Intermediate Microeconomics

COSTS

BEN VAN KAMMEN, PHD

PURDUE UNIVERSITY

No free lunch

Previously we have examined the result of a firm using inputs in a production process . . . that output is produced.

We saw the concept of the *isoquant map*, which indicated that there are many different input combinations that will result in the same quantity of output.

If the inputs were free, there would be no point in choosing which combination to use.

- But employing inputs is not free for the firm.

Since it has to pay its workers and its suppliers of capital, the firm is interested in the combination of inputs that will <u>minimize</u> its <u>costs</u>.

“Prices” of labor and capital

Suppliers of labor (workers) are paid a wage rate per unit of time they devote to working for the firm.

- Denote the wage rate as (w).

Suppliers of capital are paid a rental rate for each unit of capital they let the firm use.

- Denote the rental rate as (v).

The wage rate times the number of labor hours used and the rental rate times the number of “machine hours” used by the firm represent its costs.

The firm’s costs

When a firm uses two inputs (capital and labor), its costs are:

$$Total\ Costs\ (TC) = w * L + v * K.$$

This is the firm’s cost function.

The firm will choose L and K in such a way as to minimize TC for any given level of output.

w and v as opportunity costs

The wage rate is the amount per hour that the worker could get in his next-best employment opportunity.

- The firm has to pay him at least this much to prevent him from taking the other job.
- But not more because they wouldn't be minimizing costs.

Similarly the rental rate is the most that any *other* firm would be willing to pay to rent the machine, i.e., the machine's next-best employment opportunity.

- You can characterize rental rate in this way regardless of whether or not the firm owns the machine.
  - If it owns its capital, the firm is implicitly foregoing rent by using it for its own production process, and
  - If it rents its capital, it is explicitly laying out the rental price to the capital's owner.

Isocost lines

Remember the term *isoquant*? Isocost is its counterpart from the cost function.

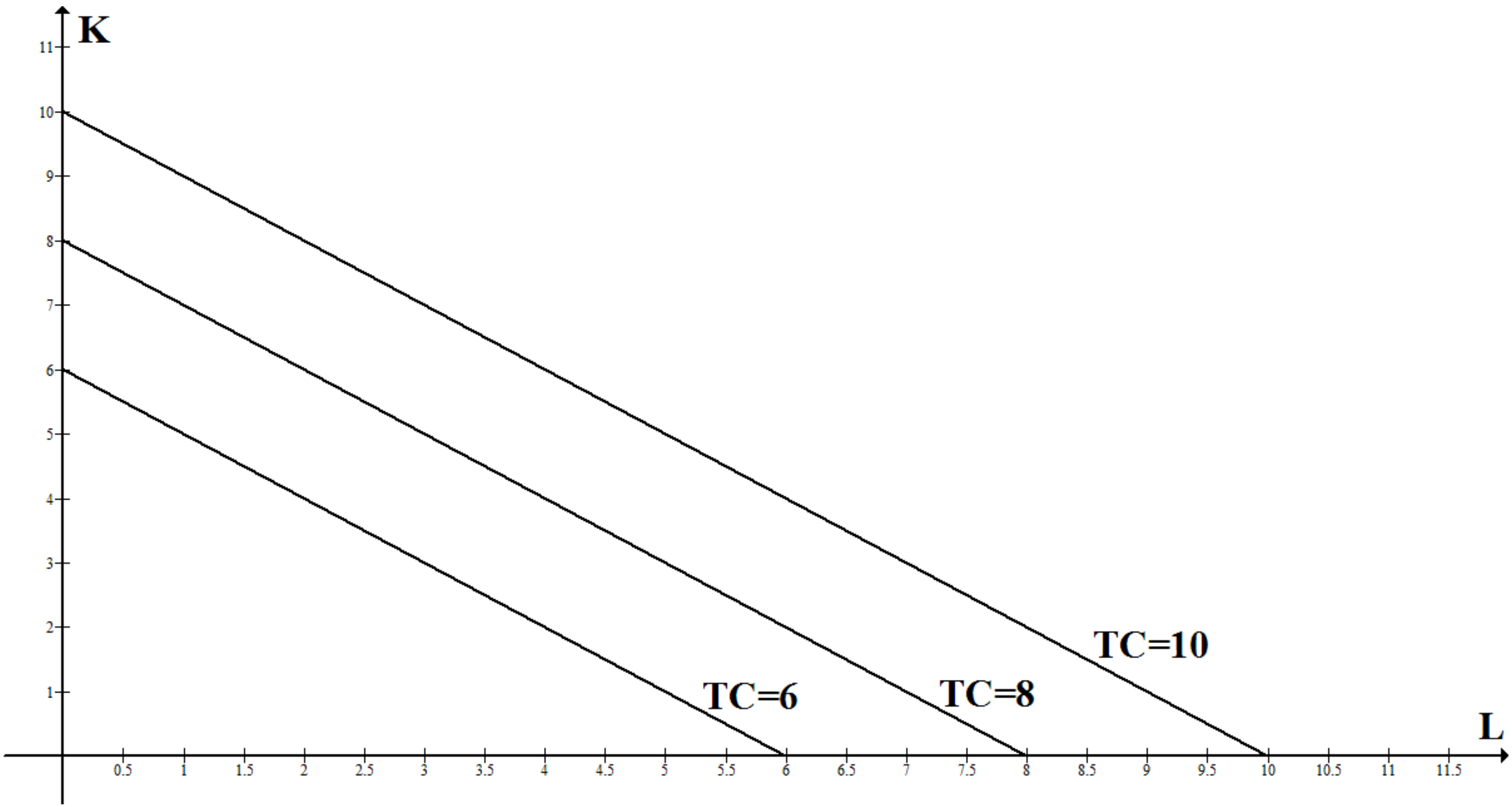
- This is a set of all input combinations that have the same cost.

