Interconnect Parasitic Extraction

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Thanks to J. White, A. Nardi, W. Kao, L. T. Pileggi, Zhenhai Zhu

Outline

- Introduction to parasitic extraction
- Resistance extraction
- Capacitance extraction
- Inductance and impedance (RLC) extraction

Introduction to Parasitic Extraction

Introduction

- Interconnect: conductive path
- Ideally: wire only connects functional elements (devices, gates, blocks, …) and does not affect design performance
- This assumption was approximately true for “large” design, it is unacceptable for DSM designs

Introduction

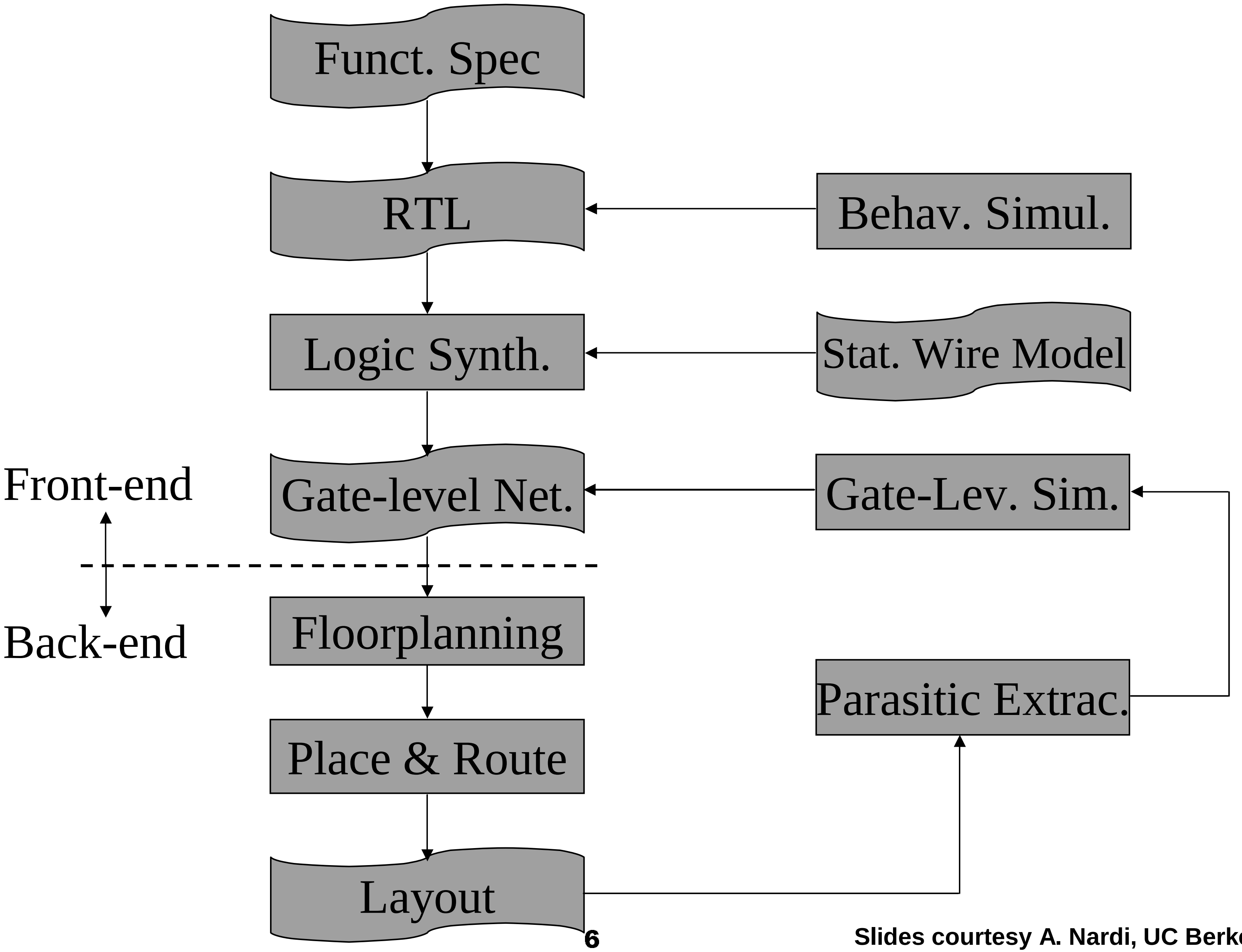
- Real wire has:
  - Resistance
  - Capacitance
  - Inductance

- Therefore wiring forms a complex geometry that introduces capacitive, resistive and inductive parasitics. Effects:
  - Impact on delay, energy consumption, power distribution
  - Introduction of noise sources, which affects reliability

⇒ To evaluate the effect of interconnects on design performance we have to model them

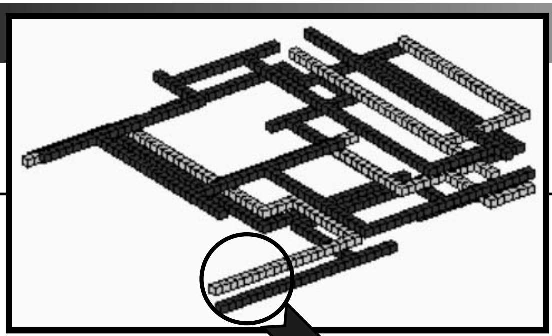
Slides courtesy A. Nardi, UC Berkeley

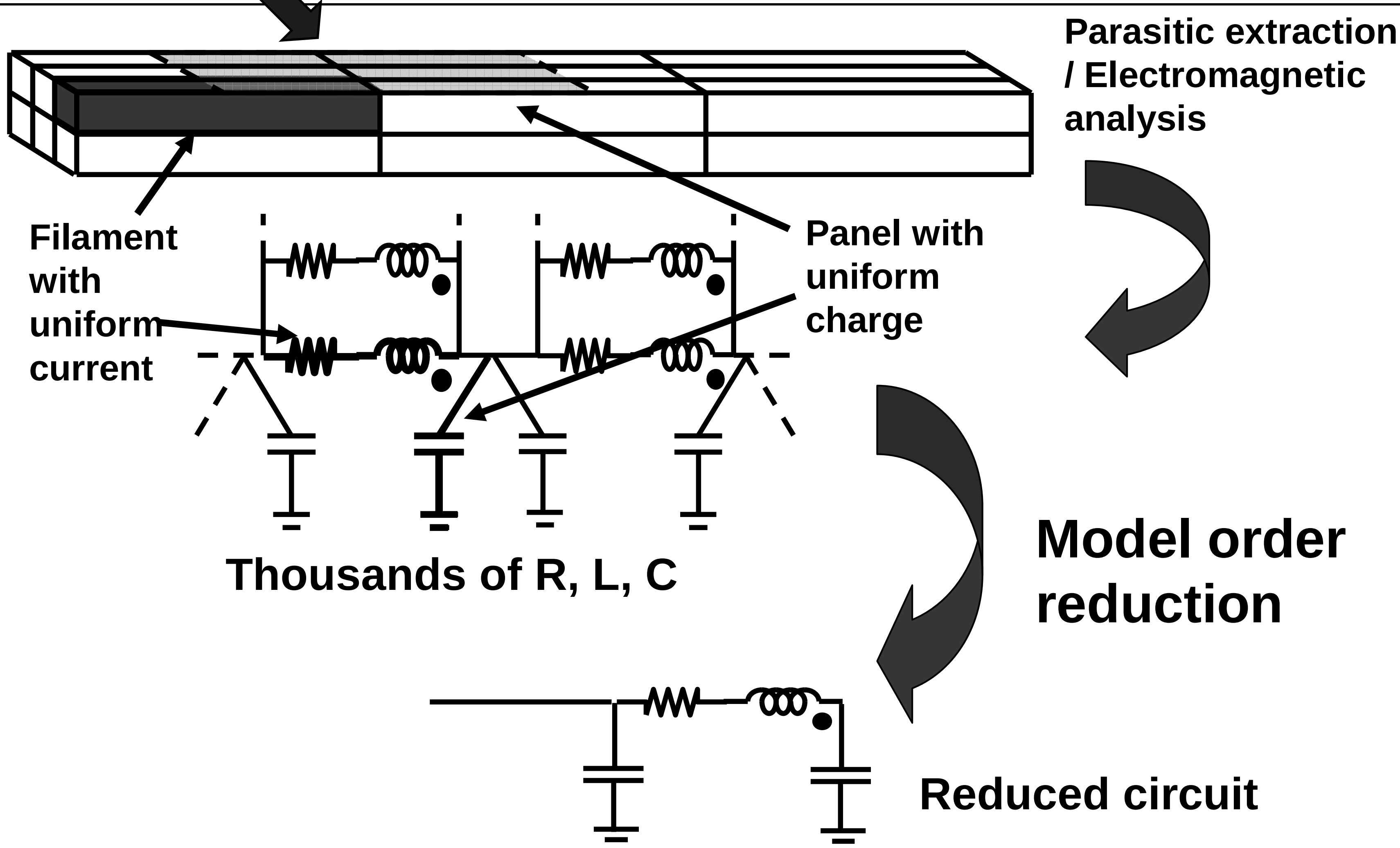
Conventional Design Flow



Slides courtesy A. Nardi, UC Berkeley

From electro-magnetic analysis to circuit simulation





Challenges for parasitic extraction

- Parasitic Extraction
  - As design get larger, and process geometries smaller than 0.35μm, the impact of wire resistance, capacitance and inductance (aka parasitics) becomes significant
  - Give rise to a whole set of signal integrity issues
- Challenge
  - Large run time involved (trade-off for different levels of accuracy)
  - Fast computational methods with desirable accuracy

Resistance Extraction

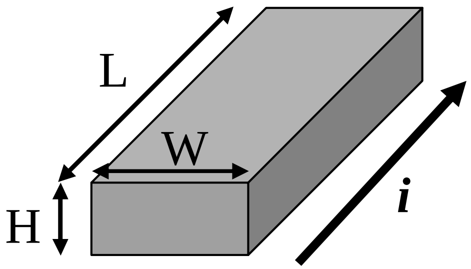
Outline

- Introduction to parasitic extraction
- Resistance extraction
  - Problem formulation
  - Extraction techniques
  - Numerical techniques
  - Other issues
- Capacitance extraction
- Inductance and RLC extraction

Resistance extraction

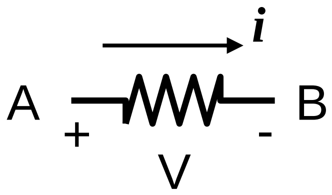
- Problem formulation
  - A simple structure

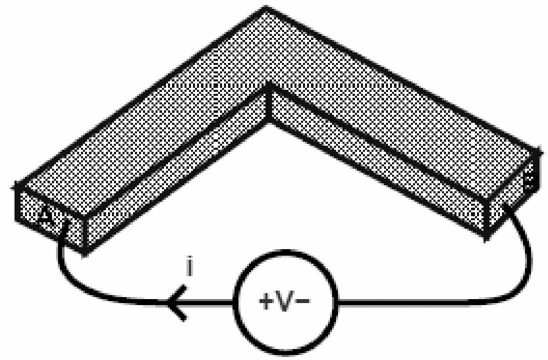
$$R = \frac{V}{i} = \frac{\rho L}{S} = \frac{\rho L}{HW}$$



  - Two-terminal structure

$$R = \frac{V}{i}$$

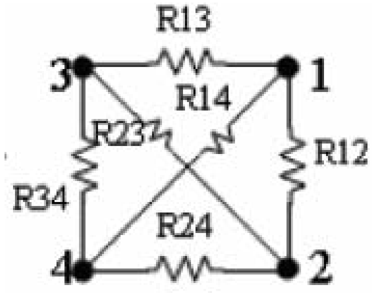


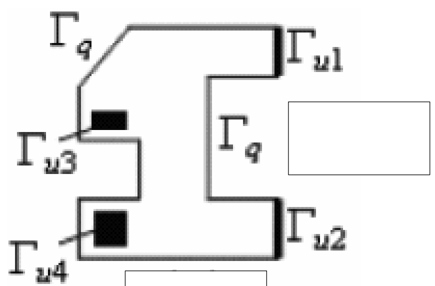


It's a single R value

  - Multi-terminal (port) structure

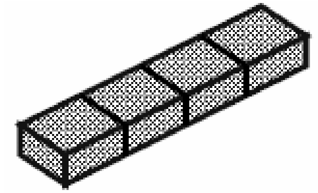
NxN *R* matrix;
diagonal entry is undefined



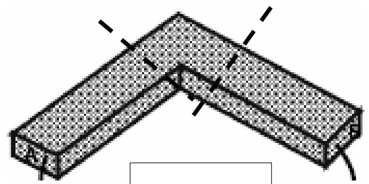


Resistance extraction

- Extraction techniques
  - Square counting $R = R_{\square} \cdot \frac{L}{W}$



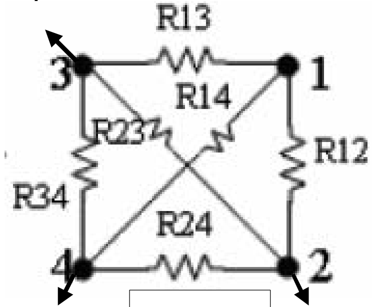
  - Analytical approximate formula
    - For simple corner structure

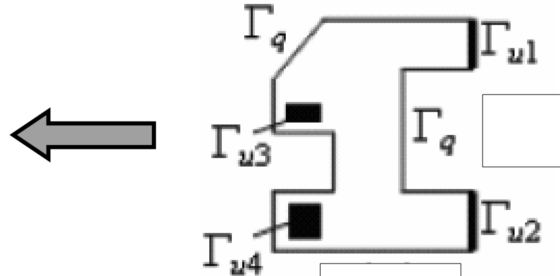


  - 2-D or 3-D numerical methods
    - For multi-terminal structure; current has irregular distribution
    - Solve the steady current field for $\boldsymbol{i}$ under given bias voltages
    - Set $V_1 = 1$, others all zero.

flowing-out current $i_{1k} = \frac{1}{R_{1k}}$

Repeating it with different settings





Resistance extraction

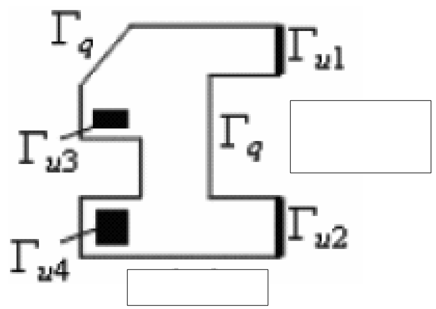
- Extraction techniques – numerical method
  - How to calculate the flowing-out current ? Field solver
  - Field equation and boundary conditions

Laplace equation inside conductor:

$$\nabla \cdot \sigma \nabla u = 0 \Longrightarrow \nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

divergence ***E***

Boundary conditions:



port surface Γ_{uk}: u is known

Normal component is zero; current can not flow out

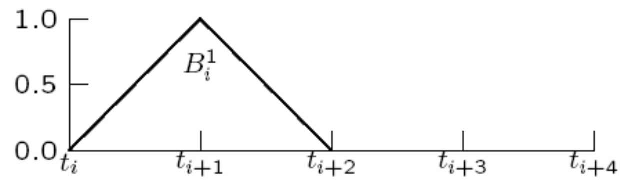
other surface: $E_n = \frac{\partial u}{\partial n} = 0$

The BVP of Laplace equation becomes solvable

$$\frac{1}{R_{jk}} = \int_{\Gamma_{uk}} \sigma \cdot \frac{\partial u}{\partial n} \, d\Gamma$$

Resistance extraction

- Numerical methods for resistance extraction
  - Methods for the BVP of elliptical PDE: $\nabla^2 u = 0$
  - Finite difference method
    - Derivative -> finite difference: $\frac{\partial^2 u}{\partial x^2} \rightarrow \frac{u_{i+1,j,k} - 2u_{i,j,k} + u_{i-1,j,k}}{(\Delta x)^2}$
    - Generate sparse matrix; for ODE and PDE
  - Finite element method
    - Express solution with local-support basis functions $\sum_{i=1}^{n} x_i \phi_i(t)$
    - construct equation system with Collocation or Galerkin method
    - Widely used for BVP of ODE and PDE

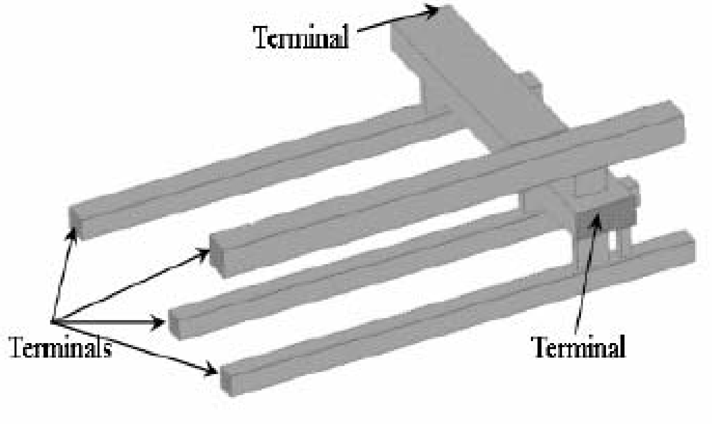


  - Boundary element method
    - Only discretize the boundary, calculate boundary value
    - Generate dense matrix with fewer unknowns

For elliptical PDE

Resistance extraction

- Where are expensive numerical methods needed ?
  - ☐ Complex onchip interconnects:
    - Wire resistivity is not constant
    - Complex 3D geometry around vias



  - ☐ Substrate coupling resistance in mixed-signal IC

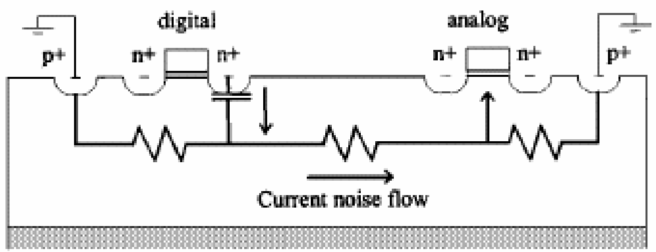


Fig. 1. Current is injected into the substrate and flows to other parts of the circuit.

