Ionizing Radiation

Chapter 1

F.A. Attix, Introduction to Radiological Physics and Radiation Dosimetry

Introduction

- Radiological physics studies ionizing radiation and its interaction with matter
- Began with discovery of x-rays, radioactivity and radium in 1890s
- Special interest is in the energy absorbed in matter
- Radiation dosimetry deals with quantitative determination of the energy absorbed in matter

Ionizing radiation

- By general definition ionizing radiation is characterized by its ability to excite and ionize atoms of matter
- Lowest atomic ionization energy is ~ eV, with very little penetration
- Energies relevant to radiological physics and radiation therapy are in keV – MeV range

Types and sources of ionizing radiation

- γ-rays: electromagnetic radiation (photons) emitted from a nucleus or in annihilation reaction
  - Practical energy range from 2.6 keV (K_α from electron capture in $^{37}_{18}Ar$) to 6.1 and 7.1 MeV (γ-rays from $^{16}_{7}N$)
- x-rays: electromagnetic radiation (photons) emitted by charged particles (characteristic or bremsstrahlung processes). Energies:
  - 0.1-20 kV "soft" x-rays
  - 20-120 kV diagnostic range
  - 120-300 kV orthovoltage x-rays
  - 300 kV-1 MV intermediate energy x-rays
  - 1 MV and up megavoltage x-rays

Types and sources of ionizing radiation

- Fast electrons (positrons) emitted from nuclei (β-rays) or in charged-particle collisions (δ-rays). Other sources: Van de Graaf generators, linacs, betatrons, and microtrons
- Heavy charged particles emitted by some radioactive nuclei (α-particles), cyclotrons, heavy particle linacs (protons, deuterons, ions of heavier elements, etc.)
- Neutrons produced by nuclear reactions (cannot be accelerated electrostatically)

Types of interaction

- ICRU (The *International Commission on Radiation Units* and Measurements; established in 1925) terminology
- *Directly ionizing radiation*: by charged particles, delivering their energy to the matter directly through multiple Coulomb interactions along the track
- *Indirectly ionizing radiation*: by photons (x-rays or γ-rays) and neutrons, which transfer their energy to charged particles (two-step process)

Description of ionizing radiation fields

- To describe radiation field at a point P need to define non-zero volume around it
- Can use *stochastic* or *non-stochastic* physical quantities

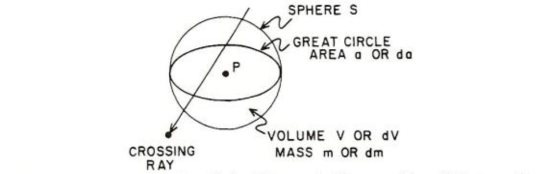


FIGURE 1.1. Characterizing the radiation field at a point *P* in terms of the radiation traversing the spherical surface *S*.

Stochastic quantities

- Values occur randomly, cannot be predicted
- Radiation is random in nature, associated physical quantities are described by probability distributions
- Defined for finite domains (non-zero volumes)
- The expectation value of a stochastic quantity (e.g. number of x-rays detected per measurement) is the mean of its measured value for infinite number of measurements

$$\overline{N} \to N_e \ \text{ for } n \to \infty$$

Stochastic quantities

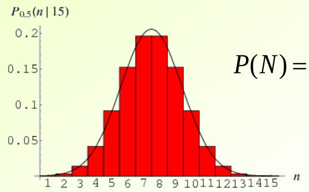
- For a "constant" radiation field a number of x-rays observed at point P per unit area and time interval follows Poisson distribution
- For large number of events it may be approximated by normal (Gaussian) distribution, characterized by standard deviation σ (or corresponding percentage standard deviation S) for a single measurement

$$\sigma = \sqrt{N_e} \cong \sqrt{\overline{N}}$$

$$S = \frac{100\sigma}{N_e} = \frac{100}{\sqrt{N_e}} \cong \frac{100}{\sqrt{\overline{N}}}$$