Since the arguments in the cost function are the same as those of the production function, we can graph them in the same space.

In the following example, the wage and rental rate are equal:

$$w = v.$$

Isocost lines (graphical)



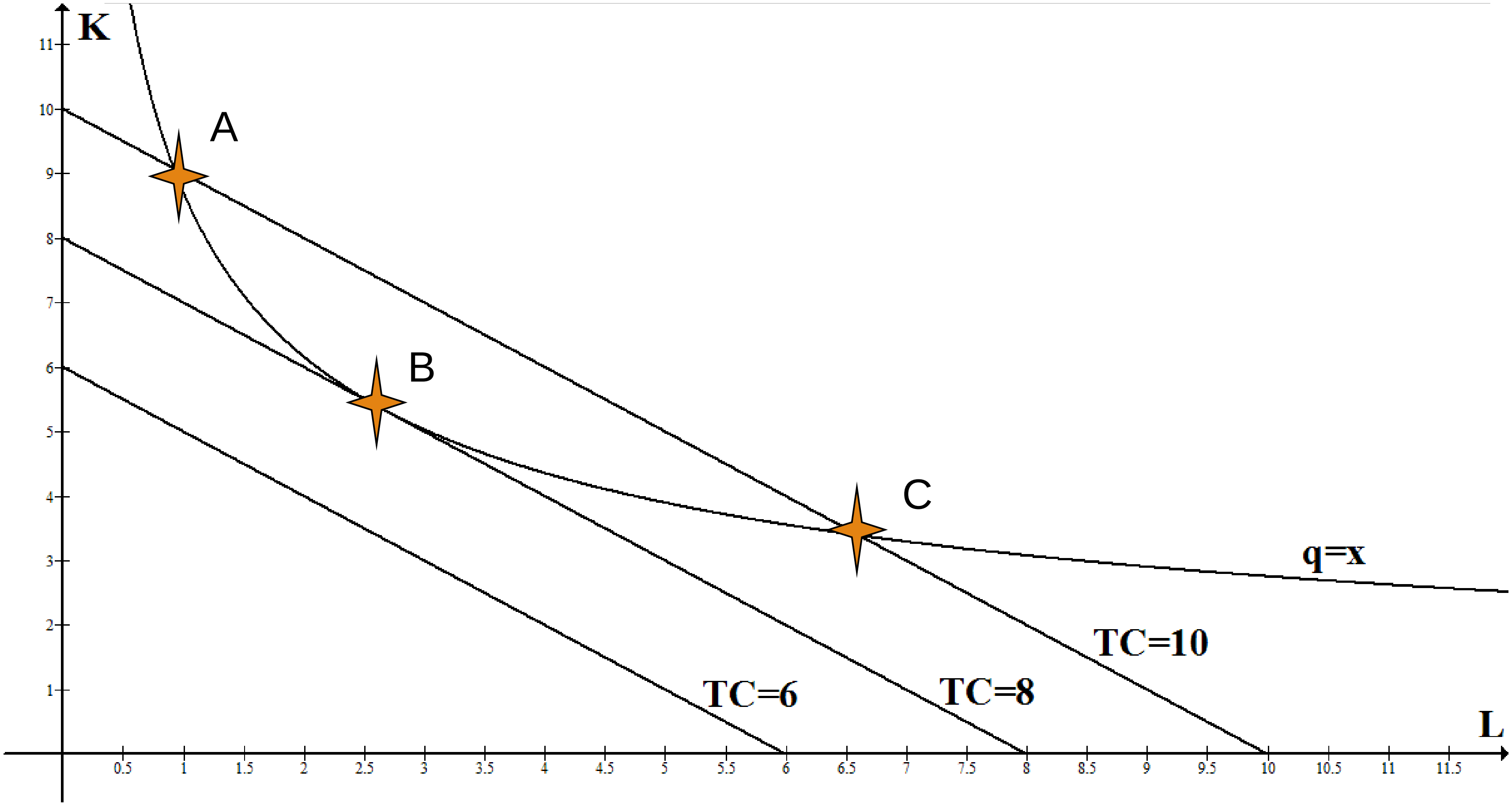
Minimizing TC

Which of these three cost levels ($6, $8, or $10) is the lowest for which x units can be produced?

The production function is still given by,

$$q = K^{\frac{2}{3}} L^{\frac{1}{3}}.$$

Isoquant for q=x



Answer

Quantity *x* is simply not feasible when the costs are limited to \$6.

