

Integration By U-Substitution

Academic Resource Center



Definition

 Integrals which are computed by change of variables is called U-substitution.

 In this we have to change the basic variable of an integrand (like 'x') to another variable (like 'u').

• This make the integral easy to determine.



Why U-Substitution

- It is one of the simplest integration technique.
- It can be used to make integration easier.
- It is used when an integral contains some function and its derivative, when

$$I = \int f(x) f^{1}(x)$$

Let u= f(x)
du=f'(x) dx



Why U-Substitution

Hence the integration becomes

$$I = \int u du$$
$$= \frac{u^2}{2} + c$$
$$= \frac{f^2(x)}{2} + c$$



When to use U-Substitution

- We have **function** and its **derivative** together.
- Where by use of simpler methods like POWER RULE, CONSTANT MULTIPLE RULE etc its difficult to solve integration.



When not to use U-Substitution

• If you fail to see such a pair of quantities, abandon this method.



Example 1

• Lets Compute the following integral

$$I = \int e^{x^2} (2x) dx$$

Put $u=x^2$

du=2xdx



Example 1(continued)

• the *indefinite* integral becomes

$$I = \int e^{u} du$$
$$= e^{u} + c$$
$$= e^{x^{2}} + c$$



Example 2

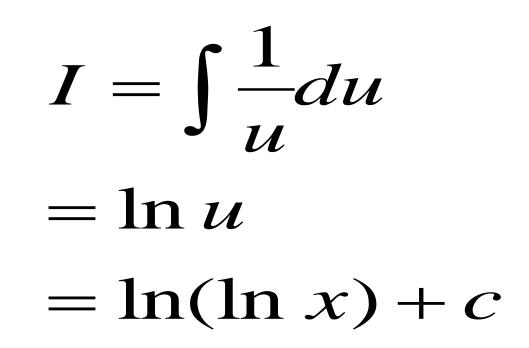
- Consider the Following Example
 1
 - $I = \int \frac{1}{x \ln x} dx$
 - Put lnx=u 1/x dx=du

Hence the given Integral becomes



Example 2(Continued)

• Hence the given Integral becomes





Example 3

• Consider the following Integral

$$I = \int \cos(2x) \sin(2x) dx$$



Example 3(Continued)

• Hence the given Integral becomes

$$I = \int \frac{1}{2} u du$$
$$= \frac{1}{2} \left(\frac{u^2}{2} \right) + c$$
$$= \frac{u^2}{4} + c$$
$$= \frac{\sin^2 2x}{4} + c$$



Definite Integral Using U-Substitution

- When evaluating a definite integral using usubstitution, one has to deal with the limits of integration.
- So by substitution, the limits of integration also change, giving us new Integral in new Variable as well as new limits in the same variable.
- The following example shows this.



Eample4 (Definite Integral)

• Consider the following Integral

$$I = \int_{0}^{1} x^{2} (x^{3} + 1)^{4} dx$$
$$Put \rightarrow x^{3} + 1 = u$$
$$3x^{2} dx = du$$
$$x^{2} dx = \frac{1}{3} du$$

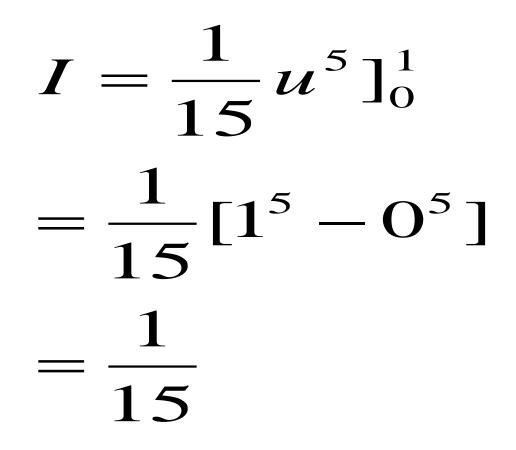


Example 4(Continued)

 Hence the definite integral becomes When x=0, u=0 When x=1,u=1 Hence = $\frac{1}{3}\int_{0}^{1}u^{4}du$ $=\frac{1}{3}\left[\frac{u^{5}}{5}\right]_{0}^{1}$



Example4(continued)





Practice Problems

$$1)I = \int \cos^{3}(x) \sin(x) dx$$

$$2)I = \int_{0}^{2} -2x\sqrt{4 - x^{2}} dx$$

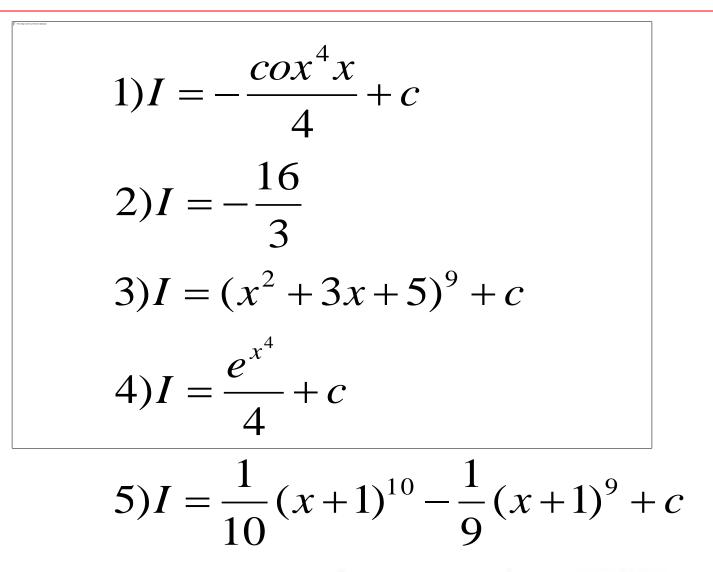
$$3)I = \int 9(x^{2} + 3x + 5)^{8}(2x + 3) dx$$

$$4)I = \int x^{3} e^{x^{4}} dx$$

$$5)I = \int x(x + 1)^{8} dx$$



Answer To Practice Problems





Important Tips for Practice Problem

- If you see a function and its derivative put function=u e.g. in question 1 put sinx=u and then solve.
- Same is the case with question 2 and 3.
- For question 2 Put **4**-**x**²=**u** and then solve.
- For question 3 Put x²+3x+5=u and then solve.
- For question 4 Put **x**⁴=**u** and then solve.
- For question 5 power rule fails because there is additional x.



Important Tips for Practice Problem

- So we can **reduce** the integral in such a way so that **power rule** works by using substitution.
- So in question 5 put x+1=u and then solve the integral in variable u by using simple power rule.