Analysing The Disc Brake Squeal: Review And Summary

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ABSTRACT: - This paper reviews the most important research works that was carried out on disc brake squeal and the factors affecting it. The paper starts with reviewing the experimental works conducted to analyze disc brake squeal. The next section focuses on review the simulation of disc brake system with finite element software in order to predict the squeal condition. The sequel was investigated by using modal participation method and complex eigenvalue analysis. The paper finished with a brief review of the available literature and also gives a result summery. The results of reading this paper will give the researcher a comprehensive collaboration between the theoretical and experimental works, beside that it will guide the researcher to find his research objective and problem statement easy. Understanding brake squeal and friction-induced noise requires complicated analysis due to complexity of brake system. The complete analysis can be done first by studying the brake friction relation at low speed and compare it with the experimental works. Second study the brake noise at high frequency and its relation with the contact stiffness. Third study the effect of changing the contact pressure and contact angle.

INDEX TERMS:- Squeal, Mode shape, Eigenvalue, Eigenvector, Finite element method, out-of-plane, Stick-sip, Diametral nodes

1 EXPERIMENTAL ANALYSIS OF DISC BRAKE SQUEALS

1.1 BEAM ON DISC TECHNIQUE

Understanding and preventing brake squeal was the main object of previous experimental studies. In a pioneering study, Masayuki and Mikio [1] used the beam on disc test with dry friction to study the brake noise. Their experimental apparatus used a beam on disc technique with accelerometers to measure the disc brake system vibration. The beam-on-disc consists of a cantilever beam which represents the pad while the rotation disc represents the disc brake rotor. The beam and the disc are pressed against each other by a weight. They classified the friction noise to rubbing and squeal. They found that the noise was caused by the lateral vibration of the rod only. They also found that when the friction coefficient is small the vibration is small for that the rubbing noise has low level. The results showed that the rubbing noise can be changed to squeal noise due to the wear during the sliding work to change the surface roughness. Masayuki and Mikio [2] studied the effect of contact angle on the squeal using beam on disc system with variable angle of the rod. Their results showed that when the rod angle is in the same direction with the disc rotation, rubbing and squeal occurs and the vibration increases with increasing the rod angle. Tworzdlo and Oden [3] studied the instability of friction in a mechanical system. A pin on disc apparatus was used to represent the brake interface surfaces. They investigated that the oscillation of the system is due to mode coupling at high-frequencies and stick slip motion in low frequencies. A jump for the beam occurs in two typical situations: in the case of high amplitude, self-excited oscillation and at the very beginning of the sliding after the static contact of two surfaces (slip after stick).

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They found that self-excited oscillation occurs when the natural frequencies of the normal and rotational vibration of the slider are close to each other in presence of friction force. Tuchinda, et al and Tarter "[4], [5]" used pin-on-disc system in order to study how squeal noise can be generated in disc brake. The two components (pin and disc) were coupled together by using coulomb friction. The model showed that instability can occur when one of the natural frequencies of the pin becomes close to the natural frequencies of the disc. Giannini, et al. [6] used beam on disc system to identify the key parameters controlling the squeal. They showed that the squeal occurs when the frequencies of the individual parts are close to each other. Squeal does not require the stick-slip limit cycle to create and does not generally affect by changing the relative velocity. Akayc, et al. [7] worked on approaching the experimental setup of brake noise to much simpler model than the commercial disc brake. The model provides a possibility of repeatable measurement of squeal occurrence. This model is consisting of simplified experimental rigs (beam-on-disc), figure 1.

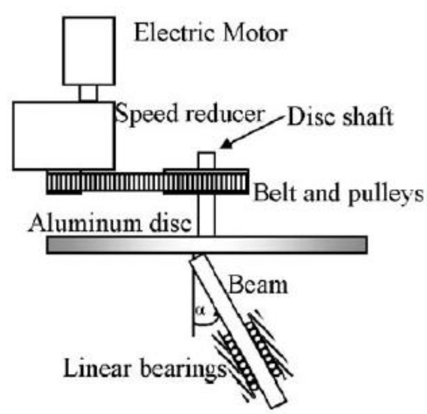

Figure 1 schematic of beam-on-disc, Akayc et al. [7]

The cantilever beam is mounted on a sliding platform that moves on two linear bearings, allowing the cantilever beam to be pre-loaded against the disc with a specified normal load. The angel between the beam and the disc is close to 45 degree. This angle is allows the friction force to excite easily the bending vibration of the beam. They found that squeal frequency is always coincident with a natural frequency of the coupled system and not with the frequency of the free rotor (this explains the small discrepancy between squeal frequency and free rotor natural frequency).

1.2 HOLOGRAPHIC INTERFEROMETRY

Fieldhouse and Newcomb [8] used a holographic image to measure the displacement of the disc and analyse the free mode of the disc/pad at self-generated noise. They providied visual images of the system in order to understand the mechanism involved, as shown in Figure 2. They showed that the entire squeal achieve when the disc in one of the diametral mode. The squeal noise was found to be close to the natural frequency of the brake component.

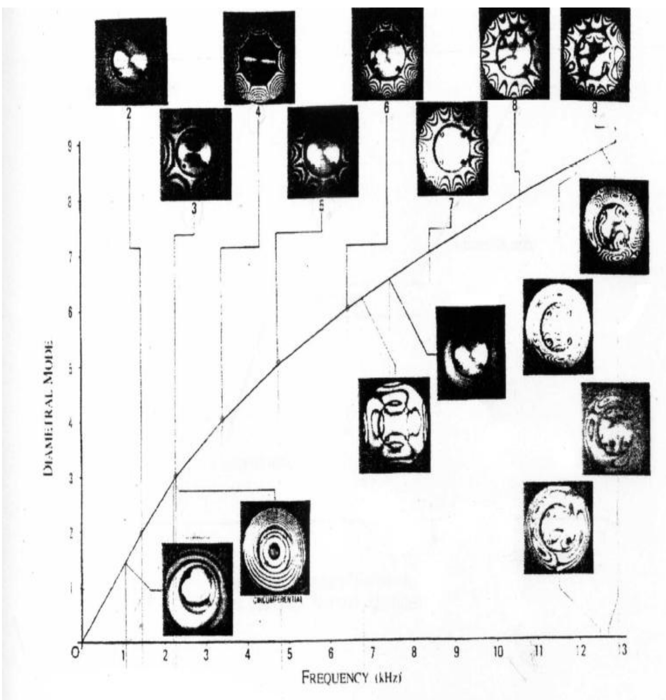

Figure 2 Photographs of the holographic reconstruction showing disc mode shape versus frequency, Fieldhouse [8]