- The isocost for \$6 and the isoquant never even touch.
- The firm is simply not productive enough to produce *x* units for \$6.

Look at points A and C. These are two different input combinations with which the firm could produce *x* units, while spending \$10.

- But these input choices are inefficient . . .

Because the firm can also produce *x* units at the lower cost of \$8. See point B.

- This point of tangency between the isoquant curve and isocost line represents the cost minimizing choice of inputs.

Tangency revisited

Recall from earlier lectures that when two functions are tangent, their slopes are the same at that point.

Also recall that the slope of any isoquant is the RTS (Rate of Technical Substitution).

Now what is the slope of the isocost line?

- $TC = wL + vK$ can be re-written, $K = \frac{TC}{v} - \frac{wL}{v}$.
- This is a linear function, so its slope is $\left|\frac{w}{v}\right|$.

$\frac{\Delta K}{\Delta L} = \frac{w}{v}$, i.e., the price ratio.

At the cost minimizing point, the <u>ratio of the marginal products equals the input price ratio</u>.

Cost minimization

Condition for cost minimization:

$$\frac{MP_L}{MP_K} = \frac{w}{v}.$$

Or, re -arranging, we have:

$$\frac{MP_L}{w} = \frac{MP_K}{v}.$$

The marginal product per dollar spent is the same for both inputs in the optimum.

This gives another way of demonstrating why points A and C are non-optimal.

- At A, $\frac{MP_L}{MP_K} > \frac{w}{v}$, and
- At C, $\frac{MP_L}{MP_K} < \frac{w}{v}$. So the condition is violated.

The shape of the cost function

From earlier microeconomics classes, you are probably familiar with functions that relate quantity of output to total cost. E.g.,

$$TC(q) = 5q.$$

These we can construct from the condition on the previous slide and some algebra.

Let's use a generalized version of the Cobb-Douglas production function (1) and the cost function (2).

1. $q = K^{\alpha} L^{\beta}$, where $(\alpha + \beta)$ are > or = or < to 1.
2. $TC = wL + vK$.

Cost and production related

The cost-minimizing condition is, again,

$$\frac{MP_L}{MP_K} = \frac{w}{v}.$$

First solve for the RTS:

$$MP_L = \frac{\partial q}{\partial L} = \beta K^{\alpha} L^{\beta-1}.$$

$$MP_K = \frac{\partial q}{\partial K} = \alpha K^{\alpha-1} L^{\beta}.$$

$$\frac{MP_L}{MP_K} = \beta K^{\alpha} L^{\beta-1} \div \alpha K^{\alpha-1} L^{\beta}$$

(after simplifying) $RTS = \left(\frac{\beta}{\alpha}\right)\left(\frac{K}{L}\right).$

Set this equal to $\left(\frac{w}{v}\right)$ for optimization.

Cost and production

$$RTS = \left(\frac{\beta}{\alpha}\right)\left(\frac{K}{L}\right)$$

This expression yields, alternately,

- a function for optimal K as a function of L and
- one for optimal L as a function of K:

$$K = L\left(\frac{w}{v}\right)\left(\frac{\alpha}{\beta}\right) \text{ and}$$

$$L = K\left(\frac{v}{w}\right)\left(\frac{\beta}{\alpha}\right).$$

Cost and production (cont'd)

Substituting one of these into the production function at a time enables you to write the quantity of output as a function of <u>one</u> input.

Substituting the condition is like ensuring that the firm chooses the other input optimally:

$$q(K) = K^{\alpha}\left[K\left(\frac{v}{w}\right)\left(\frac{\beta}{\alpha}\right)\right]^{\beta} \text{ or}$$

$$q(L) = \left[L\left(\frac{w}{v}\right)\left(\frac{\alpha}{\beta}\right)\right]^{\alpha} L^{\beta}.$$

Cost and production (cont'd)

Collecting the terms in the functions from the last slide, we have:

$$q(K) = K^{(\alpha+\beta)}\left(\frac{v\beta}{w\alpha}\right)^{\beta} \text{ and}$$

$$q(L) = \left(\frac{w\alpha}{v\beta}\right)^{\alpha} L^{(\alpha+\beta)}.$$

Solving for the quantity of input maps the desired quantity of output to the necessary amount of input, still ensuring the optimal choice for the remaining input:

$[q(w\alpha/v\beta)^{\beta}]^{[1/(\alpha+\beta)]} = K$ and

$[q(v\beta/w\alpha)^{\alpha}]^{[1/(\alpha+\beta)]} = L.$

Input requirements and total cost

$$\left[q\left(\frac{w\alpha}{v\beta}\right)^{\beta}\right]^{\left(\frac{1}{\alpha+\beta}\right)} = K \text{ and}$$

$$\left[q\left(\frac{v\beta}{w\alpha}\right)^{\alpha}\right]^{\left(\frac{1}{\alpha+\beta}\right)} = L.$$

Finally substituting these optimal “input demands” into the cost function enables us to get rid of the L and K terms, replacing them with *q*’s:

$$TC = wL + vK$$

$$TC = w\left[q\left(\frac{v\beta}{w\alpha}\right)^{\alpha}\right]^{\left(\frac{1}{\alpha+\beta}\right)} + v\left[q\left(\frac{w\alpha}{v\beta}\right)^{\beta}\right]^{\left(\frac{1}{\alpha+\beta}\right)}.$$

