ENCE 710
Design of Steel Structures

VII. Chapter 8 - Torsion

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Introduction

Following subjects are covered:

- Pure torsion
- Shear center
- Torsional differential equation
- Torsional stresses
- Analogy between torsion and plane bending
- Open vs closed thin-wall sections

Reading:

- Chapters 8 of Salmon & Johnson
- AISC Design Guide 9 – Torsional Analysis of Structural Steel Members

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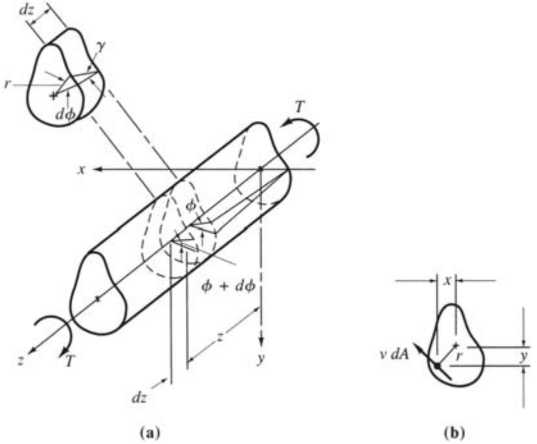
Torsion of a Prismatic Shaft



$$T = \int_A r^2 \frac{d\Phi}{dz} G dA = GJ \frac{d\Phi}{dz} = GJ\Phi'$$

(S & J 8.2.5)

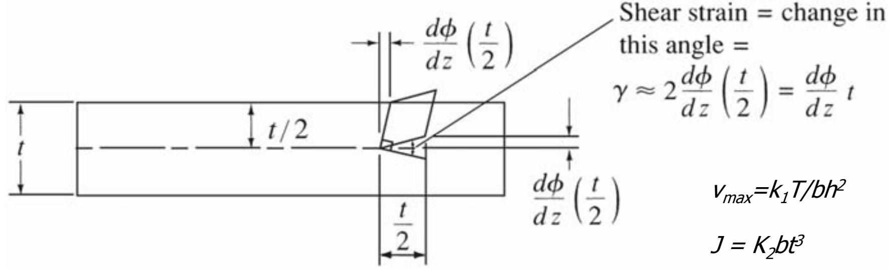
$$\tau_t = \gamma G = Gr\frac{d\Phi}{dz} = \frac{Tr}{J}$$

(S & J 8.2.6)

Torsion of Homogeneous Sections

- For Circular Section w/diameter t
  - J = polar moment of inertia = $\pi t^4/32$
  - $\tau_t = 16\ T/\pi t^3$ (S & J 8.2.8)
- For rectangular section w/thickness t
  - $\tau_t = T\ t/J$ (S & J 8.2.11)
  - where torsional constant
  - $J = K_2 b t^3$ (S & J 8.2.13)
  - K_2 (S & J Table 8.2.1)
- For I-shaped, Channel, and Tee Section
  - $J = \Sigma\ 1/3\ bt^3$ (S & J 8.2.14)

Torsion of a Rectangular Section



Shear strain = change in this angle =

$$\gamma \approx 2\frac{d\phi}{dz}\left(\frac{t}{2}\right) = \frac{d\phi}{dz}t$$

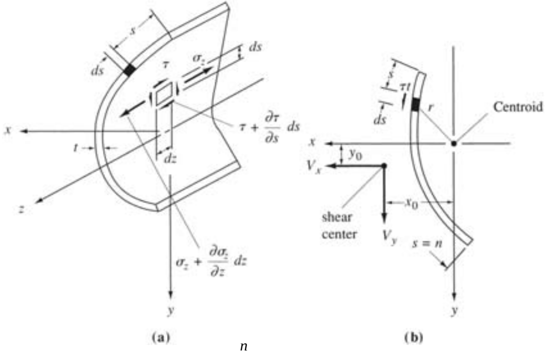
$v_{max}=k_1T/bh^2$

$J = K_2bt^3$

TABLE 8.2.1 Values of k_1 and k_2 for Eqs. 8.2.12 and 8.2.13

b/t	1.0	1.2	1.5	2.0	2.5	3.0	4.0	5.0	∞
k_1	4.81	4.57	4.33	4.07	3.88	3.75	3.55	3.44	3.00
k_2	0.141	0.166	0.196	0.229	0.249	0.263	0.281	0.291	0.333

