Interaction models – the checklist manifesto(s)

Papers discussed:
Hainmueller, Mummolo and Xu, 2016
Brambor, Clark and Golder, 2006

Jason Windawi

Interactions in linear models

A way of measuring the conditional effect of context on the relationship between a focal independent variable and an outcome

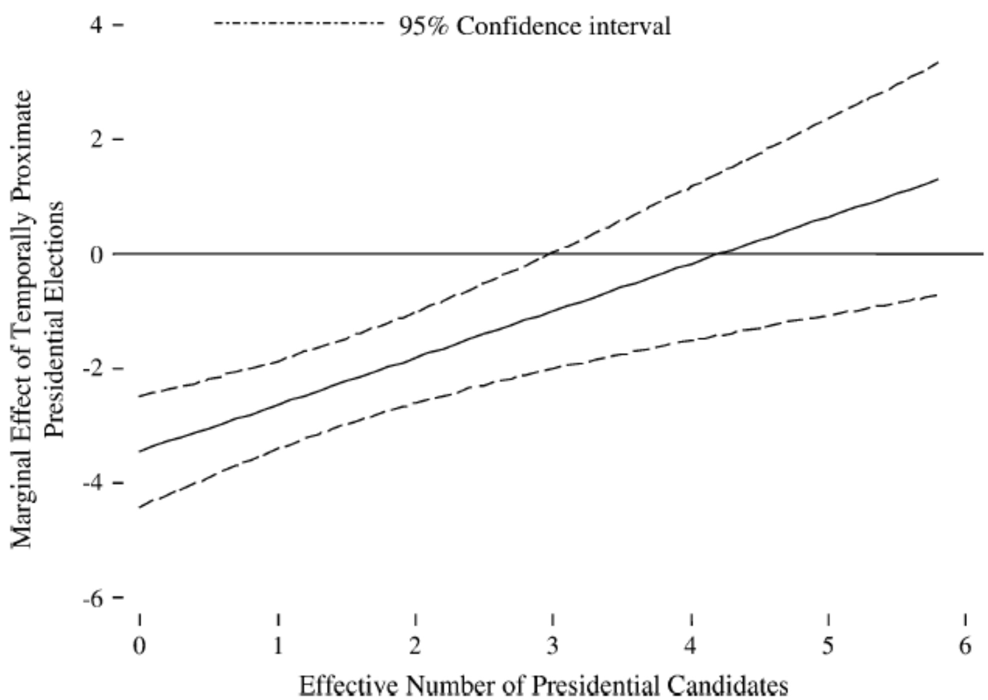
- *How does the effect of treatment D on outcome Y vary given moderator X?*

$$Y = \mu + \alpha D + \eta X + \beta (D \cdot X) + \epsilon$$

“State of the art” – Brambor, Clark and Golder (2006)

A **checklist** for empirical analysis using linear interaction models:

1. Include all constitutive terms
2. Don’t interpret constitutive terms/coefficients as unconditional marginal effects
3. Calculate <u>and plot</u> substantively meaningful marginal effects and standard errors



Sociology?

- Breznau (2015) vs. Brooks & Manza (2006)

Table 4. Models of Overall Welfare State Effort

	Model 1		Model 2	
Independent Variables	Coefficient	Standard Error	Coefficient	Standard Error
Constant	16.42*	(7.73)	2.58	(8.59)
Year	.36*	(.12)	.36*	(.08)
Per Capita GDP	–1.04*	(.18)	–.66*	(.20)
Unemployment	.18	(.23)	.55*	(.24)
Aged Population	.50	(.35)	.31	(.33)
Women's LFP	.24*	(.09)	.30*	(.08)
Political Institutions	1.84*	(.48)	.77	(.59)
Religious Party Control	—	—	.08*	(.02)
Left Party Control	—	—	.02	(.02)
Social Policy Preferences	3.70*	(.90)	2.65*	(.69)
Social Policy Prefs × Liberal Democracy	–2.35*	(.92)	–1.77*	(.71)
R^2	.78		.86	

Note: Entries are unstandardized coefficients (robust-cluster standard errors in parentheses). N = 43.
* $p < .05$ (two-tailed tests).

Liberal Democracy?

Hainmueller et al. (2016)

Two problems with the literature post-Brambor:

1. Failure to meet assumptions of a linear interaction effect (LIE)

$$Y = \mu + \alpha D + \eta X + \beta(D \cdot X) + \epsilon$$

$$ME_D = \frac{\partial Y}{\partial D} = \alpha + \beta X$$

2. Potential lack of common support (for both treatment D and moderator X) necessary

New checklist!

Hainmueller et al. recommend adding the following diagnostics to the Brambor checklist:

1. Scatterplots
2. Binning estimator
3. Kernel estimator

Simulated data

$$Y_i = 5 - 4X_i - 9D_i + 3D_iX_i + \epsilon_i, \qquad i = 1, 2, \cdots, 200.$$

$$X_i \overset{\text{i.i.d.}}{\sim} \mathcal{N}(3, 1)$$

$$\epsilon_i \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, 4)$$

$$D_i \overset{\text{i.i.d.}}{\sim} Bernoulli(0.5), \qquad D_i \overset{\text{i.i.d.}}{\sim} \mathcal{N}(3, 1)$$

$$ME_D = -9 + 3X_i$$

Diagnostic 1: Binary Treatment (D)