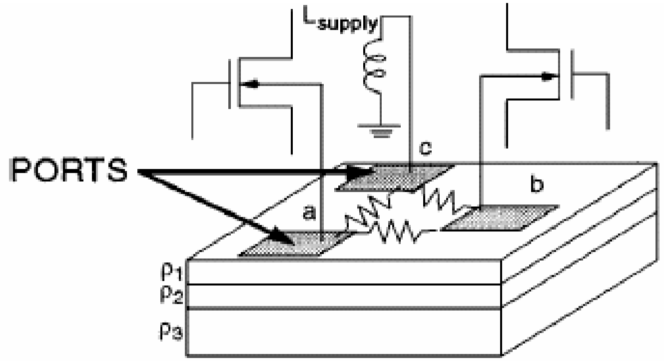


Fig. 2. Substrate ports (nodes) created to connect extracted substate resistive network to the circuit.

Resistance extraction

- All these methods calculate DC resistance
  - Suitable for analysis of local interconnects, or analysis under lower frequency
  - High frequency: R of simple geometry estimated with skin depth; R of complex geometry extracted along with L
- Reference
  - W. Kao, C-Y. Lo, M. Basel and R. Singh, “Parasitic extraction: Current state of the art and future trends,” *Proceedings of IEEE*, vol. 89, pp. 729-739, 2001.
  - Xiren Wang, Deyan Liu, Wenjian Yu and Zeyi Wang, "Improved boundary element method for fast 3-D interconnect resistance extraction," *IEICE Trans. on Electronics*, Vol. E88-C, No.2, pp.232-240, Feb. 2005.

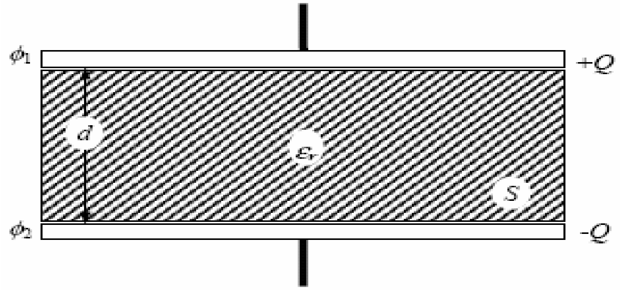
Capacitance Extraction

Outline

- Introduction to parasitic extraction
- Resistance extraction
- Capacitance extraction
  - Fundamentals and survey
  - Volume discretization method
  - Boundary element method
  - Future issues
- Inductance and RLC extraction

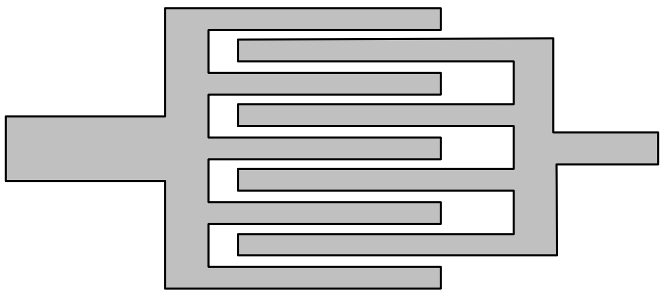
Capacitance extraction

- Problem formulation
  - A parallel-plate capacitor
    - Voltage: $V = \phi_1 - \phi_2$
    - Q and –Q are induced on both plates; Q is proportional to V
    - The ratio is defined as C: C=Q/V
    - If the dimension of the plate is large compared with spacing *d*,

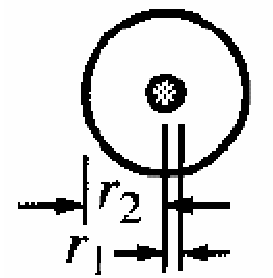


$$C = \frac{\varepsilon_0 \varepsilon_r S}{d}, \qquad \varepsilon_0 = \frac{1}{4\pi \times 9 \times 10^9} = 8.85 \times 10^{-12} C^2 / N \cdot m^2$$

  - Other familiar capacitors



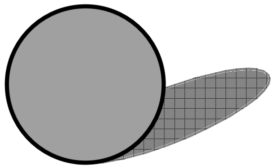
interdigital capacitor



Coaxial capacitor

Capacitance extraction

- Problem formulation
  - Capacitance exists anywhere !
  - Single conductor can have capacitance
    - Conductor sphere $C = 4\pi\varepsilon_0 R$



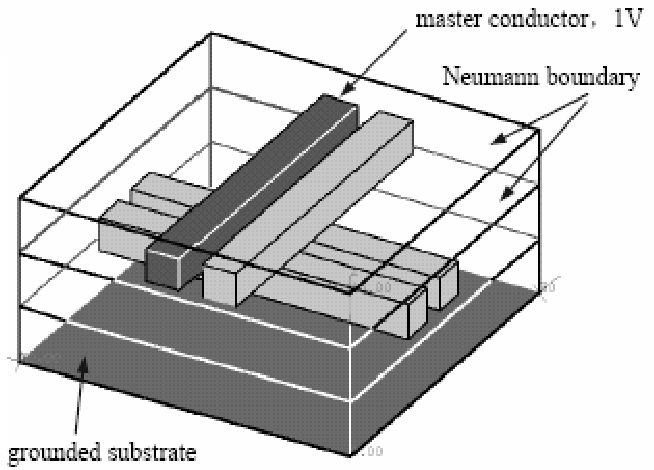
  - N-conductor system, capacitance matrix is defined:

$$[Q]=[C]\cdot[U] \qquad Q_i = \sum_{j=1}^{N} C_{ij}U_j , \qquad i=1, 2, \ldots, N,$$

C_{ij} $(i\neq j)$ **Coupling capacitance**

C_{ii} **Total capacitance**

U_j **Electric potential**



Capacitance extraction

- Interconnect capacitance extraction
  - Only simple structure has analytical formula with good accuracy
  - Different from resistance, capacitance is a function of not only wire's own geometry, but its environments
  - All methods have error except for considering the whole chip; But electrostatic has locality character

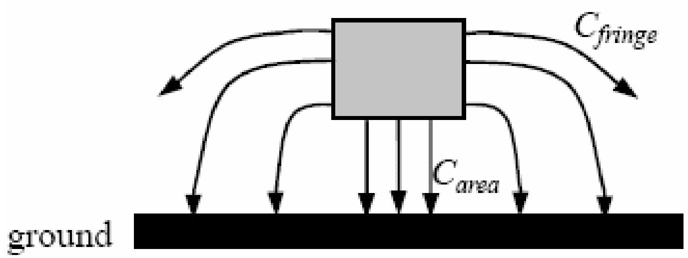
Shield; window

Stable model

  - Technique classification:
    - analytical and 2-D methods

C /unit length

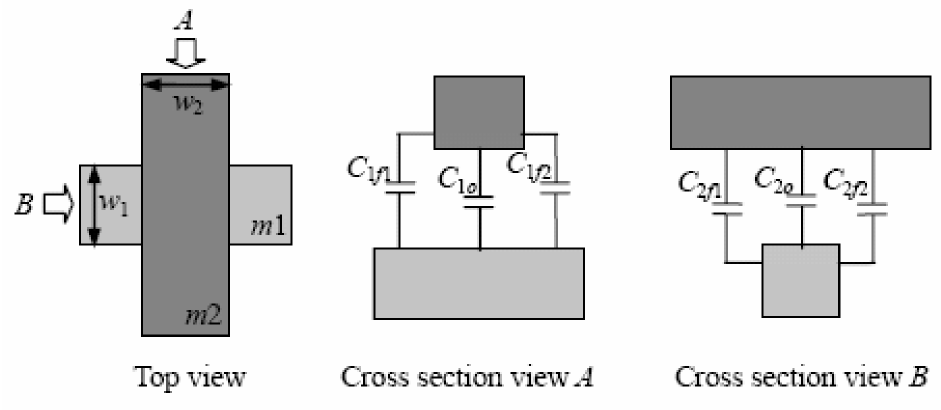
$$C = C_{area} + C_{fringe} = \frac{\varepsilon \cdot w}{d} + \frac{2\pi\varepsilon}{\log(d / H)}$$



2-D method ignores 3-D effect, using numerical technique to solve cross section geometry

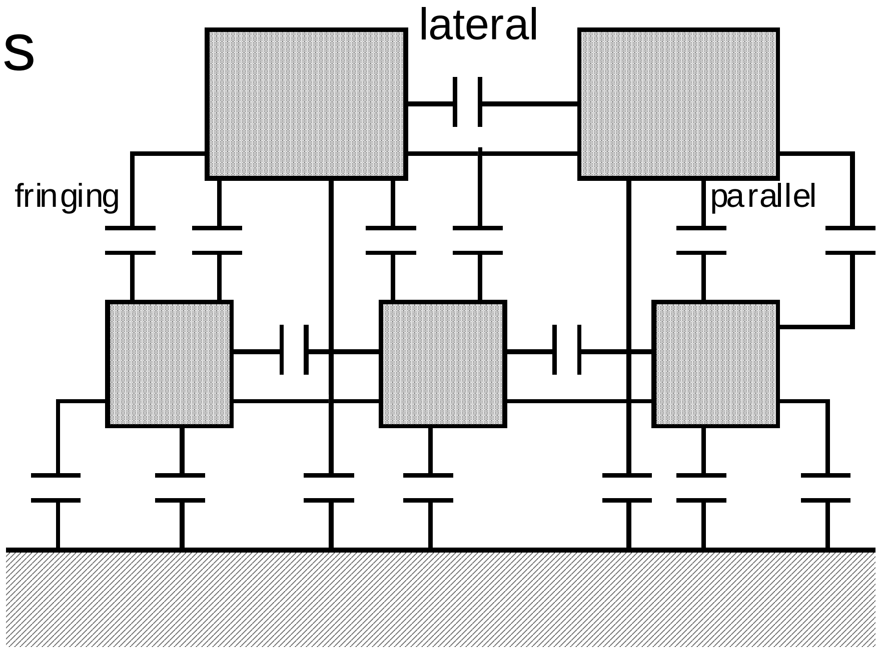
Capacitance extraction

- Interconnect capacitance extraction
  - analytical and 2-D methods
  - 2.5-D methods



$$C_{m1,\,m2}=C_A\times w_1+(C_B\text{-}C_{2o})\times w_2$$

Error > 10%

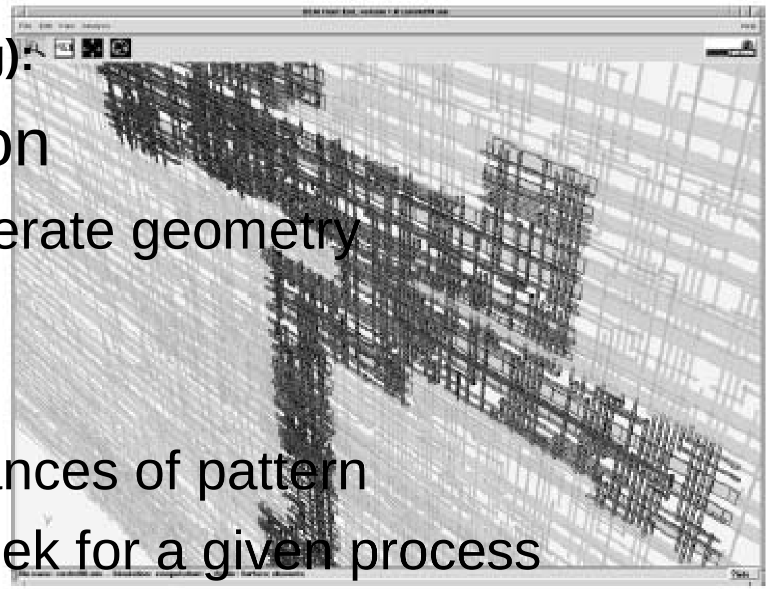


From "Digital Integrated Circuits", 2nd Edition, Copyright 2002 J. Rabaey et al.

  - Commercial tools
    - Task: full-chip, full-path extraction
    - Goal: error ≤10%, runtime ~ overnight for given process

Capacitance extraction

- Interconnect capacitance extraction
  - Commercial tools**(pattern-matching):**
  - Geometric parameter extraction
    - According to given process, generate geometry patterns and their parameters
  - Build the pattern library
    - Field solver to calculate capacitances of pattern
    - This procedure may cost one week for a given process
  - Calculation of C for real case
    - Chop the layout into pieces
    - Pattern-matching
    - Combine pattern capacitances
  - Error: pattern mismatch, layout decomposition



Cadence - Fire & Ice

Synopsys - Star RCXT

Mentor - Calibre xRC

Capacitance extraction

- 3-D numerical methods
  - Model actual geometry accurately; highest precision
  - Shortage: capacity, running time
  - Current status: widely investigated as research topic; used as library-building tool in industry, or for some special structures deserving high accuracy
- Motivation
  - The only golden value
  - Increasing important as technology becomes complicated
  - Algorithms for C extraction can be directly applied to R extraction; even extended to handle L extraction

Capacitance extraction

- Technology complexity
  - □ Dielectric configuration
    - Conformal dielectric
    - Air void
    - Multi-plane dielectric
  - □ Metal shape and type
    - Bevel line
    - Trapezoid cross-section
    - Floating dummy-fill

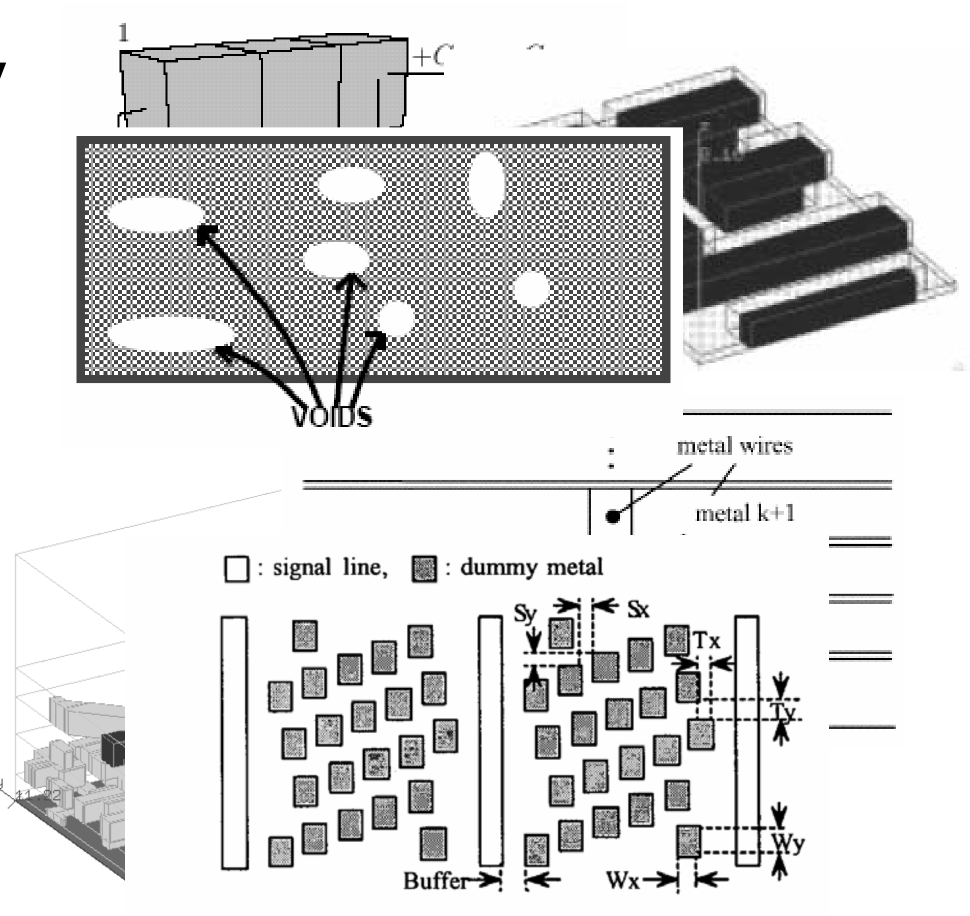


Fig. 3. The floating dummy-fills of the dot-array type (Top view).