Stresses on Thin-wall Open Sections in Bending



Shear Center – forces acting through the shear center will cause no torsional stresses

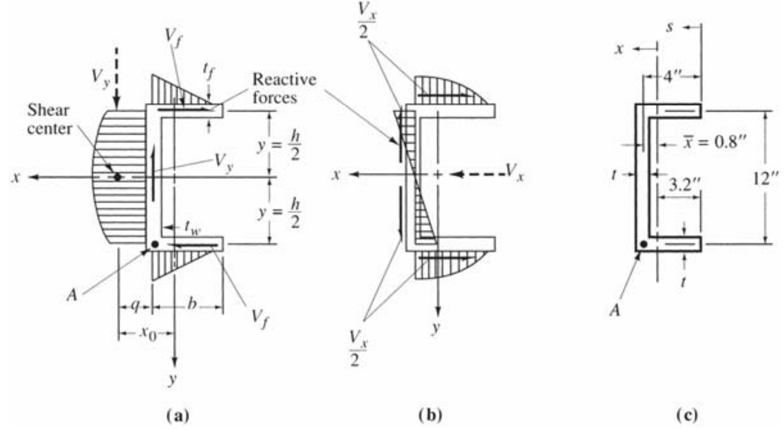
$$\int_o^n (\tau t)rds = 0 \qquad \text{(S \& J 8.4.1)}$$

$$V_y x_0 - V_x y_0 = \int_o^n (\tau t)rds = 0 \qquad \text{(S \& J 8.4.2)}$$

Shear Center

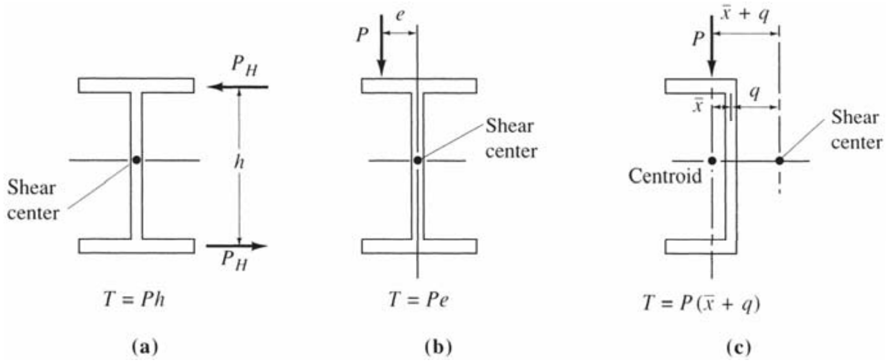
- a. Y-axis let $V_y = 0$
  - $$y_0 = -\frac{1}{V_x}\int_o^n (\tau t)rds \qquad \text{(S \& J 8.4.3)}$$
  - $$\tau t = \frac{V_x}{I_x I_y - I_{xy}^2}\left[I_{xy}\int_o^s ytds - I_x\int_o^s xtds\right]$$
- b. X-axis let $V_x = 0$
  - $$x_o = \frac{1}{V_y}\int_o^n (\tau t)rds \qquad \text{(S \& J 8.4.4)}$$
  - $$\tau t = \frac{-V_y}{I_x I_y - I_{xy^2}}\left[I_y\int_o^s ytds - I_{xy}\int_o^s xtds\right]$$

