Predictive Modeling Using Logistic Regression

Step-by-Step Instructions

This document is accompanied by the following Excel Template

- IntegrityM Predictive Modeling Using Logistic Regression in Excel Template.xlsx

Two datasets are used to run predictive modeling based on prior information:

- Training dataset - This dataset includes both historical and current data with distinction of the outcomes – coded 1 for "Yes" and 0 for "No". Steps 1 to 7 use the dataset to assess the relative importance of indicators/variables to the outcome (Weights) and to determine the strength of their association with the outcome (P-values).
- Scoring dataset - new data on individuals/entities/IDs used to compute the probability of outcome.

Step 1:

Input your analytical file; in our example, we downloaded data from the Office of Inspector General List of Excluded Individuals and Entities (LEIE) and from the Centers for Medicare and Medicaid Services (CMS) Public Use File (PUF). Columns A to G of the spreadsheet "Predictive Model" display the data.

The analytical file includes columns A to G. Column A displays the provider identifier and columns B to G show summary information on the variables per provider inputted to the analysis, including the response variable informing if the provider was excluded or not (coded "1' or "0" respectively). The working dataset consists of information stored in cells B2 to G92.

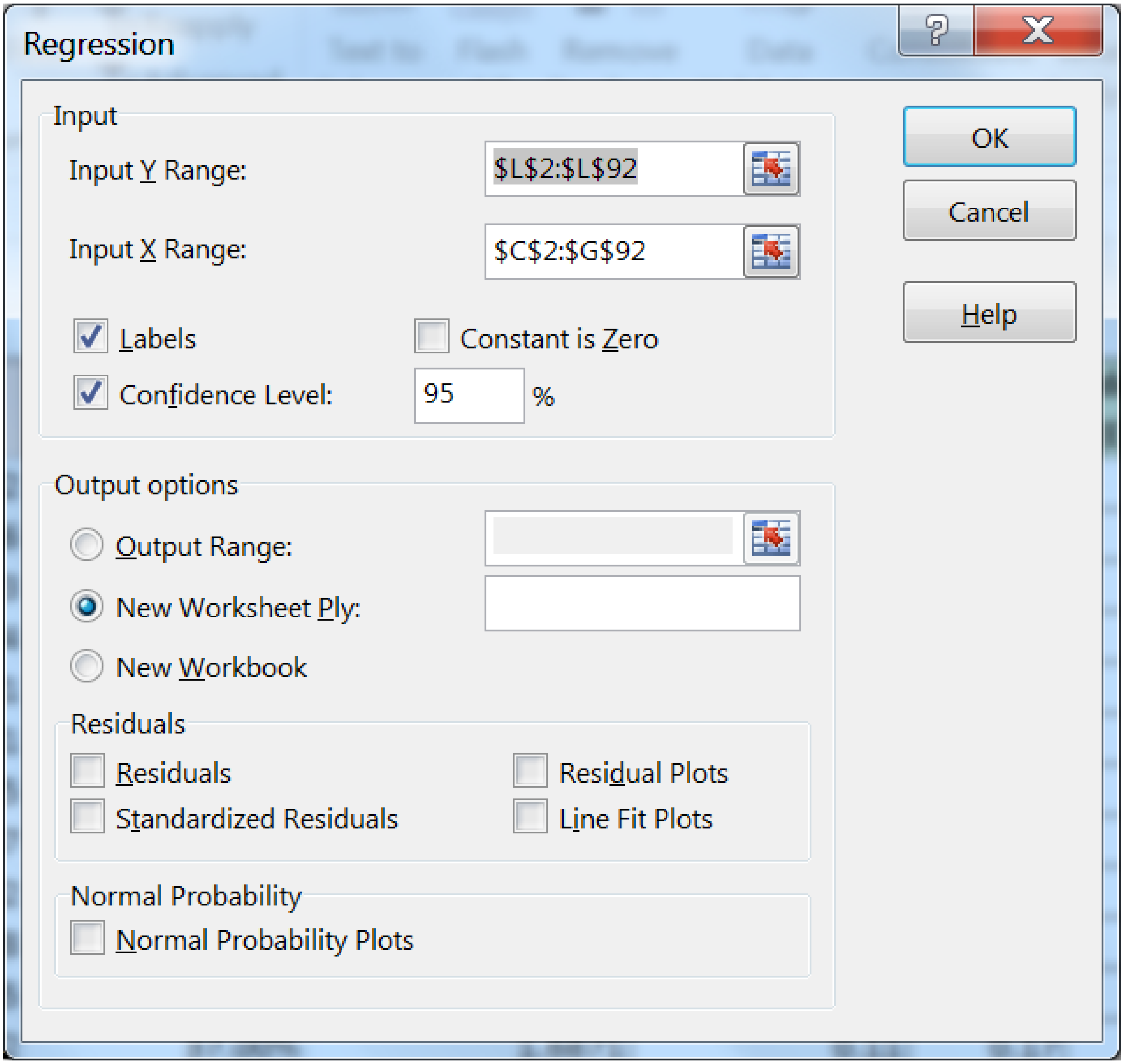
Steps 2 and 3:

Copy values from column H (RANDBETWEEN Values) to column I (P (E)) – this is necessary because of the volatile nature of the formula in column H, which will values changed every time the spreadsheet is refreshed. The rest of the columns in Step 3 will be computed automatically by the formulas embedded in them (Column J (Odds Ratio), Colum K (If Odds Ratio < 0.1 set it to 0.1) and Column L (Log (Odds Ratio))

- The Excel function RANDBETWEEN assigns random probability numbers to the providers according to their original classification ("1" or "0") - drawing from the range 0.50 to 0.99 for providers coded 1 (excluded) and from the range 0.01 to 0.49 for providers coded 0 (active in the health care program)
- Note that the RANDBETWEEN function requires as input values integers (numbers) that define *the bottom and the top values* from which a random number will be drawn. The values will be stored in column H, "RANDBETWEEN Values". The assigned values need to be transformed to probability numbers by dividing the function output number by 100 according to the provider exclusion status and the following Excel operation:
- for excluded providers, =RANDBERWEEN(50, 99)/100;
- for non-excluded (active) providers, =RANDBERWEEN(1, 49)/100;

<u>Step 4</u>:

- Step 4.1:
  - Run the Linear Regression Model by using the Data Analysis tool of Excel as shown in the screenshot below to obtain the Initial weights (coefficients) of the variables/indicators (in our example, 5 variables). The regression input Y Range (response variable) is the “Log(Odds)”, column L; and the five indicators in Input X Range are all values in “PMT per Bene”, “Services per Bene”, “Average Age of Beneficiaries”, “Beneficiary cc diab percent” and “Average HCC Risk Score of Beneficiary” – columns C to G. The Regression Model results will generate a new tab – labeled in our example “Step 4 - Reg Initial Values”.
- Step 4.2:
  - Copy the coefficients (weights) in column B from the regression model output to the Coefficients Table (in our example, the table includes cells T3 to T8 in column T of the spreadsheet “Predictive Model”).



<u>Step 5:</u>

Update the formula in Column N (variable labeled L) if you have more or less than the 5 variables that we have in our example. This formula includes the initial values of the weights from the Coefficients Table (Column T in our example) multiplied by the corresponding indicators (columns C to G in our example). Changes in the set of summation terms in the cases of 4 or 6 variables would be

- 4 variables, formula =T$3 + T$4*C4 + T$5*D4 + T$6*E4 + T$7*F4 ~~+ T$8*G4~~
- 6 variables, formula = T$3 + T$4*C4 + T$5*D4 + T$6*E4 + T$7*F4 + T$8*G4+T$9*H4