Stochastic quantities

- Normal (Gaussian) distribution is described by probability density function $P(x)$
- Mean $\overline{N}$ determines position of the maximum, standard deviation σ defines the width of the distribution



$$P(N) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left(N-\overline{N}\right)^2/\left(2\sigma^2\right)}$$

Stochastic quantities

- For a given number of measurements n standard deviation is defined as

$$\sigma' = \frac{\sigma}{\sqrt{n}} = \sqrt{\frac{N_e}{n}} \cong \sqrt{\frac{\overline{N}}{n}}$$

$$S' = \frac{100\sigma'}{N_e} = \frac{100}{\sqrt{nN_e}} \cong \frac{100}{\sqrt{n\overline{N}}}$$

- $\overline{N}$ will have a 68.3% chance of lying within interval $\pm\sigma'$ of N_e, 95.5% to be within $\pm 2\sigma'$, and 99.7% to be within interval $\pm 3\sigma'$. No experiment-related fluctuations

Stochastic quantities

- In practice one always uses a detector. An estimated precision (proximity to N_e) of any single random measurement N_i

$$\sigma \cong \left[\frac{1}{n-1}\sum_{i=1}^{n}\left(N_i - \overline{N}\right)^2\right]^{1/2}$$

$$\overline{N} = \left(\sum N_i\right)/n$$

- Determined from the data set of n such measurements

Stochastic quantities

- An estimate of the precision (proximity to N_e) of the mean value $\overline{N}$ measured with a detector n times

$$\sigma' = \sigma / \sqrt{n}$$

$$\sigma' \cong \left[\frac{1}{n(n-1)} \sum_{i=1}^{n} \left(N_i - \overline{N} \right)^2 \right]^{1/2}$$

- N_e is as correct as your experimental setup

Stochastic quantities: Example

- A γ-ray detector having 100% counting efficiency is positioned in a constant field, making 10 measurements of equal duration, Δt=100s (exactly). The average number of rays detected ("counts") per measurement is 1.00×10^5. What is the mean value of the count rate C, including a statement of its precision (i.e., standard deviation)?

$$\overline{C} = \frac{\overline{N}}{\Delta t} = \frac{1.00 \times 10^5}{100} = 1.00 \times 10^3 \text{ c/s}$$

$$\sigma'_C \cong \sqrt{\frac{\overline{C}}{n}} = \sqrt{\frac{1.00 \times 10^3}{10}} = 1 \text{ c/s}$$

$$\overline{C} = 1.00 \times 10^3 \pm 1 \text{ c/s}$$

- Here the standard deviation is due entirely to the stochastic nature of the field, since detector is 100% efficient

Non-stochastic quantities

- For given conditions the value of non-stochastic quantity can, in principle, be calculated
- In general, it is a "point function" defined for infinitesimal volumes
  - It is a continuous and differentiable function of space and time; with defined spatial gradient and time rate of change
- Its value is equal to, or based upon, the *expectation value* of a related stochastic quantity, if one exists
  - In general does not need to be related to stochastic quantities, they are related in description of ionizing radiation

Description of radiation fields by non-stochastic quantities

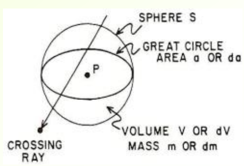
- Fluence
- Flux Density (or Fluence Rate)
- Energy Fluence
- Energy Flux Density (or Energy Fluence Rate)

Non-stochastic quantities: Fluence

- A number of rays crossing an infinitesimal area surrounding point *P*, define fluence as

$$\Phi = \frac{dN_e}{da}$$

- Units of m^{-2} or cm^{-2}



Non-stochastic quantities: Flux density (Fluence rate)

