

From nano to macro: Introduction to atomistic modeling techniques Lecture series, CEE, Fall 2005, IAP, Spring 2006

Introduction to atomistic modeling techniques: Do we need atoms to describe how materials behave?

Lecture 1



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- Introduce large-scale atomistic modeling techniques and motivate its importance for solving problems in modern engineering sciences.
- Demonstrate how atomistic modeling can be successfully applied to understand dynamical materials failure of:
 - □ Metals (Cu, Ni, Al, Fe...) and alloys (NiAl...),
 - □ Semiconductors (Si),
 - □ Thin films (of metals or other materials),
 - \Box Ceramics (Al₂O₃, SiC),
 - and biological materials (e.g. collagen) as well as natural materials (clay, C-S-H; ongoing and future studies).
- Find potential collaborations and synergies within the CEE Department and at MIT as a whole
- Target group: Undergraduate / graduate students, postdocs, faculty interested in atomistic methods and scale coupling





- Ca. 10 lectures 45-50 minutes each, with time for discussion and questions
- Clustered lectures during IAP and workshop (course 1.978, for credit)
 - □ Lectures (introduction and methods)
 - Modeling and simulation of fracture and deformation of copper (Dislocation nucleation, fracture, brittle versus ductile, comparion with theory and experiment..)
- Two UROP projects posted (fracture of silicon and modeling of collagen)
- Course material posted on the website (introductionary papers, books, etc.)

http://web.mit.edu/mbuehler/www/Teaching/LS/

Check for updates and supplementary material



- The BIG challenge to couple nano with macro
- Historical perspective: Understanding behavior of materials
- How atomistic simulations are carried out, including:
 - Definition and numerical issues
 - Time scale dilemma
 - Pre-processing and input parameters
 - Atomic interactions (potential energy surface)
 - □ Computing strategy: MD codes, parallelization, supercomputing
 - □ Analysis and visualization, data extraction
- Research examples using atomistic methods
- Discussion and conclusion: Are all atoms necessary to describe how materials behave?
- Outlook





Introduction



From nano to macro





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Chemistry

(atomic scale)





Want: Accuracy of quantum mechanics (QM) in 10²³ atom systems...

This is impossible (today and in the foreseeable future)



Historical perspective: Modeling of mechanics (behavior) of materials

Continuum

- 1500-1600s: L. da Vinci, Galileo Galilei 1700-1800: Euler, Bernoulli Beam theories, rods (partial differential equations, continuum theories)
- Continuum mechanics theories
- Development of theories of fracture mechanics, theory of dislocations (1930s)
- 1960..70s: Development of FE theories and methods (engineers)
- 1990s: Marriage of MD and FE via Quasicontinuum Method (Ortiz, Tadmor, Phillips)

- 20th century: Atoms discovered (Jean Perrin)
- MD: First introduced by Alder and Wainwright in the late 1950's (interactions of hard spheres). Many important insights concerning the behavior of simple liquids emerged from their studies.
- 1964, when Rahman carried out the first simulation using a realistic potential for liquid argon (Rahman, 1964).
- Numerical methods like DFT (Kohn-Sham, 1960s-80s)
- First molecular dynamics simulation of a realistic system was done by Rahman and Stillinger in their simulation of liquid water in 1974 (Stillinger and Rahman, 1974).
 - First fracture / crack simulations in the 1980s by Yip and others, 1990s Abraham and coworkers (large-scale MD)
- Now: MD simulations of biophysics problems, fracture, deformation are routine

Atomistic

The number of simulation techniques has greatly expanded: Many specialized techniques for particular problems, including mixed quantum mechanical - classical simulations, that are being employed to study enzymatic reactions ("QM-MM") or fracture simulations (Kaxiras and others, Buehler and Goddard).





- In atomistic simulations, the goal is to understand and model the motion of each atom in the material
- The collective behavior of the atoms allows to understand how the material undergoes deformation, phase changes or other phenomena, providing links between the atomic scale to meso/macro phenomena







 Classical MD calculates the time dependent behavior of a molecular system by integrating their equations of motion (*F*=force vector, *a*=acceleration vector)

F = ma

- The word "classical" means that the core motion of the constituent particles obeys the laws of classical mechanics
- Molecular dynamics simulations generate information at the microscopic level, which are: <u>Atomic positions, velocities, forces</u>
- The conversion of this microscopic information to macroscopic observables such as pressure, stress tensor, strain tensor, energy, heat capacities, etc., requires theories and strategies developed in the realm of <u>statistical</u> <u>mechanics</u>
- Statistical mechanics is fundamental to the study of many different atomistic systems

Important: The Ergodic hypothesis states

$$\langle A \rangle_{ensemble} = \langle A \rangle_{time}$$

Ensemble average = Time average (atomistic data usually not valid instantaneously in time and space)

