



Interactions of Ionizing Radiation with Matter

Reading Material:

Chapter 2 in Radiation Detection and Measurements,
Third Edition, G. F. Knoll, John Wiley & Sons, 1999.



Interactions of X and Gamma Rays

X-ray and Gamma Ray Interactions

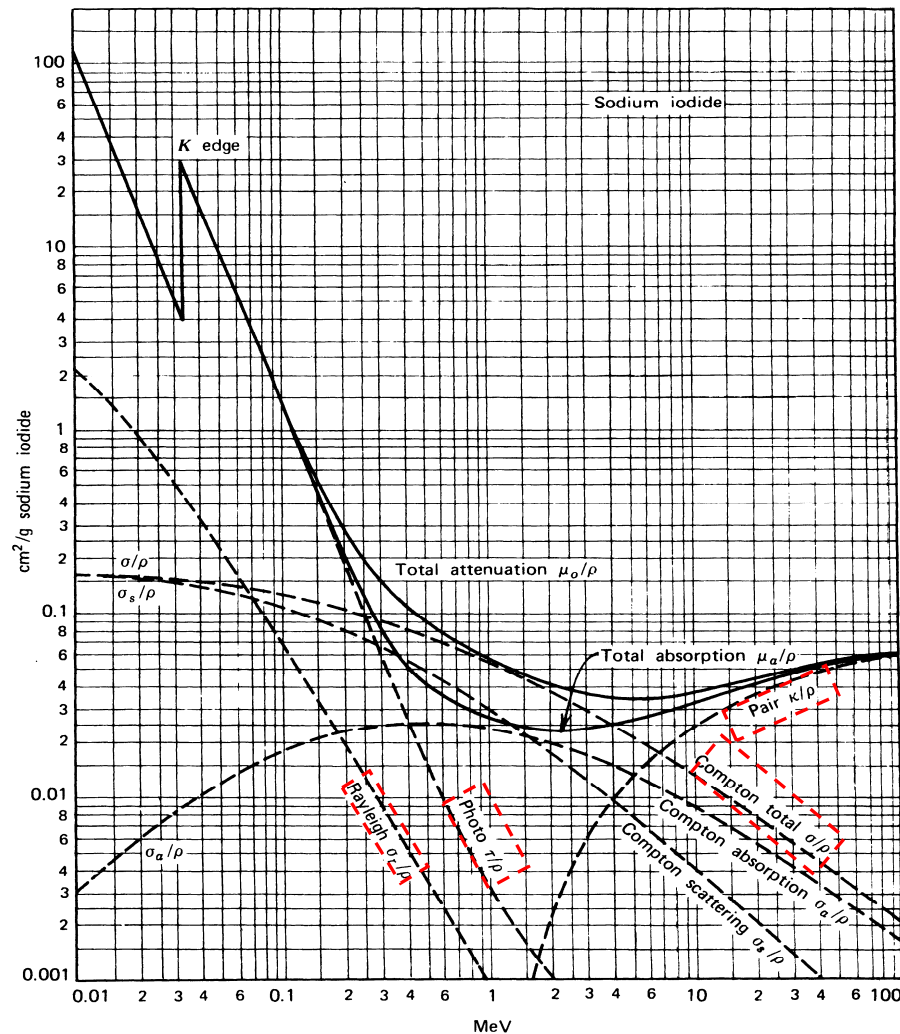
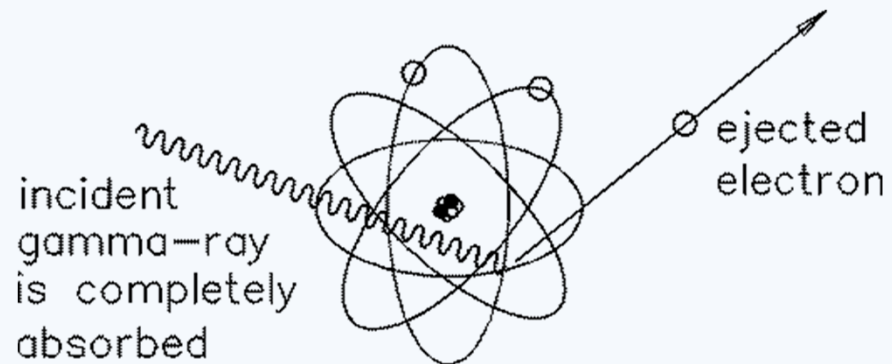
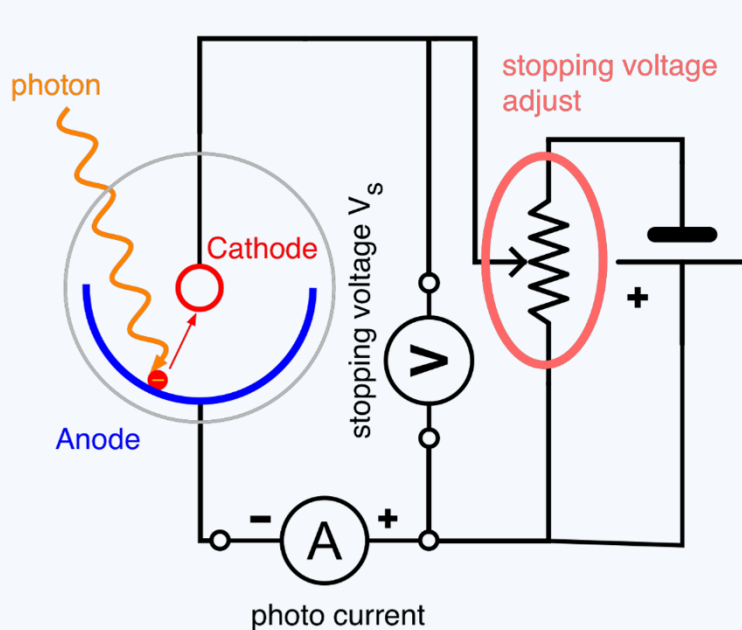


Figure 2.18 Energy dependence of the various gamma-ray interaction processes in sodium iodide. (From *The Atomic Nucleus* by R. D. Evans. Copyright 1955 by the McGraw-Hill Book Company. Used with permission.)

From Page 50, Radiation Detection and Measurements, Third Edition, G. F. Knoll, John Wiley & Sons, 1999.

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Photoelectric Effect



$$E_{e^-} = h\nu - E_b$$

h is the Planck's constant

ν is the frequency of the photon

- Photoelectric interaction is with the atom in a whole and can not take place with free electrons.
- Photoelectric effect leaves a vacancy in one of the electron shells, which leaves the atom at an excited state.

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Photoelectric Effect Cross Section

Probability of photoelectric absorption per atom is

$$\tau \propto \begin{cases} \frac{Z^4}{(h\nu)^{3.5}} & \text{low energy} \\ \frac{Z^5}{(h\nu)^{3.5}} & \text{high energy} \end{cases}$$

- The interaction cross section depends strongly on Z.
- Photoelectric effect is favored at lower photon energies.

Photoelectric Effect (2) – Absorption Edges

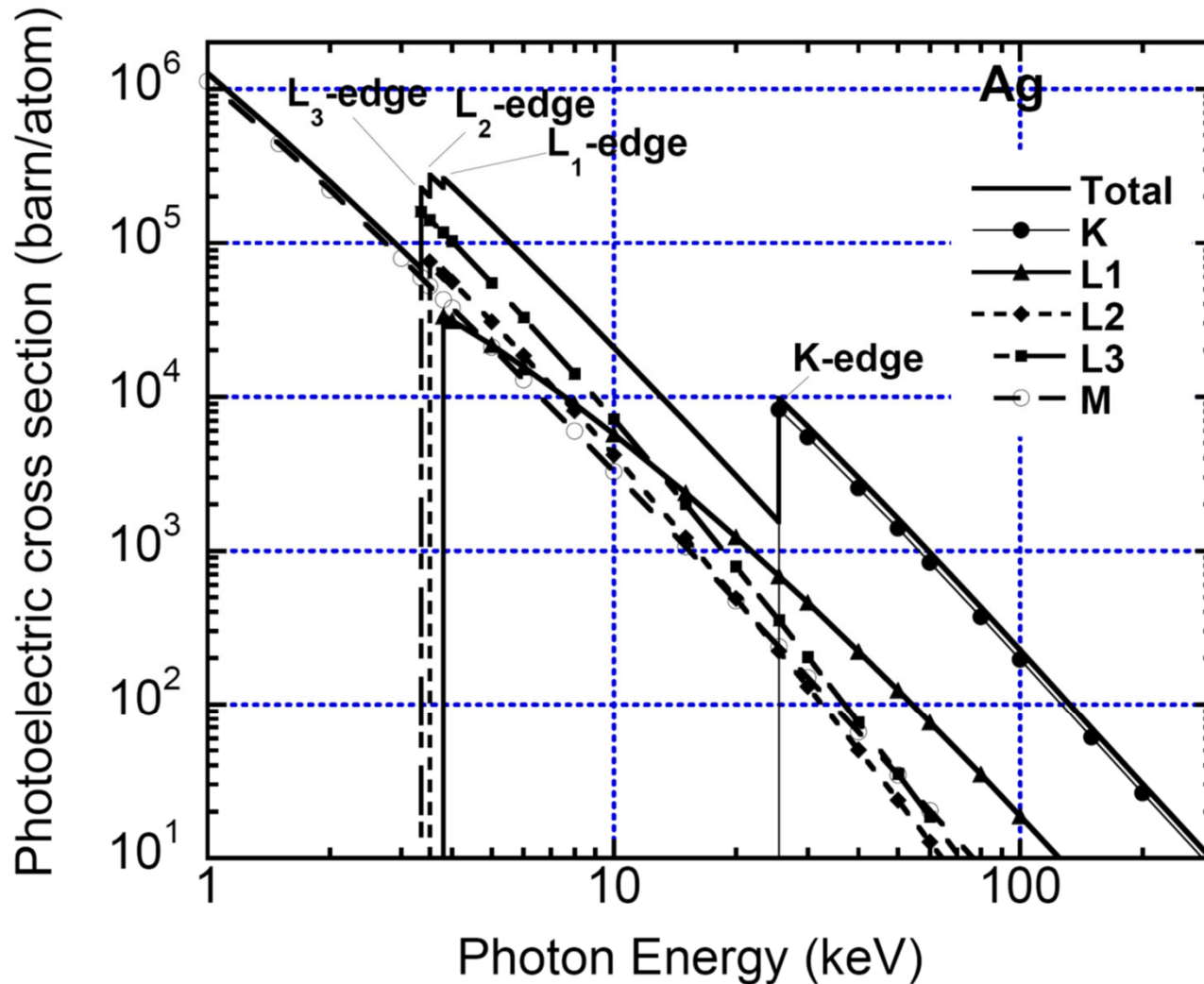
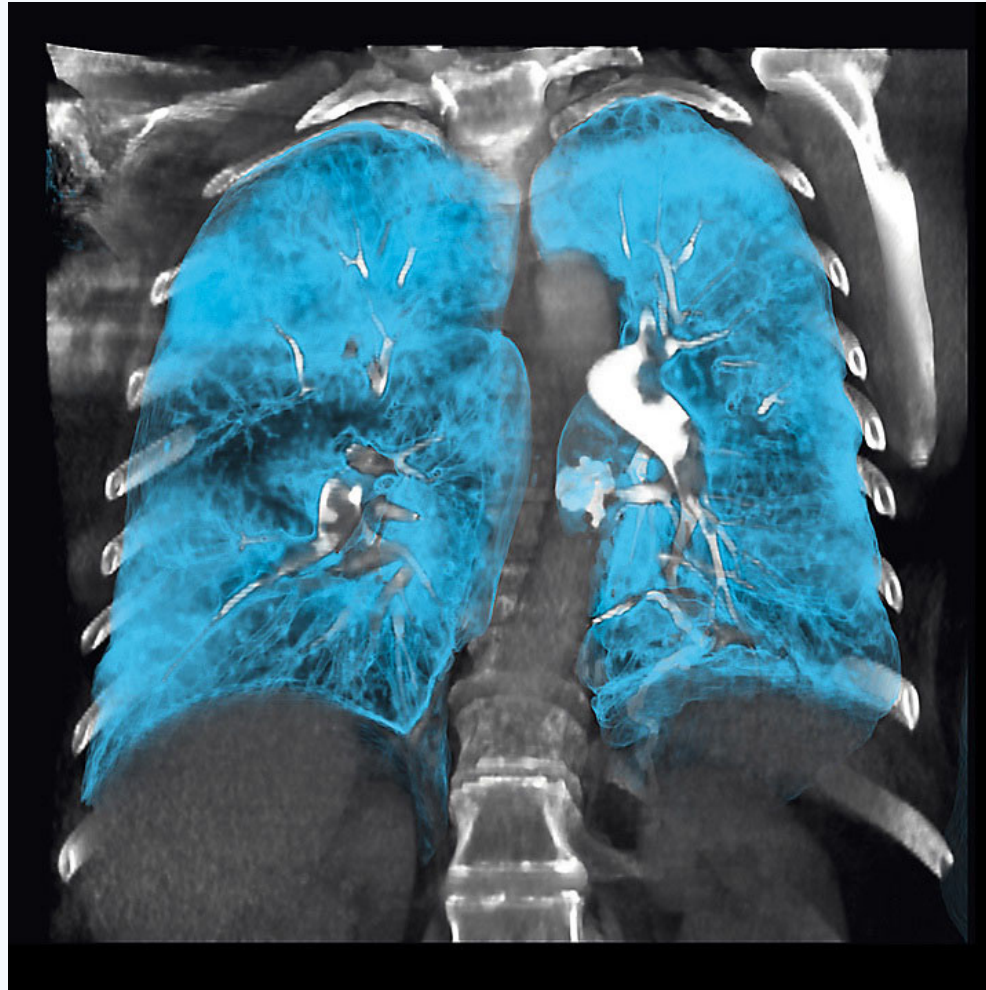


Figure 2: Total and partial atomic photoeffect of Ag.

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Photoelectric Effect (2) – Absorption Edges



Contrast enhanced CT image of the lung

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Perspective

Functional CT imaging techniques for the assessment of angiogenesis in lung cancer

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Corresponding to: Dr. Jingyun Shi, Department of Radiology, Shanghai Pulmonary Hospital, Tongji University School of Medicine, 507 Zheng Min Road, Shanghai, 200433, China. Email: shijingyun89179@126.com.

