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INTERIM REPORT ON LAGEOS MISSION ANALYSES

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INTERIM REPORT ON

LAGEOS MISSION ANALYSES

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Code 591

J. L. Cooley
Code 581.3

December 1973

GODDARD SPACE FLIGHT CENTER
Greenbelt, Maryland

ACKNOWLEDGEMENTS

We wish to thank Mr. J. L. Cooley, Mission Design Section, Mission Support and Analysis Branch of the Mission Support Computing and Analysis Division, GSFC Code 581.3 for providing the station visibility times presented in this report. Thanks also to Mr. J. J. Lynn, of Old Dominion Systems, Inc., who, under contract, provided the error analyses utilizing the using the "Navigation Analysis Program, NAP-3".

Booster performance information was obtained through the courtesy of the DELTA-Project Office, at GSFC.

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INTERIM REPORT ON
LAGEOS MISSION ANALYSES

I. INTRODUCTION

Of the parameters affecting the LAGEOS Mission and the orbit selection two error sources are found to be comparable and significant. (Order of meters) These error sources are only weakly dependent on the other parameters affecting orbit selection.

The first error source is the uncertainty in the gravity model. Current models will have to be improved by at least an order of magnitude, either before or during the mission, for the mission to succeed.

The second error source is the radiation pressure, which includes the direct radiation, albedo radiation and the "earth-shine". Methods of modeling, correcting for or a combination of both must be found for the mission to meet its goals.

It has been found that retrograde orbits increase the number of passes per day over the tracking stations at the expense of also increasing the number of time gaps when no station is observing. The significance of the result has yet to be evaluated. It also raises the question — is it an advantage to have more passes per day when the total amount of observing time is relatively unaffected?

It has also been found that with the accuracies being sought, relativistic effects are significant (55 cm/day) and must be included in orbit computation and determination systems for this mission.

Finally, if station locations are known only to 1 meter, the error in the satellite orbit is comparable (i.e. same order of magnitude) as the solar pressure induced errors.

II. PARAMETERS COMMON TO THE STUDY

A. Tracking Stations

Throughout this portion of the LAGEOS study, thirteen (13) possible laser sites have been used. No attempt has been made to approximate the real world since only major trends are desired. Therefore, all station heights were set to zero (0) and only approximate locations were used. The thirteen (13) stations follow:

Site	Location	Longitude	Latitude
Johannesburg	South Africa	28°	-25°
Kashima	Japan	140°	35°
Orroral Valley	Australia	148°	-35°
San Diego	California	242°	33°
Fort Resolution	Canada	247°	61°
Mt. Hopkins	Arizona	249°	32°
Mazatlan	Mexico	254°	23°
GSFC	Maryland	282°	39°
Arequipa	Peru	288°	-16°
Bermuda	—	294°	32°
Sao Paulo	Brazil	313°	-24°
Natal	Brazil	325°	-5°
Madrid	Spain	355°	40°

It was further assumed that satisfactory tracking could not be accomplished below 25° elevation angle from the local horizontal and that the maximum data rate was 1 point/sec.

Most of the orbits examined were for an eccentricity of 0.01 and unless otherwise stated, for a 3 day arc length.

In general, the intent of the study was to examine the character of all known perturbations which are likely to cause variations of five (5) cm or more to the orbit of the LAGEOS satellite.

B. Geopotential Model

The current study is only concerned with obtaining major trends and therefore the choice of geopotential model was not considered critical. Several models were used in order to perform the study expeditiously. The following is a list of the models and their respective uses:

Percent Coverage — Spherical Model

Total Number of Passes — SAO '66 Zonals only

Harmonic Analyses	—	$\bar{C}_{\ell,m}, \bar{S}_{\ell,m} = 10^{-5}/\ell^2$
Error Analyses	—	SAO - 1966
Miscellaneous Parts	—	SAO - 1966

C. Area/Mass

For the purposes of studying the drag and radiation pressure perturbations a constant area/mass was assumed.

$A = 2827 \text{ cm}^2$

$m = 6.82 \times 10^5 \text{ gms}$

D. Range of Inclination and Heights

A fairly broad spectrum of inclinations and heights were used in the study in order to understand the sensitivities and possible trade-offs.

Initially, inclinations of 30°, 60° and 90° were studied at heights of 4000, 6000 and 8000 km.

Later in the study, more detailed studies were made at 70° and 110° at orbit heights adjusted to produce a period of $1/n + .55$ sidereal day where n is an integer. Specifically 6.55, 7.55 and 8.55 are or will be studied.

E. Assumed Errors (Where Applicable)

The laser trackers are assumed to have a Gaussian noise with a one sigma (1σ) amplitude of 5.0 cm. In addition to the noise, the trackers were assumed to have a bias of 5.0 cm for each tracker uncorrelated with each other and uncorrelated with itself on subsequent passes.

The tracking station coordinates are assumed known to an accuracy of 1.0 meters in each of the three rectangular coordinates (u, v, w).

There is also assumed a 50 μ s (fifty microsecond) timing bias at each station again uncorrelated with all other stations and uncorrelated with itself on subsequent passes.

Finally it was assumed that the solar constant was known to ±1.0% (one percent).

III. TRACKING COVERAGE

A. Percent Coverage

Using the thirteen station tracking only above 25° elevation for a three day arc, the percentage of total tracking available was determined. It is the total time that at least one tracking station is observing to the total time available, i.e. total time/72 $\times$ 100 as a function of height and inclination. The results are shown in Figure 1. In Figure 1, the data points are connected by straight lines. No attempt has been made to account for cloud cover, in other words, perfect visibility has been assumed.

It can be seen that by using low inclinations (i.e. less than 60°) approximately a 10% increase in tracking coverage can be obtained for any height. In addition, from 5-10% increase can be obtained by increasing the height by 2000 km.

It should be pointed out that by assuming a 50% cloud cover the total spread is only 25% to 40% and that the regions of interest, i.e. 3720, 4600 and 5690 km are not overly sensitive to height or inclination to seriously affect the overall mission.

Station visibility charts are given in Figures 2(a)-(l). It is apparent that there are numerous time gaps where no station can see the satellite but conversely there are numerous times when two or more stations may be tracking simultaneously. In general, the visibility times are pretty well distributed with the effect of inclination slight. As would be expected, increasing the height increases the length of each pass over the station, increases the frequency and number of stations viewing simultaneously and of course increases the total time of visibility.

The times where there are gaps of no station tracking is shown in Figure 3. The largest time gap is approximately 1.5 hours and the number of gaps increase slightly for the retrograde orbits.

B. Total Number of Passes

The total number of passes a satellite makes over a given station per day is slightly dependent upon its nodal regression rate which in turn is controlled by the selection of the inclination. It has been implied that an increase in the number of passes/day improves the quality of orbit determination. If this is a signigicant factor, selection of retrograde orbits will improve the situation somewhat, as shown in Table 1. Comparing the 70° inclination to the 110° degree inclination it can be seen that the number of passes per day can be increased by approximately six (6) with very little change in the total amount of tracking time available.

IV. GEOPOTENTIAL RESULTS

A. Field Requirements

The Harmonic Analysis Program (HAP) was used with a geopotential field given by $\bar{C}_{\ell,m}$, $\bar{S}_{\ell,m} = 10^{-5}/\ell^2$ as a model. All terms which created perturbations of greater than 5.0 cm amplitude were retained in the study except those with beat periods in excess of 600.0 days. The first thing that is apparent due to the precision of the lasers and the sensitivity of LAGEOS is the size of the total field required even at LAGEOS heights. As can be seen, in Table 2, the total field varies from a 6×6 to a 10×10 depending mostly upon height and less sensitive to inclination. The larger field, as expected is required for the lower height. The second startling thing is the degree required for the selected coefficents to complete the field, i.e. as high as 21 in some cases. Actually these orbits are not "tuned" to avoid resonance as is evident by the rms resonant amplitude for the $i = 90°$, $h = 8000$ km orbit, i.e. 4.0 km. This orbit is near 5.0 revolutions/day. When the orbits are "tuned" the maximum degree also rises. This is because in the process of "tuning", for example the mid point between 6th and 7th order primary resonance, exact secondary resonance is occuring with the 13th order terms hence many of the high degree (~21-26), 13th order terms are perturbing the satellite. Thus in choosing the orbit not only the primary but secondary and higher resonances must also be avoided. Fortunately these higher resonances are much sharper and can easily be avoided.

The third obvious number to look at is the rms amplitude of all the resonant terms which can best be done by referring to Figure 4. This amplitude varies by an order of magnitude, i.e. $0.5 \rightarrow 5.0$ meters near the minimum which of course is the secondary resonance of the 13th order which must be avoided. But even if this could be used and assuming the field is known to about 10% accuracy the uncertainties would be 5-50 cm, or more, which implies that to accomplish the LAGEOS mission the field accuracy will have to be improved to at least 1% and when all other factors are also taken into account probably more like 0.1%. In other words, two orders of magnitude improvement.

B. Inclination Effects

Again referring to Figure 4, as mentioned above, if a low inclination is selected for the LAGEOS orbit, the accuracy of the gravity field required is approximately one order of magnitude less stringent.

More important is the slope and location of the curves at high inclinations. If the nominal orbit is 5500 ± 100 km, at $i = 30°$ the rms amplitude varies from 0.6 m to 1.4 m but at $i = 110°$ it varies from 5.8 m to 14.9 m.

C. Height Variations

As mentioned above, the total field requirements are reduced as the height is increased but not as rapidly as might have been hoped for. This is mainly due to extreme sensitivity resulting from the assumed precision of the tracking. Since many of the selected coefficients required have never been determined, a rationale must be established which will justify the mission and the chosen height in spite of these apparent difficulties.

V. BOOSTER PERFORMANCE

A. 2900+

Booster information has been obtained from the GSFC-Delta Project Office. The vehicle currently under consideration is the 2900+ where the initial "2" in the designation identifies the "straight eight" Thor and the second digit "9" indicates that nine (9) Castor II solid rocket motors are strapped to the Thor booster. The third digit "0" indicates the Delta stage configured for the smaller 5' diameter shroud, adequate for LAGEOS and the fourth digit "0" indicates a two stage vehicle. This booster is used to put the satellite in the appropriate transfer ellipse to the chosen height and the "+" indicates that a — to be selected — apogee kick motor (AKM) must be added to the booster to circularize the final orbit. The payload performance that might be expected from this system is given in Figure 5. It is seen that the approximate payload is between 450 to 710 kg. The reason for the payload maximizing at near 90° inclination is a peculiarity of range safety at PMR which requires dog-legs and net loss of payload performance.

B. Injection Errors

The apogee kick motor (AKM) if required, introduces rather large injection errors in the final orbit. The speed imparted by the AKM is accurate to about 1% of the total ΔV. For a total ΔV of 1.0 km/s the error would be $\delta V = 10.0$ m/s. For a nominal orbit height of 8000 km, eccentricity = 0.0 would introduce an eccentricity error of $\delta e = \pm 0.004$, and a period uncertainty $\delta P = \pm 1.6$ (minutes). The period error is equivalent to a frequency error $\delta f = \pm 0.03$ cycles/day. The period error and eccentricity error combine to give perigee height uncertainties of $\delta hp = \pm 111.0$ km.

The AKM also has a tip off uncertainty at separation sufficient to create an uncertainty in the direction of the total velocity vector of $\delta\gamma$ = ±0°5 (degrees). For the same nominal orbit as above this represents δhp = ±124 km and δe = ±0.01.

In the selection of the preferred LAGEOS orbit, injection errors such as those above must be taken into account for the high inclination orbits (i.e. 110 > i > 60°). However, for inclinations near 30°, launched from ETR, it may be possible to attain nearly the same payload without any AKM. Hence, the totally guided injection would reduce the errors drastically. Such a case should be studied if there is any possibility of flying the mission at the lower inclinations.

VI. ERROR ANALYSES

Table 3 summarizes the results of numerous error analyses made by J. J. Lynn of Old Dominion Systems, Inc. using the Navigation Analysis Program (NAP). The table presents the maximum uncertainty, 3(a) and 3(b), and the minimum uncertainty, 3(c) and 3(d), obtainable over the three day (72 hours) arc. The uncertainties are given in the H-L-C coordinate system where H is height or radial direction, L is along track and C is cross track. The individual error sources head the columns but the noise is also included with the individual error. 3(a) and 3(b) contain identical information as do 3(c) and 3(d) but the order of presentation has been changed from constant height and varying inclination, (a) and (c), to constant inclination and varying height, (b) and (d).