Channel of Example 8.4.1

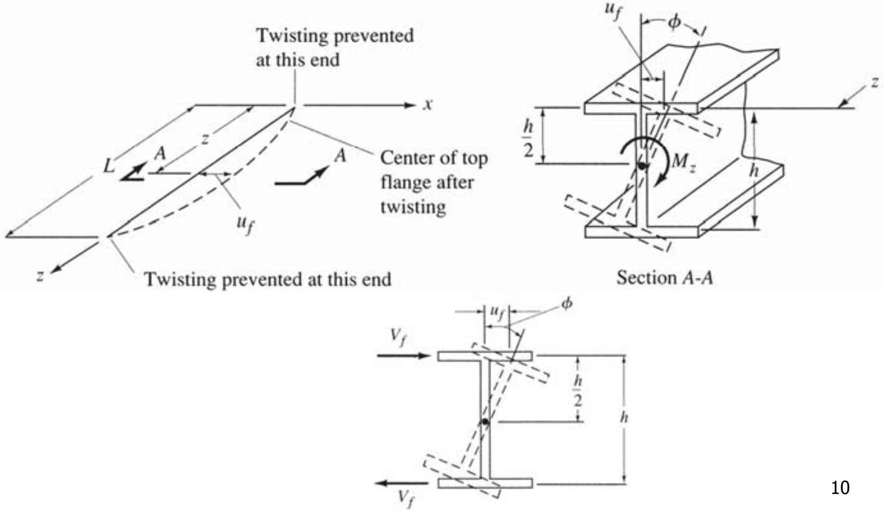


See handout for both (a) Force method and (b) Numerical method

Common Torsional Loadings



Torsion and Warping of an I-shaped Section



Solution to the Torsional Differential Equation

- Pure Torsion (Resisting moment of an unrestrained cross section)
- Warping Torsion (Resisting moment of a restrained cross section)
- Total Torsional Resisting Moment
    - $M = G J \Phi' - E C_w \Phi'''$
- Solution to the differential Equation

a. homogeneous solution – $\Phi_n = A e^{mz}$
b. Particular solution

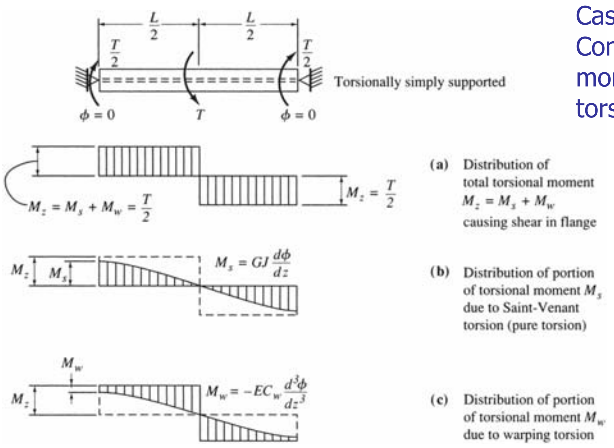
- $\Phi_n = A \sinh \lambda z + B \cosh \lambda z + C$
- $\Phi_p = C_1 + C_2 z + C_3 z^2 + \ldots\ldots$
- Loading Condition
    - Constant
    - Uniformly distributed
    - Linearly varying
- Boundary Conditions

$\Phi = 0$	No rotation	Pinned or fixed end
$\Phi' = 0$	Section cannot warp	Fixed end
$\Phi'' = 0$	Section can warp freely	Pinned or free end

Torsional Stresses

- Torsional Stresses
    - Pure Torsional Shear Stresses $\tau_t = Gt\Phi'$
    - Warping Shear Stresses $\tau_{ws} = - ES_{ws}\Phi''' / t$
    - Warping Normal Stresses $\sigma_{ws} = EW_{ns}\Phi''$
- Torsional Properties
    - J = Torsional Constant (in^4)
    - C_w = Warping Constant (in^6)
    - W_{ns} = normalized warping function at pt. S (in^2)
    - S_{ws} = Warping statical moment (in^4)
    - Q = Statical moment (in^3)