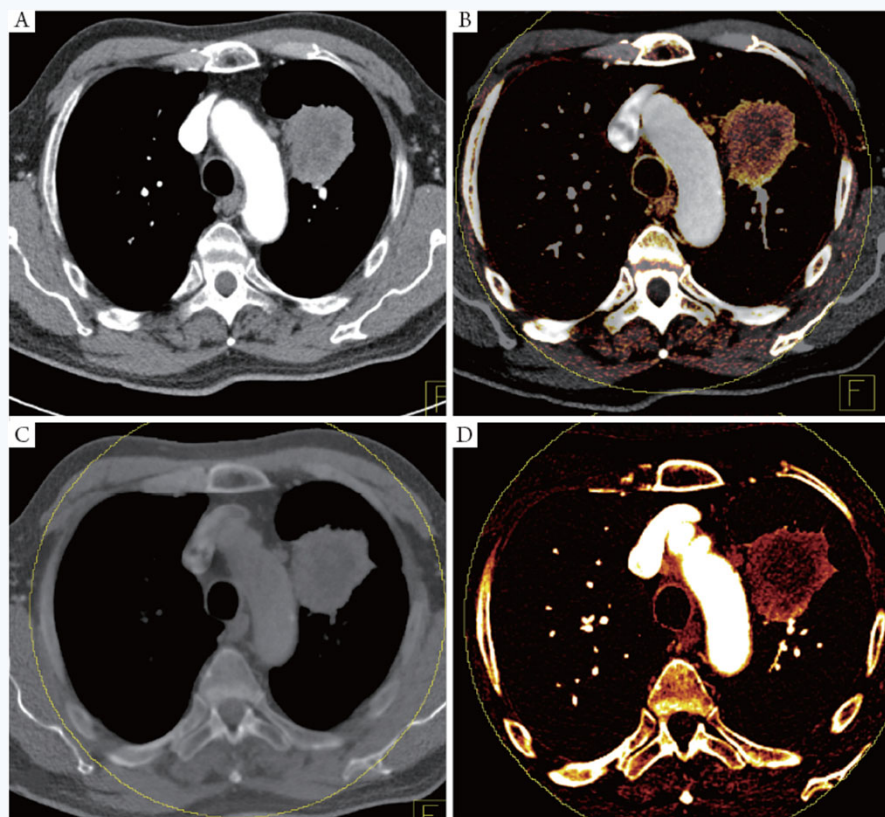


Figure 1 Contrast enhanced calculated "virtual 120 kV" dual energy CT image of a 56-year-old male patient with an adenocarcinoma of the left upper lobe (A). Fused (B) and isolated selective iodine perfusion maps (D) revealed hyper-perfusion of the peripheral tumor margins with less iodine uptake in the central areas. The corresponding virtual non-contrast CT image (C) shows hypo-attenuation of the hypo-perfused central tumor area indicating less tumor vitality.

What is Compton Scattering?

The differential scattering cross section ($d\sigma$) – the probability of a photon scattered into a unit solid angle around the scattering angle θ , when passing normally through a layer of material containing one electron per unit area.

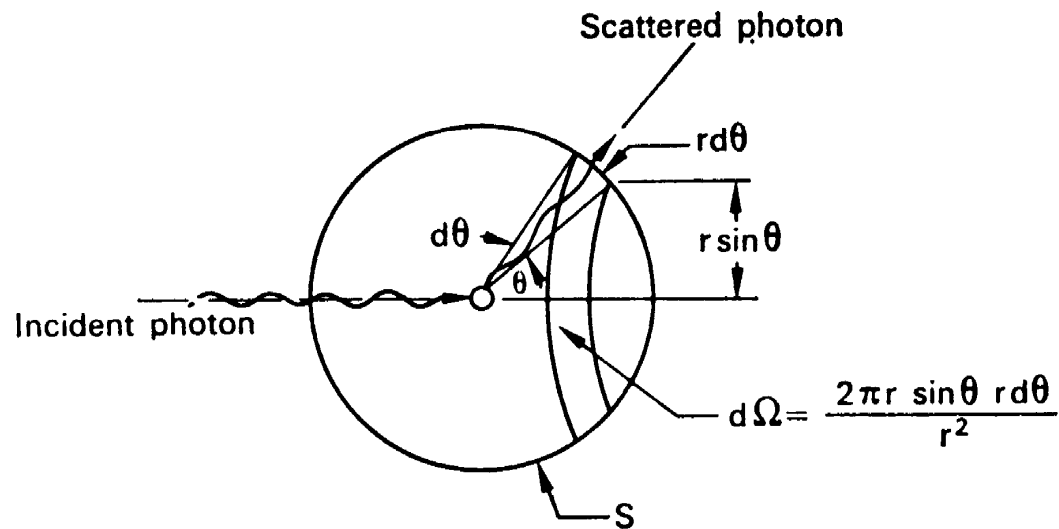


FIG. 5.15. Compton scattering diagram to illustrate differential scattering cross section. S is a sphere of unit radius whose center is the scattering electron.

Energy Transfer in Compton Scattering

If we assume that the electron is free and at rest, the scattered gamma ray has an energy

$$h\nu' = \frac{h\nu}{1 + \frac{h\nu}{m_0 c^2} (1 - \cos(\theta))},$$

Initial photon energy, ν : photon frequency,
 $h = 6.62607015 \times 10^{-34}$ meter \cdot kilogram/second, (Planck's constant)

mass of electron $m_0 c^2$ Scattering angle θ

and the photon transfers part of its energy to the electron (assumed to be at rest), which is known as a recoil electron. Its energy is simply

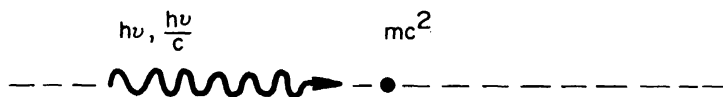
$$E_{recoil} = h\nu - h\nu' = h\nu - \frac{h\nu}{1 + \frac{h\nu}{m_0 c^2} (1 - \cos(\theta))}$$

The one-to-one relationship between scattering angle and energy loss!!

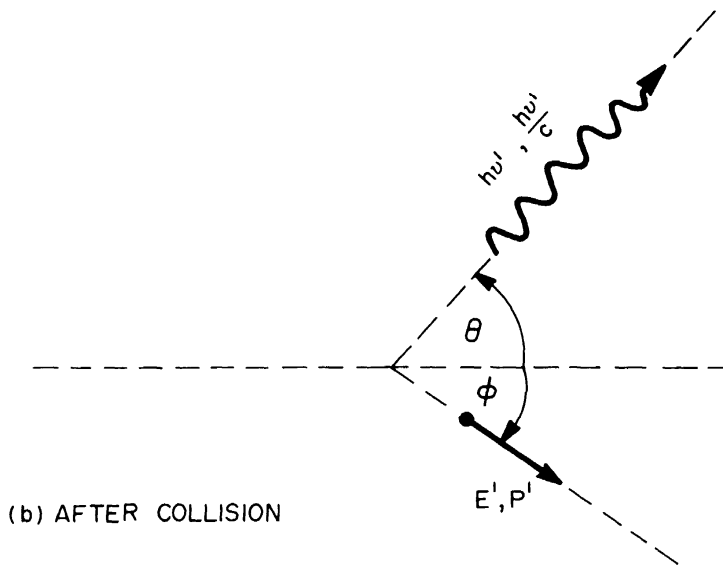
Basic Kinematics in Compton Scattering

The energy transfer in Compton scattering may be derived as the following:

- ☞ Assuming that the electron binding energy is small compared with the energy of the incident photon – elastic scattering.
- ☞ Write out the conservation of energy and momentum:



(a) BEFORE COLLISION



(b) AFTER COLLISION

Conservation of energy

$$h\nu + mc^2 = h\nu' + E'$$

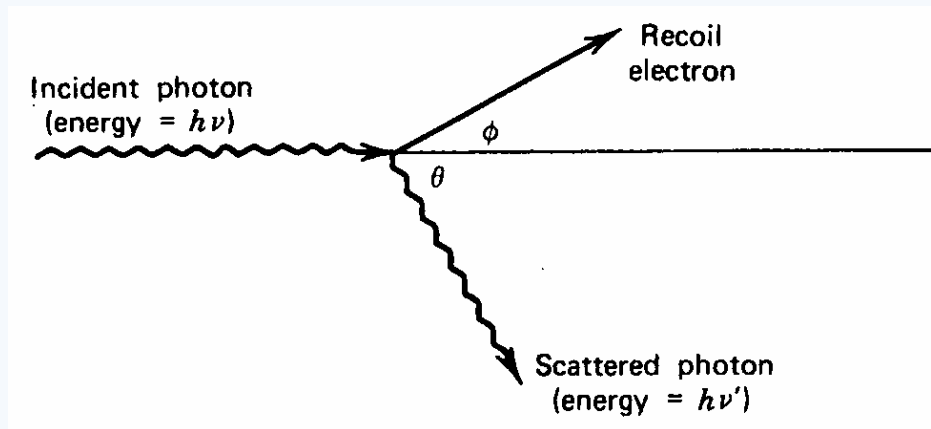
Conservation of momentum

$$\frac{h\nu}{c} = \frac{h\nu'}{c} \cos \theta + P' \cos \phi$$

$$\frac{h\nu'}{c} \sin \theta = P' \sin \phi$$

Compton Scattering with Non-stationary Electrons – Doppler Broadening

$$E_{recoil}(\theta) = h\nu - h\nu' = h\nu - \frac{h\nu}{1 + \frac{h\nu}{m_0c^2}(1 - \cos\theta)}$$



$$h\nu' = \frac{h\nu}{1 + \frac{h\nu}{m_0c^2}(1 - \cos(\theta))} \pm \sigma(h\nu')$$

Energy Transfer in Compton Scattering

If we assume that the electron is free and at rest, the scattered gamma ray has an energy

$$h\nu' = \frac{h\nu}{1 + \frac{h\nu}{m_0 c^2} (1 - \cos \theta)},$$

Initial photon energy, ν : photon frequency
Scattering angle
mass of electron

and the photon transfers part of its energy to the electron (assumed to be at rest before the collision), which is known as the recoil electron. Its energy is simply

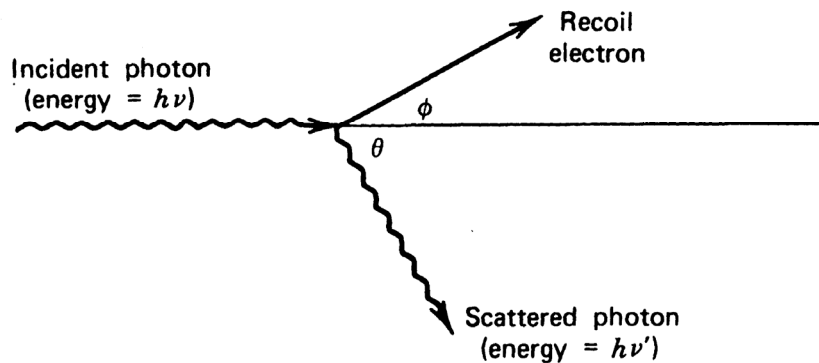
$$E_{recoil} = h\nu - h\nu' = h\nu - \frac{h\nu}{1 + \frac{h\nu}{m_0 c^2} (1 - \cos(\theta))}$$

assuming the binding energy of the electron is negligible.

In the simplified elastic scattering case, there is an one-to-one relationship between scattering angle and energy loss!!

Derivation of the Relationship Between Scattering Angle and Energy Loss

The relation between energy the scattering angle and energy transfer are derived based on the conservation of energy and momentum:



$$\vec{p}_{h\nu} + \vec{p}_e = \vec{p}_{h\nu'} + \vec{p}_{e'}$$

$$E_{h\nu} + E_e = E_{h\nu'} + E_{e'}$$

Are those terms truly zero?

Compton Scattering with Non-stationary Electrons – Doppler Broadening

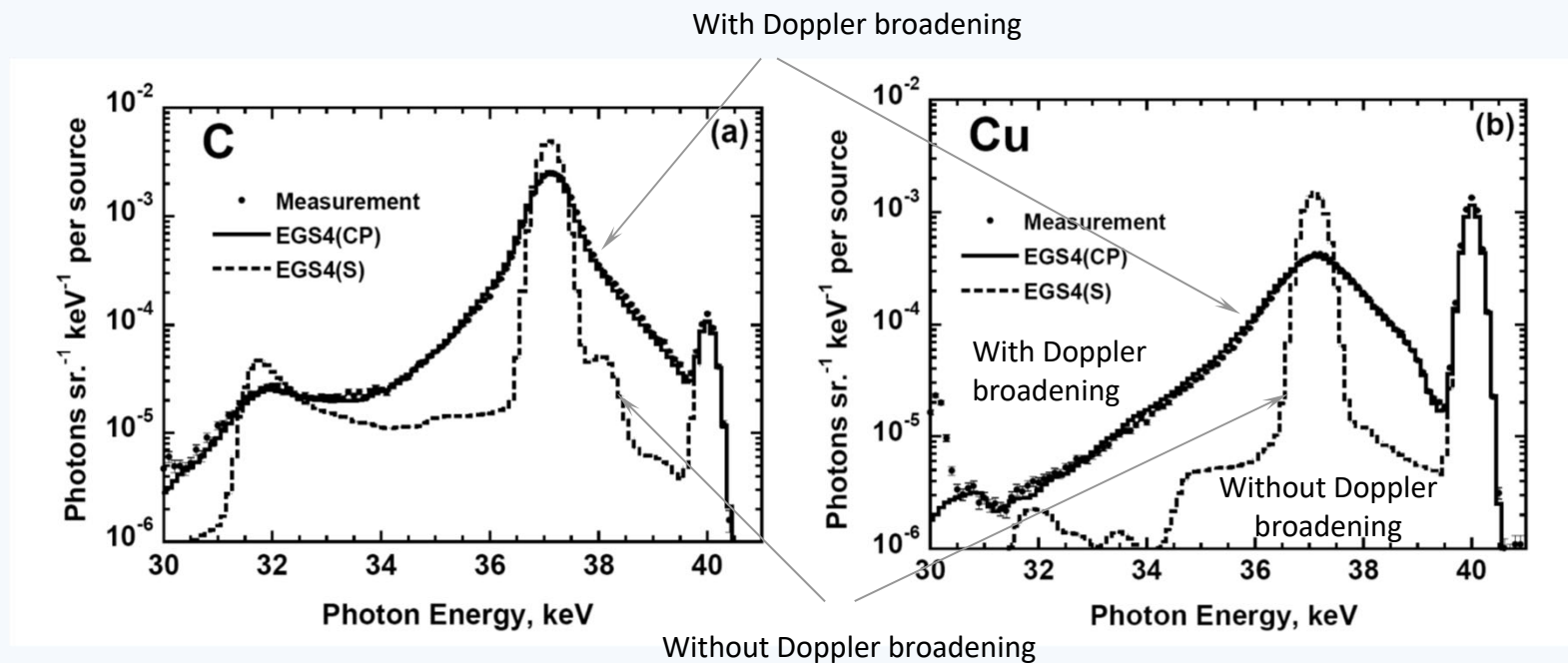
- ☞ It is so far assumed that (a) the electron is free and stationary and (b) the incident photon is unpolarized.
- ☞ When an incident photon is reflected by a non-stationary electron, for example an bond electron, an extra uncertainty is added to the energy of the scattered photon. This extra uncertainty is called Doppler broadening.

$$h\nu' = \frac{h\nu}{1 + \frac{h\nu}{m_0 c^2} (1 - \cos(\theta))} \pm \sigma(h\nu')$$

The one-to-one relationship between scattering angle and energy loss holds only when incident photon energy is far greater than the bonding energy of the electron...

