find all the zeros and explain, orally or in written format, your answer in terms of the Fundamental Theorem of Algebra.

Skill Development	
Skill:	Show that the FTA is true for quadratic equations. Find all solutions (zeros) for higher order equations.
What do I teach?:	Procedural
How do I teach?:	Factor polynomial to product of prime binomials/trinomials and use Zero Product Property
CFU Questions:	Use the Fundamental Theorem of Algebra to help identify the roots of the polynomials: $x^3 - 2x^2 + 4x - 8$ $x^3 + x^2 - x - 1$ $x^4 + x^3 + 4x^2 - 4x$

A-REI 4. a. Use the method of completing the square to transform any quadratic equation in *x* into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b

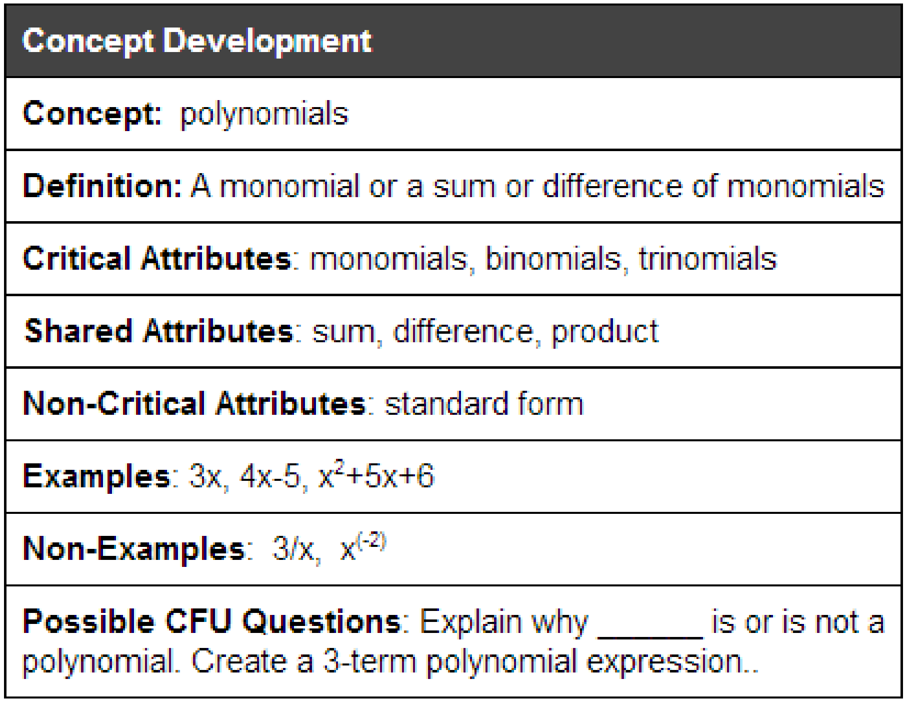
Concept Development
Concept: quadratic equation
Definition: an equation that can be written in the form of $f(x)= ax^2 + bx +c$ where a, c, b are real numbers and a can not be zero.
Critical Attributes: solving the quadratic equation with different methods and discriminant
Shared Attributes: factoring,
Non-Critical Attributes:
Examples: $f(x)= 3x^2 + 4x + 8$
Non-Examples: $f(x)= 2x + 4$
Possible CFU Questions: Explain how you solved the quadratic equation.

Skill Development
Skill: Students solve and explain why they choose the method
Procedural or Declarative: procedural and declarative
Process, Procedure, Steps: Given several quadratic equation students should be able to identify and use the best method to solve the equation. Discriminant: Use $b^2 - 4ac$ to determine how many solutions and what type of solutions a quadratic equation will have.
Details:
Possible CFU' Questions: Explain why you chose a particular method (factoring, completing the square, quadratic formula) to solve a quadratic equation.

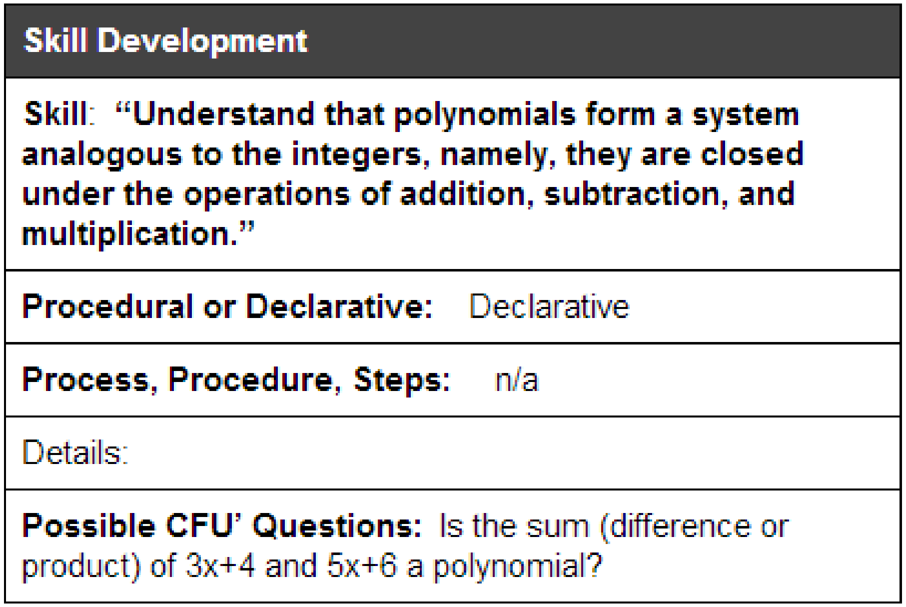
Students may solve by factoring, completing the square, and using the quadratic formula. The zero product property is used to explain why the factors are set equal to zero. Students should relate the value of the discriminant to the type of root to expect. A natural extension would be to relate the type of solutions to $ax^2 + bx + c = 0$ to the behavior of the graph of $y = ax^2 + bx + c$.

Value of Discriminant	Nature of Roots	Nature of Graph
$b^2–4ac = 0$	One real root	One x-intercept
$b^2–4ac > 0$	Two real roots	Two x-intercepts
$b^2–4ac < 0$	No real root	Does not intersect x-axis

A-APR 1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials

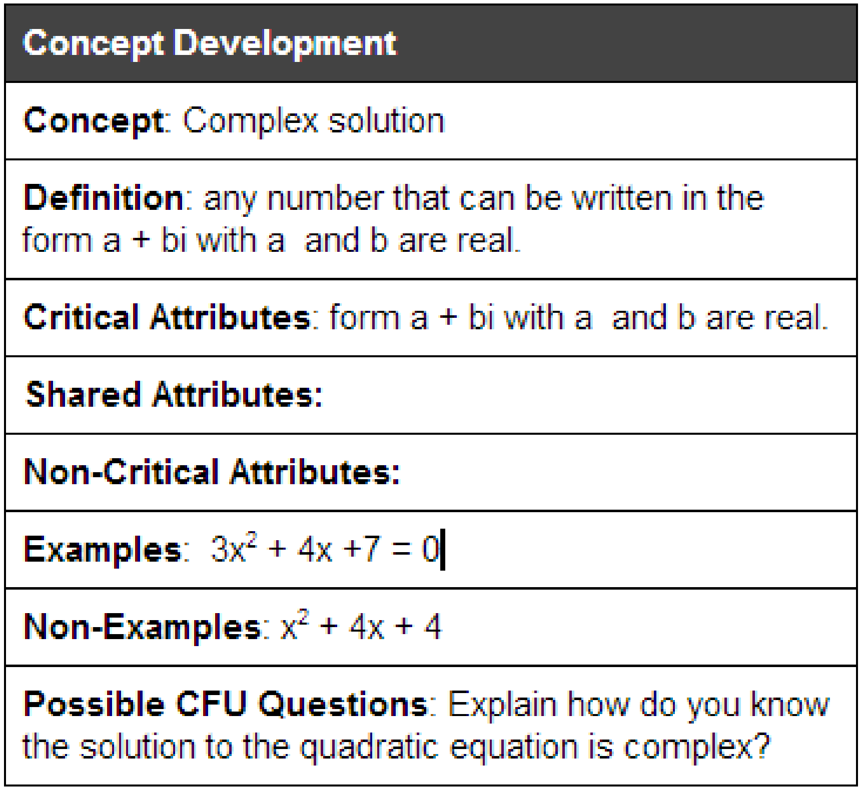


Concept Development
Concept: polynomials
Definition: A monomial or a sum or difference of monomials
Critical Attributes: monomials, binomials, trinomials
Shared Attributes: sum, difference, product
Non-Critical Attributes: standard form
Examples: 3x, 4x-5, x^2+5x+6
Non-Examples: 3/x, $x^{(-2)}$
Possible CFU Questions: Explain why ______ is or is not a polynomial. Create a 3-term polynomial expression..

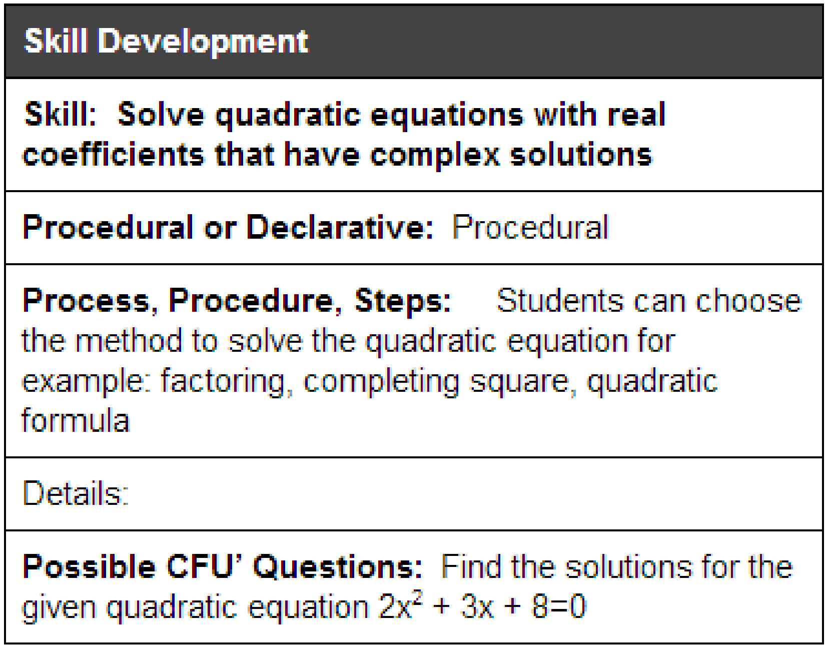


Skill Development
Skill: **"Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication."**
Procedural or Declarative: Declarative
Process, Procedure, Steps: n/a
Details:
Possible CFU' Questions: Is the sum (difference or product) of 3x+4 and 5x+6 a polynomial?

N.CN.7 Solve quadratic equations with real coefficients that have complex solutions.



Concept Development
Concept: Complex solution
Definition: any number that can be written in the form a + bi with a and b are real.
Critical Attributes: form a + bi with a and b are real.
Shared Attributes:
Non-Critical Attributes:
Examples: $3x^2 + 4x + 7 = 0$
Non-Examples: $x^2 + 4x + 4$
Possible CFU Questions: Explain how do you know the solution to the quadratic equation is complex?



Skill Development
Skill: Solve quadratic equations with real coefficients that have complex solutions
Procedural or Declarative: Procedural
Process, Procedure, Steps: Students can choose the method to solve the quadratic equation for example: factoring, completing square, quadratic formula
Details:
Possible CFU' Questions: Find the solutions for the given quadratic equation $2x^2 + 3x + 8=0$

Examples:

- Within which number system can $x^2 = – 2$ be solved? Explain how you know.
- Solve $x^2 + 2x + 2 = 0$ over the complex numbers.

Find all solutions of $2x^2 + 5 = 2x$ and express them in the form *a* + *bi.*

N.CN.1 Know there is a complex number *i* such that $i^2 = \sqrt{-1}$, and every complex number has the form *a* + *bi* with *a* and *b*, real.

N.CN.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers

Concept Development

Concept: complex number

Definition: any number that can be written in the form a + bi with a and b are real.

Critical Attributes: i = $\sqrt{-1}$ or i^2 = -1, a + bi

Shared Attributes: solutions to a quadratic function, real part, imaginary part

Non-Critical Attributes: a and b real numbers

Examples: 3+2i, 0+4i, 2+0i

Non-Examples: 5 4i

Possible CFU Questions: Explain what a complex number is.

Resources: http://ccssmath.org/?page_id=2030

Example:

- Simplify the following expression. Justify each step using the commutative, associative and distributive properties. (3 - 2*i*)(-7 + 4*i*)

Solutions may vary; one solution follows:

$(3 - 2i)(-7 + 4i)$
$3(-7 + 4i) -2i(-7 + 4i)$ Distributive Property
$-21 + 12i + 14i - 8i^2$ Distributive Property
$-21 + (12i + 14i) - 8i^2$ Associative Property
$-21 + i(12 + 14) - 8i^2$ Distributive Property
$-21 + 26i - 8i^2$ Computation
$-21 + 26i - 8(-1)$ $i^2 = -1$
$-21 + 26i + 8$ Computation
$-21 + 8 + 26i$ Commutative Property
$-13 + 26i$ Computation

Skill Development

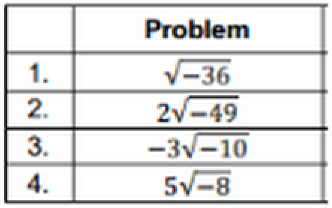
Skill: Know there is a complex number i.
Know every complex number has the form a+bi, with a and b representing real numbers.

Procedural or Declarative: Declarative

Process, Procedure, Steps:

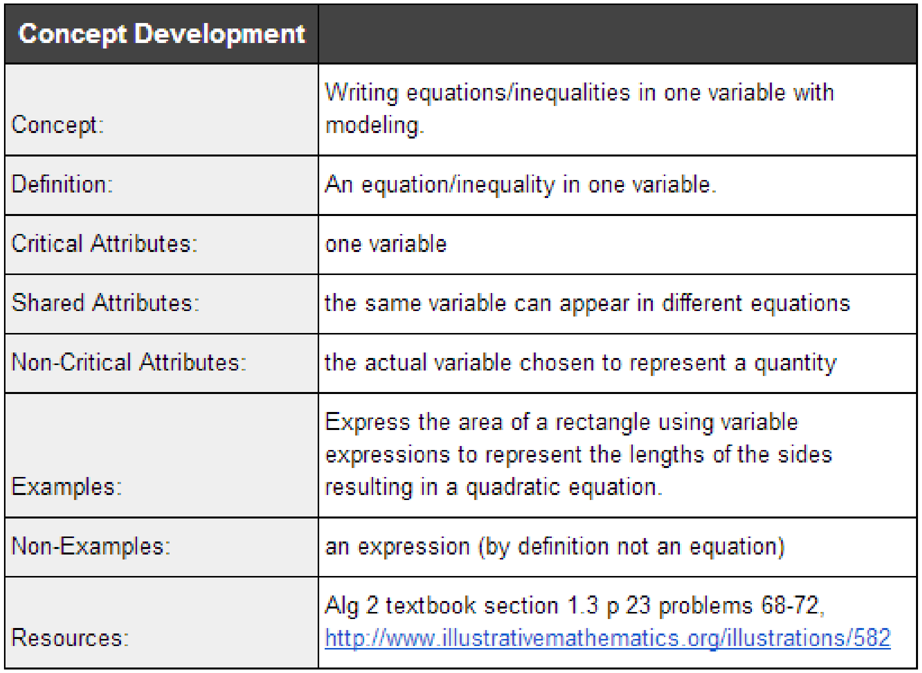
Details:

Possible CFU' Questions: Simplify:

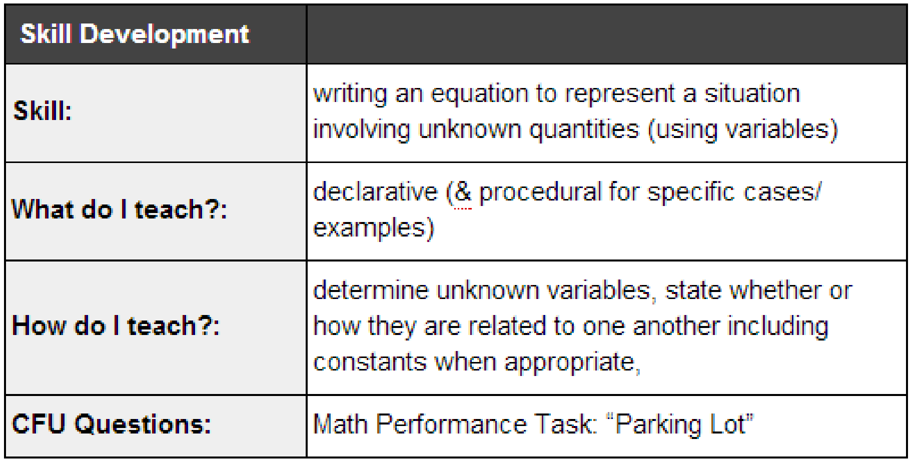


	Problem
1.	$\sqrt{-36}$
2.	$2\sqrt{-49}$
3.	$-3\sqrt{-10}$
4.	$5\sqrt{-8}$

A.CED.1 Create equations and inequalities in one variable and use them to solve problems. ***Include equations arising from linear and quadratic functions, and simple rational and exponential functions.***



Concept Development	
Concept:	Writing equations/inequalities in one variable with modeling.
Definition:	An equation/inequality in one variable.
Critical Attributes:	one variable
Shared Attributes:	the same variable can appear in different equations
Non-Critical Attributes:	the actual variable chosen to represent a quantity
Examples:	Express the area of a rectangle using variable expressions to represent the lengths of the sides resulting in a quadratic equation.
Non-Examples:	an expression (by definition not an equation)
Resources:	Alg 2 textbook section 1.3 p 23 problems 68-72, http://www.illustrativemathematics.org/illustrations/582



Skill Development	
Skill:	writing an equation to represent a situation involving unknown quantities (using variables)
What do I teach?:	declarative (& procedural for specific cases/ examples)
How do I teach?:	determine unknown variables, state whether or how they are related to one another including constants when appropriate,
CFU Questions:	Math Performance Task: “Parking Lot”

Examples:

- Given that the following trapezoid has area 54 cm^2, set up an equation to find the length of the base, and solve the equation.

Lava coming from the eruption of a volcano follows a parabolic path. The height *h* in feet of a piece of lava *t* seconds after it is ejected from the volcano is given by $h(t)= -t^2 + 16t + 936$. After how many seconds does the lava reach its maximum height of 1000 feet?

A.CED.3 *(Unit 15) (Unit 6)*

Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

Concept Development	
Concept:	Interpret solutions as viable or nonviable options in a modeling context.
Definition:	Constraints are Domain or Range restrictions to solutions.
Critical Attributes:	The model has at least one constraint.
Shared Attributes:	Linear Programming: Feasible Region, Critical Points.
Non-Critical Attributes:	The number of constraints; the set of values of the constraint(s).
Examples:	See Resources
Non-Examples:	All solutions are possible
Resources:	Alg 2 textbook sect 2.8 p 132-137, section 2.3 p94, 95. Linear Programming on p 174-176.

Skill Development	
Skill:	Interpreting the constraint in the context of the model
What do I teach?:	Procedural
How do I teach?:	Once the expression is determined that represents the model, identify any constraints that exist.
CFU Questions:	The number of individuals infected by a virus can be

Integrated Math 3 Course Standard and Resource Guide

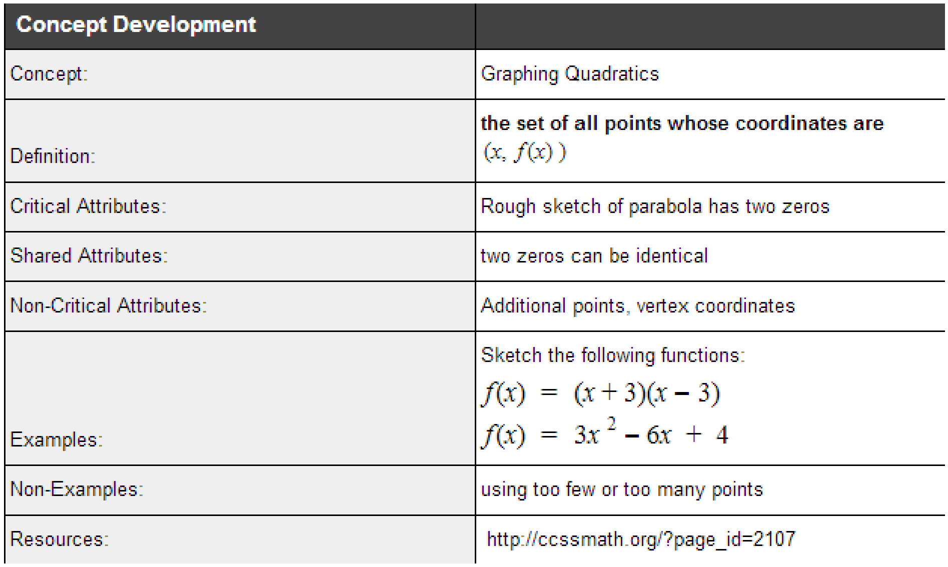
Quadratics, Polynomials, and Other Functions

UNIT 2 : Graphing Quadratics

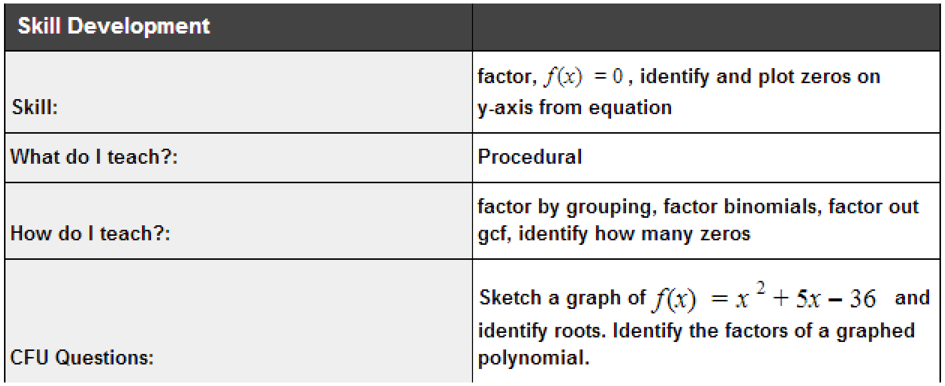
Overview	**Solve quadratic functions in various forms**

Priority standard

A. APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. Students will be able to use the zeros to construct a rough graph of the function defined by the polynomial.