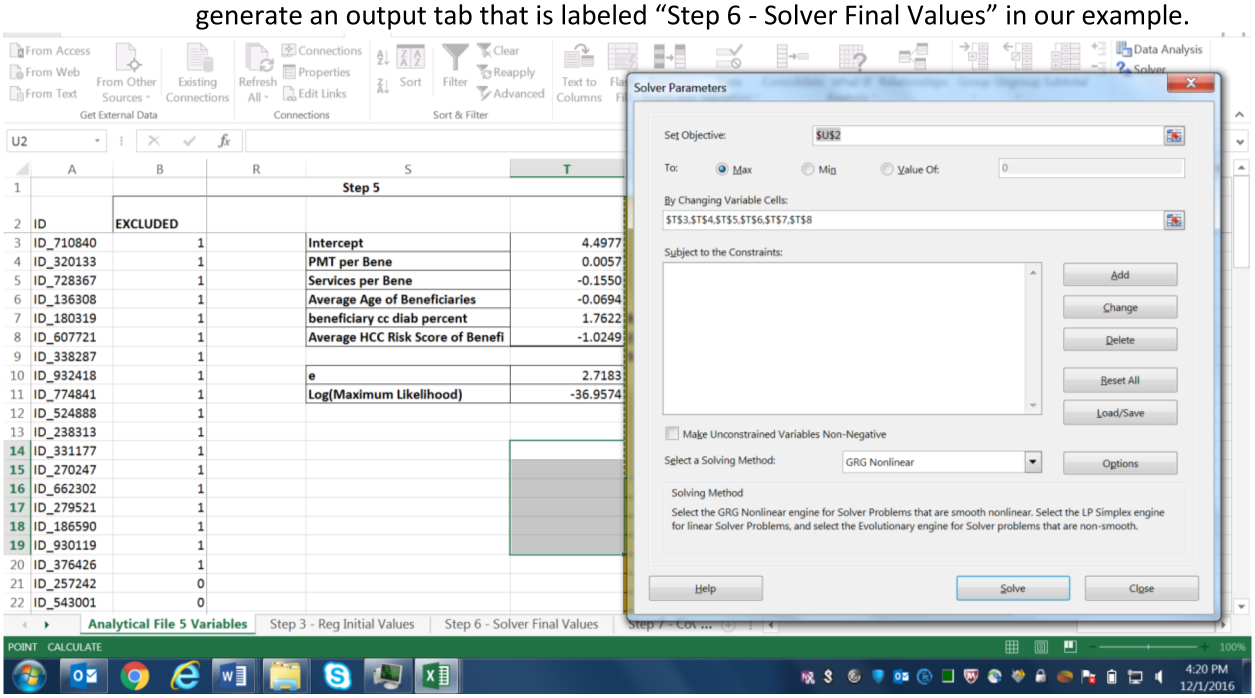
Note: the mathematical constant “e” (=2.7183 in cell T10 in our example) is required to compute the results in column O (labeled eL = "e to power L").

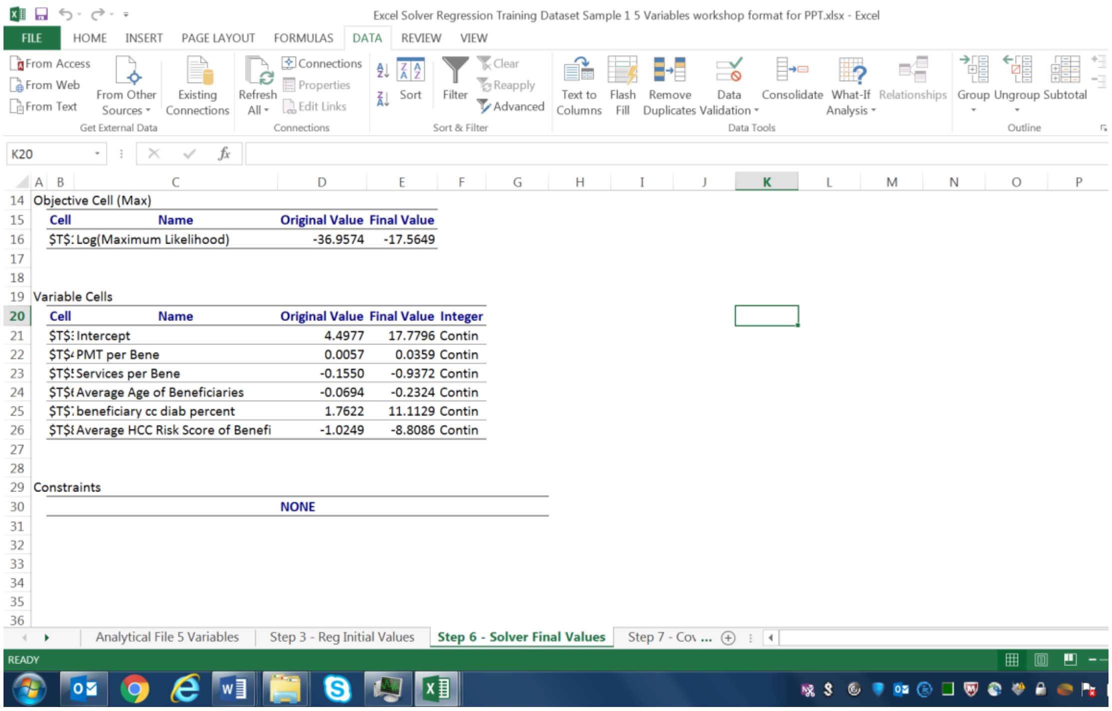
The values of column P (**P(X) = eL/(1 + eL)**) and column Q (**Y*ln[P(X)] + (1 – Y)*ln[1 – P(X)]**) will be updated automatically (where **Y = 1** for excluded providers and **Y = 0** for active providers) by the embedded formulas.

