Integrated Math 3 Module 5 Honors Modeling with Functions Ready, Set, Go! Homework Solutions

Adapted from

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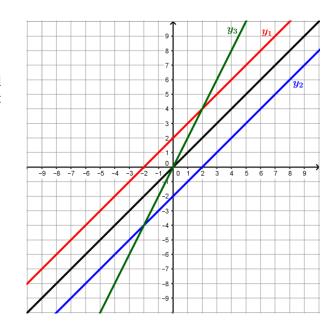
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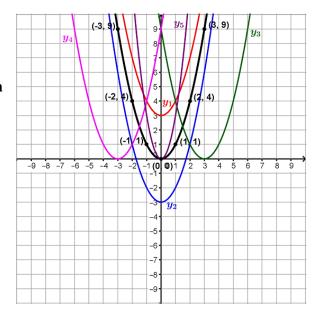
Ready

Topic: Transformations

- 1. Graph the following linear functions on the grid. The equation y = x has been graphed for you. For each new equation, explain what the number 2 does to the graph of y = x. Pay attention to the *y*-intercept, the *x*-intercept, and the slope. Identify what changes in the graph and what stays the same.
 - a. $y_1 = x + 2$ *y*-intercept changes, slope stays the same
 - b. $y_2 = x 2$ *y*-intercept changes, slope stays the same
 - c. $y_3 = 2x$ slope changes, *y*-intercept stays the same
- 2. Graph the following quadratic functions on the grid. The equation $y = x^2$ has been graphed for you. For each new equation, explain what the number 3 does to the graph of $y = x^2$. Pay attention to the *y*-intercept, the *x*-intercept(s), and the rate of change. **Identify what changes in the graph and what stays the same**.
 - a. $y_1 = x^2 + 3$ rate of change stays the same; all points translate up 3 units
 - b. $y_2 = x^2 3$ rate of change stays the same; all points translate down 3 units
 - c. $y_3 = (x 3)^2$ rate of change stays the same; all points translate right 3 units
 - d. $y_4 = (x + 3)^2$ rate of change stays the same; all points translate left 3 units
 - e. $y_5 = 3x^2$ rate of change changes; vertex stays in the same location.



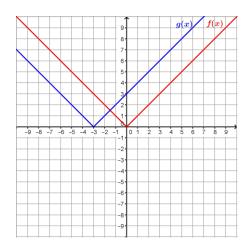




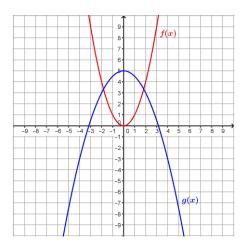
Topic: Transformations on parent functions.

Sketch the graph of the parent function and the graph of the transformed function on the same set of axes.

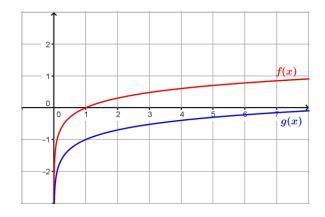
3. f(x) = |x| and g(x) = |x+3|



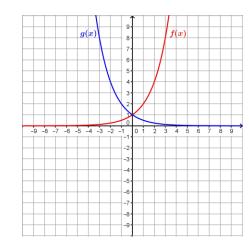
5. $f(x) = x^2$ and $g(x) = -\frac{1}{2}x^2 + 5$



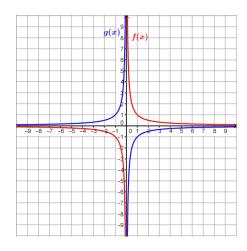
7. $f(x) = \log x$ and $g(x) = -1 + \log x$



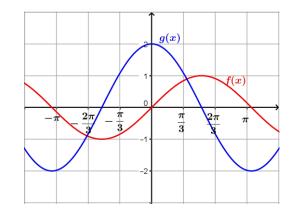
4. $f(x) = 2^x$ and $g(x) = 2^{-x}$



6. $f(x) = \frac{1}{x}$ and $g(x) = -\frac{1}{x}$



8. $f(x) = \sin x$ and $g(x) = 2\sin\left(x + \frac{\pi}{2}\right)$



Go

Topic: Evaluating functions

Find the function values: f(-2), f(0), f(1), f(3). Indicate if the function is undefined for a given value of x.