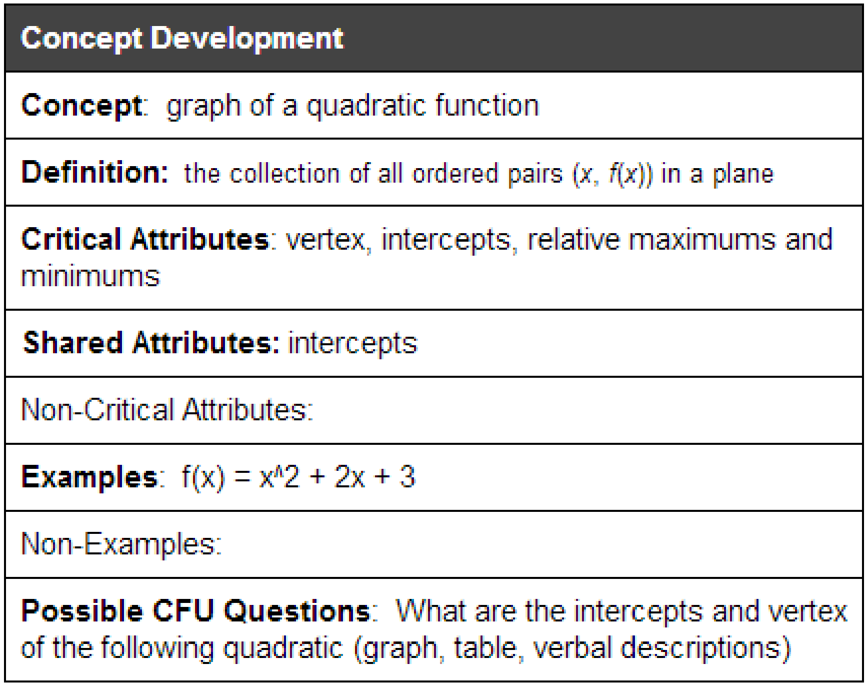
Concept Development	
Concept:	Graphing Quadratics
Definition:	the set of all points whose coordinates are $(x, f(x))$
Critical Attributes:	Rough sketch of parabola has two zeros
Shared Attributes:	two zeros can be identical
Non-Critical Attributes:	Additional points, vertex coordinates
Examples:	Sketch the following functions: $f(x) = (x+3)(x-3)$ $f(x) = 3x^2 - 6x + 4$
Non-Examples:	using too few or too many points
Resources:	http://ccssmath.org/?page_id=2107



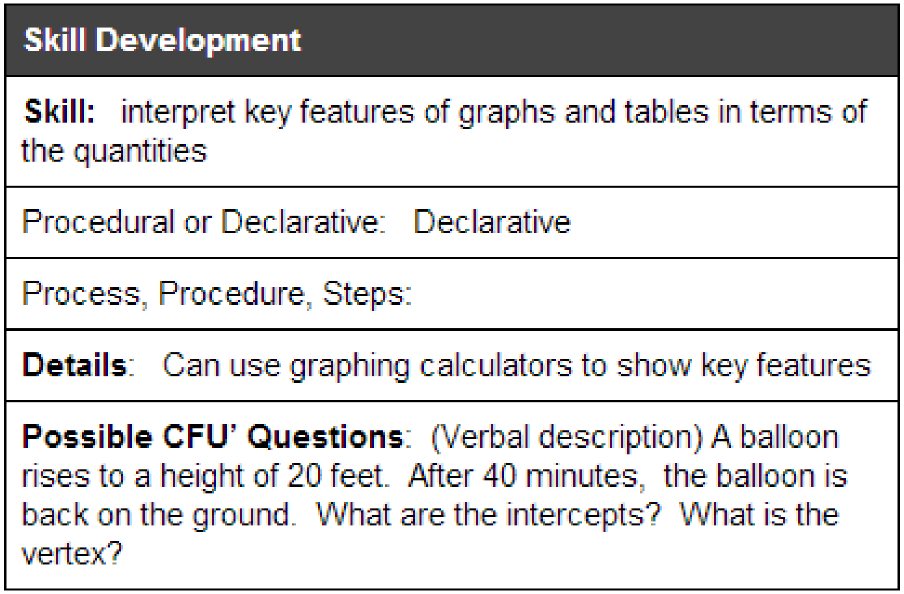
Skill Development	
Skill:	factor, $f(x) = 0$, identify and plot zeros on y-axis from equation
What do I teach?:	Procedural
How do I teach?:	factor by grouping, factor binomials, factor out gcf, identify how many zeros
CFU Questions:	Sketch a graph of $f(x) = x^2 + 5x - 36$ and identify roots. Identify the factors of a graphed polynomial.

Supporting standards

F.IF.4
For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

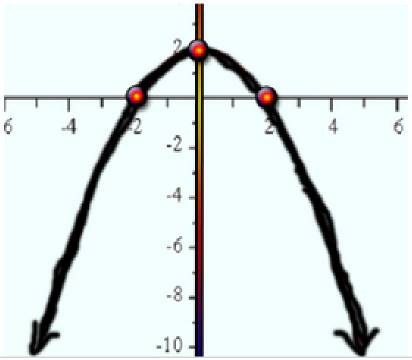


Concept Development
Concept: graph of a quadratic function
Definition: the collection of all ordered pairs (*x*, *f*(*x*)) in a plane
Critical Attributes: vertex, intercepts, relative maximums and minimums
Shared Attributes: intercepts
Non-Critical Attributes:
Examples: f(x) = x^2 + 2x + 3
Non-Examples:
Possible CFU Questions: What are the intercepts and vertex of the following quadratic (graph, table, verbal descriptions)



Skill Development
Skill: interpret key features of graphs and tables in terms of the quantities
Procedural or Declarative: Declarative
Process, Procedure, Steps:
Details: Can use graphing calculators to show key features
Possible CFU' Questions: (Verbal description) A balloon rises to a height of 20 feet. After 40 minutes, the balloon is back on the ground. What are the intercepts? What is the vertex?

F.IF.7a (part 1 and part 2 below)
Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
a. Graph linear and quadratic functions and show intercepts, maxima, and minima."
c) Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

Concept Development	
Concept:	Graphing (Quadratics)
Definition:	the set of all points whose coordinates are (x,f(x))
Critical Attributes:	Sketch of parabola has a vertex and two critical points
Shared Attributes:	Critical points are zeros. Symmetry about axis of symmetry (one side could be y-intercept, for example)
Non-Critical Attributes:	Additional points
	Given: $y = -(1/2)(x+2)(x-2)$, sketch the graph. 

Skill Development	
Skill:	Find vertex, axis of symmetry, max, min, zeros
What do I teach?:	Procedural
How do I teach?:	Find the vertex(vertices), identify axis of symmetry, find the zeros
CFU Questions:	Graph the function: $f(x) = 2x^2 + 7x + 3$ Identify key features: vertex, y-intercept, x-intercepts, line of symmetry, and end behavior.

F.IF.8

Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

(Math 2 standard)

F.IF.8.a

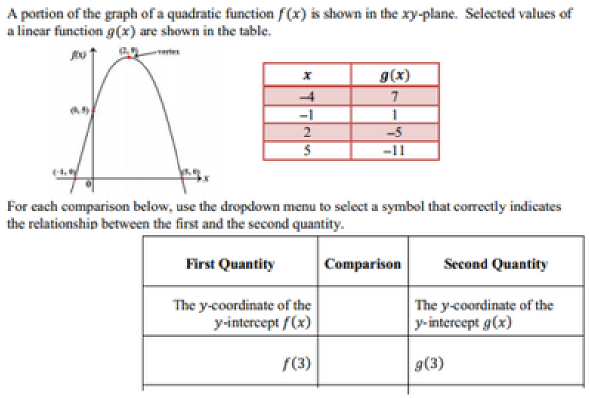
Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

Concept Development	
Concept:	Equivalent forms of functions
Definition:	functions that have the same solution set
Critical Attributes:	Must be equivalent for all values of x.
Shared Attributes:	Zeros, extreme values, end behavior. Equivalence.
Non-Critical Attributes:	values of maxima/minima change from function to function...
Examples:	Find the x-intercepts of $f(x) = -3(x - 2)^2 +3$
Non-Examples:	Find the x-intercepts of $f(x) = (x + 2)(x - 5)$
Resources:	http://www.illustrativemathematics.org/illustrations/640

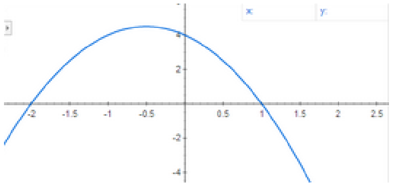
Skill Development	
Skill:	Complete the square. Completely factor.
What do I teach?:	Procedural
How do I teach?:	Std to Vertex: use completing the square Vertex to Std: distribute and combine like terms
CFU Questions:	Change the following function from standard form to vertex form. "in context" example: Suppose $h(t) = -5t^2 + 10t + 3$ is an expression giving the height of a diver above the water (in meters), t seconds after the diver leaves the springboard. How high above the water is the springboard? Explain how you know. When does the diver hit the water? At what time on the diver's descent toward the water is the diver again at the same height as the springboard? When does the diver reach the peak of the dive?

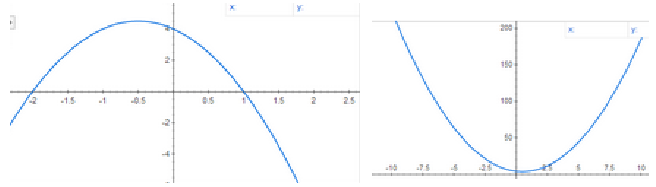
F.IF.9

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

<table>
<tr><th>Concept Development</th><th></th></tr>
<tr><td>Concept:</td><td>compare or contrast two different functions given in two different forms</td></tr>
<tr><td>Definition:</td><td>"forms" given: algebraically, graphically, numerically in tables, and verbally</td></tr>
<tr><td>Critical Attributes:</td><td>each function must be represented in a different form</td></tr>
<tr><td>Shared Attributes:</td><td></td></tr>
<tr><td>Non-Critical Attributes:</td><td>the two functions must be same type (ex: both quadratic).</td></tr>
<tr><td>Examples:</td><td>Compare this quadratic function expressed algebraically with a graphed quadratic function.

A portion of the graph of a quadratic function $f(x)$ is shown in the xy-plane. Selected values of a linear function $g(x)$ are shown in the table.

<table>
<tr><th>x</th><th>$g(x)$</th></tr>
<tr><td>–4</td><td>7</td></tr>
<tr><td>–1</td><td>1</td></tr>
<tr><td>2</td><td>–5</td></tr>
<tr><td>5</td><td>–11</td></tr>
</table>
For each comparison below, use the dropdown menu to select a symbol that correctly indicates the relationship between the first and the second quantity.
<table>
<tr><th>First Quantity</th><th>Comparison</th><th>Second Quantity</th></tr>
<tr><td>The y-coordinate of the y-intercept $f(x)$</td><td></td><td>The y-coordinate of the y-intercept $g(x)$</td></tr>
<tr><td>$f(3)$</td><td></td><td>$g(3)$</td></tr>
</table>
</td></tr>
<tr><td>Non-Examples:</td><td>What is the behavior of these two graphs? Identify key features, including maximum, minimum, intercepts and end behavior in your description.</td></tr>
</table>

<table>
<tr><th>Skill Development</th><th></th></tr>
<tr><td>Skill:</td><td>Graph functions, identify ordered pairs, identify independent and dependent variables from given data</td></tr>
<tr><td>What do I teach?:</td><td>Declarative</td></tr>
<tr><td>How do I teach?:</td><td>Compare/contrast two functions of same type (ex: quadratic or exponential) at specific points</td></tr>
<tr><td>CFU Questions:</td><td>Identify the similarities and differences between the two polynomial functions below:
A.) $f(x) = -2x^2 - 2x + 4$


B.) TABLE Quadratic (x|-2,1,3,4) | (f(x)|-8,-5,7,16)</td></tr>
<tr><td></td><td>
</td></tr>
<tr><td>Resources:</td><td>http://www.illustrativemathematics.org/illustrations/1279</td></tr>
</table>

F.IF.5

Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. ***For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.***

Concept Development
Concept: domain
Definition: the set of "input" or argument values for which the function is defined
Critical Attributes: the set of "input"
Shared Attributes:
Non-Critical Attributes:
Examples: Domain for a maximum area function is always positive
Non-Examples:
Possible CFU Questions: Using real world applications explain why or why not the domain of this function________ makes sense?

Skill Development
Skill: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes
Procedural or Declarative: procedural
Process, Procedure, Steps: Use different type of graphs to represent real world applications.
Details:
Possible CFU' Questions: Identify the domain of the graph

F.BF.3

Identify the effect on the graph of replacing f(x) by f(x) + k, kf(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Concept Development	
Concept:	Transformations of parabolas.
Definition:	the operation of changing (as by rotation or mapping) one configuration or expression into another in accordance with a mathematical rule; *especially* : a change of variables or coordinates in which a function of new variables or coordinates is substituted for each original variable or coordinate
Critical Attributes:	type of function stays same (quadratic, etc)
Shared Attributes:	location of k always has same transformation effect on ANY kind of function (exponential, absolute value,...)
Non-Critical Attributes:	f(x) could be g(x) or y...
Examples:	Use the equation to answer the question $y = f(x + A) + B$ Describe how each parameter (A and B) affects the graph of the function $y = x^2$. Include specific information about how positive and negative values affect the graph for each parameter in your answer.
Non-Examples:	if k=1 or zero: y=1x+0 has no transformation
Resources:	http://ccssmath.org/?page_id=2195

Skill Development	
Skill:	Graph quadratic functions in standard form, intercept form, and vertex form
What do I teach?:	Declarative
How do I teach?:	Use technology to produce a variety of graphs to investigate the effects of *k* on the functions and have students find patterns that can be generalized to describe transformations of functions.
CFU Questions:	Describe the graphical differences between the two functions. f(x)= 2(x+3)+1 and g(x)=5(x-1)+2

Integrated Math 3 Course Standard and Resource Guide

Quadratics, Polynomials, and Other Functions

UNIT 3 : Higher Order Polynomials

Overview	**Solving and Graphing higher order polynomials**

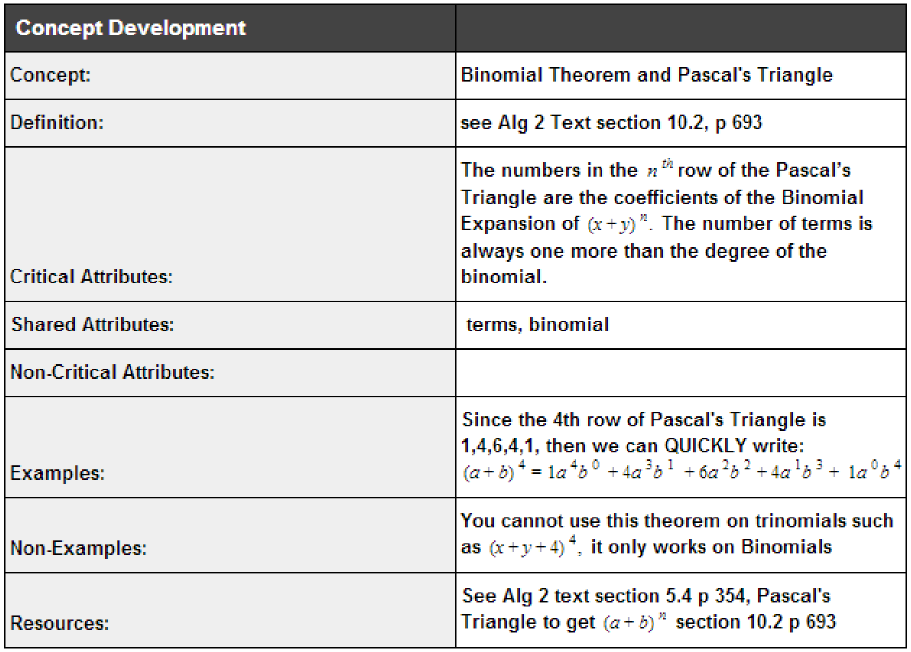
Priority standard

A. APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

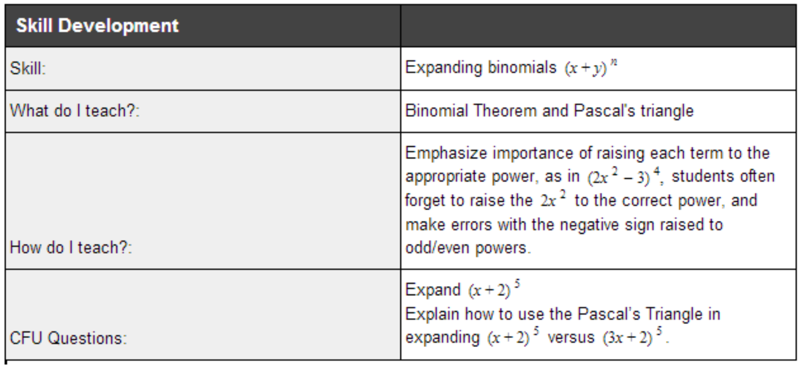
Supporting standards

A.APR.5

(+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.

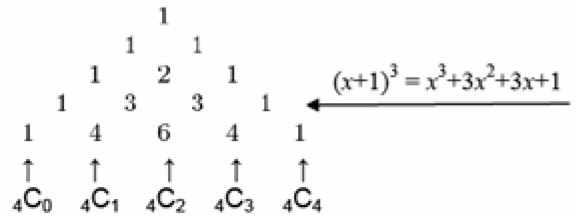


Concept Development	
Concept:	Binomial Theorem and Pascal's Triangle
Definition:	see Alg 2 Text section 10.2, p 693
Critical Attributes:	The numbers in the n^{th} row of the Pascal's Triangle are the coefficients of the Binomial Expansion of $(x+y)^n$. The number of terms is always one more than the degree of the binomial.
Shared Attributes:	terms, binomial
Non-Critical Attributes:	
Examples:	Since the 4th row of Pascal's Triangle is 1,4,6,4,1, then we can QUICKLY write: $(a+b)^4 = 1a^4b^0 + 4a^3b^1 + 6a^2b^2 + 4a^1b^3 + 1a^0b^4$
Non-Examples:	You cannot use this theorem on trinomials such as $(x+y+4)^4$, it only works on Binomials
Resources:	See Alg 2 text section 5.4 p 354, Pascal's Triangle to get $(a+b)^n$ section 10.2 p 693



Skill Development	
Skill:	Expanding binomials $(x+y)^n$
What do I teach?:	Binomial Theorem and Pascal's triangle
How do I teach?:	Emphasize importance of raising each term to the appropriate power, as in $(2x^2-3)^4$, students often forget to raise the $2x^2$ to the correct power, and make errors with the negative sign raised to odd/even powers.
CFU Questions:	Expand $(x+2)^5$ Explain how to use the Pascal's Triangle in expanding $(x+2)^5$ versus $(3x+2)^5$.

Examples:

- Use Pascal's Triangle to expand the expression $(2x-1)^4$.
- Find the middle term in the expansion of $(x^2+2)^{18}$.
- 

A.APR.1
Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Concept Development	
Concept:	Operations of Addition, Subtraction, Multiplication on Polynomials. "Closure" applies.
Definition:	Closure: Like terms when you add, subtract, or multiply polynomials, you get another polynomial.
Critical Attributes:	Combining "like terms," using distributive property.
Shared Attributes:	Other functions, such as radicals, can also have like terms. $x^{1/2}+x^{1/2}$
Non-Critical Attributes:	Order in which you list terms (ascending, descending...)
Examples:	$x^2+4x^2=$ $\left(4x^2+5x+6\right)\left(3x^2+5x\right)(3x+2)=$
Non-Examples:	x^2+y^2 are not like terms $\left(x^3\right)\left(x^3\right)$ is NOT x^9 , (x+3)(x+3) is not x^2+9
Resources:	http://ccssmath.org/?page_id=2103, Alg 2 Textbook section 5.3 p 346-352.

Skill Development	
Skill:	**Add, subtract, multiply polynomials.**
What do I teach?:	**Declarative and Procedural**
How do I teach?:	**Identify and combine like terms.**
CFU Questions:	**1. Simplify:** $\left(x^3+5x^2+3x-2\right)+\left(x^4-3x^3+7x-1\right)$ $\left(x^3+5x^2+3x-2\right)-\left(x^4-3x^3+7x-1\right)$ $\left(x^3+5x^2+3x-2\right)\left(x^4-3x^3+7x-1\right)$ **2. When you subtract two polynomials, you _____ (sometimes, always, never) get another polynomial.** **3. Circle all of the following operations that are closed operations on the set of polynomials: addition, subtraction, multiplication, division of polynomials.**

A.APR.2

Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).