<u>Step 6</u>:

- Step 6.1:
  - Update the summation of the Log (Maximum Likelihood) if you have more rows than our example:
    - If you have 100 rows in your data rather than the 90 rows of our example, change the formula from =SUM(Q3:Q92) to =SUM(Q3:Q102)

- Step 6.2:
  - Run the Excel Solver tool to obtain the set of final weights that define the relative importance of the indicators/variables to the outcome. The Excel Solver will work on the set of initial weights (previously generated by the logistic regression) to update the Coefficients Table with a set of final weights that maximizes the likelihood of obtaining the data (outcome variable and indicators) actually observed. The Excel Solver will generate an output tab that is labeled “Step 6 - Solver Final Values” in our example.





Objective Cell (Max)

Cell	Name	Original Value	Final Value
T:	Log(Maximum Likelihood)	-36.9574	-17.5649

Variable Cells

Cell	Name	Original Value	Final Value	Integer
T:	Intercept	4.4977	17.7796	Contin
T‹	PMT per Bene	0.0057	0.0359	Contin
T!	Services per Bene	-0.1550	-0.9372	Contin
T(	Average Age of Beneficiaries	-0.0694	-0.2324	Contin
T:	beneficiary cc diab percent	1.7622	11.1129	Contin
T{	Average HCC Risk Score of Benefi	-1.0249	-8.8086	Contin

Constraints

NONE

Note: the signs of the regression coefficients indicate whether additional units of the associated variables increase (positive sign) or decrease (negative sign) the probability that the analyzed provider will reach the target outcome (joining the OIG Exclusion List in this example).

For example, the average payment per beneficiary (positively associated with the probability of exclusion) for excluded providers $342.52 as opposed to $214.26 for non-excluded (active) providers; on the other hand, the average risk score per beneficiary (negatively associated with the probability of exclusion) is 1.1632 for excluded providers in contrast with 1.7361 for non-excluded providers.

<u>Step 7:</u>

Steps 7.1 to 7.7 determine the strength of the association (p-values) of each Indicator/variable (explanatory variable) with the outcome variable (the probability of being excluded from the Medicare program in our example) by estimating the covariance matrix of the model.

- <u>Step 7.1:</u>
  - Construct an Excel table having a row per provider and a column per variable *without including the provider identifier*; add to the table a column of “1s” (intercept). In our example the table includes columns A to F of the spreadsheet “Step 7 - Covariance Matrix” (intercept plus five variables - “PMT per Bene”, “Services per Bene”, “Average Age of Beneficiaries”, “Beneficiary cc diab percent” and “Average HCC Risk Score of Beneficiary”).
- <u>Step 7.2</u>:

- Copy values from column P (labeled P(X) = eL/ (1 + eL)) from tab "Predictive Model" to column G (labeled P(X) = eL/ (1 + eL)) in tab "Step 7 - Covariance Matrix". Values in column H (labeled (1 - P(X))) and column I (labeled P(X)*(1 - P(X))) will be updated automatically by the embedded formulas.

