Introduction to Acoustics

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INTRODUCTION TO ACOUSTICS

- Sound and Noise
- Sound waves
  - Frequency, wavelength and wavespeed
  - Point sources
  - Sound power and intensity
- Wave reflection
  - Standing waves
- Measures of sound
  - Decibels

SOUND AND NOISE

Sound - is what we hear: "audible air vibrations".

- human ear responds to sound in the range 20 Hz to 20000 Hz (= cycles/s)
- lower frequencies: ‘infra-sound’
- higher frequencies: ‘ultra-sound’

Noise - is unwanted sound, disturbing, annoying.

- involves a subjective assessment of the sound.
- everyone responds differently to sounds.
- want measures of sound that reflect its acceptability to an ‘average’ person

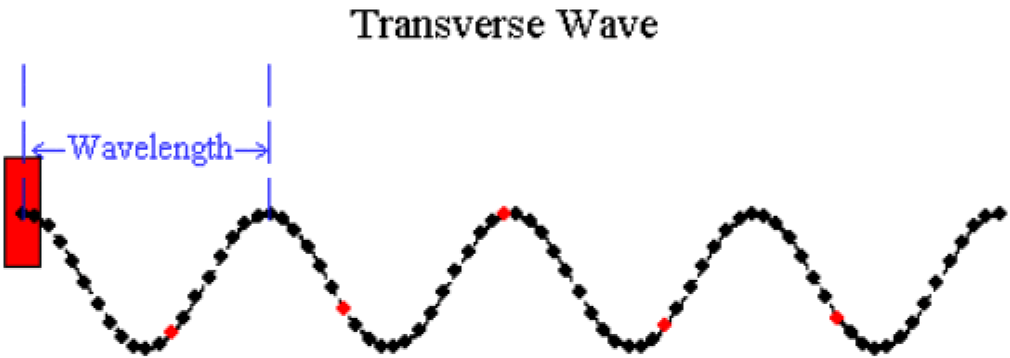
Acoustics

- acoustics is simply the study of sound
  [*particular meaning*: properties of rooms]

Sound waves

SOUND WAVES

- Sound is a wave phenomenon.
- Waves in strings, beams, plates etc are *transverse* waves.
- Particle motion is transverse to direction of wave propagation.
- Waves transfer *energy* without transferring matter.



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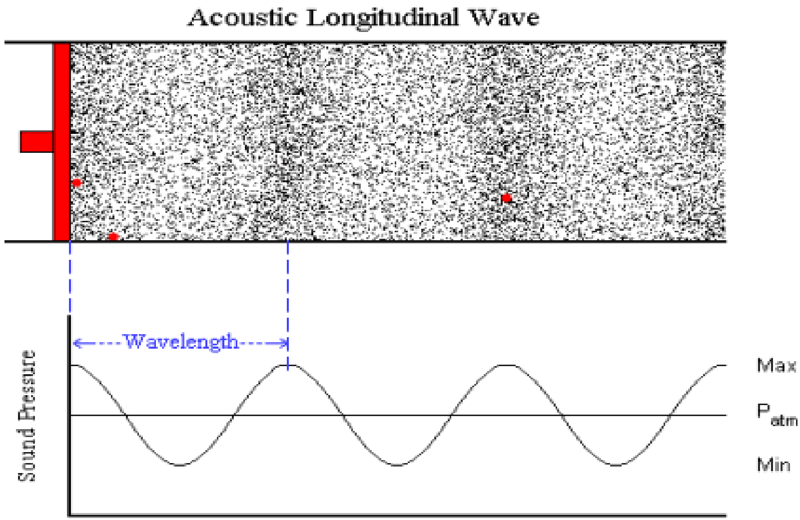
SOUND WAVES

- Sound waves are *longitudinal* waves.
- Fluid is compressed/expanded as ‘particles’ move.



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SOUND QUANTITIES

Sound involves a small time-varying disturbance of

- the **density** of a fluid from its equilibrium value,
- the **pressure** of a fluid from its equilibrium value,
- oscillatory movements (**vibration**) of the fluid ‘particles’.

We usually measure **sound pressure**.

NB **Sound pressures** are typically less than 1 Pa (Nm^{-2}).
At 100 Pa they would be literally deafening.
Atmospheric pressure = 1.013×10^5 Pa (1 bar).

FREQUENCY AND WAVELENGTH

The **frequency**, f determines how rapidly the sound changes with *time*.

The *distance* between adjacent peaks is called the **wavelength**, λ.