The total cost curve

(after factoring)

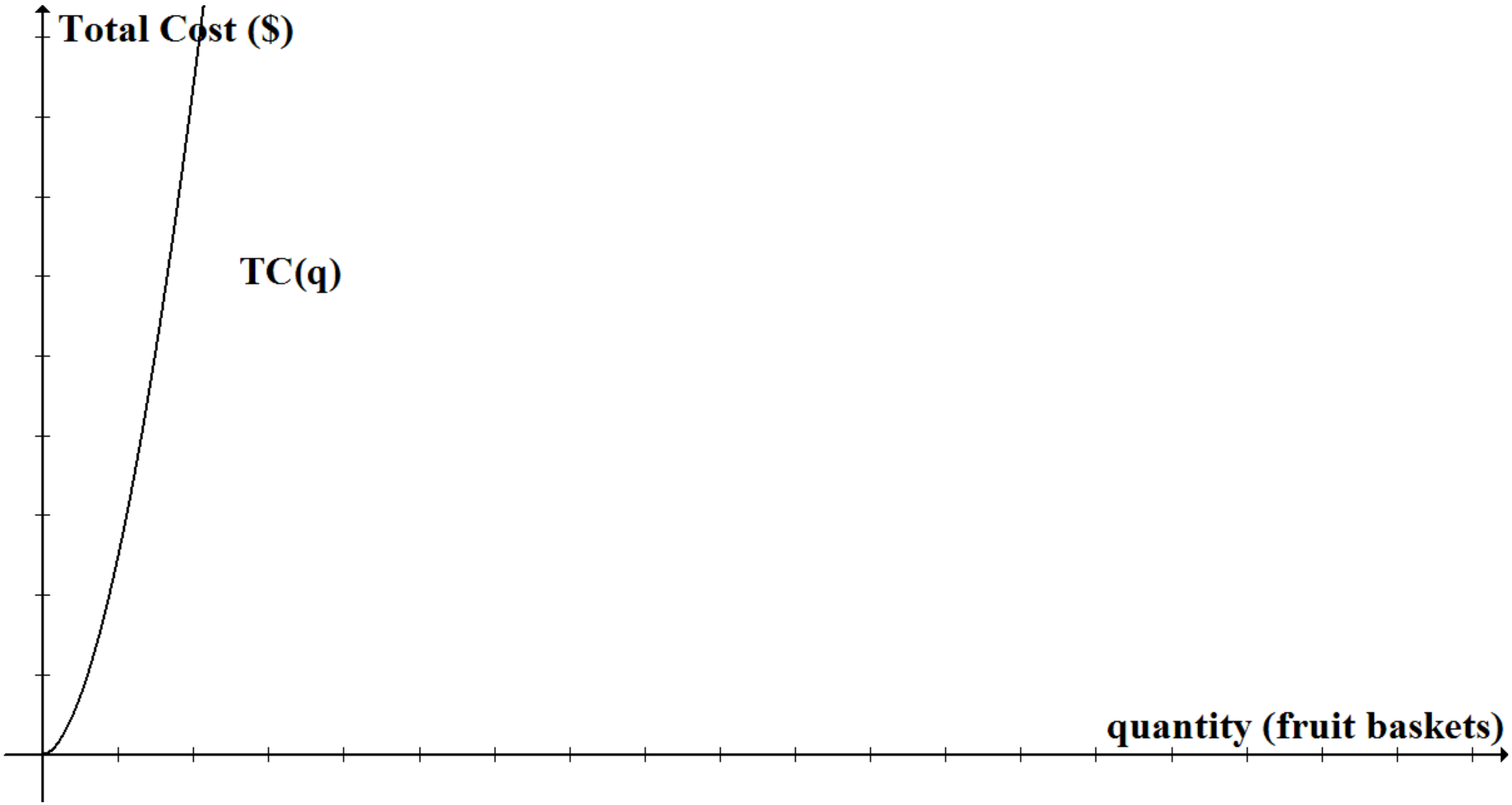
$$TC = q^{\left(\frac{1}{\alpha+\beta}\right)}\left[w^{\left(\frac{\beta}{\alpha+\beta}\right)}\left(\frac{v\beta}{\alpha}\right)^{\left(\frac{\alpha}{\alpha+\beta}\right)} + v^{\left(\frac{\alpha}{\alpha+\beta}\right)}\left(\frac{w\alpha}{\beta}\right)^{\left(\frac{\beta}{\alpha+\beta}\right)}\right].$$

And we're finished. This gives total cost as a function of the quantity produced alone.