- An increment in fluence over an infinitesimally small time interval

$$\varphi = \frac{d\Phi}{dt} = \frac{d}{dt}\left(\frac{dN_e}{da} \right)$$

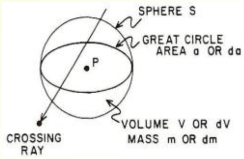
- Units of $m^{-2}\,s^{-1}$ or $cm^{-2}\,s^{-1}$
- Fluence can be found through integration:

$$\Phi(t_0, t_1) = \int_{t_0}^{t_1} \varphi(t)\,dt$$

Non-stochastic quantities: Energy fluence

- For an expectation value R of the energy carried by all the N_e rays crossing an infinitesimal area surrounding point P, define energy fluence as

$$\Psi = \frac{dR}{da}$$



- Units of J m^{-2} or erg cm^{-2}
- If all rays have energy E

$$R = EN_e$$

$$\Psi = E\Phi$$

Differential distributions

- More complete description of radiation field is often needed
- Generally, flux density, fluence, energy flux density, or energy fluence depend on all variables: θ, β, or E
- Simpler, more useful differential distributions are those which are functions of only one of the variables

Differential distributions by energy and angle of incidence

- Differential flux density as a function of energy and angles of incidence: distribution

$$\varphi'(\theta, \beta, E)$$

- Typical units are m^{-2} s^{-1}sr^{-1}keV^{-1}
- Integration over all variables gives the flux density:

$$\varphi = \int_{\theta=0}^{\pi} \int_{\beta=0}^{2\pi} \int_{E=0}^{E_{max}} \varphi'(\theta, \beta, E) \sin\theta d\theta d\beta dE$$

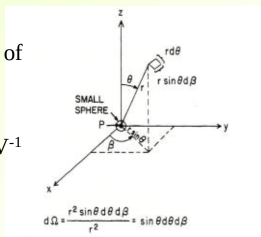


FIGURE 1.2. Polar coordinates. The element of solid angle is dΩ

(x,y,z) –> (r,θ,β)

Differential distributions: Energy spectra

- If a quantity is a function of energy only, such distribution is called the *energy spectrum* (e.g. $\varphi'(E)$)
- Typical units are m^{-2} s^{-1}keV^{-1} or cm^{-2} s^{-1}keV^{-1}
- Integration over angular variables gives flux density spectrum

$$\varphi'(E) = \int_{\theta=0}^{\pi} \int_{\beta=0}^{2\pi} \varphi'(\theta, \beta, E) \sin\theta d\theta d\beta$$

- Similarly, may define energy flux density $\psi'(E)$

Differential distributions: Energy spectrum example

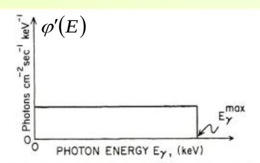


FIGURE 1.3a. A flat spectrum of photon flux density φ′(E).

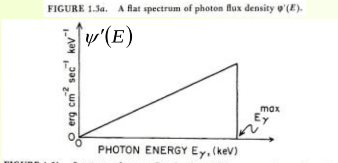


FIGURE 1.3b. Spectrum of energy flux density ψ′(E) corresponding to Fig. 1.3a.

- A "flat" distribution of photon flux density
- Energy flux density spectrum is found by

$$\psi'(E) = E\varphi'(E)$$

Typically units for E are joule or erg, so that [ψ'] = Jm^{-2}s^{-1}keV^{-1}

Example: Problem 1.8

An x-ray field at a point P contains 7.5x10^8 photons/(m^2-sec-keV), uniformly distributed from 10 to 100 keV.

a) What is the photon flux density at P?
b) What would be the photon fluence in one hour?
c) What is the corresponding energy fluence, in J/m^2 and erg/cm^2?

Example: Problem 1.8

Energy spectrum of a flux density $\varphi'(E)$ =7.5x10^8 photons/m^2-sec-keV

a) Photon flux density

$$\varphi = \varphi'(E)\cdot(E_{\max} - E_{\min}) =$$
$$7.5\times10^{8}\cdot 90 = 6.75\times10^{10}\ \text{photons/m}^2\text{s}$$