Compton Scattering with Non-stationary Electrons – Doppler Broadening

Comparison of the energy spectra for the photons scattered by C and Cu samples. $E_{\text{hv}}=40\text{keV}$, $\theta=90$ degrees



The Doppler broadening is stronger in Cu than in C because of the Cu electrons have greater bonding energy.

Energy Transfer in Compton Scattering

- ➡ The maximum energy carried by the recoil electron is obtained by setting θ to 180° ,

$$E_{\max} = \frac{2h\nu}{2 + mc^2/h\nu}$$

- ➡ The maximum energy transfer is exemplified by the Compton edge in measured gamma ray energy spectra.

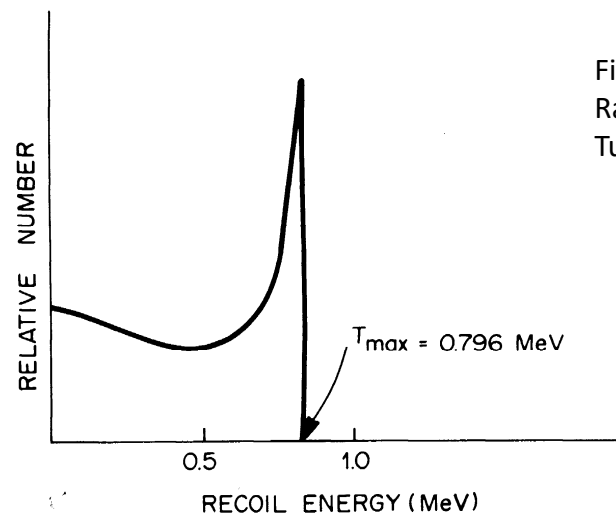


Figure from Atoms, Radiation, and Radiation Protection, James E Turner, p180.

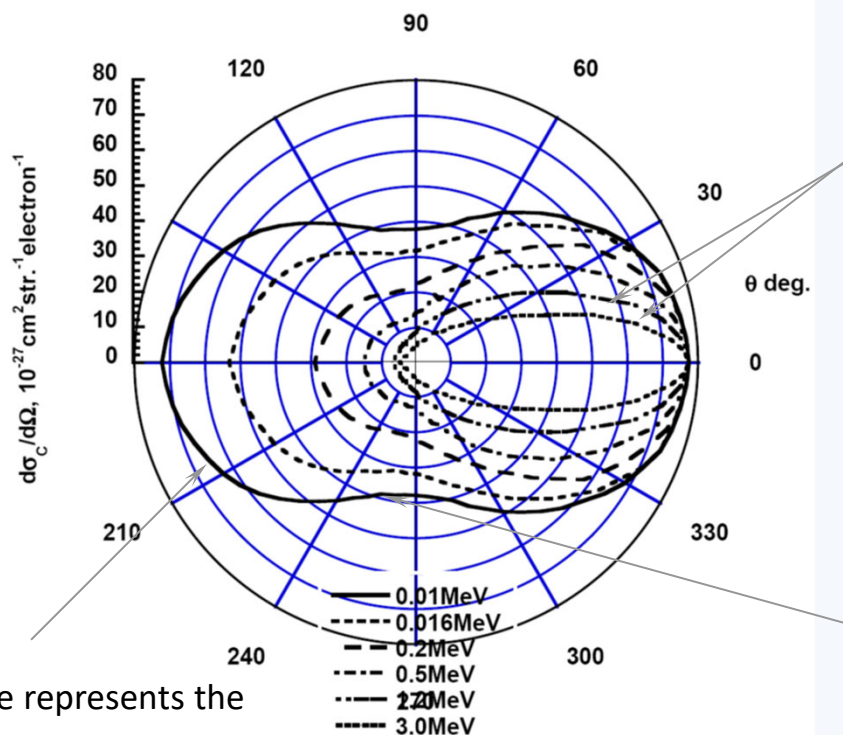
FIGURE 8.5. Relative number of Compton recoil electrons as a function of their energy for 1-MeV photons.

Angular Distribution of the Scattered Gamma Rays

Klein-Nishina formula:

$$\frac{d\sigma}{d\Omega}(\theta) = Zr_e^2 \left(\frac{1}{1 + \alpha(1 - \cos(\theta))} \right)^2 \left(\frac{1 + \cos^2(\theta)}{2} \right) \left(1 + \frac{1 + \cos^2(\theta)}{(1 + \cos^2(\theta))[1 + \alpha(1 - \cos(\theta))]} \right), \quad \alpha = \frac{h\nu}{m_0c^2}$$

Probability of Compton scattering within a unit solid angle around a scattering angle θ .



Radial distance represents the differential cross section.

Incident photons with higher energy tend to scatter with smaller angle (forward scattering)

Incident photons with lower energy (a few hundred keV) have greater chances of undergoing large angle scattering (back scattering)

Application of the Klein-Nishina Formula (1)

Klein-Nishina formula can be used to calculate the expected energy spectrum of recoil electrons as the following:

$$\frac{d\sigma}{dE_{recoil}} = \frac{d\sigma}{d\Omega} \frac{d\Omega}{d\theta} \frac{d\theta}{dE_{recoil}}$$

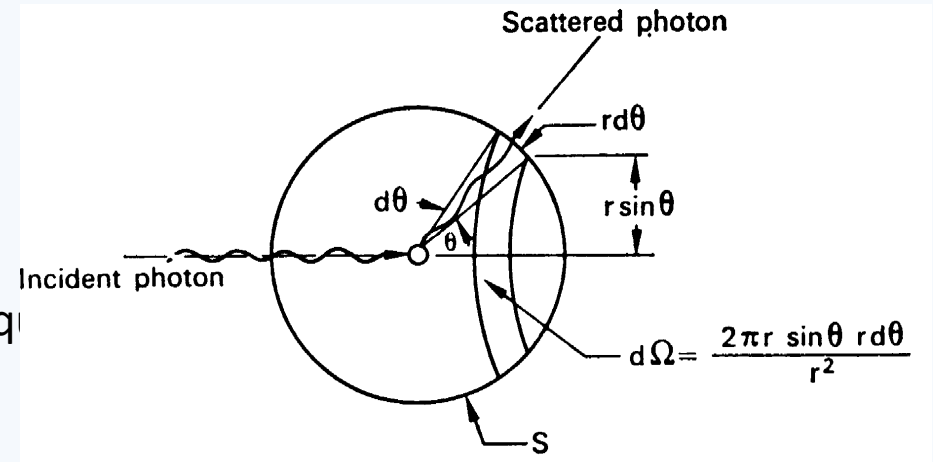
and the probability that a recoil electron possesses an energy between $E_{recoil} - \Delta E/2$ and $E_{recoil} + \Delta E/2$ is

$$\propto \Delta E \cdot \frac{d\sigma}{dE_{recoil}}$$

Application of the Klein-Nishina Formula (1)

$$\frac{d\sigma}{dE_{recoil}} = \frac{d\sigma}{d\Omega} \frac{d\Omega}{d\theta} \frac{d\theta}{dE_{recoil}}$$

The terms on the right hand side of the equation are relationships,



$$\frac{d\sigma}{d\Omega}(\theta) = r_e^2 \left(\frac{1}{1 + \alpha(1 - \cos \theta)} \right)^2 \left(\frac{1 + \cos^2 \theta}{2} \right) \left(1 + \frac{\alpha^2(1 - \cos \theta)^2}{(1 + \cos^2 \theta)[1 + \alpha(1 - \cos \theta)]} \right),$$

$$E_{recoil} = h\nu - h\nu' = h\nu - \frac{h\nu}{1 + \frac{h\nu}{m_0 c^2}(1 - \cos \theta)} \Rightarrow \frac{d\theta}{dE_{recoil}} = -\frac{m_0 c^2}{E_{recoil}^2 \sin \theta}$$

$$d\Omega = 2\pi \sin \theta d\theta \Rightarrow \frac{d\Omega}{d\theta} = 2\pi \sin \theta$$

The expected energy spectrum of recoil electrons can be evaluated numerically using these relationships.

Angular Distribution of the Scattered Gamma Rays

The differential scattering cross section per electron – the probability of a photon scattered into a unit solid angle around the scattering angle θ , when passing normally through a very thin layer of scattering material that contains one electron per unit area.

$$\frac{d\sigma}{d\Omega}(\theta) = r_e^2 \left(\frac{1}{1 + \alpha(1 - \cos \theta)} \right)^2 \left(\frac{1 + \cos^2 \theta}{2} \right) \left(1 + \frac{\alpha^2(1 - \cos \theta)^2}{(1 + \cos^2 \theta)[1 + \alpha(1 - \cos \theta)]} \right) (m^2 sr^{-1})$$

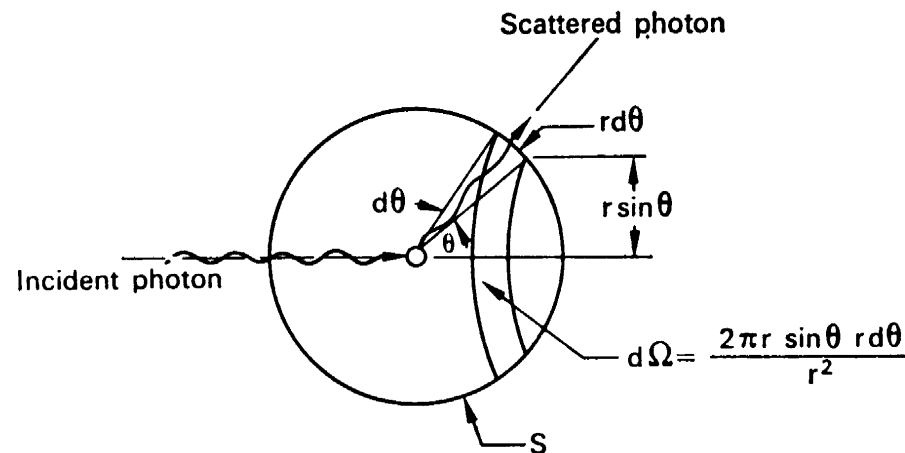
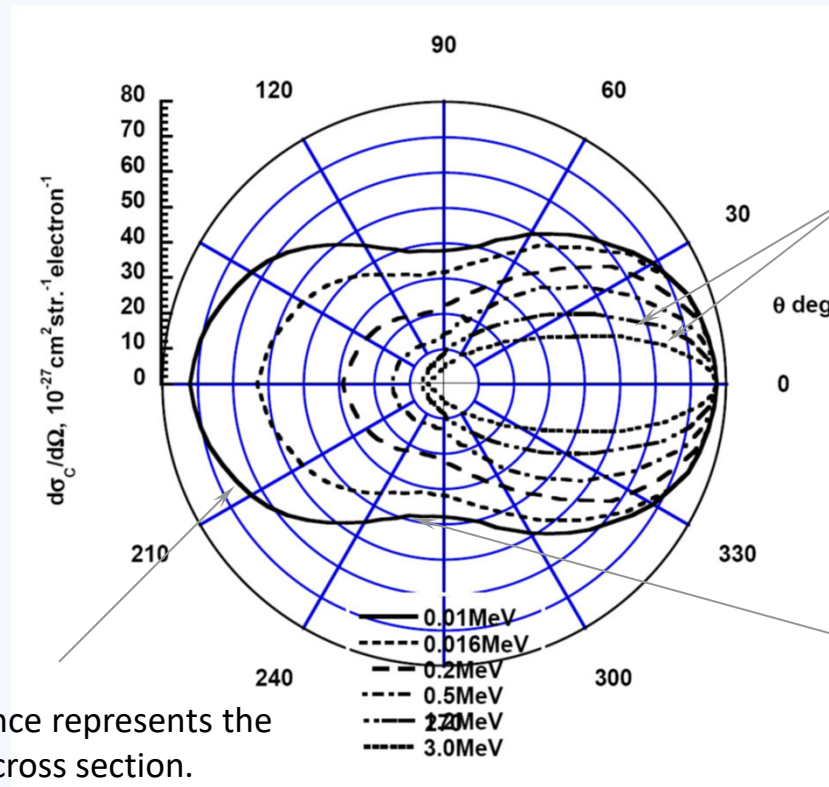


FIG. 5.15. Compton scattering diagram to illustrate differential scattering cross section. S is a sphere of unit radius whose center is the scattering electron.

Angular Distribution of the Scattered Gamma Rays



Incident photons with higher energies tend to scatter with smaller angles (forward scattering).

Incident photons with lower energy (a few hundred keV) have greater chance of undergoing large angle scattering (back scattering).

The higher the energy carried by an incident gamma ray, the more likely that the gamma ray undergoes forward scattering ...

Total Compton Collision Cross Section

Compton Collision Cross Section is defined as the total cross section per electron by Compton scattering. It can be derived by integrating the differential cross section over the 4π solid angle.

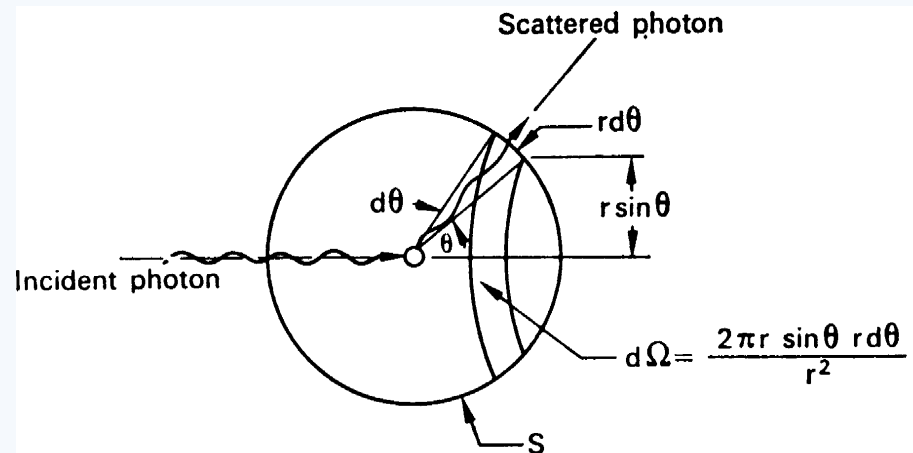
Since

$$d\Omega = 2\pi \sin \theta d\theta$$

The total scattering cross section is

$$\sigma = 2\pi \int \frac{d\sigma}{d\Omega} \sin \theta d\theta \text{ m}^2$$

Note that the Compton collision cross section is given in unit of m^2 .



Application of the Klein-Nishina Formula (1)

- ➡ The energy distribution of the recoil electrons derived using the Klein-Nishina formula is closely related to the energy spectrum measured with “small” detectors.