Torsional Case of Example 8.5.1



Case of Example 8.5.1. Concentrated torsional moment at midspan; torsionally simply supported

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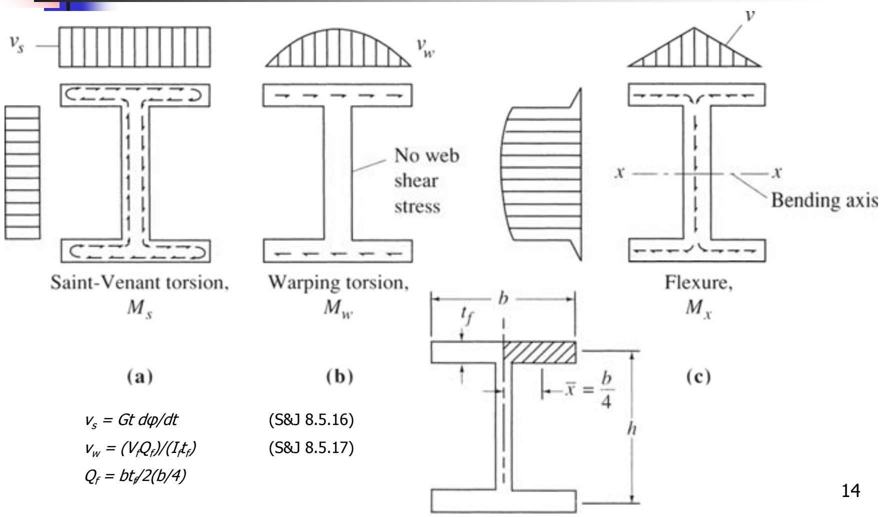
Direction and Distribution of Shear Stress in I-shaped Sections

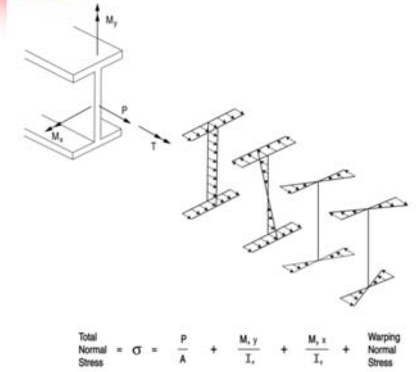


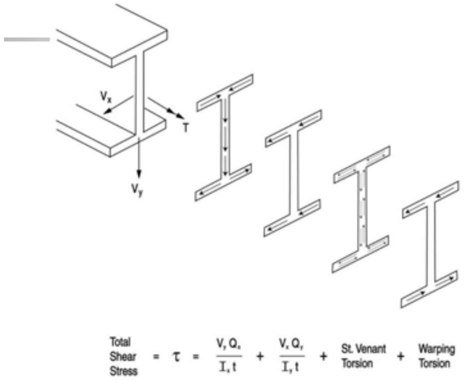
$v_s = Gt\ d\varphi/dt$ (S&J 8.5.16)

$v_w = (V_fQ_f)/(I_ft_f)$ (S&J 8.5.17)

$Q_f = bt_f/2(b/4)$

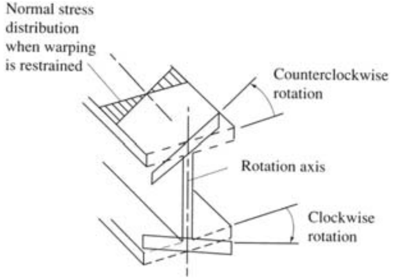
Normal and Shear Stresses of an Open Section