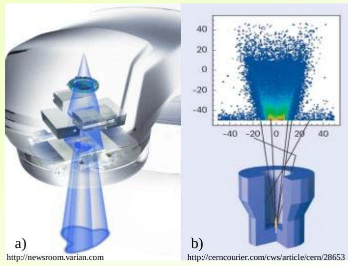
b) The photon fluence in one hour

$$\Phi(t = 1\,\text{hour}) = \varphi\cdot\Delta t =$$
$$6.75\times10^{10}\cdot 3600 = 2.43\times10^{14}\ \text{photons/m}^2$$

c) The corresponding energy fluence, in J/m^2 and erg/cm^2

$$\Psi = \Delta t\cdot\int_{E=10}^{E=100}\varphi'(E)\cdot E dE = \Delta t\cdot\varphi'\cdot\frac{E^2}{2}\Big|_{10}^{100} =$$
$$3600\cdot 7.5\times10^{8}\cdot\frac{1}{2}\left(100^2 - 10^2\right) = 1.336\times10^{16}\ \text{keV/m}^2 =$$
$$1.336\times10^{16}\cdot 1.602\times10^{-16} = 2.14\,\text{J/m}^2 = 2.14\times10^{3}\ \text{erg/cm}^2$$

Differential distributions: Angular distributions



Azimuthal symmetry: a) accelerator beam after primary collimator; b) brachytherapy surface applicator

- Full differential distribution integrated over energy leaves only angular dependence
- Often the field is symmetrical with respect to a certain direction, then only dependence on polar angle θ or azimuthal angle β

Summary

- Types and sources of ionizing radiation
  - γ-rays, x-rays, fast electrons, heavy charged particles, neutrons
- Description of ionizing radiation fields
  - Due random nature of radiation: expectation values and standard deviations
  - Non-stochastic quantities: fluence, flux density, energy fluence, energy flux density, differential distributions

Quantities for Describing the Interaction of Ionizing Radiation with Matter

Chapter 2

F.A. Attix, Introduction to Radiological Physics and Radiation Dosimetry

Introduction

- Need to describe interactions of ionizing radiation with matter
- Special interest is in the energy absorbed in matter, absorbed dose – delivered by directly ionizing radiation
- Two-step process for indirectly ionizing radiation involves kerma and absorbed dose

Definitions

- Most of the definitions are by ICRU
  - ICRU Report 33, Radiation quantities and units, 1980
  - Revised several times (the latest: ICRU Report 85 Fundamental quantities and units for ionizing radiation, 2011)
- Energy transferred by indirectly ionizing radiation leads to the definition of kerma
- Energy imparted by ionizing radiation leads to the definition of absorbed dose
- Energy carried by neutrinos is ignored

Energy transferred

- ε_{tr} - *energy transferred* in a volume V to charged particles by indirectly ionizing radiation (photons and neutrons)
- *Radiant energy R* – the energy of particles emitted, transferred, or received, excluding rest mass energy
- Q - energy delivered from *rest mass* in V (positive if m→E, negative for E→m)

Energy transferred

- The energy transferred in a volume V

$$\varepsilon_{tr} = \left(R_{in}\right)_{\underset{\text{uncharged}}{u}} - \left(R_{out}\right)_u^{nonr} + \sum Q$$

- $\left(R_{out}\right)_u^{nonr}$ does not include radiative losses of kinetic energy by charged particles (bremsstrahlung or in-flight annihilation)
- Energy transferred is only the kinetic energy received by charged particles

Kerma

- Kerma K is the energy transferred to charged particles per unit mass

$$K = \frac{d\left(\varepsilon_{tr}\right)_e}{dm} \equiv \frac{d\varepsilon_{tr}}{dm}$$

- Includes radiative losses by charged particles (bremsstrahlung or in-flight annihilation of positron)
- Excludes energy passed from one charged particle to another
- Units: 1 Gy = 1 J/kg = 10^2 rad = 10^4 erg/g

Relation of kerma to energy fluence for photons

- For mono-energetic photon of energy E and medium of atomic number Z, relation is through the mass energy-transfer coefficient:

$$K = \Psi \cdot \left(\frac{\mu_{tr}}{\rho}\right)_{E,Z}$$

- For a spectrum of energy fluence $\Psi'(E)$

$$K = \int_{E=0}^{E=E\max} \Psi'(E) \cdot \left(\frac{\mu_{tr}}{\rho}\right)_{E,Z} dE$$