SEPARATION/HETEROGENEITY

- Divide data into cases by <u>treatment</u>

LINEARITY

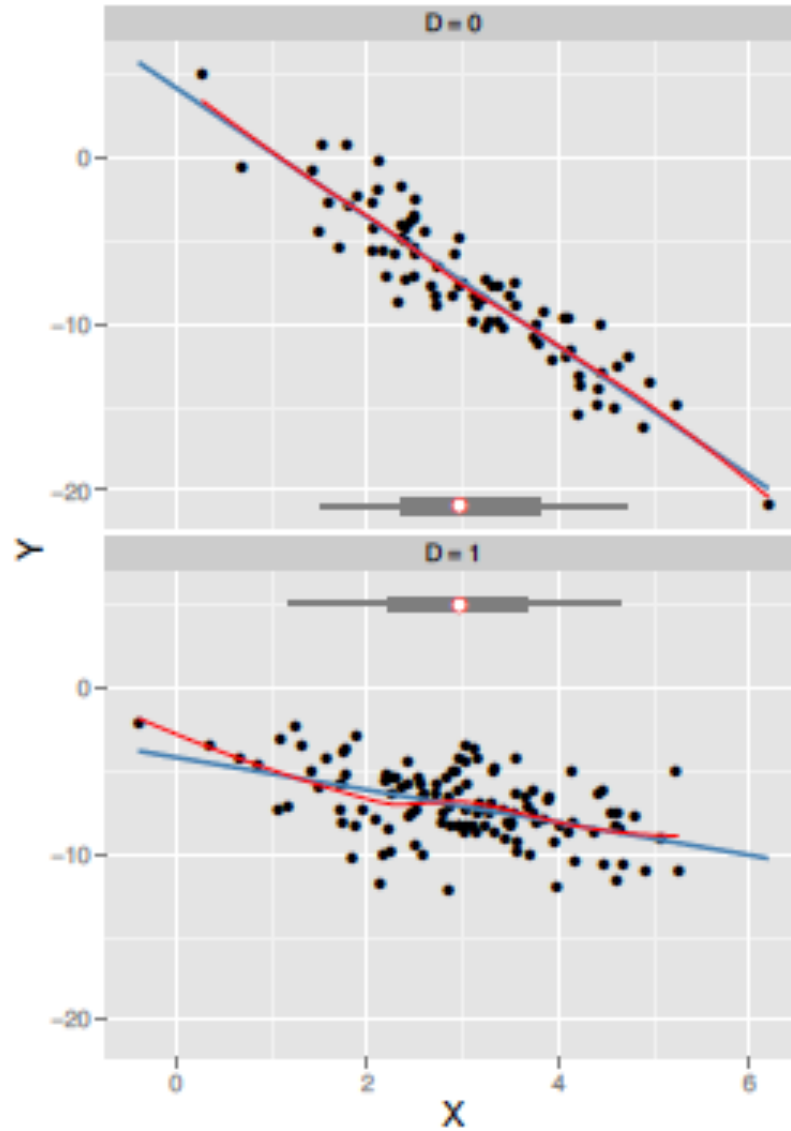
- Does the distribution of results indicate a linear relationship?

  LOESS (red) vs. regression (blue)

SUPPORT

- Is there sufficient common support?

  Box plot of distribution of X



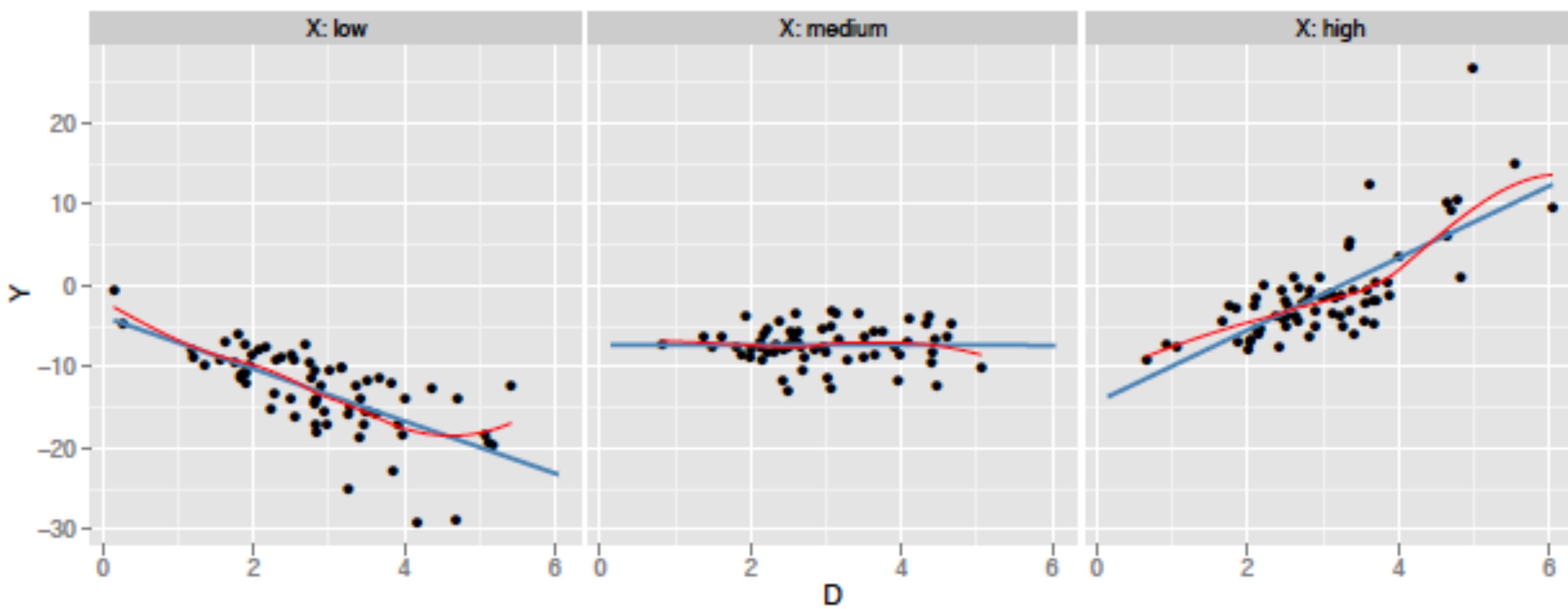
Diagnostic 1: Continuous Treatment (D)