9. f(x) = |x + 5|10. f(x) = |x - 2|11. f(x) = x|x|f(-2) = 3, f(0) = 5,f(-2) = 4, f(0) = 2,f(-2) = -4, f(0) = 0,f(1) = 6, f(3) = 8f(1) = 1, f(3) = 1f(1) = 1, f(3) = 9

12.
$$f(x) = 3^x$$
13. $f(x) = 3^{x+2}$ 14. $f(x) = (3^x) + x$ $f(-2) = \frac{1}{9}, f(0) = 1,$ $f(-2) = 1, f(0) = 9,$ $f(-2) = -\frac{18}{9}, f(0) = 1,$ $f(1) = 3, f(3) = 27$ $f(1) = 27, f(3) = 243$ $f(1) = 4, f(3) = 30$

15.
$$f(x) = \frac{x}{x}$$

 $f(-2) = \mathbf{1}, f(0) = \mathbf{und},$
 $f(-2) = \mathbf{1}, f(3) = \mathbf{1}$
 $f(1) = \mathbf{1}, f(3) = \mathbf{1}$
16. $f(x) = \frac{x}{(x-4)}$
 $f(-2) = \frac{1}{3}, f(0) = \mathbf{0},$
 $f(-2) = \frac{1}{3}, f(0) = -\mathbf{0},$
 $f(1) = -\mathbf{1}, f(3) = -\mathbf{1}$
 $f(1) = -\mathbf{1}, f(3) = -\mathbf{1}$

18.
$$f(x) = \log_3 x$$
19. $f(x) = \log_7(7)^x$ 20. $f(x) = x \log_{10} 1000$ $f(-2) = und, f(0) = und,$ $f(-2) = -2, f(0) = 0,$ $f(-2) = -6, f(0) = 0,$ $f(1) = 0, f(3) = 1$ $f(1) = 1, f(3) = 3$ $f(1) = 3, f(3) = 9$

Ready

Topic: Function boundaries.

1. The black solid curve in the graph at the right shows the graph of $f(x) = \sin x$.

Write the equation of the dashed line labeled g(x).

$$g(x) = 1$$

Write the equation of the dotted line labeled h(x).

h(x) = -1

List everything you notice about these three graphs.

g(x) touches all the maximum points on f(x) and h(x) touches all the minimum points on f(x).

2. The black solid curve in the graph at the right shows the graph of $f(x) = \sin x$.

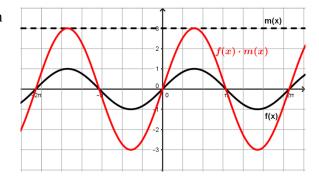
Write the equation of the dashed line labeled m(x).

m(x) = 3

Sketch the graph of $f(x) \cdot m(x)$ on the same grid.

What is the equation of $f(x) \cdot m(x)$? **3** sin *x*

Would the line y = -3 also be a boundary line for your sketch of $f(x) \cdot m(x)$? Explain. Yes it would be a boundary line because it would hit all the minimum points.





g(x)

f(x)

h(x)

3. The black solid curve in the graph at the right shows the graph of $f(x) = \sin x$.

Write the equation of the dashed line labeled b(x).

 $\boldsymbol{b}(\boldsymbol{x}) = -\boldsymbol{3}$

Sketch the graph of $f(x) \cdot b(x)$ on the same grid.

```
What is the equation of f(x) \cdot b(x)?
-3sin x
```

Would the line y = 3 also be a boundary line for your sketch of $f(x) \cdot b(x)$? Explain. Yes it would be a boundary line because it hits all the maximum points.

How does the graph of $f(x) \cdot b(x)$ differ from the graph of $f(x) \cdot m(x)$ from question 2? **The graphs are reflections over the** *x***-axis**.