- They are the challenges, even for 3-D field solver

Capacitance extraction

- 3-D numerical methods – general approach
  - Set voltages on conductor; solve for Q_i

$$Q_i = \sum_{j=1}^{N} C_{ij} U_j \, , \qquad i=1, 2, \ldots, N,$$

Solve the electrostatic field for *u*, then $Q_i = \int_{\Gamma_i} \varepsilon \cdot \frac{\partial u}{\partial n}$

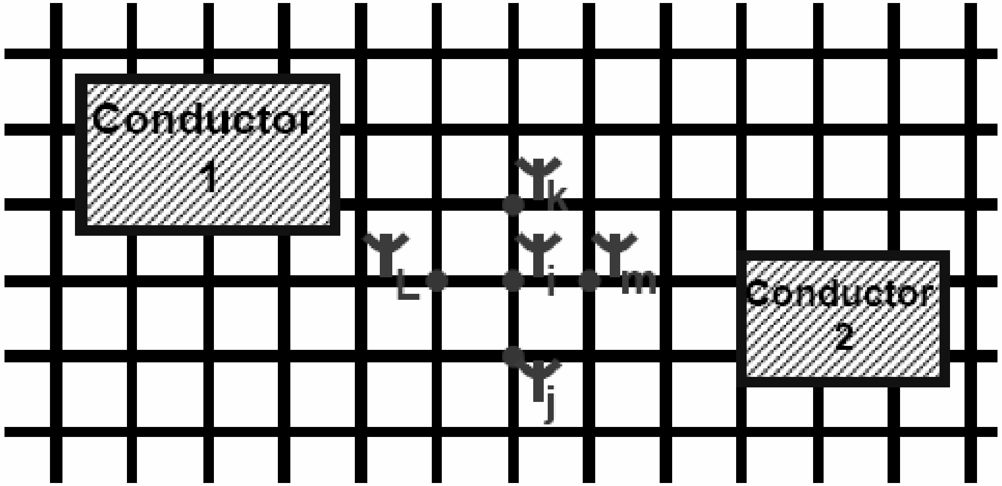
  - Global method to get the whole matrix
- Classification
  - Volume discretization: FDM, FEM
  - Boundary integral (element) method
  - Stochastic method
  - Others – semi-analytical approaches

Raphael's RC3 - Synopsys
SpiceLink, Q3D – Ansoft

FastCap, HiCap, QBEM

QuickCap - Magma

Volume Methods - Finite-Differences/Finite-Elements

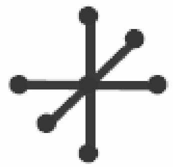


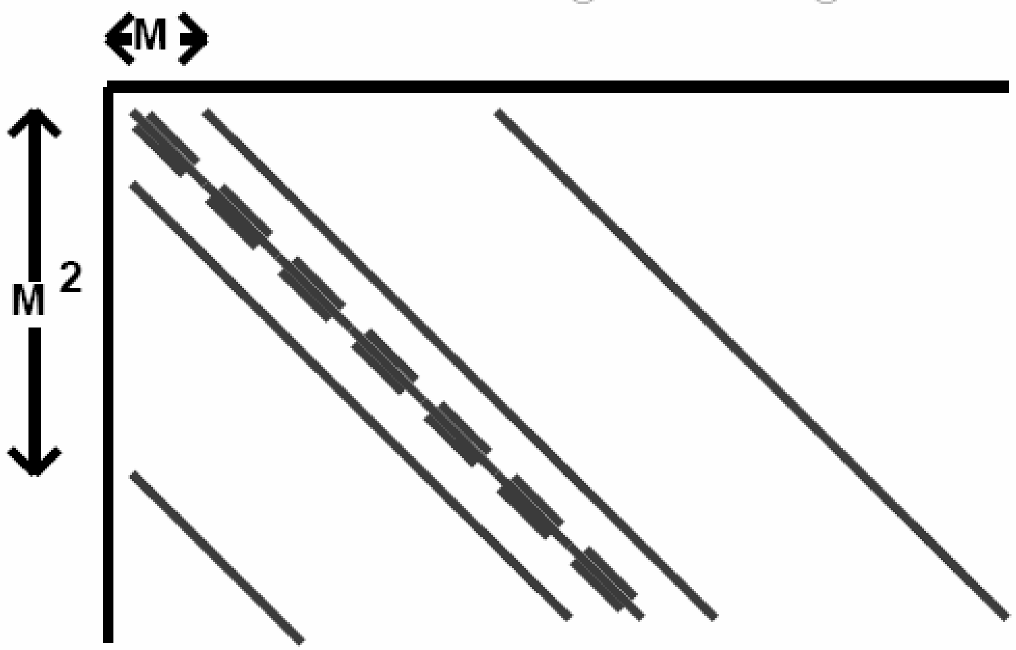
Solve Laplace's equation, $\nabla \cdot \epsilon \nabla \Psi = 0$ in the conductor exterior.

- Approximate derivatives by finite-differences.
- Conductors provide potential boundary conditions (e.g., 1 on conductor 1, zero on conductor 2).
- 2-D example

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} \approx \frac{\frac{\Psi_m - \Psi_i}{x_m - x_i} - \frac{\Psi_i - \Psi_L}{x_i - x_L}}{0.5\left((x_m - x_i) + (x_i - x_L)\right)} + \frac{\frac{\Psi_k - \Psi_i}{y_l - y_i} - \frac{\Psi_i - \Psi_j}{y_i - y_j}}{0.5\left((y_l - y_i) + (y_i - y_L)\right)}$$

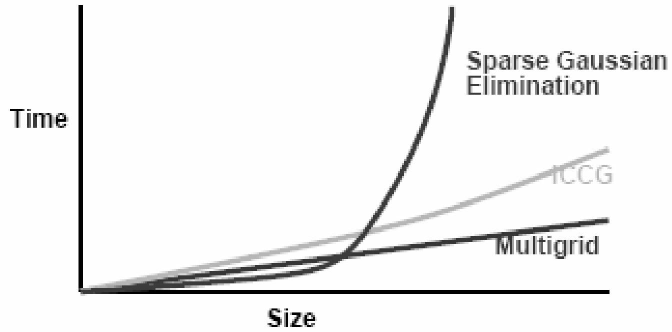
Volume Methods generate sparse matrices

- One equation for each grid node 
- In 3-D, each equation involves at least 7 variables
  - Up-Down for $\frac{\partial}{\partial z}$, Left-Right for $\frac{\partial}{\partial x}$, Backward-Forward for $\frac{\partial}{\partial y}$
- Sparse matrix for an $M \times M \times M$ grid is Large



Slides courtesy J. White, MIT

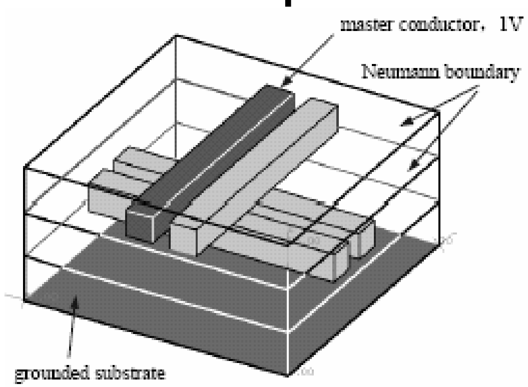
Matrix Solution Methods.

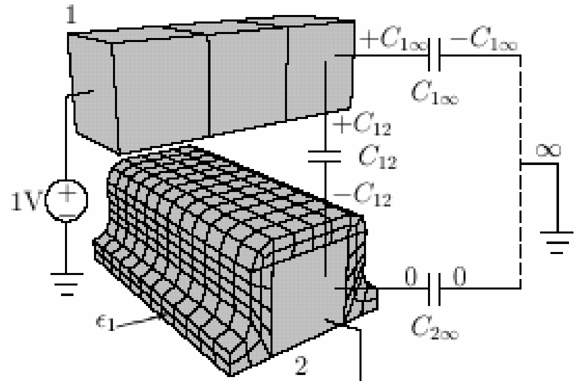


- Sparse Gaussian Elimination
  - Direct, complicated data structures, existing packages.
  - Order N^2 time and storage.
- Incomplete Cholesky Conjugate-Gradient Method (ICCG)
  - Iterative, easy to program, fast when ground planes nearby.
  - Order $NlogN$ time and Order N storage.
- Multigrid methods
  - Iterative, complicated for general grids, fast convergence.
  - Order N time and storage.

Capacitance extraction

- Volume methods
  - □ What's the size of simulation domain ?
  - □ Two kinds of problem: finite domain and infinite domain



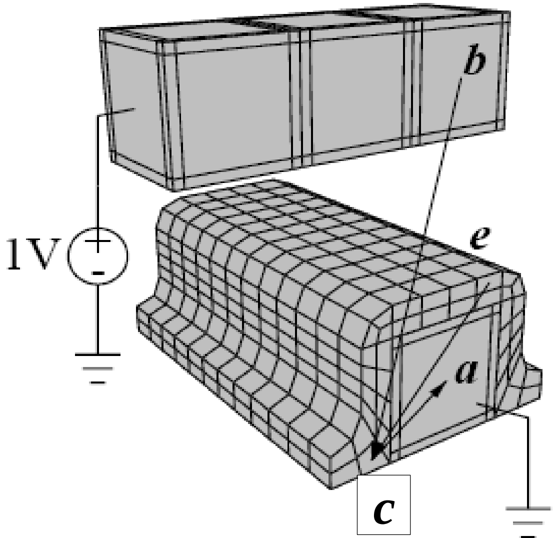


  - □ Which one is correct ? both in most time
    - 3-D extraction is not performed directly on a "real" case
    - In the chopping & combination procedure, both models used
    - Because of attenuation of electric field, the results from two models can approach to each other

Because of its nature, volume methods use finite-domain model

Integral Formulation Example

inside alg. of FastCap



- Influence of charge on panel c at the center of panel a is

$$\frac{q_c}{A_c}\int_{\text{panel } c}\frac{1}{r_{ac}}\,dA.$$

- Potential at panel a is sum of all contributions:

$$v_a = \cdots + q_c\left(\frac{1}{A_c}\int_{\text{panel } c}\frac{1}{r_{ac}}\,dA\right) + \cdots + q_b\left(\frac{1}{A_b}\int_{\text{panel } b}\frac{1}{r_{ab}}\,dA\right) + \cdots.$$

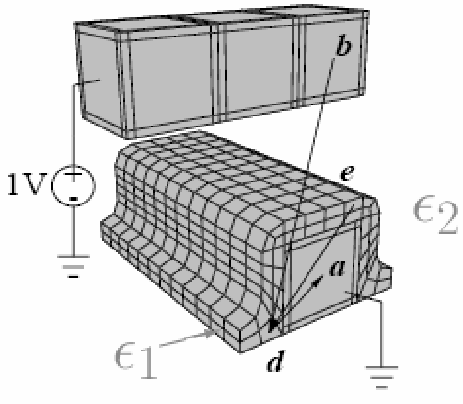
MoM (method of moment)

Method of virtual charge

Indirect boundary element method

Slides courtesy J. White, MIT

Include the Effects of the Dielectric Interfaces



- Dielectric panel d's charge contributes to v_a, as did b and c.
- To force $0 = \epsilon_1 E_{n1} - \epsilon_2 E_{n2}$ at panel d's center:

Polarized charge

$$0 = \cdots + q_e \left[(\epsilon_1 - \epsilon_2) \frac{\partial}{\partial \hat{n}} \frac{1}{A_e} \int_{\text{panel } e} \frac{1}{r_{de}} dA \right]$$
$$+ \cdots + q_b \left[(\epsilon_1 - \epsilon_2) \frac{\partial}{\partial \hat{n}} \frac{1}{A_b} \int_{\text{panel } b} \frac{1}{r_{db}} dA \right] + \cdots .$$

Slides courtesy J. White, MIT

Pack into Matrices

$$\begin{bmatrix} v_1 \\ \vdots \\ v_{n_p} \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1n} \\ \vdots & \vdots & & \vdots \\ P_{n_p 1} & P_{n_p 2} & \cdots & P_{n_p n} \\ E_{n_p+1,\,1} & E_{n_p+1,\,2} & \cdots & E_{n_p+1,\,n} \\ \vdots & \vdots & & \vdots \\ E_{n1} & E_{n2} & \cdots & E_{nn} \end{bmatrix} \begin{bmatrix} q_1 \\ \vdots \\ q_{n_p} \\ q_{n_p+1} \\ \vdots \\ q_n \end{bmatrix}$$

$$P_{ij} \triangleq \frac{1}{A_j} \int_{\text{panel } j} \frac{1}{r_{ij}}\, dA;$$

$$E_{ij} \triangleq \frac{\partial}{\partial \hat{n}} \frac{1}{A_j} \int_{\text{panel } j} \frac{1}{r_{ij}}\, dA, \quad i \neq j.$$

Slides courtesy J. White, MIT

Solve $A\vec{x} = \vec{b}$ System

where $A = \begin{bmatrix} P \\ E \end{bmatrix}$ is a (# of panels)×(# of panels) dense matrix.

- Direct methods like Gaussian Elimination require n^3 operations.
- Iterative methods such as GMRES requires n^2 operations.
- Both direct and iterative methods require n^2 storage.

Slides courtesy J. White, MIT

Solve $A\vec{x} = \vec{b}$ Iteratively

1. Guess charges, $\tilde{\vec{x}}$.
2. Calculate $A\tilde{\vec{x}}$—costs $O(n^2)$.
3. If $A\tilde{\vec{x}}$ does not equal b, fix $\tilde{\vec{x}}$.

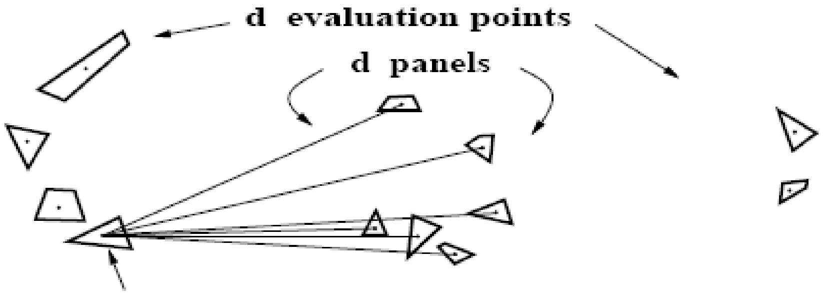
Speed Up $A\vec{x}$ Product

- Computing $A\vec{x}$ is equivalent to computing n potentials and electric fields from n charges.
- Accelerate matrix-vector products using potential approximations.
- Save Memory by not forming A

Slides courtesy J. White, MIT

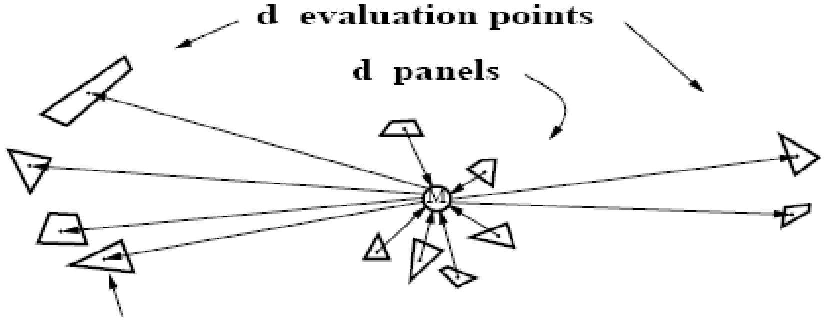
The Multipole Idea

Direct Potential Evaluation



- Computing d potentials due to d panels costs d^2 operations.