Note: From steps 7.3 to 7.6, remember to highlight the destination cells (range of cells) BEFORE completing the formula with Shift-Control-Enter

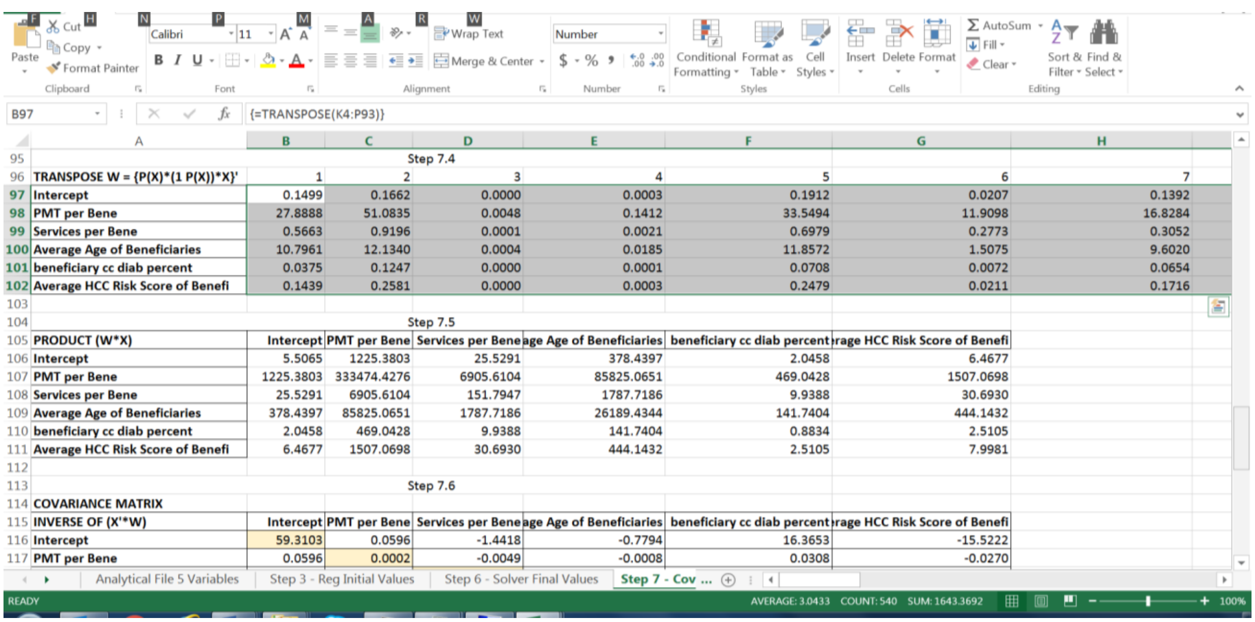
Note: The destination of the transpose operation needs to be highlighted (cells B97 to CM102 in our example) before the re-scaled table (cells K4 to P93) is transposed. Operations with matrices (tables) in Excel are explained in "Notes on Matrix Operations in Excel" by Ronald Larsen, http://www.eng.auburn.edu/~clemept/CEANALYSIS_SPRING2011/matrixoperations_notes.pdf

- Step 7.3:
  - Multiply every row of the table created in step 7.1 (cells A4 to G93 in our example) by the values in column I (P(X)*(1 - P(X))), to generate a re-scaled table stored in cells K4 to P93. This multiplication is accomplished in row 4 by the Excel function "=I4*(A4:F4)", which is copied and pasted to all rows from 5 to 93.

Note: Remember to highlight the destination cells (range of cells) BEFORE completing the formula with Shift-Control-Enter

- Step 7.4:
  - Transpose the re-scaled table stored in cells K4 to P93 using the Excel matrix function "TRANSPOSE(re-scaled table)" to generate the transposed table stored in cells B97 to CM102 in our example.

Note: Remember to highlight the destination cells (range of cells) BEFORE completing the formula with Shift-Control-Enter



Step 7.4

TRANSPOSE W = {P(X)*(1 P(X))*X}'	1	2	3	4	5	6	7
Intercept	0.1499	0.1662	0.0000	0.0003	0.1912	0.0207	0.1392
PMT per Bene	27.8888	51.0835	0.0048	0.1412	33.5494	11.9098	16.8284
Services per Bene	0.5663	0.9196	0.0001	0.0021	0.6979	0.2773	0.3052
Average Age of Beneficiaries	10.7961	12.1340	0.0004	0.0185	11.8572	1.5075	9.6020
beneficiary cc diab percent	0.0375	0.1247	0.0000	0.0001	0.0708	0.0072	0.0654
Average HCC Risk Score of Benefi	0.1439	0.2581	0.0000	0.0003	0.2479	0.0211	0.1716