Frequency and wavelength are related by the **wavespeed**, c.

$$c = f\lambda$$

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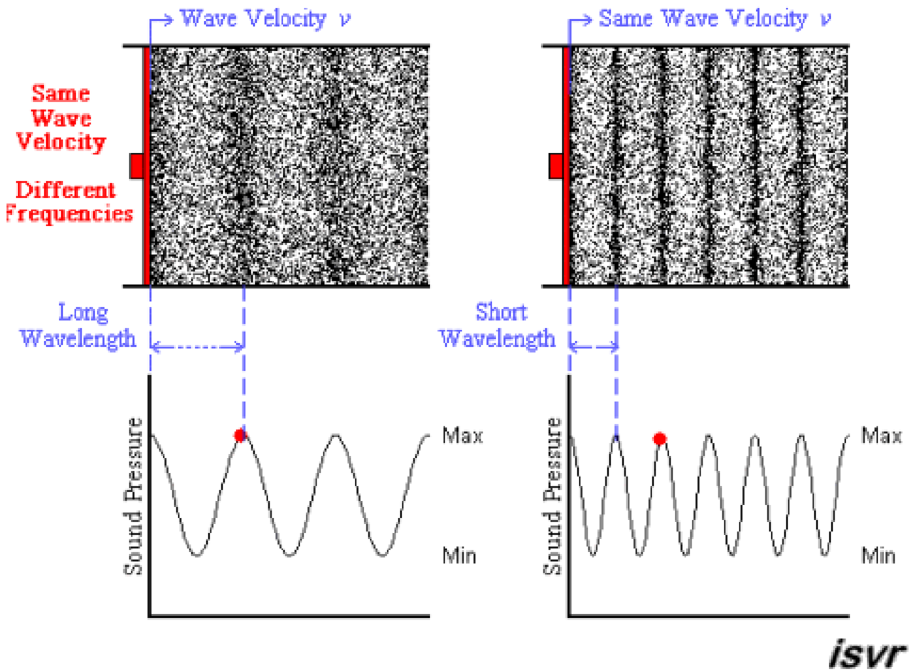
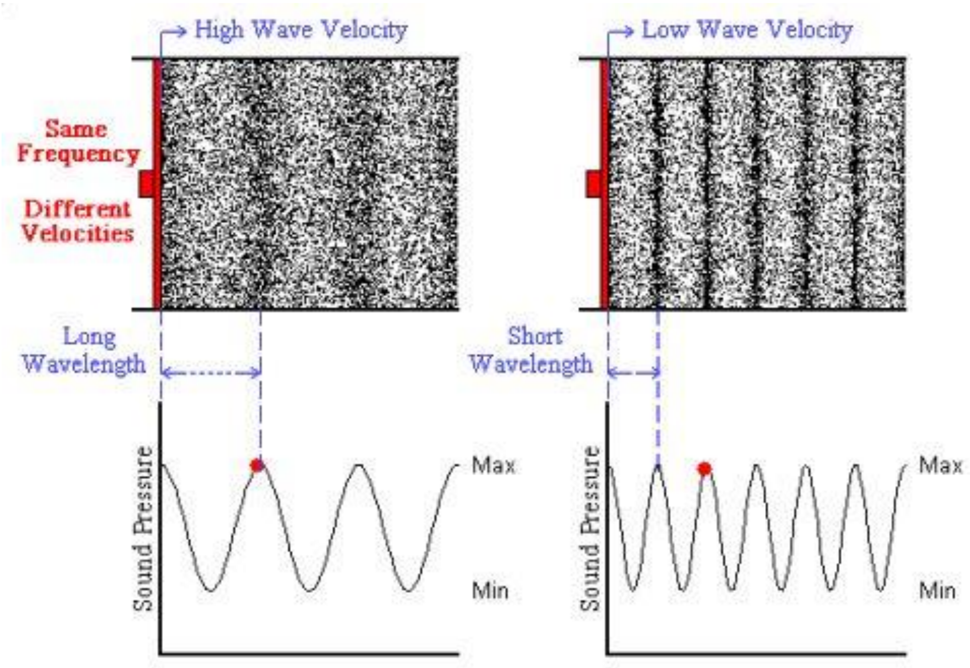
Two acoustic waves with different frequencies but the same wavespeed. The wavelength is halved when the frequency is doubled.



$$c = f\lambda$$

Two acoustic waves with same frequency but the different wavespeed. The wavelength is halved when the wavespeed is halved.



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HARMONIC FLUCTUATIONS

A basic building block of acoustic analysis is a harmonic (or single frequency) fluctuation, in this case of a sound pressure:

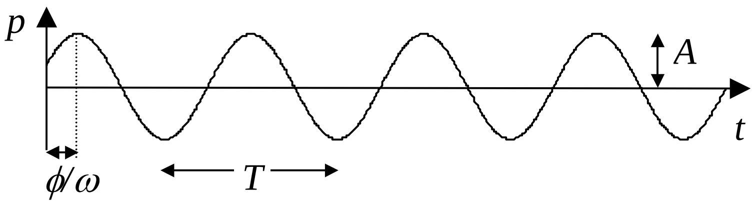
$$p(t) = A\cos(\omega t - \phi) \qquad (2)$$

This has ***amplitude*** A (real value) and ***circular frequency*** ω (rad/s).