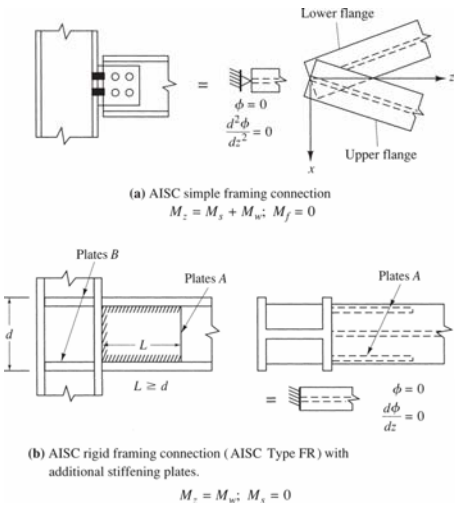




- Total normal stress: a combination of axial stress, major axis bending stress, lateral bending stress, and warping normal stress (left).
- Total shear stress is the sum of vertical shear stress, horizontal shear stress, St. Venant torsional shear stress (generally relatively small), and warping shear stress (right).

Warping of Cross-section

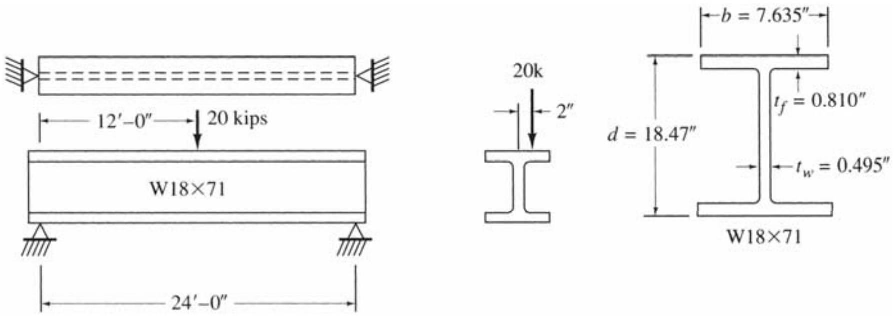




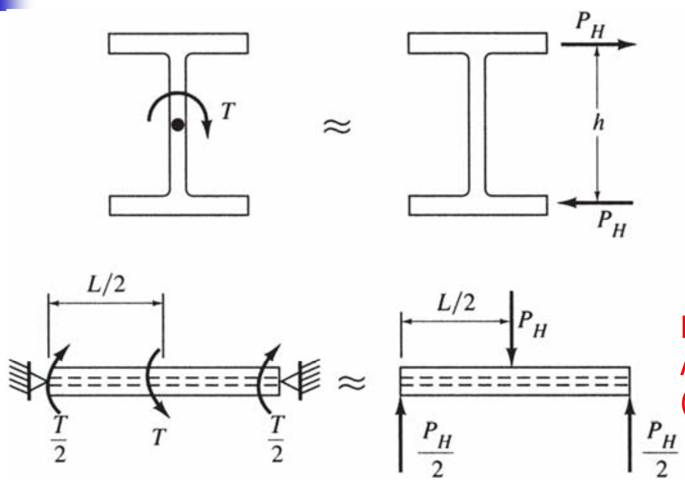
(a) AISC simple framing connection $M_z = M_s + M_w;\ M_f = 0$

(b) AISC rigid framing connection (AISC Type FR) with additional stiffening plates. $M_z = M_w;\ M_s = 0$

Data for Example 8.5.2

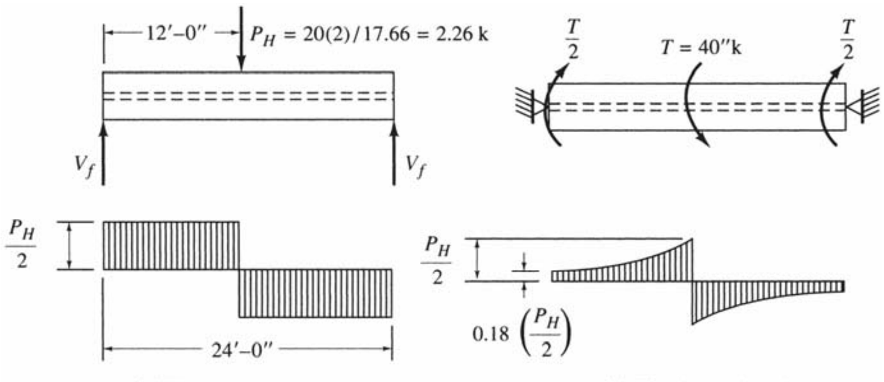


Analogy between Flexure and Torsion



Flexure Analogy (S&J 8.6)