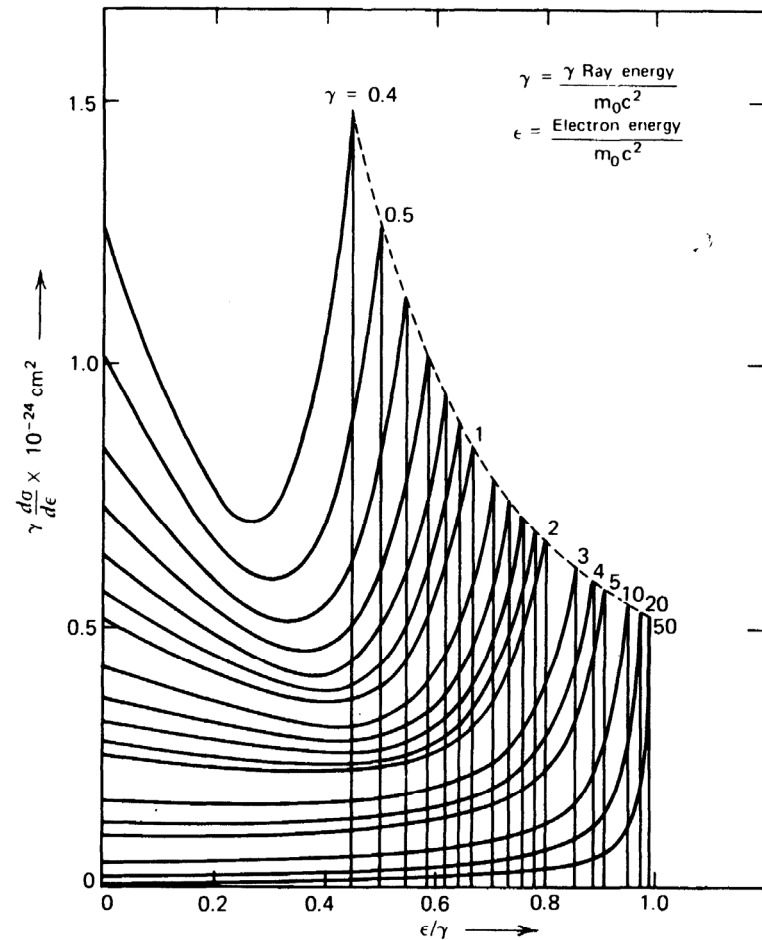


Figure 10.1 Shape of the Compton continuum for various gamma-ray energies. (From S. M. Shafroth (ed.), *Scintillation Spectroscopy of Gamma Radiation*. Copyright 1964 by Gordon & Breach, Inc. By permission of the publisher.)

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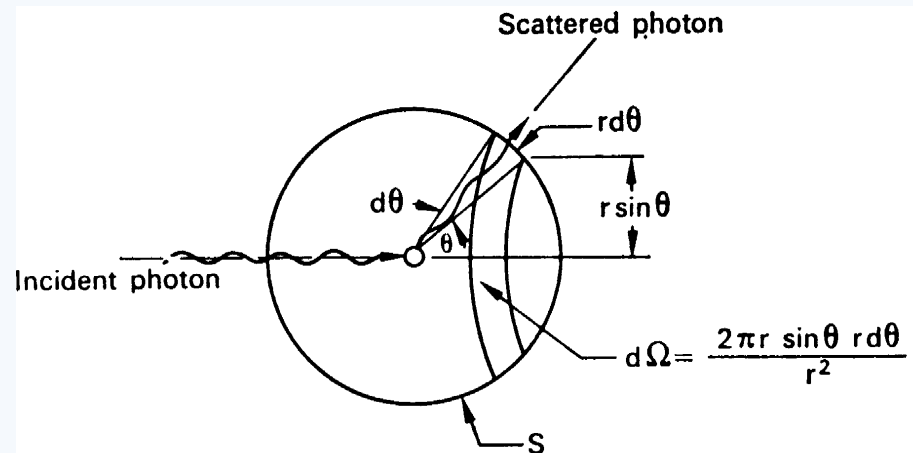
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The differential scattering cross section per electron – the probability of a photon scattered into a unit solid angle around the scattering angle θ , when passing normally through a very thin layer of scattering material that contains one electron per unit area.

$$\frac{d\sigma}{d\Omega}(\theta) = r_e^2 \left(\frac{1}{1 + \alpha(1 - \cos \theta)} \right)^2 \left(\frac{1 + \cos^2 \theta}{2} \right) \left(1 + \frac{\alpha^2(1 - \cos \theta)^2}{(1 + \cos^2 \theta)[1 + \alpha(1 - \cos \theta)]} \right) (m^2 sr^{-1})$$

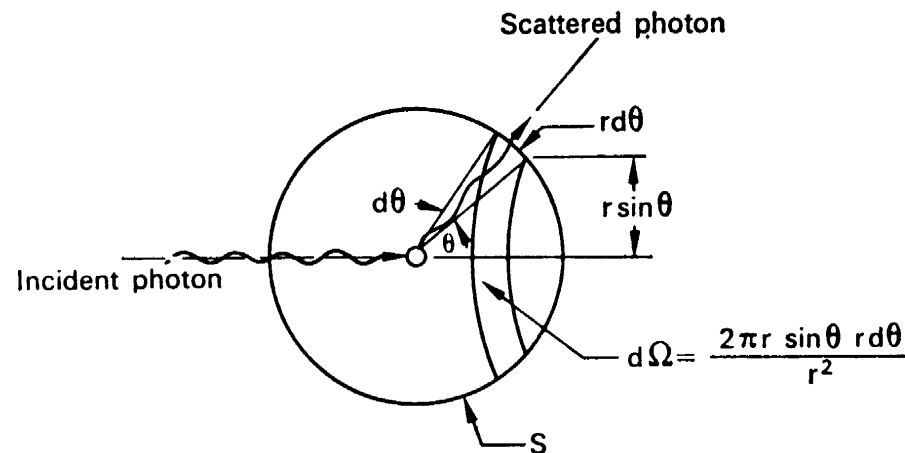


FIG. 5.15. Compton scattering diagram to illustrate differential scattering cross section. S is a sphere of unit radius whose center is the scattering electron.

Question 2:

If we know the total Compton scattering cross section

$$\sigma = 2\pi \int \frac{d\sigma}{d\Omega} \sin \theta d\theta \text{ m}^2$$

How do we derive the linear attenuation coefficient of gamma rays through Compton scattering?

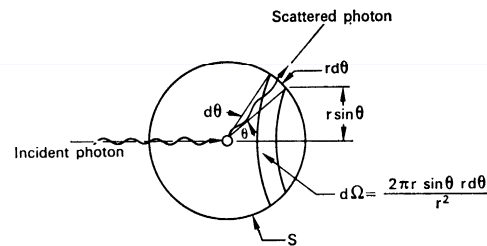


FIG. 5.15. Compton scattering diagram to illustrate differential scattering cross section. S is a sphere of unit radius whose center is the scattering electron.

The linear attenuation coefficient is the probability of a gamma ray's Compton scattering in the material while traveling through a unit distance.

Application of the Klein-Nishina Formula (3)

The differential Compton cross section given by the Klein-Nishina Formula can also be related to another important parameter for gamma ray dosimetry – the linear attenuation coefficient.

Suppose that the total number of atoms per m^3 in the absorber is N and the atomic number is Z , the electron density in the absorber is NZ .

Therefore the product

$$\sigma_{linear} = NZ\sigma_{compton} \quad (cm^{-1})$$

and

$$\sigma_{compton} = \int \frac{d\sigma}{d\Omega} d\Omega = \int \frac{d\sigma}{d\Omega} \frac{d\Omega}{d\theta} d\theta = 2\pi \int \frac{d\sigma}{d\Omega} \sin\theta d\theta \quad (cm^2)$$

$$\frac{d\sigma}{d\Omega}(\theta) = r_e^2 \left(\frac{1}{1 + \alpha(1 - \cos\theta)} \right)^2 \left(\frac{1 + \cos^2\theta}{2} \right) \left(1 + \frac{\alpha^2(1 - \cos\theta)^2}{(1 + \cos^2\theta)[1 + \alpha(1 - \cos\theta)]} \right) (m^2 sr^{-1})$$

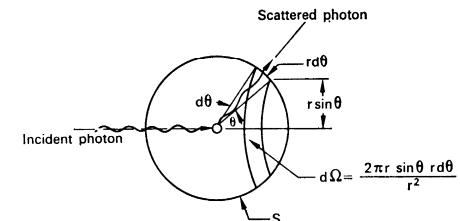


FIG. 5.15. Compton scattering diagram to illustrate differential scattering cross section. S is a sphere of unit radius whose center is the scattering electron.

Application of the Klein-Nishina Formula (3)

$$\sigma_{linear} = NZ\sigma_{compton}$$

has the unit of m^{-1} , is

- ☞ The total cross section of all electrons “seen” by a photon while traversing a unit distance in the absorber, or
- ☞ The total probability that the photon will interact with the absorber through Compton scattering while traversing a unit distance in the absorber.

Compton Scattering for Imaging?

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The CGRO Mission (1991 - 2000)

The Compton Gamma Ray Observatory was the second of NASA's Great Observatories. Compton, at 17 tons, was the heaviest astrophysical payload ever flown at the time of its [launch](#) on **April 5, 1991** aboard the space shuttle Atlantis. Compton was safely deorbited and [re-entered](#) the Earth's atmosphere on **June 4, 2000**.

Compton had four instruments that covered an unprecedented six decades of the [electromagnetic spectrum](#), from 30 keV to 30 GeV. In order of increasing spectral energy coverage, these instruments were the Burst And Transient Source Experiment ([BATSE](#)), the Oriented Scintillation Spectrometer Experiment ([OSSE](#)), the Imaging Compton Telescope ([COMPTEL](#)), and the Energetic Gamma Ray Experiment Telescope ([EGRET](#)). For each of the instruments, an improvement in sensitivity of better than a factor of ten was realized over previous missions.



The Observatory was named in honor of [Dr. Arthur Holly Compton](#), who won the Nobel prize in physics for work on scattering of high-energy photons by electrons - a process which is central to the gamma-ray detection techniques of all four instruments.

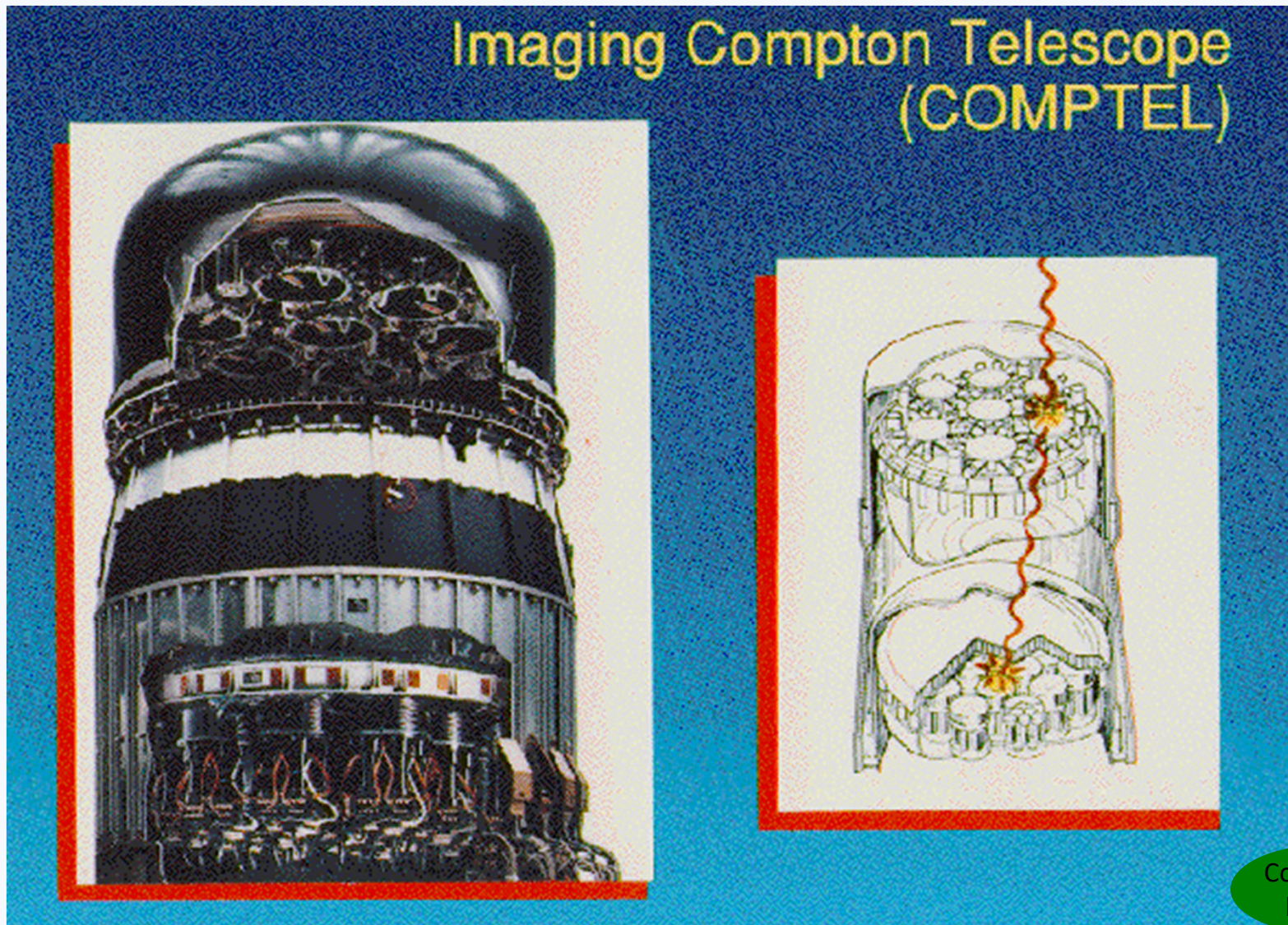
[CGRO Observation Timelines](#)

If you have a question about CGRO, please contact us via the [Feedback form](#).

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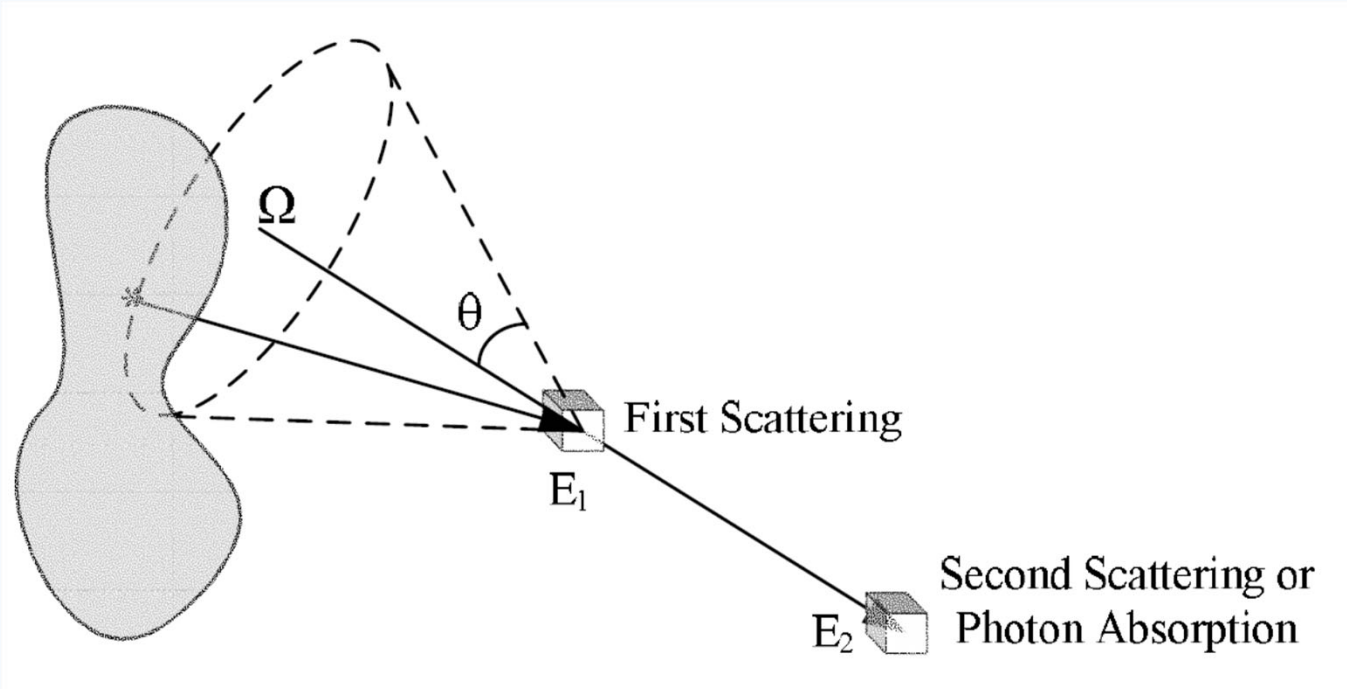
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Compton Scattering for Imaging?