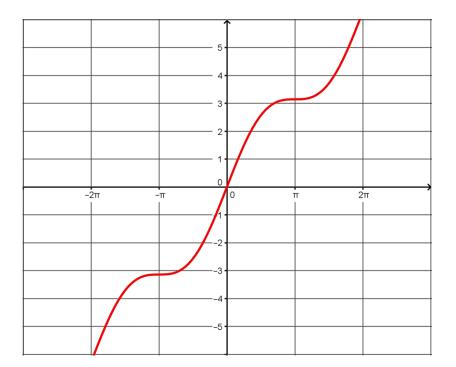
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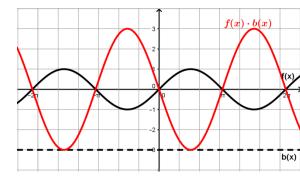
Topic: Combining functions

4.
$$f(x) = x$$
 $g(x) = \sin x$ $h(x) = f(x) + g(x)$

Some values for f(x) and g(x) are given. Fill in the values for h(x). Then graph $h(x) = x + \sin x$ with a smooth curve.

x	f(x)	g(x)	h(x)
-2π	-6.28	0	-6.28
$-\frac{3\pi}{2}$	-4.71	1	-3.71
-π	-3.14	0	-3.14
$-\frac{\pi}{2}$	-1.57	-1	-2.57
0	0	0	0
$\frac{\pi}{2}$	1.57	1	2.57
π	3.14	0	3.14
$\frac{3\pi}{2}$	4.71	-1	3.71
2π	6.28	0	6.28



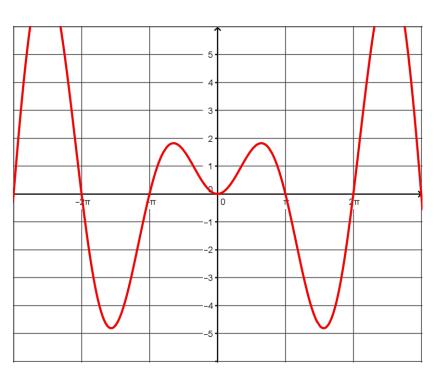


6

5. f(x) = x $g(x) = \sin x$

Now graph $k(x) = f(x) \cdot g(x)$ or $k(x) = x \cdot \sin x$

x	f(x)	g(x)	k(x)
-2π	-6.28	0	0
$-\frac{3\pi}{2}$	-4.71	1	-4.71
$-\pi$	-3.14	0	0
$-\frac{\pi}{2}$	-1.57	-1	1.57
0	0	0	0
$\frac{\pi}{2}$	1.57	1	1.57
π	3.14	0	0
$\frac{3\pi}{2}$	4.71	-1	-4.17
2π	6.28	0	0



Match the equations with the appropriate graph. Describe the features of the graph that helped you match the equations.

- 6. f(x) = |x² 4| Key features:
 F; always positive & without the absolute value function, it is a parabola with a vertex at (0, -4).
- 9. $d(x) = (10 x^2) + 5 \sin x$ Key features: A; *y*-intercept is 10.
- 7. $g(x) = -x + 5 \sin x$ Key features: B; Sine curve following the line y = -x

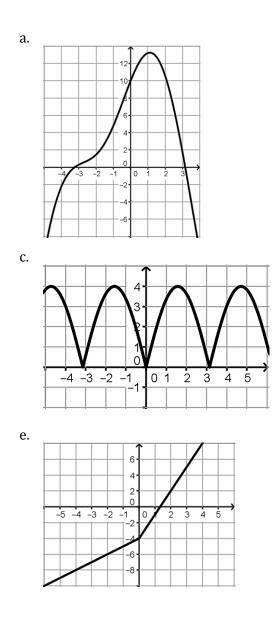
D; amplitude in the middle

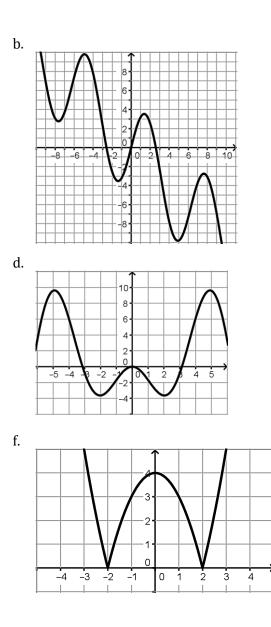
10. $w(x) = -x \cdot 2 \sin x$

of the graph is 2.