The noise was assumed random (in the observed laser range) with an amplitude of 5.0 cm. The solar pressure error was assumed to be 1% of the pressure coefficient. $\delta C_{0,0}$ = 1.D-6 was the bias uncertainty assumed and $\delta C_{2,0}$ = 1.D-8. Station location bias errors were considered a one meter (1.0 m) in each of the u-v-w coordinates uncorrelated with each other but correlated with itself on subsequent passes. The laser range was assumed to be biased (δr_β) to 5.0 cm uncorrelated with each other and uncorrelated with itself on subsequent passes and finally the timing bias at each station was assumed to be 50 μs (microsecond) again uncorrelated with each other and uncorrelated with itself on subsequent passes.

The errors due to albedo and earthshine have not been included as yet but it has been implied that they should be covered by a 10% error in the solar pressure constant. If this is true errors ten times larger (10X) than those shown in Table 3 should be expected. This would make radiation pressure a very significant uncertainty.

It has been argued that the uncertainty in $C_{0,0}$ would only be a scale factor that gets washed out in the orbit determination process and therefore should not contribute but if the assumed uncertainty is reasonable that error is considerable. We have used a relative error of 1.D-5 and the difference in the SAO model and an NWL model is 3.D-6.

The major cause for concern is that, except for the noise, all the errors are significant at the 5.0 cm level. It is apparent that LAGEOS will push the state of the art.

VII. MISCELLANEOUS PERTURBATIONS

A. Drag

In order to get a feeling for the sensitivity of LAGEOS to the drag perturbation, a numerical integration was performed for 11.25 days using the program model atmosphere with a nominal drag coefficient and repeated with the drag coefficient increased by a factor of ten (10X). At the end of the 11.25 days, the magnitude of the position separation vector was 3.0 mm, and the final periods differed by 15.0 nanoseconds. The model contained in the program has a density of 3.61D-13 gm/cc @ 1500 km height and 1.0-20 gm/cc @ 13,000 km and interpreted linearly in a log-log sense. No claim is made for the accuracy of the model but it is reasonable to assume it is correct within two orders of magnitude. Hence drag should not be a significant problem for LAGEOS.

The program integration interval was verified by doubling the interval normally used and this introduced a difference in the magnitude of the position vectors of 8.0 mm. It is therefore assumed that the normal interval (1/128 hrs) is adequate for the purposes of this study.

B. Sun and Moon

A similar test to that of the drag was done with the sun as the only perturbation and then repeated with the sun and moon as the only perturbations. The results are shown in Figure 6. The sun perturbation does not make a complete cycle in the time spanned but in the 11.25 days the period increased approximately 0.33 milliseconds and the amplitude of the moon's perturbation is 5.6 milliseconds peak-to-peak. If the masses are assumed to be known to at least five places, neither of these perturbations will detract from the LAGEOS mission.

C. Relativity

With the accuracy of lasers and LAGEOS it will be necessary to include relativity in the computation and determination of orbits. For a circular orbit at a height of 5690 km the relativistic advance of perigee is approximately 55.0 cm/da but if it is assumed known to 1% it should not detract from the mission.

VIII. SUMMARY

Both the gravity model and the radiation pressure carry comparable uncertainties which threaten the success of the LAGEOS mission. The study indicates gravity models will require improvement of at least an order of magnitude and preferably two orders of magnitude if possible. It may turn out that the LAGEOS mission will be the only method of obtaining the required improvement.

The radiation pressure problem is less amenable to solution and perhaps it should be suitably modelled and solved for during the mission.

In addition, though not a problem, relativistic forces must be included in the computational programs due to the extreme accuracies being considered.

Finally, all other uncertainties examined in the study, thus far, do not pose a serious problem for the mission.

However, timing errors of the assumed 50 microseconds may be a bit optimistic. If so, timing errors would be comparable to the radiation pressure and gravity model errors.

The decision to either accept $i = 30°$ or rule it out should be made as soon as possible. The injection errors resulting from the apogee kick motor required for the high inclination orbits places the extra stress on the accuracy of the gravity model required since the nominal orbit must be biased far enough off the optimum orbit to avoid possible resonances. In these regions, the model uncertainty sensitivities are rising rapidly.

On the other hand, with a low inclination, it may be possible to have a guided injection. If $i = 30°$ is a possible choice, this possibility should be investigated before the remainder of the work is undertaken.

At the present time two orbit heights are under major consideration for LAGEOS. One at approximately 4000 km and the other at about 6000 km. Prof. Kaula, in a letter dated Nov. 9, 1973, feels that a height of 3700 km at most would be the most useful for the determination of tectonic motions on a scale of 1000 km or less. However, he intends to pursue the matter in further error analyses. The lower height has the advantage of higher mass (610 kg as opposed to 420 kg) capability from the booster. This reduces the sensitivity to radiation pressure errors. It also has the disadvantages of decreased tracking coverage (50% as opposed to 64%, assuming no cloud cover) and an increased sensitivity to gravity model errors.

IX. FUTURE WORK

A. Continue Error Analyses

It is planned to continue the error analyses similar to the above by looking at the effects of additional error sources, e.g., to add albedo and earthshine to the system. As the choice of orbits narrows, increased detail will be executed.

B. Increase Emphasis on Low Altitude

Interest in the lower satellite height, i.e. 3720 km, has been renewed. More detailed analyses of the lower height will be instituted and an effort made to assess the trade-offs between the improved geometry at the lower height versus the improved tracking coverage at the higher heights.

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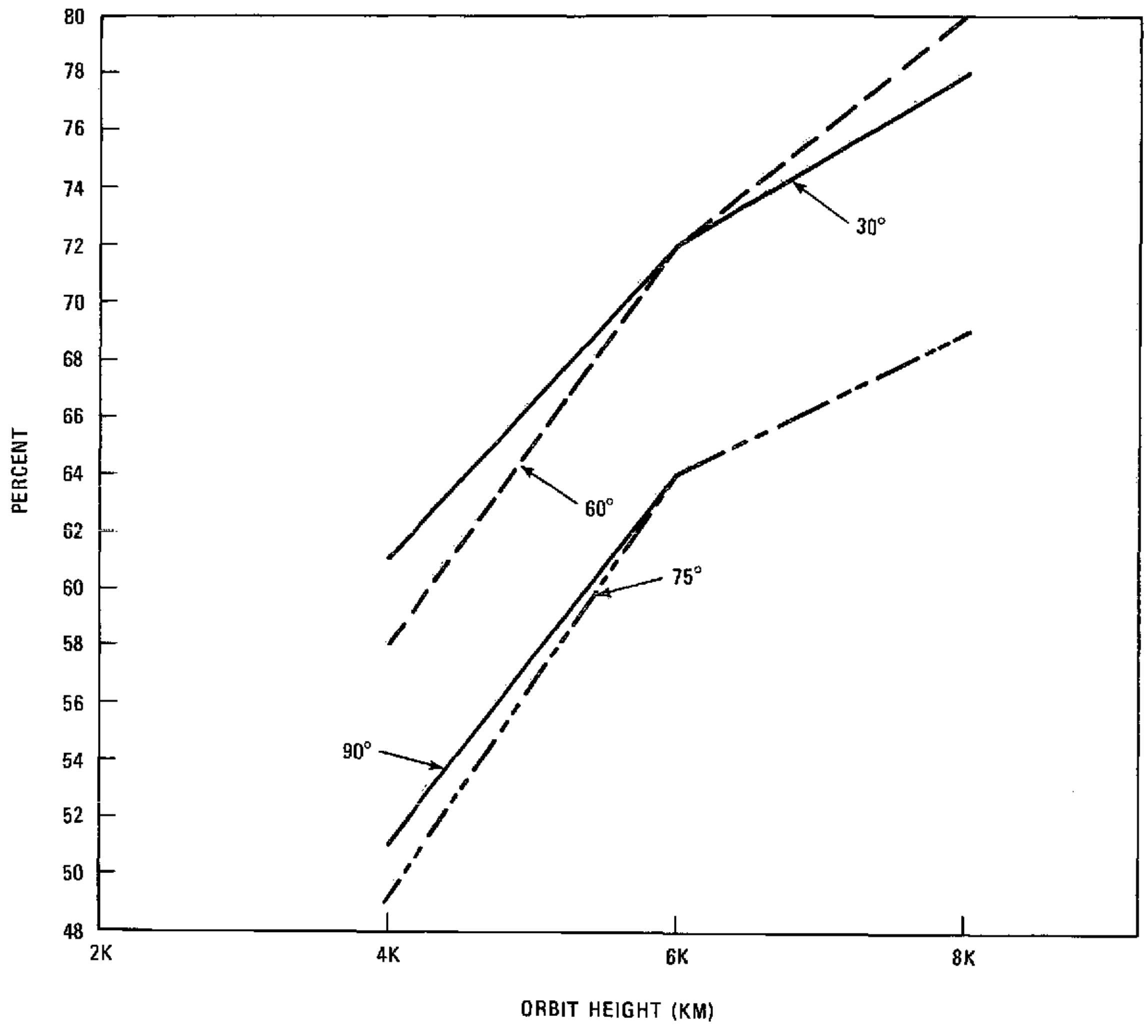
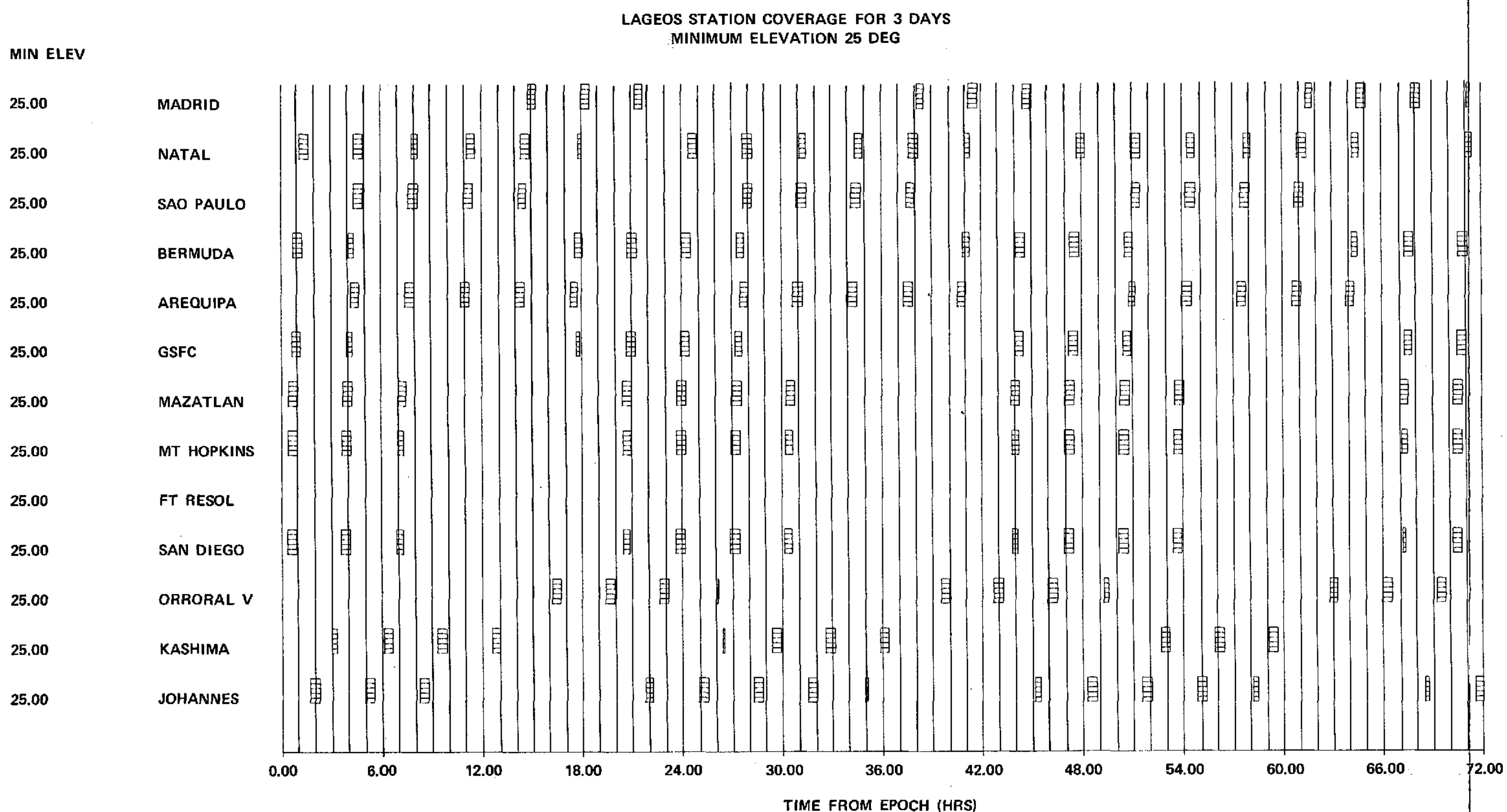