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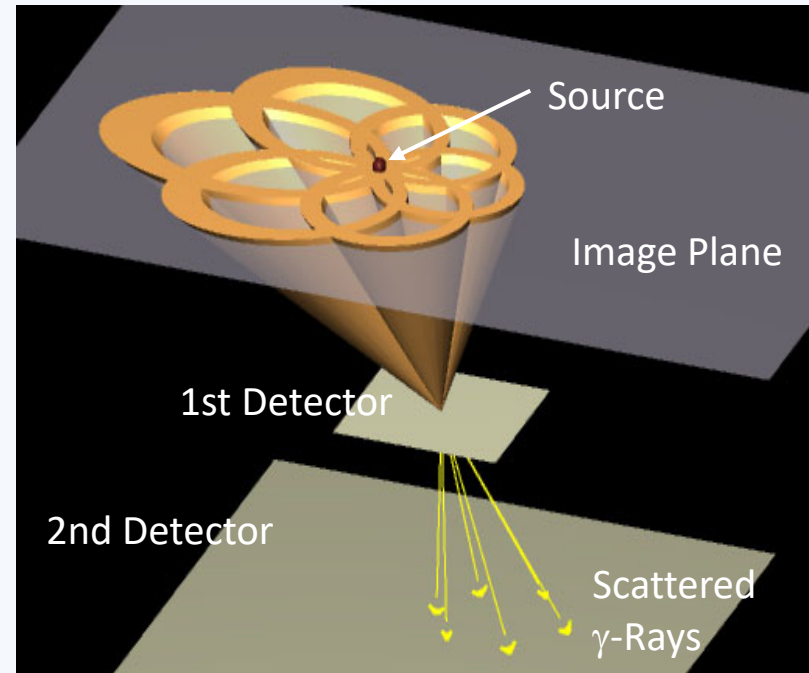
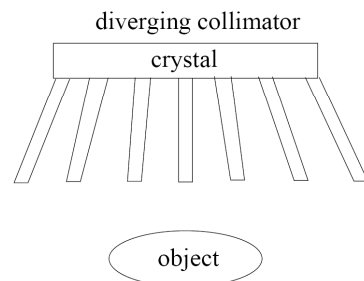
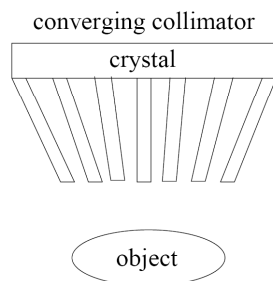
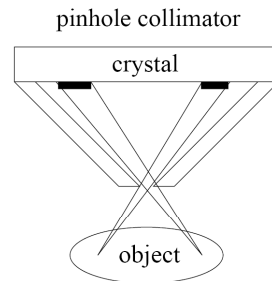
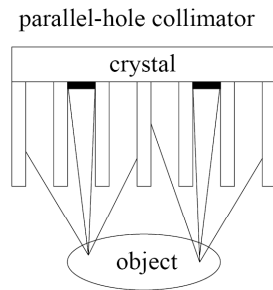
Principle of Compton Imaging



$$\cos \theta = 1 + \frac{m_e c^2}{E_0} - \frac{m_e c^2}{E_0 - E_1}$$

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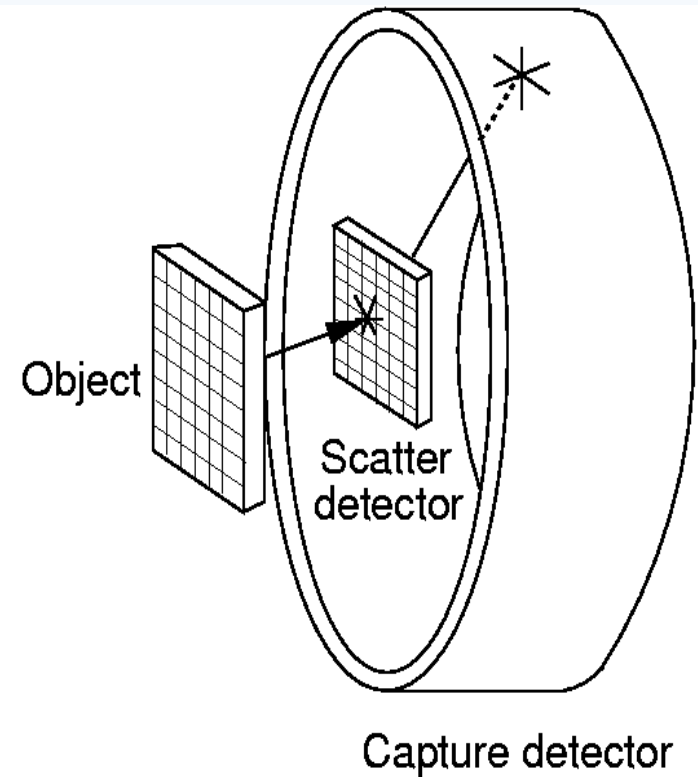
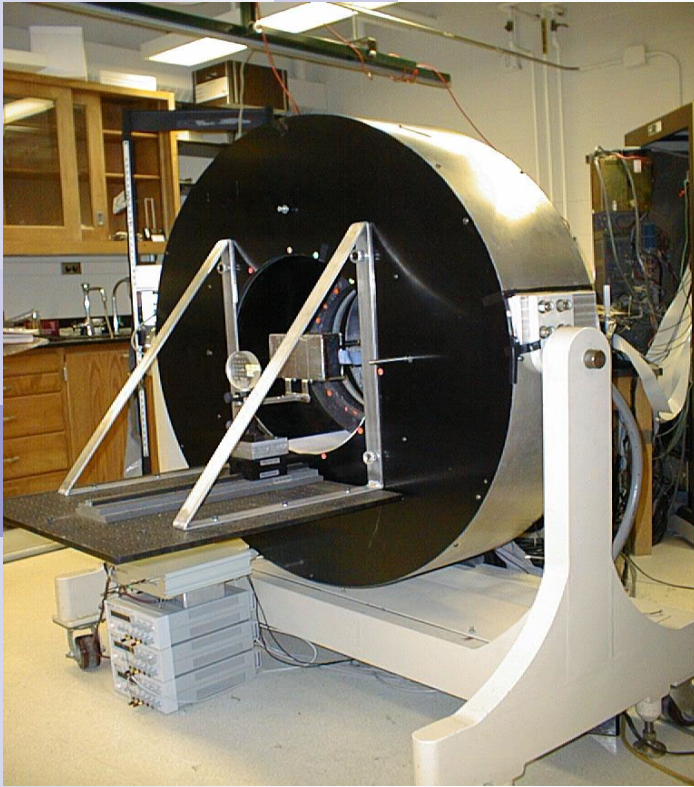
Principle of Compton Imaging



- No need for mechanical collimation – much higher detection efficiency.
- Requires the first detector to have a very good energy resolution.

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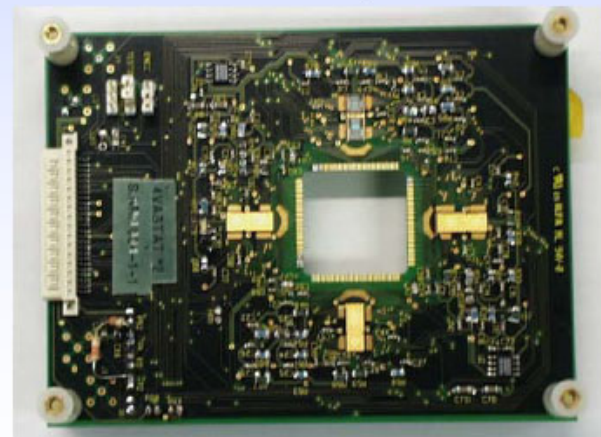
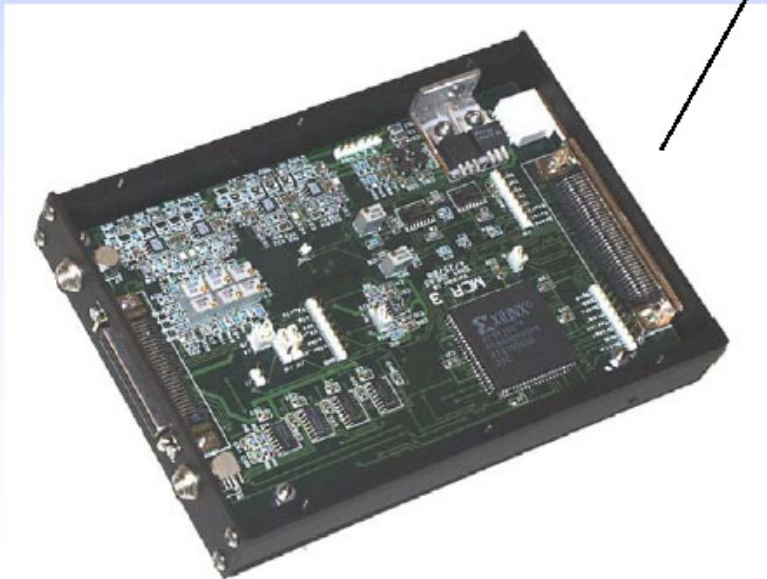
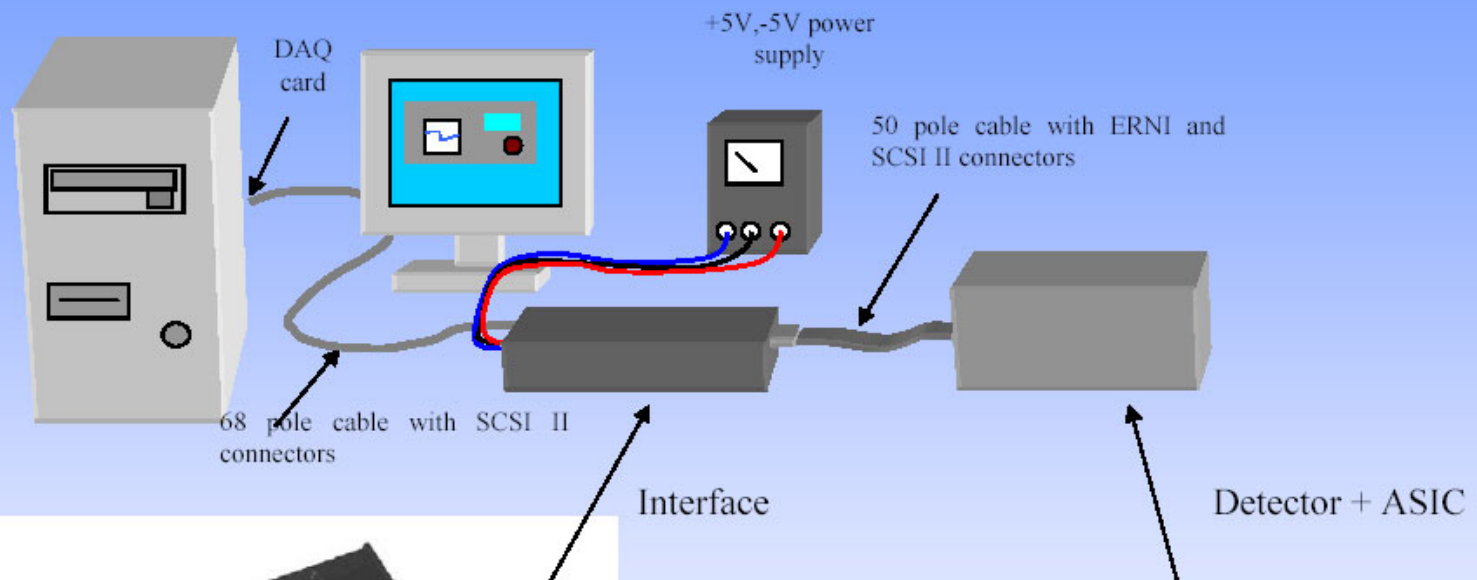
Experimental Compton Camera