Key features:

- 8. h(x) = 4|sin x| Key features:
 C; sine curve that is always positive.
- 11. r(x) = (2x 4) + |x|Key features: **E**; contains the line y = 2x - 4 left of the *y*-axis.





The chart below names five families of functions and the parent function. The parent is the equation in its simplest form. In the right hand column is a list of key features of the functions in random order. Match each key feature with the correct function. A key feature may relate to more than one function.

Family	Parent(s)	Key Features
11. Linear <u>(E. H. I. P)</u>	y = x	a. The ends of the graph have the same behavior.b. The graphs may have a horizontal asymptote and a vertical asymptote.c. The graph only has a horizontal asymptote.
12. Quadratic <u>(A. F. K. P)</u>	$y = x^2$	 d. These functions either have both a local maximum and minimum or no local maximum and minimum. e. The graph is usually defined in terms of its slope and <i>y</i>-intercept. f. The graph has either a maximum or a minimum but not both.
13. Cubic (<u>D, H, L, P)</u>	$y = x^3$	 g. As <i>x</i> approaches -∞, the function values approach the <i>x</i>-axis. h. The ends of the graph have opposite behavior. i. The rate of change of this graph is constant.
14. Exponential <u>(C. G. J. M)</u>	$y = 2^{x}$ $y = 3^{x}$ Etc.	 j. The rate of change of this graph is constantly changing. k. This graph has a linear rate of change. l. These functions are of degree 3.
15. Rational <u>(B. D. G. J. N)</u>	$y = \frac{1}{x}$	 m. The variable is an exponent. n. These functions contain fractions with a polynomial in both the numerator and denominator. p. The constant will always be the <i>y</i>-intercept.

Ready

Topic: Evaluating functions

Evaluate each function. Simplify your answers when possible. State *undefined* when applicable.

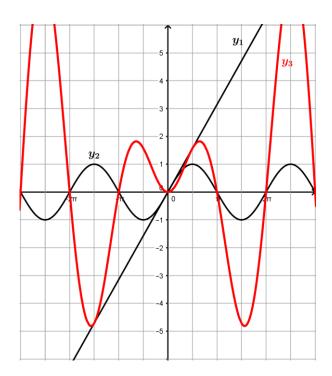
1.	$f(x) = x^2 - 8x$ a. $f(0)$ 0								$f(x+2)$ $x^2 - 4x - 12$
2.	$g(x) = \frac{3x-5}{x}$ a. $g(-1)$								
	8						und		$\frac{6x+7}{2x+4}$
3.					$h\left(\frac{11\pi}{6}\right)$ $-\frac{1}{2}$				$h\left(\cos^{-1}\left(\frac{1}{2}\right)\right), x < \pi$ $\frac{\sqrt{3}}{2}$
4.	$w(x) = \tan x$ a. $w(\pi)$	b.	$W\left(\frac{3\pi}{2}\right)$	C.	$W\left(\frac{7\pi}{6}\right)$	d.	$W\left(\frac{3\pi}{4}\right)$	е. и	$v\left(\cos^{-1}\left(-\frac{1}{2}\right)\right)$, $x < \pi$
	0		und		$\frac{\sqrt{3}}{3}$		-1	-	$-\sqrt{3}$

10

Topic: Dampening functions

Two functions are graphed. Graph a third function by multiplying the two functions together. Use the table of values to assist you. It may help you to change the function values to decimals.