In 1993, Fieldhhous and Newcomb [9] used a double pulsed laser holographic technique, which was developed to allow simultaneous recording of three orthogonal visual images of vibrating brake system. Visual images of vibration characteristics of noisy brake showed the disc to be in bending mode with diametral modes. The pad was seen to vibrate in a variety of modes such as bending, torsion and often a combination of both. The pad and flutter (disc) was shown to make significant contribution to make system noise. The researchers found that the introduction of asymmetry into the rotor was a solution to inhibit the formation of symmetrical diametral mode of vibration and the trendy to generate noise. In 1999, Fieldhouse [10] used technique of holographic interferometry to record the modes of the vibration and found that increasing the contact pressure resulted in increasing frequency over a specific range. The research showed that the preferred excitation frequency of any disc brake may be related to the free mode frequency of the disc. The tendency to generate noise frequency often less than the free mode frequency of the disc for the same mode order. The mode shape under the pad will be compressed if two antinodes are held, causing the free antinodes to expand and results in a lower frequency. If one anti-node is held below the pad it will expand in order to occupy the space available. This causes the free antinodes to compact and generate higher frequency. An increase in pad length would not significantly affect the pad/rotor interface pressure distribution. The experiment showed that the maximum pad effective length is varying between 80% -100% of full pad length. Talbot and Feildhous [11] captured holographic images and created animated 3-dimensional images of brake system during the squeal. The results showed that the displacement variation of the disc at fix radius was a sin wave during the squeal. Fieldhouse [12] analysed the noise by using holographic interferometry of two brake system with different volumes. The noise occurs as a result of coupling natural frequency of the individual brake parts when those frequencies are close together. The researchers indicated that anti-node positioned at the centre of the pad at higher frequency whereas it is a node in the case of lower. Filnt and Hald [13] studied the existence and direction of travelling waves in the squealing disc brakes by using acoustic holography. Acoustic holography measures the radiation sound pressure. They used the microphone to measure the sound during the squeal. The results showed that the nodal position rotated 180 degree in clockwise direction during one period of oscillation. This means that there is a travelling wave in the opposite direction of rotation of the disc and the motion of the node is not due to the rotation of the disc.

1.3 PULSE ELECTRONIC SPECKLE PATTERN INTERFEROMETRY

The works based on holographic interferometry technique was continues until another optical technique, electronic speckle pattern interferometry (ESPI), was developed. ESPI device automatically fire a laser and provide the complete deformation map of the component under test and can capture the squealing signal. Dantec Dynamics Company used EPSI to study the brake squeal which was illustrated in Figure 3. They measured the brake disc while it was rotating with the pulsed ESPI system. The result indicated that three dimensional ESPI was very useful for the analysis of the dynamic behaviour of disc brake in highly dynamic processes.

Figure 3 Dantec Dynamics Company apparatus, 3D Pulse ESPI system, http://www.dantecdynamics.com/Default.aspx?ID=1603

Chen et al. [14] used EPSI technique to obtain the mode shapes of a disc brake when it was squealing. This work investigated the radial mode and friction process which contribute in brake squeal noise. The researchers showed that when the system has higher vibration amplitude, it will have a higher tendency to be operated in an unstable region. If the transverse mode of the rotor has the same resonant frequency of radial mode the squeal noise will amplify. René et al [15] used camera beside the ESPI to capture the disc mode shape as René et al [16] used the same apparatus on drum brake.

1.4 Laser doppler vibrometer (LDV)

McDaniel et al [17] applied a LDV to scan the disc at the normal velocity of a shaker-excited stationary brake system and they found that the resonant behaviour was associated with squeal mechanism. The LDV is an instrument that was used with a non-contact to measure the vibration of the disc surface. The laser beam from the LDV was directed to the surface of disc, and due to the motion of the disc surface the laser doppler frequency changed, however the vibration amplitude and frequency were extracted from this alter. They understand from the findings that the rotor was responsible for the most noise and primary radiator to the sound since the rotor area is much larger than other component. These methods have two advantages, it does not require difficult task in the laboratory and it allows measuring the frequency on a stationary rotor. Svend et al. [18] applied LDV on the brake and they found additional benefit of this method that the frequency range of measure can be quite high. This includes high spatial resolution in the measurement and faster than ESPI. Chen et al. [19] used LDV to measure the rotor diametral. Their result indicated that the coupling of the in-plane rotor modes and diametral mode of the rotor was responsible for the generation of high frequency squeal. Claus Thomas [20] carried out vibration analysis of squealing brake system under running condition. These experimental approaches used accelerometer, laser-interferometry and acoustic camera. The diametral mode shape with zero pad pressure was investigated for entire brake systems at different frequencies.

1.5 Reduce brake squeal method

Join Flint [21] analysed the effect of a rubber coated steel plate- a shim- on the backing plate of a brake pad during the brake squeal. The results illustrate that the thick constrained shims layer gives high damping at low frequencies, while thin layer give high damping at high frequencies. Kung, S. - W. et al [22] made many modifications on disc brake system to reduce the brake squeal by focusing on solution to reduce the stiffness of the rotor. This is accomplished by a reduction in the young’s modulus of the rotor material. The simple modification was done by changing the amount of the graphite in the cast iron to shift the natural frequency of the rotor, so the young’s modulus of the rotor is 96GPa as opposed to 120 GPA. The result showed that changing the rotor material may decouple the modal interaction and eliminate dynamic instability. Fieldhouse and Beveridge [23] studied the effect of non-chamfered and chamfered pads on the frequency. The elastic fix shims was proven as useful at higher frequencies over 6000 Hz but it was not so effective at the lower frequencies of around 2000Hz. They investigated the effects of the calliper angel, pressures and temperatures on the brake squeal for two types of friction material. From the result it is seen that the brake squeal become quite at angle 166 degree and above. Gouya and Nishiwaki [24] found that in every doublet mode there is a critical contact span angle. Homffman CT [25] studied the reduce of brake squeal by using a viscoelastic material (damping material) on the back of the back plate of the pads and found it can effect in reducing squeal when the pad in bending vibration.