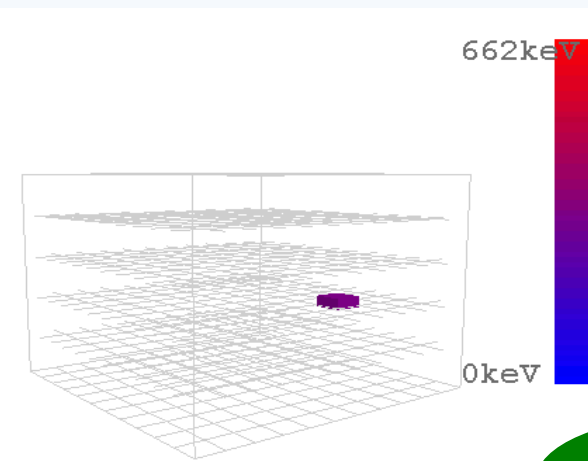
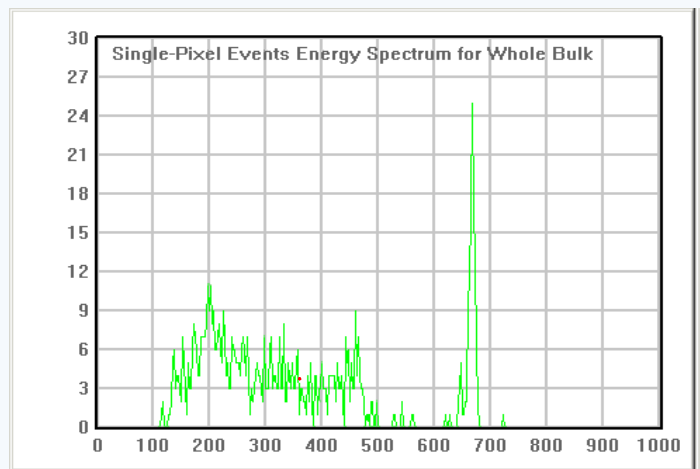
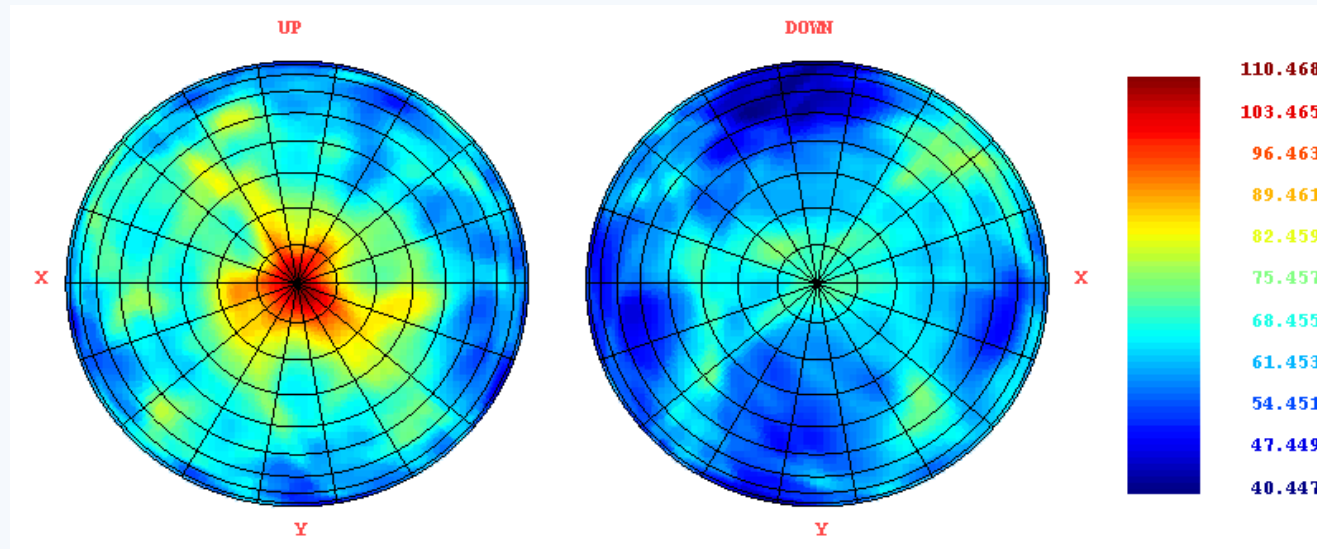
Developed by the Detector Physics Group, University of Michigan, lead by Neal Clinthorne and Les. Rogers

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Detector system



Imaging Spectroscopy



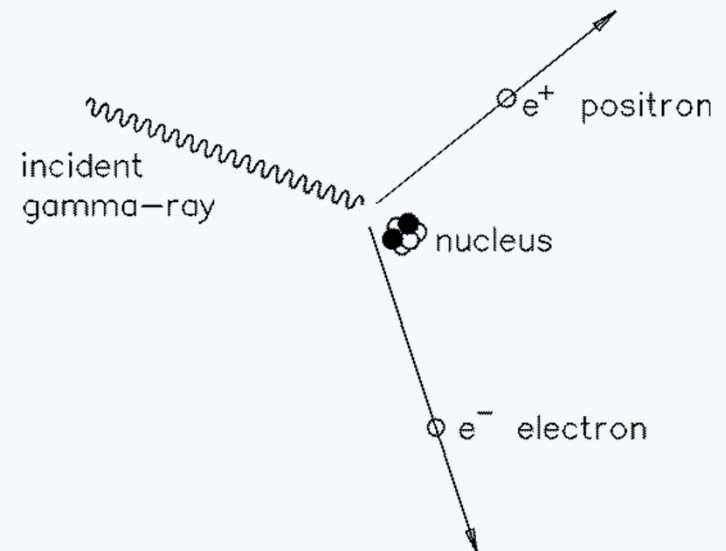
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Pair Production

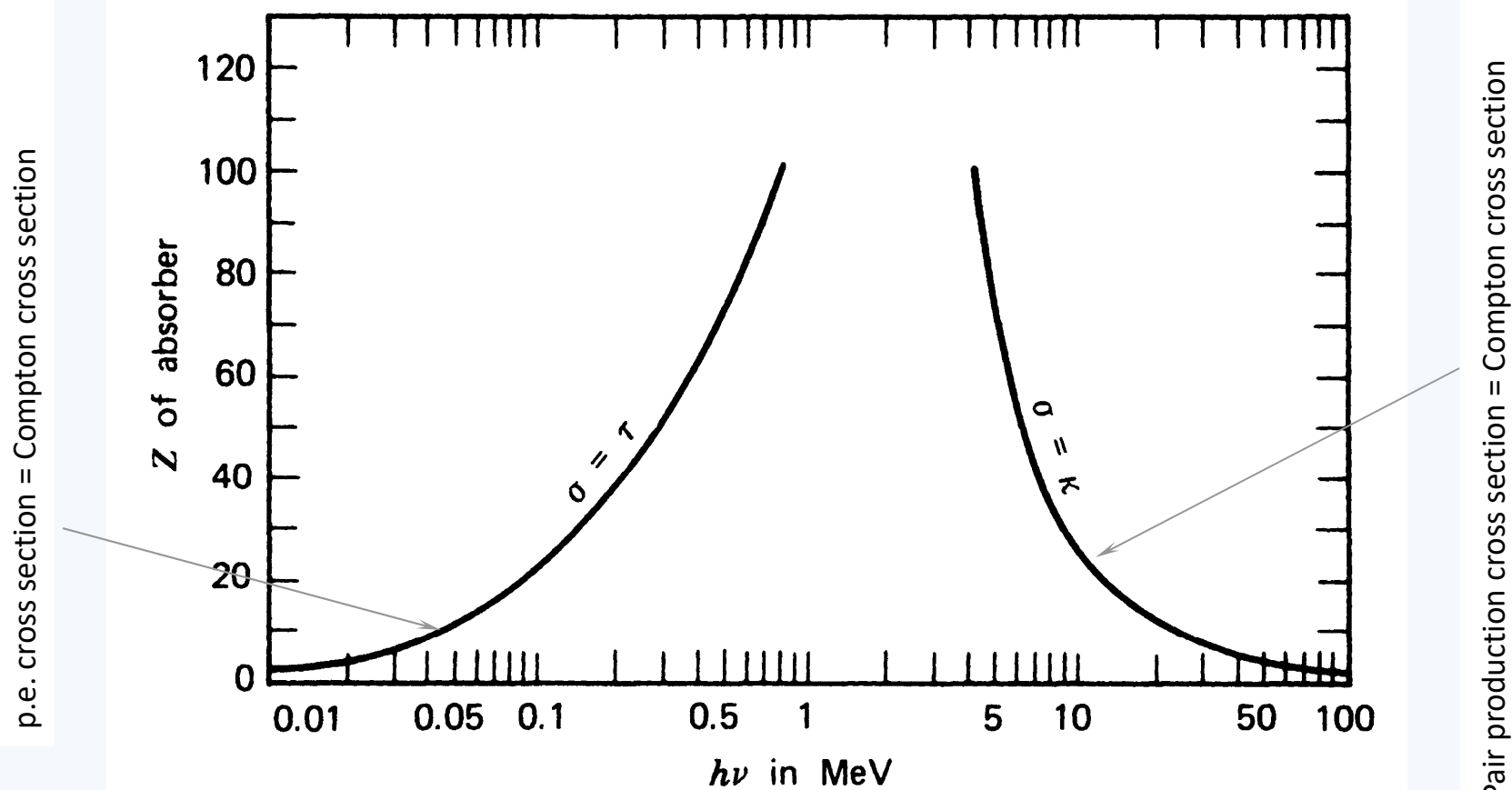
- Pair production refers to the creation of an electron-positron pair by an incident gamma ray in the vicinity of a nucleus.
- The minimum energy required is

$$E_{\gamma} \geq 2m_e c^2 + \frac{2m_e^2 c^2}{m_{nucleus}} \approx 2m_e c^2 = 1.022 MeV$$

- The process is more probable with a heavy nucleus and incident gamma rays with higher energies.
- The positron will soon annihilates with ordinary electrons near by and produces two 511keV gamma rays.



The Relative Importance of the Three Major Type of X and Gamma Ray Interactions



Page 52, Chapter 2 in Radiation Detection and Measurements, Third Edition, G. F. Knoll, John Wiley & Sons, 1999.

X-ray and Gamma Ray Interactions

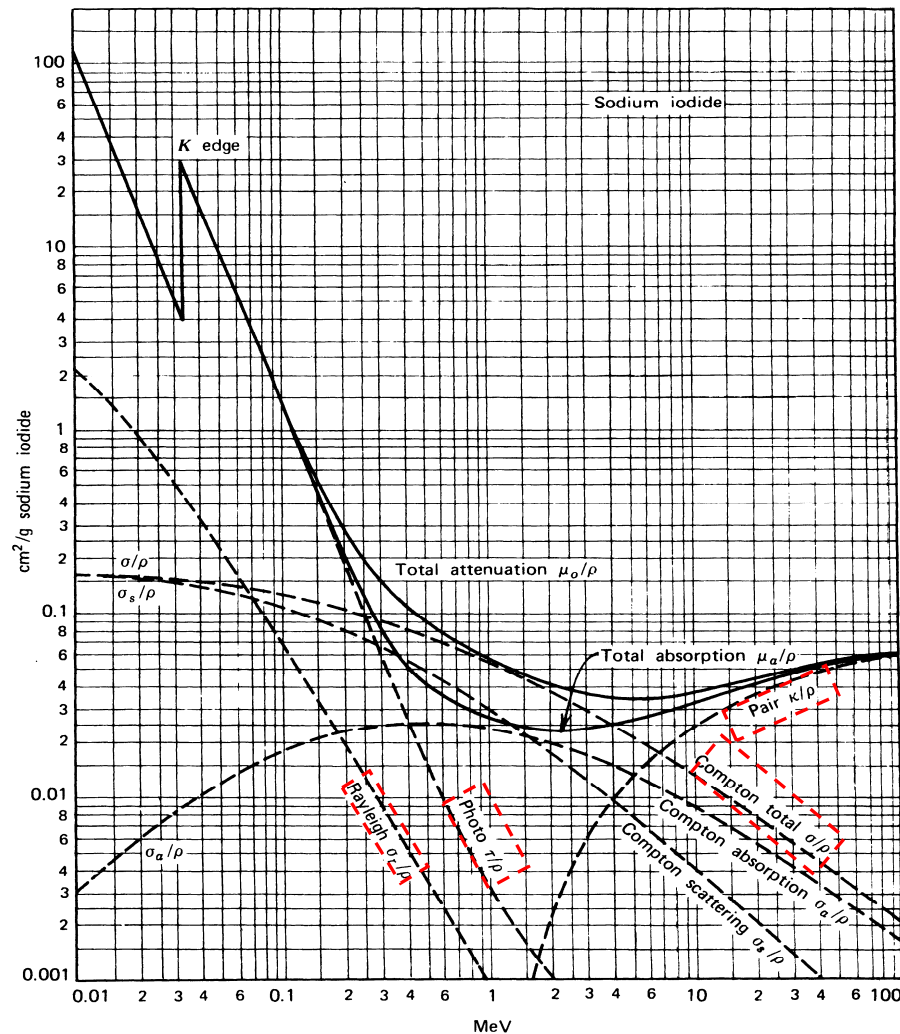


Figure 2.18 Energy dependence of the various gamma-ray interaction processes in sodium iodide. (From *The Atomic Nucleus* by R. D. Evans. Copyright 1955 by the McGraw-Hill Book Company. Used with permission.)

From Page 50, Radiation Detection and Measurements, Third Edition, G. F. Knoll, John Wiley & Sons, 1999.

Covered in lecture



Case Study I – Dual Energy CT

What are the challenges for CT imaging?

- Compton scattering is the predominant interaction for x-ray attenuation.
- Compton scattering depends on Z (or electron density)
- There are many materials of interest that have similar electron density.

If we solely rely on Compton scattering as the contrast mechanism for X-ray imaging, we are measuring the attenuation factor that is a **linear** function of the density ... This may not be sufficient for many cases ...

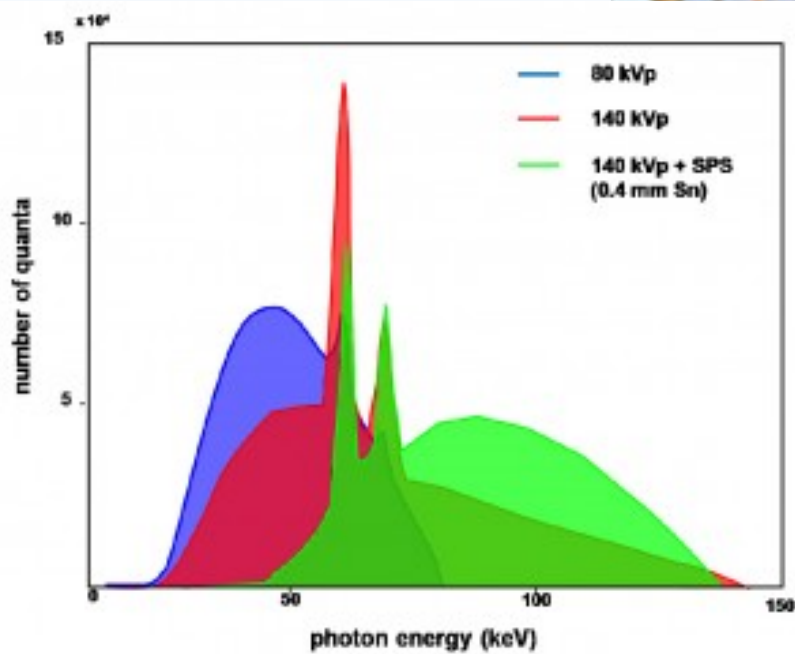
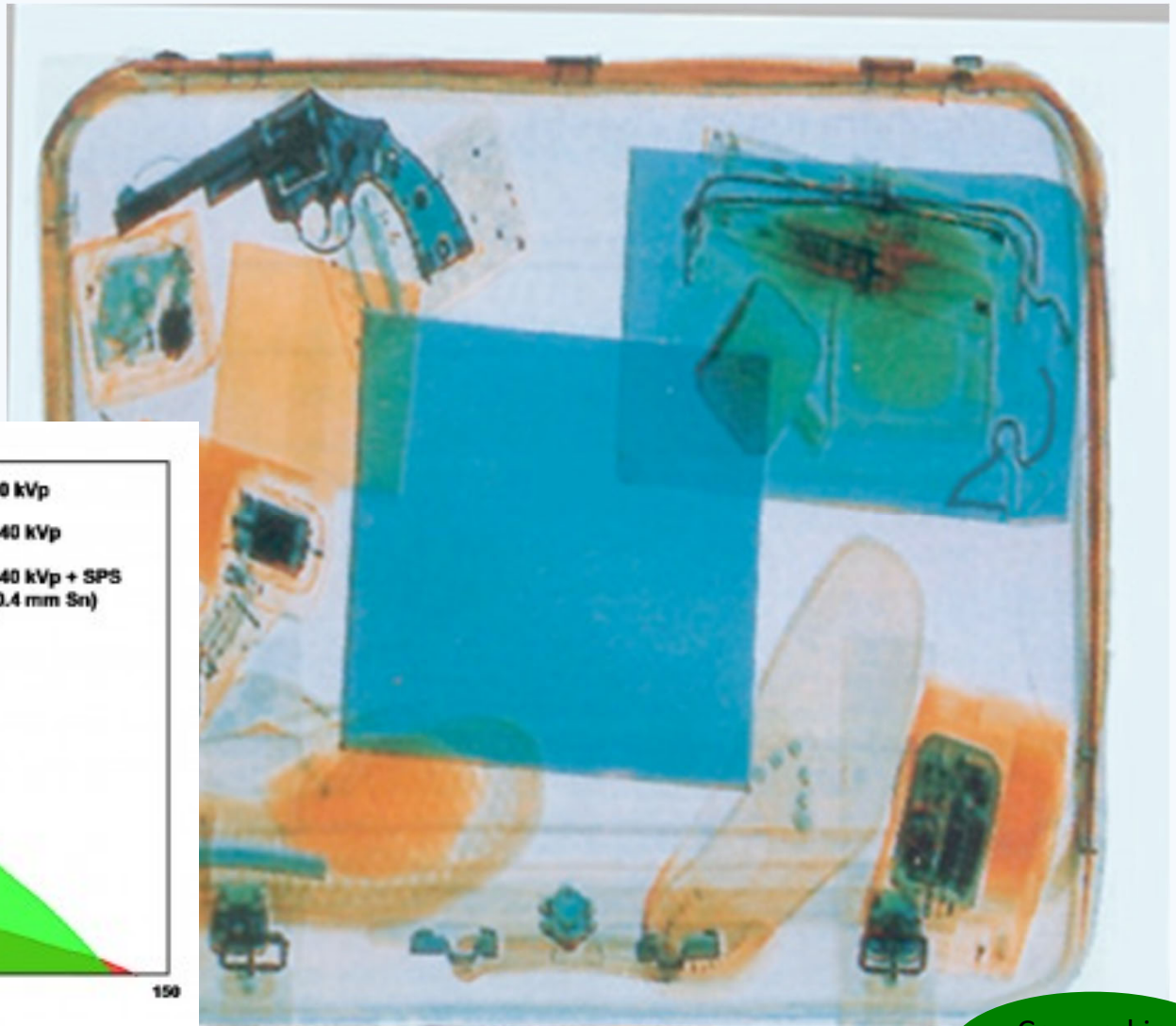
Classification of Photon Interactions