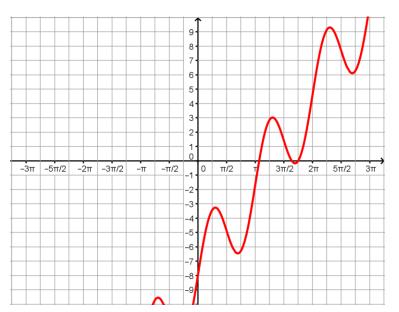
5.				
	x	$y_1 = x$	$y_2 = \sin x$	$y_3 = x \cdot \sin x$
	-2π	-6.283	0	0
	$-\frac{3\pi}{2}$	-4.712	1	-4.712
	$-\pi$	-3.14	0	0
	$-\frac{\pi}{2}$	-1.57	-1	1.57
	0	0	0	0
	$\frac{\pi}{2}$	1.57	1	1.57
	π	3.14	0	0
	$\frac{3\pi}{2}$	4.712	-1	-4.712
	2π	6.283	0	0



Topic: Graphing products and sums of functions.

For each set of functions below, graph the indicated operation. Be sure to use the features of each individual function to help you complete the graph.

6. f(x) = 2x - 8 and $g(x) = 3\sin(2x)$. Graph f(x) + g(x).



7. $f(x) = 8 \cdot \left(\frac{1}{2}\right)^x$ and $g(x) = \cos(4x)$. Graph $f(x) \cdot g(x)$.



Go

Topic: Measures of central tendency (Mean, median, mode)

During salary negotiations for teacher pay in a rural community, the local newspaper headlines announced: **Greedy Teachers Demand More Pay!** The article went on to report that teachers were asking for a pay hike even though district employees, including teachers, were paid an average of \$70,000 per year, while the average annual income for the community was calculated to be \$55,000 per household. The 65 school teachers in the district responded by declaring that the newspaper was spreading false information.

Use the table below to explore the validity of the newspaper report.

Job Description	Number Having Job	Annual Salary
Superintendent	1	\$258,000
Business Administrator	1	\$250,000
Financial Officer	1	\$205,000
Transportation Coordinator	1	\$185,000
District secretaries	5	\$ 55,000
School Principals	5	\$200,000
Assistant Principals	5	\$175,000
Guidance Counselors	10	\$ 85,000
School Nurse	5	\$ 83,000
School Secretaries	10	\$ 45,000
Teachers	65	\$ 48,000
Custodians	10	\$ 40,000

Which measure of central tendency (mean, median, mode) do you think the newspaper used to report the teachers' salaries? Justify your answer.
 They used the mean since the mean is \$69,605.04 and the median and mode are both \$48,000.
 The mean is skewed away from the salary of teachers due to higher up positions in the district.

9. Which measure of central tendency do you think the teachers would use to support their argument? Justify your answer.

The median would support their argument since the salary of teachers is the median of the data. The median is lower than the annual income for the community.

10. Which measure gives the clearest picture of the salary structure in the district? Justify.

Mean gives the clearest picture of the salary structure because it shows how skewed the salaries are in favor of administrators (away from teachers' salaries).

11. Make up a headline for the newspaper that would be more accurate.

Answers may vary.

Ready

Topic: Recognizing operations on a variable

Each function below contains 2 functions. One of the functions will be "inside" the other. Identify the "inner" function as u by writing u =_____. Then substitute u into the outer function so that the new function is of the form h(u).

Example:

Given: $h(x) = 5x^3$

I can see two functions acting on *x*. First, *x* is being cubed and then x^3 is multiplied by 5. So if $u = x^3$, then $5x^3 = 5u$. Therefore, h(u) = 5u.