Multipole Potential Evaluation



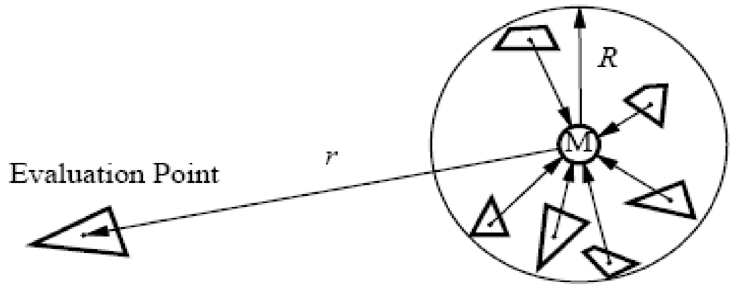
- Multipole Approximations compute d potentials due to d panels is order d operations.

Multipole expansion with order l

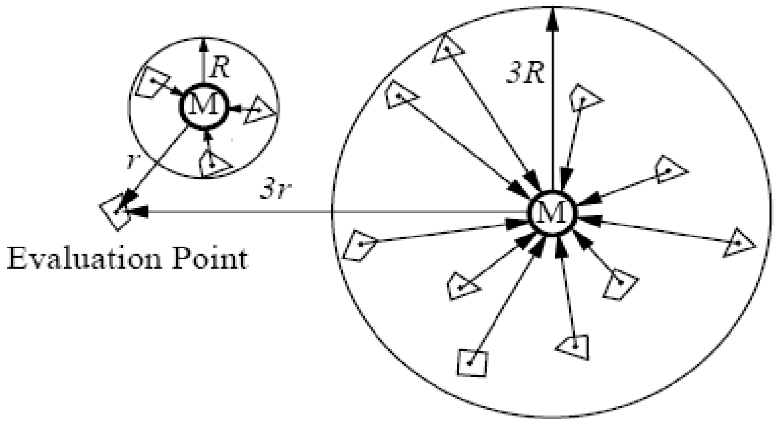
l=0: $$\frac{\sum_{i=1}^{n_2} q_i}{r_j},$$

Slides courtesy J. White, MIT

The Multipole Difficulty

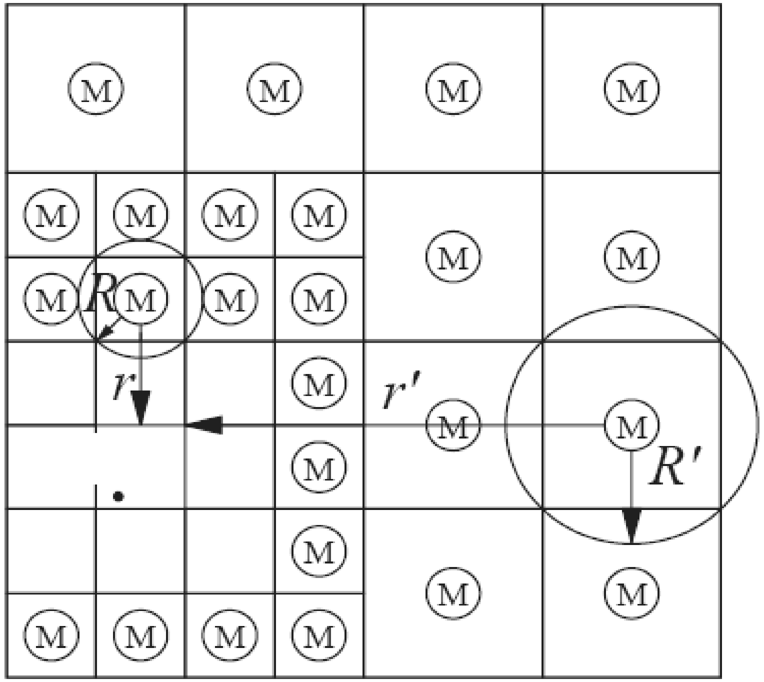


$$\text{Evaluation Error} \leq K\left(\frac{R}{r}\right)^{order+1}$$



$$\text{Evaluation Error} \leq K\left(\frac{R}{r}\right)^{order+1} = K\left(\frac{3R}{3r}\right)^{order+1}$$

Multipole Algorithm Hierarchy



- Bounded error:

$$\text{Error} \leq K \left(\frac{R}{r}\right)^{order+1} \leq K \left(\frac{1}{2}\right)^{order+1}$$

- $order = 2$ gives good results.
- Order n operations for all n potential evaluations.

Slides courtesy J. White, MIT

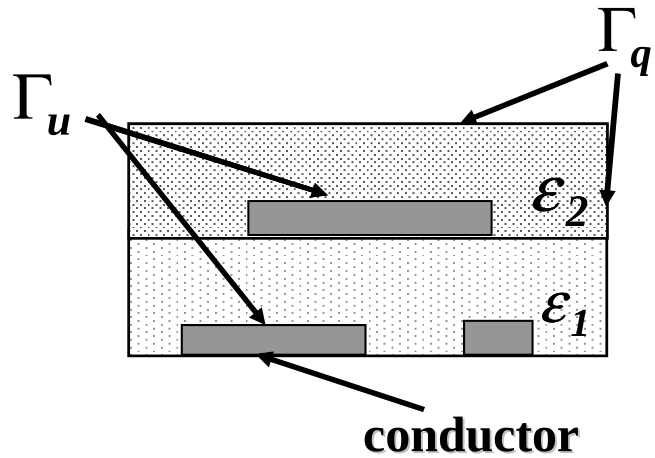
Capacitance extraction

- 3-D numerical methods – direct BEM
  - Field equation and boundary conditions

Laplace equation in dielectric region:

$$\nabla \cdot \varepsilon \nabla u = 0 \Longrightarrow \nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

divergence ***E***



Boundary conditions:

conductor surface Γ_u : u is known

Neumann boundary: $E_n = \frac{\partial u}{\partial n} = 0$

Finite-domain model, Nuemann boundary condition

Inside alg. of QBEM

Capacitance extraction

- 3-D numerical methods – direct BEM

Scalar field

  - Green's Identity $\int_{\Omega}(u\nabla^2 v - v\nabla^2 u)d\Omega = \oint_{\Gamma}(u\frac{\partial v}{\partial \boldsymbol{n}} - v\frac{\partial u}{\partial \boldsymbol{n}})d\Gamma$
  - Free-space Green's function as weighting function
  - The Laplace equation is transformed into the BIE:

$$c_s u_s + \int_{\partial\Omega_i} q_s^* u \; d\Gamma = \int_{\partial\Omega_i} u_s^* q \; d\Gamma$$

***s* is a collocation point**

***u_s^** is the fundamental solution of Laplace equation**

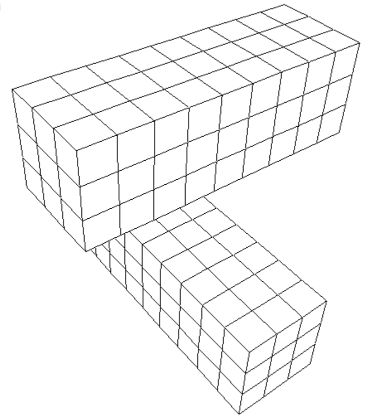
$$u_s^*(r) = \frac{1}{4\pi r}$$

More details:

C. A. Brebbia, ***<u>The Boundary Element Method for Engineers</u>***, London: Pentech Press, 1978

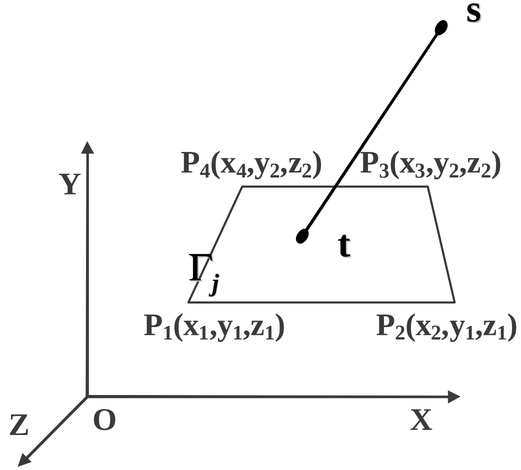
Direct BEM for Cap. Extraction

- Discretize domain boundary
  - Partition quadrilateral elements with constant interpolation
  - Non-uniform element partition
  - Integrals (of kernel $1/r$ and $1/r^3$) in discretized BIE:



$$\boxed{c_s u_s} + \sum_{j=1}^{N} \left(\int_{\Gamma_j} q_s^* d\Gamma \right) u_j = \sum_{j=1}^{N} \left(\int_{\Gamma_j} u_s^* d\Gamma \right) q_j$$

  - Singular integration
  - Non-singular integration
    - Dynamic Gauss point selection
    - Semi-analytical approach improves computational speed and accuracy for near singular integration



Direct BEM for Cap. Extraction

- Write the discretized BIEs as:

$$\boldsymbol{H}^i \cdot \boldsymbol{u}^i = \boldsymbol{G}^i \cdot \boldsymbol{q}^i, \quad (i=1, \ldots, M)$$

Compatibility equations along the interface

$$\begin{cases} \varepsilon_a \cdot \partial u_a / \partial \boldsymbol{n}_a = -\varepsilon_b \cdot \partial u_b / \partial \boldsymbol{n}_b \\ u_a = u_b \end{cases}$$

$$\boldsymbol{A}\boldsymbol{x} = \boldsymbol{f}$$

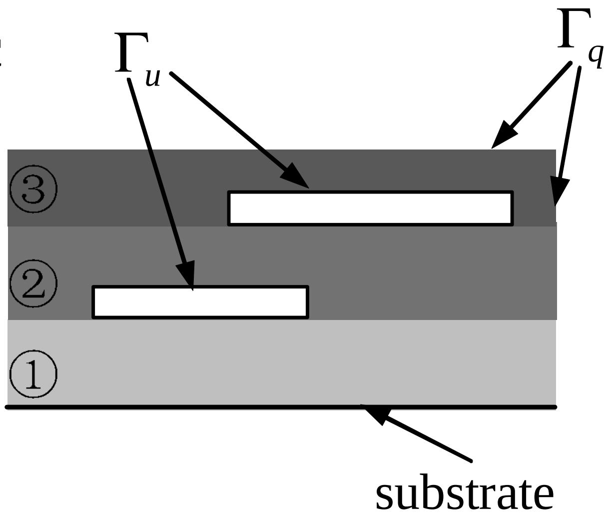
- **Non-symmetric large-scale matrix A**
- **Use GMRES to solve the equation**
- **Charge on conductor is the sum of *q***

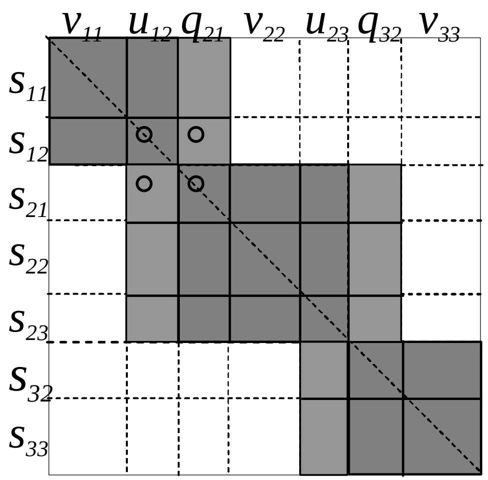
For problem involving multiple regions, matrix *A* exhibits sparsity!

Fast algorithms - QMM

- Quasi-multiple medium method
  - In each BIE, all variables are within same dielectric region; this leads to sparsity when combining equations for multiple regions

3-dielectric structure





Population of matrix *A*

QMM !

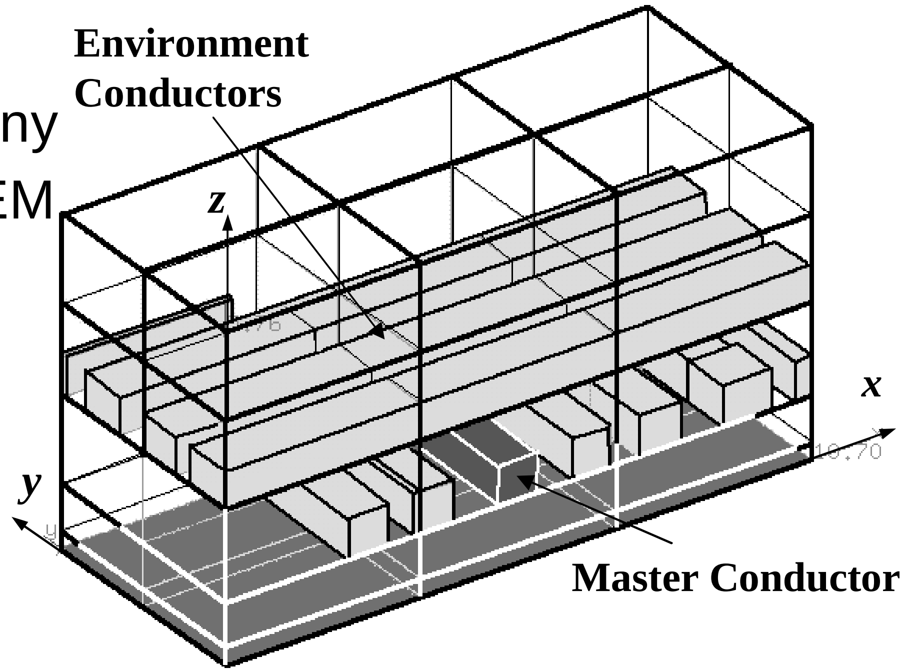
  - Make fictitious cutting on the normal structure, to enlarge the matrix sparsity in the direct BEM simulation.
  - With iterative equation solver, sparsity brings actual benefit.

Fast algorithms - QMM

- QMM-based capacitance extraction
  - Make QMM cutting
  - Then, the new structure with many subregions is solved with the BEM
- Time analysis
  - while the iteration number dose not change a lot

$$t \propto Z$$

  - *Z*: number of non-zeros in the final coefficient matrix ***A***



A 3-D multi-dielectric case within finite domain, applied 3×2 QMM cutting

Guaranteed by efficient matrix organization and preconditioned GMRES solver

FastCap vs. QBEM

Contrast

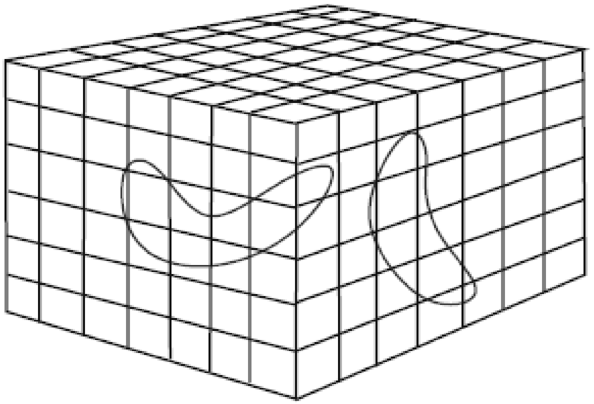
	FastCap	**QBEM**
Formulation	Single-layer potential formula	Direct boundary integral equation
System matrix	Dense	Dense for single-region, otherwise sparse
Matrix degree	N, the number of panels	A little larger than N
Acceleration	Multipole method: less than N^2 operations in each matrix-vector product	QMM method -- maximize the matrix sparsity: much less than N^2 operations in each matrix-vector product
Other cost	Cube partition and multipole expansion are expensive	Efficient organizing and storing of sparse matrix make matrix-vector product easy

- **Resemblance:**
  - **boundary discretization**
  - **stop criterion of 10^{-2} in GMRES solution**
  - **similar preconditioning**
  - **almost the same iteration number**

Volume-based vs. Surface-based formulations

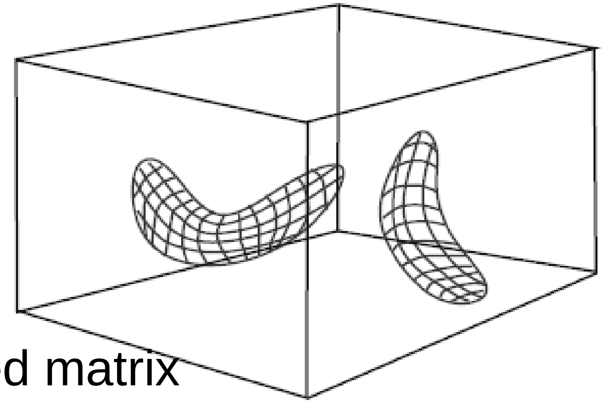
Finite-difference/Finite-Element

- Mesh & solve for entire physical domain.
- Sparse matrix problems.
- Order Volume N Multigrid Solvers.