The parameter ϕ is the ***phase angle****.*

The ***frequency*** *is given by* $f = \omega/2\pi$ (Hz).

The ***period*** *of oscillation is* $T = 1/f = 2\pi/\omega$ *(sec).*



The pressure reaches its maximum value A at time $t = (2n\pi + \phi)/\omega$.

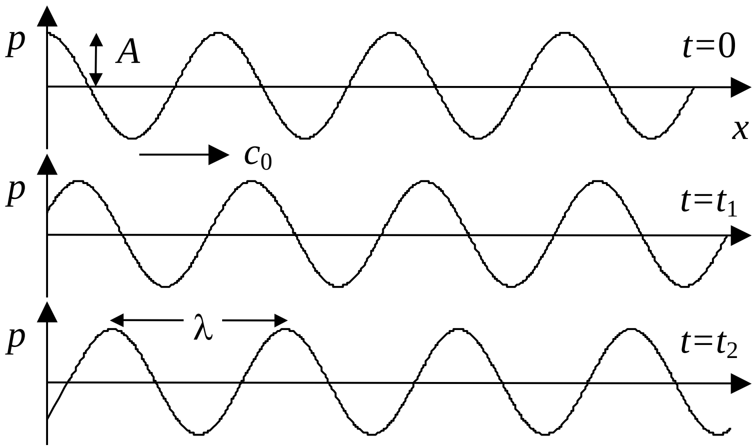
Harmonic fluctuations are a special case of ***periodic*** *fluctuations.*

HARMONIC WAVES

For a **harmonic** fluctuation of circular frequency ω (see eq. (2)), the positive travelling wave solution *must* have the form

$$p(t) = A\cos(\omega\{t - x / c_0\} + \varphi)$$

The time variation is given by the frequency ω; similarly the spatial variation repeats itself when $x' = x + 2\pi n\, c_0/\omega$. Thus the wavelength is given by $\lambda = 2\pi\, c_0/\omega = c_0 / f$.



WAVES AT A SINGLE FREQUENCY: COMPLEX EXPONENTIAL REPRESENTATION

At a single frequency, the acoustic pressure can be expressed as

$$p(x,t) = f(t - x/c) = \mathrm{Re}\{p_{pk} e^{j\omega(t-x/c)}\}$$

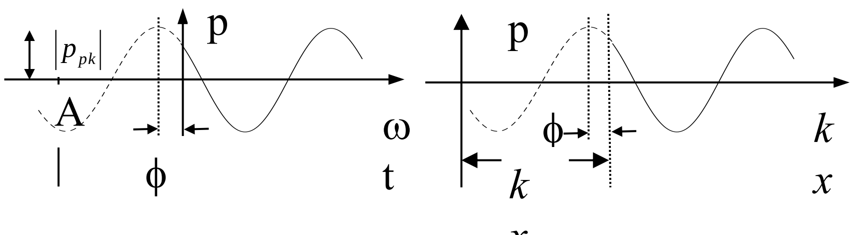
where Re{ } means the "real part of" and p_{pk} is a complex number.

Since the *wavenumber* $k = \omega / c$

$$p(x,t) = \mathrm{Re}\{p_{pk} e^{j(\omega t - kx)}\}$$

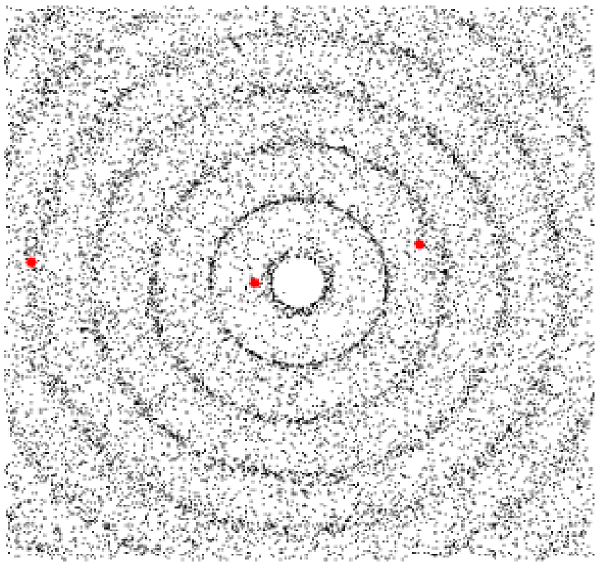
Using $e^{j\phi} = \cos\theta + j\sin\theta$ and since $p_{pk} = |p_{pk}| e^{j\phi}$

$$\boxed{p(x,t) = |p_{pk}| \cos(\omega t - kx + \phi)}$$



POINT SOURCES

A loudspeaker in a sealed box acts like a point source. Sound spreads out in three dimensions from the source.