SEPARATION/HETEROGENEITY

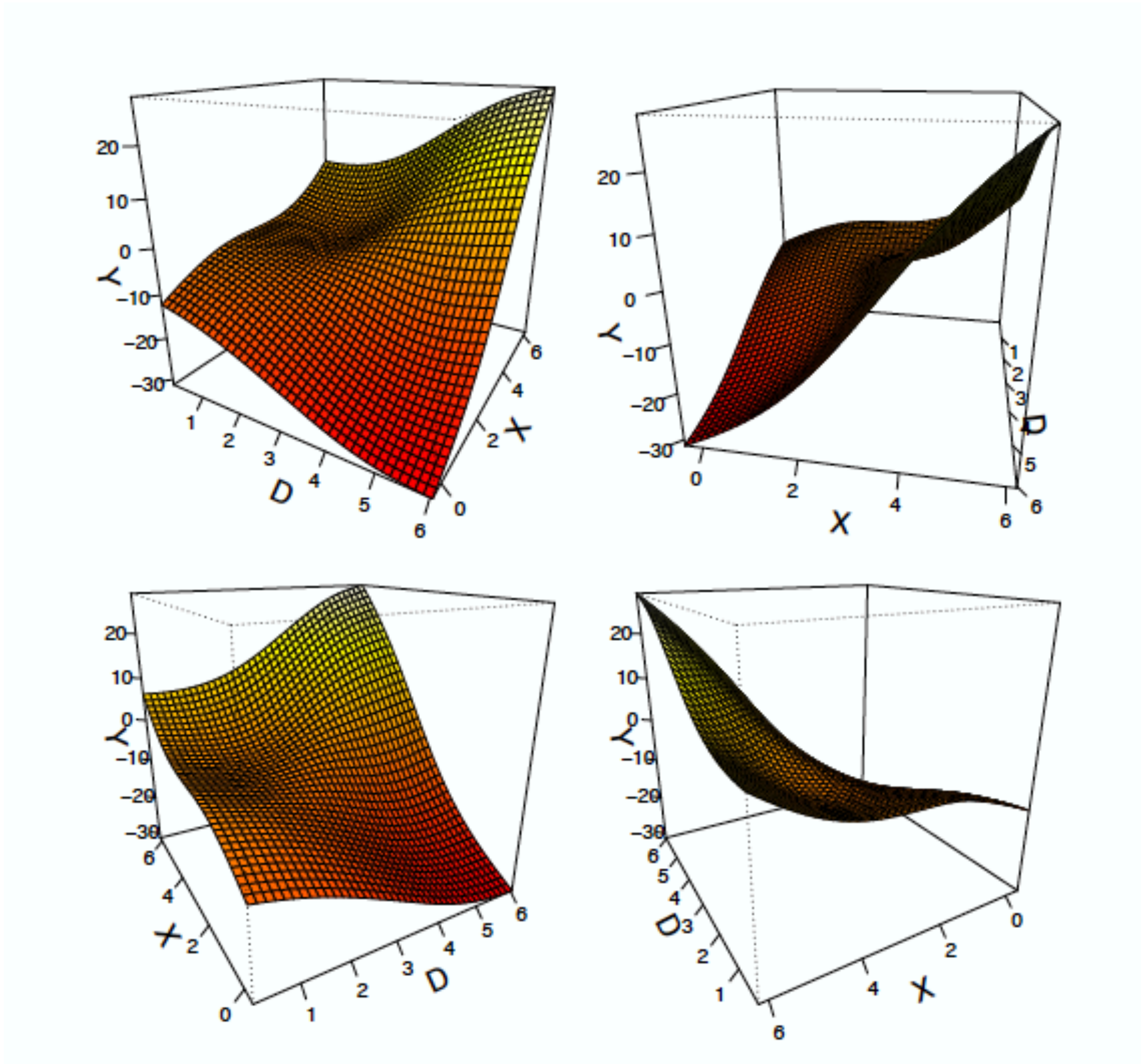
- Divide data into three bins by <u>moderator</u>

LINEARITY?

SUPPORT?