2 Analysis of squeal by using finite element method

The finite element is the tool for modelling disc brake system and providing a new insight into the problem of brake squeal. FEM allows accurate representation of complex geometries and boundary condition. The finite element method has been employed by the researchers in the brake squeal study. One of the uses of finite element method is to investigate the modes and the natural frequency of the brake rotor with complex eigenvalue then they used modal participation method in order to analyse the contribution of each part of the brake system in generating the squeal. The third section is analysing the squeal by using beam-disc system. Then study the two direction response due to the self-excited vibration.

2.1 Predicting squeal with complex eigenvalue

Liles [26] used solid elements to build his finite element model for each brake component and carried out modal testing on these components. Friction was added into the model as a geometric coupling. A complex eigenvalue formulation was derived for the system. The complex eigenvalue was constructed by solving the equation of the motion and consist of two parts. The first is real indicating to the stability while the second was imaginary indicating to the damped frequency. He constructed the friction stiffness matrix using relative displacements between mating surfaces. Kido et al. [27] studied the relation between the squeal propensity and the ratio of the eigenvalues. They showed in the finite element results that higher ratio of eigenvalue increased the tendency to squeal. Blashchke et al [28] used the complex eigenvalue analysis to detect the unstable mode of the system. They modelled the rotor and pad interface to create the matrix of the contact and used it inside the equation of motion. The contact matrix made both the mass matrix and the stiffness matrix to be a non-symmetric, so that the eigensolution became complex. A cure (one of the brake parts) was proposed joining the system in order to add some lumped masses to the drum. The benefit of that method was to add the effect of the braking pressure, rotation velocity and temperature to this analysis. Nack [29] used complex eigenvalue in his study on brake squeal. The aim of complex eigenvalue was to shift those complex eigenvalues that appear in the right-hand side of the plane (root locus diagram) to the left-hand side of the same plane. He also presented a method to add the friction stiffness to his vehicle modal inside FE software and use eigenvalue analysis to determine the necessary condition for the system to become unstable and grow into a state of limit cycles. Kung et al. [30] used complex eigenvalue to investigate the effect of contact friction on the squeal. The results showed that major noise frequencies are higher than 5 kHz. The cause was mostly due to the involvement of tangential modes. The shear mode would contribute to squeal problem at frequency 12.5 kHz (that mode is outside the audible mode). If the diametral mode be coincided with the longitudinal mode, the noise would line up. Zhang et al. [31] found by using complex eigenvalue that not all the unstable mode squeal but an unstable mode was just one necessary condition for squeal. Their result also showed that if damping included in the model, some unstable modes became stable while some modes appear at very low friction coefficient. Chung et al. [32] presented a new process to analyses brake squeal by applying modal domain analysis using FEM to provide a new method instead of complex eigenvalue called virtual design process. Kung et al. [33] used complex eigenvalue to analyse disc brake squeal. The brake pressure was iterated form 0.69MPa to 2.76MPa (brake pressure range) and the result was plotted in complex plane. Material damping and friction damping between the lining material and the rotor was added to the system. Complex eigenvalue was found in the same previous iteration pressure and it was found that the number of unstable mode was decreased. The researchers showed that the major noise frequency appeared at a frequency higher than 5KHz due to involve tangential mode and shear mode. Lou [34] used complex eigenvalue to study disc brake squeal. He was applied coulomb's friction at the contact interface and produced unsymmetrical contact matrix which yield complex eigenvalue. His analysis showed that any complex eigenvalue with a positive real part indicated an unstable mode, which may results in squeal. In his work Lou searched for the mode responsible about the squeal and the percentage of the propensity (eigenvalue with positive real part). He found that the bigger propensity appeared with higher unstable mode. Cao et al. [35] modeled the disc brake component using FE. The disc was modelled as a thin plate. The pads mate with different spatial area as the disc rotates and vibrates. The disc brake and the stationary component are studied as a moving load problem. Due to the asymmetry of the real brake there are no double frequencies in the finite element model of the real disc. A linear complex value was derived for the friction-induced vibration of the disc brake in order to present the unstable frequency. Joe et al. [36] used complex eigenvalue to investigate the dynamic instability of the brake system. If the real part of an eigenvalue is positive, the corresponding imaginary part was thought to be possible squeal frequency. The analysis indicates that modal coupling is responsible for disc brake squeal due to the friction force. The friction is introduced by altering the system stiffness matrix to be unsymmetrical. Higher friction coefficients always tend to make two modes merging and form unstable complex mode. Increasing the stiffness of the lining material causes the system to become more unstable. Increasing the length of the pad causes the system to become more unstable, as increase the thickness of the lining material. Increasing the disc thickness, the unstable mode above 10000Hz become unstable. The squeal at 5000Hz and below is due to the modal coupling while above 10000Hz is due to the modal splitting. Fritz et al. [37] used FEM to compute complex eigenvalue technique. They found that by adding damping to the system the real parts of eigenvalue get shift to the negative value. The results showed that when the system is stable and the friction coefficients increase the real part remains zero but the frequency of the stable mode tend to get closer. Dai et al. [38] suppressed brake noise by using FE and found positive complex eigenvalue which produce unstable modes where the positive eigenvalue indicate the propensity towards generation of squeal noise. The researchers developed a sufficiently reliable brake dynamic modelling tool that could be applied to evaluate the effects of pad structural design parameters on brake squeal generation and avoid trial-and-error approaches to address brake noise concerns, which are time consuming and costly. The authors proposed theory based on a double-pin on disc formulation as shown in Figure 4.

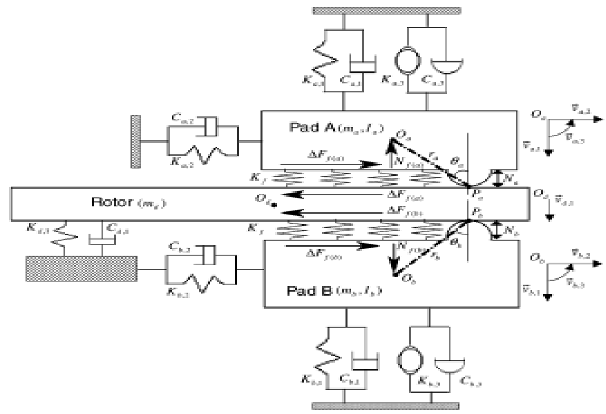

Figure 4 Schematic of the proposed lumped parameter model of the rotor–pad assembly, Dai, Y. et al[38]

In this proposed contact model, the pads are pivoted against a pair of rigid contact pins designated as Pa and Pb. The effect of the pin radius from pivoted centre on the degree of instability was studied. Increasing the radius will increase the degree of instability. The effect of contact angle between the pin and the disc face Ɵa and Ɵb affected by various factor, including length of the lining, chamfer geometry and nature of the load distributed. The effect of friction coefficient of the lining rotor was investigated for inner and outer pad. As friction increases, some of the modal frequencies start to merge toward each other. Figure 5 shows the instability versus friction coefficient of inner and outer pad. The propensities of squeal increasing with increase friction coefficient because the higher coefficients of friction cause the variable friction force to excite greater number of unstable modes.