C



- Verlet algorithm
- Leap-frog algorithm
- Beeman's algorithm
- Velocity Verlet (popular)

Update of positions

$$r(t+\delta t) = r(t) + v(t)\delta t + \frac{1}{2}a(t)\delta t^2$$

Update of velocities

$$v(t+\delta t) = v(t) + \frac{1}{2} \left[a(t) + a(t+\delta t) \right] \delta t$$

- Algorithms to control the temperature of a system, pressure, stress, etc. exist (e.g. Nosé-Hoover, Berendson, etc.)
- NVE, NVT, NPT calculations
- Most calculations in mechanics field are NVE (nonequilibrium phenomena such as fracture)

F (use
$$a=F/m$$
)
r, v, a
F (use $a=F/m$)
F (use $a=F/m$)



Calculate timely evolution of large number of particles (integrate using Velocity Verlet, for example)



Time scale range of MD: Picoseconds to several nanoseconds

Timescale dilemma: No matter how many processors (how powerful the computer), can only reach nanoseconds: <u>can not parallelize time</u>



- Very high strain rates in fracture or deformation (displacement km/sec)
- Limited accessibility to diffusional processes or any other slow mechanisms
- Unlike as for the scale problem (ability to treat more atoms in a system) there is no solution in sight for the time scale dilemma
- MD has to be applied very carefully while considering its range of validity (window, niche: fracture ideal, since cracks move at km/sec)
- When valid, MD is very powerful and nicely complements experiment and theory, but it has limitations which need to be understood

km/sec		yes	yes w/ limitations	no
	Fracture in model materials	×		
	Fracture in real materials		×	
	GB diffusion at high temperatures		×	
	GB diffusion at low temperatures			×
	Plasticity in model materials	×		
	Plasticity in real materials		×	
		1	1	(Buehler, 20

http://www.fz-juelich.de/nic-series/volume23/frenkel.pdf See also article by Art Voter *et al.* on the time scale dilemma





- Monte Carlo (MC) techniques and alike have been developed to overcome some of the limitations of dynamical (MD) atomistic calculations
- Instead of integrating the EOM, MC performs a random walk to measure properties: Randomly probing the geometry of the molecular system (configuration space, acceptance depends on "cost function")
- MC enables modeling of diffusion and other "slow" processes (slow compared to the time scale of atomic vibrations)
- There exist many different flavors, including
 - Classical MC (no information about dynamics, only about mechanisms and steady state properties, e.g. thermodynamical variables)
 - □ Kinetic MC (get information about dynamics)
 - □ Advanced MD methods (marriage between MC and MD, e.g. Temp. Acc. Dyn.)
 - Bias potentials (e.g. restraints) to facilitate specific events by reducing the barriers
- Generally, MC techniques require more knowledge about the system of interest than MD

http://www.fz-juelich.de/nic-series/volume23/frenkel.pdf

D. Frenkel and B. Smit Understanding Molecular Simulations: from Algorithms to Applications, Academic Press, San Diego, 2nd edition (2002).

http://www.ccl.net/cca/documents/molecular-modeling/node9.html



Example: Measuring the average depth of the Charles River





Classical grid-based quadrature scheme:

Discretize problem and perform measurements at grid points



Monte Carlo:

Perform random walk through the river; measurements are performed only at accepted locations





- Atomistic or molecular simulations (molecular dynamics, MD) is a <u>fundamental approach</u>, since it considers the basic building blocks of materials as its smallest entity: <u>Atoms</u>
- At the same, time, molecular dynamics simulations allow to model materials with dimensions of several hundred nanometers and beyond: Allows to study deformation and properties, mechanisms etc. with a very detailed "computational microscope", thus <u>bridging through various scales</u> from "nano" to "macro" possible by DNS
- Sometimes, MD has been referred to as a "first principles approach to understand the mechanics of materials" (e.g. dislocations are "made" out of atoms...)
- With the <u>definition of the interatomic potentials</u> (how atoms interact) all materials properties are defined (endless possibilities & challenges...)





DFT or Empirical or Semi-empirical...

> http://www.sr.bham.ac.uk/x mm/images/structures/spher espring_300_248.jpg

First principles description of mechanics: Dislocations carry plasticity in metals













- The fundamental input into molecular simulations, in addition to structural information (position of atoms, type of atoms and their velocities/accelerations) is provided by definition of the interaction potential (equiv. terms often used by chemists is "force field")
- MD is very general due to its formulation, but hard to find a "good" potential (extensive debate still ongoing, choice depends very strongly on the application)
- Popular: <u>Semi-empirical or empirical</u> (fit of carefully chosen mathematical functions to reproduce the energy surface...)