Concept Development	
Concept:	Remainder Theorem
Definition:	If p(x) is a polynomial and r is the remainder when p(x) is divided by (x-a), then r = p(a). Also, if the remainder is zero, then (x-a) is a factor of p(x).
Critical Attributes:	Polynomials must be in descending order with zero coefficients, ex: $3x^2 + 0x + 5$. Know that, in rational expressions, the denominator is the divisor.
Shared Attributes:	Like division of whole numbers, polynomial division can have remainders (which are useful in graphing and in finding roots).
Non-Critical Attributes:	
Examples:	Divide $\left(x^3 + 5x^2 - 7x + 2\right)$ by $x - 2$ using long division, noting that the remainder equals f(2)
Non-Examples:	Can't use remainder theorem with higher order divisors, like $x^2 - 4$
Resources:	http://ccssmath.org/?page_id=2105 Alg 2 Textbook section 5.5 p 362-368

Skill Development	
Skill:	Long division of polynomials, synthetic division which is a special case of division of polynomials where $(x - a)$ must be the divisor
What do I teach?:	Procedural Skill
How do I teach?:	Long Division: (algorithm analogous to steps for long division with integers) 1. Arrange polynomials in descending order with zero coefficients, ex: $3x^2 + 0x + 5$ 2. Know that, in rational expressions, the denominator is the divisor. Synthetic Division: only works when $(x - a)$ is the divisor
CFU Questions:	1. Using the Remainder Theorem, decide whether (x - 5) and (x + 2) are factors of the polynomial $f(x) = 2x^3 - 5x^2 - 28x + 15$ 2. Divide $\left(x^3 + 5x^2 - 7x + 2\right)$ by $x - 2$ using long division, noting that the remainder equals f(2) 3. When is it appropriate to use synthetic division? 4.Describe and correct the error in synthetic division to divide $x^3 - 5x + 3\ by\ x - 2$. *(see textbook p 366 problems #19, 20)*

A.APR.3

Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Concept Development	
Concept:	Division of polynomials gives us a tool for finding roots, and vice versa.
Definition:	Each factor gives us an x-intercept, when set equal to zero.
Critical Attributes:	if f(x)=0, then x is a real zero for the polynomial
Shared Attributes:	one or more zeros may exist
Non-Critical Attributes:	solutions that are imaginary may exist but are not used at this point
Examples:	Sketch: $f(x) = (x + 5)(x - 1)(x - 3)$. Zeros are at x=-5, +1, +3, with function going up on right, down on left end. Sketch: $f(x) = -(x+2)^3(x-3)$. Zeros are at x=-2, +3, with function going down on right, up on left end. Function goes through the x axis at triple root x=-2.
Non-Examples:	If (x-2)(x+3) = 5, zeros are not x = 2 and x = -3. (The right side of the equation MUST be zero!)
Resources:	Alg 2 text section 5.4 p 353-359, section 5.7 p 379-386, section 5.8 p 387-392, and PreCalc text for end-behavior, multiple roots, etc. http://ccssmath.org/?page_id=2107

Skill Development	
Skill:	Plot rough graph.
What do I teach?:	Procedural
How do I teach?:	Plot all roots on x axis, picture end behavior, deal with multiple roots of odd degree crossing axis and even degree returning in same direction. (We do not care about actual values of y at relative min/max).
CFU Questions:	1. Sketch a graph of f(x)=(x-2)(x+3)(x+1) and identify roots. 2. Identify the factors of a graphed polynomial.

A.APR.6

Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x),and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.

Concept Development	
Concept:	"Equivalent Expressions" is the general concept. This is a specific set of cases which is re-writing rational expressions using division or factoring to find quotient and remainder.
Definition:	(See objective & standard above.)
Critical Attributes:	quotient, divisor, remainder (which might include r(x)=0)
Shared Attributes:	divisor is always the denominator of the remainder.
Non-Critical Attributes:	remainder could be zero
Examples:	Rewrite $\frac{x^2+2x-4}{x-2}$. Solution by inspection: $x+4+\frac{4}{x-2}$
Non-Examples:	$\frac{2x-4}{x^2-2}$
Resources:	Alg 2 Textbook section 5.5 p 362-368 http://ccssmath.org/?page_id=2113

Skill Development	
Skill:	Factoring and simplifying rational expressions and long division of polynomials.
What do I teach?:	Procedural
How do I teach?:	When given a rational expression, you can simplify it by using long (or synthetic division), where the denominator becomes the divisor. Any remainder is written as the numerator over the divisor.
CFU Questions:	Divide: $\frac{4x^3+x^2-3x+7}{x-1}$. To solve it, rewrite as: x-1 $)\overline{4x^3+x^2-3x+7} = 4x^2+5x+2+\frac{9}{x-1}$

N.CN.8

(+) Extend polynomial identities to the complex numbers.

Concept Development	
Concept:	Completely factoring polynomials to include imaginary roots.
Definition:	Complete linear factorization: in every factor, x has degree of one.
Critical Attributes:	Imaginary/Complex factors always occur in conjugate pairs, ex: (x+2i)(x-2i).
Shared Attributes:	Real-factored polynomials.
Non-Critical Attributes:	Greatest Common Factor
Examples:	$x^3+5x^2+8x+3=(x+3)(x-(-1+i))(x-(-1-i))$
Non-Examples:	x^2+4 is not a linear factor, (x+2i)(x-2i) is completely factored.
Resources:	(fyi: from Pre-Calc text, not in Alg 2 text). See framework. http://ccssmath.org/?page_id=2103,

Skill Development	
Skill:	**Factor Polynomial**
What do I teach?:	**Procedural**
How do I teach?:	**Polynomials need to be factored completely include imaginary roots**
CFU Questions:	**Determine linear factors of x^2+16 over the complex number system.**

N.CN.9

(+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Concept Development	
Concept:	Fundamental Theorem of Algebra (FTA) and its Corollary
Definition:	If f(x) is a polynomial of degree n (n>0) then f(x)=0 has at least one solution in the set of complex numbers. Corollary: number of solutions equals the degree, n.
Critical Attributes:	The degree of polynomial will match the number of linear factors.
Shared Attributes:	Find all zeros of polynomials from linear factorization
Non-Critical Attributes:	Some solutions could be real or imaginary. or a combination of both.
Examples:	How many solutions does the equation $x^3 + 5x^2 + 4x + 20 = 0$ have? Justify your answer.
Non-Examples:	You can't use FTA to find solutions to non-polynomials, like $x^{\frac{1}{2}} - 2^x = 0$.
Resources:	http://ccssmath.org/?page_id=2046 Alg 2 text section 5.7 p 379-386

Skill Development	
Skill:	**Show that the FTA is true for quadratic equations. Find all solutions (zeros) for higher order equations.**
What do I teach?:	**Procedural**
How do I teach?:	**Factor polynomial to product of prime binomials/trinomials and use Zero Product Property**
CFU Questions:	**Use the Fundamental Theorem of Algebra to help identify the roots of the polynomials:** $x^3 - 2x^2 + 4x - 8$ $x^3 + x^2 - x - 1$ $x^4 + x^3 + 4x^2 - 4x$

A.SSE.2

Use the structure of an expression to identify ways to rewrite it.

Concept Development	
Concept:	Structure of Expressions
Definition:	Writing equivalent expressions, specifically completely factoring polynomials, using previously-learned techniques.
Critical Attributes:	Must be same as the original expression.
Shared Attributes:	Factor
Non-Critical Attributes:	Type of Polynomial
Examples:	$x^4 - y^4 = (x^2)^2 - (y^2)^2$ $x^2 - 5 = (x - \sqrt{5})(x + \sqrt{5})$
Non-Examples:	$u^2 + 4u + 3$
Resources:	http://ccssmath.org/?page_id=2091 Alg 2 text, section 5.7. Also see PreCalculus text.

Skill Development	
Skill:	Completely factor polynomials.
What do I teach?:	Procedural
How do I teach?:	rewrite polynomials as product of prime factors
CFU Questions:	Factor: $x^4 + 4x^2 + 3$ Extension/Challenge question: Ciera says that factoring $5^{2x} + 4(5^x) + 3$ is really easy. Show and explain what she knows. Solution: rewrite $(5^x)^2 + 4(5^x) + 3$ and let u= 5^x then $u^2 + 4u + 3$...

Quadratics, Polynomials, and Other Functions

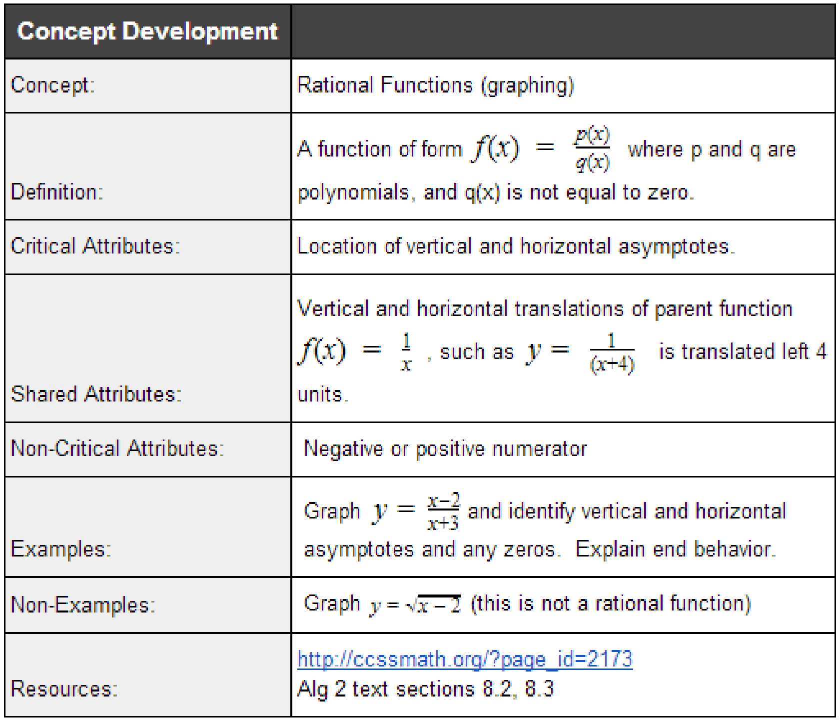
UNIT 4 : Rational

Overview	**Solving and graphing rational functions**

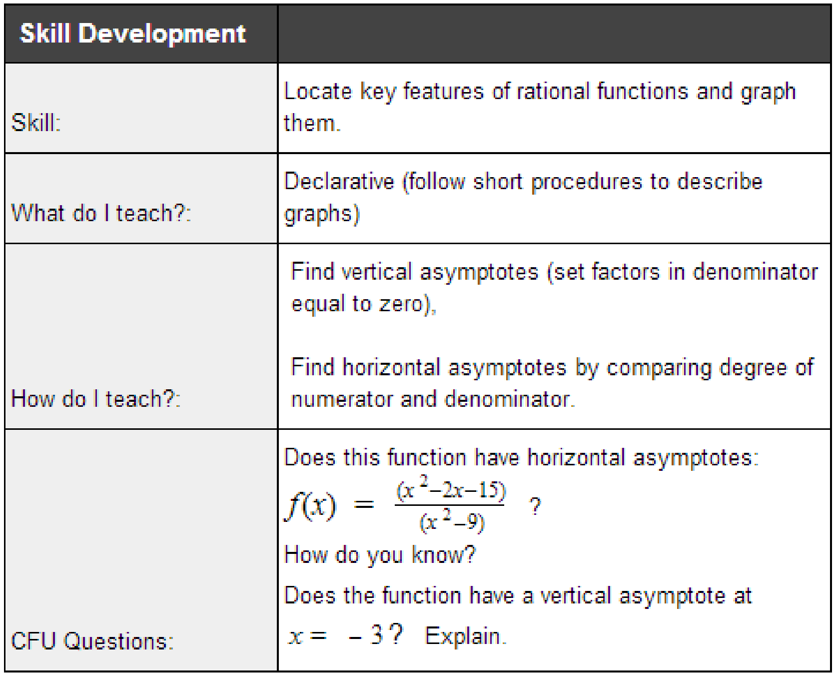
Priority standard

F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases

- **(+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.**



Concept Development	
Concept:	Rational Functions (graphing)
Definition:	A function of form $f(x) = \frac{p(x)}{q(x)}$ where p and q are polynomials, and q(x) is not equal to zero.
Critical Attributes:	Location of vertical and horizontal asymptotes.
Shared Attributes:	Vertical and horizontal translations of parent function $f(x) = \frac{1}{x}$, such as $y = \frac{1}{(x+4)}$ is translated left 4 units.
Non-Critical Attributes:	Negative or positive numerator
Examples:	Graph $y = \frac{x-2}{x+3}$ and identify vertical and horizontal asymptotes and any zeros. Explain end behavior.
Non-Examples:	Graph $y = \sqrt{x-2}$ (this is not a rational function)
Resources:	http://ccssmath.org/?page_id=2173 Alg 2 text sections 8.2, 8.3



Skill Development	
Skill:	Locate key features of rational functions and graph them.
What do I teach?:	Declarative (follow short procedures to describe graphs)
How do I teach?:	Find vertical asymptotes (set factors in denominator equal to zero), Find horizontal asymptotes by comparing degree of numerator and denominator.
CFU Questions:	Does this function have horizontal asymptotes: $f(x) = \frac{(x^2-2x-15)}{(x^2-9)}$? How do you know? Does the function have a vertical asymptote at $x = -3$? Explain.

Supporting standards

F.BF.3
Identify the effect on the graph of replacing f(x) by f(x) + k, kf(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Concept Development	
Concept:	Transformations of rational functions.
Definition:	transformations include translations (vertical/horizontal shifts), stretching
Critical Attributes:	has a numerator and a denominator.
Shared Attributes:	location of k always has same transformation effect on ANY kind of function (exponential, absolute value, ...)
Non-Critical Attributes:	f(x) could be g(x) or y ...
Examples:	$y = \frac{1}{(x+3)} - 4$ is $y = \frac{1}{x}$ shifted 3 left (horizontal shift) and down 4 (vertical shift).
Non-Examples:	if k=1 or zero: $y = \frac{1}{(x+0)}$ has no transformation
Resources:	http://ccssmath.org/?page_id=2195 Alg 2 text 8.2, 8.3

Skill Development	
Skill:	Graphing using transformations and identifying the transformation by comparing two graphs.
What do I teach?:	Procedural (graphing) and Declarative (describing)
How do I teach?:	Beginning with the parent function, graph the new function based on the transformation and state what the transformation is given two graphs.
CFU Questions:	1.Sketch $y = \frac{1}{(x-3)}$ 2. Contrast graphs of $y = \frac{1}{(x-4)}$ and $y = \frac{4}{(x-4)}$ Describe the transformation.

A.APR.7

(+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions

Concept Development	
Concept:	Operations of Addition, Subtraction, Multiplication, Division on Rational expressions. "Closure" applies when denominator is nonzero.
Definition:	Closure: when you add, subtract, or multiply, divide rational expressions, you get another rational expression.
Critical Attributes:	Use same properties as fractions.
Shared Attributes:	Other functions, such as radicals, can also have like terms. $x^{\frac{1}{2}} + x^{\frac{1}{2}}$
Non-Critical Attributes:	Negative or positive numerator
Examples:	$\frac{1}{x} + \frac{3}{(2-x)} = \_\_?$ $\frac{1}{x} - \frac{3}{(2-x)} = \_\_?$ $\frac{x^2y^3}{4x^8y^2} = \_\_?$ $\frac{x^2-4}{x^2+4x+4} = \_\_?$
Non-Examples:	$x^2 + y^2$ are not like terms, $(x^3)(x^3)$ is NOT x^9 (x+3)(x+3) is not $x^2 + 9$
Resources:	http://ccssmath.org/?page_id=2115 See Alg 2 Text sections 8.4, 8.5

Skill Development	
Skill:	Add, subtract, multiply divide rational expressions.
What do I teach?:	Procedural
How do I teach?:	Compare to operations with regular fractions.
CFU Questions:	Perform the operation on the rational expressions. 1. $\frac{x+1}{x^2+4x+4} - \frac{6}{x^2-4}$ 2. $\frac{6x^2+x-15}{4x^2} + \frac{2x+5}{2x}$

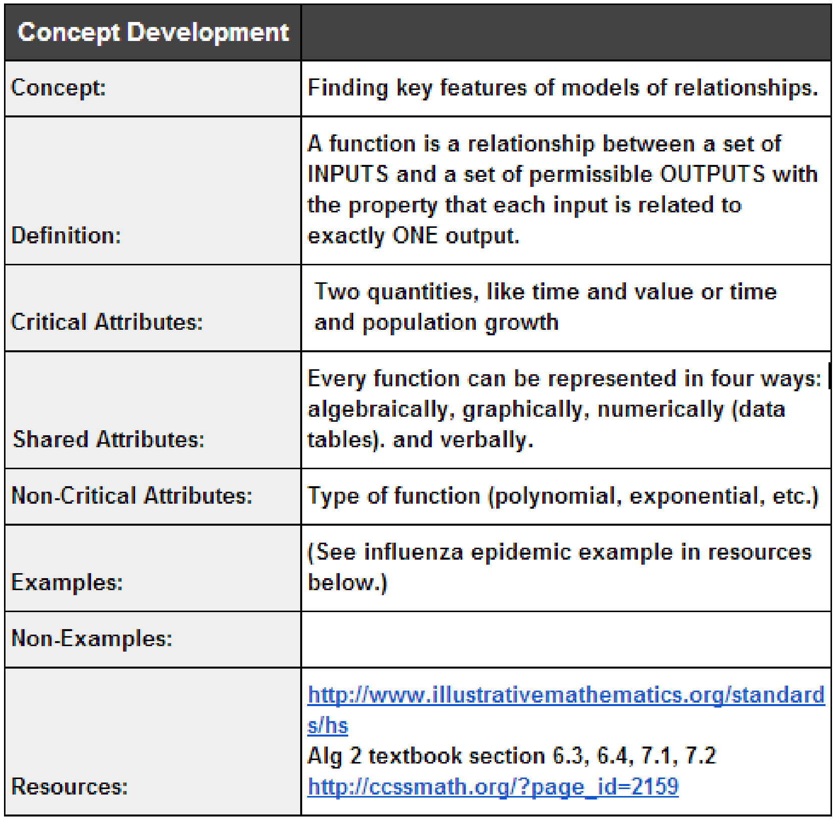
A.REI.2
Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Concept Development	
Concept:	Extraneous solutions
Definition:	A solution that emerges from the process of solving the problem but is not a valid solution to the original problem.
Critical Attributes:	Solution that results in denominator equal to zero
Shared Attributes:	
Non-Critical Attributes:	You might have zero, one, or many solutions.
Examples:	Solve $\frac{6}{(x-3)} = \frac{8x^2}{(x^2-9)} - \frac{4x}{(x+3)}$ and identify all solutions including extraneous solutions if any and explain. $Ans:\ x = \frac{3}{2}$ is solution, $x = -3$ is extraneous. (see p 591)
Non-Examples:	$\frac{x-4}{5} + \frac{x-3}{6} = 1$, x=4 and x=3 are not solutions.
Resources:	http://ccssmath.org/?page_id=2127 See Alg 2 text section 8.6 p 589

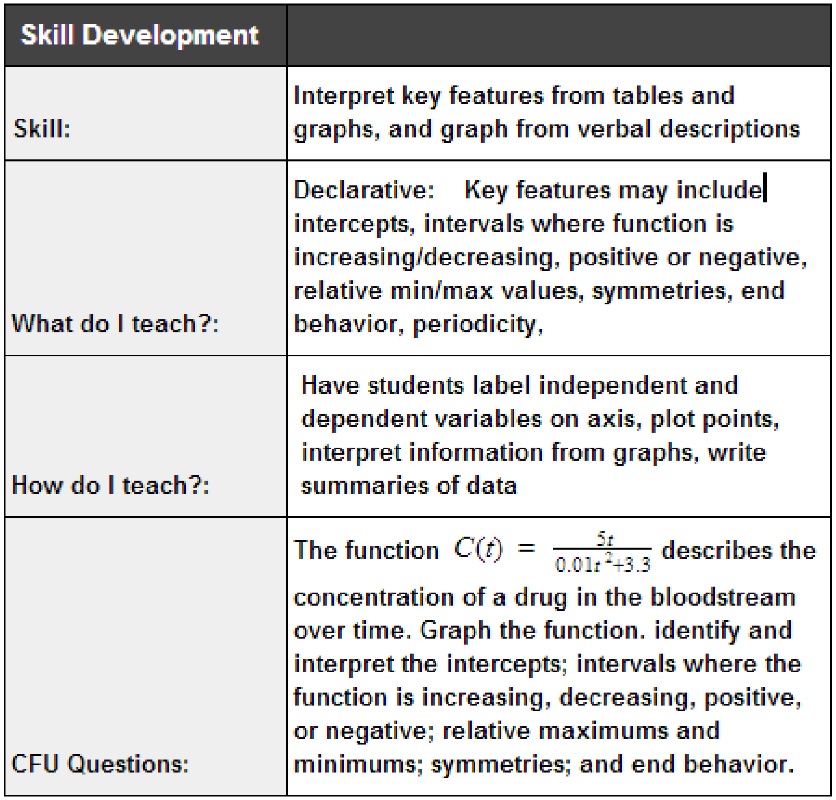
Skill Development	
Skill:	Solve rational equations.
What do I teach?:	Procedural
How do I teach?:	Techniques depend on problem. Least Common denominator, Analogous to solving linear equations, but need to be aware of extraneous solutions.
CFU Questions:	Alg 2, Example 6 on pg. 592

F.IF.4
For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

(Include rational, square root, cube root; emphasize selection of appropriate models)



Concept Development	
Concept:	Finding key features of models of relationships.
Definition:	A function is a relationship between a set of INPUTS and a set of permissible OUTPUTS with the property that each input is related to exactly ONE output.
Critical Attributes:	Two quantities, like time and value or time and population growth
Shared Attributes:	Every function can be represented in four ways: algebraically, graphically, numerically (data tables). and verbally.
Non-Critical Attributes:	Type of function (polynomial, exponential, etc.)
Examples:	(See influenza epidemic example in resources below.)
Non-Examples:	
Resources:	http://www.illustrativemathematics.org/standards/hs Alg 2 textbook section 6.3, 6.4, 7.1, 7.2 http://ccssmath.org/?page_id=2159



Skill Development	
Skill:	Interpret key features from tables and graphs, and graph from verbal descriptions
What do I teach?:	Declarative: Key features may include intercepts, intervals where function is increasing/decreasing, positive or negative, relative min/max values, symmetries, end behavior, periodicity,
How do I teach?:	Have students label independent and dependent variables on axis, plot points, interpret information from graphs, write summaries of data
CFU Questions:	The function $C(t) = \frac{5t}{0.01t^2+3.3}$ describes the concentration of a drug in the bloodstream over time. Graph the function. identify and interpret the intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.