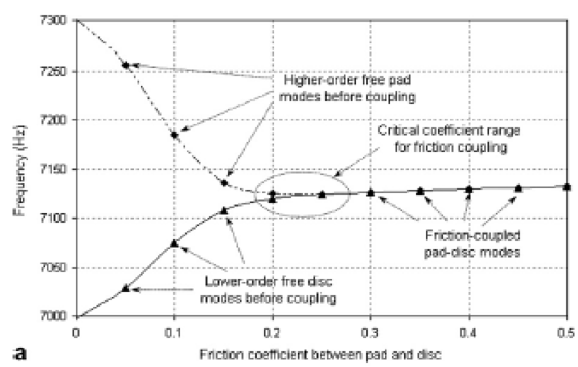

Figure 5 Friction coefficient as a function for the frequency, Dai, Y. et al [38]

2.2 PREDICTING SQUEAL WITH MODAL PARTICIPATION METHOD

Kung et al [22] used modal participation method for calculating the brake component participation percentage in generating the squeal, in order to define the system squeal mode shape. Component participation was made for each part of the brake system in order to conduct each part contributes in brake squeal. From the results, it was seen that the rotor was one of the brake parts that has major responsibility for brake squeal. NASTRAN Software was used to calculate the modal participation factor based on MAC (Modal Assurance Criteria)

Algorithm. The purpose of their work was to demonstrate a systematic approach of modal participation analysis as a post proceeding tool for the complex eigenvalue method. The authors found that the first circumferential (tangential) mode of the rotor was very close to the squeal frequency and he suspected that the circumferential mode causes the squeal. Kung et al. [22] computed the modal participation factors of brake component to find a ways of suppressing unwanted modes and thus reducing the possibility of a particular squeal frequency.

2.3 Study the effect of asymmetric by using FEM

Ghesquiere and Castel [39] represented the disc and the two pads by three degree of freedom system model including the contact pressure in order to get the natural frequency of the system. The friction was applied between the pads and the disc. The effect of the friction led the mode shape to have a different aspect, and lose its symmetry. The equation of the motion had mass and stiffness matrix, in case of adding friction or changing the brake system symmetry shape, the stiffness matrix will lose it symmetry. The effect of the friction led the mode shape to have a different aspect, and they lose their symmetry. Their result showed that the frequency grows faster after third diametral mode because the contact stiffness counter of the disc displacement and thus the system stiffness energy grow. When the friction coefficient is increased from 0 to 0.7, the coupling mode shapes that appear are not totally symmetric or totally anti-symmetric. Lee and Yoo [40] analysed the effect of changing shoe from uniform cross-section to non-uniform cross-section on the brake squeal. In this study, the drum and the shoes are assumed as a uniform ring and non-uniform arches respectively. The non-uniform cross-section built by changing the shapes of the shoes partially. The influences of brake design parameters upon the squeal investigated by theoretical analysis. The effect of this change verified through noise dynamometer tests. The effect of the asymmetry of the drum, which built by additional mass, was presented. The increase of the cross-section area and the decrease of the bending stiffness of the shoe are advantageous to the reduction of the squeal. The lumped masses were attached to the drum for the asymmetry, and the influences of the masses were analysed with the coefficient of friction of 1.0 which is much higher than that of the critical value 0.37. Lou and Wu [34] studied the disk brake squeal due to unstable friction-induced vibration using the ABLE algorithm FE software. Coulomb Friction and normal displacement were applied in the area of contact for that the stiffness matrix becomes unsymmetrical. Unsymmetrical matrix led to generate complex eigenvalue. The researchers found that when brake squeal occurs, the rubbing surfaces do not stick to each other and the relative sliding speed is always unidirectional. The eigenvalues and the eigenvectors are found in each individual component by the FEM. The friction springs element was applied between the nodes at the pad–rotor interfaces to simulate the frictional mechanism. Asymmetric distribution of the pressure between the rotor and piston-pad interface made Abu Bakar et al. [41] to study about the contact pressure distribution. The asymmetry of this pressure is due to the friction force between the surfaces. The authors made modification on the pad and/or at the piston and the back plate to make the life of the pad longer, reduce the wear and eliminate the squeal. The pressure distribution at rotor and piston pad interface was examined with different rotor speed in order to achieve uniform pressure distribution. The interface pressure distribution of the pad under the piston was conducted with three different rotation velocities. The FE software showed when the rotor at the rest the pressure distributed is symmetric about the centre line of the pad while when the disc rotates the pressure distribution no longer symmetric and the high pressure occurs at the leading side of the pad. Increasing the thickness of the back-plate could produce desirable pressure distribution. It is found that by making right connection between the piston and pack plate the contact area can increase and the contact pressure distributed uniformly thus the squeal reduced.

2.4 Beam-on-disc model

Massi and Baillet [42] developed and compared two different models: a linear model, used to predict squeal frequencies through a complex eigenvalues analysis and a non-linear model for a contact problems with friction between deformable bodies. The nonlinear model was applied with Plast3 software while the linear model was applied with ANSYS software and the results of both models were compared. The researcher demonstrated the agreement of the two models and the possibility of using the linear model approach instead of the non-linear model. The contact elements were implemented in the FE software using matrix27 as a contact element. The results of ANSYS software was carried out to investigate the mode shape which was responsible for the squeal, figure 6.