Step 7.5

PRODUCT (W*X)	Intercept	PMT per Bene	Services per Bene	age Age of Beneficiaries	beneficiary cc diab percent	rage HCC Risk Score of Benefi
Intercept	5.5065	1225.3803	25.5291	378.4397	2.0458	6.4677
PMT per Bene	1225.3803	333474.4276	6905.6104	85825.0651	469.0428	1507.0698
Services per Bene	25.5291	6905.6104	151.7947	1787.7186	9.9388	30.6930
Average Age of Beneficiaries	378.4397	85825.0651	1787.7186	26189.4344	141.7404	444.1432
beneficiary cc diab percent	2.0458	469.0428	9.9388	141.7404	0.8834	2.5105
Average HCC Risk Score of Benefi	6.4677	1507.0698	30.6930	444.1432	2.5105	7.9981

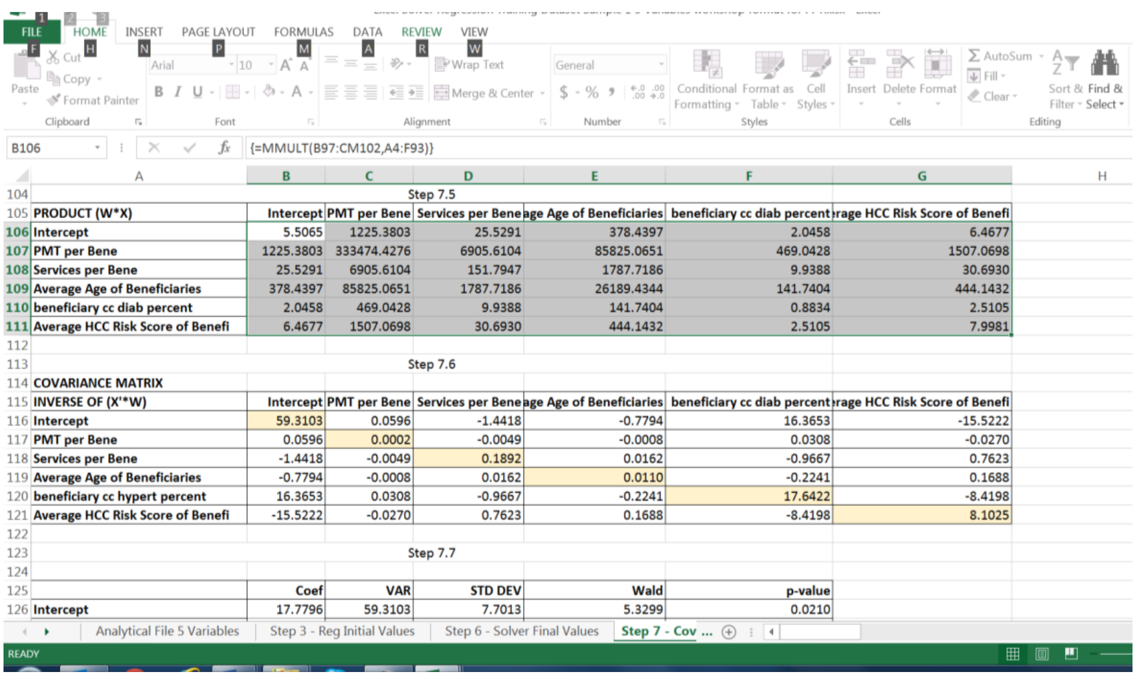
Step 7.6

COVARIANCE MATRIX

INVERSE OF (X'*W)	Intercept	PMT per Bene	Services per Bene	age Age of Beneficiaries	beneficiary cc diab percent	rage HCC Risk Score of Benefi
Intercept	59.3103	0.0596	-1.4418	-0.7794	16.3653	-15.5222
PMT per Bene	0.0596	0.0002	-0.0049	-0.0008	0.0308	-0.0270

- Step 7.5:
    - Multiply the transpose of the re-scaled table, computed in the previous step and stored in cells B97 to CM102, by the original (not re-scaled) table stored in cells A4 to F93 using the Excel matrix function "MMULT(transpose of the re-scaled table, original table)" to create a product table stored in cells B106 to G111.