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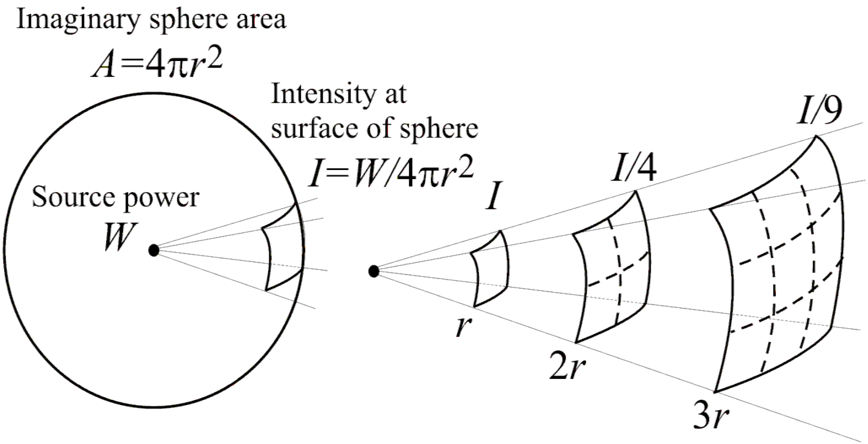
SOUND POWER AND INTENSITY

- Sound waves transmit energy. The total sound energy emitted by a source per second is its **sound power**, W (in Watts).
- The **sound intensity** is the sound power per unit area $I = W/A$.

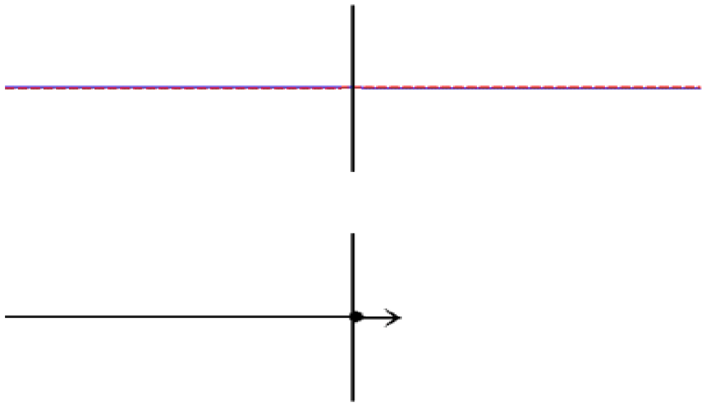


- For a point source the intensity reduces with the square of the distance (“inverse square law”).
- Far from the source the sound pressure is proportional to the square root of intensity, so $p \propto 1 / r$

Wave reflection

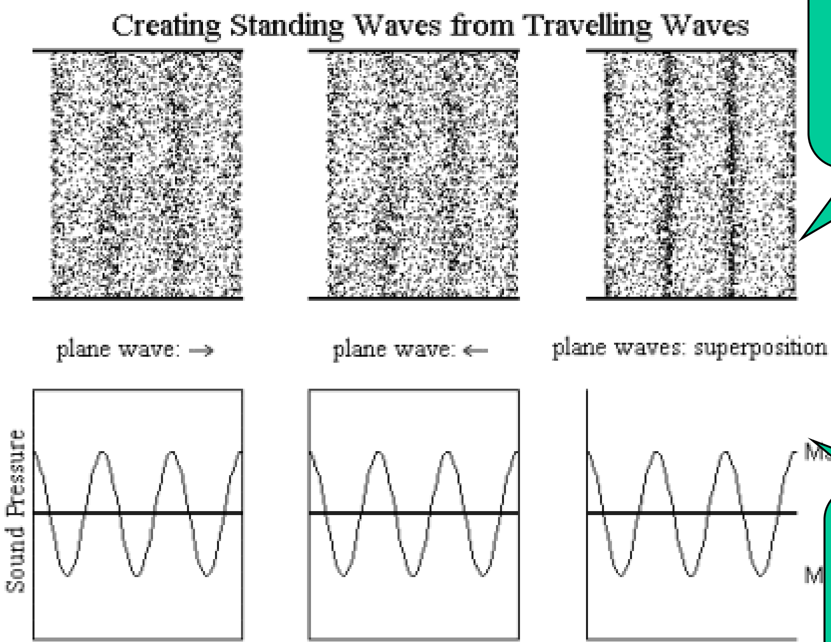
WAVE REFLECTION

- When waves meet a boundary they are reflected.
- Example of a wave in a string: actual string motion made up of ‘incident wave’ and ‘reflected wave’.



STANDING WAVES

- Standing waves may be created from two waves of equal amplitude and frequency travelling in opposite directions (e.g. reflections between two hard walls).