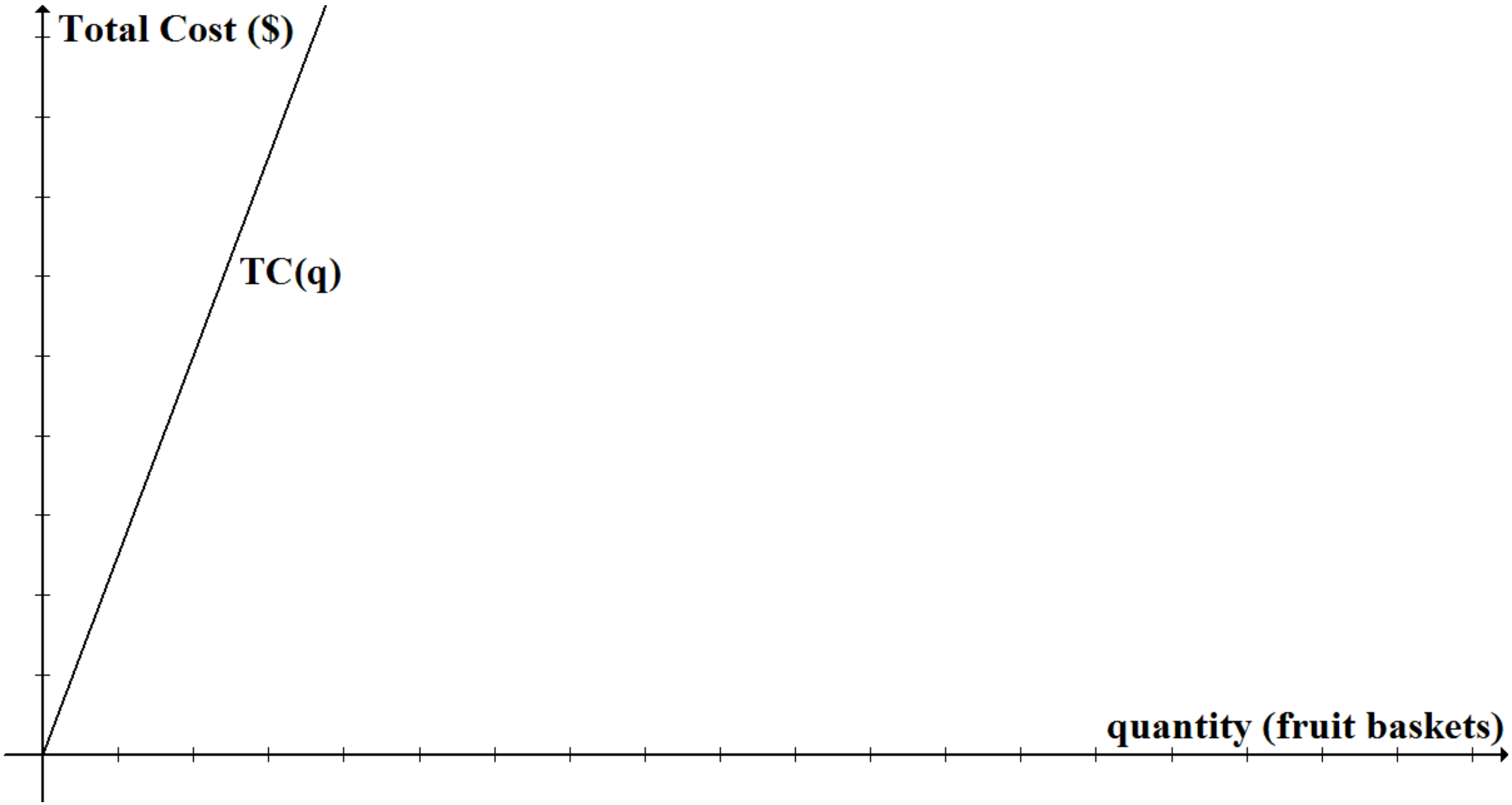
- The whole term in brackets is production function parameters and the input prices (which the firm cannot control).

We can now ask about the shape of the TC curve.