Note: Remember to highlight the destination cells (range of cells) BEFORE completing the formula with Shift-Control-Enter.



	A	B	C	D	E	F	G
104				Step 7.5			
105	PRODUCT (W*X)	Intercept	PMT per Bene	Services per Bene	age Age of Beneficiaries	beneficiary cc diab percent	rage HCC Risk Score of Benefi
106	Intercept	5.5065	1225.3803	25.5291	378.4397	2.0458	6.4677
107	PMT per Bene	1225.3803	333474.4276	6905.6104	85825.0651	469.0428	1507.0698
108	Services per Bene	25.5291	6905.6104	151.7947	1787.7186	9.9388	30.6930
109	Average Age of Beneficiaries	378.4397	85825.0651	1787.7186	26189.4344	141.7404	444.1432
110	beneficiary cc diab percent	2.0458	469.0428	9.9388	141.7404	0.8834	2.5105
111	Average HCC Risk Score of Benefi	6.4677	1507.0698	30.6930	444.1432	2.5105	7.9981
112							
113				Step 7.6			
114	COVARIANCE MATRIX						
115	INVERSE OF (X'*W)	Intercept	PMT per Bene	Services per Bene	age Age of Beneficiaries	beneficiary cc diab percent	rage HCC Risk Score of Benefi
116	Intercept	59.3103	0.0596	-1.4418	-0.7794	16.3653	-15.5222
117	PMT per Bene	0.0596	0.0002	-0.0049	-0.0008	0.0308	-0.0270
118	Services per Bene	-1.4418	-0.0049	0.1892	0.0162	-0.9667	0.7623
119	Average Age of Beneficiaries	-0.7794	-0.0008	0.0162	0.0110	-0.2241	0.1688
120	beneficiary cc hypert percent	16.3653	0.0308	-0.9667	-0.2241	17.6422	-8.4198
121	Average HCC Risk Score of Benefi	-15.5222	-0.0270	0.7623	0.1688	-8.4198	8.1025
122							
123				Step 7.7			
124							
125		Coef	VAR	STD DEV	Wald	p-value	
126	Intercept	17.7796	59.3103	7.7013	5.3299	0.0210	

Step 7.6:

- Compute the inverse of the table created in the step 7.5 using the Excel matrix function "MINVERSE(product table)" to create the covariance matrix, which is stored in cells B116 to G121. The highlighted main diagonal values as shown below are the variances of the variables to be used in Step 7.7.

Note: Remember to highlight the destination cells (range of cells) BEFORE completing the formula with Shift-Control-Enter

B116 {=MINVERSE(B106:G111)}

	A	B	C	D	E	F	G
113				Step 7.6			
114	COVARIANCE MATRIX						
115	INVERSE OF (X'*W)	Intercept	PMT per Bene	Services per Bene	age Age of Beneficiaries	beneficiary cc diab percent	rage HCC Risk Score of Benefi
116	Intercept	59.3103	0.0596	-1.4418	-0.7794	16.3653	-15.5222
117	PMT per Bene	0.0596	0.0002	-0.0049	-0.0008	0.0308	-0.0270
118	Services per Bene	-1.4418	-0.0049	0.1892	0.0162	-0.9667	0.7623
119	Average Age of Beneficiaries	-0.7794	-0.0008	0.0162	0.0110	-0.2241	0.1688
120	beneficiary cc hypert percent	16.3653	0.0308	-0.9667	-0.2241	17.6422	-8.4198
121	Average HCC Risk Score of Benefi	-15.5222	-0.0270	0.7623	0.1688	-8.4198	8.1025
122							