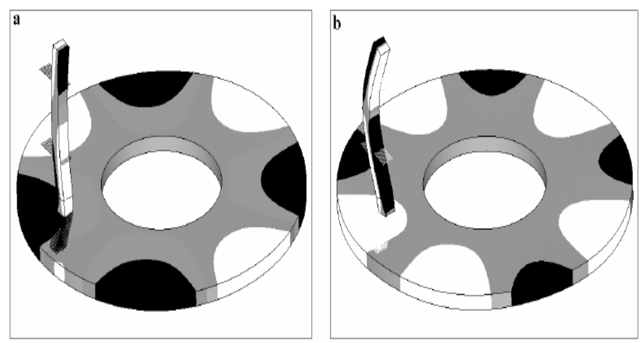

Figure 6 (a) Unstable mode at 3500 Hz; (b) Unstable mode at 5000 Hz, Massi and Baillet [42]

From figure 6 the effect of contact on the node and anti-node appeared clearly. In the frequency of 3500 Hz, the contact prevented the anti-node from appearing while in the frequency of 5000 Hz the contacts prevent the node to appear.

2.5 Changing the mechanical properties of disc brake system

ILM et al. [43] studied the behaviour of changing the friction material by using FEM. The FEM was used to predict the deformation of the friction material and also the pressure distribution across the contact area with the rotor. The results showed that the nodal reaction force distribution over the node of the contact between the disc and the pad under the piston pressure. The maximum value of the nodal force found to be at the centreline of the pad. The pad mode shape showed either bending or torsion mode during the squeal. Ouyanget al. [44] presented a model to represent brake system. The disc modelled using thin plate. They found that the instability are highlight depend on the disc speed. Furthermore, they found that the damping can reduce the squeal noise. Mezian [45] used explicit dynamic finite element software to study the instability generated by the friction. He used coulomb friction in the contact interface. The result showed separation (periodic shock phenomenon) in the contact region indicates to the instability control by stick-slip separation wave. The result showed that the pad was the reason to generate the instability. In order to find the mode responsible about the squeal the two surfaces (pad and disc contact surfaces) was assumed to be in stuck when the mode shape was conducted. Poisson's ratio was involved in this study and found its effect on the instability. The result showed that Poisson's ratio can transfer the system from unstable to stable region. The impact force generated from stick-slip separation wave generates the squeal. This wave gives raise to the frequency of the system. Liu et al. [46] studied the effect of rotating velocity of the disc, the friction coefficient of the contact interactions between the pad and the disc, the stiffness of the disc, and the stiffness of the back plates of the pad, on the disc squeal. Their findings showed that the pad bending was responsible about the squeal. The squeal can be reduced by decreasing the friction coefficient, increasing the stiffness of the disc, using damping material on the back plates of the pads, and modifying the shape of the brake pads. The disc stiffness can be changed by varying young's modulus and the disc thickness. They also investigated that reducing the friction will eliminate brake squeal but this will reducing brake efficiency. Increasing the hydraulic pressure increase the propensity to produce the squeal because this would generate higher friction coefficient between the pad and the disc. Fritz et al. [47] studied the effect of damping on brake squeal by using FE model. The researchers found that if damping was distributed equally on the modes involved in the mode coupling phenomenon, a shift of the real part-friction coefficient curves towards the negative real parts achieved but if the damping was spread non-equally over the coupling modes, a shifting and a smoothing effect can be seen on coalescence curves. If the damping ratio between the two modes is sufficient, an increase in damping tends to make the brake unstable with a lower value of the friction coefficient. Mario et al. [48] examined the effect of the operation parameter (friction coefficient, brake pressure and brake temperature) to find the unstable frequencies under different conditions. The result showed that the brake temperature can change the coupling frequency. They found that increasing the temperature induces a decrease in the young's modulus for the friction material and also loss factor showed slight increase while the friction coefficient decrease. Junior used complex eigenvalue to study the instability for the previous parameters. They found that adding insulator to the system increases the degree of instability because the stiffness of the pad increases.

3 STICK-SLIP WITH DRY FRICTION

Stick-slip vibration is self-excited oscillation induced by dry friction. The resistance against the beginning of the motion from the state of the rest called stick mode while the resistance against of an existence motion called slip mode. Stick-slip motion was generated by the variation of friction coefficient and can be introduced by the difference between the coefficient of the kinetic and static friction. The change of friction coefficient was due to the vibration of contact area, or the alternative of kinetic friction coefficient as sliding speed changes. Friction was postulated by Leonardo da Vinci (1452-1519). He recognized that the friction force was proportional to the applied normal load. The friction was assumed to be independent from area of contact, and that the frictional force was directly proportional to the applied normal load. Coulomb recognized that the force applied to the static body would not cause the body to slide unless it exceeded the static friction. Bowden and Tabor [49] used the term of stick slip for the first time when describing the relative motion of two surfaces in contact. They mentioned that the system governed by static friction while the surface in stick and by kinetic when the surface in slip. Chen [14] found that the friction force may also induce fundamental frequencies and their harmonics, in which the fundamental frequencies are synchronized to the resonant frequencies of dynamic system. However, there are a lot of the parameters affecting the stick-slip friction which should be considered such as, material properties, geometry of contact surfaces, surface asperities, surface chemistry, sliding speed, temperature, and normal load. In this thesis only two parameters are consider to be changed which are the sliding speed and normal load. Ichiba and Nagasawa [50] identified six natural squeal modes and the vibration of the rotor, back plate and lining for each mode by using accelerometer. They confirmed that the exciting energy was generated by fluctuating in friction force. The fluctuating in friction force is due to fluctuation in surface contact pressure. They also found that the squeal frequently varies during a single rotation of the rotor. When squeal occurs, the systems resonance is large, however the squeal does not accord when, there is no resonance. The higher the coefficients of friction, the higher it causes the squeal. Brake squeal can be defined as a self-excited vibration caused by fluctuations in the friction forces at the pad-rotor interfaces, Lou [34]. This section include three subjects: the first is stick slip in one direction in order to analyse the effect of this relation of the pad disc system. The second is about stick slip motion with two degree of freedom. Increase the degree of freedom is going to increase the accuracy of the result. The third part deals with conducting the stick slip motion in two directions which can be represented in some cases the normal movement of the pad on the disc.