- 1. Would the answer in the example have been different if you were given $(5x)^3$? Explain $u = 5x, (5x)^3 = u^3$
- 2. $h(x) = (x 6)^2$ u = x - 63. $h(x) = \tan(x + 4)$ u = x + 44. $h(x) = \sqrt[3]{(2x - 7)}$ u = 2x - 7
 - $h(u) = (x 6)^2 = u^2$ $h(u) = \tan u$ $h(u) = \sqrt[3]{u}$
- 5. h(x) = -9(x + 5) u = x + 5 h(u) = -9u6. $h(x) = \frac{5}{x^2}$ $u = \frac{1}{x^2} \text{ or } x^2$ $h(u) = 5u \text{ or } \frac{5}{u}$ $h(u) = u^4$

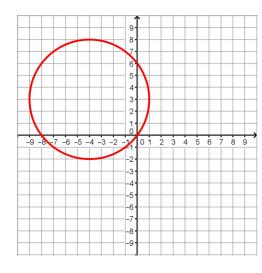


Identify the type of conic section (parabola, circle, ellipse, or hyperbola). Then graph the conic section.

8.
$$(x+4)^2 + (y-3)^2 = 25$$

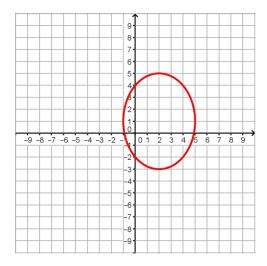
9.
$$(y+1)^2 = 12(x-2)$$

Circle

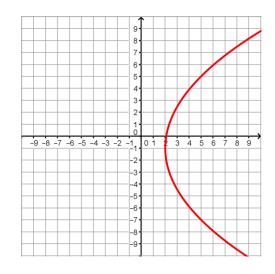


$$10.\,\frac{(x-2)^2}{9} + \frac{(y-1)^2}{16} = 1$$

Ellipse

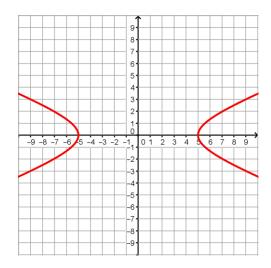


Parabola



$$11.\,\frac{x^2}{25} - \frac{y^2}{4} = 1$$

Hyperbola



Topic: Creating formulas for composite functions

Note:
$$f(g(x)) = (f \circ g)(x)$$

12. Let $f(x) = 2x^2 - 4$ and $g(x) = 5x$. Find:
a. $(f \circ g)(x)$ b. $(g \circ f)(1)$ c. $f(f(-2))$ d. $g(g(-1))$
 $50x^2 - 4$ -10 28 -25
13. Let $f(x) = \frac{8}{x-3}$ and $g(x) = \frac{15}{x+1}$ Find:
a. $f(g(x))$ b. $g(f(x))$ c. $(f \circ f)(x)$ d. $(g \circ g)(x)$

$$\frac{8(x+1)}{-3x+12} \qquad \frac{15(x-3)}{x+5} \qquad \frac{8(x-3)}{-3x+17} \qquad \frac{15(x+1)}{x+16}$$

14. Use the functions in question 13 to find:

a. (f	• g)(2)	b.	$(g \circ f)(-5)$
4			undefined

15. Use the functions in question 13. Describe the domains for:

a.	$(f \circ g)(x)$	b. $(g \circ f)(x)$	c. $(f \circ f)(x)$	d.	$(g \circ g)(x)$
	$x \neq 4, -1$	$x \neq 3, -5$	$x \neq 3, \frac{17}{3}$		$x \neq -1, -16$

16. What makes the domain for each composition different? **Denominator cannot equal 0.**

Go

Topic: Writing equations of polynomials given the degree and the roots

Write the equation of the polynomial with the given features. Hint: Use the roots and leading coefficient to first write the function in factored form $f(x) = a(x - p_1)(x - p_2)(x - p_3)$...