Table 1: Physical Properties of Several Materials

Material	Chemical Compo.	Density (g/cm³)	Effective Z	Linear Attenuation Coefficients (cm ⁻¹)						μ _{30keV} : μ _{80keV} : μ _{300keV}
				30keV		80keV		300keV		
				Comp : Phot Total		Comp : Phot Total		Comp : Phot Total		
Iron	Fe	7.6	26	3.14	59	1.29	3.1	0.78	0.055	74 : 5.2 : 1
				62		4.4		0.84		
Aluminium	Al	2.7	13	0.69	2.35	1.246	0.288	0.28	1.6x10 ⁻³	11 : 5.5 : 1
				3.05		1.53		0.28		
Polyethylene	(CH ₂) _n	~1.2	3.75	0.266	0.059	0.217	0.0022	0.146	3.1x10 ⁻⁵	2.2 : 1.5 : 1
				0.325		0.219		0.146		
Wood	C ₂₈ H ₄₂ N	~0.9	4.05	0.195	0.047	0.158	0.0018	0.11	2.5x10 ⁻⁵	2.2 : 1.45 : 1
				0.242		0.16		0.11		
Cocaine	C ₁₇ H ₂₁ O ₄ N	~1.2	4.77	0.256	0.093	0.204	0.0036	0.137	5.0x10 ⁻⁵	2.55 : 1.51 : 1
				0.349		0.207		0.137		
TNT	C ₇ H ₅ O ₆ N ₃	1.64	6.14	0.344	0.179	0.268	0.007	0.179	9.8x10 ⁻⁵	2.9 : 1.54 : 1
				0.523		0.275		0.179		
RDX	C ₃ H ₆ O ₉ N ₆	~1.50	6.39	0.319	0.19	0.246	0.0075	0.164	1.1x10 ⁻⁴	3.1 : 1.54 : 1
				0.509		0.253		0.164		

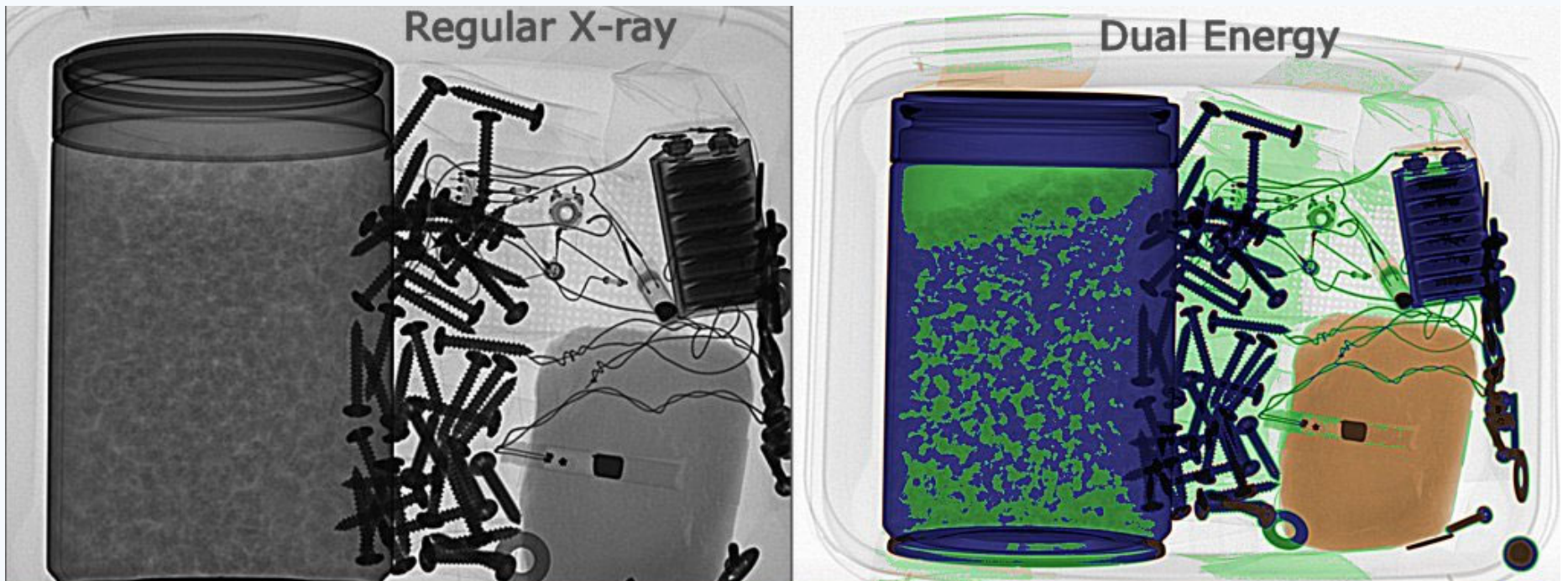
Covered in lecture

Airport X-ray Inspection



Covered in
lecture

Case Study I – Dual Energy CT



Organic explosive material is easily visible in Orange, metallic components in Blue and soft inorganic materials in Green

<http://www.vidisco.com/node/338>

Classification of Photon Interactions

Type of interaction Interaction with:	Absorption	Scattering	
		Elastic (Coherent)	Inelastic (Incoherent)
Atomic electrons	Photoelectric effect $\sigma_{pe} \begin{cases} \sim Z^4 (L.E.) \\ \sim Z^5 (H.E.) \end{cases}$	Rayleigh scattering $\sigma_R \sim Z^2 (L.E.)$	Compton scattering $\sigma_C \sim Z$
Nucleus	Photonuclear reactions $(\gamma, n), (\gamma, p),$ photofission, etc. $\sigma_{ph.n.} \sim Z$ $(h\nu \geq 10\text{MeV})$	Elastic nuclear scattering $(\gamma, \gamma) \sim Z^2$	Inelastic nuclear scattering (γ, γ')
Electric field surrounding charged particles	Electron-positron pair production in field of nucleus, $\sigma_{pair} \sim Z^2$ $(h\nu \geq 1.02\text{MeV})$		
	Electron-positron pair production in electron field, $\sigma_{trip} \sim Z^2$ $(h\nu \geq 2.04\text{MeV})$		
	Nucleon-antinucleon pair production $(h\nu \geq 3\text{ GeV})$		

Note the difference!

Covered in lecture



Energy Loss Mechanisms for Fast Electrons

Tracks of Beta Particles in Absorbing Medium

- Since beta particles have the same mass as the orbital electrons, they are easily scattered during collision and therefore follow tortuous paths in absorbing medium.
- The electrons are “wondering” more significantly near the end of their tracks.
- Energy-loss interactions are more sparsely distributed at the beginning of the track.

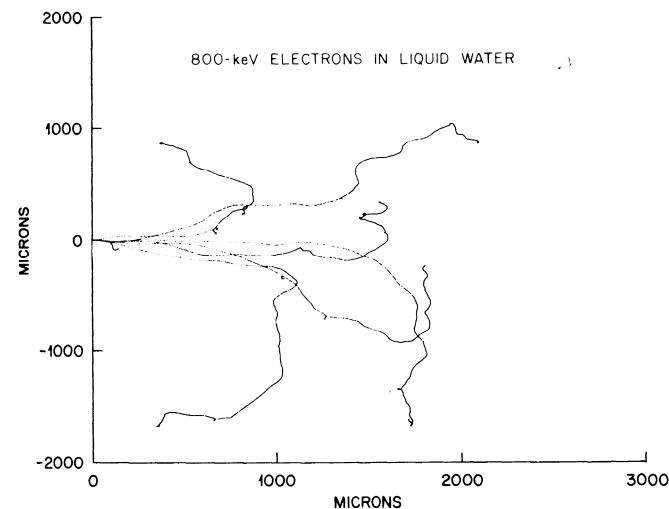


FIGURE 6.7. Calculated tracks (projected into the plane of the figure) of 800-keV electrons in water. Each electron starts moving horizontally toward the right from the point 0 on the vertical axis.

Figure from Atoms, Radiation, and Radiation Protection, James E Turner, p150

Specific Energy Loss of Fast Electrons

- The specific energy loss of electrons by excitation and ionization is

$$-\left(\frac{dE}{dx}\right)_c = \frac{2\pi e^4 N Z}{m_e v^2} \left[\left(\ln \frac{m_e v^2 E}{2I^2 (1 - \beta^2)} - (\ln 2)(2\sqrt{1 - \beta^2} - 1 + \beta^2) \right) + (1 - \beta^2) + \frac{1}{8} \left(1 - \sqrt{1 - \beta^2} \right)^2 \right], \quad \beta \equiv \frac{v}{c}$$

Effective atomic number of the absorbing material

No. of atoms per unit volume

Electron velocity

Specific Energy Loss of Fast Electrons

- The linear specific energy loss through Bremsstrahlung is

Specific energy loss by Bremsstrahlung increases with particle energy.

$$-\left(\frac{dE}{dx}\right)_r = \frac{NEZ(Z+1)e^4}{137m_e^2c^4} \left(4 \ln \frac{2E}{m_e c^2} - \frac{4}{3}\right)$$

more important for higher-Z materials, such as lead and tungsten.

Energy Loss by Bremsstrahlung

- For beta particles to stop completely , the fraction of energy loss by Bremsstrahlung process is approximately given by

$$f_{\beta} = 3.5 \times 10^{-4} Z E_m,$$

where f_{β} = the fraction of the incident beta energy converted into photons,
 Z = atomic number of the absorber,
 E_m = maximum energy of the beta particle, MeV.

Specific Energy Loss of Fast Electrons

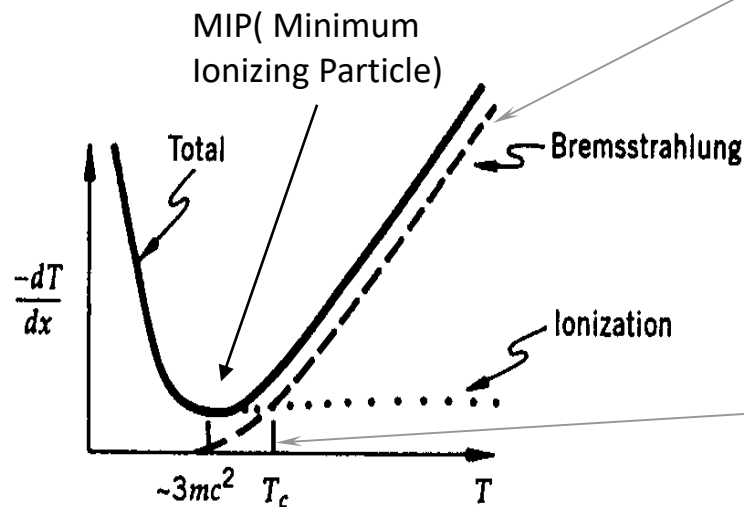
- The total specific energy loss of electrons is

$$\left(\frac{dE}{dx}\right)_{tot} = \left(\frac{dE}{dx}\right)_{coulomb} + \left(\frac{dE}{dx}\right)_{radiation}$$

- The ratio of specific energy loss is

$$\frac{(dE/dx)_r}{(dE/dx)_c} \cong \frac{EZ}{700}$$

Bremsstrahlung is the favored process at higher electron energies and for high-Z materials.