Figure 1. LAGEOS Tracking Coverage, 13 Stations, Elev. $\geq$ 25°, 10/25/73



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TRAJECTORY ANALYSIS AND GEODYNAMICS DIVISION
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Figure 2(a). Station Visibility Times Chart

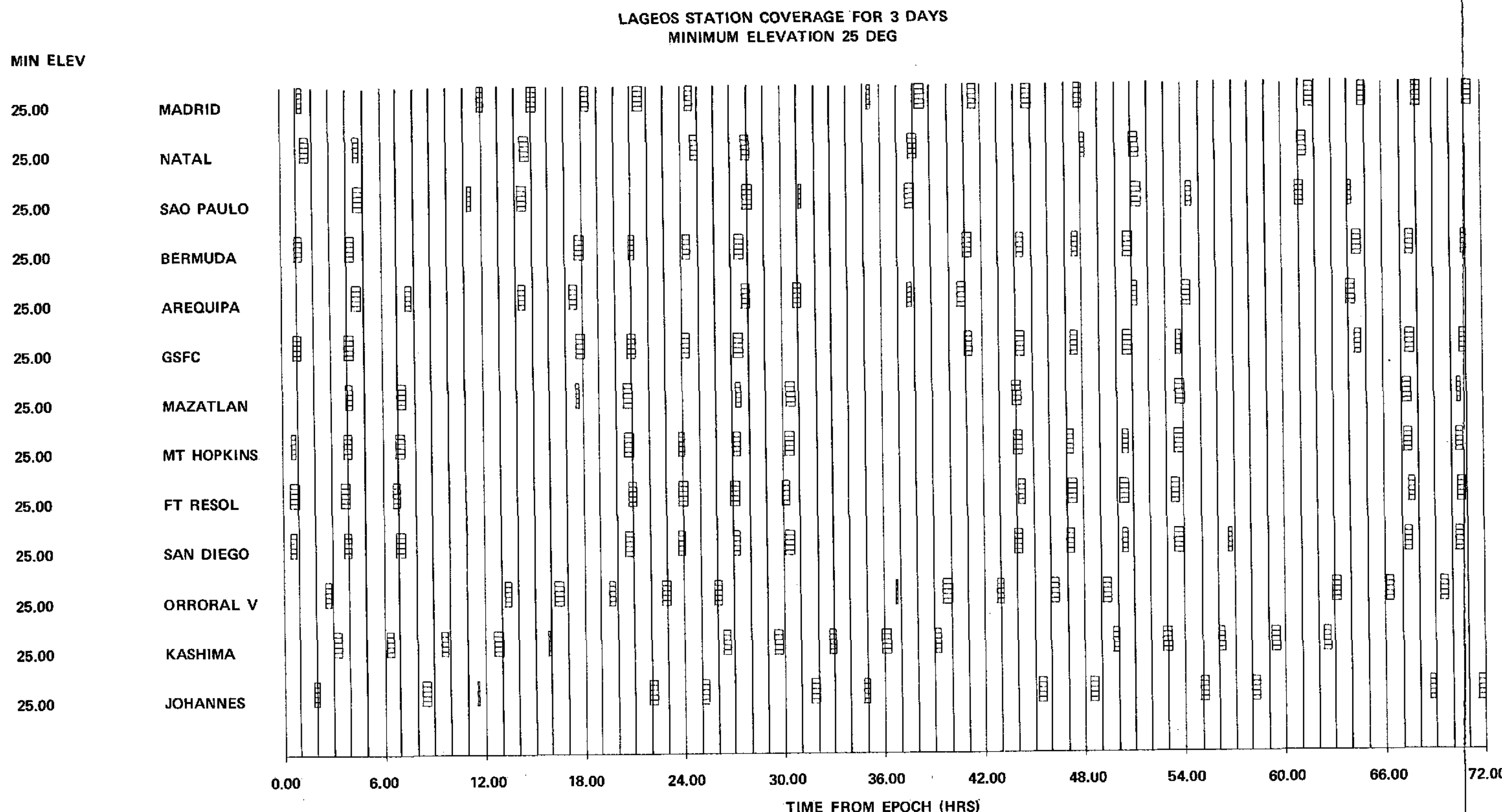

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Figure 2(b). Station Visibility Times Chart

LAGEOS STATION COVERAGE FOR 3 DAYS
MINIMUM ELEVATION 25 DEG

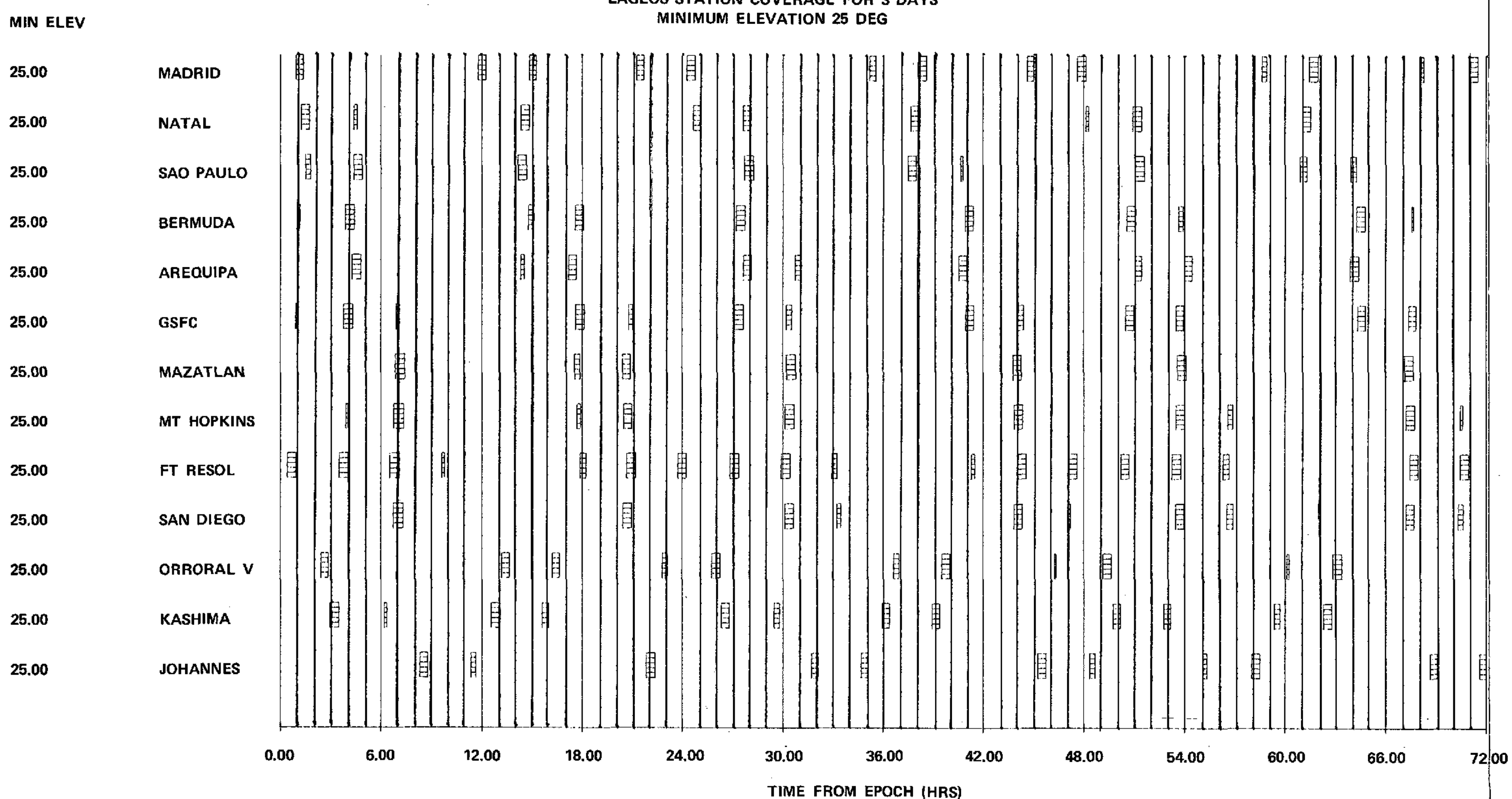

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Figure 2(c). Station Visibility Times Chart

LAGEOS STATION COVERAGE FOR 3 DAYS
MINIMUM ELEVATION 25 DEG

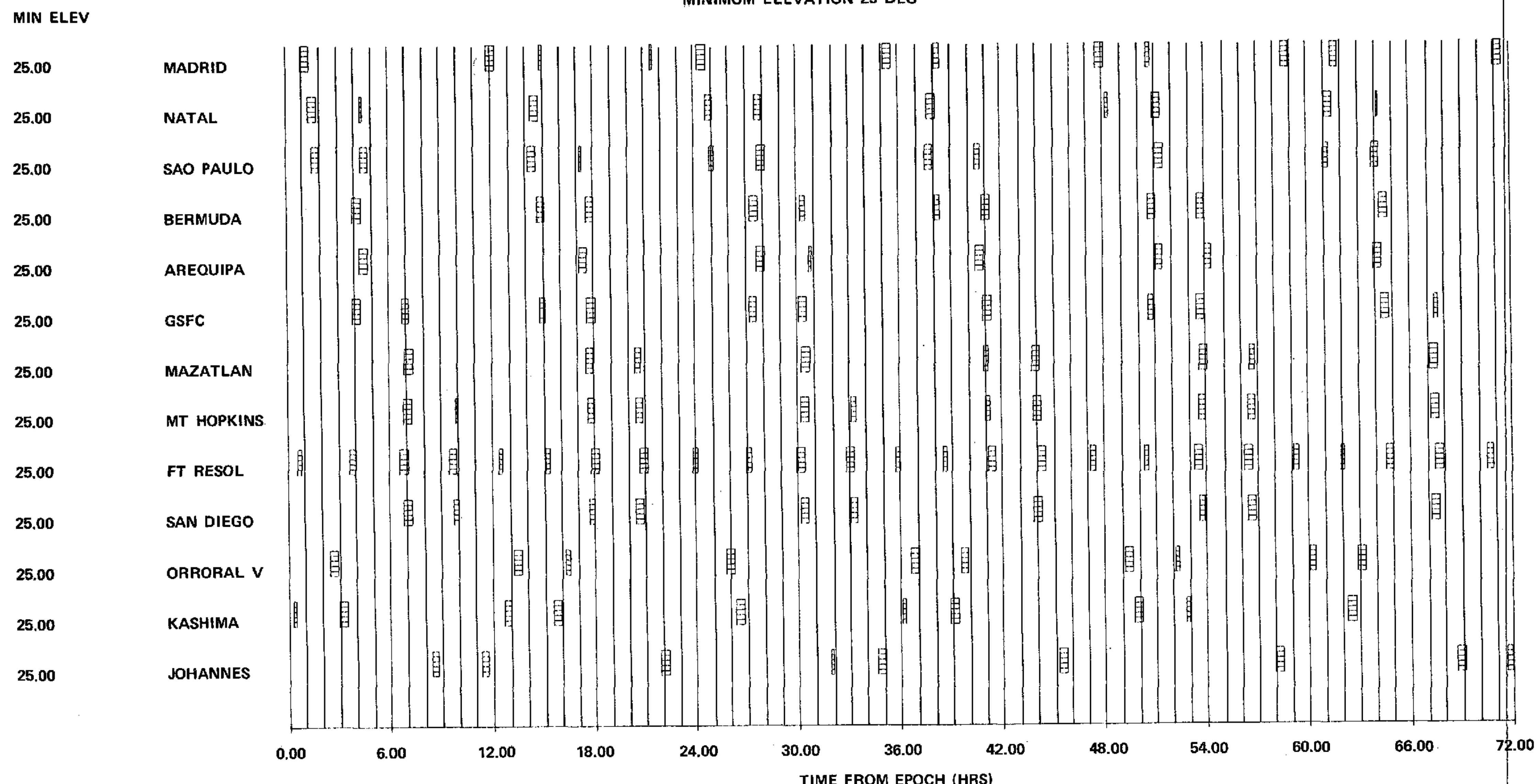

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Figure 2(d). Station Visibility Times Chart