Alternative Diagnostic 1: Generalized Additive Model



Diagnostic 2: Binning Estimator

1. Separate continuous moderator X into bins (recommend 3)

$$G_1 = \begin{cases} 1 & X < \delta_{1/3} \\ 0 & otherwise \end{cases}, \quad G_2 = \begin{cases} 1 & X \in [\delta_{1/3}, \delta_{2/3}) \\ 0 & otherwise \end{cases}, \quad G_3 = \begin{cases} 1 & X \geq \delta_{2/3} \\ 0 & otherwise \end{cases}$$

2. Establish evaluation points x_j , $j = 1, 2, 3$

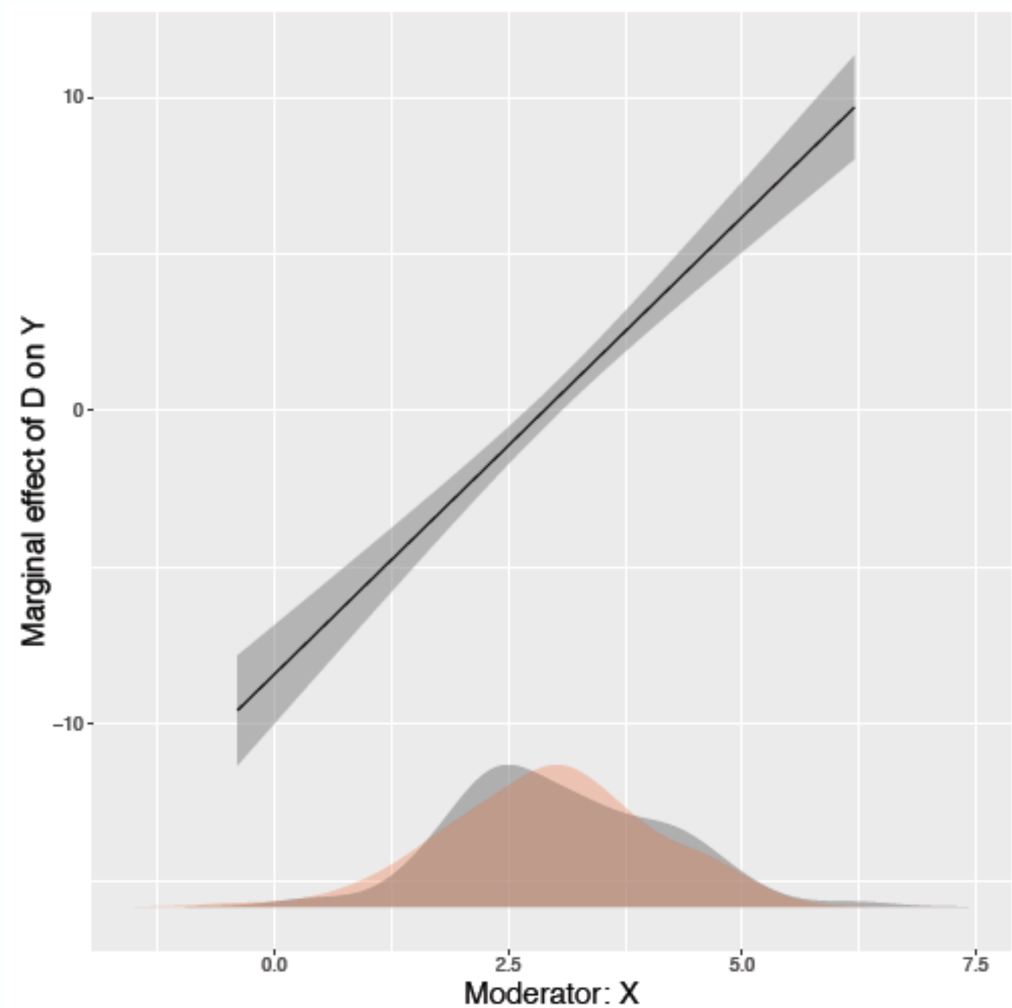
3. Estimate coefficients using evaluation points

$$Y = \sum_{j=1}^{3} \left\{ \mu_j + \alpha_j D_i + \eta_j (X - x_j) + \beta_j (X - x_j) D \right\} G_j + \gamma Z + \epsilon$$