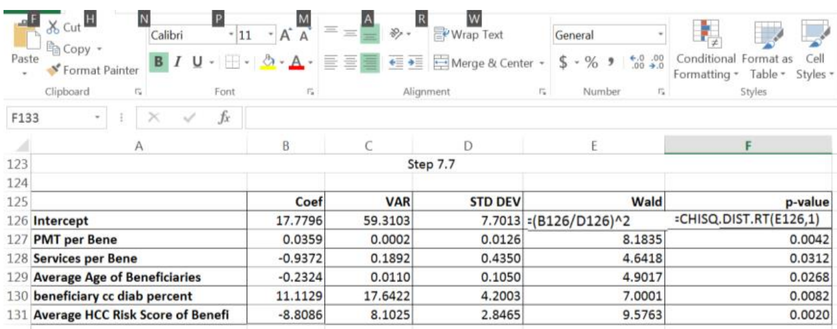
<u>Step 7.7:</u>

- Copy the coefficients (weights) computed by the Excel Solver from the cells T3 to T8 (in our example) from tab "Predictive Model" to the cells B126 to B131 of tab "Step 7 - Covariance Matrix". The final weights are also found in the Excel Solver output tab.
- Copy the variances of the variables from the main diagonal of the covariance matrix – cells B116, C117, D118, E119, F120 and G121 - to cells C126 to C131 in column C of tab "Step 7 - Covariance Matrix".

Note: the next steps will calculate new values automatically using the weights and variances inputted in the previous steps.

- The standard deviations are computed using the Excel function "SQRT(variance)" applied to values in cells C126 to C131 and store the results in cells D126 to D131 in our example.
- The Wald Chi-square statistic for each variable is computed as the squared value of the ratio (coefficient/standard deviation) and stored it in cells E126 to 131 under the label "Wald" in our example.
- The strength of the association "p-value" is computed using the Excel function "CHISQ.DIST.RT(Wald,1)", where the second input to the function - the number 1 - is the number of "degrees of freedom" in this case. The p-values are stored in cells F126 to F131.

Note: The P-value of each variable assesses the strength of the association between the variable and the outcome. The lower the p-value the stronger the association between variables. The standard rule is to consider p-values <= 5% (0.05) as indicative of statistically significant association. In our example, all variables were found to be strongly associated with the outcome.



	A	B	C	D	E	F
123			Step 7.7			
124						
125		Coef	VAR	STD DEV	Wald	p-value
126	Intercept	17.7796	59.3103	7.7013	=(B126/D126)^2	=CHISQ.DIST.RT(E126,1)
127	PMT per Bene	0.0359	0.0002	0.0126	8.1835	0.0042
128	Services per Bene	-0.9372	0.1892	0.4350	4.6418	0.0312
129	Average Age of Beneficiaries	-0.2324	0.0110	0.1050	4.9017	0.0268
130	beneficiary cc diab percent	11.1129	17.6422	4.2003	7.0001	0.0082
131	Average HCC Risk Score of Benefi	-8.8086	8.1025	2.8465	9.5763	0.0020

The screenshot below reproduces the results of the predictive model analysis using the same data but implemented with the statistical package SAS, which generates exactly the same results as the ones obtained using Excel.

SAS OUTPUT

Analysis of Maximum Likelihood Estimates

Parameter	DF	Estimate	Standard Error	Wald Chi-Square	Pr > ChiSq
Intercept	1	17.7796	7.7013	5.3299	0.0210
PMT_per_Bene	1	0.0359	0.0126	8.1835	0.0042
Services_per_Bene	1	-0.9372	0.4350	4.6418	0.0312
Average_Age_of_Benef	1	-0.2324	0.1050	4.9017	0.0268
beneficiary_cc_diab_	1	11.1129	4.2003	7.0001	0.0082
Average_HCC_Risk_Sco	1	-8.8086	2.8465	9.5763	0.0020

<u>Step 8:</u>

Steps 8 to 10 use a new dataset of active (unclassified as excluded) providers to score their probability of joining the OIG Exclusion List

- Step 8.1:
  - Input your Scoring dataset - our example used a sample of 100 non-excluded providers. The dataset included the 5 indicators and used the final weights to score their probability of being excluded. The final weights are inputted in columns K to P in our example.
- Step 8.2:
  - The formulas embedded in columns G to I will generate values based on the data inputted.

Note: the value of the mathematical constant "**e**" (= 2.7183, copied to the cell N3) is used to compute values in column G (labeled L).

The probabilities of exclusion P(X) of provider in row 2 are generated by using the formula "=H2/(1 + H2)", where the values of H2 are computed in the previous operation.