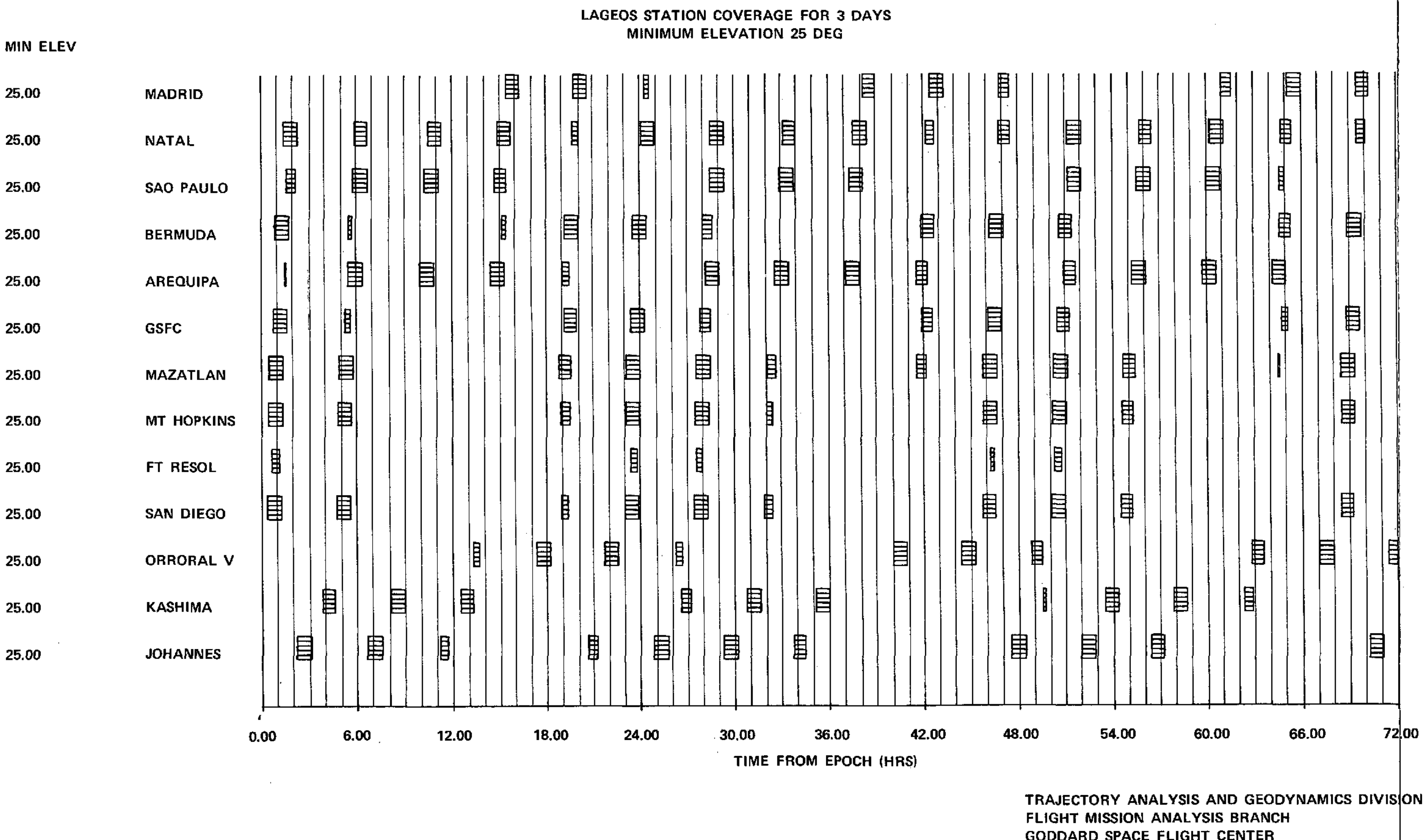

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Figure 2(e). Station Visibility Times Chart

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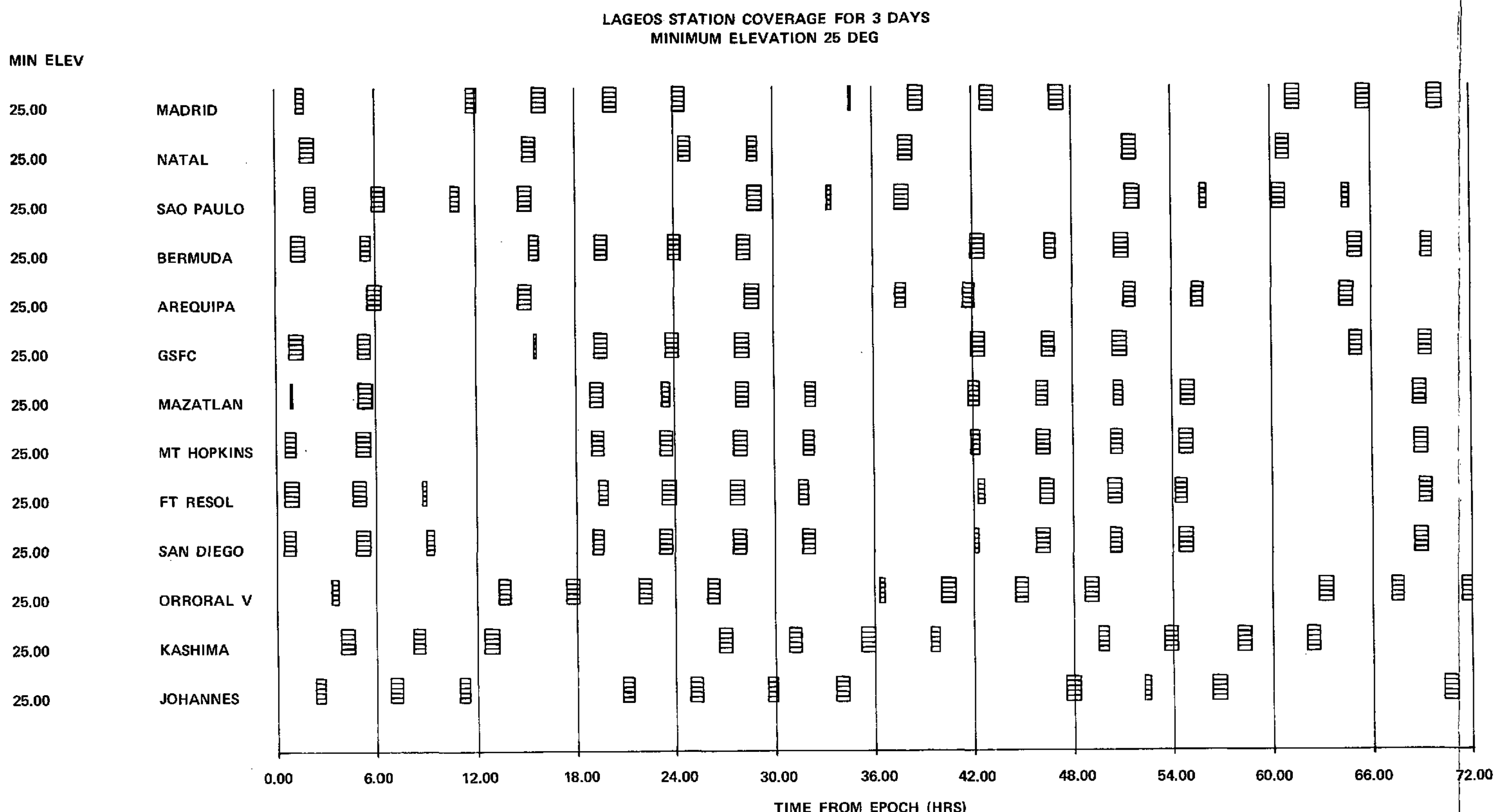

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Figure 2(f). Station Visibility Times Chart

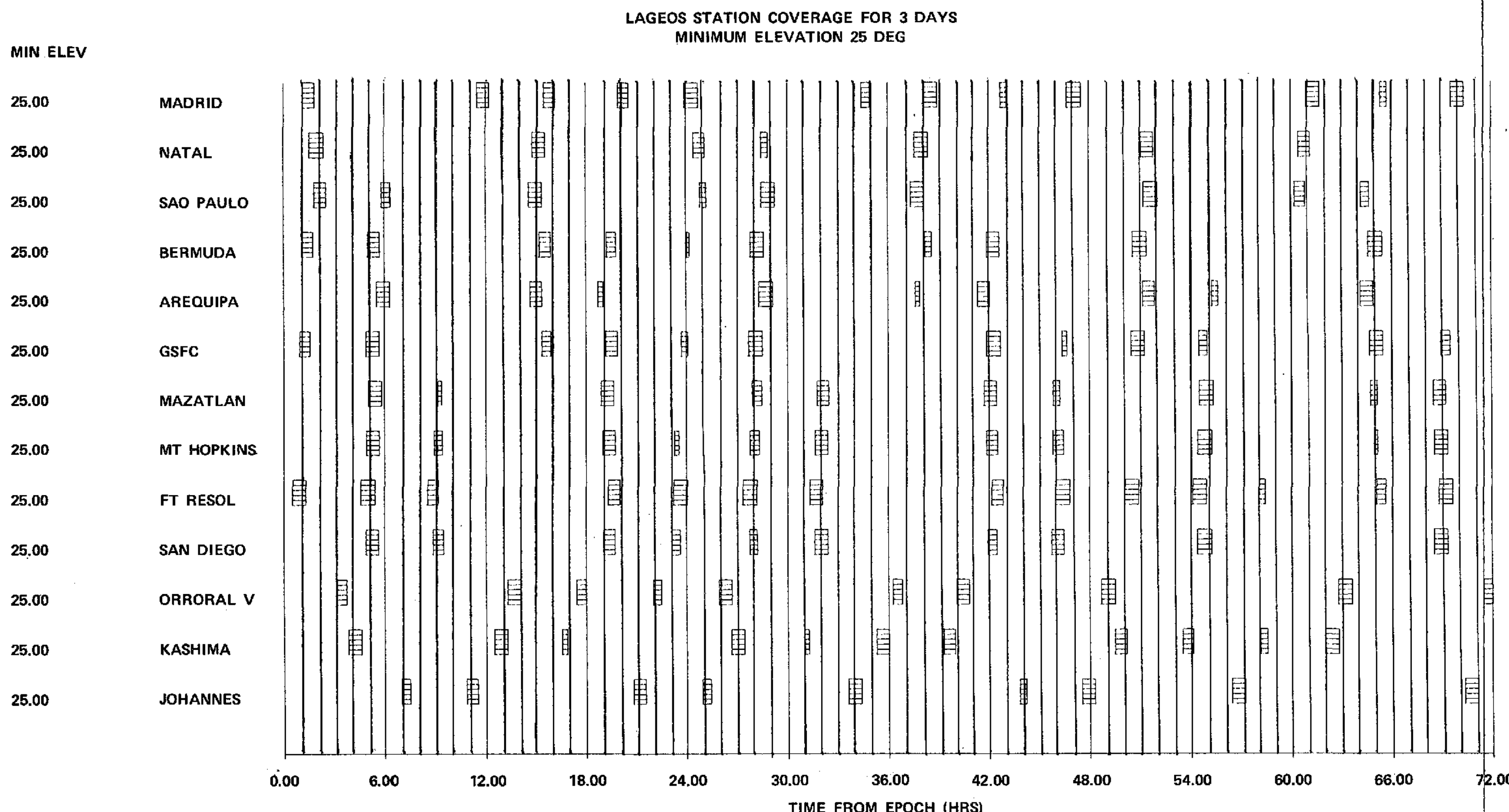

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Figure 2(g). Station Visibility Times Chart