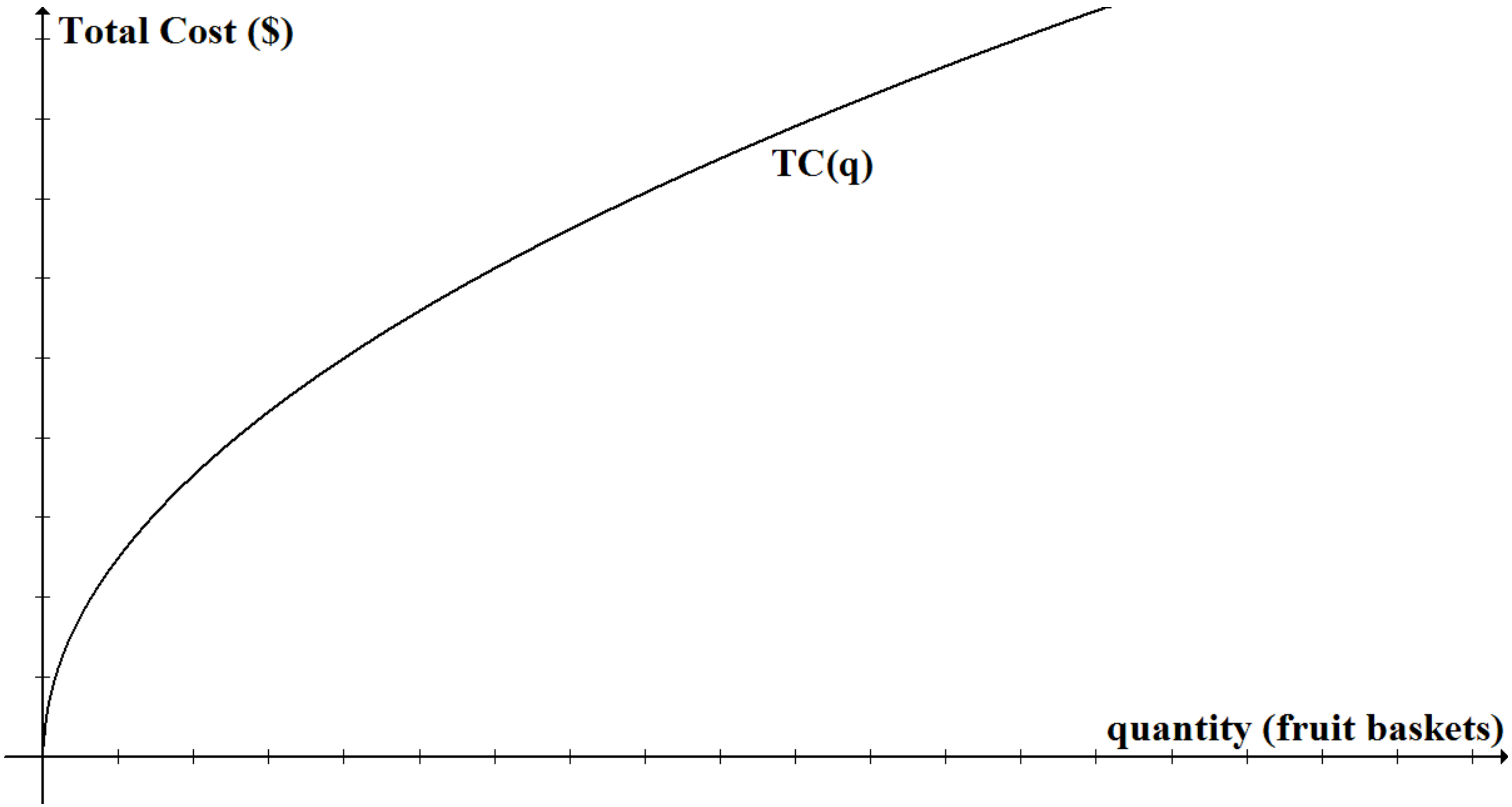
Increasing marginal cost?



Constant marginal cost?



Decreasing marginal cost?



Shape of TC curve and marginal cost

It is customary for Total Cost curves to slope upward because producing more always requires the firm to pay more costs.

But will the slope be increasing, constant, or decreasing?

This depends directly on the returns to scale for the production function.

Recall the TC function from the previous slide:

$$TC = q^{\left(\frac{1}{\alpha+\beta}\right)}\left[w^{\left(\frac{\beta}{\alpha+\beta}\right)}\left(\frac{v\beta}{\alpha}\right)^{\left(\frac{\alpha}{\alpha+\beta}\right)} + v^{\left(\frac{\alpha}{\alpha+\beta}\right)}\left(\frac{w\alpha}{\beta}\right)^{\left(\frac{\beta}{\alpha+\beta}\right)}\right]$$

Differentiating this with respect to q gives the marginal cost (MC).

Marginal cost

Marginal Cost: The additional cost of producing one more unit of output.

- The partial derivative of the Total Cost function.

For simplicity, let's call the big term in brackets in the$TC(q)$ function "λ".

So $TC(q) = \lambda q^{\left(\frac{1}{\alpha+\beta}\right)}$.

Differentiating with respect to q gives us:

$$\frac{\partial TC}{\partial q} = \left[\frac{\lambda}{\alpha + \beta}\right] * q^{\left(\frac{1-\alpha-\beta}{\alpha+\beta}\right)} = MC.$$

MC and returns to scale

$$MC = \left[\frac{\lambda}{\alpha + \beta}\right] * q^{\left(\frac{1-\alpha-\beta}{\alpha+\beta}\right)}$$

If this expression gets larger when *q* gets larger, you have increasing marginal cost.

If it gets smaller when q gets larger, decreasing marginal cost.