Note that the particles are stationary at the edges

But the pressure amplitude is maximum

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SUPERPOSITION OF WAVES

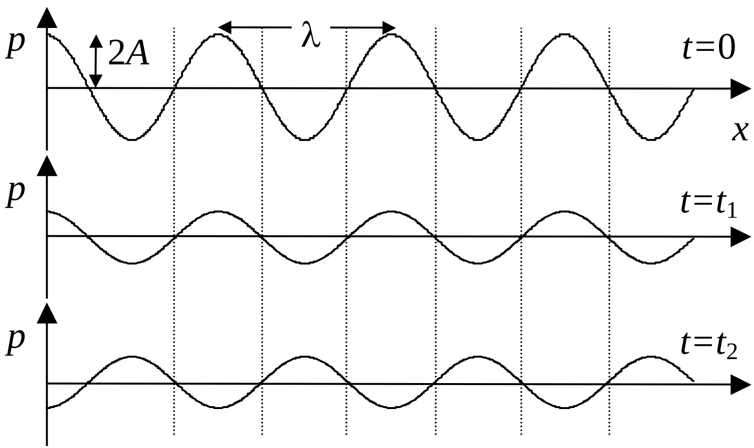
With forward and backward travelling waves at frequency ω the pressure is given by

$$p(x,t) = A_1 \cos(\omega\{t - x/c_0\} + \varphi_1) + A_2 \cos(\omega\{t + x/c_0\} + \varphi_2)$$

For the special case $A_1 = A_2$ *and* $\varphi_1 = \varphi_2$ *[= 0],* the pressure has the form

$$p(x,t) = 2A_1 \cos(\omega t)\cos(\omega x/c_0)$$

which is a **standing wave** of wavelength $\lambda = c_0 / f$.



PARTICLE VELOCITY

In a **travelling** wave, the **particle velocity** is in phase with the pressure. In a positive **travelling** harmonic wave (see eq. (5)) it satisfies

$$u(x,t) = \frac{A_1}{\rho_0 c_0}\cos(\omega\{t - x/c_0\} + \varphi_1) \quad \text{or} \quad u = \frac{p}{\rho_0 c_0}$$

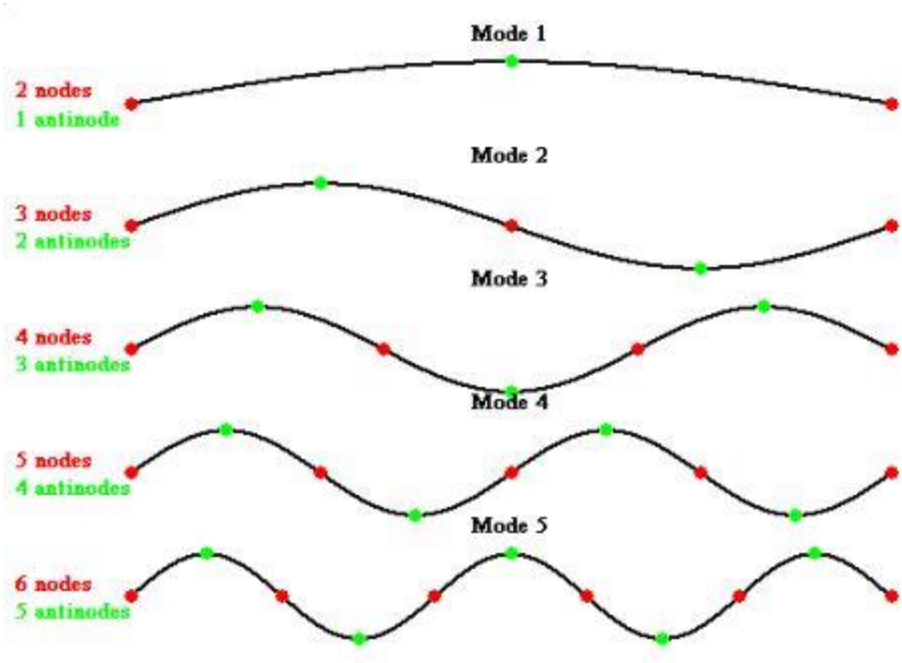
In a **negative** travelling wave, $u = -p/\rho_0 c_0$. *NB* $\rho_0 c_0$ *is called the* ***characteristic acoustic impedance*** *of the fluid (*$\rho_0 c_0$ *= 415 for air at 20*^{0}*C).*

For a **standing** wave the particle velocity is maximum at the positions and times when the pressure is zero and vice versa,

$$u(x,t) = \frac{2A_1}{\rho_0 c_0}\sin(\omega t)\sin(\omega x/c_0)$$

STANDING WAVES

- Examples of standing waves in a string.
- Number of nodal points / anti-nodes increases with frequency.