Comparison of lateral shear on flange due to warping torsion with that from simple lateral flexure analogy



(a) Flexure analogy shear

(b) Warping torsion shear

Example 8.6.1
By Flexural Analogy

f_b+f_{bw}=31.9 ksi

$v+v_s$=4.17 ksi

$v+v_s+v_w$=5.32 ksi

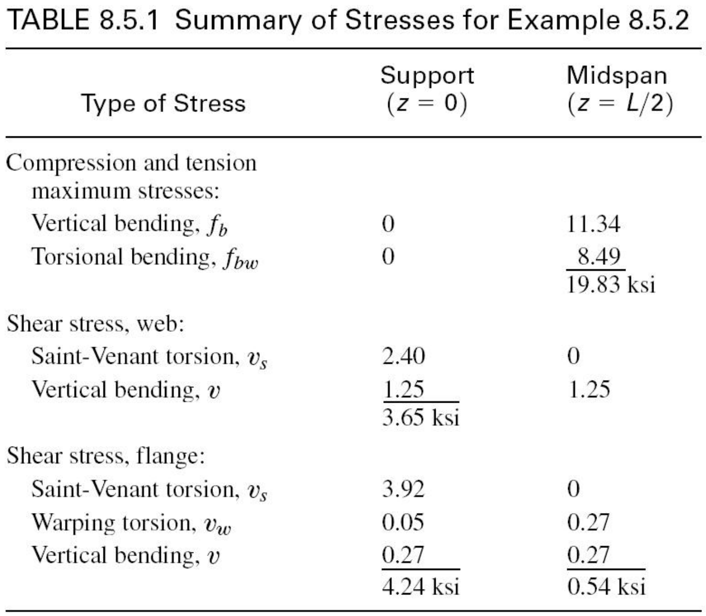
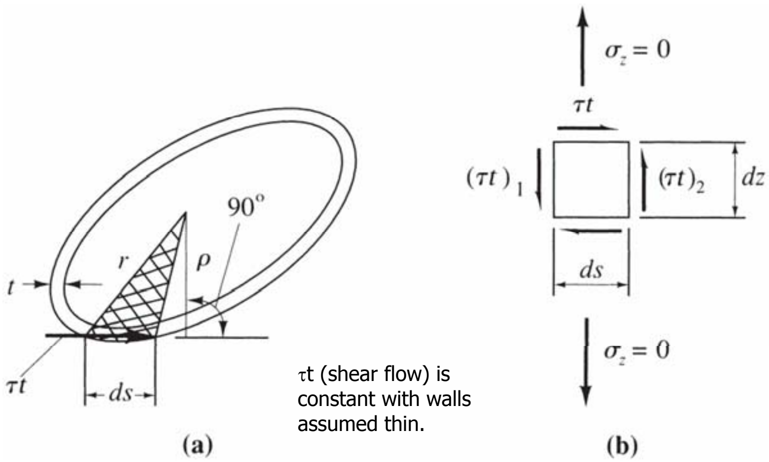