Boundary elements

- Mesh & solve for only surface unknowns.
- Dense matrix problems.
- Order Surface N Fast Solvers.



Sparse blocked matrix
each block is dense

Which Extraction Method Is Best? Depends.

- Need the Self Capacitance of a Complicated Net?
  - Floating Random Walk Methods.
- Need Accurate Small Coupling Capacitances?
  - Fast Solvers for Integral Formulations.
- Have Field Dependent Dielectrics?
  - Volume Methods.

QuickCap

FastCap
FFTCap
QBEM

Raphael

Slides courtesy J. White, MIT

Capacitance extraction

- Future issues
  - Improve speed and accuracy for complex process
  - Make field solver suitable for full-chip or full-path extraction task
  - Parallelizability
  - Rough surface effect – stochastic integral equation solver
  - Process variation (multi-corner) # pattern becomes larger
  - Consider DFM issues (dummy-fill, OPC, etc)

Capacitance extraction

Reference

[1] W. Kao, C-Y. Lo, M. Basel and R. Singh, “Parasitic extraction: Current state of the art and future trends,” *Proceedings of IEEE*, vol. 89, pp. 729-739, 2001.

[2] Wenjian Yu and Zeyi Wang, “Capacitance extraction”, in *Encyclopedia of RF and Microwave Engineering* , K. Chang [Eds.], John Wiley & Sons Inc., 2005, pp. 565-576.

[3] K. Nabors and J. White, FastCap: A multipole accelerated 3-D capacitance extraction program, *IEEE Trans. Computer-Aided Design*, 10(11): 1447-1459, 1991.

[4] Y. L. Le Coz and R. B. Iverson, “A stochastic algorithm for high speed capacitance extraction in integrated circuits,” *Solid State Electronics*, 35(7): 1005-1012, 1992.

[5] J. R. Phillips and J. White, “A precorrected-FFT method for electrostatic analysis of complicated 3-D structures,” *IEEE Trans. Computer-Aided Design*, 16(10): 1059-1072, 1997

[6] W. Shi, J. Liu, N. Kakani and T. Yu, A fast hierarchical algorithm for three-dimensional capacitance extraction, *IEEE Trans. Computer-Aided Design*, 21(3): 330-336, 2002.

[7] W. Yu, Z. Wang and J. Gu, Fast capacitance extraction of actual 3-D VLSI interconnects using quasi-multiple medium accelerated BEM, *IEEE Trans. Microwave Theory Tech.*, 51(1): 109-120, 2003.

[8] W. Shi and F. Yu, A divide-and-conquer algorithm for 3-D capacitance extraction, *IEEE Trans. Computer-Aided Design*, 23(8): 1157-1163, 2004.

Inductance Extraction

Outline

- Basic
  - Two laws about inductive interaction
  - Loop inductance
- Onchip inductance extraction
  - Partial inductance & PEEC model
  - Frequency-dependent LR extraction - *FastHenry*
- Inductance or full-wave extraction with BEM
  - Maxwell equations & assumptions
  - Boundary element method

Two inductive laws

- Ampere's Law

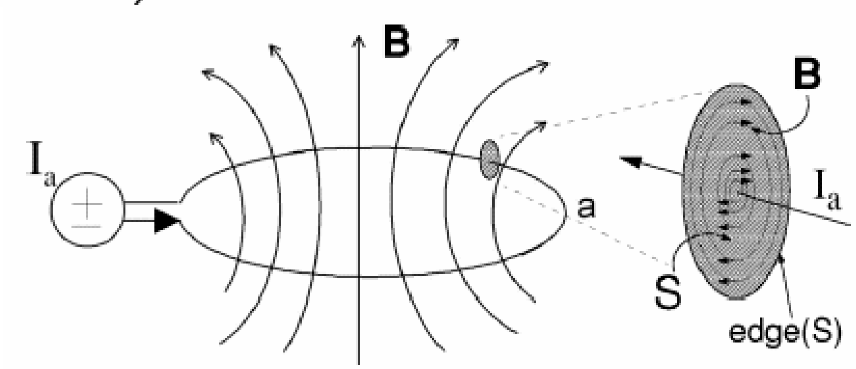
Curl operator $\vec{\nabla} \times \vec{B} = \mu \vec{j} + \mu\varepsilon \frac{d\vec{E}}{dt}$

Magnetic field created by: currents in conductor loop, time-varying electric fields

Integral form (derived via Stokes' Law):

$$\oint_{\mathrm{edge(S)}} \vec{B} \cdot d\vec{l} = \mu \int_{\mathrm{S}} \left(\vec{j} + \varepsilon \frac{d\vec{E}}{dt} \right) \cdot d\vec{S}$$

For 1D wire, field direction predicted with *right-hand rule*



$\varepsilon \frac{d\vec{E}}{dt}$ Displacement current density

AC current flowing through capacitor

Two inductive laws

- Ampere's Law (cont'd)

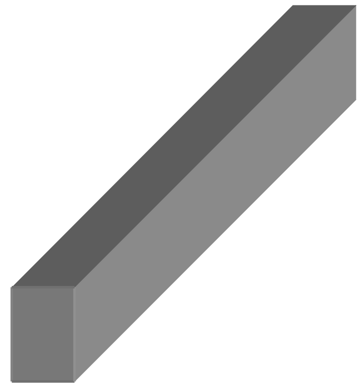
$$\vec{\nabla} \times \vec{B} = \mu \vec{j} + \mu\varepsilon \frac{d\vec{E}}{dt}$$

Current density Displacement current density

For IC, the second term is usually neglected

0.13μm technology:

Transistor switching current: 0.3mA
minimal spacing of conductor: 0.13 μm
maximal voltage difference: 2V
minimal signal ramp time: 20ps



0.13×0.26

$$\text{Ratio} = \frac{(0.3\times10^{-3})/(0.13\times0.26\times10^{-12})}{3\times8.9\times10^{-12}\times(2/(0.13\times10^{-6}))/(20\times10^{-12})} = \frac{1200}{2.6}$$

Quasi-static assumption: $\vec{\nabla} \times \vec{B} \approx \mu \vec{j}$

Decouples inductive and capacitive effects in circuit

Two inductive laws

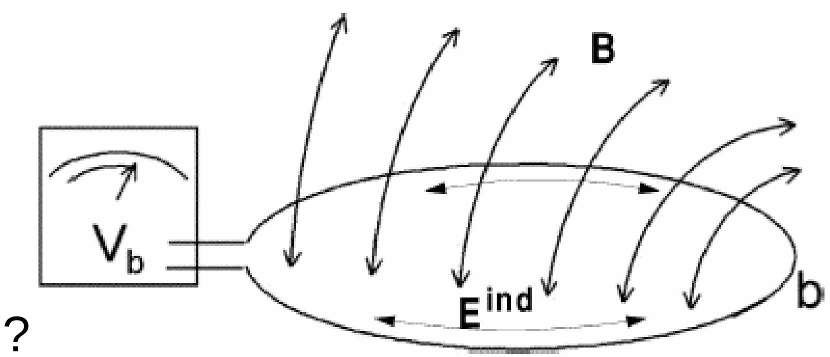
- Faraday's Law

Time-varying magnetic field creates induced electric field

$$\oint_b \vec{E}_{\text{ind}} \cdot d\,\vec{l} = -\frac{d\Phi}{dt} \quad \text{with} \quad \Phi = \int_{\text{Area of b}} \vec{B} \cdot d\,\vec{S}\,, \qquad \nabla \times \vec{E}_{\text{ind}} = -\frac{\partial \vec{B}}{\partial t}$$

Magnetic flux

This induced electric field exerts force on charges in **b**



E$_{\text{ind}}$ is a different field than the *capacitive* electric field ***E***$_{\text{cap}}$:

$$\vec{\nabla} \cdot \vec{E}_{\text{cap}} = \frac{\rho}{\varepsilon}$$

How about curl?

Both electric fields have force on charge !

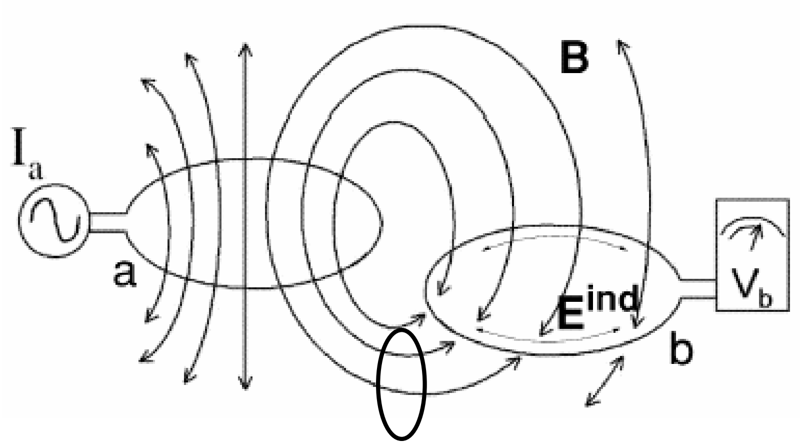
Two inductive laws

■ Faraday's Law (cont'd)

Induced voltage along the victim loop:

$$V_b^{\mathrm{ind}} = -\oint_b \vec{E}_{\mathrm{ind}} \cdot d\,\vec{l}$$

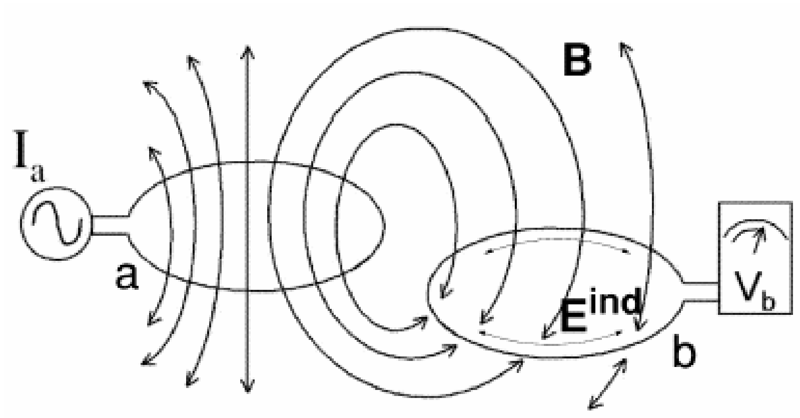
Orientation of the loop with respect to the $\boldsymbol{E}_{\mathrm{ind}}$ determines the amount of induced voltage.



Magnetic field effect on the orthogonal loop can be zero !

That's why the partial inductive couplings between orthogonal wires becomes zero

Loop inductance



■ Three equations

$$\vec{\nabla} \times \vec{B} \approx \mu \, \vec{j}$$

$$\oint_b \vec{E}_{\text{ind}} \cdot d\,\vec{l} = -\frac{d\Phi}{dt} \quad \text{with} \quad \Phi = \int_{\text{Area of b}} \vec{B} \cdot d\,\vec{S}$$

$$V_b^{\text{ind}} = -\oint_b \vec{E}_{\text{ind}} \cdot d\,\vec{l}$$

All linear relationships

Relationship between time-derivative of current and the induced voltage is linear as well:

$$V_b^{\text{ind}} = L_{ba} \frac{dI_a}{dt} \quad \text{with} \quad I_a = \int_{\text{Crossection of a}} \vec{j} \cdot d\,\vec{S}$$

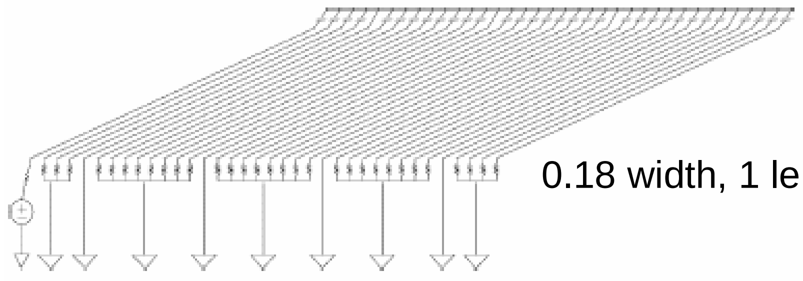
Mutual inductance; self inductance if a=b

$$L_{ba} = \frac{\Phi_b}{I_a}$$

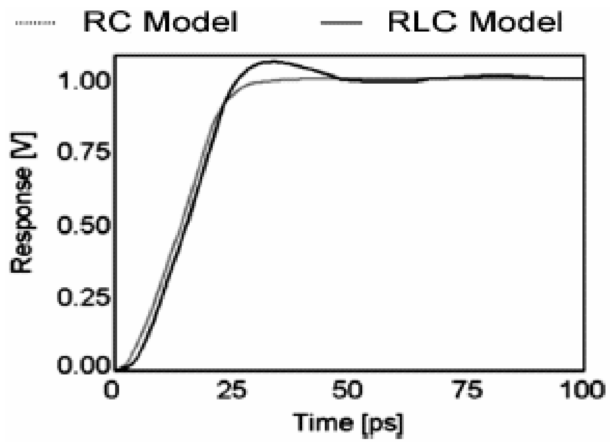
There are inductors in IC as components of filter or oscillator circuits;
There are also inductors not *deliberately designed* into IC, i.e. **parasitic** inductance

Onchip interconnect inductance

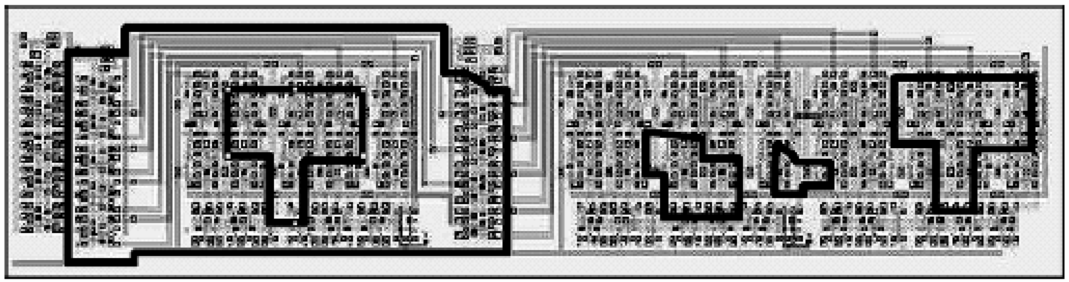
- Parasitic inductive effect
  - An example
  - Ringing behavior
  - 50% delay difference is 17%
- Model magnetic interaction
  - "chicken-and-egg" problem