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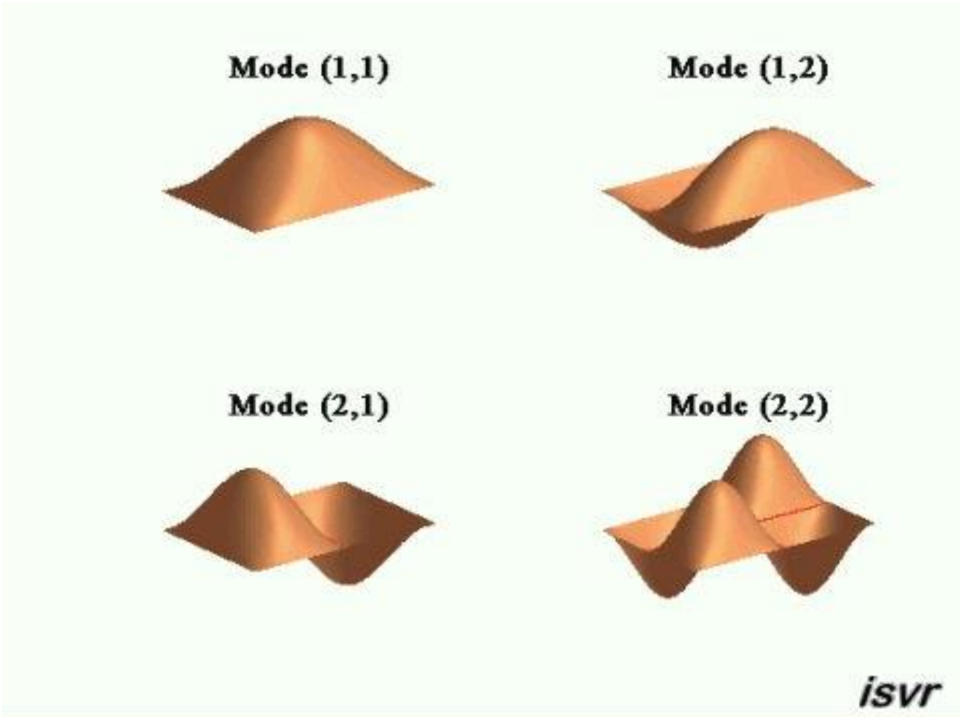
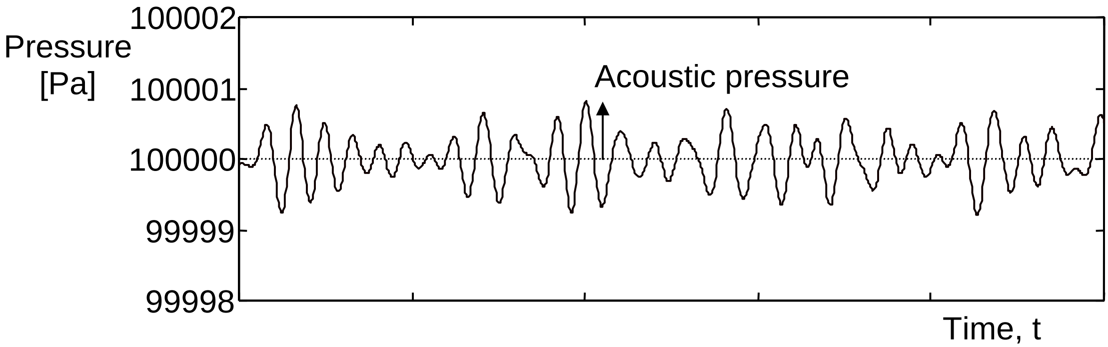
STANDING WAVES

- Examples of standing waves in a membrane.
- Modes defined by number of anti-nodal lines in each direction.



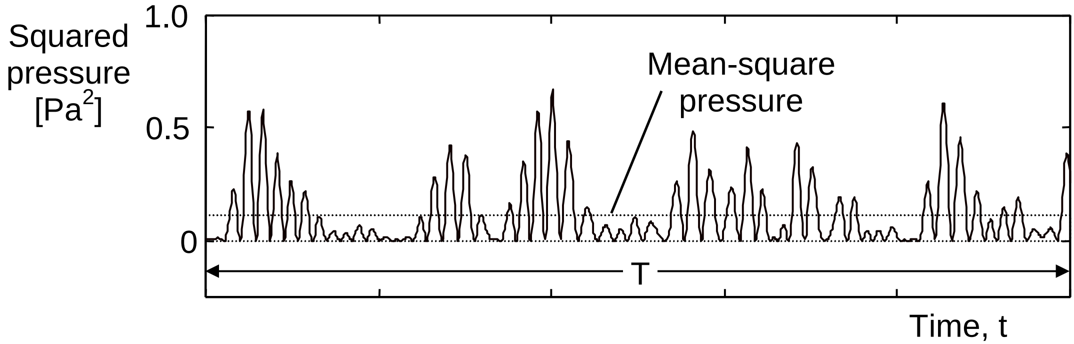
Measures of sound

MEASURES OF SOUND



- Average pressure is equal to atmospheric pressure.
- Small fluctuations around this value are acoustic pressure.

MEASURES OF SOUND



- Average the *squared* acoustic pressure (**mean-square**):

$$\overline{p^2} = \frac{1}{T} \int_{t_1}^{t_1+T} p^2(t)\, dt$$

- Square root of this is the **root mean square** (rms) pressure.
- It can depend strongly on the averaging time.