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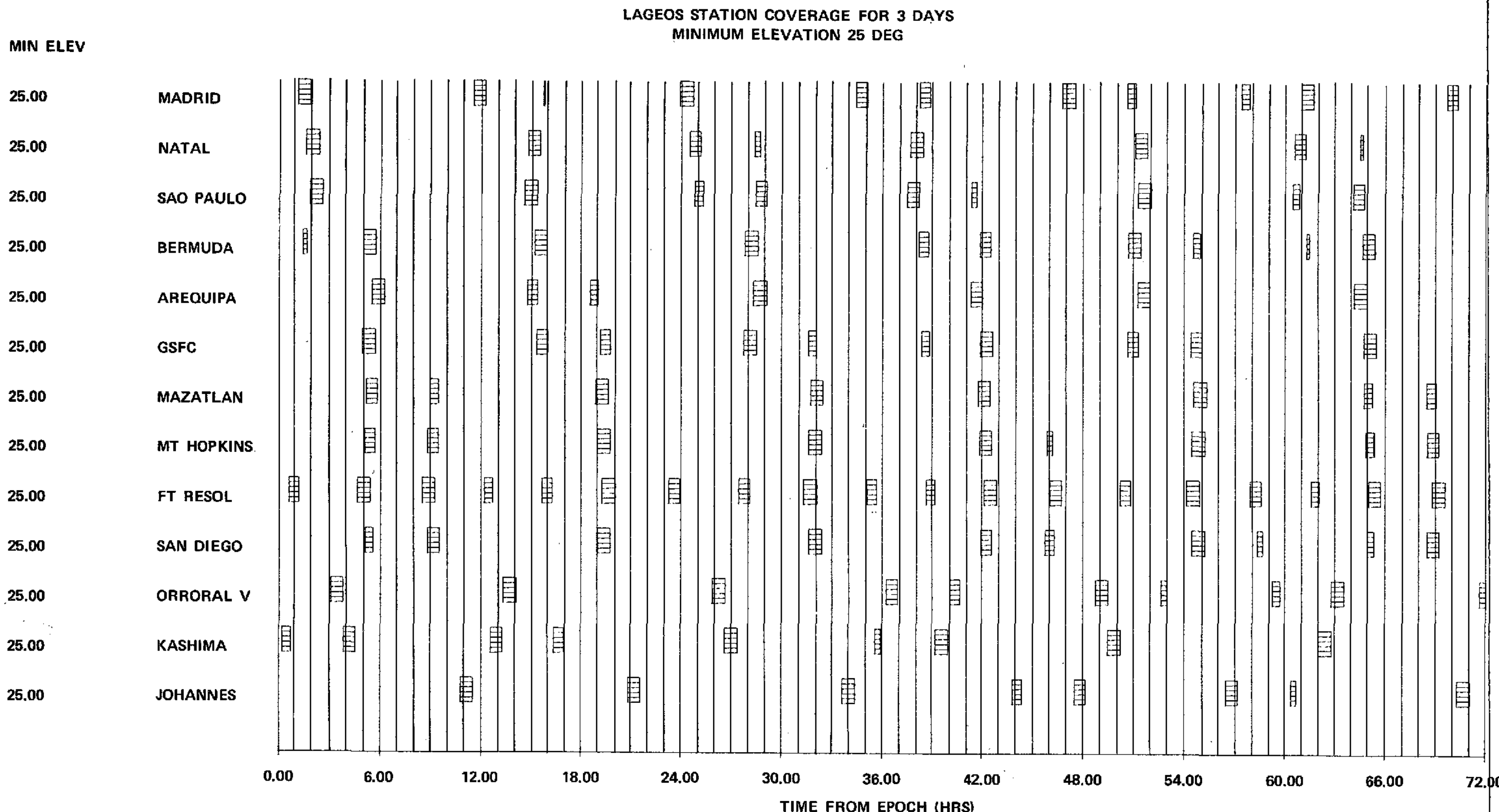

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Figure 2(h). Station Visibility Times Chart

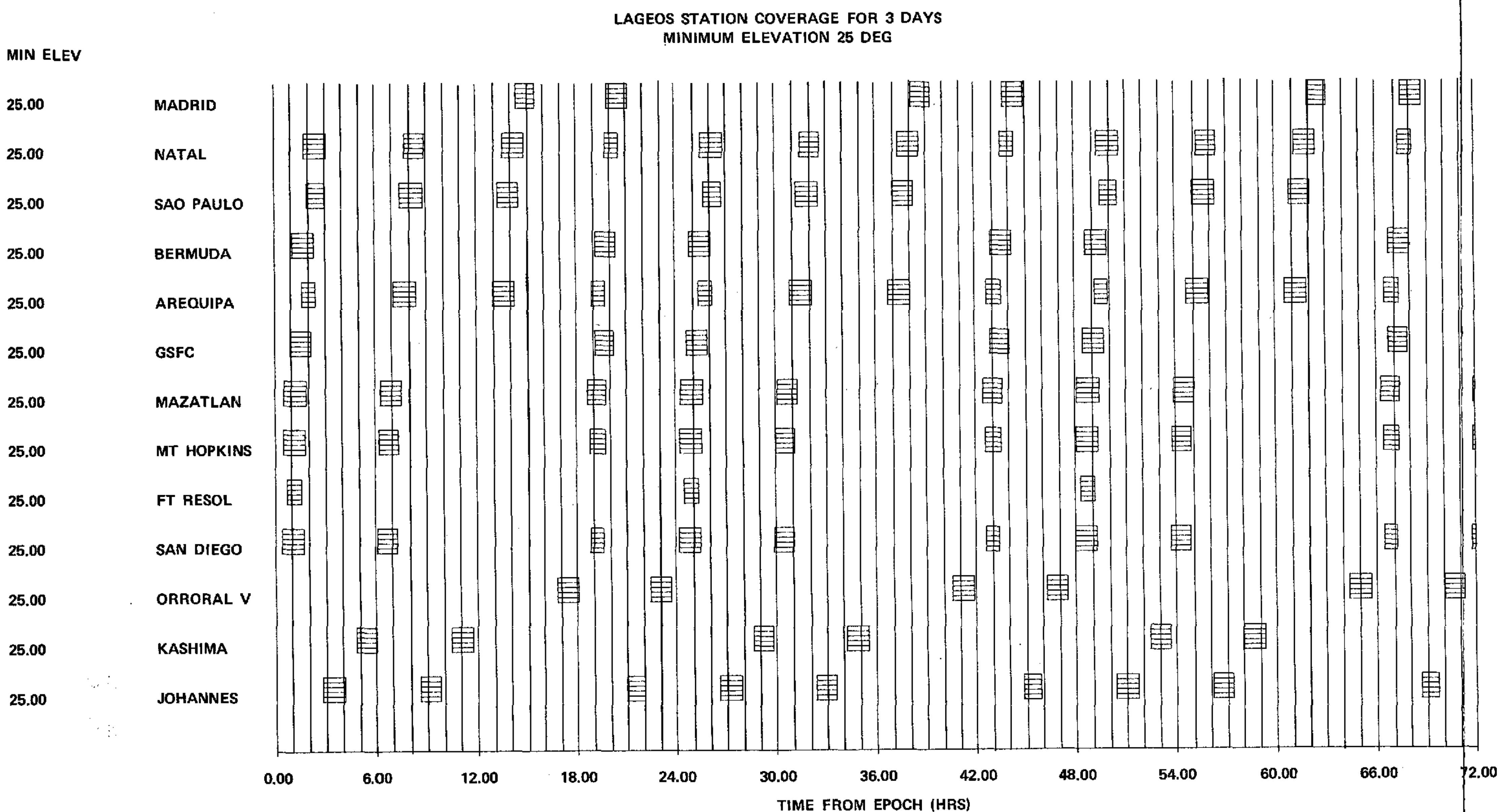

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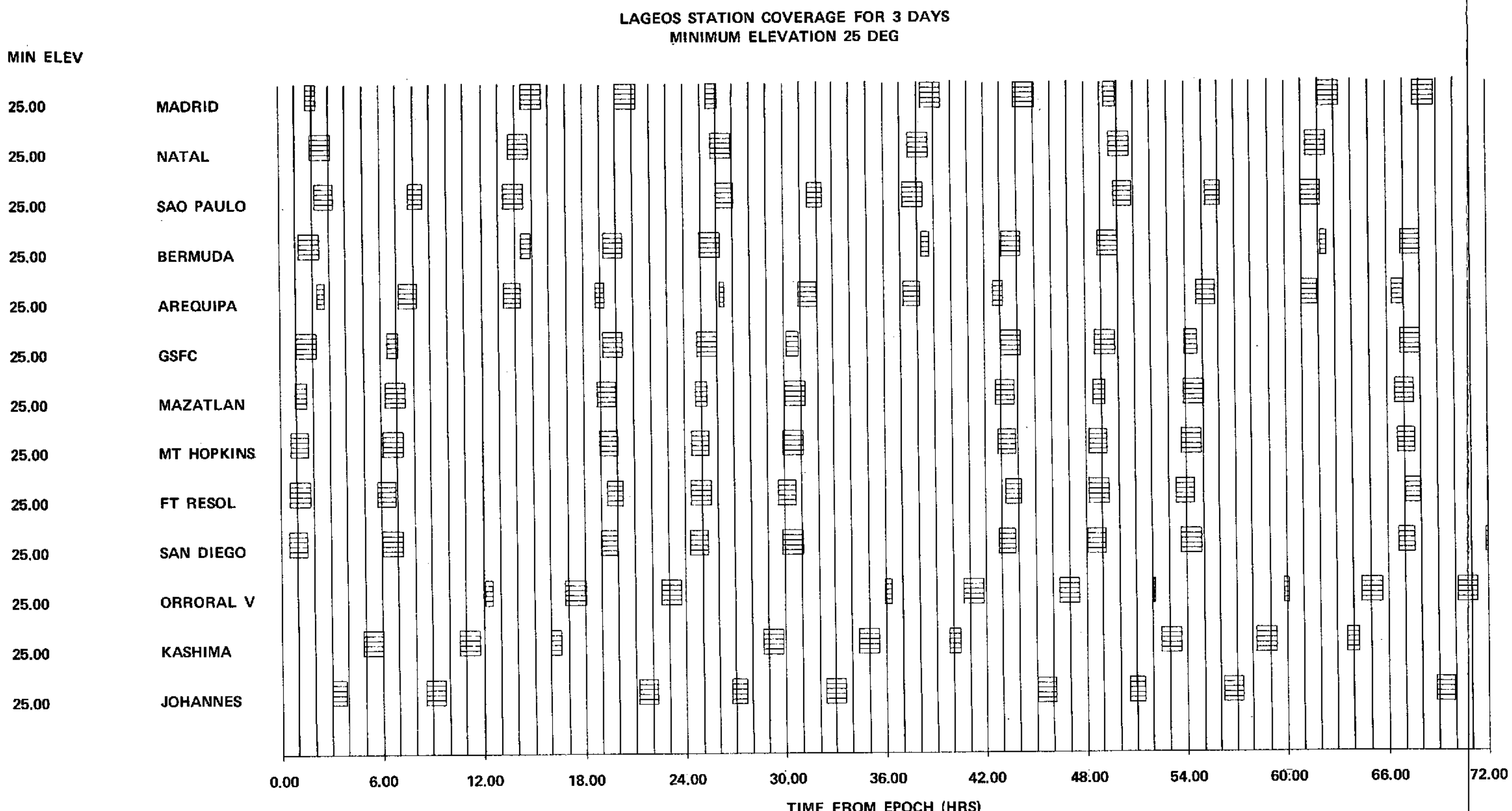
Figure 2(i). Station Visibility Times Chart