Integrated Math 3 Course Standard and Resource Guide

Quadratics, Polynomials, and Other Functions

UNIT 5 : Exploring other functions

Overview	**Solving radical equations, graph parent functions with transformations, and solve system of equations**

Priority standard

A.REI.2

Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

- **radical**

Concept Development	
Concept:	Extraneous solutions
Definition:	A solution that emerges from the process of solving the problem but is not a valid solution to the original problem.
Critical Attributes:	Radicals with even roots (fraction exponents with even denominators) have domain limitations (radicand must be non-negative).
Shared Attributes:	
Non-Critical Attributes:	You might have zero, one, or many solutions.
Examples:	p 454 Ex 5 Solve: $x + 1 = (7x + 15)^{\frac{1}{2}}$ -- has one extraneous solution. p 457 #44. Explain how you can tell that $(x + 4)^{\frac{1}{2}} =- 5$ has no solutions. p 454 Ex 4 Solve $(x+2)^{\frac{3}{4}} - 1 = 7$.
Non-Examples:	See p 456 #32, #33
Resources:	http://ccssmath.org/?page_id=2127 See Alg 2 text section 6.6 p 452

Skill Development	
Skill:	Solve radical equations.
What do I teach?:	Procedural
How do I teach?:	• Analogous to solving linear equations, but need to be aware of extraneous solutions • Techniques depend on problem: raise both sides of equation to the reciprocal of exponent • Always check every apparent solution in the ORIGINAL equation
CFU Questions:	Solve and eliminate extraneous solutions if they arise. $3x^{\frac{3}{2}} = 375$ $x - \frac{1}{2} = \sqrt{\frac{1}{4}x}$

F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

- **Graph_________.**
  - **square root**
  - **cube root**
  - **piecewise-defined functions {maybe including step functions?}**
  - **absolute value functions.**

Concept Development	
Concept:	Rough sketch (of various non-linear functions)
Definition:	Rough sketch is a drawing that shows the main features of a graph
Critical Attributes:	Critical points
Shared Attributes:	Critical points could be zeros; Symmetry about axis of symmetry (one side could be y-intercept, for example)
Non-Critical Attributes:	
Examples:	$y = \|x\|,\ g(x) = x^{\frac{1}{2}}, f(x) = x^{\frac{1}{3}},$ h(x)=int(x)
Non-Examples:	It is not necessary to plot several points once the general behavior of the graph is determined.
Resources:	http://ccssmath.org/?page_id=2165 p123 Section 2.7, p 446 section 6.5, see pre Calc book for integer and step functions

Skill Development	
Skill:	Graphing the remaining non-linear functions
What do I teach?:	Procedural (plotting points, rough sketch) and Declarative (recognizing and describing the transformation)
How do I teach?:	Parent functions and basic transformations (with or without calculators at this time).
CFU Questions:	Graph the function and identify key features: $g(x) = \|x + 3\| - 2$ compare and contrast the graphs of $h(x) = -x^{1/3}$ and $f(x) = x^{1/3}$

F.BF.3

Identify the effect on the graph of replacing f(x) by f(x) + k, kf(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Concept Development	
Concept:	Transformations of remaining non-linear functions.
Definition:	transformations include translations (vertical/horizontal shifts) and dilations (stretching)
Critical Attributes:	recognizing parent function shapes of new functions: square root, cube root, abs value, piece-wise.
Shared Attributes:	location of k always has same transformation effect on ANY kind of function (exponential, quadratic value,...)
Non-Critical Attributes:	
Examples:	$y = 3\|x+1\| - 5$ $g(x) = -2(x-2)^{\frac{1}{2}}$ $f(x) = (x+2)^{\frac{1}{3}} - 2$ $h(x) = [[x-2]]$ h(x)=int(x-2)
Non-Examples:	1. Given f(x) + k and f(x+k), if k=0, then no transformation exists. 2. Given k f(x) and f(kx), if k=1, then no transformation exists.
Resources:	p123 Section 2.7, p 446 section 6.5, see pre Calc book for integer and step functions

Skill Development	
Skill:	Graphing using transformations and identify the transformation by comparing two graphs.
What do I teach?:	Declarative and procedural
How do I teach?:	Beginning with the parent function, graph the new function (using technology) based on the transformation and state what the transformation is given two graphs.
CFU Questions:	Describe the graphical relationships between the two functions. 1. $t(x) = 2\|x+1\|$ *and* $v(x) = \frac{1}{2}\|x-3\| - 1$ 2. $h(x) = \sqrt{x+1}$ *and* $k(x) = \frac{1}{2}\sqrt{x-3} - 1$

Teaching Note: This standard (A.REI.11) could be moved to Quarter 3 (wk 8 & 9) in modeling unit.

A.REI.11
Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Concept Development	
Concept:	Solving Systems of equations
Definition:	On a graph of two functions, the intersection(s), if any exist, are the solutions to a system of equations.
Critical Attributes:	$f(x) = g(x)$
Shared Attributes:	systems of inequalities
Non-Critical Attributes:	a system may have no, one, or many solutions
Examples:	Given two equations, identify the type of function, determine the possibilities for intersections, and then graph to confirm your predicted solution(s).
Non-Examples:	avoid the common error: if an ordered pair satisfies one equation, it may not represent a solution to the system since it may not be a solution to the other equations in the system
Resources:	http://ccssmath.org/?page_id=2149

Skill Development	
Skill:	Solving two equations for possible intersections and finding them algebraically.
What do I teach?:	Procedural
How do I teach?:	1. graphing calculator or other technology 2. substitution, elimination methods for solving systems
CFU Questions:	1. Draw sketches where a quadratic function intersects an absolute value function at 4 points, 3..., 2, 1, 0. 2. How many liters of a 70% alcohol solution must be added to 50L of a 40% alcohol solution to produce a 50% alcohol solution? 3. Given the following equations determine the *x* value that results in an equal output for both functions. $f(x) = 3x - 2$ $g(x) = (x + 3)^2 - 1$

Integrated Math 3 Course Standard and Resource Guide

Mathematical Modeling

UNIT 6

Overview	**Inverse Functions**

F.BF.1 Write a function that describes a relationship between two quantities.
b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.”*
(+) c – Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

Concept Development	
Concept:	Function
Definition	A relation between a set of inputs and a set of outputs with the property that each input is related to exactly one output.
Critical Attributes:	Variables must be defined
Shared Attributes:	functions, relationships
Non-Critical Attributes:	the particular variable chosen to represent a quantity may vary
Examples:	3x + 4y = 8
Non-Examples:	x = 10
Possible CFU Questions:	Is x = 5 a function? Is y = 6 a function?
Resources:	http://ccssmath.org/?page_id=2189

Skill Development	
Skill:	Combining function with arithmetic operations
Procedural or Declarative:	Procedural
Process, Procedure, Steps:	When working through word problems show students how to combine to functions with addition, subtraction, multiplication, and division to form another function.
Possible CFU’ Questions:	The total revenue for a company is found by multiplying the price per unit by the number of units sold minus the production cost. The price per unit is modeled by $p(n) = -0.5n^2 + 6$, where n represents the number of units sold. Production cost is modeled by $c(n) = 3n + 7$. Write the revenue function.

F.BF.3

Identify the effect on the graph of replacing f(x) by f(x) + k, kf(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Concept Development	
Concept:	Transformations of exponential functions. $y = a\,b^{cx-h} + k$
Definition:	transformations include translations (vertical/horizontal shifts), dilations
Critical Attributes:	exponential function
Shared Attributes:	location of k always has same transformation effect on ANY kind of function (exponential, absolute value,...)
Non-Critical Attributes:	f(x) could be g(x) or y...
Examples:	$y = e^{x+3} - 4$ is $y = e^{x}$ shifted 3 left (horizontal shift) and down 4 (vertical shift).
Non-Examples:	if $k = 1$ or zero: $y = 3^{1x+0}$ has no transformation
Resources:	http://ccssmath.org/?page_id=2195 Alg 2 text 7.2, 7.1

Skill Development	
Skill:	**Transform exponential functions.**
What do I teach?:	**Procedural (graphing) and Declarative (describing)**
How do I teach?:	**Suggest using tables and electronic tools (graphing calculator) to see transformation relationships.**
CFU Questions:	**Describe the graphical relationship between the two functions.** $f(x) = 2^{x} + 7$ **and** $g(x) = 2^{x+1} + 7$

F. BF. 4 Find inverse functions. and:

a. solve an equation in the form f(x)=c for a simple function f that has an inverse and write an expression for the inverse

(+) b – Verify by composition that one function is the inverse of another.

(+) c – Read values of an inverse function from a graph or a table, given that the function has an inverse.

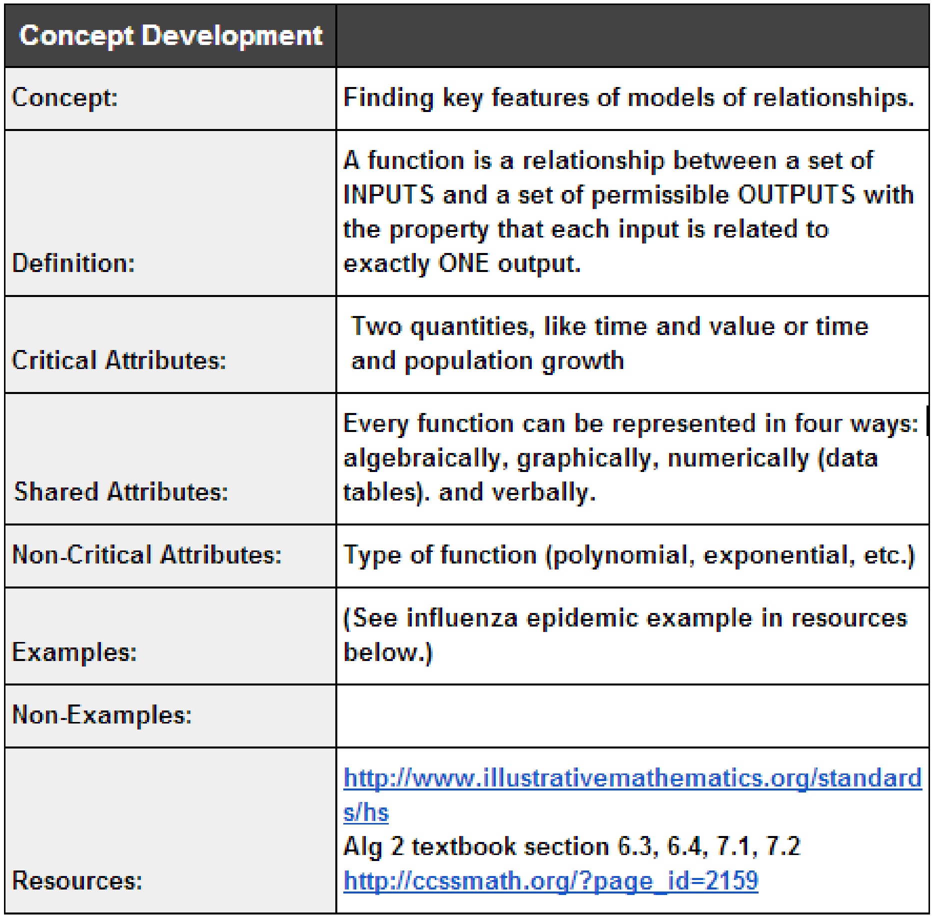
Concept Development	
Concept:	inverse function
Definition:	An inverse relation interchanges the input and output values of the original relation. If both the original relation and the inverse relation are functions, then the two functions are called inverse functions. Functions f and g are inverses of each other provided f(g(x))=g(f(x))=x. The function g is denoted by f^{-1} is read as "f inverse."
Critical Attributes:	one to one
Shared Attributes:	some functions can be inverse functions with a constrained domain
Non-Critical Attributes:	function type or degree can vary
Examples:	f(x)= 3x+4 has an inverse of $f^{-1}(x) = \frac{x-4}{3}$
Non-Examples:	$f^{-1}(x)$ does not equal $\frac{1}{f(x)}$
Resources:	http://ccssmath.org/?page_id=2199 Alg 2 text section 6.4, and 7.4 for logs as inverse of exponentials; http://www.illustrativemathematics.org/standards/hs

Skill Development	
Skill:	Find inverse functions.
What do I teach?:	Procedural
How do I teach?:	Using algebraic rules of manipulation: 1. switch x and y roles, and then solve for y 2. when using a model, avoid confusion by not switching variables, but instead just solve for the desired variable in terms of the other(s)
CFU Questions:	1. For the following functions, find the inverse if it exists: $f(x) = \frac{2x+5}{x-7}$, $g(x) = 3(2^{x}) + 1$, $h(x) = \sqrt{x+5} - \sqrt{x+1}$ 2. The average price P (in dollars) for a National Football League ticket can be modeled by $P = 35t^{0.192}$ where t is the number of years since 1995. Find the inverse model that gives time as a function of the average ticket price.

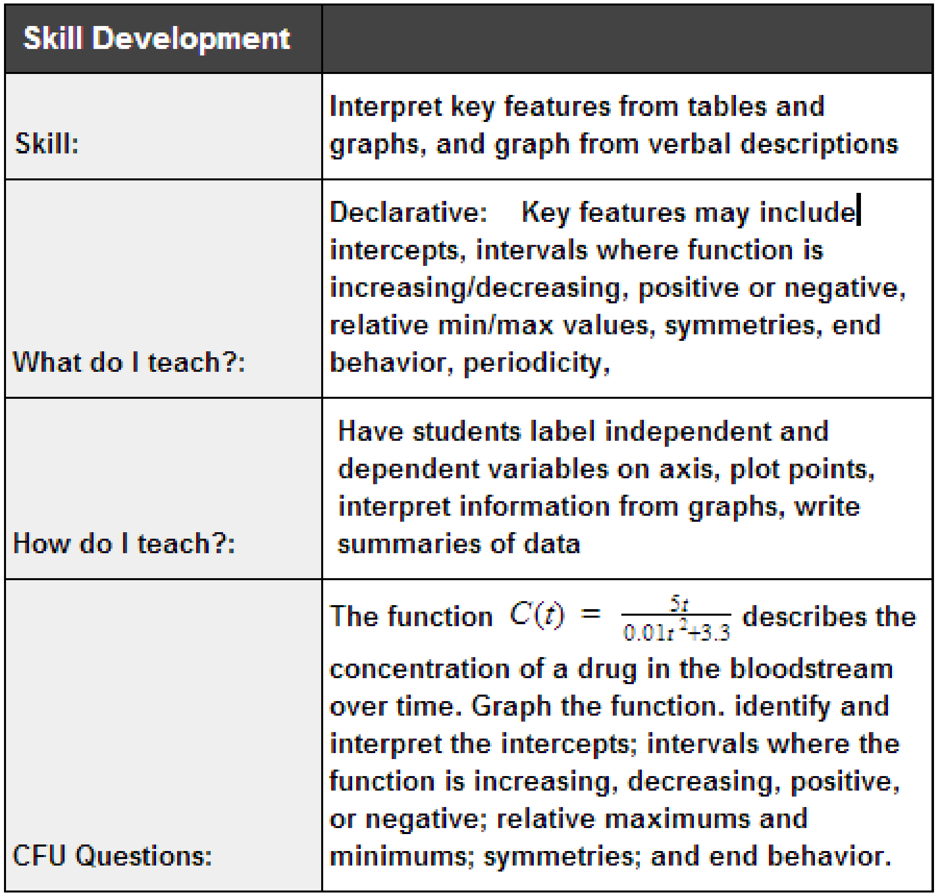
F.IF.4

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity

(Include rational, square root, cube root; emphasize selection of appropriate models)



Concept Development	
Concept:	Finding key features of models of relationships.
Definition:	A function is a relationship between a set of INPUTS and a set of permissible OUTPUTS with the property that each input is related to exactly ONE output.
Critical Attributes:	Two quantities, like time and value or time and population growth
Shared Attributes:	Every function can be represented in four ways: algebraically, graphically, numerically (data tables). and verbally.
Non-Critical Attributes:	Type of function (polynomial, exponential, etc.)
Examples:	(See influenza epidemic example in resources below.)
Non-Examples:	
Resources:	http://www.illustrativemathematics.org/standards/hs Alg 2 textbook section 6.3, 6.4, 7.1, 7.2 http://ccssmath.org/?page_id=2159



Skill Development	
Skill:	Interpret key features from tables and graphs, and graph from verbal descriptions
What do I teach?:	Declarative: Key features may include intercepts, intervals where function is increasing/decreasing, positive or negative, relative min/max values, symmetries, end behavior, periodicity,
How do I teach?:	Have students label independent and dependent variables on axis, plot points, interpret information from graphs, write summaries of data
CFU Questions:	The function $C(t) = \frac{5t}{0.01t^2+3.3}$ describes the concentration of a drug in the bloodstream over time. Graph the function. identify and interpret the intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.

F.IF.7 (Unit 4, Part1)

Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases

- **Graph logarithmic functions, showing intercepts and end behavior.**

Concept Development	
Concept:	Graphing Logarithmic Function
Definition:	Rough sketch is a general approximation of what the graph looks like. A logarithm is defined as: Let b and y be positive numbers with $b \neq 1$. The logarithm of y with base b is denoted by $log_{b}y = x$ if and only if $b^{x} = y$. The expression $log_{b}y$ is read as "log base b of y."
Critical Attributes:	Rough sketch of critical points: x- or y-intercept and vertical or horizontal asymptote.
Shared Attributes:	x-intercepts.
Non-Critical Attributes:	Base could be any real number greater than 0.
Examples:	Rate of growth or decay. $y = log_{3}x,\ g(x) = log_{\frac{1}{2}}(x - 3) + 2,\ h(t) = ln(t)$
Non-Examples:	using too few or too many points
Resources:	See Alg 2 text sections 7.4 https://docs.google.com/a/muhsd.org/document/d/1QB2C0NSoTZvfxHTd1zN97KLhfiJHcMN4HpFrCfV2oNE/edit

Skill Development	
Skill:	Graphing logarithmic functions
What do I teach?:	Procedural
How do I teach?:	Use table and plot points then sketch rough graph. Or students use technology (web-based application or standard graphing calculator) to plot graphs.
CFU Questions:	Graph the function: $y = log_{2}(x + 3) + 1$. Provide and appropriate viewing window where key features are visible. Describe the end behavior and identify the asymptote.

F. BF. 5 (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Concept Development	
Concept:	logarithm
Definition:	A function $y=\log_b x$,where *b* is any number such that $b>0,\ b\neq 1,\ \text{and}\ x>0$, $y=\log_b x$ is equivalent to $x=b^y$

This standard represents an application of the students' understanding of the relationship between logarithms and exponents.

Skill Development	
Skill:	Understand inverse relationship of exponential and logarithmic functions
What do I teach?:	Declarative
How do I teach?:	You can graph exponential and logarithmic functions and show the line of symmetry, plot points and switch them, or calculate $f(f^{-1}(x))$ and prove that it equals x. All of those should be enough evidence to support the fact that exponentials and logarithms are inverses.
CFU Questions:	How do you know that two functions are inverses of each other?

F.LE.4 *
For exponential models, express as a logarithm the solution to $(ab)^{ct} = d$ where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology.