0.18 width, 1 length



Far end of active line



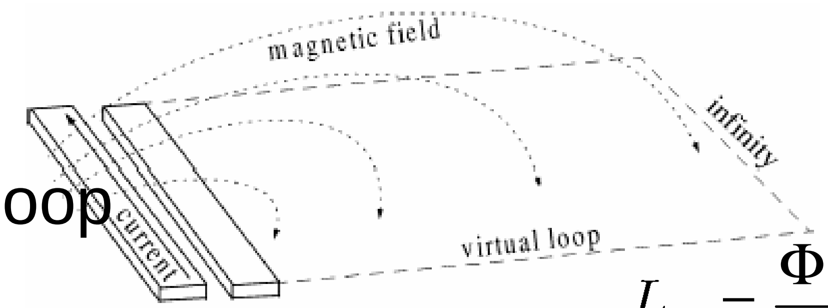
Possible current loop: $O(N^2)$

Generate L coefficients for all loop pairs is impractical ! $O(N^4)$

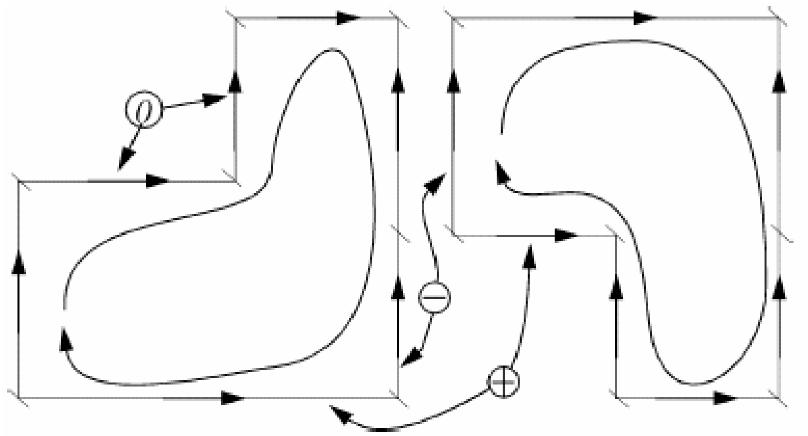
"Many of these loop couplings is negligible due to little current; but in general we need to solve for them to make an accurate determination"

Onchip interconnect inductance

- Partial inductance model
  - Invented in 1908; introduced to IC modeling in 1972
  - Definition: magnetic flux created by the current through the virtual loop which victim segment forms with infinity
  - Loop L is sums of partial L's of segments forming loop



$$L_{ba} = \frac{\Phi_b}{I_a}$$



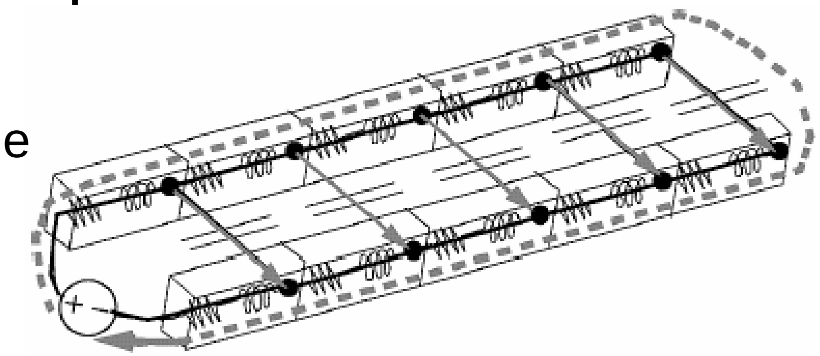
$$L_{ab,\,\mathrm{loop}} = \sum_i \sum_j S_{ij} L_{ij,\,\mathrm{partial}} \quad \text{with} \quad S_{ij} = \pm 1$$

S_{ij} are -1 if exactly one of the currents in segments i and j is flowing opposite to the direction assumed when computing $L_{ij,\,\mathrm{partial}}$

Onchip interconnect inductance

- Partial inductance model
  - Partial inductance is used to represent the loop interactions without prior knowledge of actual loops
  - Contains all information about magnetic coupling
- PEEC model
  - Include partial inductance, capacitance, resistance
  - Model IC interconnect for circuit simulation
  - Has sufficient accuracy up to now

A two-parallel-line example

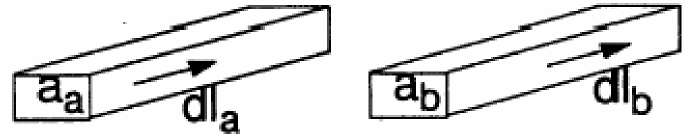


Onchip inductance extraction

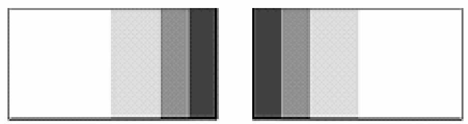
- To calculate partial inductance
  - Formula for two straight segments:

$$L_{ab,\,\text{partial}} = \frac{\mu}{4\pi}\frac{1}{a_a a_b}\int_{a_a}\int_{l_a}\int_{a_b}\int_{l_b}\frac{d\vec{l}_a \cdot d\vec{l}_b}{|\vec{r}_a - \vec{r}_b|}\,da_a\,da_b$$

Assumptions:
current evenly distributed



  - Analytical solution is quite involved even for simple geometry
  - Numerical solution, such as Gaussian quadrature can be used, but much more time-consuming
  - How about high-frequency effects ?
    - Skin effect; proximity effect
    - Path of least impedance -> least loop L



Signal line & its return

Inductance extraction

- Related research directions
  - Design solution to cope with inductive effects
    - Limited current loop; inductive effect is reduced, or easy to be analyzed (calculating partial L is costly)

Simplify the problem

  - Use partial inductance (PEEC model)
    - Consider issues of circuit simulation
    - Inductance brings dense matrix to circuit simulation; both extraction and simulation is expensive, if possible

No good locality as C

Approaches of matrix sparsification

$$\begin{bmatrix} C & \\ & L \end{bmatrix}\begin{bmatrix} \dot{V} \\ \dot{I} \end{bmatrix} + \begin{bmatrix} G & -A^T \\ A & R \end{bmatrix}\begin{bmatrix} V \\ I \end{bmatrix} = U_s$$

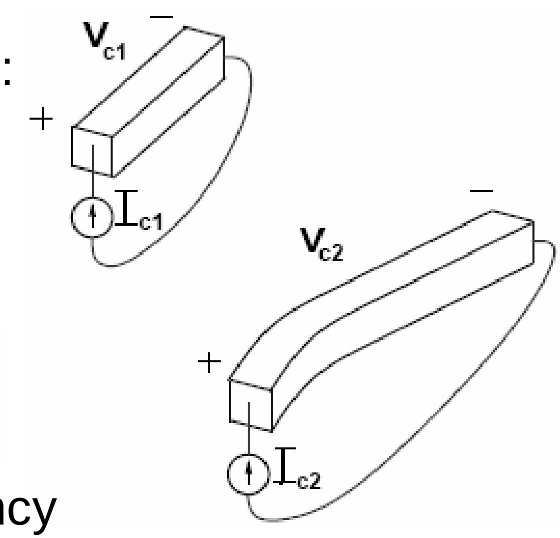
  - Inductance extraction considering high-frequency
    - Beyond the onchip application
    - MQS, EMQS, full-wave simulation

No ***L*** explicitly; just ***Z***

Frequency-dependent LR extraction

- High frequency consideration
  - nonuniform current distribution affects R
  - Extract R and partial L together
  - Capacitive effects analyzed separately (MQS)
  - Due to the interaction of magnetic field, values of L and R both rely on environments, like capacitance
  - Problem formulation:

Terminal pairs:



Impedance extraction:

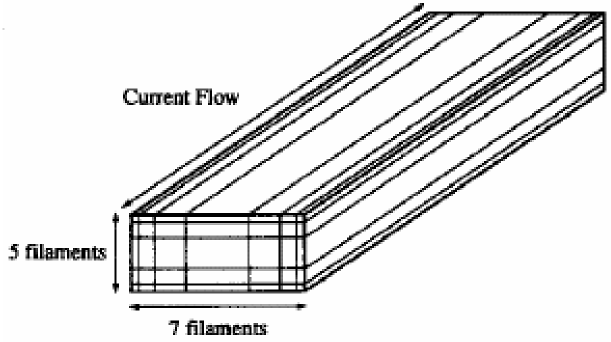
$$\begin{bmatrix} Z_{11}(\omega) & Z_{12}(\omega) \\ Z_{21}(\omega) & Z_{22}(\omega) \end{bmatrix} \begin{bmatrix} I_{c1} \\ I_{c2} \end{bmatrix} = \begin{bmatrix} V_{c1} \\ V_{c2} \end{bmatrix}$$

$$\begin{bmatrix} R_{11}(\omega) + j\omega L_{11}(\omega) & R_{12}(\omega) + j\omega L_{12}(\omega) \\ R_{21}(\omega) + j\omega L_{21}(\omega) & R_{22}(\omega) + j\omega L_{22}(\omega) \end{bmatrix} \begin{bmatrix} I_{c1} \\ I_{c2} \end{bmatrix} = \begin{bmatrix} V_{c1} \\ V_{c2} \end{bmatrix}$$

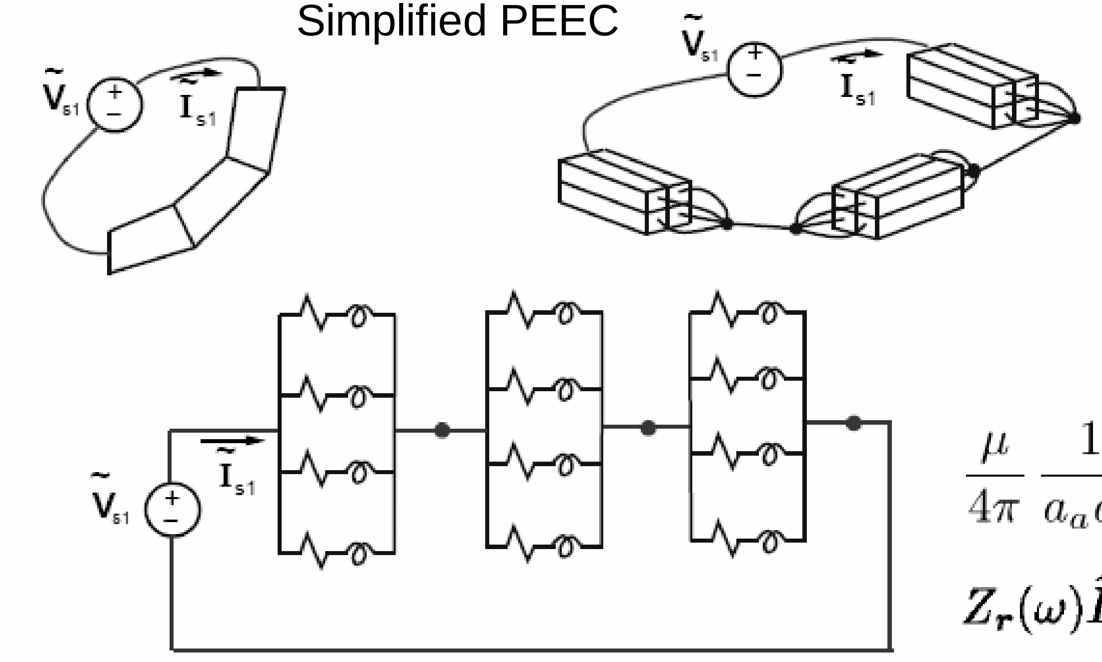
Mutual resistance becomes visible at ultra-high frequency

Frequency-dependent LR extraction

- FastHenry of MIT
  - Two assumptions: MQS; terminal pairs with known current direction
  - Partitioned into filaments, current distributed evenly



Simplified PEEC



$$\boldsymbol{Z}\cdot\boldsymbol{I}_b=\boldsymbol{V}_b$$

$$\boldsymbol{Z}=\boldsymbol{R}+jw\boldsymbol{L}$$

$$R_{ii}=\frac{l_i}{\sigma a_i}\qquad L_{ij}=\frac{\mu}{4\pi}\frac{1}{a_a a_b}\int_{a_a}\int_{l_a}\int_{a_b}\int_{l_b}\frac{d\vec{l}_a\cdot d\vec{l}_b}{|\vec{r}_a-\vec{r}_b|}\,da_a\,da_b$$

$$Z_r(\omega)\tilde{I}_s(\omega)=\tilde{V}_s(\omega)$$

Solve circuit equation !

Frequency-dependent LR extraction

- FastHenry of MIT
  - Nodal analysis: $AZ^{-1}A^t\tilde{\Phi}_n = I_s$
  - Avoid forming Z^{-1}: $\begin{bmatrix} Z & -A^t \\ A & 0 \end{bmatrix}\begin{bmatrix} I_b \\ \tilde{\Phi}_n \end{bmatrix} = \begin{bmatrix} 0 \\ I_s \end{bmatrix}$
  - Mesh-based approach

A: incidence matrix

Inverse of a dense matrix !

Much larger system

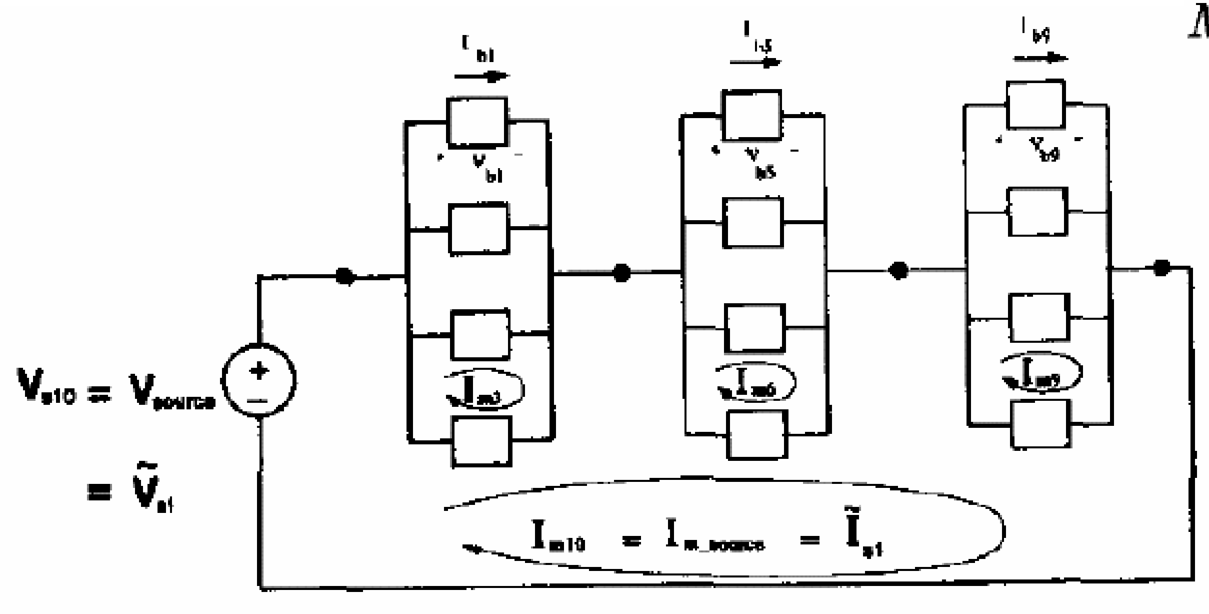
M: mesh matrix

$MV_b = V_s$

$M^t I_m = I_b$

$\longrightarrow MZM^tI_m = V_s$

$m = b' - n + 1$

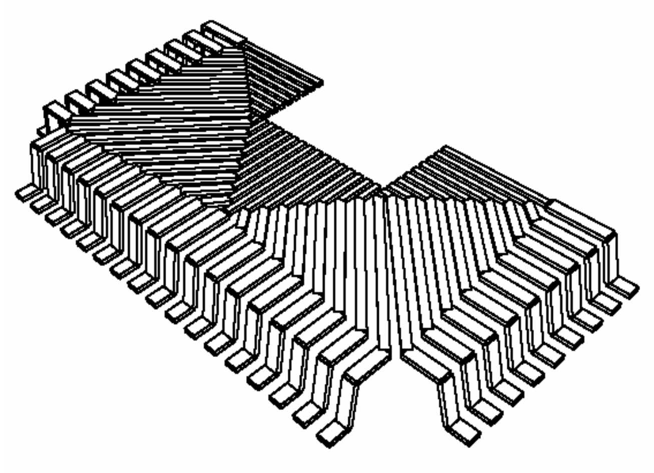


GMRES can be used to solve this system, with given V_s

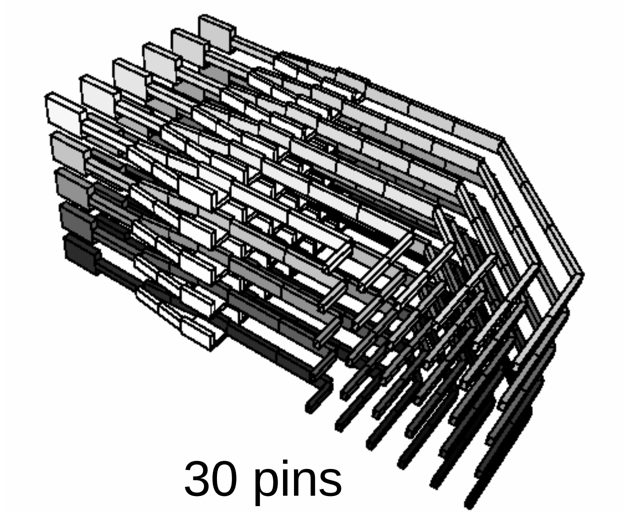
$$\boldsymbol{Y}_r \boldsymbol{V}_s = \boldsymbol{I}_s \qquad \boldsymbol{Z}_r = \boldsymbol{Y}_r^{-1}$$

Frequency-dependent LR extraction

- FastHenry of MIT
  - ☐ To solve: $MZM^t I_m = V_s$.   Multiple right-hand sides
  - ☐ Multipole acceleration; preconditioning techniques



35 pins

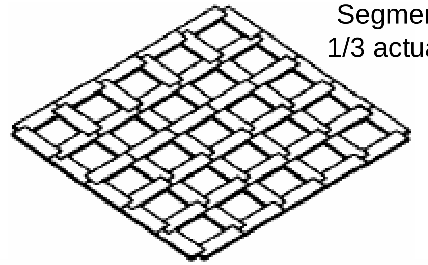


30 pins

  - ☐ Application: package, wide onchip wires (global P/G, clock)
  - ☐ Shortage: computational speed   Field solver !
    model inaccuracy; substrate ground plane

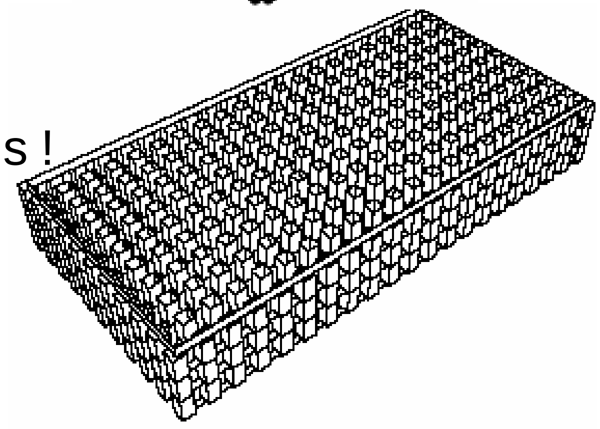
Problems of FastHenry

Segments are
1/3 actual width



- Lossy substrate discretization
  - Current direction is not clear
  - Ground plane
  - Multilayer substrate

Huge # of unknowns !