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MEAN ANOMALY	0.0000

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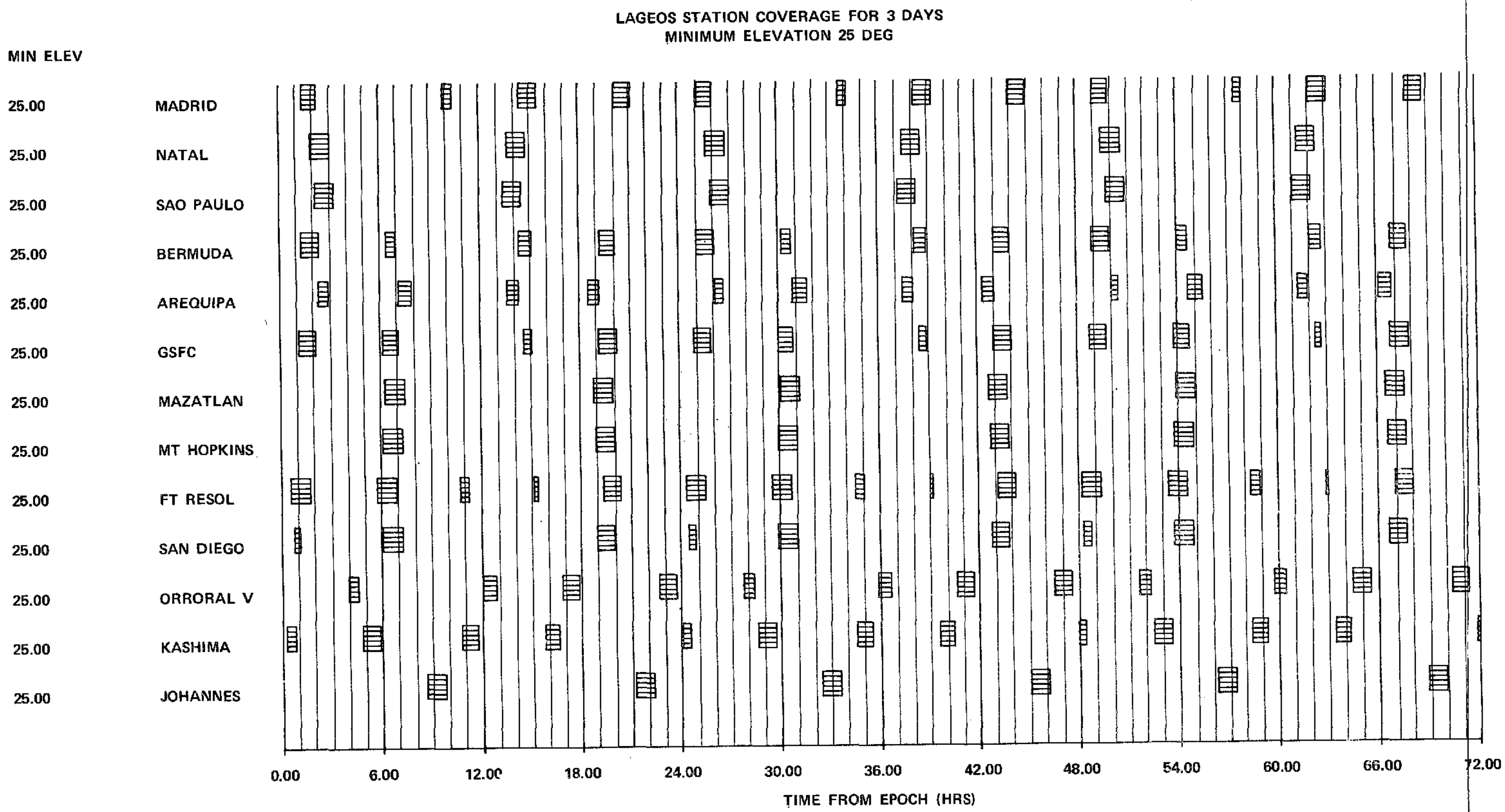
Figure 2(j). Station Visibility Times Chart



EPOCH

JAN 21 1976 0 HRS 0 MIN 0.000 SEC

STATE VECTOR AT EPOCH

SEMI-MAJOR AXIS	14378.00
ECCENTRICITY	0.01000
INCLINATION	74.9999
RIGHT ASC OF ASD NODE	299.9996
ARGUMENT OF PERIGEE	0.0000
MEAN ANOMALY	0.0000

TRAJECTORY ANALYSIS AND GEODYNAMICS DIVISION
FLIGHT MISSION ANALYSIS BRANCH
GODDARD SPACE FLIGHT CENTER

Figure 2(k). Station Visibility Times Chart

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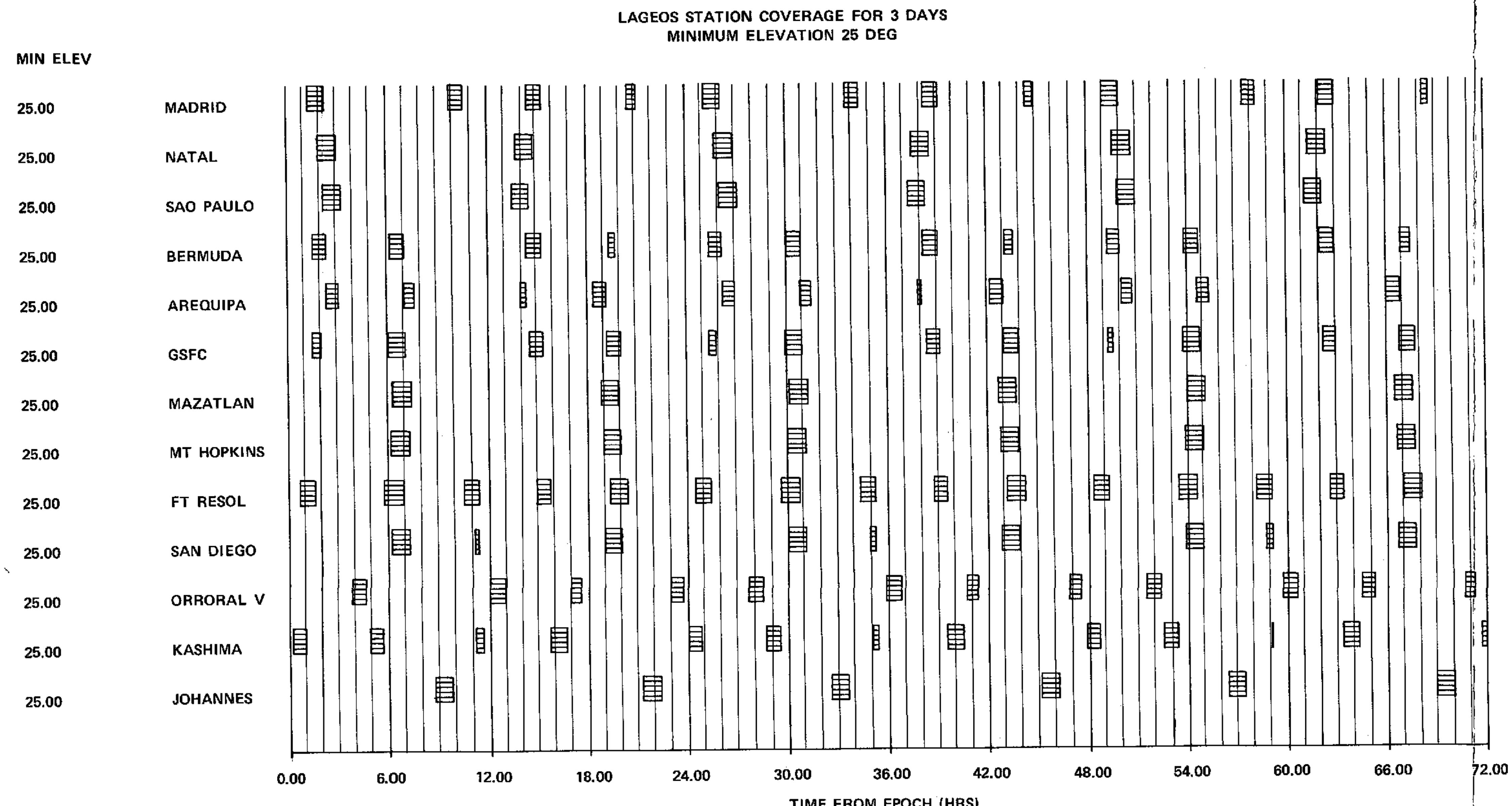
FOLDOUT FRAME

FOLDOUT FRAME



EPOCH

JAN 21 1976 0 HRS 0 MIN 0.000 SEC

STATE VECTOR AT EPOCH

SEMI-MAJOR AXIS	14378.00
ECCENTRICITY	0.01000
INCLINATION	89.9999
RIGHT ASC OF ASD NODE	299.9996
ARGUMENT OF PERIGEE	0.0000
MEAN ANOMALY	0.0000