Energy-transfer coefficient

- Linear energy-transfer coefficient μ_{tr} , units of m^{-1} or cm^{-1}
- Mass energy-transfer coefficient $\left(\frac{\mu_{tr}}{\rho}\right)_{E,Z}$, units of m^2/kg or cm^2/g
- Set of numerical values, tabulated for a range of photon energies, Appendix D.3

Relation of kerma to fluence for neutrons

- Neutron field is usually described in terms of fluence rather than energy fluence
- Kerma factor is tabulated instead of kerma (units are rad cm^2/neutron, Appendix F)

$$\left(F_n\right)_{E,Z} = \left(\frac{\mu_{tr}}{\rho}\right)_{E,Z} \cdot E$$

- For mono-energetic neutrons

$$K = \Phi \cdot \left(F_n\right)_{E,Z}$$

Components of Kerma

- Energy received by charged particles may be spent in two ways
  - *Collision* interactions – local dissipation of energy, ionization and excitation along electron track
  - *Radiative* interactions, such as bremsstrahlung or positron annihilation, carry energy away from the track
- Kerma may be subdivided in two components, collision and radiative:

$$K = K_c + K_r$$

- When kerma is due to neutrons, resulting charged particles are much heavier, $K=K_c$

Collision Kerma

- Subtracting radiant energy emitted by charged particles R_u^r from energy transferred results in *net* energy transferred locally

$$\varepsilon_{tr}^{net} = \varepsilon_{tr} - R_u^r =$$
$$(R_{in})_u - (R_{out})_u^{nonr} - R_u^r + \sum Q$$

- Now collision kerma can be defined

$$K_c = \frac{d\varepsilon_{tr}^{net}}{dm}$$

Mass energy-absorption coefficient

- Since collision kerma represents energy deposited (absorbed) locally, introduce mass energy-absorption coefficient. For mono-energetic photon beam

$$K_c = \Psi \cdot \left(\frac{\mu_{en}}{\rho}\right)_{E,Z}$$

- Depends on materials present along particle track before reaching point P

Mass energy-absorption coefficient

- For low Z materials and low energy radiative losses are small, therefore values of μ_{tr} and μ_{en} are close

γ-ray Energy (MeV)	$100\,(\mu_{tr} - \mu_{en})/\mu_{tr}$		
	$Z = 6$	29	82
0.1	0	0	0
1.0	0	1.1	4.8
10	3.5	13.3	26

Kerma rate

- Kerma rate at point P and time *t*

$$\dot{K} = \frac{dK}{dt} = \frac{d}{dt}\left(\frac{d\varepsilon_{tr}}{dm}\right)$$

- Units of J/(kg s), erg/(g s), or rad/s
- Knowing kerma rate, kerma

$$K(t_0, t_1) = \int_{t_0}^{t_1} \dot{K}(t)\,dt$$

Absorbed dose

- Energy imparted by ionizing radiation to matter of mass m in volume V

$$\varepsilon = \underbrace{(R_{in})_u - (R_{out})_u}_{\text{due to uncharged}} + \underbrace{(R_{in})_c - (R_{out})_c}_{\text{due to charged}} + \sum Q$$

- Absorbed dose is defined as

$$D = \frac{d\varepsilon}{dm}$$

- Units: 1 Gy = 1 J/kg = 10^2 rad = 10^4 erg/g

Absorbed dose

- D represents the energy per unit mass which remains in the matter at P to produce any effects attributable to radiation
- The most important quantity in radiological physics
- Absorbed dose rate:

$$\dot{D} = \frac{dD}{dt} = \frac{d}{dt}\left(\frac{d\varepsilon}{dm}\right)$$

Example 1

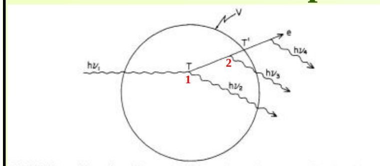


FIGURE 2.1a. Illustration of the concepts of energy imparted, energy transferred, and net energy transferred for the case of a Compton interaction followed by bremsstrahlung emission (Attix, 1983).