4. Plot all the things (new checklist)

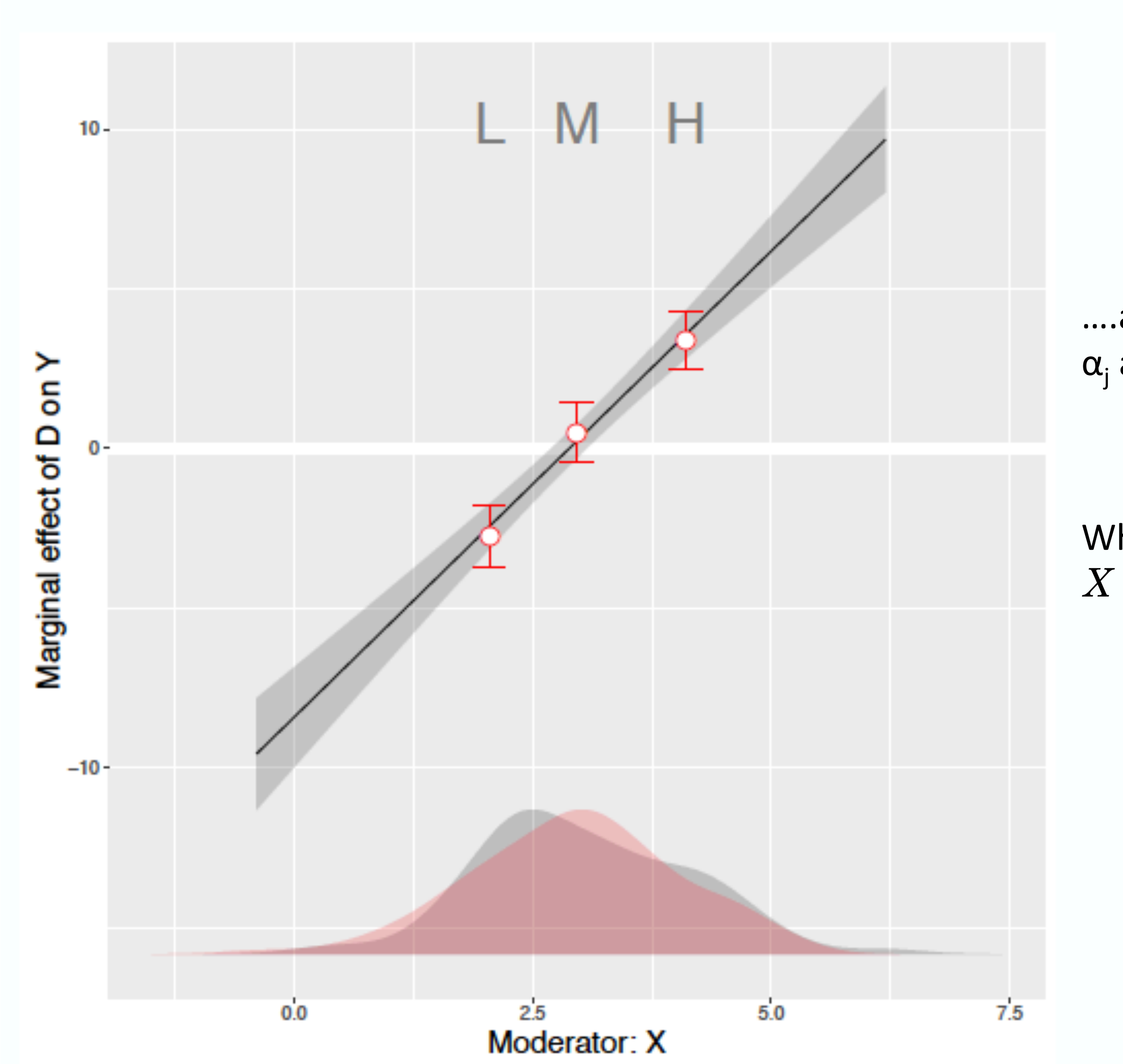
Diagnostic 2: Plotting the Binning Estimator

Start with output from standard linear interaction model per Brambor et al…



….add graphic to show support for X

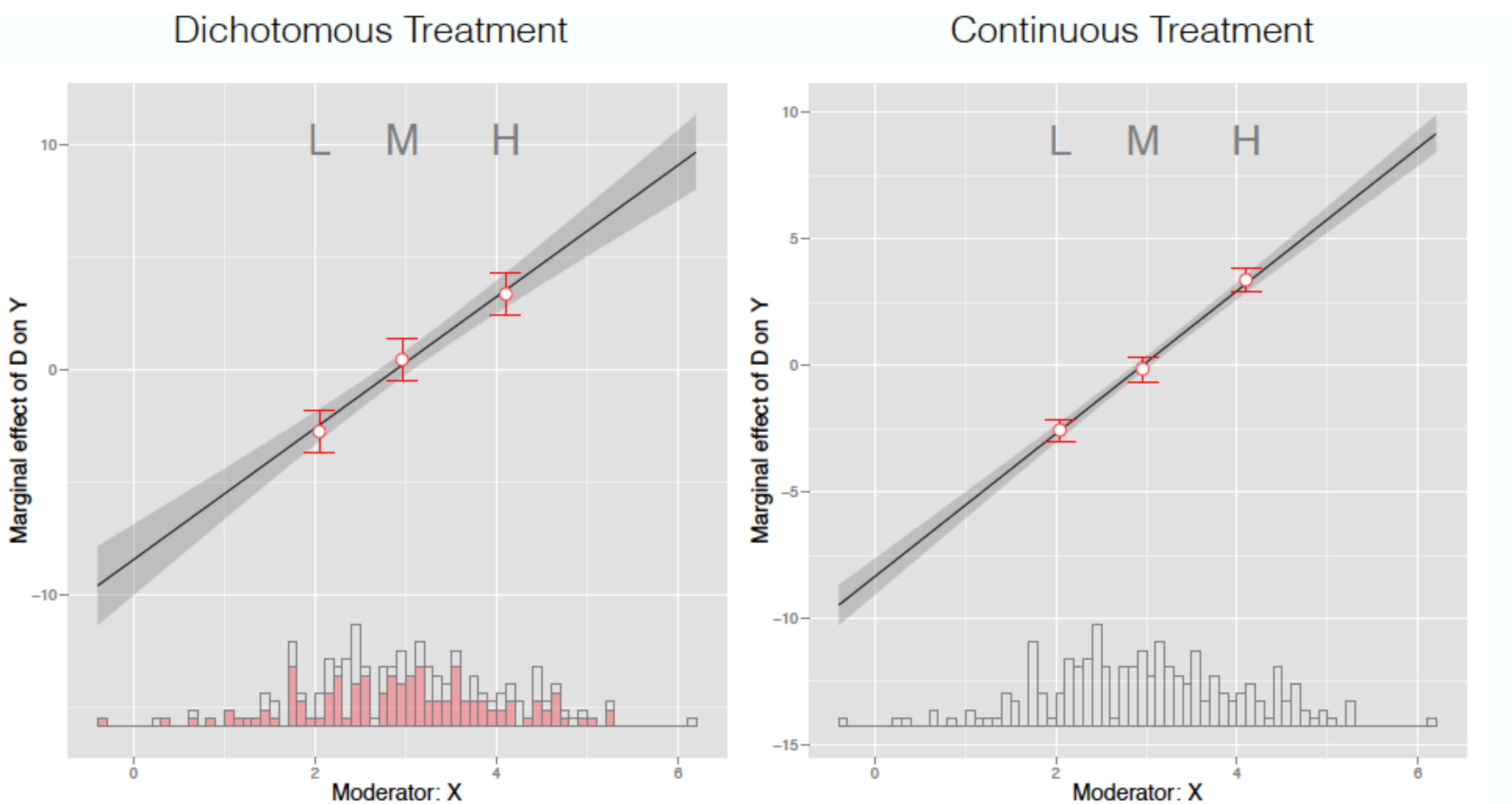
Diagnostic 2: Plotting the Binning Estimator