TRAJECTORY ANALYSIS AND GEODYNAMICS DIVISION
FLIGHT MISSION ANALYSIS BRANCH
GODDARD SPACE FLIGHT CENTER

Figure 2(1). Station Visibility Times Chart

2

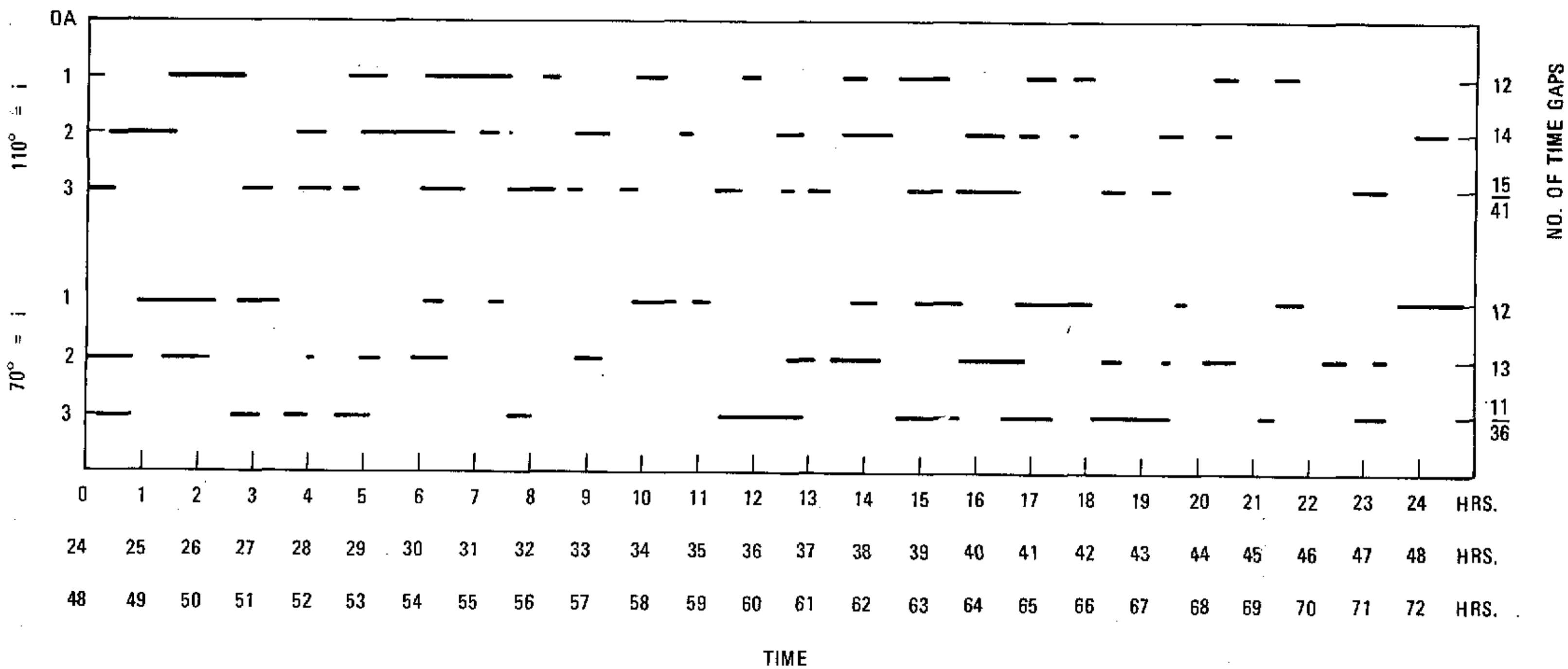


Figure 3. Time Gaps of Zero Tracking, h = 5690 (km), 11/12/73

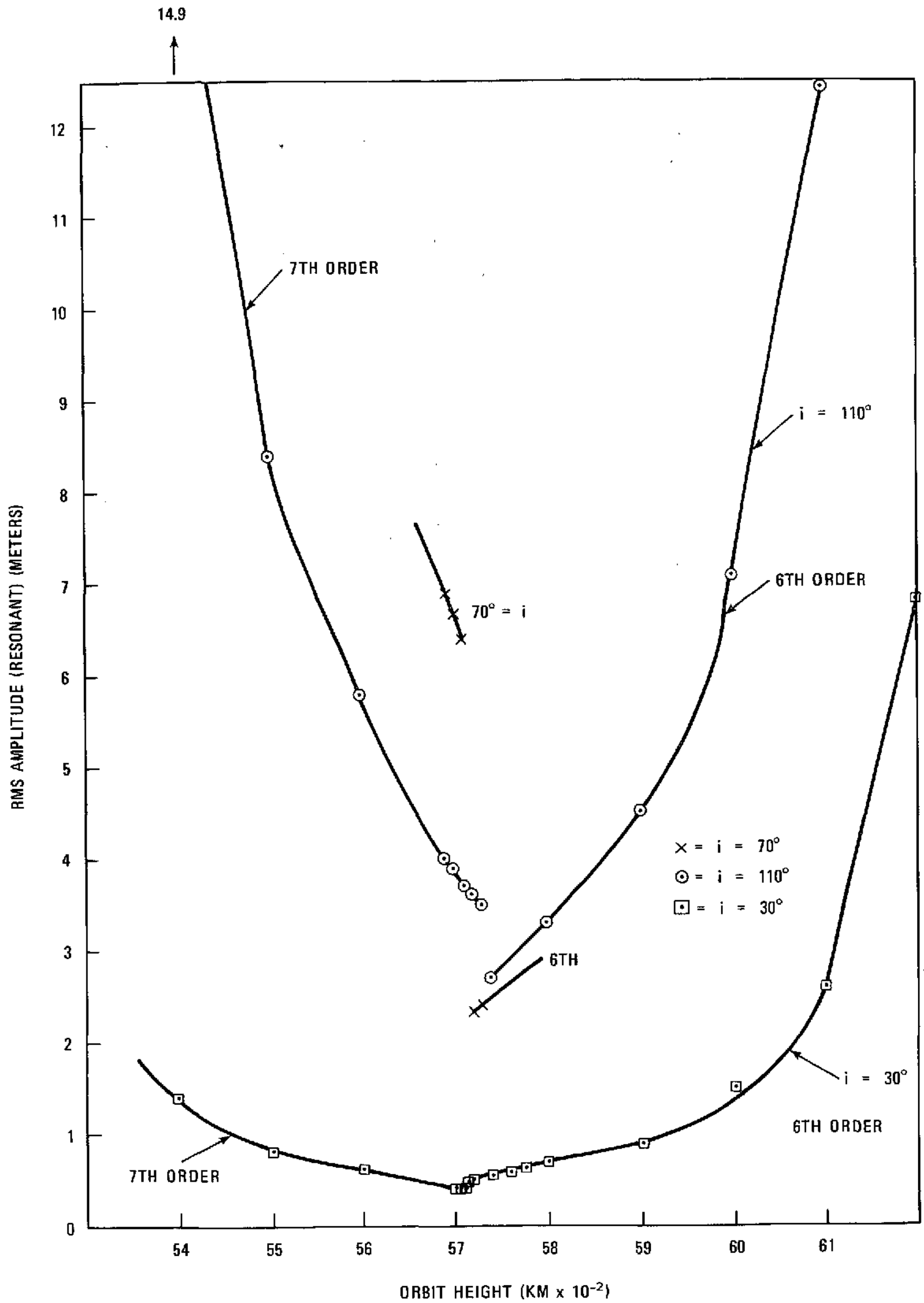


Figure 4. RMS Amplitude (Resonant), i = 110°, 11/19/73

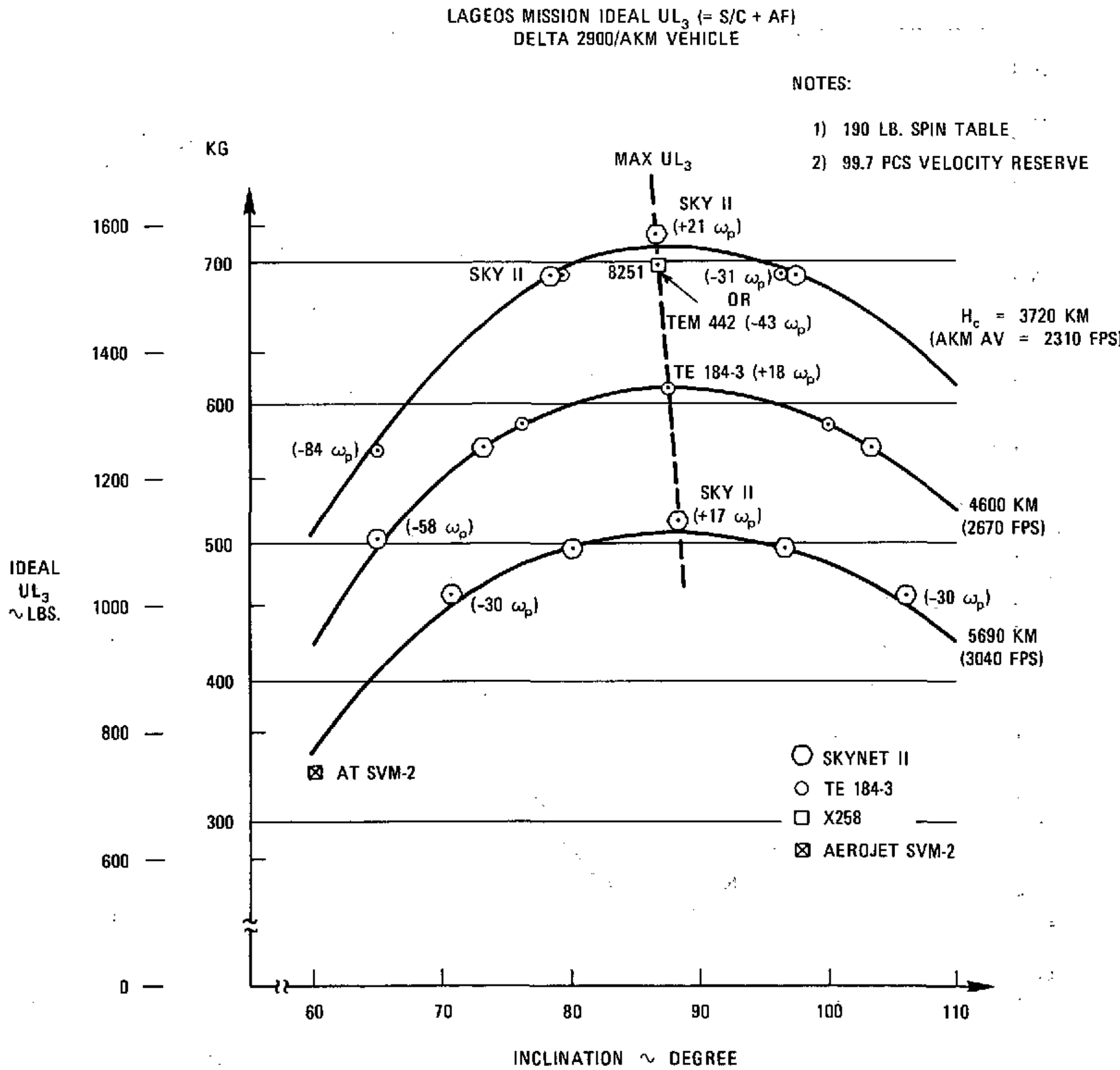


Figure 5. DELTA Payload Performance

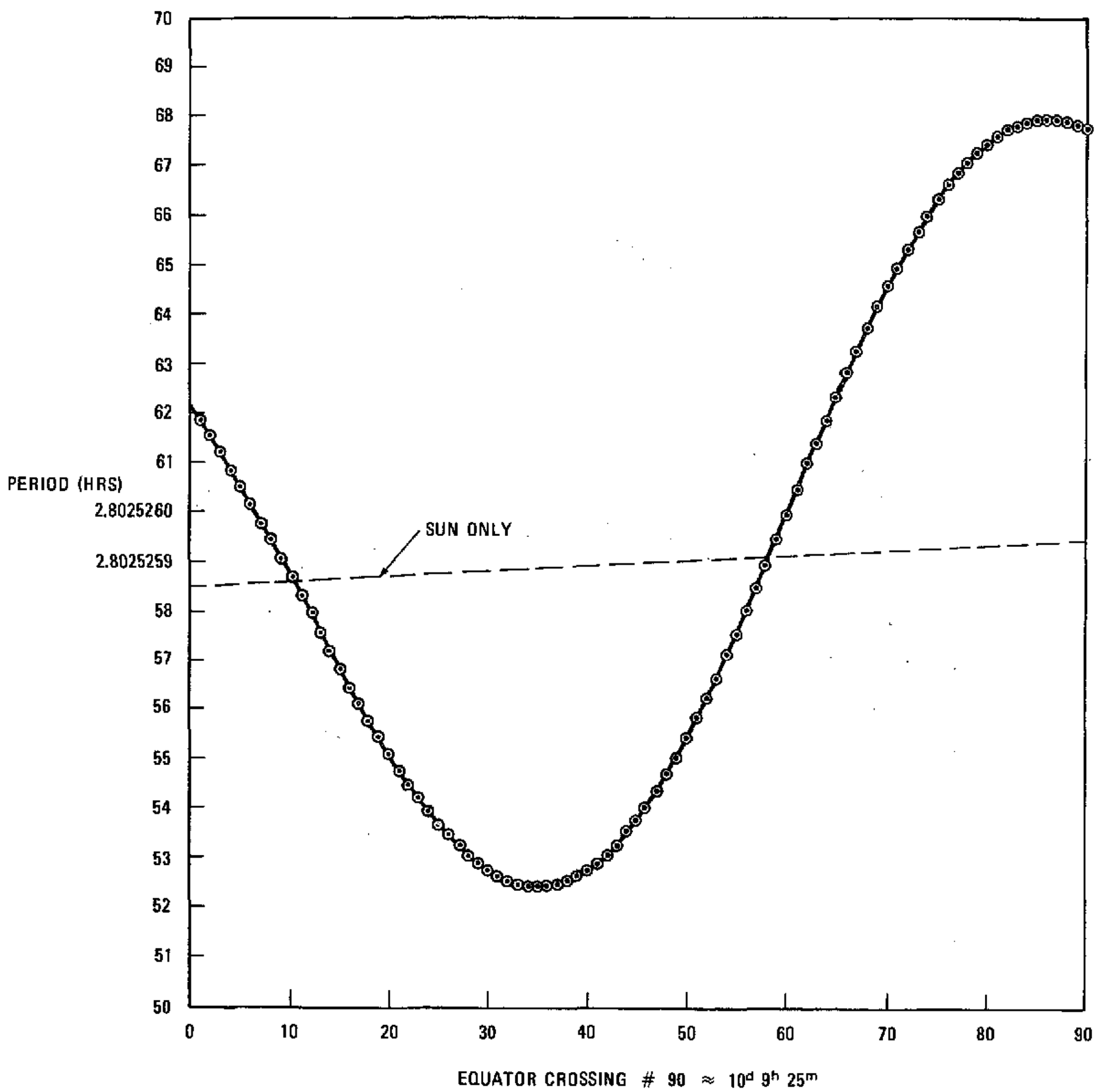


Figure 6. Sun and Moon Only Perturbations, 7/10/73

Table 1

LAGEOS Passes/Day
11/8/73

Inclination	070°				110°			
				Total				Total
Day	1	2	3		1	2	3	
JOH	3	4	3	10	3	4	3	10
KAS	4	4	4	12	4	5	5	14
ORR	4	5	3	12	5	4	5	14
SAN	4	4	4	12	5	4	5	14
FOR	5	4	4	13	5	4	5	14
HOP	2	4	4	10	5	3	4	12
MAZ	3	3	5	11	3	3	4	10
GOD	5	3	3	11	5	5	4	14
ARE	2	2	3	7	2	2	5	9
BER	5	4	3	12	4	4	4	12
SAO	4	3	3	10	5	3	4	12
NAT	3	2	3	8	3	3	4	10
MAD	5	4	3	12	4	5	4	13
Total				140 ~47/day				158 ~53/day
Total Time	$48^{h}.88$				$48^{h}.06$			

Table 2(a)

HAP Results
11/27/73

h (km)	Res. Order	Total Fld.	Max. Deg.	Beat Per (day)	RMS Res. Ampl. (m)	RMS Non-Res. Ampl. (m)	Cycles/Day
30° Inclination							
4000	8	9	17	6.1	2.4	294	8.215
5400	7	8	17	-4.2	1.4	333	6.794
5500	7	8	17	-3.1	.8	336	6.708
5600	7	8	17	-2.4	.6	340	6.624
5700	7	8	20	-2.0	.4	339	6.542
5705	7	8	22	-2.0	.42	339	6.538
5710	7	8	24	-2.0	.41	334	6.534
5715	6	8	23	2.0	.49	336	6.530
5720	6	8	24	2.0	.50	335	6.526
5740	6	8	18	2.1	.54	339	6.510
5760	6	8	17	2.1	.57	340	6.494
5780	6	8	17	2.2	.62	342	6.478
5800	6	8	17	2.3	.7	342	6.462
5900	6	8	17	2.8	.9	344	6.383
6000	6	8	15	3.6	1.5	346	6.306
6100	6	7	15	4.9	2.6	349	6.230
6200	6	7	15	7.7	6.8	351	6.156
8000	5	7	17	54.9	593.	378	5.037

Table 2(b)