3.1 STICK-SLIP ONE DEGREE OF FREEDOM

Brockley and Ko [51] studied friction-induced vibration theoretically by using one degree of freedom. The system was simple one degree of freedom structure with a nonlinear excitation term driven by the frictional force between the block and the moving belt. They classified friction-induced vibration into; quasi-harmonic and stick-slip vibration. The work emphasis on developing a system which gives uniform friction-induced vibration could be measured accurately. The researchers found that at certain damping value the vibration decreases. McMillan, A.J. [52] used block attached to a rigid wall by a simple spring and dashpot to understand the phenomenon of squeal. The aim of this work was to develop a dynamic system to analyse the brake squeal. The researcher presented the interface deformation schematic to show the deformation at stick-slip state. Leine et al. [53] studied stick-slip model for one degree of freedom system. The model consists of a set of ordinary non-stiff differential equations so the system can be solved with any standard ODE-solver. This work was a switch model was presented to simulate stick-slip vibrations. They present the shooting method to calculate the limit cycle. Thomson [54] studied the relation between the friction induced self-excited oscillation and high frequency external excitation. The researcher used traditional mass-on-moving-belt model. The aim of the study was to reduce or totally suppress friction-induced vibrations by using high frequency excitation. The author conducted the typical response by using his derivative equation when there is no external harmonic excitation. He found that since the velocity of the mass must change continuously with the time, the maximum and minimum displacement of the mass should appear during the slip phase. When adding high-frequency harmonic to the equation the response will be same as figure 7. The response in the figure diminished with the time while the mass showed tiny oscillating about the non-zero equilibrium position. The result showed that the presences of fast vibration smoothness the discontinuity of dry friction and cancels the negative slop and prevent self-excited oscillations from building up.

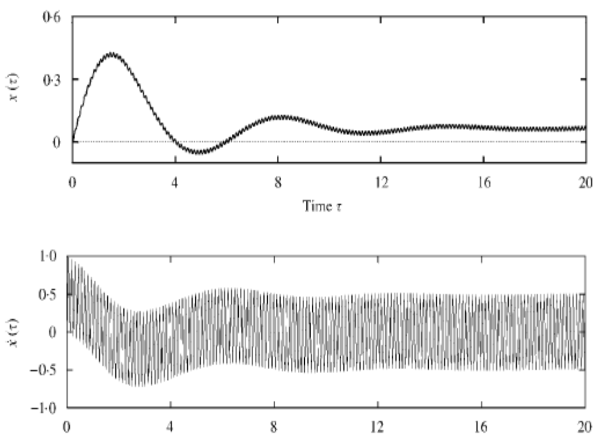

Figure 7 Responses in the presence of high-frequency external excitation, ω=50, Thomson [54]

Pilipchuk et al. [55] presented analytical model of disc-pad system and assumed equal normal forces on both sides of the disc and thus equal friction forces, as in Figure 8. The model was built based on Sprag-slip theory developed by Spurr. The pads were pressed against the rotating disc under the action of the piston forces. The friction force was not uniformly distributed over the entire contact area between the pad and the disc surface. However, the friction force exerts a moment on each pad and causes the normal force to redistribute therefore there is no contact with the rear edge of the pad, as illustrated in Figure 9.

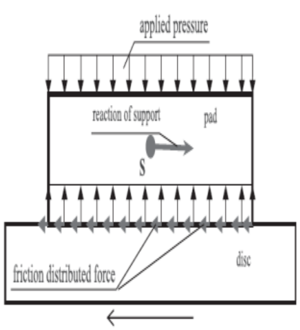

Figure 8 Non-equilibrium sum of the moment with respect to the point S, Pilipchuk et al. [55]

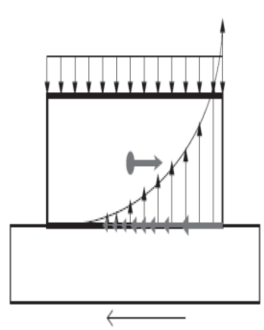

Figure 9 Possible equilibrium distributions of force and moment, Pilipchuk et al [55]

Expression represents the friction relative velocities value was investigated. The curve looks to be close to the Stribeck curve as shown in Figure 10.

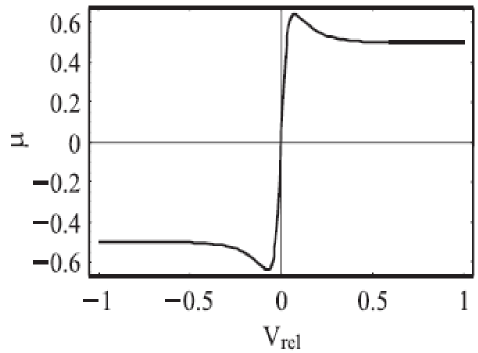

Figure 10 Stribeck-curve, the relative velocity versus friction coefficient for stick slips case, Pilipchuk et al. [55]

Thomsen and Fildin [56] used the classic mass-on-moving-belt for studying friction-induced vibration. The friction force which applied between the mass and moving belt; first decrease then increase with the relative velocity. They found that during the stationary oscillation the mass will never exceed the belt velocity. The oscillation cycle and oscillation amplitude was studied as a function of excitation speed. As the excitation speed increases the oscillation cycle decreases while stick-slip amplitude increase. Stein et al. [57] simulated and analysed general single degree of freedom oscillator system with viscous damper and dry friction in order to model dry friction. The macro-slip approach was represented in them mathematical model. The researchers used two approaches to describe the dry friction macro-slip model; signum function approach and stick slip approach. It was found that stick-slip approach described the reality better than signum approach because signum approach completely distorts the acceleration response signal. Chatterjee [58] introduced a new method of controlling friction-driven self-excited vibration. A single degree of freedom oscillator on moving belt was used to represent the model. The control law was derived by using Lyapunov's second method. An approximated method for estimating the critical value of the control parameter was proposed. The researcher discussed the control law of modulating the normal load to achieve the stability of static equilibrium of the system. It was showed from the phase-plane plot that as the duration amplitude of stick-slip limits cycle decreased as the value of normal load modulation factor increased. Hetzler et al. [59] used a simple friction oscillator, mass-on-moving-belt to find the stability and local bifurcation behaviour due to exponential decay of friction-relative velocity. The power spectral density for the in-plane disc motion and the acoustic signal did not match the dominant spectral frequency of the in-plane of the pad. Groan only appeared at low speed where the decay of friction characteristic was investigated. Kang et al. [60] studied stick-slip oscillations for one degree of freedom system with energy translate to the system as a non-linear friction curve. They divided the response according to the frequency into two forms; mode merged and mode-separated oscillations. The result showed that the period of oscillation and the stick interval are proportional to the normal load but both decrease with increasing the belt velocity. Popp and Stelter [61] studied two continuos model of stick-slip. The aim of the research was to find a simple mechanical model had dry friction and could transit from regular to chaotic model. Even though the researchers used two friction models, the phase plane appeared to be equal for both friction models. The results showed that the velocity of the belt was greater than the velocity of the block so that the direction of sliding was squeal. The friction force always appears to have the same direction of the belt motion. Even though the friction force was greater than moving in opposite direction of the belt, the velocity of the slip was smaller.