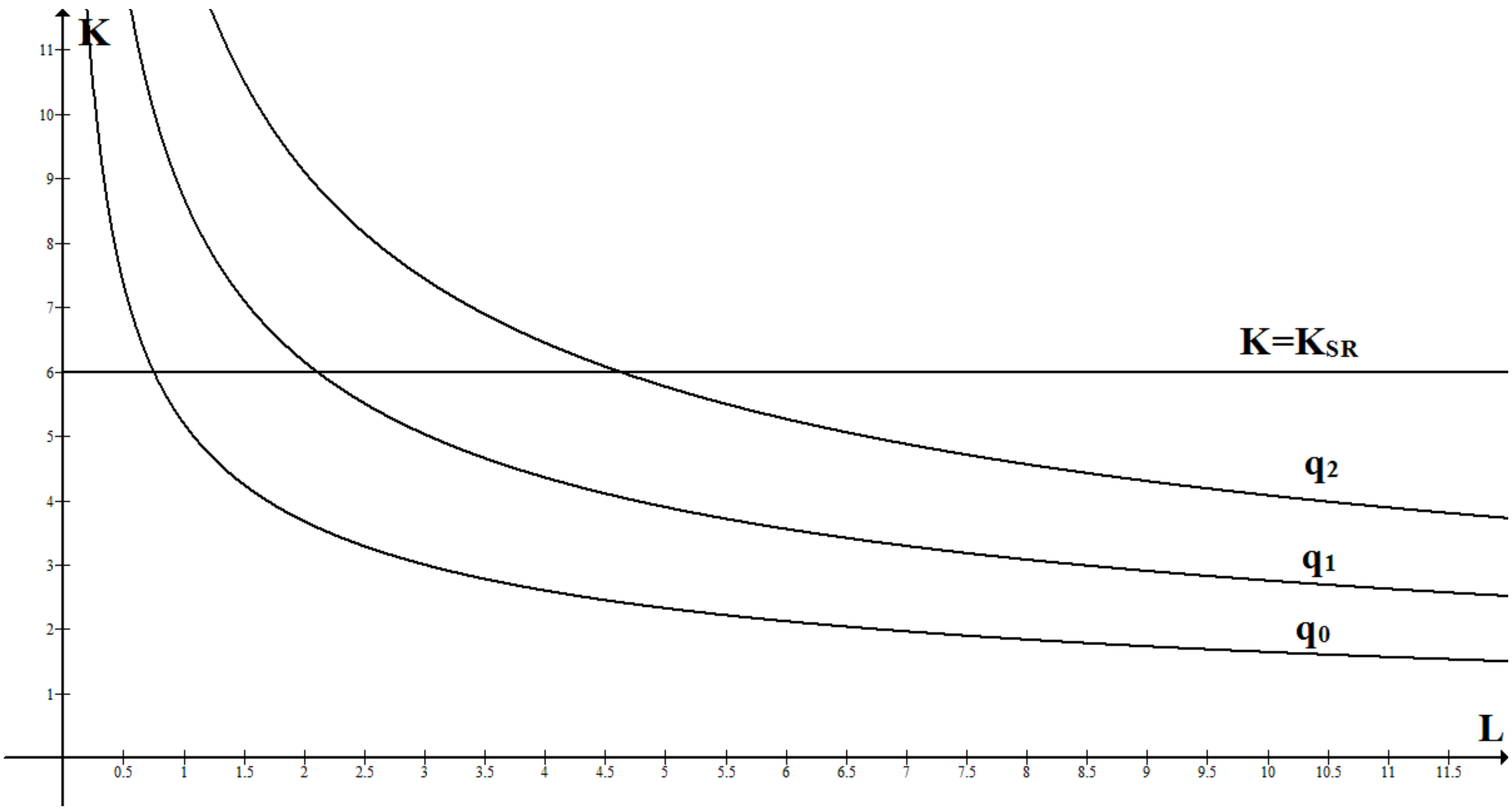
In the special case where $(\alpha + \beta) = 1$, the exponent becomes zero, so the marginal cost is just $\left[\frac{\lambda}{\alpha+\beta}\right] * 1$, i.e., constant for all values of q.

MC and returns to scale

So constant returns to scale in the production function implies constant marginal cost in the cost function.

Production Function	Cost Function
Decreasing Returns to Scale	Increasing MC
Constant Returns to Scale	Constant MC
Increasing Returns to Scale	Decreasing MC

Input choice in the short run



In the short run, capital stock is fixed at the level K_{SR}.

Short and long run

Short Run: (SR) The period of time in which a firm must consider some inputs to be fixed in making its decisions.

Long Run: (LR) The period of time in which a firm may consider all of its inputs to be variable in making its decisions.

As discussed earlier, the short run is characterized by having a fixed stock of capital and variable labor input.