HAP Results
11/27/73

h (km)	Res. Order	Total Fld.	Max. Deg.	Beat Per (day)	RMS Res. Ampl. (m)	RMS Non-Res. Ampl. (m)	Cycles/Day
60° Inclination							
4000	8	10	15	5.9	23.6	153	8.211
5800	6	8	16	2.3	4.2	161	6.460
5900	6	8	14	2.8	5.6	162	6.381
6000	6	8	14	3.6	8.2	160	6.304
6100	6	8	11	4.9	15.4	161	6.228
6200	6	8	11	7.7	34.6	162	6.154
8000	5	7	9	55.7	3841.	160	5.035
70° Inclination							
5690	7	8	17	-2.1	6.9	202	6.548
5700	7	8	20	-2.1	6.7	202	6.540
5710	7	8	22	-2.0	6.4	201	6.531
5720	6	8	26	2.0	2.3	210	6.523
5730	6	8	22	2.0	2.4	202	6.515
75° Inclination							
4000	8	10	21	5.7	15.5	452	8.210
6000	6	8	12	3.6	4.1	217	6.303
8000	5	6	9	53.4	4589.	217	5.035

Table 2(c)

HAP Results
11/27/73

h (km)	Res. Order	Total Fld.	Max. Deg.	Beat Per (day)	RMS Res. Ampl. (m)	RMS Non-Res. Ampl. (m)	Cycles/Day
80° Inclination							
5690	7	8	18	-2.1	9.4	228	6.547
90° Inclination							
4000	8	9	21	5.4	10.6	485	8.210
5690	7	7	18	-2.1	9.7	242	6.547
5800	6	7	16	2.3	2.7	243	6.459
5900	6	7	14	2.8	3.2	244	6.380
6000	6	7	14	3.5	4.5	244	6.303
6100	6	7	12	4.8	7.9	245	6.227
6200	6	7	12	7.4	18.7	245	6.153
8000	5	6	9	49.1	4025.	244	5.035
100° Inclination							
5690	7	8	18	-2.1	5.8	229	6.547

Table 2(d)

HAP Results
11/27/73

h (km)	Res. Order	Total Fld.	Max. Deg.	Beat Per (day)	RMS Res. Ampl. (m)	RMS Non-Res. Ampl. (m)	Cycles/Day
110° Inclination							
5400	7	8	11	-4.5	14.9	204	6.791
5500	7	8	14	-3.3	8.4	205	6.705
5600	7	8	16	-2.6	5.8	206	6.622
5690	7	8	16	-2.2	4.0	205	6.548
5700	7	8	18	-2.1	3.9	204	6.540
5710	7	8	18	-2.1	3.7	205	6.531
5720	7	8	20	-2.0	3.6	204	6.523
5730	7	8	22	-2.0	3.5	202	6.515
5740	6	8	22	2.0	2.7	203	6.507
5800	6	8	16	2.2	3.3	206	6.459
5900	6	8	14	2.7	4.5	206	6.380
6000	6	8	11	3.4	7.2	205	6.303
6100	6	8	11	4.6	12.4	206	6.228

Table 3(a)

NAP Results
H-L-C Max. 12/3/73

I	H (km)	Noise (cm)			Solar Press.			$C_{0,0}$			$C_{2,0}$			Station Location			δr_β			δt_β			All		
30°	4 K	.11	.53	.45	4.8	20.8	.67	351	96.2	628	2.7	12.5	202	2.0	6.7	9.5	1.1	2.6	6.4	2.6	10.9	1.6	351	97.2	629
60°		.11	.61	.35	5.3	17.2	.35	447	242	369	1.4	11.7	201	2.2	5.7	5.5	1.1	2.6	2.8	2.4	10.4	1.4	447	243	372
90°		.12	.66	.39	5.1	16.0	.47	462	252	25.5	2.8	2.0	.39	2.4	5.2	5.5	1.1	2.6	.45	2.3	10.2	1.1	462	252	25.8
0°	6 K	.09	.65	.27	6.5	24.5	.49	414	275	75.6	1.9	1.6	8.0	.31	9.1	1.2	.18	1.3	.67	.19	10.6	.37	414	277	75.6
30°		.09	.52	.45	6.5	20.7	.99	451	226	719	2.5	10.1	133	2.0	7.3	10.6	1.1	2.3	7.3	1.8	9.1	1.6	451	226	719
60°		.09	.56	.35	6.6	20.2	.79	547	394	573	2.2	10.5	135	2.3	6.3	6.1	1.2	2.6	3.3	1.7	8.7	1.6	547	394	573
90°		.10	.65	.40	6.8	22.4	.46	562	400	18.9	3.6	4.1	.40	2.6	6.1	6.5	1.3	3.1	.48	1.7	9.0	1.4	562	400	19.4
0°	8 K	.08	.62	.27	8.1	31.3	.66	481	512	19.0	1.5	1.8	1.8	.14	9.6	.53	.10	1.9	.35	.09	8.2	.31	481	512	19.1
30°		.09	.56	.50	8.1	26.1	1.3	542	428	1003	2.0	5.2	88.9	2.1	8.3	14.0	1.1	2.8	9.2	1.3	7.8	1.7	543	428	1004
60°		.08	.59	.39	8.2	23.0	1.2	655	618	899	1.2	10.7	86.4	2.4	6.8	8.9	1.3	2.9	4.6	1.3	8.0	2.0	655	618	899
90°		.09	.64	.45	8.4	25.7	.75	670	709	267	2.0	1.8	.45	2.7	6.5	10.0	1.3	3.0	2.5	1.2	8.5	1.6	670	709	267

Table 3(b)

NAP Results
H-L-C Max. 12/3/73

I	H (km)	Noise (cm)			Solar Press.			$C_{0,0}$			$C_{2,0}$			Station Location			δr_β			δt_β			All		
0°	6 K	.09	.65	.27	6.5	24.5	.49	414	275	75.6	1.9	1.6	8.0	.31	9.1	1.2	.18	1.3	.67	.19	10.6	.37	414	277	75.6
0°	8 K	.08	.62	.27	8.1	31.3	.66	481	512	19.0	1.5	1.8	1.8	.14	9.6	.53	.10	1.9	.35	.09	8.2	.31	481	512	19.1
30°	4 K	.11	.53	.45	4.8	20.8	.67	351	96.2	628	2.7	12.5	202	2.0	6.7	9.5	1.1	2.6	6.4	2.6	10.9	1.6	351	97.2	629
30°	6 K	.09	.52	.45	6.5	20.7	.99	451	226	719	2.5	10.1	133	2.0	7.3	10.6	1.1	2.3	7.3	1.8	9.1	1.6	451	226	719
30°	8 K	.09	.56	.50	8.1	26.1	1.3	542	428	1003	2.0	5.2	88.9	2.1	8.3	14.0	1.1	2.8	9.2	1.3	7.8	1.7	543	428	1004
60°	4 K	.11	.61	.35	5.3	17.2	.35	447	242	369	1.4	11.7	201	2.2	5.7	5.5	1.1	2.6	2.8	2.4	10.4	1.4	447	243	372
	6 K	.09	.56	.35	6.6	20.2	.79	547	394	573	2.2	10.5	135	2.3	6.3	6.1	1.2	2.6	3.3	1.7	8.7	1.6	547	394	573
	8 K	.08	.59	.39	8.2	23.0	1.2	655	618	899	1.2	10.7	86.4	2.4	6.8	8.9	1.3	2.9	4.6	1.3	8.0	2.0	655	618	899
90°	4 K	.12	.66	.39	5.1	16.0	.47	462	252	25.5	2.8	2.0	.39	2.4	5.2	5.5	1.1	2.6	.45	2.3	10.2	1.1	462	252	25.8
	6 K	.10	.65	.40	6.8	22.4	.46	562	400	18.9	3.6	4.1	.40	2.6	6.1	65	1.3	3.1	.48	1.7	9.0	1.4	562	400	19.4
	8 K	.09	.64	.45	8.4	25.7	.75	670	709	267	2.0	1.8	.45	2.7	6.5	10.0	1.3	3.0	2.5	1.2	8.5	1.6	670	709	267

Table 3(c)

NAP Results
H-L-C Min. 12/3/73

I	H (km)	Noise (cm)			Solar Press.			$C_{0,0}$			$C_{2,0}$			Station Location			δr_β			δt_β			All		
30°	4 K	.09	.28	.34	.09	.35	.35	338	.32	1.1	.40	.29	.50	1.1	5.0	3.0	.10	.32	.52	.13	8.1	.57	339	11.3	12.6
60°		.10	.27	.32	.11	.32	.33	245	1.2	.53	.10	.31	.39	.69	3.2	5.1	.15	.31	.58	.13	8.1	.53	245	10.7	12.2
90°		.11	.31	.32	.11	.36	.36	231	.43	.36	.12	.31	.33	.21	.47	1.2	.19	.37	.40	.17	8.7	.52	231	9.8	4.5
0°	6 K	.09	.35	.27	.09	.39	.27	405	.36	.32	1.4	.36	.27	.13	8.8	.40	.09	.36	.28	.09	10.2	.27	405	14.2	1.1
30°		.08	.27	.34	.08	.32	.37	370	.36	1.5	.14	.29	.38	.89	5.7	3.4	.09	.34	.44	.10	7.1	.48	370	9.6	17.1
60°		.08	.27	.33	.09	.33	.35	276	.75	3.4	.09	.29	.35	.58	3.5	5.6	.10	.31	.47	.10	7.3	.38	276	8.8	7.6
90°		.10	.65	.40	.10	.34	.35	261	.59	.38	.10	.30	.34	.17	.53	.78	.13	.33	.40	.12	7.9	.46	262	8.9	5.1
0°	8 K	.08	.34	.27	.08	.37	.27	463	.52	.31	1.2	.34	.27	.09	9.0	.33	.08	.36	.28	.09	7.8	.28	463	1.3	.62
30°		.08	.28	.35	.08	.35	.35	407	.33	1.9	.09	.30	.49	.82	6.6	4.9	.15	.40	1.3	.11	6.2	.71	407	10.0	15.5
60°		.08	.28	.34	.08	.31	.37	296	1.8	3.4	.08	.28	.39	.55	4.1	7.4	.21	.36	1.6	.10	6.7	.71	296	9.6	15.9
90°		.09	.31	.36	.09	.36	.38	281	.75	2.1	.09	.31	.36	.35	.84	5.0	.24	.43	1.4	.12	7.7	1.7	281	9.1	6.1

Table 3(d)

NAP Results
H-L-C Min. 12/3/73

I	H (km)	Noise (cm)			Solar Press.			$C_{0,0}$			$C_{2,0}$			Station Location			δr_β			δt_β			All		
0°	6 K	.09	.35	.27	.09	.39	.27	405	.36	.32	1.4	.36	.27	.13	8.8	.40	.09	.36	.28	.09	10.2	.27	405	14.2	1.1
	8 K	.08	.34	.27	.08	.37	.27	463	.52	.31	1.2	.34	.27	.09	9.0	.33	.08	.36	.28	.09	7.8	.28	463	1.3	.62
30°	4 K	.09	.28	.34	.09	.35	.35	339	.32	1.1	.40	.29	.50	1.1	5.0	3.0	.10	.32	.52	.13	8.1	.57	339	11.3	12.6
	6 K	.08	.27	.34	.08	.32	.37	370	.36	1.5	.14	.29	.38	.89	5.7	3.4	.09	.34	.44	.10	7.1	.48	370	9.6	17.1
	8 K	.08	.28	.35	.08	.35	.35	407	.33	1.9	.09	.30	.49	.82	6.6	4.9	.15	.40	1.3	.11	6.2	.71	407	10.0	15.5
60°	4 K	.10	.27	.32	.11	.32	.33	245	1.2	.53	.10	.31	.39	.69	3.2	5.1	.15	.31	.58	.13	8.1	.53	245	10.7	12.2
	6 K	.08	.27	.33	.09	.33	.35	276	.75	3.4	.09	.29	.35	.58	3.5	5.6	.10	.31	.47	.10	7.3	.38	276	8.8	7.6
	8 K	.08	.28	.34	.08	.31	.37	296	1.8	3.4	.08	.28	.39	.55	4.1	7.4	.21	.36	1.6	.10	6.7	.71	296	9.6	15.9
90°	4 K	.11	.31	.32	.11	.36	.36	231	.43	.36	.12	.31	.33	.21	.47	1.2	.19	.37	.40	.17	8.7	.52	231	9.8	4.5
	6 K	.10	.65	.40	.10	.34	.35	261	.59	.38	.10	.30	.34	.17	.53	.78	.13	.33	.40	.12	7.9	.46	262	8.9	5.1
	8 K	.09	.31	.36	.09	.36	.38	281	.75	2.1	.09	.31	.36	.35	.84	5.0	.24	.43	1.4	.12	7.7	1.7	281	9.1	6.1