RANGE OF SOUND AMPLTIDES

- The human ear is sensitive to a very large range of pressure amplitudes:
  - threshold of hearing at 1000 Hz: 2×10^{-5} Pa
  - rustling leaves: 2×10^{-4} Pa
  - conversation: 0.01 Pa
  - noise inside a vehicle at idle: 0.1 Pa
  - noise inside a vehicle at 120 km/h: 1 Pa
  - threshold of feeling: 20 Pa
  - threshold of pain: 200 Pa

- The ear responds approximately logarithmically to sound. A doubling in pressure amplitude corresponds to a fixed change in ‘loudness’. Therefore we usually use a logarithmic scale to define sound amplitudes.

DECIBELS

- General definition of a decibel:

$$Level = 10\log_{10}\left(\frac{quantity}{reference\ value}\right)$$

'Level' => decibels

Deci => 10 x bels

Decibels are a relative quantity

Quantity can be anything

DECIBELS FOR SOUND PRESSURE

- **Sound pressure level**:

Mean-**square** pressure

$$L_p = 10\log_{10}\left(\frac{\overline{p^2}}{p_{ref}^2}\right) = 20\log_{10}\left(\frac{p_{rms}}{p_{ref}}\right)$$

Relative to reference pressure

Alterative definition using rms pressure

- p_{ref} is usually 2×10^{-5} Pa. Then we write: **dB re 2×10^{-5} Pa.**

RANGE OF SOUND AMPLTIDES

- The large range of pressure amplitudes corresponds to a much smaller range of dB values:
  - threshold of hearing at 1000 Hz: $2x10^{-5}$ Pa – **0 dB**
  - rustling leaves: $2x10^{-4}$ Pa – **20 dB**
  - conversation: 0.01 Pa – **54 dB**
  - noise inside a vehicle at idle: 0.1 Pa – **74 dB**
  - noise inside a vehicle at 120 km/h: 1 Pa – **94 dB**
  - threshold of feeling: 20 Pa – **120 dB**
  - threshold of pain: 200 Pa – **140 dB**

SOUND POWER AND INTENSITY LEVEL

- **Sound power level**:

$$L_W = 10\log_{10}\left(\frac{W}{W_{ref}}\right)$$

  - W_{ref} is usually 10^{-12} W. Then we write: **dB re 10^{-12} W**.

- **Sound intensity level**:

$$L_I = 10\log_{10}\left(\frac{I}{I_{ref}}\right)$$

  - I_{ref} is usually 10^{-12} W/m^2. Then we write: **dB re 10^{-12} W/m^2**.

RELATION BETWEEN PRESSURE AND INTENSITY

- In general there is **no direct relationship** between sound intensity and sound pressure. For example in a standing wave the time-averaged intensity is zero but the pressure is not.

- For the *special case* of a plane wave, $I = \dfrac{\overline{p^2}}{\rho_0 c_0}$

  $\rho_0 c_0$ is the **characteristic specific acoustic impedance** of the fluid ($\rho_0 c_0$ = 415 for air at 20^0C).

- I_{ref} and p_{ref} are chosen so that $L_{\text{p}} \approx L_{\text{I}}$

PROPERTIES OF DECIBELS

- From the properties of logarithms:

Source & transfer function

$$\log(AB) = \log(A) + \log(B) \;\rightarrow\; 10\log_{10}(AB) = 10\log_{10}(A) + 10\log_{10}(B)$$

Double power -> 3 dB

$$\log(2A) = \log(A) + \log(2) = \log(A) + 0.3 \;\rightarrow\; 10\log_{10}(2A) = 10\log_{10}(A) + 3$$

e.g. speed dependence

$$\log(A^n) = n\log(A) \;\rightarrow\; 10\log_{10}(A^n) = 10n\log_{10}(A)$$

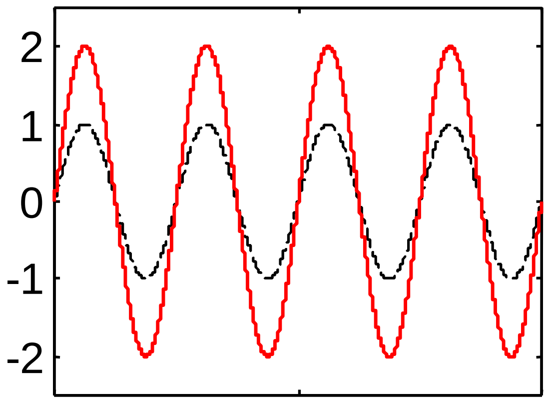
$$\log(A + B) = \ldots \;\rightarrow\; 10\log_{10}(A + B) = \ldots$$

‘dB addition’