	Degree of Polynomial	Given Roots (you may have to determine others):	Leading Coefficient	Equation in Standard Form:
17.	3	−2, 1, and −1	3	$3x^3 + 6x^2 - 3x - 6$
18.	4	(2 + <i>i</i>), 4, 0	1	$x^4 - 8x^3 + 21x^2 - 20x$
19.	5	1 multiplicity 2, —1 multiplicity 2, and 3	-1	$-x^5 + 3x^4 + 2x^3 - 6x^2 - x + 3$
20.	4	$(3-i),\sqrt{2}$	-2	$-2x^4 - 4x^2 - 24x + 40$

Ready

Topic: Using a table to find the value of a composite function

Use the table to find the indicated function values.

x	f(x)	g(x)
-2	2	3
-1	1	-2
0	3	-24
1	-1	-1
2	0	-8
3	19	0

Example: Find g(f(1)). f(1) = -1f(1) = -1g(-1) = -2Therefore, g(f(1)) = -2

- 1. f(g(3)) 2. f(g(1)) 3. g(f(-2)) 4. g(f(-1))

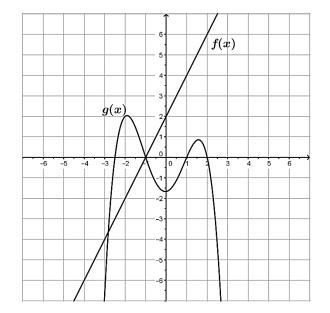
 3
 1
 -8
 -1

 5. g(f(0)) 6. g(g(-2)) 7. f(f(0)) 19
- 8. Do the graphs of f(x) and g(x) ever intersect each other? How do you know?

Yes because f(1) = g(1)

Use the graph at the right to find the indicated values.

9. f(g(-2))6 10. f(g(-1))11. f(g(0.5))0 6





Identify the type of conic section (parabola, circle, ellipse, or hyperbola). Then graph the conic section.

13.
$$\frac{(y-3)^2}{25} - \frac{(x+2)^2}{4} = 1$$

14. $(x-4)^2 + (y+5)^2 = 9$

-9 -8 -7 -6 -5 -4 -3 -2 -1

9 8 7

6.

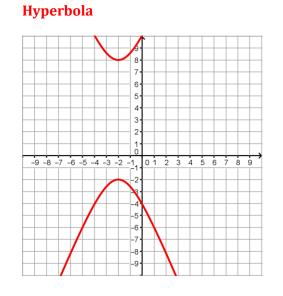
5 4 3

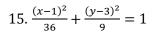
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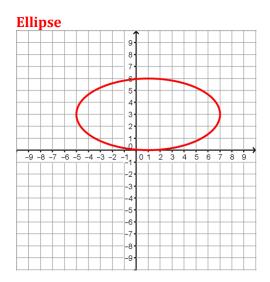
-2 -3 -4 -5 -6

-7 -8 0123456789

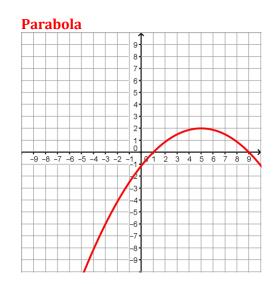
Circle







16. $(x-5)^2 = -8(y-2)$



Topic: Creating a composite function given its components

Let $f(x) = x^2$, g(x) = 5x, and $h(x) = \sqrt{x} + 2$. Express each function as a composite of f, g and/or h. 17. $A(x) = x^4$ 18. $C(x) = 5x^2$ 19. P(x) = x + 2 f(f(x)) g(f(x)) h(f(x))20. $R(x) = 5\sqrt{x} + 10$ 21. Q(x) = 25x22. $M(x) = 25x^2$ g(h(x)) g(g(x)) f(g(x))23. $P(x) = \sqrt{x^2 + 2} + 2$

23.
$$D(x) = \sqrt{\sqrt{x} + 2} + 2$$

 $h(h(x))$
24. $B(x) = x + 4\sqrt{x} + 4$
 $f(h(x))$
 $f(h(x))$
 $h(g(x))$

Go

Topic: Finding the zeros of a polynomial and rational equations

Solve for all of the values	of <i>x</i> . Identify any restrictions on <i>x</i> .	
26. $x^2 + 6 = 5x$	27. $5x^3 = 45x$	$28. x^4 - 26x^2 + 25 = 0$
x = 3.2	x = +3.0	x = +5, +1
$\lambda = 3, \Delta$	$\lambda = \pm 3, 0$	$\lambda = \pm 3, \pm 1$