Concept Development	
Concept:	Logarithms and exponents are inverse functions.
Definition:	Let b and y be positive numbers with $b \neq 1$. The logarithm of y with base b is denoted by $log_b y = x$ if and only if $b^x = y$. The expression $log_b y$ is read as "log base b of y."
Critical Attributes:	base b is positive real number such that $b \neq 1$
Shared Attributes:	variables and constants, positive values
Non-Critical Attributes:	----------------
Examples:	**Newton's Law of Cooling:** $T = (T_0 - T_R)e^{-rt} + T_R$, Defined as initial temperature T_0 , temperature T after t minutes, where T_R is the surrounding temperature and r is the substance's cooling rate http://www.illustrativemathematics.org/standards/hs
Resources:	http://ccssmath.org/?page_id=2221 Alg 2 text section 7.5, 7.6

Skill Development	
Skill:	Use logarithms to solve exponential equations. Use exponents to solve logarithms.
What do I teach?:	Use equivalence of logs and exponents (p 515, 517), properties of exponents (p 330) and of logarithms (p 499, 507, 508). Teach bases 2, 10, and e.
How do I teach?:	Use technology, so students can see graphs and tables to investigate exponents and logarithms
CFU Questions:	Convert $log_2(\frac{1}{16}) = -4$ to exponential form. Expand using logarithmic properties $ln(\frac{3x^2}{y+1})$

F.LE.4.1

Prove simple laws of logarithms. CA *

Concept Development	
Concept:	Logarithms and exponents are inverse functions.
Definition:	Let b and y be positive numbers with $b \neq 1$. The logarithm of y with base b is denoted by $log_{b}y = x$ if and only if $b^{x} = y$. The expression $log_{b}y$ is read as "log base b of y."
Critical Attributes:	base b is positive real number such that $b \neq 1$
Shared Attributes:	variables and constants, positive values
Non-Critical Attributes:	----------------
Examples:	**Newton's Law of Cooling:** $T = (T_0 - T_R)e^{-rt} + T_R$, Defined as initial temperature T_0 , temperature T after t minutes, where T_R is the surrounding temperature and r is the substance's cooling rate http://www.illustrativemathematics.org/standards/hs
Resources:	http://ccssmath.org/?page_id=2221 Alg 2 text section 7.5, 7.6

Skill Development	
Skill:	Prove simple laws of logarithms.
What do I teach?:	$\log A - \log B = \log \frac{A}{B}$ $\log A - \log B = \log \frac{A}{B}$ $\log A^n = n \log A$
How do I teach?:	Use the properties of exponents to help the students understand and/or use technology to investigate some example to show that the properties are equal.
CFU Questions:	1. Simplify $3 \log x - \log x^2$. 2. Condense to express as a single logarithm: $\log_3(x+5) + \log_3(x-5) - 4\log_3(2)$ 3. Expand to express as a multiple of logarithms: $\ln\left(\frac{(x+5)^6(x^2-4)^7}{(x^3-5)^8}\right)$

F.LE.4.2

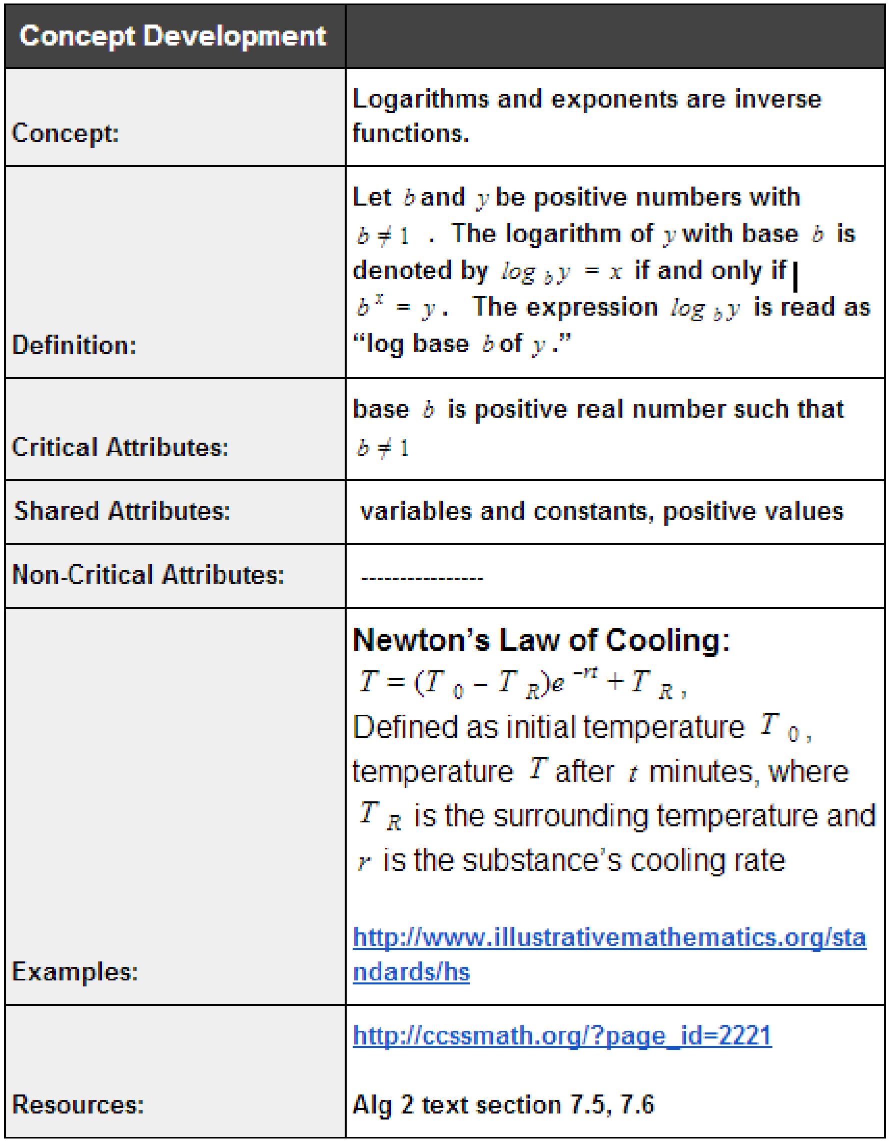
Use the definition of logarithms to translate between logarithms in any base. CA *

Concept Development	
Concept:	Logarithms and exponents are inverse functions.
Definition:	Let b and y be positive numbers with $b \neq 1$. The logarithm of y with base b is denoted by $\log_b y = x$ if and only if $b^x = y$. The expression $\log_b y$ is read as "log base b of y."
Critical Attributes:	base b is positive real number such that $b \neq 1$
Shared Attributes:	variables and constants, positive values
Non-Critical Attributes:	----------------
Examples:	**Newton's Law of Cooling:** $T = (T_0 - T_R)e^{-rt} + T_R$, Defined as initial temperature T_0 , temperature T after t minutes, where T_R is the surrounding temperature and r is the substance's cooling rate http://www.illustrativemathematics.org/standards/hs
Resources:	http://ccssmath.org/?page_id=2221 Alg 2 text section 7.5, 7.6

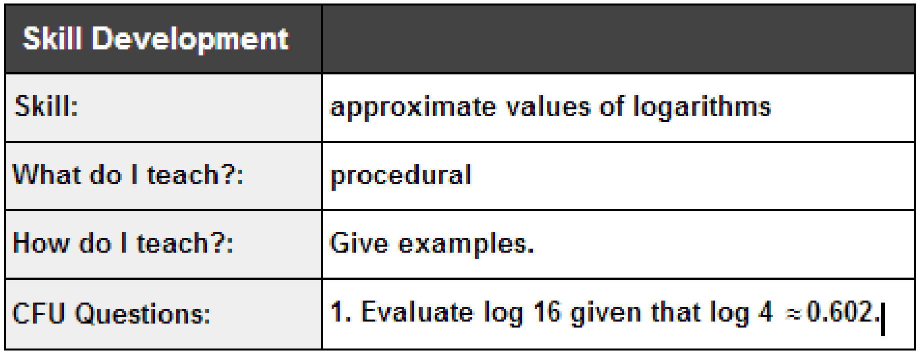
Skill Development	
Skill:	Use logarithms to solve exponential equations. Use exponents to solve logarithms.
What do I teach?:	Use equivalence of logs and exponents (p 515, 517), properties of exponents (p 330) and of logarithms (p 499, 507, 508). Teach bases 2, 10, and e.
How do I teach?:	Show examples that apply the rules of logs. **Find x.** $log_3 8 = x$ **Rewrite as** $8 = 3^x$ **Now take log of both sides of eq:** $log8 = log3^x$ **Apply prop of log:** $log8 = x\ log3$ **Isolate variable** x**:** $\frac{log 8}{log\ 3} = x$ **Then conclude:** $\frac{log 8}{log\ 3} = x = log_3 8$
CFU Questions:	1. Find $log_2 30$ using a calculator or table. 2. Graphene problem https://www.illustrativemathematics.org/illustrations/1569

F.LE.4.3

Understand and use the properties of logarithms to simplify logarithmic numeric expressions and to identify their approximate values. CA *



Concept Development	
Concept:	Logarithms and exponents are inverse functions.
Definition:	Let b and y be positive numbers with $b \neq 1$. The logarithm of y with base b is denoted by $\log_b y = x$ if and only if $b^x = y$. The expression $\log_b y$ is read as "log base b of y."
Critical Attributes:	base b is positive real number such that $b \neq 1$
Shared Attributes:	variables and constants, positive values
Non-Critical Attributes:	----------------
Examples:	**Newton's Law of Cooling**: $T = (T_0 - T_R)e^{-rt} + T_R$, Defined as initial temperature T_0 , temperature T after t minutes, where T_R is the surrounding temperature and r is the substance's cooling rate http://www.illustrativemathematics.org/standards/hs
Resources:	http://ccssmath.org/?page_id=2221 Alg 2 text section 7.5, 7.6



Skill Development	
Skill:	approximate values of logarithms
What do I teach?:	procedural
How do I teach?:	Give examples.
CFU Questions:	1. Evaluate log 16 given that log 4 ≈ 0.602.

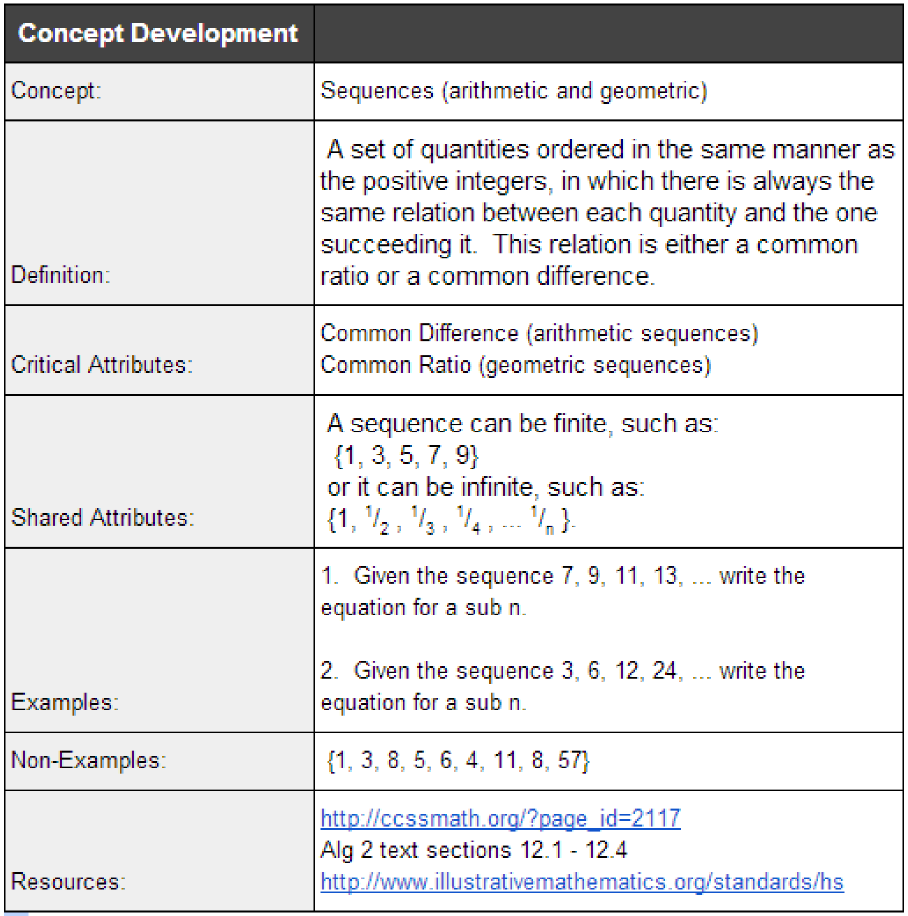
Integrated Math 3 Course Standard and Resource Guide

Mathematical Modeling

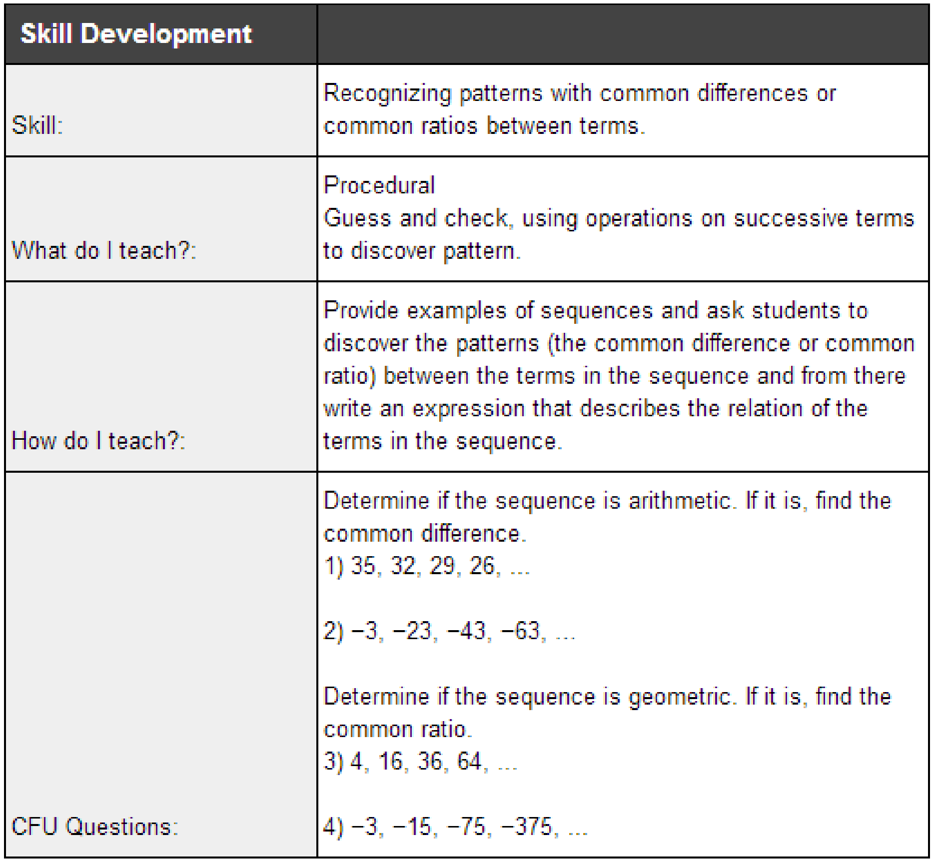
UNIT 7:

Overview	**Arithmetic and Geometric Sequences**

A.CED.1
Create equations and inequalities in one variable including ones with absolute value and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. CA



Concept Development	
Concept:	Sequences (arithmetic and geometric)
Definition:	A set of quantities ordered in the same manner as the positive integers, in which there is always the same relation between each quantity and the one succeeding it. This relation is either a common ratio or a common difference.
Critical Attributes:	Common Difference (arithmetic sequences) Common Ratio (geometric sequences)
Shared Attributes:	A sequence can be finite, such as: {1, 3, 5, 7, 9} or it can be infinite, such as: $\{1, {}^{1}/_{2}, {}^{1}/_{3}, {}^{1}/_{4}, \ldots {}^{1}/_{n}\}$.
Examples:	1. Given the sequence 7, 9, 11, 13, ... write the equation for a sub n. 2. Given the sequence 3, 6, 12, 24, ... write the equation for a sub n.
Non-Examples:	{1, 3, 8, 5, 6, 4, 11, 8, 57}
Resources:	http://ccssmath.org/?page_id=2117 Alg 2 text sections 12.1 - 12.4 http://www.illustrativemathematics.org/standards/hs



Skill Development	
Skill:	Recognizing patterns with common differences or common ratios between terms.
What do I teach?:	Procedural Guess and check, using operations on successive terms to discover pattern.
How do I teach?:	Provide examples of sequences and ask students to discover the patterns (the common difference or common ratio) between the terms in the sequence and from there write an expression that describes the relation of the terms in the sequence.
CFU Questions:	Determine if the sequence is arithmetic. If it is, find the common difference. 1) 35, 32, 29, 26, ... 2) −3, −23, −43, −63, ... Determine if the sequence is geometric. If it is, find the common ratio. 3) 4, 16, 36, 64, ... 4) −3, −15, −75, −375, ...

F.IF.6

Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

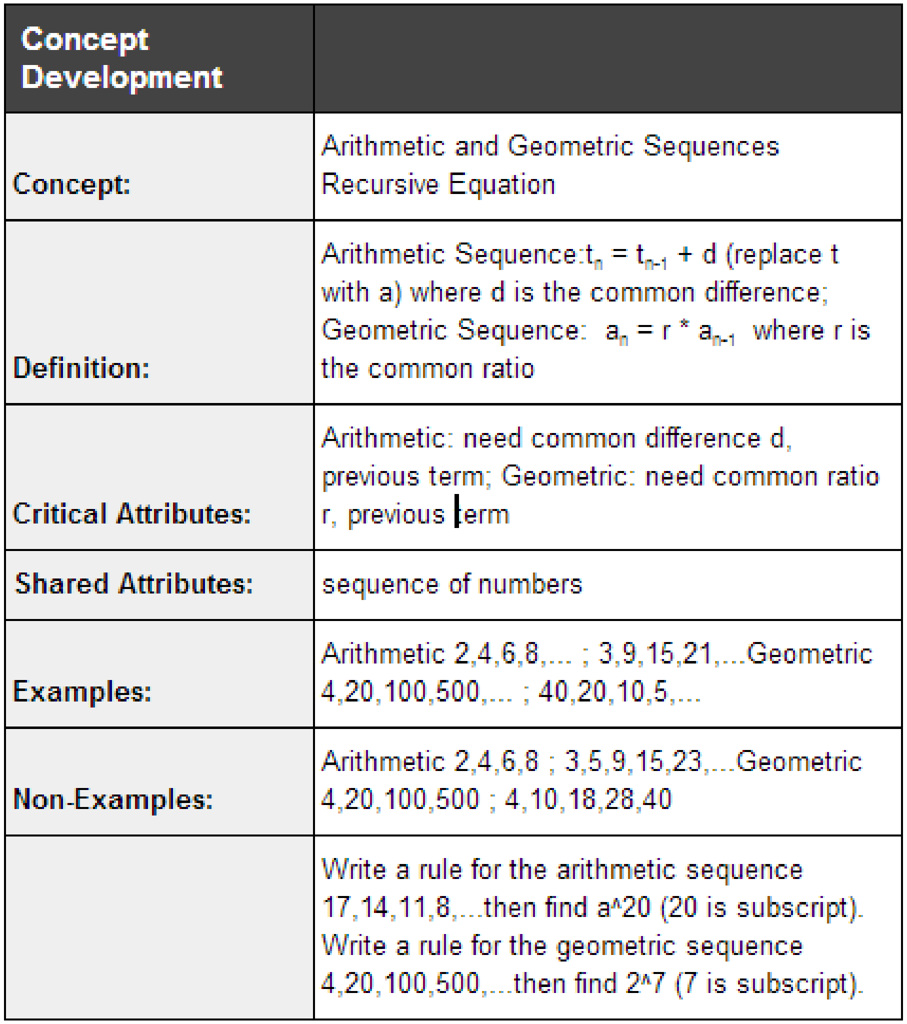
Concept Development	
Concept:	Average Rate of Change
Definition:	Average Rate of Change is a process that calculates the amount of change in one item divided by the corresponding amount of change in another.
Critical Attributes:	interval, variables, graph or table of values
Shared Attributes:	slope of lines, unit values
Non-Critical Attributes:	actual values,
Examples:	1.) (see "Garbage Trucks" Performance task): 2.) Mathemafish Population http://www.illustrativemathematics.org/illustrations/686
Resources:	http://ccssmath.org/?page_id=2163

Skill Development	
Skill:	Calculate rate of change
What do I teach?:	Procedural (calculation) and Declarative (interpreting meaning from the rates in context)
How do I teach?:	Choose an interval and calculate rate of change (slope of line connecting the endpoints of the chosen interval).
CFU Questions:	1.) The following table shows the average daylight hours in Alaska for each month. Months are represented by the number of months after January. Month: 0, 2, 4, 6, 8, 10 Daylight Hours: 5.7, 10.4, 16.9, 19.2, 14.3, 8.5 Calculate the average rate of change from March to September.