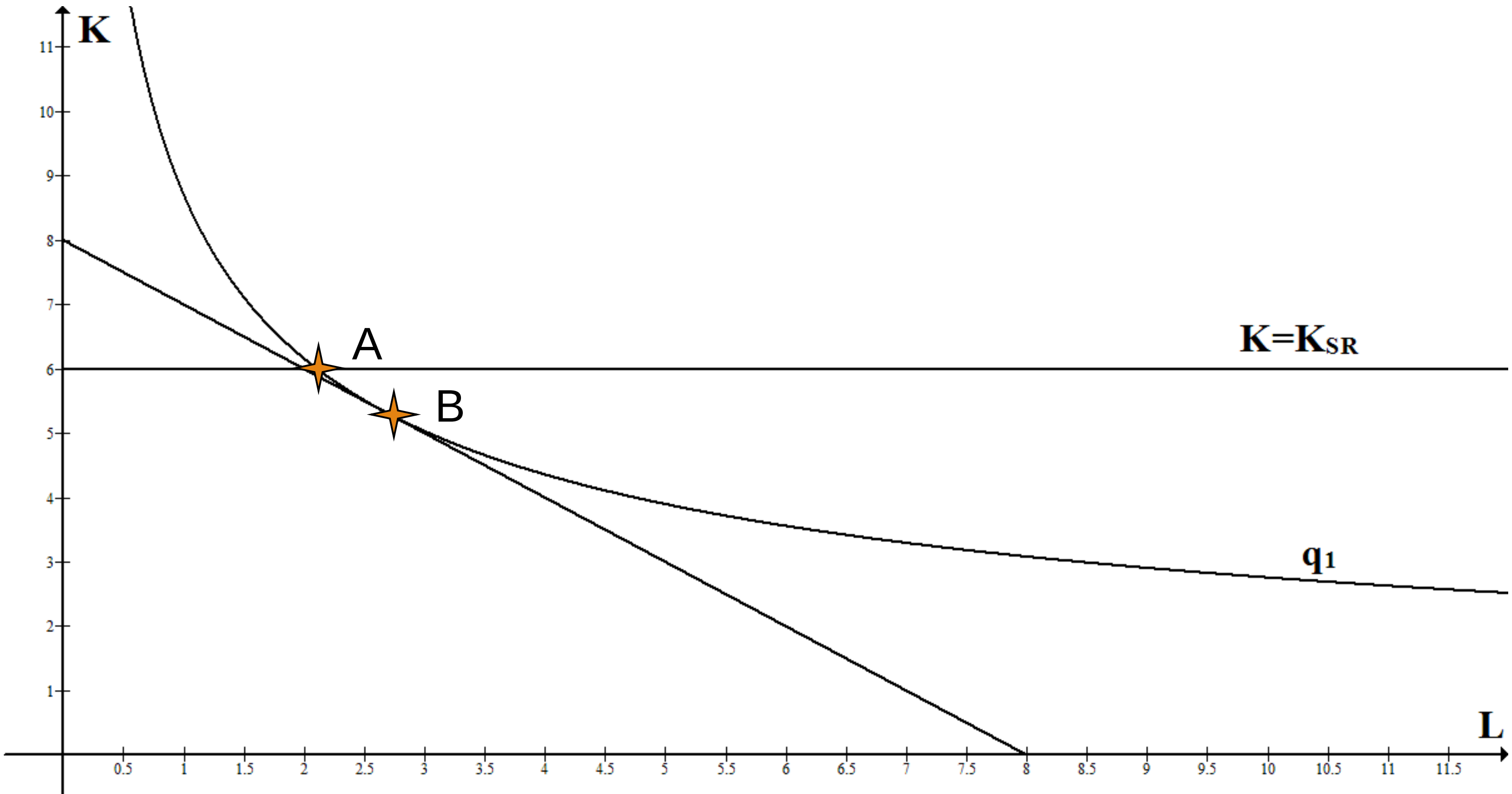
Fixed and variable costs

When an input is fixed, the amount the firm spends on that input is also fixed. When it is variable, the amount spend is also variable.

Fixed Costs: (FC) Costs associated with inputs that are fixed in the short run.

Variable Costs: (VC) Costs associated with inputs that can be varied in the short run.

Non-optimal input choices in SR



Firm produces at B in long run but cannot decrease K below K_{SR} in short run. It has to produce with “too much” capital, given the input price ratio (pt. A).

Short run marginal cost (SMC)

Elaborating on the idea of fixed capital in the short run, let's say that $K_{SR} = 8$.

So now the short run production function only depends on L:

$$q(L) = 4L^{\frac{1}{3}}, \text{or}$$
$$L = \left(\frac{q}{4}\right)^3 = \frac{q^3}{64}$$

The cost function also depends only on L:

$$TC = wL + 8v.$$

Short run marginal cost (SMC)

Substituting the production function into the cost function gives TC as a function of q:

$$TC = \frac{wq^3}{64} + 8v \ldots \text{with marginal cost,}$$

$$MC = \frac{3wq^2}{64}$$

For the sake of concreteness, call the wage $12. MC becomes,

$$MC = \left(\frac{9}{16}\right) q^2.$$

We have increasing marginal cost in the short run, since there is diminishing marginal product of labor.

Summary

Employing capital and labor requires the firm to pay a rental rate and a wage rate, respectively for each unit employed in production.

The sum of these products of input prices and input quantities gives the firm’s total costs.

Firms attempt to minimize their costs for any given level of output by choosing a combination of labor and capital.

Summary

Isocost lines can be graphed in the same space (L, K) as production isoquants.

Isocost curves have a slope equal to the input price ratio $\left(\frac{w}{v}\right)$.

Costs are minimized for a given output level when the isocost and isoquant are tangent.

- $RTS = \frac{w}{v}$.

Summary

The shape of the total cost (TC) curve depends on the returns to scale of the production function.

Marginal cost (MC) is the first derivative of TC with respect to quantity.

- It is the additional cost incurred by producing one more unit of output.

In the short run the capital stock is fixed, so firms usually have to make constrained (sub-optimal) production decisions.

Conclusion

The TC and MC functions we derived in this lesson are the result of firms' cost minimization decisions.

- They tell us the cheapest way of producing a given level of output.

We have not discussed the firm's decision of how much output to produce.

This decision is governed by the firm's ultimate objective: <u>profit maximization</u>.

In order to examine this objective, the <u>revenue</u> side of the firm's "balance sheet" needs to be discussed. <u>Revenue</u> is the next topic to which we turn.