ADDING DECIBELS – COHERENT SOURCES

- ‘Coherent’ sources are directly related. The combined sound pressure level depends on the relative phase.
- Examples: components at engine firing frequency and its harmonics from the engine, intake and exhaust are coherent.
- e.g. two single frequency sources with same magnitude:

two sources **in phase**: $|p| = p_1 + p_2$ **$L_p = L_1 + 6$ dB**



ADDING DECIBELS – COHERENT SOURCES

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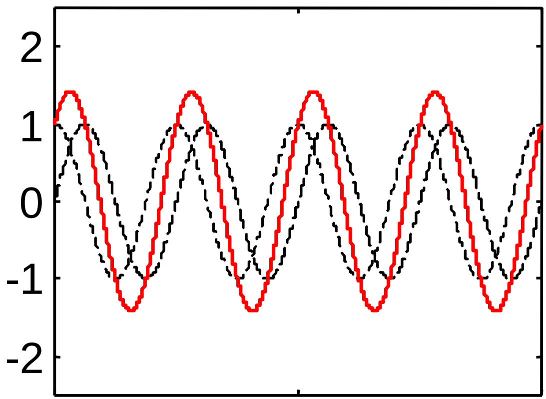
- Examples: components at engine firing frequency and its harmonics from the engine, intake and exhaust are coherent.
- e.g. two single frequency sources with same magnitude:

two sources **90^0 out of phase**:

$L_p = L_1 + 3$ **dB**

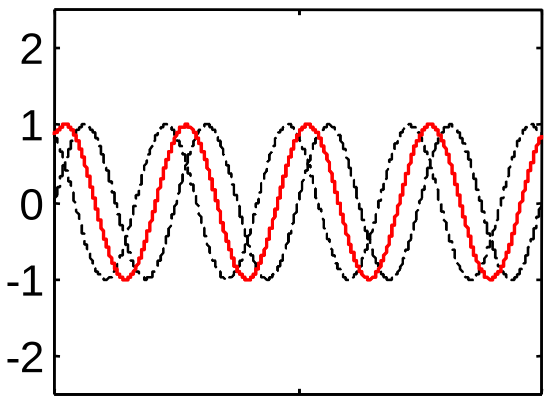


ADDING DECIBELS – COHERENT SOURCES

- ‘Coherent’ sources are directly related. The combined sound pressure level depends on the relative phase.
- Examples: components at engine firing frequency and its harmonics from the engine, intake and exhaust are coherent.
- e.g. two single frequency sources with same magnitude:

two sources **120^0 out of phase**:

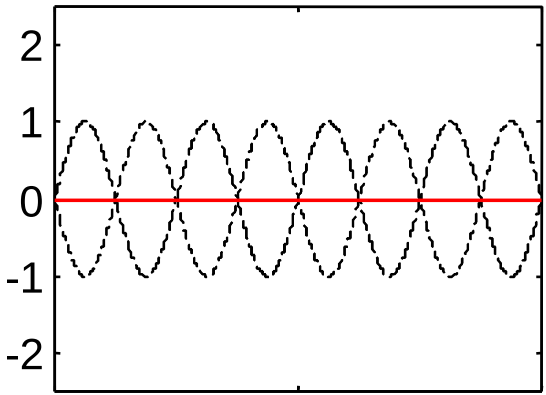
$L_p = L_1 + 0$ **dB**



ADDING DECIBELS – COHERENT SOURCES

- ‘Coherent’ sources are directly related. The combined sound pressure level depends on the relative phase.
- Examples: components at engine firing frequency and its harmonics from the engine, intake and exhaust are coherent.
- e.g. two single frequency sources with same magnitude:

two sources **180^0 out of phase**: $|p| = p_1 - p_2$ **$L_p = -\infty$ dB**



ADDING DECIBELS – INCOHERENT SOURCES

- 'Incoherent' sources are unrelated.
- e.g. sound at different frequencies, independent random signals
- In this case we add their **mean-square pressures**.

$$\overline{p^2} = \overline{p_1^2} + \overline{p_2^2}$$

- Or in decibels:

$$L_\Sigma = 10\log_{10}\left(10^{L_1/10} + 10^{L_2/10} + \ldots\right)$$

ADDING DECEIBELS - EXAMPLE

- Suppose we have two sources that by themselves produce sound pressure levels of **68** and **72** dB. What is the combined sound pressure level?

  (a) 140 dB?

  (b) 70 dB?

  (c) 73.5 dB?

  (d) 76.2 dB?

$$L_{\Sigma} = 10\log_{10}\left(10^{L_1/10} + 10^{L_2/10} + \ldots\right)$$

ADDING DECIBELS - SUMMARY

(1) *Almost all practical cases*

- sound in different frequency bands...
- sound from sources at different frequencies...
- sound from unrelated (uncorrelated) random sources...
- sound from correlated sources in terms of a band-average at high enough frequencies...

.... add the mean-*square* values

(2) *A few special cases*

- sound from the same source coming via different paths (e.g. reflections)...
- sound from sources which are directly related (correlated) i.e. come from the same process (e.g. engine firing frequency)...

.... add the pressures accounting for phase