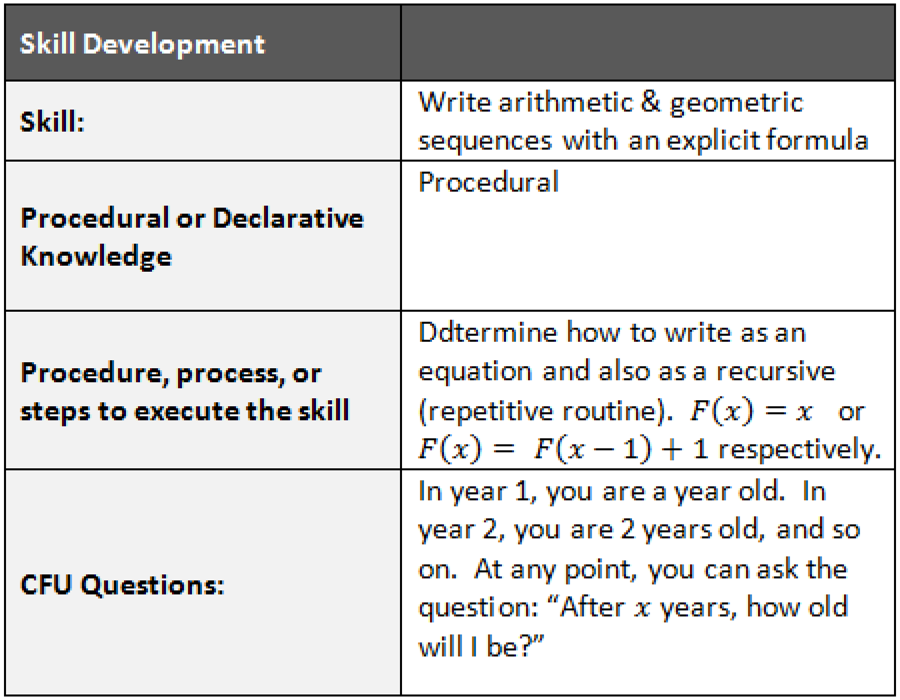
Month	0	2	4	6	8	10
Daylight Hours	5.7	10.4	16.9	19.2	14.3	8.5

F.BF.2 (review from math 1)

Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.



Concept Development	
Concept:	Arithmetic and Geometric Sequences Recursive Equation
Definition:	Arithmetic Sequence:$t_n = t_{n-1} + d$ (replace t with a) where d is the common difference; Geometric Sequence: $a_n = r * a_{n-1}$ where r is the common ratio
Critical Attributes:	Arithmetic: need common difference d, previous term; Geometric: need common ratio r, previous term
Shared Attributes:	sequence of numbers
Examples:	Arithmetic 2,4,6,8,... ; 3,9,15,21,...Geometric 4,20,100,500,... ; 40,20,10,5,...
Non-Examples:	Arithmetic 2,4,6,8 ; 3,5,9,15,23,...Geometric 4,20,100,500 ; 4,10,18,28,40
	Write a rule for the arithmetic sequence 17,14,11,8,...then find a^20 (20 is subscript). Write a rule for the geometric sequence 4,20,100,500,...then find 2^7 (7 is subscript).



Skill Development	
Skill:	Write arithmetic & geometric sequences with an explicit formula
Procedural or Declarative Knowledge	Procedural
Procedure, process, or steps to execute the skill	Ddtermine how to write as an equation and also as a recursive (repetitive routine). $F(x) = x$ or $F(x) = F(x-1) + 1$ respectively.
CFU Questions:	In year 1, you are a year old. In year 2, you are 2 years old, and so on. At any point, you can ask the question: “After x years, how old will I be?”

A.SSE.4
Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.

Concept Development	
Concept:	Finite geometric series
Definition:	The expression formed by adding the terms of a geometric sequence is a called a geometric series. The sum of the first n terms in a geometric series is denoted by S_n. a_1 represents the first term and r represents the common ratio.
Critical Attributes:	Common ratio $\neq 1$
Examples:	http://www.illustrativemathematics.org/illustrations/1283
Non-Examples:	2, 4, 6, 8, 10, 12, 14
Resources:	http://ccssmath.org/?page_id=2101 Algebra 2 text section 12.3 and 12.4

Skill Development	
Skill:	Derive and Calculate sum of geometric series.
What do I teach?:	Procedural
How do I teach?:	Show derivation through examples. After several examples, students are then brought to conclude the general formula and can then apply it in situations.
CFU Questions:	1. In 1990, the total box office revenue at U.S. movie theaters was about $5.02 billion. From 1990 through 2003, the total box office revenue increased by about 5.9% per year. a.) Write a rule for the total box office revenue a_n (in billions of dollars) in terms of the year. Let $n = 1$ represent 1990. b.) What was the total box office revenue at U.S. movie theaters for the entire period 1990-2003? 2. Write 0.333... as an infinite geometric series. Represent this series using summation notation. Find the sum.

Mathematical Modeling

UNIT 8: *Modeling with Systems of Equations/Inequalities*

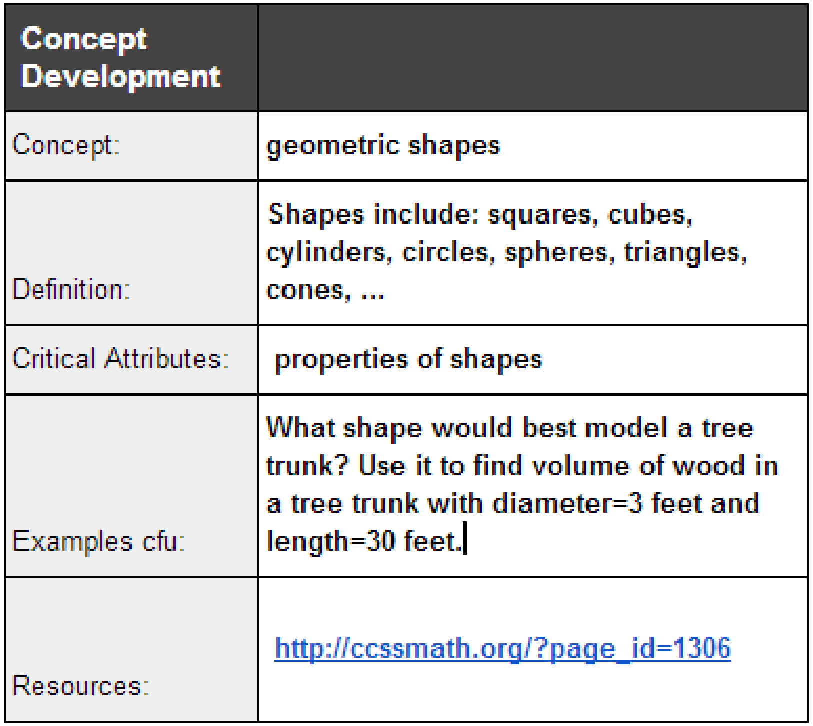
Overview	***Additional modeling with systems of equations/inequalities if needed.***

Mathematical Modeling
UNIT 9

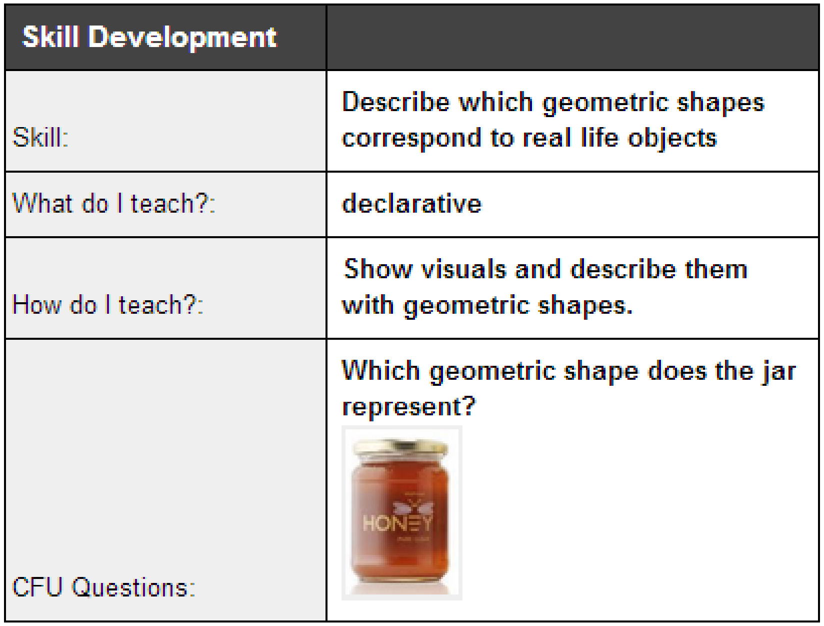
Overview	**Apply geometric concepts in modeling situations.**

G.MG.1

Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

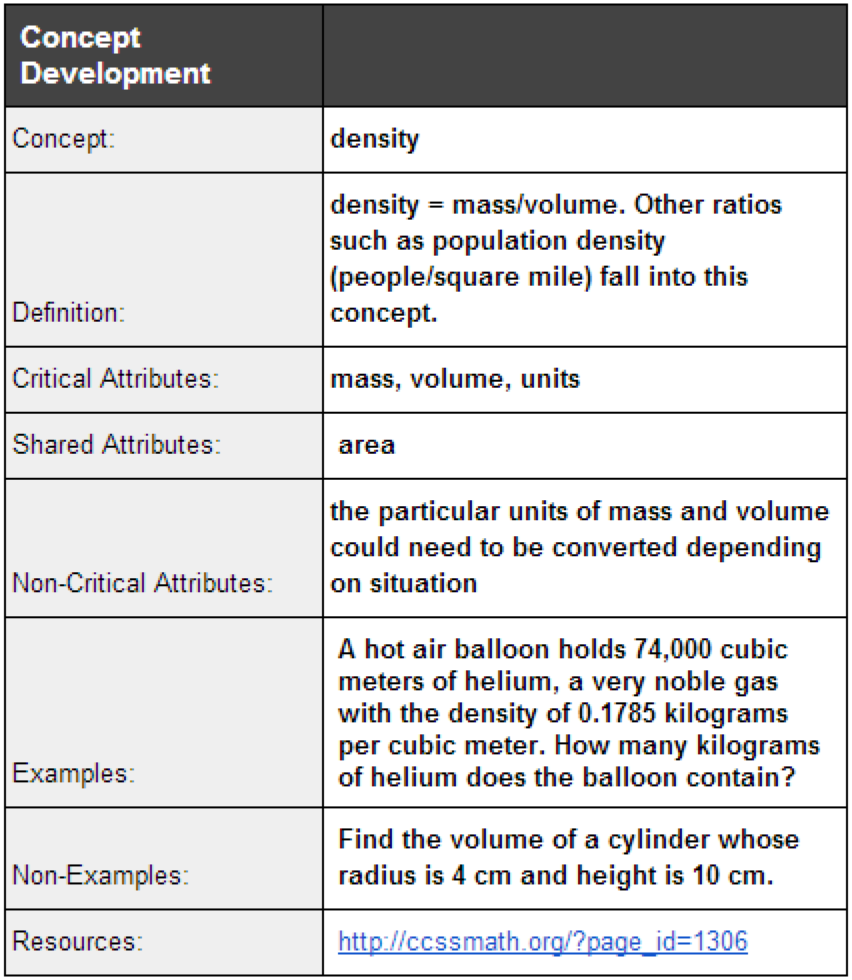


Concept Development	
Concept:	geometric shapes
Definition:	Shapes include: squares, cubes, cylinders, circles, spheres, triangles, cones, ...
Critical Attributes:	properties of shapes
Examples cfu:	What shape would best model a tree trunk? Use it to find volume of wood in a tree trunk with diameter=3 feet and length=30 feet.
Resources:	http://ccssmath.org/?page_id=1306

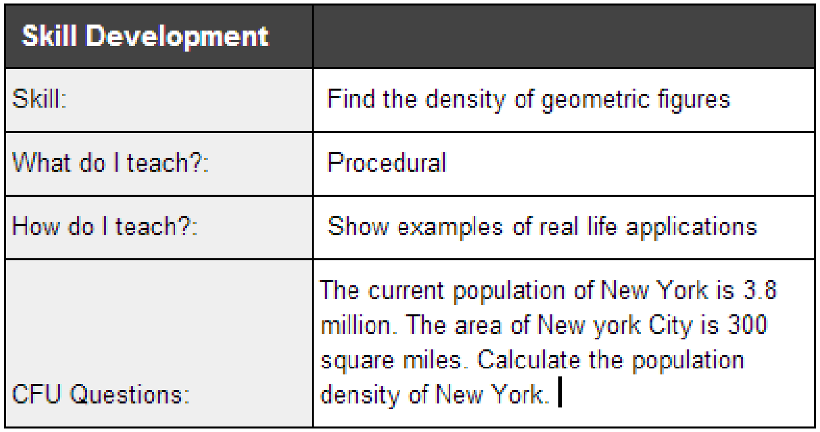


Skill Development	
Skill:	Describe which geometric shapes correspond to real life objects
What do I teach?:	declarative
How do I teach?:	Show visuals and describe them with geometric shapes.
CFU Questions:	Which geometric shape does the jar represent?

G.MG.2
Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).



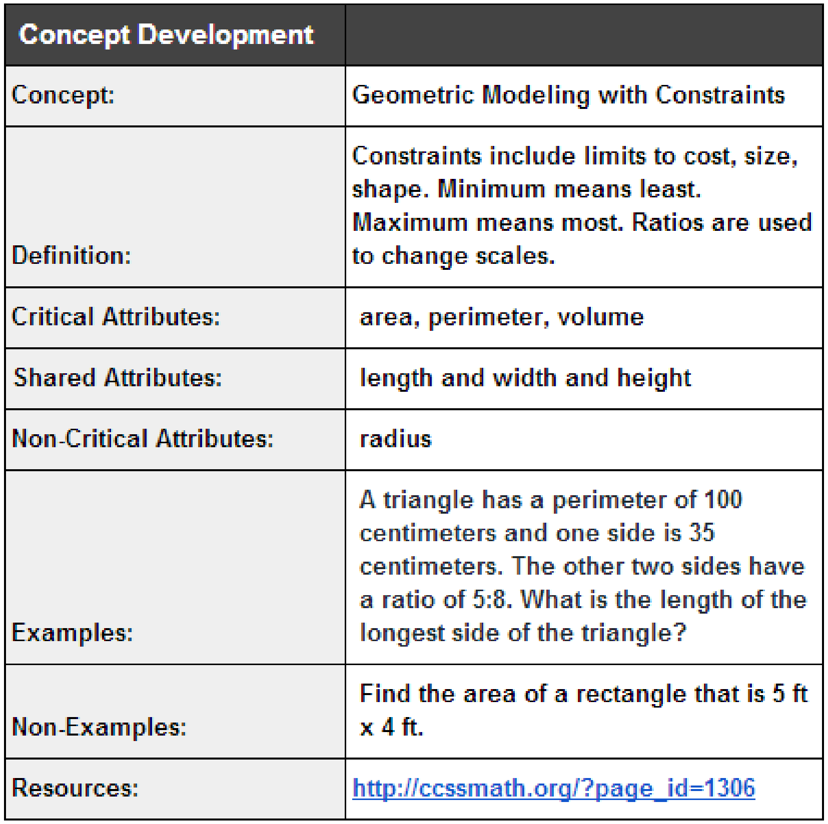
Concept Development	
Concept:	**density**
Definition:	**density = mass/volume. Other ratios such as population density (people/square mile) fall into this concept.**
Critical Attributes:	**mass, volume, units**
Shared Attributes:	**area**
Non-Critical Attributes:	**the particular units of mass and volume could need to be converted depending on situation**
Examples:	**A hot air balloon holds 74,000 cubic meters of helium, a very noble gas with the density of 0.1785 kilograms per cubic meter. How many kilograms of helium does the balloon contain?**
Non-Examples:	**Find the volume of a cylinder whose radius is 4 cm and height is 10 cm.**
Resources:	http://ccssmath.org/?page_id=1306



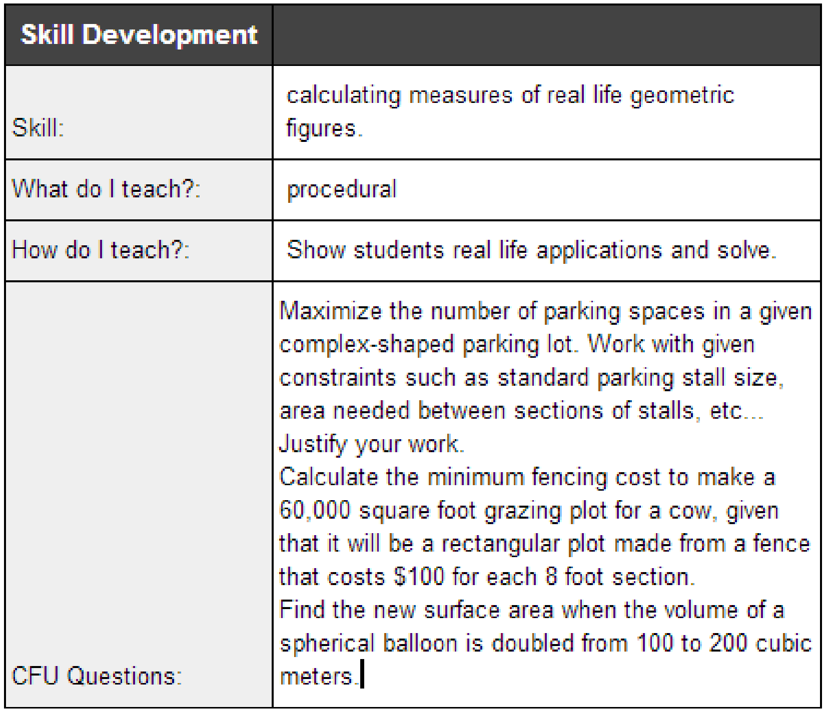
Skill Development	
Skill:	Find the density of geometric figures
What do I teach?:	Procedural
How do I teach?:	Show examples of real life applications
CFU Questions:	The current population of New York is 3.8 million. The area of New york City is 300 square miles. Calculate the population density of New York.

G.MG.3

Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).



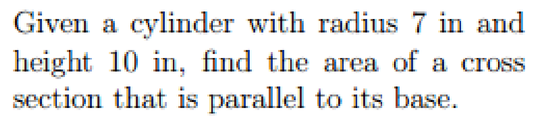
Concept Development	
Concept:	Geometric Modeling with Constraints
Definition:	Constraints include limits to cost, size, shape. Minimum means least. Maximum means most. Ratios are used to change scales.
Critical Attributes:	area, perimeter, volume
Shared Attributes:	length and width and height
Non-Critical Attributes:	radius
Examples:	A triangle has a perimeter of 100 centimeters and one side is 35 centimeters. The other two sides have a ratio of 5:8. What is the length of the longest side of the triangle?
Non-Examples:	Find the area of a rectangle that is 5 ft x 4 ft.
Resources:	http://ccssmath.org/?page_id=1306



Skill Development	
Skill:	calculating measures of real life geometric figures.
What do I teach?:	procedural
How do I teach?:	Show students real life applications and solve.
CFU Questions:	Maximize the number of parking spaces in a given complex-shaped parking lot. Work with given constraints such as standard parking stall size, area needed between sections of stalls, etc... Justify your work. Calculate the minimum fencing cost to make a 60,000 square foot grazing plot for a cow, given that it will be a rectangular plot made from a fence that costs $100 for each 8 foot section. Find the new surface area when the volume of a spherical balloon is doubled from 100 to 200 cubic meters.

G.GMD.4

Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Concept Development	
Concept:	Cross Sections
Definition:	A cross section is the face created by slicing an object.
Critical Attributes:	Cross Sections
Shared Attributes:	faces
Non-Critical Attributes:	slices could be in any of several directions, ie: parallel to x or y axis.
Examples:	 Given a cylinder with radius 7 in and height 10 in, find the area of a cross section that is parallel to its base.
Non-Examples:	Find the volume of sphere whose radius is 6 cm.
Resources:	http://ccssmath.org/?page_id=1306

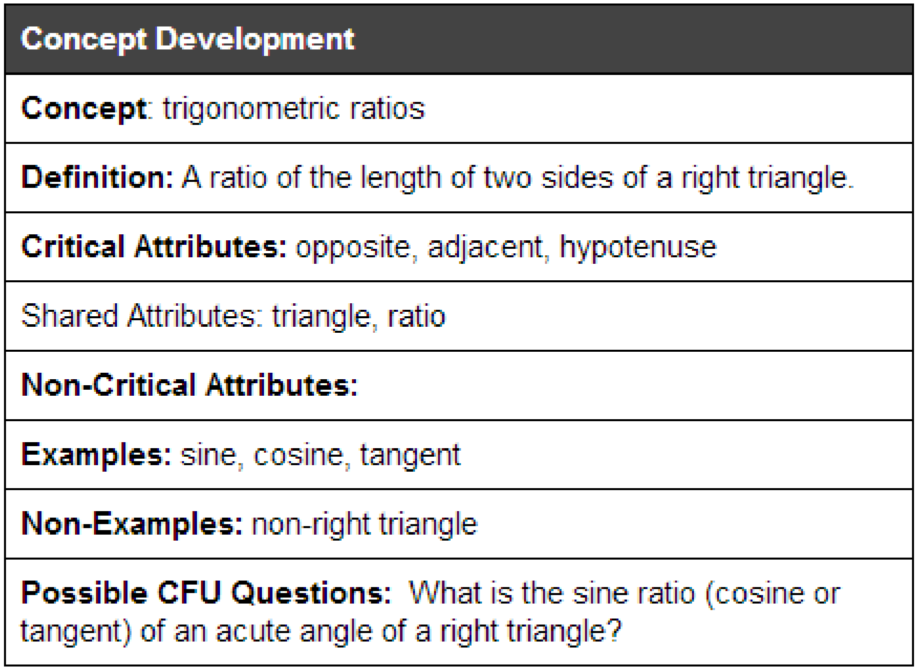
Skill Development	
Skill:	identify cross sections of 2D and 3D figures
What do I teach?:	declarative
How do I teach?:	Use visuals with videos and demonstrations.
CFU Questions:	Demonstrate how you could slice an octahedron to create a triangle, a square, a rhombus that is not a square. Find the volume of a cone created by rotating an equilateral triangle with perimeter = 36 meters.octahedron to create a triangle, a square, a rhombus that is not a square.