3.2 STICK-SIP WITH MULTI-DEGREE OF FREEDOM

Ouyang et al. [62] studied the effect of the resonances parameter on the system stability by using an elastic system. The elastic system consist of two spring-dashpot in the transverse and in-plane (circumferential) directions, and a common point mass as illustrated in figure 11.

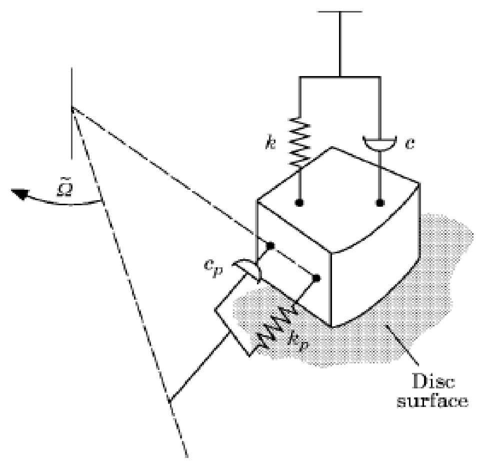

Figure 11 Mass with in-plane and transverse motion, Ouyang et al. [62]

The elastic system (mass) rotated around an annular disc with friction had a negative slope versus velocity. In the article, the speed was considered high enough to prevent sticking but it was in the range of the negative friction-velocity relationship. The mass continues to slide in one direction around the disc without sticking because the friction force never reverses its direction. They investigated that the transverse damper tends to destabilize all the additional resonance while the disc in-plane damper stabilize all the circumstances resonance. They plotted a region where the system can operate as a stable system. Ouyang et al. [63] studied the vibration of an elastic slider (mass) at the in-plane and out-of-plane of flexible disc and the vibrations of the disc itself. Numerical method was used to solve the two coupled equations of the motion, which was derived depends on his model. The slider system consists of a transverse mass-spring-damper, attached through an in-plane to a drive point which rotates at a constant speed. Dry friction acts between the slider system and the disc. The static friction coefficient was higher than the dynamic friction. The authors found that when transverse (out-of-plane) motion of the slider system becomes so violent the total normal force became negative or several times larger than the initial normal load. A vibration solution was studied at different parameters values by varying one parameter while keeping other parameters constant. The result showed that increasing the rotating speed made the vibrations larger or more unstable. Damping can effectively reduce the magnitude of the vibrations. The in-plane stiffness of the slider can reduce the magnitude of transverse vibration at some values while increasing the transverse vibration at other values. The in-plane stiffness had a critical value at which the system became unstable. Shin et al. [64] analysed friction induced vibration by using nonlinear model of two-degree of freedom. They investigated the effect of limit cycle size to demonstrate the nonlinear dynamics produce squeal state. The researchers believed that the size of the limit cycle was more important than an existence of the limit cycle. The researchers present stick-slip motions for various values of negative gradient of dynamic friction coefficient considering. Increase the value of the gradient increase the limit cycle size. It was found that the size of the limit cycle decrease as the damping of both pad and disc increase. When the damping of the pad was increased only, the size of the limit cycle corresponding to the pad decrease, while for the disc increase. Shin et al presented the friction relative velocity formula and used it for his study. The values for stick-slip model were chosen to be in line with Leine et al [53] values to have a basis for a comparison the results. The researchers found that the coupling stiffness between rotor and pad varies due to the growth of friction layers at the contact surfaces. Addition of damping at the contact stabilized the system faster than adding damping to the pad or rotor. In a case of a non-unity mass ratio (pad mass/disc mass) and negligible damping, the size of the instability increased with a small increase in stiffness coupling. When the mass ratio was unity and friction coefficients were constants, the system motion changed significantly with a slight change in coupling stiffness. Systems with high damping values and nonlinear friction characteristics stabilized quickly with a slight increase in coupling stiffness. Baleri et al. [65] studied stick-slip vibration by using two large circle discs. The objective was to gain a better understanding of the stick-slip phenomenon and develop a mathematical presentation that can be used for modelling. The system developed with finite element software to obtain the distribution of the contact pressure at the interface between the discs. It was seen that the maximum contact pressure was at the load application point. Different combination of driving speed, applied load, loading, loading radius and spring stiffness was applied. The result showed that the amplitude of stick-slip motion decrease when either the applied load increase or the spring stiffness degrees. Increase the total load showed increase the stick-slip amplitude, and hence the period of stick-slip. Manish Paliwal et al. [66] extended the model which was presented by Shin et al [64] by incorporating the stiffness layer which represents the formation of friction layers on the contact surface as in figure 12.

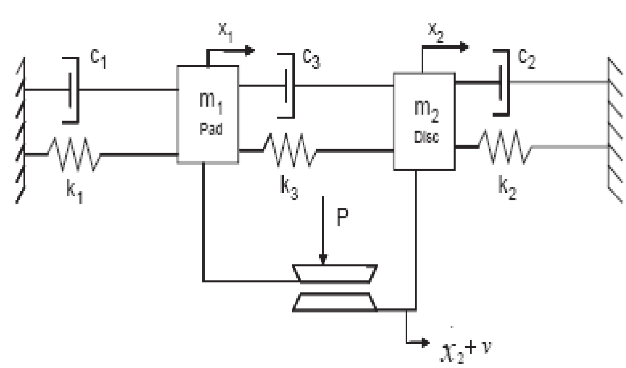

Figure 12 Two-degree of freedom system, Paliwal et al [66]

The pad and rotor disc are modeled as single-degree-of-freedom system which are connected through a sliding friction interface and interfacial coupling stiffness. They found that the coupling of the modes occurs when those natural frequencies were close to each other. Khizgiyayev, S.V. [67] studied self-excited oscillation of a two-mass with dry friction as illustrated in the figure 13.