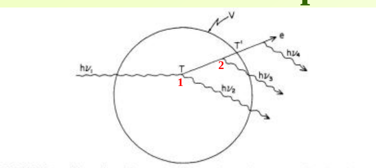
$$(R_{in})_u = h\nu_1$$
$$(R_{out})_u = (R_{out})_u^{nonr} = h\nu_2$$
$$(R_{in})_c = 0, \quad Q = 0$$
$$(R_{out})_c = h\nu_3 + T'$$
$$R_u^r = h\nu_3$$

Energy imparted $\quad \varepsilon = (R_{in})_u - (R_{out})_u + (R_{in})_c - (R_{out})_c + \sum Q$

Energy transferred $\quad \varepsilon_{tr} = (R_{in})_u - (R_{out})_u^{nonr} + \sum Q$

Net energy transferred $\quad \varepsilon_{tr}^{net} = (R_{in})_u - (R_{out})_u^{nonr} - R_u^r + \sum Q$

Example 1



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$$(R_{in})_c = 0, \quad Q = 0$$
$$(R_{out})_c = h\nu_3 + T'$$
$$R_u^r = h\nu_3$$

$$\varepsilon = h\nu_1 - (h\nu_2 + h\nu_3 + T') + 0$$
$$\varepsilon_{tr} = h\nu_1 - h\nu_2 + 0 = T$$
$$\varepsilon_{tr}^{net} = h\nu_1 - h\nu_2 - h\nu_3 + 0$$
$$= T - h\nu_3$$

Example 2

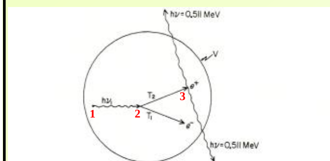


FIGURE 2.1b. Example involving γ-ray emission, pair production, and positron annihilation (Attix, 1983).

Positron has no excess kinetic energy to transfer to photons after annihilation

$$(R_{in})_u = (R_{in})_c = (R_{out})_c = R_u^r = 0$$
$$(R_{out})_u = (R_{out})_u^{nonr} = 2h\nu = 1.022\,\mathrm{MeV}$$
$$\sum Q = h\nu_1 - \underbrace{2m_0c^2}_{\text{pair production}} + \underbrace{2m_0c^2}_{\text{annihilation}} = h\nu_1$$
$$\varepsilon = \varepsilon_{tr} = \varepsilon_{tr}^{net} = h\nu_1 - 1.022\,\mathrm{MeV} = T_1 + T_2$$

Example 3

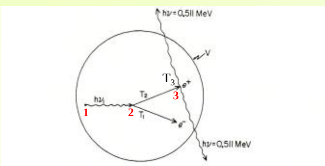


FIGURE 2.1b. Example involving γ-ray emission, pair production, and positron annihilation (Attix, 1983).

- Positron transfers excess kinetic energy T_3 to photons after annihilation.
- It generates radiative loss from charged-particle kinetic energy
- Affects ε and ε_{tr}^{n} by subtraction of T_3

$$(R_{in})_u = (R_{in})_c = (R_{out})_c = 0$$
$$(R_{out})_u = 2h\nu + \boxed{T_3} = 1.022\,\mathrm{MeV} + T_3$$
$$(R_{out})_u^{nonr} = 2h\nu = 1.022\,\mathrm{MeV}$$
$$R_u^r = \boxed{T_3}$$
$$\sum Q = h\nu_1 - 2m_0c^2 + 2m_0c^2 = h\nu_1$$

Exposure

- Historically, was introduced before kerma and dose, measured in roentgen (R)
- Defined as a quotient

$$X = \frac{dQ}{dm}$$

- dQ is absolute value of the total charge of the ions of one sign produced in air when all electrons liberated by photons in air of mass dm are completely stopped in air
- Ionization from the absorption of radiative loss of kinetic energy by electrons is *not included*

Exposure

- Exposure is the ionization equivalent of the collision kerma in air for x and γ-rays
- Introduce mean energy expended in a gas per ion pair formed, $\overline{W}$, constant for each gas, independent of incoming photon energy
- For dry air

$$\frac{\overline{W}_{air}}{e} = \frac{33.97\ \text{eV/i.p.}}{1.602\times10^{-19}\ \text{C/electron}} \times 1.602\times10^{-19}\ \text{J/eV}$$
$$= 33.97\ \text{J/C}$$