Integrated Math 3 Course Standard and Resource Guide
Trigonometry
UNIT 10

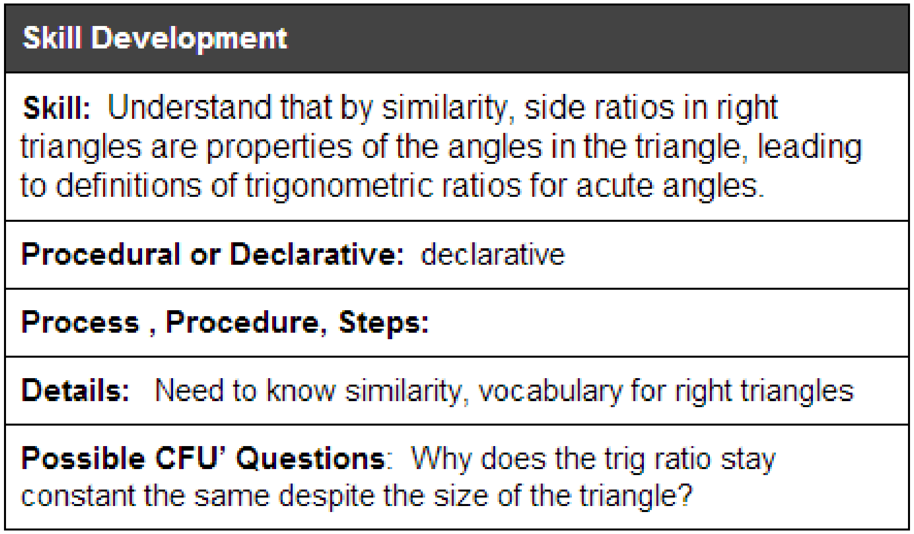
Overview	**Right Triangle Trigonometry**

G.SRT.6

Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.



Concept Development
Concept: trigonometric ratios
Definition: A ratio of the length of two sides of a right triangle.
Critical Attributes: opposite, adjacent, hypotenuse
Shared Attributes: triangle, ratio
Non-Critical Attributes:
Examples: sine, cosine, tangent
Non-Examples: non-right triangle
Possible CFU Questions: What is the sine ratio (cosine or tangent) of an acute angle of a right triangle?



Skill Development
Skill: Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
Procedural or Declarative: declarative
Process , Procedure, Steps:
Details: Need to know similarity, vocabulary for right triangles
Possible CFU' Questions: Why does the trig ratio stay constant the same despite the size of the triangle?

G.SRT.7

Explain and use the relationship between the sine and cosine of complementary angles.

Concept Development

Concept: complementary angles

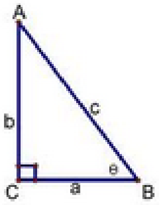
Definition: two angles whose sum is 90°.

Critical Attributes: right triangle, opposite, adjacent, hypotenuse

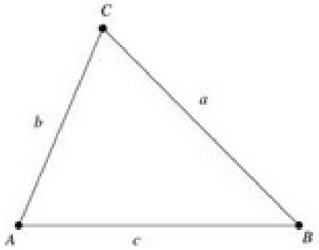
Shared Attributes: ratio

Non-Critical Attributes:

Examples: sine B =b/c; cosine B = a/c



Non-Examples: sine A =a/c; cosine A = b/c



Possible CFU Questions: Why does the sin Θ = cos (90 – Θ)?

Skill Development

Skill: Explain and use the relationship between the sine and cosine of complementary angles.

Procedural or Declarative: Declarative.

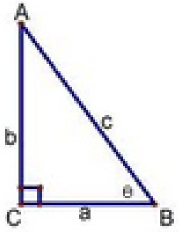
Process, Procedure, Steps:

Details: know what complementary angles and trig definitions

Possible CFU' Questions:

1. Explain the relationship between the sine A and cosine B.

2. If the sin 56° =0.829 what is cos 34°.



G.SRT.8

Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Concept Development

Concept: sine and cosine

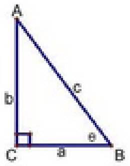
Definition: in a right triangle sine is the ratio of the opposite side to the hypotenuse and cosine is the ratio of the adjacent side and the hypotenuse.

Critical Attributes: right triangle, opposite, adjacent, hypotenuse

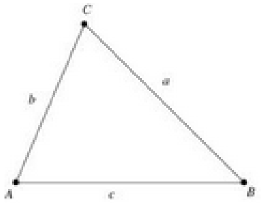
Shared Attributes: ratio

Non-Critical Attributes:

Examples: sine B =b/c; cosine B = a/c



Non-Examples: sine A =a/c; cosine A = b/c



Possible CFU Questions:) A young boy lets out 30 ft of string on his kite. If the angle of elevation from the boy to his kite is 27°, how high is the kite?

Skill Development

Skill: Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Procedural or Declarative: Procedural

Process, Procedure, Steps: Solve for an unknown using trig ratios

Details: angle of depression or elevation

Possible CFU' Questions: A ranger is on top of a 50-foot tower and spots a fire. If the angle of the depression is 30∘, how far is the fire from the base to the fire.

G.SRT.8.1

Derive and use the trigonometric ratios for special right triangles (30°, 60°, 90°and 45°, 45°, 90°). CA

Concept Development

Concept: special right triangle

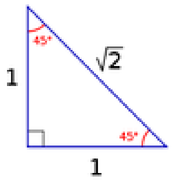
Definition: a right triangle with some regular feature that makes calculations on the triangle easier, or for which simple formulas exist. Knowing the ratios of the angles or sides of these special right triangles allows one to quickly calculate various lengths in geometric problems without resorting to more advanced methods

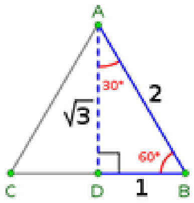
Critical Attributes: right triangle, equilateral triangle, square

Shared Attributes: ratio

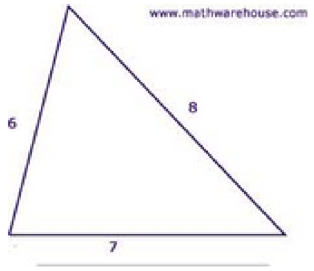
Non-Critical Attributes:

Examples:





Non-Examples:



Possible CFU Questions: Explain how to derive the special right triangle ratios for a 45-45-90 triangle.

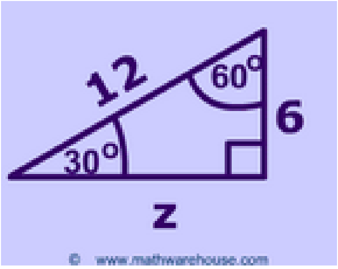
Skill Development

Skill: Use special right triangle ratios to find side lengths of special right triangles.

Procedural or Declarative: Procedural

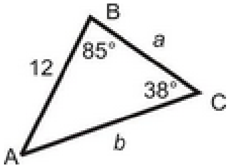
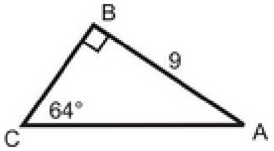
Process, Procedure, Steps: Solve for the unknown using ratios (similar triangles).

Possible CFU' Questions: Find the value of z.



G.SRT.11

(+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Concept Development	
Concept:	Law of Sines and Cosines
Definition:	Identities that are used to find missing pieces of oblique triangles
Critical Attributes:	pythagorean thm, trigonometric ratios
Examples:	Find the lengths of a and b. 
Non-examples:	Find the legth of BC 
Possible CFU:	When do you use the law of sines or the law of cosines?
Resources:	http://ccssmath.org/?page_id=1306

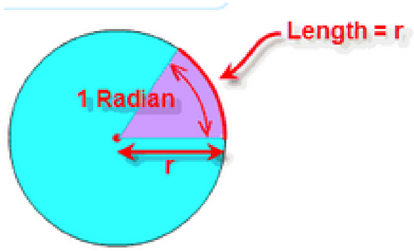
Skill Development	
Skill:	Apply the law of cosines and sines
Procedural or Declarative:	Procedural
Process, Details:	Use the formula to solve problems.
Possible CFU's	**1.** Surveyors preparing to build a bridgeAB across a ravine laid out the distance BC = 36 yards along one side of the ravine. They measured ∠B = 52° and ∠C = 48°. To the nearest yard, how long will the bridge be?

Integrated Math 3 Course Standard and Resource Guide

Trigonometry UNIT 11:

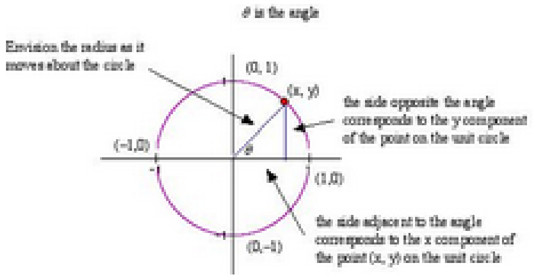
Overview	**Unit Circle**

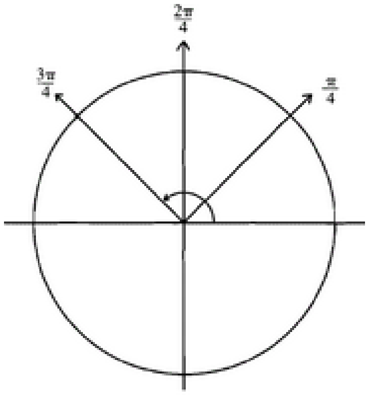
F.TF.1
Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

Concept Development	
Concept:	radian measure
Definition:	A unit of measure for angles. One radian is the angle made at the center of a circle by an arc whose length is equal to the radius of the circle.
Critical Attributes:	angle and unit circle, circumference
Examples:	
Non-examples:	
Possible CFU:	1. Find the radian measure of 30 degrees on a unit circle. 2. How many radian are in a full circle?
Resources:	http://ccssmath.org/?page_id=1304

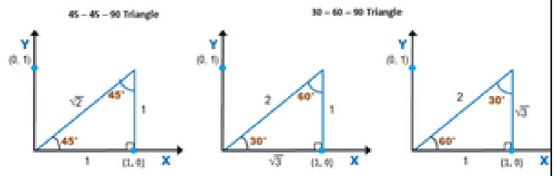
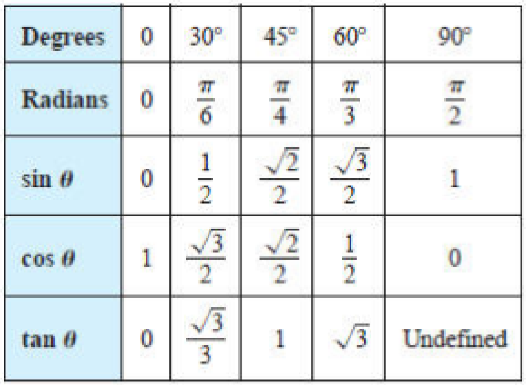
Skill Development	
skill	understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
Procedural or Declarative:	Declarative
Process, Details:	Use technology to show and demonstrate that 1 radian is equivalent to 57.3 degrees. Furthermore develop the conversion factor of $\pi/180^\circ$
Possible CFU's	Convert 120 degrees into radians.

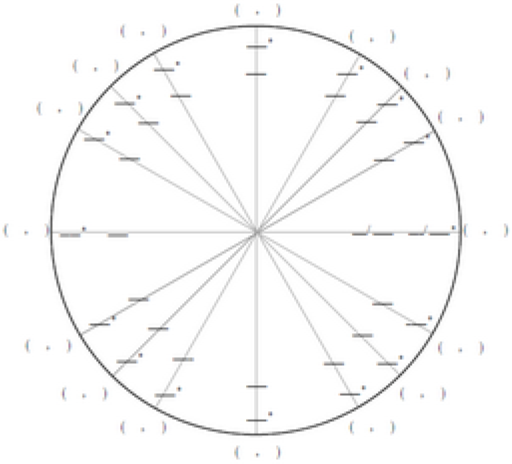
F.TF.2
Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

Concept Development	
Concept	unit circle
Definition	A unit circle is a circle that has a radius of one unit
critical attributes	coordinate plane, radian measure, trigonometric function
Examples:	
Possible CFUs	Explain why $\sin\theta = y$ and $\cos\theta = x$.
Resources:	http://ccssmath.org/?page_id=1304 http://www.themathpage.com/atrig/unit-circle.htm

Skill Development	
skill	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
Procedural or declarative	Declarative
Process	Go over (x,y) coordinates, pythagorean theorem, triangle trigonometry, quadrants and radian measure.
Possible CFU	Why is $\frac{3\pi}{4}$ in the second quadrant and explain why the sine of that angle would be positive and the cosine would be negative.  Figure 10.20

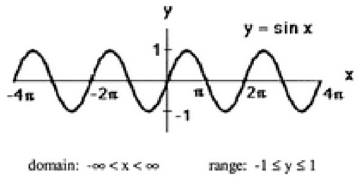
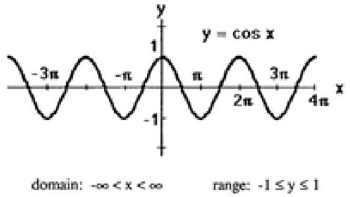
F.TF.3 (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosines, and tangent for x, π + x, and 2π – x in terms of their values for x, where x is any real number.

<table>
<tr><th>Concept Development</th><th></th></tr>
<tr><td>Concept</td><td>unit circle</td></tr>
<tr><td>Definition</td><td>A unit circle is a circle that has a radius of one unit</td></tr>
<tr><td>critical attributes</td><td>coordinate plane, radian measure, trigonometric function</td></tr>
<tr><td>Examples:</td><td>
</td></tr>
<tr><td>non-examples</td><td></td></tr>
<tr><td>Possible CFUs</td><td>Fill in the chart

<table>
<tr><th>Degrees</th><th>0</th><th>30°</th><th>45°</th><th>60°</th><th>90°</th></tr>
<tr><td>Radians</td><td>0</td><td>$\frac{\pi}{6}$</td><td>$\frac{\pi}{4}$</td><td>$\frac{\pi}{3}$</td><td>$\frac{\pi}{2}$</td></tr>
<tr><td>$\sin\theta$</td><td>0</td><td>$\frac{1}{2}$</td><td>$\frac{\sqrt{2}}{2}$</td><td>$\frac{\sqrt{3}}{2}$</td><td>1</td></tr>
<tr><td>$\cos\theta$</td><td>1</td><td>$\frac{\sqrt{3}}{2}$</td><td>$\frac{\sqrt{2}}{2}$</td><td>$\frac{1}{2}$</td><td>0</td></tr>
<tr><td>$\tan\theta$</td><td>0</td><td>$\frac{\sqrt{3}}{3}$</td><td>1</td><td>$\sqrt{3}$</td><td>Undefined</td></tr>
</table></td></tr>
<tr><td>Resources:</td><td>http://ccssmath.org/?page_id=1304
http://www.themathpage.com/atrig/unit-circle.htm</td></tr>
</table>

Skill Development	
skill	Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for x
Procedural or declarative	Procedural
Process	use special right triangles and coordinate planes on a unit circle. At this point derive the sine, cosine and tangent of π/3, π/4 and π/6.
Possible CFU	Fill in the unit circle. 

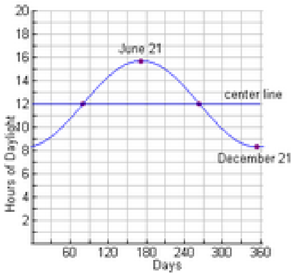
Overview	**Graph and model sinusoids**

F.TF.2.1 Graph all 6 basic trigonometric functions. CA

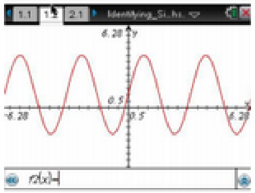
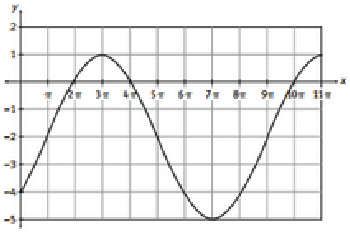
Concept Development	
Concept	trigonometric function
Definition	Trigonometric functions are commonly defined as ratios of two sides of a right triangle containing the angle, and can equivalently be defined as the lengths of various line segments from a unit circle.
critical attributes	coordinate plane, radian measure, trigonometric function
Examples:	 
Possible CFUs	On the axes from 0 to 2π, graph: $y = 2\sin(3x)$. State the amplitude, frequency and period of this graph.
Resources:	http://www.regentsprep.org/Regents/math/algtrig/ATT7/graphpractice.htm

Skill Development	
skill	graphing trigonometric functions
Procedural or declarative	Procedural
Process	using technology students can see a pattern of what happens when the amplitude, frequency, or vertical shift is changed on the equation.
Possible CFU	Given g(x) = 2 sin(2x), do the following: a. state the amplitude and EXACT period b. graph the function on the interval $(-2\pi, 2\pi)$ c. find the EXACT coordinates of the maximum using the graph d. find the EXACT coordinates of the minimum using the graph e. find the EXACT coordinates of the x-intercepts using the graph

F.TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

Concept Development	
Concept	periodic function
Definition	a function returning to the same value at regular intervals.
Critical attributes	amplitude, frequency, and midline
Examples:	The number of hours of daylight measured in one year in Ellenville can be modeled by a sinusoidal function. During 2006, (not a leap year), the longest day occurred on June 21 with 15.7 hours of daylight. The shortest day of the year occurred on December 21 with 8.3 hours of daylight. Write a sinusoidal equation to model the hours of daylight in Ellenville. 
non-example	The temperature in an office is controlled by an electronic thermostat. The temperatures vary according to the sinusoidal function: $y = 19 + 6\sin\left(\frac{\pi}{12}(x-11)\right)$ where *y* is the temperature (°C) and *x* is the time in hours past midnight. a.) What is the temperature in the office at 9 A.M. when employees come to work? b.) What are the maximum and minimum temperatures in the office?

CFU	A ferris wheel is 50 feet in diameter, with the center 60 feet above the ground. You enter from a platform at the 3 o'clock position. It takes 80 seconds for the ferris wheel to make one revolution clockwise. Find the model that gives your height above the ground at time t (t=0 when you entered).
Resources:	http://www.regentsprep.org/Regents/math/algtrig/ATT7/graphpractice3.htm http://ccssmath.org/?page_id=1304

Skill Development	
skill	Finding equations of sinusoids
Procedural/declarative	Procedural
Process details	Review the effects of amplitude, frequency, and midline on graphs with students using technology.
Possible CFU's	1. Consider this graph of a sinusoidal function [in radian measure]. Determine a function f(x) = A sin (B(x – C)) + D whose graph is the same as the one given  2. Write an equation for the graph below in terms of sine. 

Integrated Math 3 Course Standard and Resource Guide

Statistics

UNIT 13:

Overview	**Understand why two events are independent and determine independence. Understand conditional probability and find conditional probabilities. (math 2 review)**

Standards

S-CP.1.

Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).

Concept Development
Concept: events as subsets of a sample space
Definition: Sample space is a collection of all possible outcomes. Event is a collection of outcomes from a sample space
Critical Attributes: events, unions, intersection, complements
Shared Attributes:
Non-Critical Attributes:
Examples: Rolling a die S= { 1,2,3, 4, 5,6} an event can be, odd numbers= (1,3,5)
Non-Examples:
Possible CFU Questions: Describe the sample space when tossing two coins? Using the sample space find the outcomes for the event of getting two heads.

Skill Development
Skill: Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).
Procedural or Declarative: Declarative
Process, Procedure, Steps:
Details: Create and use Venn diagrams to illustrate relationships between sample spaces and events
Possible CFU’ Questions: You have a set of 10 cards numbered 1 to 10. Choose a card at random. Event A is choosing a number less than 7. Event B is choosing an odd number. Find the following events: find the intersection of A and B, find the union of A or B, find the complement of A, find the complement of B.

S-CP.2

Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

Concept Development

Concept: Independent events

Definition: Two events such that the occurrence of one event has no effect on the occurrence of the other event.

Critical Attributes: no effect

Shared Attributes: events, occurrence

Non-Critical Attributes:

Examples: Rolling a die twice.

Non-Examples: Drawing a card and drawing another card without replacement..

Possible CFU Questions: Explain why rolling a die twice is an independent event.

Skill Development

Skill: Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

Procedural or Declarative: Declarative and Procedural

Process, Procedure, Steps: Use venn diagrams or two- way tables to show P(A and B) = P(A) P(B)

Details: Students need to explain why the two events are independent

Possible CFU' Questions: When rolling two dice:

1) What is the probability of rolling a sum that is greater than 7?
2) What is the probability of rolling a sum that is odd?
3) What is the probability of rolling a sum that is greater than 7 and is odd?
4) Are the events rolling a sum greater than 7 and rolling a sum that is odd independent? Justify your answer

S-CP.3.
Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B

Concept Development

Concept: conditional probability

Definition: The probability that event B will occur given that event A has occurred.

Critical Attributes: Probability

Shared Attributes: event

Non-Critical Attributes:

Examples: Probability of drawing a club given the first was a club.

Non-Examples: Probability of drawing an ace.

Possible CFU Questions: Explain why or why not an event is conditional

Skill Development

Skill: Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B

Procedural or Declarative: Procedural and Declarative

Process, Procedure, Steps: Calculate conditional probabilities using $P(A / B) = \frac{P(A \text{ and } B)}{P(B)}$

Details: Understand that events A and B are independent if and only if they satisfy $P(A) = P(A / B)$or satisfy $P(B) = P(B / A)$

Possible CFU' Questions: Using the given information in a venn diagram or two way table calculate a conditional probability and determine if the two events are independent.