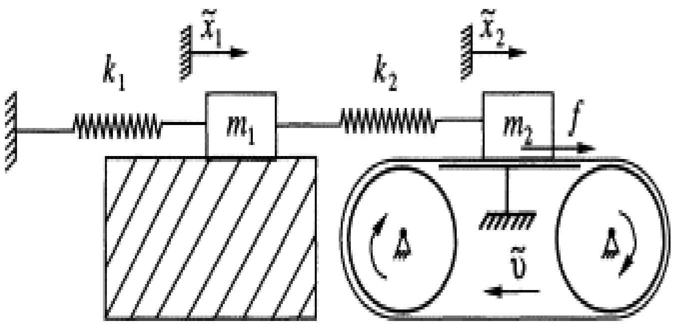

Figure 13 Stick-slip two degree of freedom, Khizgiyayev, S.V. [67]

The simulation was run with different friction coefficient in order to investigate the analytical solution of limit cycle. The result showed that in case of equal masses value there are reveres and forward limit cycle which had period increase as the ratio of friction coefficient (stick to slip) increases. The result also showed that the period of the limit cycle decrease with increase the velocity of the belt. Wagner et al. [68] reviewed the minimal models which describing the basic behaviour of disc brake squeal and found that there is still a lack in these model to describe brake squeal. The researcher discussed the models which have two degree of freedom for Shin et al, Hoffmann et al, Popp et al and Brommundt and showed them weakness in the describing brake squeal. The researcher presented them model which is consist of a rigid disc. The disc is hinged in a spherical joint in it centre of mass and supporting by rotational springs. The disc performs wobbling motion while rotating with constant speed. The general analysis of the nonlinear model is performed using Autolev Software. The result showed root locus of eigenvalue for varying speed of the disc. In the root locus plot there is a positive real part above the critical speed indicate the instability with self-excited vibration. The result also presents the critical speed as a function for the contact stiffness.

3.3 Friction induced vibration

Ibrahim [69] reviewed the mechanics of contact and friction in metal-metal and elastometer-metal contact. Both dry friction and lubrication contact were considered. The modelling of friction occurs in the mechanical system depends on material properties, geometry of sliding surfaces, surface roughness, surface chemistry, sliding speed, temperature (The heat generated from the friction is an important factor in selecting brake materials because increases the temperature, decreases the brake efficiency), normal load, mode of deformation, the presence of the wear articles, the environmental condition, direction of motion and overall shapes of the contacting surface. The fact that the kinetic coefficient of friction is less than the static one is due to the asperities could jump over the gap between asperities of the other surface. As the sliding velocity increases the force which it applied on the asperities will increases also. This increment increases the amplitude of the normal vibration which was governed by the contact stiffness and mass of the slider. As a result of increasing the amplitude, the penetration between the surfaces decreases and so on the kinematic friction force. Usually stick-slip occurs when the static friction has higher value than the kinetic friction and also when the friction relative velocity curve decreases. The stick-slip disappears when the velocity of sliding increases. A metal has the ability to flow plastically and to weld together under the load when they are in contact for adequate time. Friction is proportional to the shear strength of the asperities junction. As the normal load increase the junction area grows. However, the friction force reaches maximum value at the stick and drops rapidly during slip thus there were changes in the area of contact and surface temperature. When one surface is harder than the other, the harder surface asperities ploughing in the softer material, thus the total friction was in general composed of two components, one due to the shearing and the other due to ploughing. An applied tangential force whose magnitude is less than the friction cannot give sliding motion but will give friction at the contact interface which was going to cause the contact body to deform in shear. If completely there is no sliding between the bodies, there must be appoint at the interface where the surfaces deform without relative motion. In fact even the tangential force is less than the limiting friction force it causes a small relative motion known as micro-slip while the other surface remains in stick. The self-excited vibration does not occur if the sliding velocity lowers than the critical value. The friction may have more than one value for same relative speed depending on weather the velocity increased or decreased. The friction force during the acceleration is larger than during the deceleration. Ibrahim (1994) in his second review paper presented the following name which had some work about friction-induced vibration. Den Hartog [70] obtained an exact solution for stick-slip one degree of freedom but without a relative motion. Scinclair and Manville [71] showed that the frictional vibration can be generated from increase of the friction coefficient as slip speed decrease. The dependence of friction on sliding speed was examined by Tondl [72]. He considered mass-spring-damper on moving belt with Coulomb friction. The system was represented by nonlinear equation and two amplitudes were obtained for same mass motion. The smaller amplitudes corresponding to an unstable limit cycle, while the larger belong to a stable limit cycle. Beards [73] showed that the energy dissipation in joint is related to the friction force and relative slip between the contact surfaces. Friction in support boundary acts as a source of energy dissipation and result in variation in dynamic properties such as natural frequency. The effect of the damping on the brake element was studied by Earles and Chambers [74]. They considered pin-on-disc model and found that the disc damping reduce squeal generation, while pad damping may increase or decrease it. Rhee, et al (1989) classified brake noise into two main category; low frequencies (100-1000 Hz) and brake squeal (1000-18000 Hz).

4 Summary

It is now generally accepted that brake squeal is caused by friction induced vibration (Blashchke et al). There are two major hypotheses in explaining the phenomenon: the first is that the squeal is due to stick-slip phenomenon at the friction interface, while the second attributes that the squeal is due to the geometric coupling of the brake assembly. Complex eigenvalue analysis is in widespread use and no doubt time domain analysis will follow with increase the computational power and reductions of the cost. Experimental work on a disc brake system also indicated that the rotor is the resonant member, vibrating in transverse modes with diametral nodes, where the mode travels around the disc in a uniform rate. The rotor was responsible for the most noise and primary radiator to the sound since the rotor area is larger than other brake parts. The speed of rotation depends on the frequency and mode order. The amount of squeal increased when the natural frequencies of the pads, calliper, and brake rotor were close to each other. That closeness of these frequencies is not a necessary condition for squeal generation, because many other parameters play an important role in generating brake squeal. Many of the analysis were done by using complex eigenvalue approved that the result was gotten from FEM is close to the experiments. The circumferential mode is the mode which causing the squeal (Lee, [40]). Not all the unstable mode will squeal however, the unusable mode is just one necessary condition for squeal. In the literature review, it is noticeable that no one used the friction coefficient versus relative velocity which was gotten from stick-slip and use it in ABAQUS Software to study the response at low velocity. Also from the literature review there is no one model plate on rotating disc instead of beam on rotating disc to find the effect of system parameters on the instability at the high-frequency noise. The focus of this thesis is in the area of numerical friction modelling analysis and analysis the instability by finite element method to complement industrial testing and development of the brake system. The squealing frequencies are slightly lower than the natural frequencies of the stationary rotor. The vibration mode and frequency of squealing disc brake's rotor are influenced by the natural frequency and mode shape of stationary rotor.

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