Minimum occurs when $v > 0.96c$

The critical energy



Specific Energy Loss of Fast Electrons

- The energy for which the two terms become equal, is called the critical energy.

$$E_c \approx \frac{700}{Z} (MeV)$$

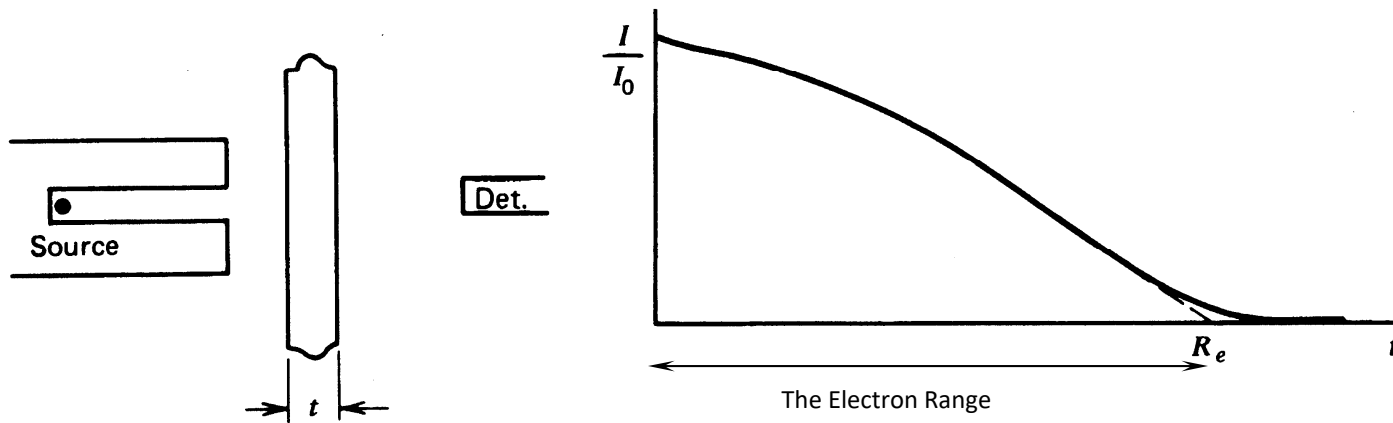
Material	Radiation Length (g/cm ²)	Critical Energy (MeV)
H ₂	63	340
Al	24	47
Ar	20	35
Fe	13.8	24
Pb	6.3	6.9
H ₂ O	36	93
NaI(Tl)	9.5	12.5
BGO	8.0	10.5

- At keV-MeV energy range, ionization is the dominant mechanism for energy loss.



Range of Fast Electrons

Range of Fast Electrons in Medium



From page 45, fig. 2.13, Radiation Detection and Measurements, Third Edition, G. F. Knoll, John Wiley & Sons, 1999.

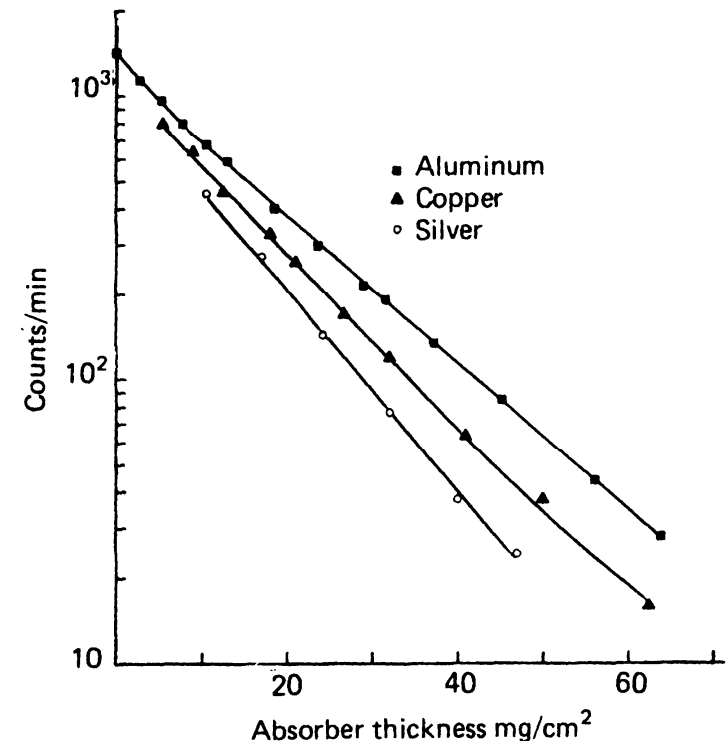
- As a very crude estimate, electron ranges tend to be about 2mm per MeV in low-density materials and 1mm per MeV in materials of moderate density.
- To a fair degree of approximation, the product of the range times the density of the absorber is a constant for different materials for electrons of equal initial energy.

Absorption of Beta Particles

- The number of beta particles emerging from an absorber of a given thickness tends to follow an exponential behavior,

$$I = I_0 e^{-nt}$$

- Note that there is no fundamental basis for interpreting this exponential behavior, as does in gamma ray attenuation.



From P.46, Radiation Detection and Measurements, Third Edition, G. F. Knoll, John Wiley & Sons, 1999.



Interaction of Positrons

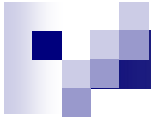
- Positron shares the major mechanism of energy loss with their negative counterparts (electrons).
- However, positrons differ significantly in the annihilation radiation process that results in 0.511MeV gamma rays.



A Useful Website

<http://physics.nist.gov/PhysRefData/contents.html>

By NIST, Physics Laboratory,



Interaction of Heavy Charged Particles



Energy Loss Mechanisms

- Heavy charged particles loss energy primarily through the ionization and excitation of atoms.
- Heavy charged particles can transfer only a small fraction of its energy in a single collision. Its deflection in collision is almost negligible. Therefore heavy charged particles travel in almost straight paths in matter, losing energy continuously through a large number of collisions with atomic electrons.
- At low velocity, a heavy charged particle may lose a negligible amount of energy in nuclear collisions. It may also pick up free electrons along its path, which reduces its net charge.
- Energetic heavy charged particle can also induce nuclear reactions.

Energy Loss Mechanisms

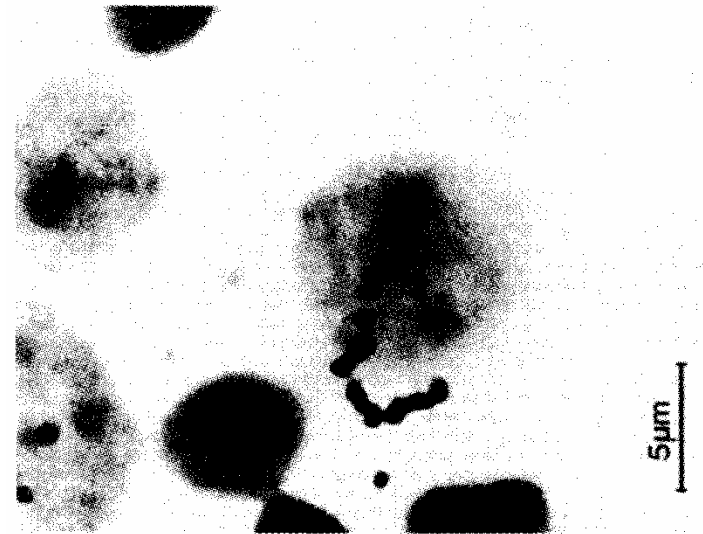
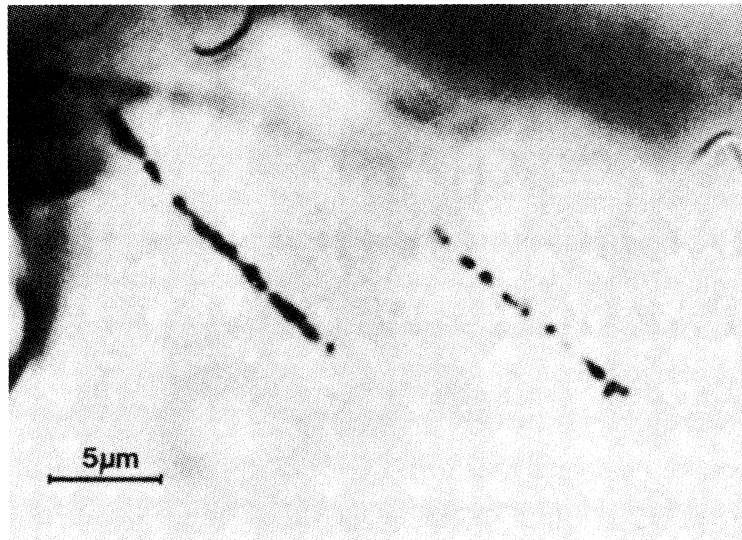


FIGURE 5.1. (Top) Alpha-particle autoradiograph of rat bone after inhalation of ^{241}Am . Biological preparation by R. Masse and N. Parmentier. (Bottom) Beta-particle autoradiograph of isolated rat-brain nucleus. The ^{14}C -thymidine incorporated in the nucleolus is located at the track origin of the electron emitted by the tracer element. Biological preparation by M. Wintzerith and P. Mandel. (Courtesy R. Rechenmann and E. Witten-dorp-Rechenmann, Laboratoire de Biophysique des Rayonnements et de Methodologie INSERM U.220, Strasbourg, France.)



Energy Loss Mechanisms

For heavy charged particles, the maximum energy that can be transferred in a single collision is given by the conservation of energy and momentum:

$$\frac{1}{2}MV^2 = \frac{1}{2}MV_1^2 + \frac{1}{2}mv_1^2$$

$$MV = MV_1 + mv_1.$$

where M and m are the mass of the heavy charged particle and the electron. V is the initial velocity of the charged particle. V_1 and v_1 are the velocities of both particles after the collision.

The maximum energy transfer is therefore given by

$$Q_{\max} = \frac{1}{2}MV^2 - \frac{1}{2}MV_1^2 = \frac{4mME}{(M + m)^2}$$

Energy Loss Mechanisms

For a more general case, which includes the relativistic effect, the maximum energy transferred by a single collision is

$$Q_{\max} = \frac{2\gamma^2 m V^2}{1 + 2\gamma m/M + m^2/M^2}$$

m: mass of a electron at rest

M: mass of the heavy charged particle

where $\gamma = 1/\sqrt{1 - \beta^2}$, $\beta = V/c$, and c is the speed of light

In extreme relativistic case, $\gamma m/M \ll 1$. So the above equation reduces to

$$Q_{\max} = 2\gamma^2 m V^2 = 2\gamma^2 m c^2 \beta^2$$

TABLE 5.1. Maximum Possible Energy Transfer, Q_{\max} , in Proton Collision with Electron

Proton Kinetic Energy E (MeV)	Q_{\max} (MeV)	Maximum Percentage Energy Transfer $100Q_{\max}/E$
0.1	0.00022	0.22
1	0.0022	0.22
10	0.0219	0.22
100	0.229	0.23
10^3	3.33	0.33
10^4	136.	1.4
10^5	1.06×10^4	10.6
10^6	5.38×10^5	53.8
10^7	9.21×10^6	92.1



Linear Stopping Power of a Medium for Heavy Charged Particles

The linear stopping power of a medium is given by the Bethe formula,

$$-\frac{dE}{dx} = \frac{4\pi k_0^2 z^2 e^4 n}{mc^2 \beta^2} \left[\ln \frac{2mc^2 \beta^2}{I(1 - \beta^2)} - \beta^2 \right].$$

In this relation

$$k_0 = 8.99 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

z = atomic number of the heavy particle,

e = magnitude of the electron charge,

n = number of electrons per unit volume in the medium,

m = electron rest mass,

c = speed of light in vacuum,

$\beta = v/c$ = speed of the particle relative to c ,

I = mean excitation energy of the medium.



Linear Stopping Power of a Medium for Heavy Charged Particles

The Bethe formula can be further simplified by substituting known constants, which gives


$$-\frac{dE}{dx} = \frac{5.08 \times 10^{-31} z^2 n}{\beta^2} \left[\ln \frac{1.02 \times 10^6 \beta^2}{I (1 - \beta^2)} - \beta^2 \right] \text{MeV cm}^{-1}$$

It is further simplified to emphasize some important quantities related to the stopping power, the “speed” of the particle β , atomic mass of the charged particle z , the number of electron per cm^3 n and the mean excitation-ionization potential I :

$$-\frac{dE}{dx} = \frac{5.08 \times 10^{-31} z^2 n}{\beta^2} [F(\beta) - \ln I] \text{MeV cm}^{-1}$$

where

$$F(\beta) = \ln \frac{1.02 \times 10^6 \beta^2}{1 - \beta^2} - \beta^2$$



Linear Stopping Power of a Medium for Heavy Charged Particles (revisited)

The linear stopping power of a medium is given by the Bethe formula,

$$-\frac{dE}{dx} = \frac{4\pi k_0^2 z^2 e^4 n}{mc^2 \beta^2} \left[\ln \frac{2mc^2 \beta^2}{I(1 - \beta^2)} - \beta^2 \right].$$

In this relation

$$k_0 = 8.99 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

z = atomic number of the heavy particle,

e = magnitude of the electron charge,

n = number of electrons per unit volume in the medium,

m = electron rest mass,

c = speed of light in vacuum,

$\beta = V/c$ = speed of the particle relative to c ,

I = mean excitation energy of the medium.

Range for Heavy Charged Particles

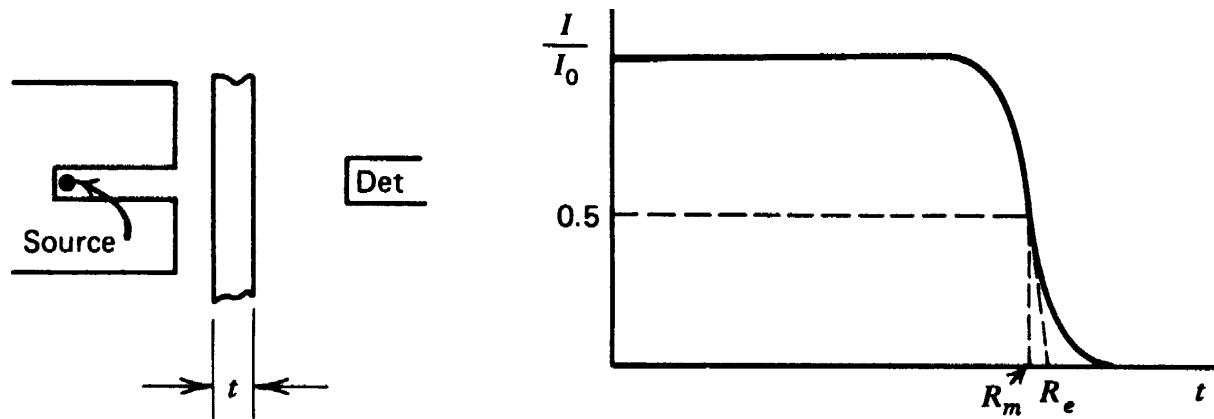


Figure 2.5 An alpha particle transmission experiment. I is the detected number of alpha particles through an absorber thickness t , whereas I_0 is the number detected without the absorber. The mean range R_m and extrapolated range R_e are indicated.

There are two related definitions of the range of heavy charged particles:

1. Mean range: the absorber thickness that reduces the alpha particle count to exactly one-half of its value in the absence of the absorber.
2. Extrapolated range: extrapolating the linear portion of the end of the transmission curve to zero.

Range for Alpha Particles

- ☞ The range of alpha particles in air (15°C, 1atm) can be approximately given by

$$\begin{aligned} R &= 0.56E, & E < 4; \\ R &= 1.24E - 2.62, & 4 < E < 8. \end{aligned}$$

where E is given in MeV and R is given in cm.

- ☞ The range of alpha particles in any other medium with a similar atomic composition can be computed from the following relationship:

$$R_m, \text{mg/cm}^2 = 0.56A^{1/3} R,$$

where A = atomic mass number of the medium,
 R = range of the alpha particle in air, cm.

- ☞ Because the effective atomic composition of tissue is not very much different from that of air, the following relationship may be used to calculate the range of alpha particles in tissue:

$$R_a \times \rho_a = R_t \times \rho_t,$$