S-CP.4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*

Concept Development

Concept: two-way frequency table

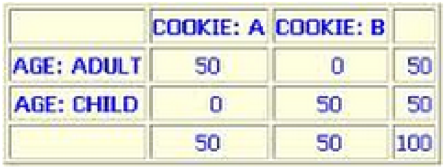
Definition: a table in which frequencies correspond to two variables

Critical Attributes: two-way

Shared Attributes: table, data

Non-Critical Attributes:

Examples:



	COOKIE: A	COOKIE: B	
AGE: ADULT	50	0	50
AGE: CHILD	0	50	50
	50	50	100

Non-Examples:

Class (Marks)	Frequency
11 - 15	2
16 - 20	3
21 - 25	3
26 - 30	5
31 - 35	6
36 - 40	6
41 - 45	3
46 - 50	2
Total	30

Possible CFU Questions: Explain why or why not this _________ is a two-way frequency table.

Skill Development

Skill: Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.

Procedural or Declarative: Procedural

Process, Procedure, Steps: construct a two-way table by inputting data on two variables making sure the columns and rows add to the same grand total.

Details:

Possible CFU' Questions: Construct a two-way frequency table. On one axis, compare grade level and on the other axis, compare the favorite fast-food hamburger place (McDonalds, Burger King, Jack in the Box, In-and-out, Carls Jr.) Find the probability that it is a sophomore who likes McDonalds? What is the probability that a students likes Burger King over anything else?

S-CP.5.
Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*

Concept Development
Concept: conditional probability
Definition:The probability that event B will occur given that event A has occurred.
Critical Attributes: independence
Shared Attributes:
Non-Critical Attributes:
Examples: Is owning a smartphone independent from grade level?
Non-Examples:
Possible CFU Questions: Explain how do you know if two events are conditional or independent.

Skill Development
Skill: Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.
Procedural or Declarative: Declarative
Process, Procedure, Steps: The most important key in this lesson is to teach students to think critically about the questions they want answers to. From this, students should be able to link their questions to the types of data they will gather. Finally, they should be able to assemble the data and infer relationships from the data using their knowledge about probabilities.
Details: students use the establish formulas in standard S.C.P .3
Possible CFU' Questions: Compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.

S-CP.6.
Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.

Concept Development

Concept: conditional probability

Definition:The probability of an event (*A*), given that another (*B*) has already occurred.

Critical Attributes: already occurred

Shared Attributes: probability, event

Non-Critical Attributes:

Examples: Find the probability you passed science given you passed math.

Non-Examples:

Possible CFU Questions: Construct a tree diagram to find the conditional probability of getting heads on the second toss given the first toss was heads

Skill Development

Skill: Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.

Procedural or Declarative: Procedural

Process, Procedure, Steps: use venn diagrams, two-way table or tree diagram to find conditional probabilities

Details:

Possible CFU' Questions: Determine the probability of getting the flu, and compare that to the probability of getting the flu given that an individual takes high doses of vitamin C

S-CP.7.
Apply the Addition Rule, P(A or B) = P(A) + P (B) – P(A and B), and interpret the answer in terms of the model.

Concept Development

Concept: Addition Rule

Definition: A statistical property that states the probability of one and/or two events occurring at the same time is equal to the probability of the first event occurring, plus the probability of the second event occurring, minus the probability that both events occur at the same time.

Critical Attributes: union, intersections,

Shared Attributes: event, probability

Non-Critical Attributes:

Examples: Probability of drawing an ace or a spade.

Non-Examples: Probability of drawing an ace and a spade.

Possible CFU Questions: Explain how to use the addition rule when two events are given.

Skill Development

Skill: Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.

Procedural or Declarative: Procedural

Process, Procedure, Steps: students use addition Rule to find the P(A or B)

Details:

Possible CFU' Questions: Find the probability of drawing an ace or a spade.

Resources:

http://ccssmath.org/
http://www.geometrycommoncore.com/index.html
https://sites.google.com/site/misterbledsoe/cc2-videos
http://www.geogebratube.org/

Integrated Math 3 Course Standard and Resource Guide

Statistics

UNIT 14:

Overview	Understand and evaluate random processes underlying statistical experiments

S.IC.1
Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

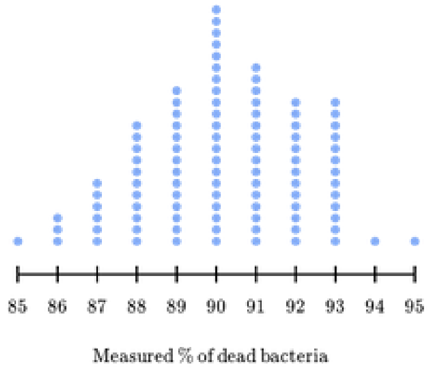
Concept Development	
Concept:	Inference
Definition:	a conclusion reached on the basis of evidence and reasoning
Critical Attributes:	random sampling, population
Examples:	A pollster wants to find out whether or not American citizens would support a candidate running for national office who wants to lower the legal drinking age from 21 to 18. They plan on doing this by sending 10,000 text messages across the entire United States to randomly selected, active, U.S. based phones with text messaging capabilities. Assume every text that is sent receives a reply. Why is this random sample, despite being truly randomly chosen, unlikely to be a good representative sample of the American population's opinion in an election?
CFU's	A fair six-sided die is randomly tossed to get a sample of 1, 1, 1, 1, 1, and 1. Is this a random sample and why?
Resources:	http://www.shmoop.com/common-core-standards/ccss-hs-s-ic-1.html#drills https://www.illustrativemathematics.org/illustrations/122

Skill Development	
Skill:	Compare and contrast methods of sampling procedures.
What do I teach?:	Declarative
How do I teach?:	Teach by describing different sampling methods and having students make their own samples.
CFU Questions:	Why is picking out different candies from a bag without looking not as effective a random sample than if you were to assign numbers to each piece of candy and let someone else pick those randomly instead?

S.IC.2
Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin lands heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?

Concept Development	
Concept:	Plausibility of a model.
Definition:	Plausible means that the model is likely to have produced certain data.
Critical Attributes:	simulation, sample, data-generated
Shared Attributes:	random
Examples:	A six-sided die is biased. To find the probability that it rolls a 6, a simulation is done by a researcher. The die is rolled 120 times and the outcome is 6 only 15 times. What does this simulation suggest?
Resources:	http://ccssmath.org/?page_id=1311 http://www.sophia.org/tutorials/simulations?pathway=ccss-math-standard-9-12sic2 https://www.khanacademy.org/search?page_search_query=s.ic.2

Skill Development	
Skill:	Compare model results with data.
Declarative/Procedural?:	Declarative

How do I teach?:	Use technology to help with setting up simulations.
CFU Questions:	Alma has developed a new kind of antibiotic that she expects to kill 90% of harmful bacteria when applied. She applied her antibiotic to a Petri dish full of bacteria, waited for it to take effect, and took a random sample of 200 bacteria. She found that 87% of them were dead. In light of the results, Alma had to test the hypothesis that the true percentage of dead bacteria is 90%. She performed 100 computer generated simulations of random samples of 200 bacteria, supposing the true percentage of dead bacteria is 90%, to find how likely it is that a sample would have 87% dead bacteria. The results of the simulations are plotted below. How do the results of the simulations affect the likelihood of the hypothesis that Alma's antibiotic kills 90%of bacteria? • The results are reasonably consistent with the hypothesis. • The results make it very unlikely that the hypothesis is correct. 

S.IC.3
Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

Concept Development	
Concept:	sampling
Definition:	ways of gathering data
Critical Attributes:	sample surveys,experiments, and observational studies
Shared Attributes:	random samples
Examples:	A scientist selects 500 smokers to test how long they can hold their breath. Not surprisingly, the smokers can't hold their breath for long. The average result was a measly 23 seconds. What kind of study was this?
Resources:	http://ccssmath.org/?page_id=2361

Skill Development	
Skill:	**Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.**
Declarative?:	**Declarative**
How do I teach?:	**Provide students with different types of sampling methods and have discussions with them in small groups and whole class.**
CFU Questions:	A pharmaceutical company is trying to figure out whether a drug called SmartiePants can make you smarter. (It also tastes like candy. The more you eat, the smarter you can get.) They prepare a double-blind study as follows: Step 1: A randomly selected pool of individuals will be brought into a clinic and evaluated for any existing health conditions that would disqualify them from the experiment. Step 2: After passing the health screening the individuals will be split up into two groups: test and controlled. Step 3: The control group will receive a placebo, but neither the clinician administering it nor the participants know this. Step 4: The treatment group will receive SmartiePants, but neither the clinician administering it nor the participants know this. Where is the mistake in this double blind study?Explain what type of sampling method is this?

S.IC.4
Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

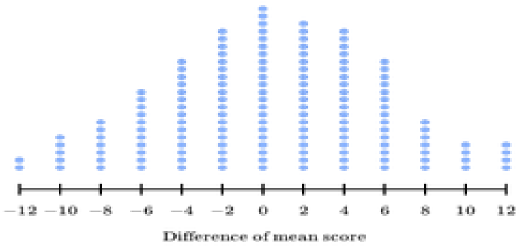
<table>
<tr><th>Concept Development</th><th></th></tr>
<tr><td>Concept:</td><td>population mean or proportion</td></tr>
<tr><td>Definition:</td><td>The statistic that estimates the parameter p,
a proportion of a population that has some property
The statistic used to give information about the population parameter μ;</td></tr>
<tr><td>Critical Attributes:</td><td>Categorical and quantitative data</td></tr>
<tr><td>Shared Attributes:</td><td>sampling</td></tr>
<tr><td>Examples:</td><td>A local candy store has found that kids prefer certain colors of candy regardless of their taste. For kids ages five to eight, the following data was collected:
<table>
<tr><td>Blue</td><td>67</td></tr>
<tr><td>Red</td><td>89</td></tr>
<tr><td>Yellow</td><td>27</td></tr>
<tr><td>Green</td><td>13</td></tr>
</table>
What is the estimated population proportion of the most preferred candy color from this sample?</td></tr>
<tr><td>Resources:</td><td>http://ccssmath.org/?page_id=2363</td></tr>
</table>

Skill Development	
Skill:	Estimate a population mean or proportion
Declarative/procedural:	Declarative
How do I teach?:	use technology and formulas
CFU Questions:	A company specializing in building robots that clean your house has found that the average amount of time kids are forced (yes, forced) to spend cleaning their houses is about 2 hours per week. If their sample size was 1000 randomly chosen kids and the standard deviation was 0.3 hours, what is the margin of error for a confidence interval of 95%?

S.IC.5
Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

Concept Development	
Concept:	experiments
Definition:	A unit is a person, animal, plant or thing which is actually studied by a researcher; the basic objects upon which the study or experiment is carried out
Critical Attributes:	treatments, populations
Shared Attributes:	randomness
Examples:	A study compares how much faster someone runs after drinking a shot of espresso compared to someone who drinks only water. If the data seems fairly different, what would be the best course of action?
Resources:	https://www.khanacademy.org/commoncore/grade-HSS-S-IC

Skill Development	
Skill:	conduct an experiment through simulation to compare parameters
Procedural/Declarative:	Procedural and Declarative
How do I teach?:	using technology to gather data from a simulation

CFU Questions:	Researchers were interested in the effect of pre-existing inappropriate highlighting of text on reading comprehension. They randomly assigned a group of 300 students to a treatment group and to a control group. Both groups were asked to answer a 38-point reading comprehension test. The text given to the treatment group had inappropriate passages highlighted, while the text of the control group wasn't highlighted at all. They found that the mean score of the treatment group is 8 points less than the mean score of the control group. Using a simulator, they re-randomized the results into two new groups and measured the difference between the means of the new groups. They repeated this simulation 150 times, and plotted the resulting differences, as given below. According to the simulations, is the result of the experiment significant? • Yes. According to the simulations, the result of the experiment is significant. • No. According to the simulations, the result of the experiment is insignificant. 

S.IC.6
Evaluate reports based on data.

Concept Development	
Concept:	data
Definition:	a collection of facts or information from which conclusions may be drawn.
Critical Attributes:	population proportion or population mean
Shared Attributes:	sampling methods
Examples:	A study samples 100 Coca-Cola drinkers and finds that 99 of them really dislike the taste of the new cola drink. What inference can be drawn from this?
Resources:	http://ccssmath.org/?page_id=2367 https://www.khanacademy.org/search?page_search_query=s.ic.4

Skill Development	
Skill:	Students will be able to evaluate reports based on data
Procedural/Declarative:	Declarative
How do I teach?:	Provide students with different types of reports which can include graphs and tables.
CFU Questions:	A nutritionist had a hypothesis that eating a single banana an hour before a marathon (a 42-km run) can improve performance and reduce running time. To test her hypothesis, she randomly assigned a group of 360 men about to participate in a marathon to two groups. One group was instructed to eat a single banana an hour before the race, and the other group was instructed to eat nothing during the few hours before the race. After the race was done, she compared the average running times of the two groups. The nutritionist found that the average running time of the group who ate a banana was 5 minutes shorter than the running time of the group who hadn't. Based on some re-randomization simulations, she concluded that the result is significant and not due to the randomization of the groups. What valid conclusions can be made from this result?

If time permits and you want to challenge students, these last two standards may be introduced.

S.MD.6
(+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).

S.MD.7
(+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

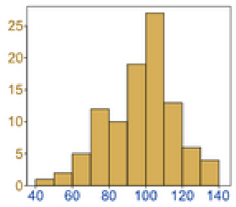
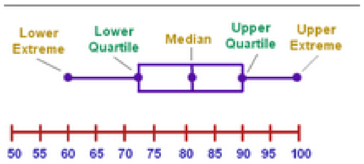
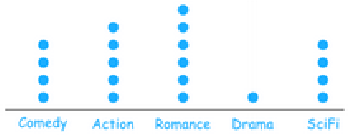
Integrated Math 3 Course Standard and Resource Guide

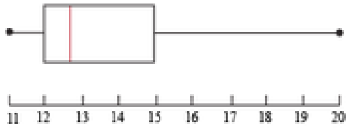
Statistics

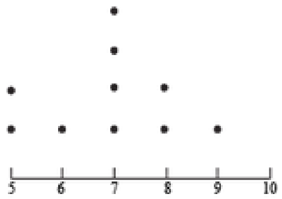
UNIT 15:

Overview	Summarize, represent, and interpret data on a single count or measurement variable.

S.ID.1 (math 1) Represent data with plots on the real number line (dot plots, histograms, and box plots).

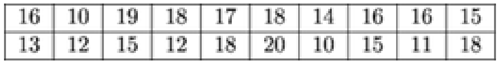
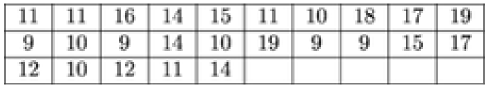
Concept Development	
Concept:	data
Definition:	facts or information used usually to calculate, analyze, or plan something
Critical Attributes:	dot plots, histogram, and box plots
Shared Attributes:	graph
Non-Critical Attributes:	information gathered
Examples:	  

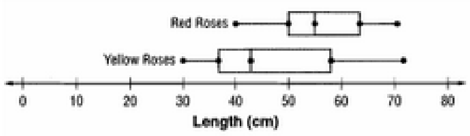
CFU	Given the following box plot, what are the median, lower, and upper quartiles? 
Resources:	http://ccssmath.org/?page_id=2333 http://www.mathsisfun.com/data/histograms.html

Skill Development	
Skill:	**Represent data with plots on the real number line**
Procedural or Declarative	**Procedural**
Procedure, process, or steps to execute the skill	**Using technology to get data show students which graph should be used for the different data sets.**
CFU Questions:	Given the following dot plot, what is the median of the data? 

S.ID.2 (math 1)

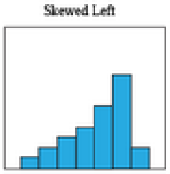
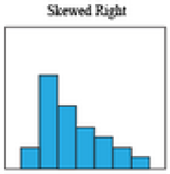
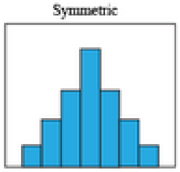
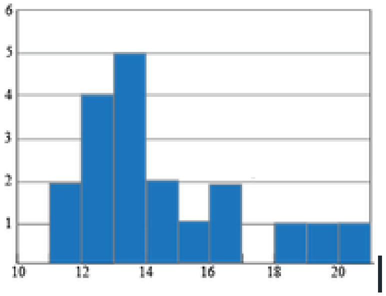
Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

<table>
<tr><th>Concept Development</th><th></th></tr>
<tr><td>Concept:</td><td>data distribution</td></tr>
<tr><td>Definition:</td><td>collection and interpretation of quantitative data</td></tr>
<tr><td>Critical Attributes:</td><td>two data sets, center and spread</td></tr>
<tr><td>Shared Attributes:</td><td>median, mean, interquartile range, standard deviation</td></tr>
<tr><td>Examples:</td><td>Below are the scores two different sections of a vocabulary quiz. Given that the distribution of scores follows a normal distribution, which section had a greater spread in the data?
Section 1: 
<table>
<tr><td>16</td><td>10</td><td>19</td><td>18</td><td>17</td><td>18</td><td>14</td><td>16</td><td>16</td><td>15</td></tr>
<tr><td>13</td><td>12</td><td>15</td><td>12</td><td>18</td><td>20</td><td>10</td><td>15</td><td>11</td><td>18</td></tr>
</table>
Section 2: 
<table>
<tr><td>11</td><td>11</td><td>16</td><td>14</td><td>15</td><td>11</td><td>10</td><td>18</td><td>17</td><td>19</td></tr>
<tr><td>9</td><td>10</td><td>9</td><td>14</td><td>10</td><td>19</td><td>9</td><td>9</td><td>15</td><td>17</td></tr>
<tr><td>12</td><td>10</td><td>12</td><td>11</td><td>14</td><td></td><td></td><td></td><td></td><td></td></tr>
</table></td></tr>
<tr><td>Non-Examples:</td><td>Find the median 5, 3, 4, 1, 2 =3</td></tr>
<tr><td>CFU</td><td>Billy-Joe Bob and Bobby-Joe Bill are having a contest to see whose chickens provide more eggs. Over the course of 10 days, the farmers each count and record the number of eggs they collect. Compare the two data sets using shape, center and spread.
Billy-Joe Bob: 28, 21, 8, 15, 6, 18, 16, 30, 25, 17
Bobby-Joe Bill: 27, 28, 15, 28, 28, 23, 20, 8, 14, 8</td></tr>
<tr><td>Resources:</td><td>http://ccssmath.org/?page_id=2335</td></tr>
</table>

<table>
<tr><th>Skill Development</th><th></th></tr>
<tr><td>Skill:</td><td>compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.</td></tr>
<tr><td>Procedural or Declarative Knowledge</td><td>Procedural</td></tr>
<tr><td>Procedure, process, or steps to execute the skill</td><td>Provide students with data set have them plot on the appropriate graph and have compare the center and spread.</td></tr>
<tr><td>CFU Questions:</td><td>Jane collected some red and yellow roses. She measured the lengths of their stems, and drew the following box plots. Write down the median lengths of both the yellow and red roses to the nearest centimeter.

Which color rose would you buy for a 40 cm tall vase?</td></tr>
</table>

S.ID.3 (math 1)

Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

Concept Development	
Concept:	Data
Definition:	facts or information used usually to calculate, analyze, or plan something
Critical Attributes:	shape, center, spread, outliers
Shared Attributes:	graphs
Non-Critical Attributes:	information gathered
Examples:	Skewed Left  Skewed Right  Symmetric 
CFU	Given the following histogram, how can we describe the shape of the data? 
Resources:	http://ccssmath.org/?page_id=2337 http://www.mathsisfun.com/data/histograms.html

Skill Development	
Skill:	**Interpret differences in shape, center, and spread in the context of the data sets**
Procedural or Declarative	Declarative
Procedure, process, or steps to execute the skill	**Provide students with visuals of different types of graphs and show students how to describe data distribution in terms of shape.**
CFU Questions:	**1. The mean of a data set is 12 and the median is 10. What shape is the data?** **2. Given the data points 18, 14, 12, 14, 11, 11, 19, 20, 16, and 11, which values would be considered outliers?**

S.ID.4

Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Concept Development	
Concept:	Normal Distribution, z-score.
Definition:	A normal distribution is modeled by a bell-shaped curve called a normal curve that is symmetric about the mean. Z-scores correspond to the number of standard deviations that the x-value lies above or below the mean xbar.
Critical Attributes:	properties of the normal distribution with the mean and standard deviation
Shared Attributes:	Standard Deviation, Mean, z-scores.
Examples:	The grades on a math midterm at Gardner Bullis are normally distributed with μ=76 and σ=4.5. Daniel scored 64 on the exam. Find the z-score for Daniel's exam grade. Round to two decimal places.
Resources:	Alg 2 Textbook section 11.1 p 744-748 and section 11.3 p757-762 https://www.khanacademy.org/search?page_search_query=s.id.4 http://ccssmath.org/?page_id=2339

Skill Development	
Skill:	Students should be able to complete normal distribution calculations. Use properties of normal distributions to draw conclusions.
What do I teach?:	Know the properties of the normal distribution. Find z-values.
How do I teach?:	can use technology or table
CFU Questions:	1. What is the relation between the z score and the standard deviation? 2. You purchased 10 baskets of strawberries at the local farmer's market and counted the number of strawberries in each basket. Based on your purchases, do you think the number of strawberries in a basket is normally distributed? 20 24 20 22 21 19 17 15 20 22