....add estimates of α_j at x_j with 95% CIs

Why? For bin estimator:
$X = x_j \Rightarrow ME(x_j) = \alpha_j$

Diagnostic 2: Simulation results



Diagnostic 3: Kernel estimator

- Draws on Li and Racine (2010)'s semi-parametric, variable-coefficient model
  - Designed to accommodate both dichotomous and continuous variables
  - Designed to capture variation in coefficient(s) of interest while address shortcomings of approaches relying on separation/binning

Diagnostic 3: Kernel estimator

Assumed Model

$$Y = f(X) + g(X)D + \gamma(X)Z + \epsilon$$

Kernel

$$K\left(\frac{X_i - x_0}{h}\right)$$

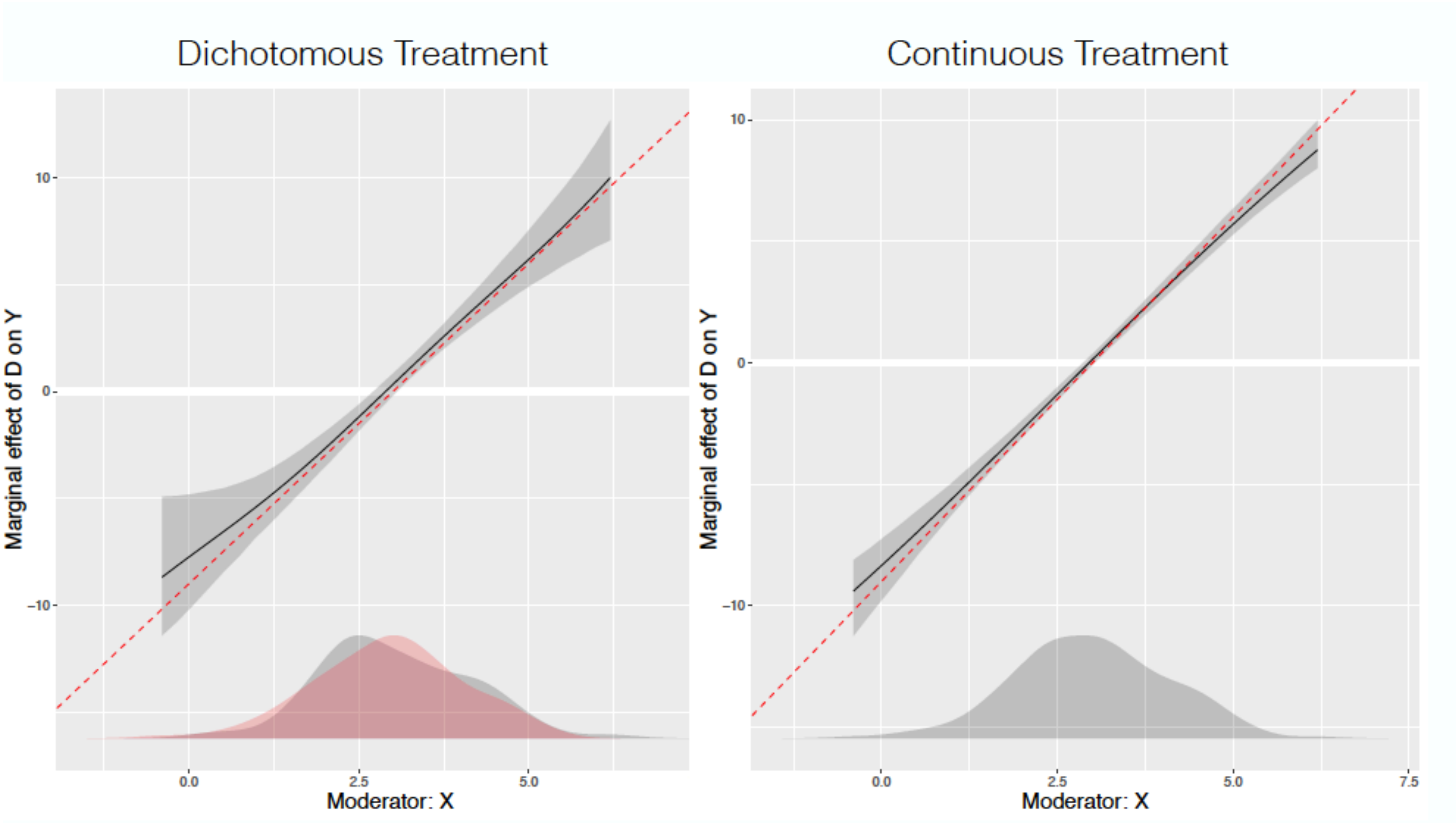
$$L = \sum_i^N \left\{ \left[Y_i - \tilde{\mu} - \tilde{\alpha} D_i - \tilde{\eta}(X_i - x_0) - \tilde{\beta} D_i (X_i - x_0) - \tilde{\gamma} Z_i \right]^2 K\left(\frac{X_i - x_0}{h}\right) \right\}$$

