International mathematical olympiad*

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Today mathematical competitions are very popular with primary and secondary school students and there are many countries all around the world where they are regularly organised. There are several rounds and a lot of students are included, especially at the beginning rounds. The best students from the previous round have the right to continue on the higher level of competition. The final level for the secondary school student competitors is the International Mathematical Olympiad (IMO). The team for the IMO from Croatia is determined at the National Competition which is held in May.

The first mathematical competitions were organised in Hungary in 1894, and in Romania in 1898. Mathematical competitions in Croatia for the secondary school students started in 1959 and next year the first Federal Competition was held, which was then organised every year until 1991.

Romania was the initiator of the first international competition. The idea of organizing it came from the Romanian mathematician *Tiberiu Roman* in 1956, and mathematics is still his great love, although he is 83 years old. After detailed preparations the first International Mathematical Olympiad was held in Romania in 1959, as well as the second one in 1960. At the beginning only the following countries from the Eastern Europe participated: Bulgaria, East Germany, Czechoslovakia, Hungary, Poland, Romania and the USSR. In 1963 Yugoslavia participated for the first time, and after that new and new countries from Europe arrived. The first Olympiad in Yugoslavia took place in Cetinje, Montenegro, in 1967, and the second one in Belgrade in 1977. 21 countries took part at that 19 th IMO. Cuba was the first non-European country which participated at the 13 th IMO, in 1971, and it was the host country in 1987. Australia participated for the first time at the 22 nd IMO in 1981, and was the host country in 1988, when the 200 th anniversary of Europeans inhabiting that continent was. In 1980 the IMO was not organized, and only some local olympiads were held. In 1993 Croatia as well as Bosnia and Herzegovina, Macedonia and Slovenia became regular members of IMO.

In 1999 the 40 th IMO was organized in Romania, and it was the fifth one held in this country (the previous ones had been held in 1959, 1960, 1969, 1978). During the last few years there where about 80 countries and 450 contestants at the IMO.

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A few words from the Regulations of the International Mathematical Olympiad

The aims of the IMO are:

– to discover, encourage, and challenge mathematically gifted young people in all countries;

– to foster friendly relations between mathematicians of all countries;

– to create opportunities for the exchange of the information on school syllabuses and practice throughout the world.

Participation and Responsibility

The contestants should not have been formally enrolled at a university or any equivalent post-secondary institution and on the first day of contest they should not be older than 20 years.

The Organizers of the IMO cover all official expenses for Leaders, Deputy Leaders and Student Contestants, including meals and accommodation, for the period of the official program. The Organizers cover the costs of all contest activities.

Deputy Leaders are responsible for the conduct of their students during the whole period of the IMO.

Proposals for Problems

Each participating country is invited to submit up to six proposed problems, with solutions, to the Problem Selection Committee, until the deadline given by the Organizers.

Proposed problems should cover various fields of the pre-university mathematics and should be of varying degrees of difficulty.

The Problem Selection Committee will prepare a short list of at least 25 problems and no more than 30 problems with their solutions, for consideration by the Jury.

Jury Regulations

The International Jury shall consist of all Leaders of participating countries and a Chairman appointed by the IMO Organizing Committee.

The Jury should confirm that the total number of prizes will not exceed one half of the number of the Student Contestants; that the established number of prizes will be distributed as the first, second and third prizes in a ratio approximating 1:2:3 as closely as possible.

A certificate of honorable mention shall be awarded to each Student Contestant who does not receive a prize and who has obtained full marks on at least one question.

Special prizes shall be awarded for a complete solution of outstanding merit only.

The Jury approves the translations of the chosen problems into all required languages.

Contest Regulations

Each of two contest periods lasted four and a half hours.

The only instruments permitted during the contest are writing and mechanical drawing instruments.

During the first half hour of each examination period each Student Contestant may submit, *on a specially provided note a paper*, a written question for consideration by the Jury.

The Coordinators together with the Leader and the Deputy Leader of each country decide on the scores during a coordination session.

The Leader of the country which submitted the problem, will for each of the six problems, respectively, verify the coordination of the solution given by Student Contestants from the host country.

A breaf overlook on the IMO

The official beginning of the IMO is the first Jury meeting and three days later is the official arrival of student contestants with their Deputy Leaders. Every participating country has its representative in the Jury. It has to prepare the problems for the competition. The problems have been divided in four groups: algebra, geometry, number theory and combinatorics. Six problems have to be chosen, three for the first day and three for the second day. The first problem should be the easiest, the second one more difficult than the first and the third, the most difficult of the first day problems. The first problem of the second day should be the easiest one of the second day, the second one should be more difficult than this one, and the third should be most difficult of all problems. This one is obtaining for the special prize, and it is very hard to achieve it. For the first of all the English version is prepared, and after that the versions in official languages: French, Chinese, German, Russian and Spanish. After that the Leader of every country translates the problems into the official language of his country. Each contestant may obtain a version in his own language and in one of the official ones.

The day before the first day of contest the Opening Ceremony of the IMO takes place. At that moment the Leaders have to be strictly separated from the Student Contestants and the Deputy Leaders. On the first day of contest after answering the student questions an excursion for the members of the Jury is organized. After the second day of contest excursions are prepared for the students, they can spend their time on computers and sport, but they also have a possibility to keep company with students from other countries.