Relation of exposure to energy fluence

- Exposure at a point due to energy fluence of mono-energetic photons

$$X = \Psi \cdot \left(\frac{\mu_{en}}{\rho}\right)_{E,air} \left(\frac{e}{\overline{W}}\right)_{air} =$$
$$\left(K_c\right)_{air} \left(\frac{e}{\overline{W}}\right)_{air} = \left(K_c\right)_{air} / 33.97$$

- Units of [X]=C/kg in SI

Units of exposure

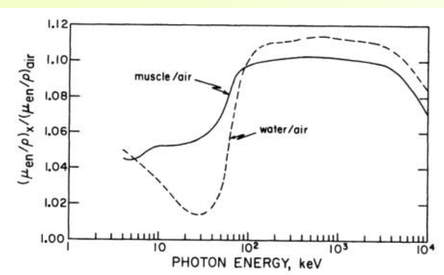
- The roentgen R is the customary unit
- The roentgen is defined as exposure producing in air one unit of esu of charge per 0.001293 g of air irradiated by the photons. Conversion

$$1R = \frac{1esu}{0.001293g} \times \frac{1C}{2.998\times10^{9}\,esu} \times \frac{10^{3}g}{1kg}$$
$$= 2.580\times10^{-4}\ \text{C/kg}$$
$$1\ \text{C/kg} = 3876\ \text{R}$$

Significance of exposure

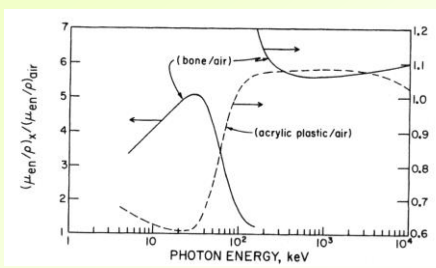
- Energy fluence is proportional to exposure for any given photon energy or spectrum
- Due to similarity in effective atomic number
  - Air can be made a tissue equivalent medium with respect to energy absorption – convenient in measurements
  - Collision kerma in muscle per unit of exposure is nearly independent of photon energy

Significance of exposure



Ratio of mass energy-absorption coefficients for muscle/air and water/air are nearly constant (within <5%) for energies from 4keV to 10 MeV

Significance of exposure



Ratio of mass energy-absorption coefficients for bone/air and acrylic/air are nearly constant for energies above 100keV

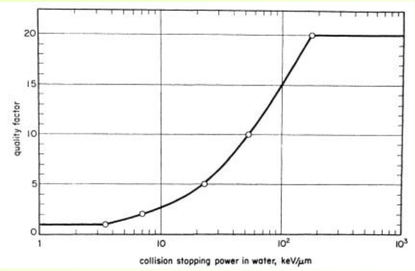
Significance of exposure

- X-ray field at a point can be characterized by means of exposure regardless of whether there is air actually located at this point
- It implies that photon energy fluence at that point is such that it would produce exposure of a stated value
- Same is applicable to kerma or collision kerma, except that reference medium (not necessarily air) has to be specified

Radiation protection quantities

- Quality factor Q – weighting factor to be applied to absorbed dose to provide an estimate of the relative human hazard of ionizing radiation
- It is based on relative biological effectiveness (RBE) of a particular radiation source
- Q is dimensionless

Radiation protection quantities



- Higher-density charged particle tracks (higher collision stopping power) are more damaging per unit dose

Radiation protection quantities

- Dose equivalent *H*, is defined as

$$H \equiv DQN$$

- Here D – dose, Q- quality factor, N-product of modifiying factors (currently=1)
- Units of H:
  - severs, *Sv*, if dose is expressed in J/kg
  - *rem*, if dose is in rad (10^{-2} J/kg)

Summary

- Quantities describing the interaction of ionizing radiation with matter
  - Kerma, components of kerma
  - Absorbed dose
  - Exposure
- Relationship with fluence and energy fluence
- Quantities for use in radiation protection
  - Quality factor Q
  - Dose equivalent H