$$\hat{g}(x_0) = \hat{a}D - \hat{\beta}D(0) = \hat{a}D = \hat{a}(x_0)$$

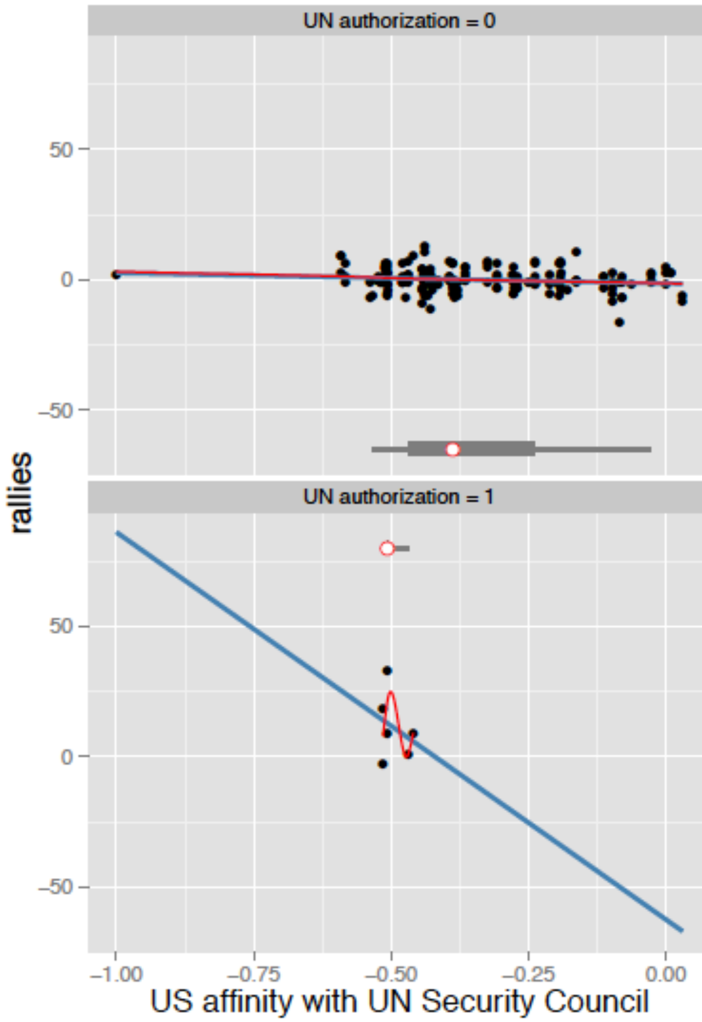
$$\Rightarrow \frac{\partial Y}{\partial D}(x_0) = \hat{a}$$