TABLE 8.5.1 Summary of Stresses for Example 8.5.2

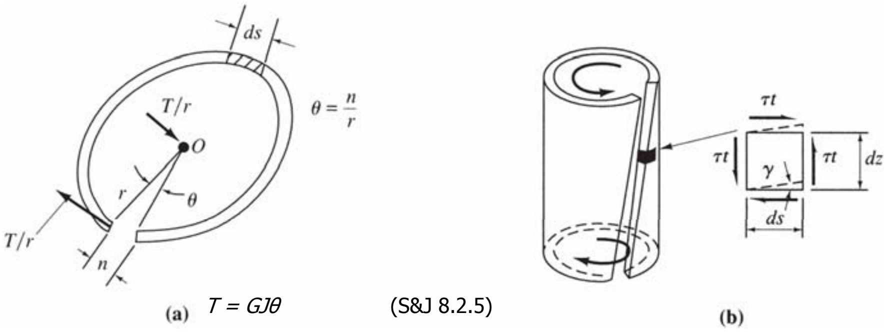
Type of Stress	Support ($z = 0$)	Midspan ($z = L/2$)
Compression and tension maximum stresses:		
Vertical bending, f_b	0	11.34
Torsional bending, f_{bw}	0	8.49
		19.83 ksi
Shear stress, web:		
Saint-Venant torsion, v_s	2.40	0
Vertical bending, v	1.25	1.25
	3.65 ksi	
Shear stress, flange:		
Saint-Venant torsion, v_s	3.92	0
Warping torsion, v_w	0.05	0.27
Vertical bending, v	0.27	0.27
	4.24 ksi	0.54 ksi

Shear Flow in a Closed Thin Wall Section



τt (shear flow) is constant with walls assumed thin.

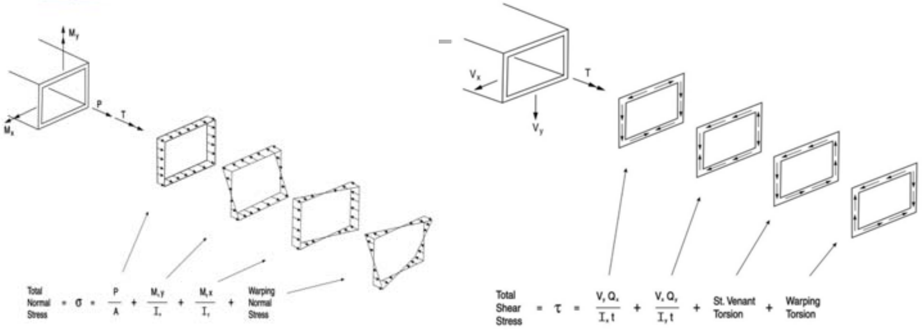
Forces on a Cut Thin-wall Section



$T = GJ\theta$ (S&J 8.2.5)

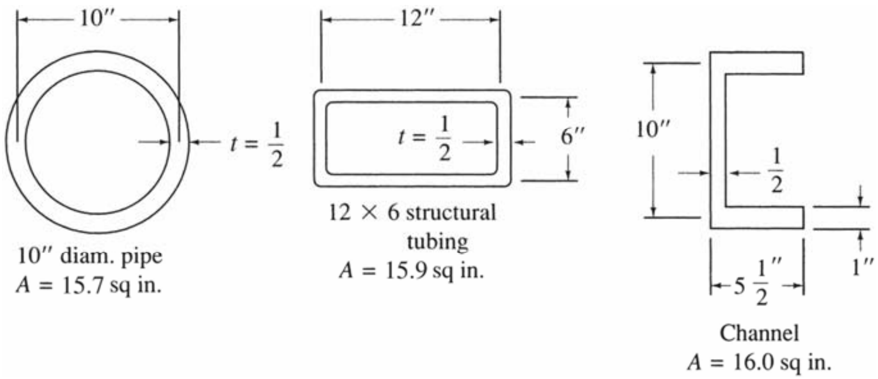
$J = 4A^2/\int_s ds/t$ (S&J 8.10.12)

Normal and Shear stresses of a Close Section



- Figure 8.9 illustrates the general box girder normal stresses which can occur in a curved or skewed box-shaped girder.
- Closed box sections are extremely efficient at carrying torsion by means of St. Venant torsional shear flow (Figure 8.10). When combined with vertical shear in the webs, this shear flow is always subtractive in one web and additive in the other.

Sections for Example 8.10.1



J=393 in^4

J=288 in^4

J=4.1 in^4