- With frequency increase
  - Filament # increases to capture skin, proximity effects
- Used only under MQS assumption
- How to improve ?
  - Surface integral formulation (BEM)

Computational
expensive !

Fundamentals of BEM

Inside alg. of FastImp of MIT

- Maxwell's equations are not in dispute
  - Governing equations

$$\nabla \times \vec{E} = -j\omega\mu\vec{H}$$ Faraday's law

$$\nabla \times \vec{H} = \vec{J} + j\omega\epsilon\vec{E}$$ Ampere's law

$$\nabla \cdot \vec{J} = -j\omega\rho$$ $$\oint_S \vec{J} \cdot d\vec{s} = -\frac{\partial}{\partial t}\int_V \rho dv$$

Two other equ's usually known:

$$\nabla \cdot (\epsilon\vec{E}) = \rho$$
$$\nabla \cdot (\mu\vec{H}) = 0$$

  - Constitutive equation for conductor

$$\vec{J} = \sigma\vec{E}$$

$$\nabla \times \nabla \times \vec{E} - \omega^2\epsilon\mu\vec{E} = -j\omega\mu\vec{J}.$$

Inside each conductor:

$$\nabla \cdot \vec{E}(\vec{r}) = 0, \quad \vec{r} \in V_i$$

$$\Longrightarrow (\nabla^2 + \omega^2\epsilon\mu)\vec{E}(\vec{r}) = j\omega\mu\vec{J}(\vec{r}), \quad \vec{r} \in V_i.$$

Vector identity: $\nabla \times \nabla \times \vec{E} = \nabla(\nabla \cdot \vec{E}) - \nabla^2\vec{E}$

Fundamentals of BEM

- Equation in each conductor

$$(\nabla^2 + \omega^2 \epsilon\mu)\vec{E}(\vec{r}) = j\omega\mu\vec{J}(\vec{r}), \quad \vec{r} \in V_i.$$

Vector Holmholtz equ.

Classification of PDE ?

General solution:

$$T\vec{E}(\vec{r}) = \int_{S_i} dS' \left(G_0(\vec{r},\vec{r}')\frac{\partial \vec{E}(\vec{r}')}{\partial n(\vec{r}')} - \frac{\partial G_0(\vec{r},\vec{r}')}{\partial n(\vec{r}')}\vec{E}(\vec{r}') \right)$$

$$T = \begin{cases} 1, & \text{if } \vec{r} \in V_i \\ 1/2, & \text{if } \vec{r} \in S_i \\ 0, & \text{otherwise} \end{cases} \qquad -j\omega\mu \int_{V_i} dV' G_0(\vec{r},\vec{r}')\vec{J}(\vec{r}')$$

With $\vec{J} = \sigma\vec{E}$ $\Longrightarrow$ $\nabla^2\vec{E}(\vec{r}) + (\omega^2\epsilon\mu - j\omega\mu\sigma_i)\vec{E}(\vec{r}) = 0, \quad \vec{r} \in V_i$

$$\frac{1}{2}\vec{E}(\vec{r}) = \int_{S_i} dS' \left(G_1(\vec{r},\vec{r}')\frac{\partial \vec{E}(\vec{r}')}{\partial n(\vec{r}')} - \frac{\partial G_1(\vec{r},\vec{r}')}{\partial n(\vec{r}')}\vec{E}(\vec{r}') \right) \quad \surd$$

$$\vec{r} \in S_i$$

$$G_1(\vec{r},\vec{r}') = \frac{e^{jk_1|\vec{r}-\vec{r}'|}}{4\pi|\vec{r}-\vec{r}'|}, \quad k_1 = -\sqrt{\omega^2\epsilon\mu - j\omega\mu\sigma_i}.$$

Fundamentals of BEM

- Equation in the homogeneous medium

$$\vec{E}(\vec{r}) = -j\omega\vec{A} - \nabla\phi(\vec{r})$$
$$= -j\omega\mu\int_V dV' G_0(\vec{r},\vec{r}')\vec{J}(\vec{r}') - \nabla\phi(\vec{r})$$

Hold anywhere

A: Magnetic potential $\nabla\times\vec{A} = \vec{B}, \quad \nabla\times\vec{E}_{\text{ind}} = -\frac{\partial B}{\partial t}$

$$T\vec{E}(\vec{r}) = \int_{S_i} dS'\left(G_0(\vec{r},\vec{r}')\frac{\partial\vec{E}(\vec{r}')}{\partial n(\vec{r}')} - \frac{\partial G_0(\vec{r},\vec{r}')}{\partial n(\vec{r}')}\vec{E}(\vec{r}')\right) - j\omega\mu\int_{V_i} dV' G_0(\vec{r},\vec{r}')\vec{J}(\vec{r}')$$

Sum for all conductors

$$\frac{1}{2}\vec{E}(\vec{r}) = \int_S dS'\left(G_0(\vec{r},\vec{r}')\frac{\partial\vec{E}(\vec{r}')}{\partial n(\vec{r}')} - \frac{\partial G_0(\vec{r},\vec{r}')}{\partial n(\vec{r}')}\vec{E}(\vec{r}')\right)$$
$$- j\omega\mu\int_V dV' G_0(\vec{r},\vec{r}')\vec{J}(\vec{r}'), \quad \vec{r}\in S_k$$

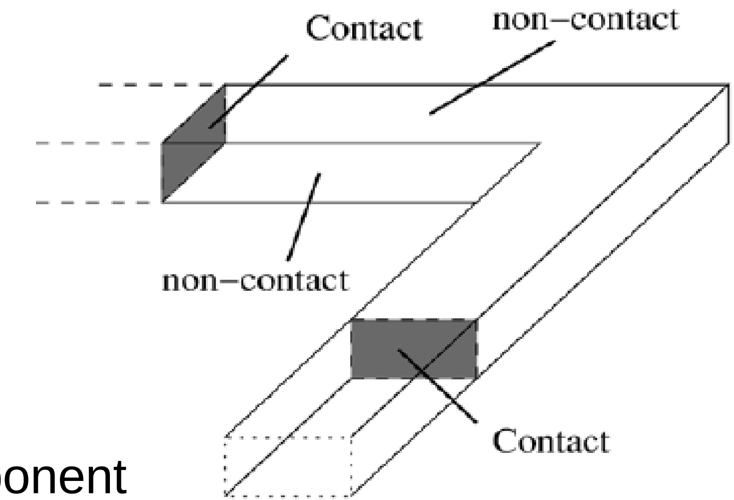
$$G_0(\vec{r},\vec{r}') = \frac{e^{jk_0|\vec{r}-\vec{r}'|}}{4\pi|\vec{r}-\vec{r}'|},$$
$$k_0 = \omega\sqrt{\epsilon\mu}$$

$$-\frac{1}{2}\vec{E}(\vec{r}) = \int_S dS'\left(G_0(\vec{r},\vec{r}')\frac{\partial\vec{E}(\vec{r}')}{\partial n(\vec{r}')} - \frac{\partial G_0(\vec{r},\vec{r}')}{\partial n(\vec{r}')}\vec{E}(\vec{r}')\right)$$
$$+ \nabla\phi(\vec{r}), \quad \vec{r}\in S_k$$

$\surd$

Fundamentals of BEM

- Boundary conditions



Contact is artificially exposed surface

NC, C $\nabla \cdot \vec{E}(\vec{r}) = 0,$ Hold due to assumption of no charge accumulation

C $\hat{t}(\vec{r}) \cdot \vec{E}(\vec{r}) = \vec{E}_t(\vec{r}) = 0$ No transversal component of current into contact

C $\hat{n}(\vec{r}) \cdot \frac{\partial \vec{E}(\vec{r})}{\partial n(\vec{r})} = \frac{\partial E_n(\vec{r})}{\partial n(\vec{r})} = 0$

NC $\hat{n}(\vec{r}) \cdot \vec{E}(\vec{r}) = E_n(\vec{r}) = \frac{j\omega\rho(\vec{r})}{\sigma}$ Here ρ is surface charge density

C $\phi(\vec{r}) = \text{constant}, \quad \vec{r} \in S_c$

$$\phi(\vec{r}) = \int_S dS' \frac{\rho(\vec{r}')}{\epsilon} G_0(\vec{r}, \vec{r}'), \quad \vec{r} \in S \qquad \surd$$

Totally 8 state variables:

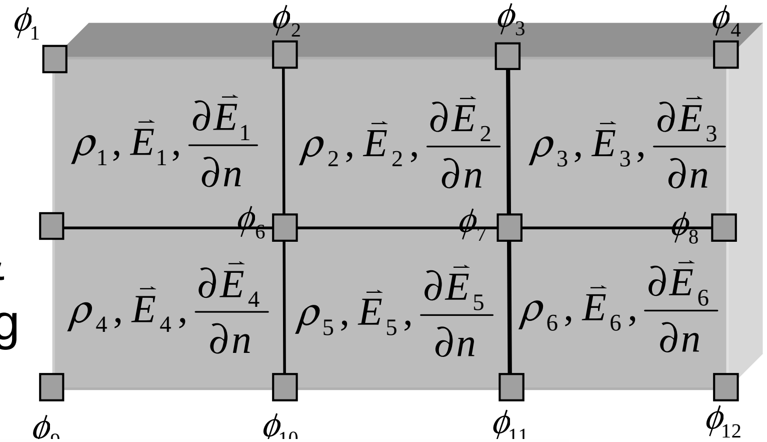
$E_x, E_y, E_z, (\partial Ex)/(\partial n),$
$(\partial Ey)/(\partial n),\ (\partial Ez)/(\partial n),$
ϕ, and ρ

Full wave simulation

Fundamentals of BEM

Fullwave analysis

Discretization & unknown setting



Equation formulation

$$
\left[\begin{array}{ccc|ccc|ccc}
P_1 & 0 & 0 & D_1 & 0 & 0 & 0 & 0 & 0 \\
0 & P_1 & 0 & 0 & D_1 & 0 & 0 & 0 & 0 \\
0 & 0 & P_1 & 0 & 0 & D_1 & 0 & 0 & 0 \\
\hline
T_{1,x}P_0 & T_{1,y}P_0 & T_{1,z}P_0 & T_{1,x}D_0 & T_{1,y}D_0 & T_{1,z}D_0 & g_{11} & g_{12} & 0 \\
T_{2,x}P_0 & T_{2,y}P_0 & T_{2,z}P_0 & T_{2,x}D_0 & T_{2,y}D_0 & T_{2,z}D_0 & g_{21} & g_{21} & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & -I\epsilon & 0 & P_0^1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -I\epsilon & P_0^2 \\
\hline
-A_x & -A_y & -A_z & C_x & C_y & C_z & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & N_x & N_y & N_z & 0 & 0 & \frac{-j\omega}{\sigma} I \\
\hline
0 & 0 & 0 & T_{1,x} & T_{1,y} & T_{1,z} & 0 & 0 & 0 \\
0 & 0 & 0 & T_{2,x} & T_{2,y} & T_{2,z} & 0 & 0 & 0 \\
\hline
N_x & N_y & N_z & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0
\end{array}\right]
\left[\begin{array}{c}
\frac{\partial E_x}{\partial n} \\ \frac{\partial E_y}{\partial n} \\ \frac{\partial E_z}{\partial n} \\
\hline
E_x \\ E_y \\ E_z \\
\hline
\phi_{\mathrm{nc}} \\ \phi_{\mathrm{c}} \\
\hline
\rho
\end{array}\right]
=
\left[\begin{array}{c}
0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ \hline 0 \\ 0 \\ \hline 0 \\ \hline 0 \\ \hline 0 \\ 0 \\ \hline 0 \\ \hline \Phi_c
\end{array}\right]
$$

Fundamentals of BEM

- Full wave
  - Complete Maxwell's equations (no assumption)
- Electro-Magneto-Quasistatics (EMQS)
  - Consider RLC
  - Ignore the displacement current
- Magneto-Quasistatics (MQS)
  - Consider RL
  - Ignore the displacement current

$$G_0 = \frac{1}{4\pi|\vec{r} - \vec{r}'|}$$ in medium equ.

$$G_1(\vec{r}, \vec{r}') = \frac{e^{jk_1|\vec{r}-\vec{r}'|}}{4\pi|\vec{r} - \vec{r}'|},$$ in conductor equ.

$$k_1 = -\sqrt{-j\omega\mu\sigma_i}$$

no $\phi(\vec{r}) = \int_S dS' \frac{\rho(\vec{r}')}{\epsilon} G_0(\vec{r}, \vec{r}'), \quad \vec{r} \in S$

Three modes all are widebanded; they behave differently at high frequencies

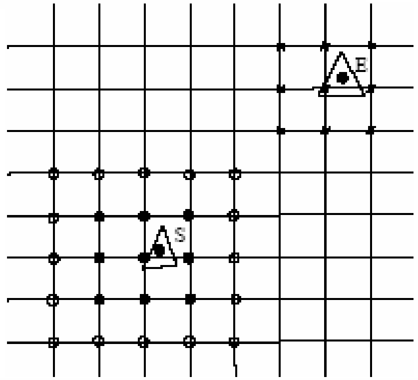
FastImp

Frequency-dependent
multiple kernels

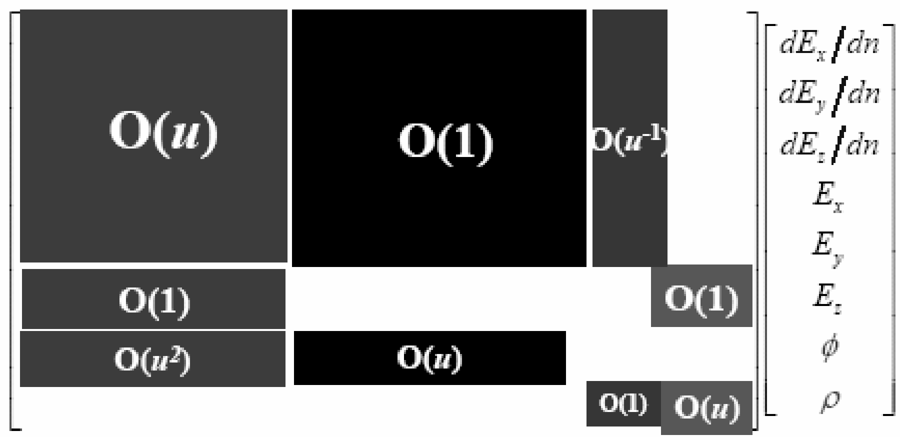
$$\left\{\begin{aligned} P_1(a,b) &= \int_{panel_b} \frac{e^{ikr(x_a,y)}}{4\pi r(x_a,y)}\,dy \\ D_1(a,b) &= \int_{panel_b} \frac{\partial}{\partial n_y}\left[\frac{e^{ikr(x_a,y)}}{4\pi r(x_a,y)}\right] dy \end{aligned}\right.$$