Diagnostic 3: Kernel estimator

Graphing $\hat{a}(x_0)$ across support of X

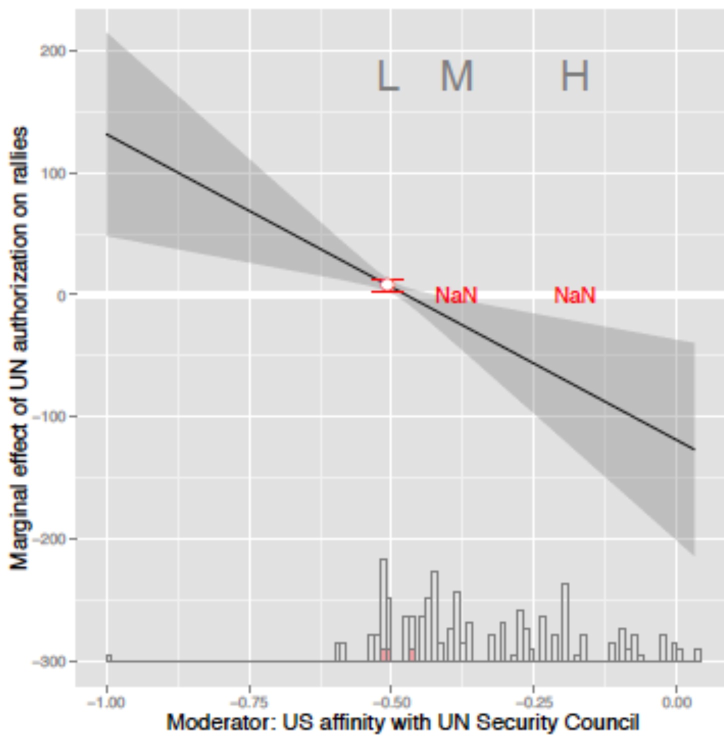


Problem replication: lack of common support*

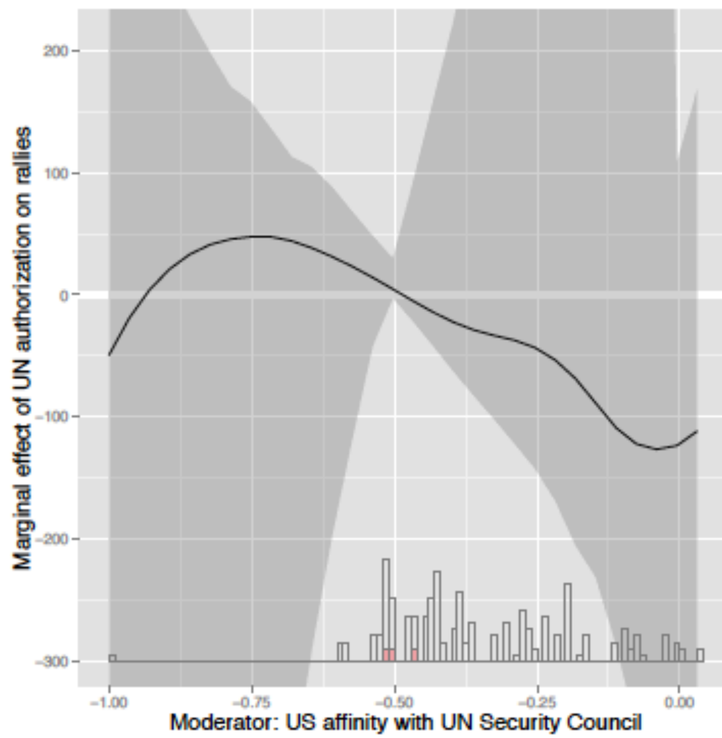


** Chapman (2009)*

Problem replication: lack of common support*



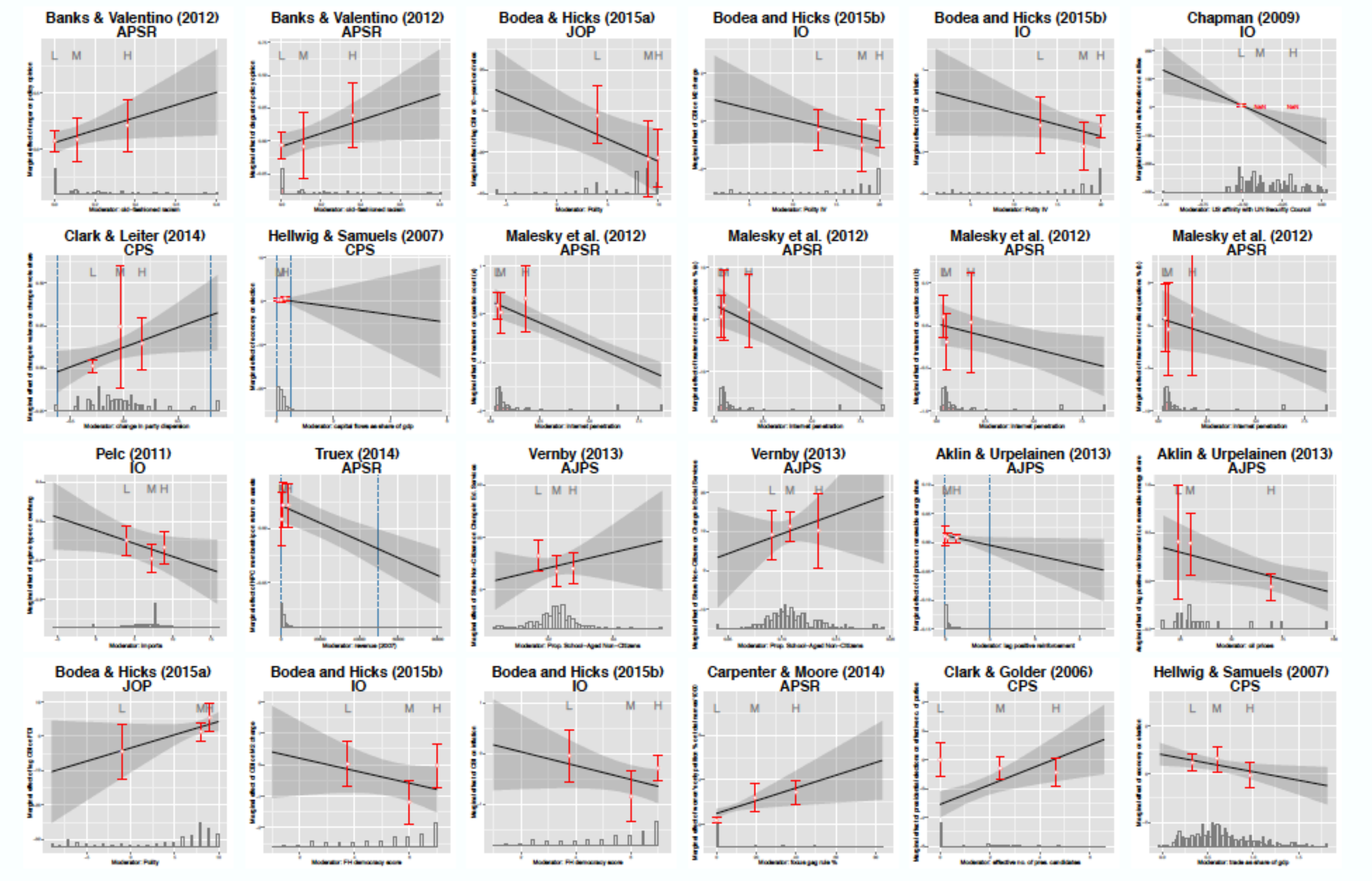
(b) Marginal Effects from Replicated Model (black line) and from Binning Estimator (red dots)



(c) Marginal Effects from Kernel Estimator

** Chapman (2009)*

Widespread problems



How widespread? A scoring system

Four possible points, one each for:

- Reject equality of marginal effects (α_j) for low and high bins
- No severe interpolation or extrapolation (includes L-kurtosis hurdle)
- Monotonic
- Fail to reject linear model in Wald test vs. binned

Scoring results

Score:	4	3	2	1
Number	4	5	10	17
Share	8.7%	10.9%	21.7%	37%

Sample: 55 replications from 22 papers in leading Politics journals