Two days after the contest Leaders and Deputy Leaders have to coordinate the solutions of the students. At the end, according to the results of the contest, the Jury decides about the prizes. An honorable mention was introduced at the Olympiad in Australia.

Up to 1991 the Student Contestants from Croatia, as members of the Yugoslavian team, obtained 7 silver and 10 bronze medals, and one honorable mention. From 1993 to 1999 they obtained 2 silver and 18 bronze medals as well as 4 honorable mentions. Our Contestants, who have been three times at the IMO are: Mladen Bestvina from Osijek (1976 – 1978), Miroslav Jurišić from Split (1993 – 1995), Andrijana Radovčić from Split (1997 – 1999) and Matija Kazalicki from Zagreb (1997 – 1999). Silver medals were won by: Damir Henč from Zagreb (1969), Mladen Bestvina (silver + bronze + silver), Pavle Padžić from Zagreb (1982), Miroslav Jurišić from Kaštel (1990), Bojan Antolović from Zagreb (1996) and Vedran Zorić from

Split (1997). The first Contestant from Croatia was Valerijan Bjelik from Vinkovci (1963).

A great majority of Student Contestants decide to study mathematics, and many of them are offered a job at the Department of Mathematics in Zagreb, or in some other faculties. Some of them go to study abroad and return home after some years, but some of them continue to work in some foreign country.

In Croatia a great care is dedicated to the preparation of student contestants as well as organising the Winter and Summer Mathematical Meetings.

Now there is a plan of organizing the IMO for 8 years in future: South Korea (2000), SAD (2001), Great Britain (2002), Japan (2003), Greece (2004), Iran (2005), Slovenia (2006) and Vietnam (2007).

Members of the Croatian Olympiad team at the 40th IMO, from the left: Andrijana Radovčić, Tomislav Pejković, Srđan Maksimović, Mislav Mišković, Miodrag Cristian Iovanov (Team leader in Romania), Marinko Jablan, Matija Kazalicki, te Ilko Brnetić and Željko Hanjš (Team leaders)

Logo of the 40$^{\text{th}}$ IMO

Every IMO has its Logo, and some of them are very interesting. Let us have a look at the Logo for the last IMO which was held in Romania.

Logo for the 40th IMO

The Logo for IMO 99 was inspired by an old and very popular elementary problem in Romanian high schools. It is called the *"five lei coin problem"*. The Romanian curency is called "leu" and its plural form is "lei". The English translation of "leu" is lion. So, this logo makes reference to the coin of five lei (lions). The legend says that, using this coin, the famous Romanian geometer Gheorghe Titeica created during the joke the following problem: three equal circles having a common point, pairwisely intersect again at three points which lie on a fourth circle equal to the previous ones.

In the given logo, the given circles are colored red, yellow and blue, which are the colors of the Romanian national flag. The forth circle is painted black and it is used to create number 40, the rank of this IMO, while the letter O from IMO helps us to obtain five circles in all, the total number of circles used in the Olympic Games.

The author of this Logo is Professor Bogdan Enescu, a former golden medallist of the 20^{th} IMO. Please, have some fun solving the *"five lei coin problem"*!

Some other International Mathematical Competitions

- Mediterranean Mathematical Competition
- Austrian-Polish Mathematical Competition
- Polish-Israel Mathematical Competition
- Ibero-American Mathematical Competition
- Asian-Pacific Mathematical Competition
- The Tournament of the Towns

Some other information about the International Mathematical Olympiads can be found on Internet, at:

http://www.math.hr/hmd

The problems from the 40^{th} IMO

First day, 16 July 1999

Problem 1. Determine all sets S of at least three points in the plane which satisfy the following condition:
for any two distinct points A and B in S, the perpendicular bisector of line segment AB is an axis of symmetry for S.

Problem 2. Let n be a fixed integer, with $b \geq 2$.

1. Determine the least constant C such that the inequality

$$\sum_{1 \leq i < j \leq n} x_i x_j (x_i^2 + x_j^2) \leq C \left(\sum_{1 \leq i \leq n} x_i \right)^4$$

   holds for all real numbers $x_1, ..., x_n \geq 0$.

2. For this constant C, determine when the equality holds.

Problem 3. Consider an $n \times n$ square board, where n is a fixed even positive integer. The board is divided into n^2 unit squares. We say that two different squares on the board are *adjacent* if they have a common side.

N unit squares on the board are marked in such a way that every square (marked or unmarked) on the board is adjacent to at least one marked square.

Determine the smallest possible value of N.

Second day, 17 July 1999

Problem 4. Determine all pairs (n, p) of positive integers such that

p is a prime,

$n \leq 2p$, and

$(p-1)^n + 1$ is divisible by n^{p-1}.

Problem 5. Two circles Γ_1 and Γ_2 are contained inside the circle Γ, and are tangent to Γ at the distinct points M and N, respectively. Γ_1 passes through the center of Γ_2. The line passing through the points of intersection of Γ_1 and Γ_2 meets Γ at A and B. The lines MA and MB meet Γ_1 at C and D, respectively.

Problem 6. Determine all functions $f : \mathbb{R} \to \mathbb{R}$ such that

$$f(x - f(y)) = f(f(y)) + x\,f(y) + f(x) - 1$$

for all $x, y \in \mathbb{R}$.

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