- Algorithms in FastImp
  - ☐ Integral calculation
    - Singular, near-singular integral
    - pFFT algorithm $A\alpha \xrightarrow{pFFT} (D + WHP)\alpha$

Direct, interpolation,
convolution, projection

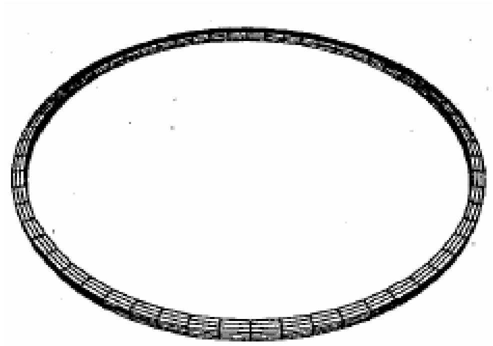


  - ☐ Scaling
    - size *u* is small
    - Improve condition number
  - ☐ Preconditioning
    - Preconditioned GMRES

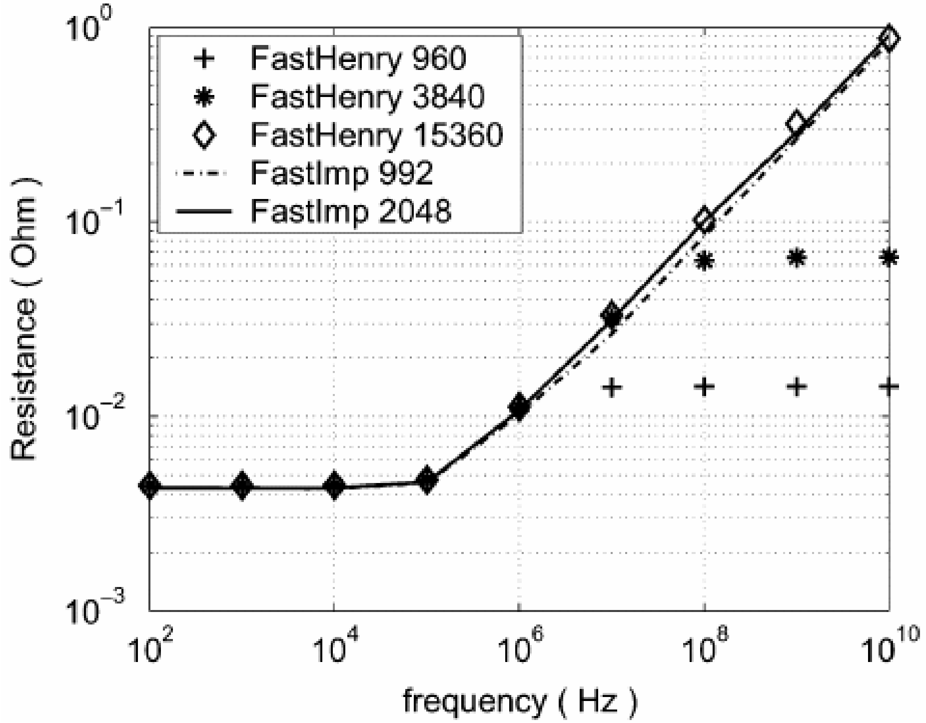


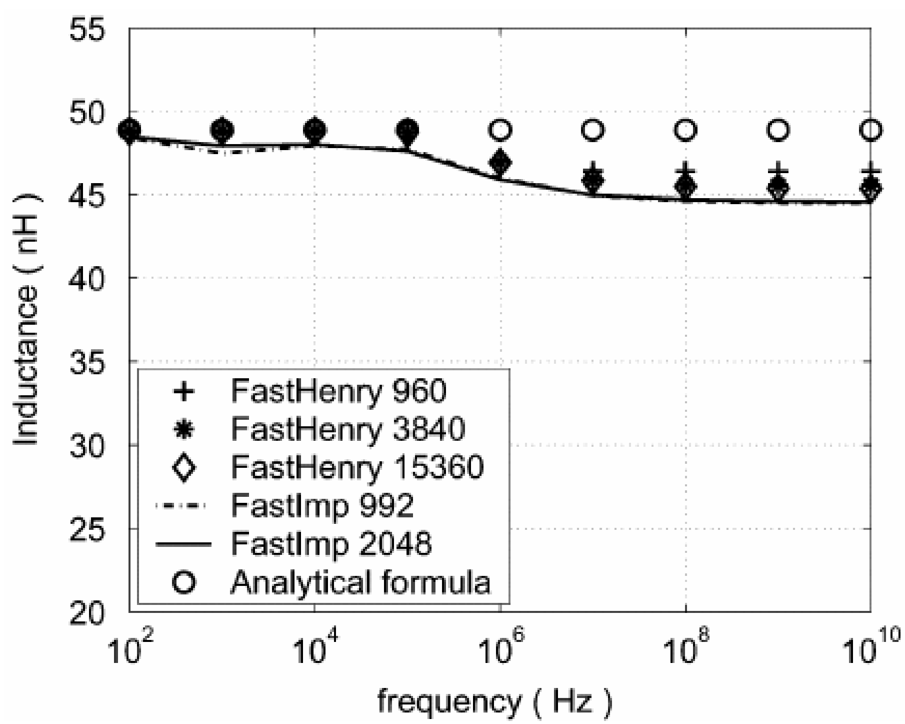
FastImp

- Experiment results
  - A ring



MQS analysis

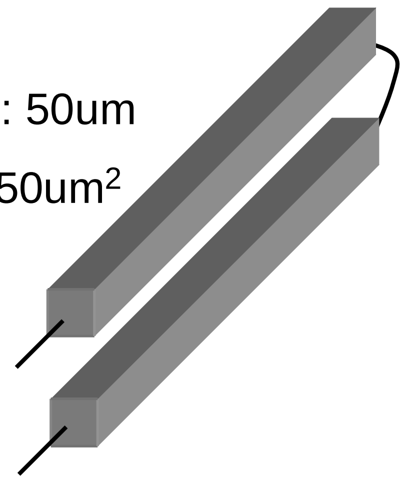




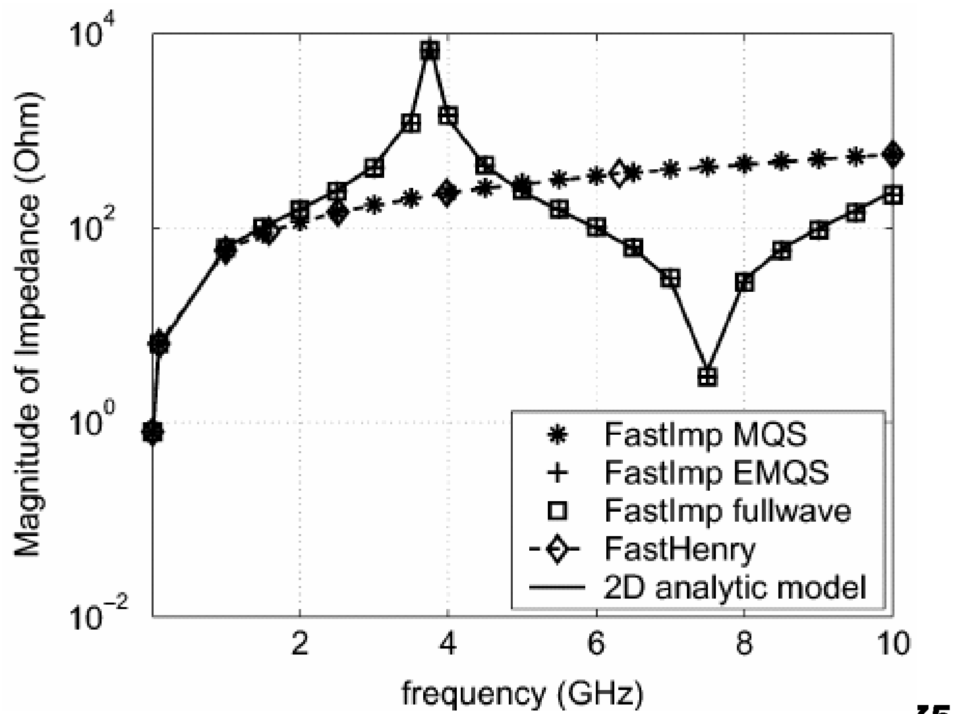
FastImp

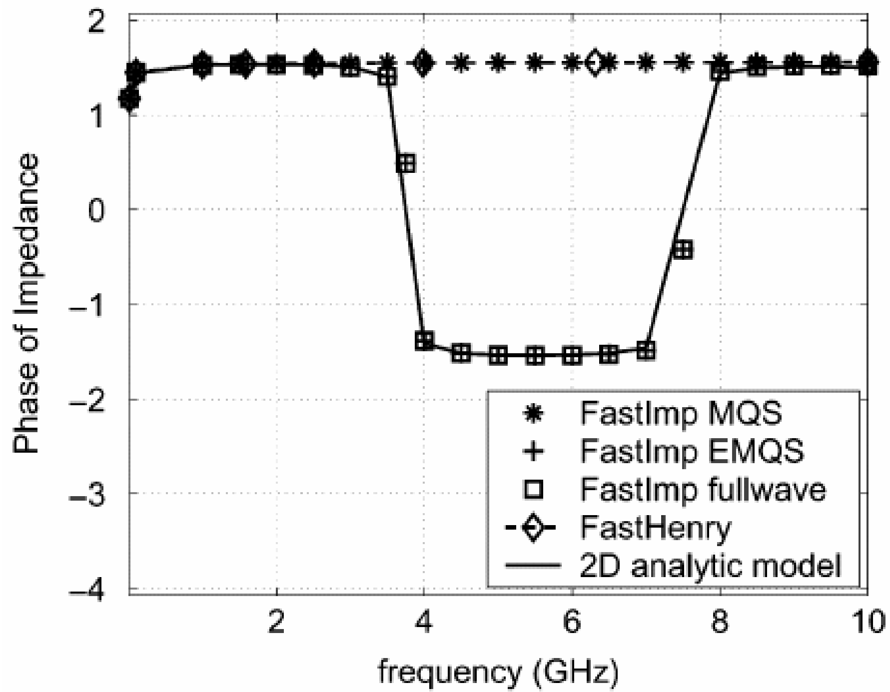
Length: 2cm; separation: 50um

Cross-section: 50x50um^2

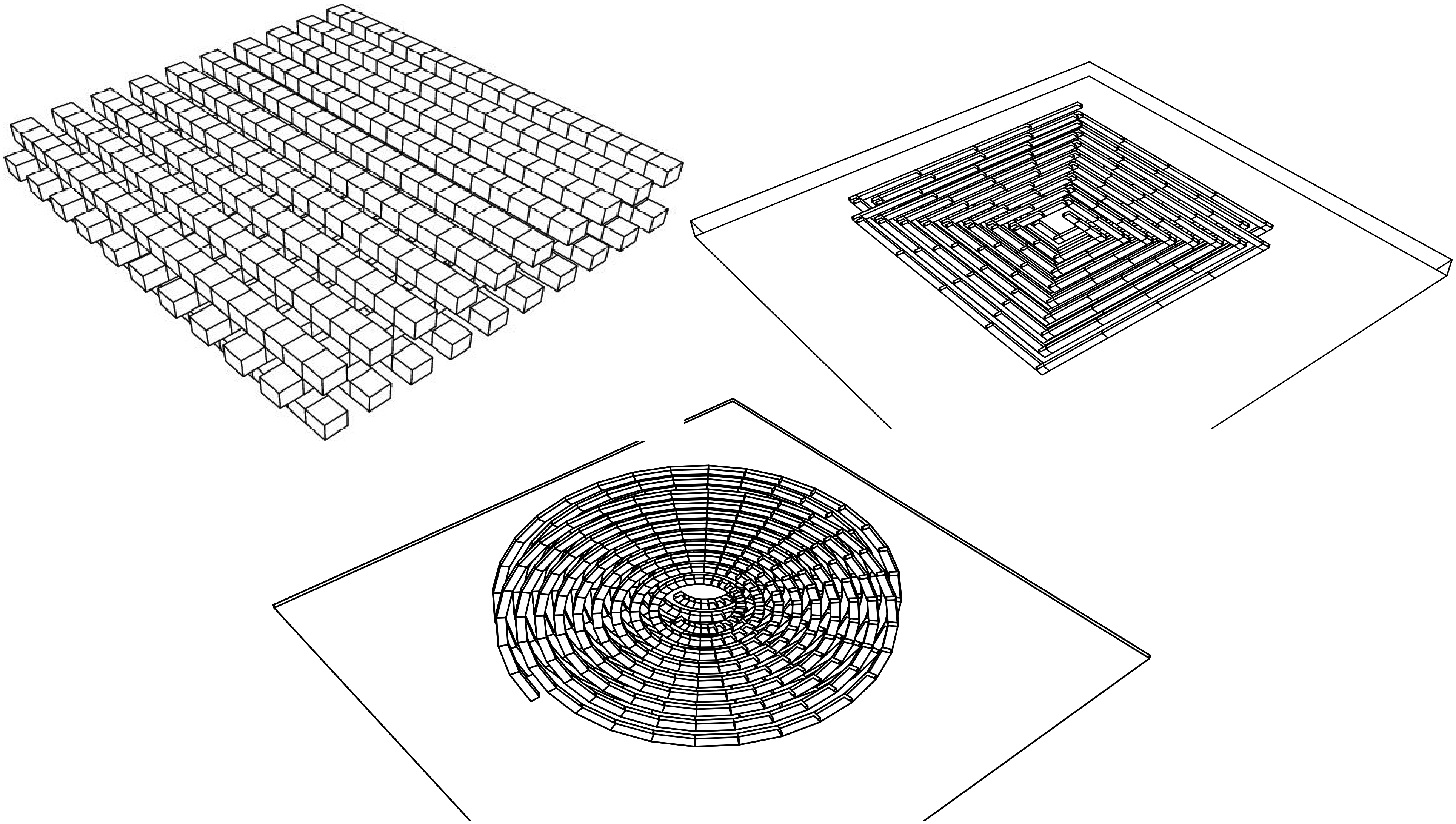


- Experiment results
  - Shorted transmission line
  - EMQS, fullwave, 2D balanced T-Line
  - MQS, FastHenry





Computational Results: Various Practical Examples



Slides courtesy Zhenhai Zhu, MIT

Computational Results: Various Practical Examples

	10x3 Buses	Stacked 9-turn Circular spirals	Stacked 8-turn Rect. spirals
FastImp	9.5 min 340Mb	68 min 642 Mb	54 min 749 Mb
Iterative	160 min 19Gb	750 min 19 Gb	590 min 22 Gb
LU	136days 19Gb	100 days 19 Gb	168 days 22 Gb

Slides courtesy Zhenhai Zhu, MIT

Inductance extraction

Reference

[1] M. W. Beattie and L. T. Pileggi, “Inductance 101: modeling and extraction,” in *Proc. Design Automation Conference*, pp. 323-328, June 2001.

[2] M. Kamon, M. J. Tsuk, and J. K. White, “Fasthenry: a multipole-accelerated 3-D inductance extraction program,” *IEEE Trans. Microwave Theory Tech*., pp. 1750 - 1758, Sep 1994.

[3] Z. Zhu, B. Song, and J. White. Algorithms in Fastimp: a fast and wide-band impedance extraction program for complicated 3-D geometries. *IEEE Trans. Computer-Aided Design*, 24(7): 981-998, July 2005.

[4] W. Kao, C-Y. Lo, M. Basel and R. Singh, “Parasitic extraction: Current state of the art and future trends,” *Proceedings of IEEE*, vol. 89, pp. 729-739, 2001.

[5] http://www.rle.mit.edu/cpg/research_codes.htm (FastCap, FastHenry, FastImp)