$29.\frac{2x-5}{x-2} - 2 = \frac{3}{x+2}$	$30.\frac{x+5}{x+8} = 1 + \frac{6}{x+1}$	$31.\frac{5}{x^2+x-6}-1=\frac{1}{x-2}$
$x \neq 2, -2$ $x = 1$	$x \neq -8, -1$ $x = -\frac{17}{3}$	$x \neq -3, 2$ $x = -4$

Ready

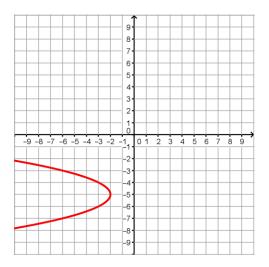
Topic: Identifying and graphing conic sections

Identify the type of conic section (parabola, circle, ellipse, or hyperbola). Then graph the conic section.

1.
$$(y+5)^2 = -(x+2)$$

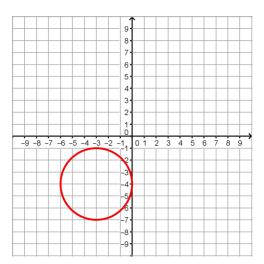
2.
$$\frac{(y+1)^2}{9} - (x-4)^2 = 1$$

Parabola

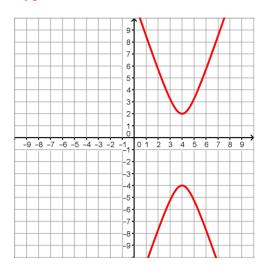


3. $(x+3)^2 + (y+4)^2 = 9$

Circle

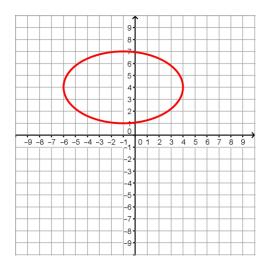


Hyperbola



4.
$$\frac{(x+1)^2}{25} + \frac{(y-4)^2}{9} = 1$$

Ellipse





Topic: Identifying the 2 functions that make up a composite function

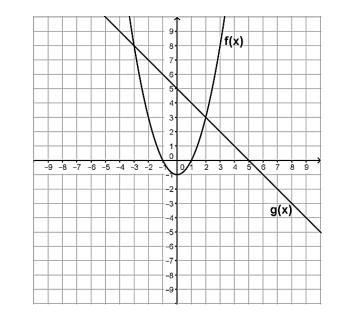
Find functions f and g so that $f \circ g = H$ 5. $H(x) = \sqrt{x^2 + 5x - 4}$ $f(x) = \sqrt{x}, g(x) = x^2 + 5x - 4$ 8. $H(x) = |5x^2 - 78|$ $f(x) = |x|, g(x) = 5x^2 - 78$ 10. $H(x) = (\tan x)^2$ $f(x) = \frac{2}{3-x^5}$ 10. $H(x) = (\tan x)^2$ $f(x) = \frac{2}{x}, g(x) = 3 - x^5$ 11. $H(x) = \tan(x^2)$ $f(x) = \tan x, g(x) = x^2$ 12. $H(x) = \sqrt{\frac{1}{6x}}$ $f(x) = \sqrt{x}, g(x) = \frac{1}{6x}$ 13. H(x) = 9(4x - 8) + 1f(x) = 9x + 1, g(x) = 4x - 8

0

Go

Topic: Finding function values given the graph

Use the graph to find all of the miss 14. $f() = 8$ 3, -3	sing values. 15. $g() = 5$ 0
16. $f() = -1$	17. $g() = 0$ 5
18. $f(-1) = $ 0	19. $g(0) = \frac{5}{5}$
20. $f(x) = g(x)$ (2, 3), (-3, 8)	21. $f(x) - g(x) =$ (2,3), (-3,8)
22. $f(x) \cdot g(x) = 0$ x = -1, 1, 5	23. $f(2) + g(2) = \frac{6}{6}$



24. f(0) - g(0) = -6