Parameters



Lennard-Jones

$$\phi(r) = 4\epsilon_0 \left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^6 \right]$$

Or more sophisticated potentials (multi-body potentials EMT, EAM, TB...) 6

Training of Force Fields: Hydrocarbon-Pt interactions





ReaxFF can describe different C-Pt bonding modes



Challenge: Coupling of various scales From QM to Macroscale







Example: Potentials for metals



$$\phi_{i}(r) = \sum_{j=1}^{N_{i}} \phi_{ij}(r_{ij}) \qquad \begin{array}{l} \text{http://phycomp.technion.ac.il/~phycomp.technion.a$$

Pair potentials Good for gases, but don't describe metallic bonding well

Lennard-Jones 12-6

 $C_{\mathrm{L2}} \neq C_{44}$

$$\phi_{ij}(r_{ij}) = D\{1 - \exp[-\beta(r_{ij} - r_0)]\}^2$$

Morse



Quality varies: Good for copper, nickel, to some extend for aluminum ...

M. S. Daw and M. I. Baskes, Phys. Rev. B **29**, 6443 (1984); S. M. Foiles, M. I. Baskes, and M. S. Daw, Phys. Rev. B **33**, 1986. M. W. Finnis and J. E. Sinclair, Philos. Mag. A **50**, 45 (1984).

K. W. Jacobsen, J. K. Nørskov and M. J. Puska, Phys. Rev. B 35, 7423 (1987).

Nano-meso-macro transition: Biopolymer



Fully atomistic (MD) (based on QM)

Plasticity,... of single fiber assemblies (cross-links)



Mesoscale (parametrization) Long-range, short-range

(Buehler, to be published)

+H₂O skin

Nano-meso-macro transition: Biopolymer



time

Quantum mechanic

(first principles)

Concurrent coupling

length

FF training

FF training

to leave out & what information to transport (and how)?



Interatomic potential concepts, materials and simulation codes



QM (not much material specific): DFT (electronic structure information), codes: JAGUAR, GAUSSIAN, GAMES, CPMD...

Electron FF: Electrons as particles (Gaussians moving according to classical EOMs), codes: CMDF

Tight binding: Orbitals, semi-empirical, has fitting parameter obtained from QM (codes: EZTB and many more)

ReaxFF: Bridge between QM and empirical FFs (charge flow)

EAM: Metals, alloys; semi-empirical expressions (QM derived); Codes: IMD, LAMMPS, XMD and many others

MEAM: Silicon, metals and other covalently dominated materials (codes: IMD, CMDF)

Tersoff: Bond order potentials (covalent systems), simple

Organic force fields (harmonic): Proteins, organics etc., CHARMM, DREIDING, AMBER (codes: NAMD, GROMACS, CHARMM...) Pair potentials: Noble gases (Ar) or model materials

Less accuracy does not mean less science can be done

computational effort

.⊆

ecrease

C

Concurrent versus hierarchical multi-scale simulations





"finer scales train coarser scales"







Combine atomistic regions embedded in continuum region

(Buehler et al., 2006)



Virial stress:

$$\sigma_{ij} = \frac{1}{2} \sum_{\alpha,\beta} \left(-\frac{1}{r} \frac{\partial \phi}{\partial r} r_i r_j \mid_{r=r_{\alpha\beta}} \right)$$

where r_i is the projection of the interatomic distance vector **r** along coordinate *i*.

- We only consider the force part, excluding the part containing the effect of the velocity of atoms (the kinetic part).
- It was recently shown by Zhou *et al.* that the virial stress including the kinetic contribution is not equivalent to the mechanical Cauchy stress.
- The virial stress needs to be averaged over space and time to converge to the Cauchy stress tensor.

D.H. Tsai. Virial theorem and stress calculation in molecular-dynamics. J. of Chemical Physics, 70(3):1375–1382, 1979.

Min Zhou, A new look at the atomic level virial stress: on continuum-molecular system equivalence, Royal Society of London Proceedings Series A, vol. 459, Issue 2037, pp.2347-2392 (2003)

Jonathan Zimmerman *et al.*, Calculation of stress in atomistic simulation, MSMSE, Vol. 12, pp. S319-S332 (2004) and references in those articles by Yip, Cheung, . © 2005 Markus J. Buehler, CEE/MIT





Atomic virial strain

$$q_{ij}^{l} = \frac{1}{N} \sum_{k=1}^{N} \left(\frac{\Delta x_{i}^{kl} \Delta x_{j}^{kl}}{r_{0}^{2}} \right) \qquad b_{ij}^{l} = \frac{N}{\lambda} q_{ij}^{l} = \frac{1}{\lambda} \sum_{k=1}^{N} \left(\frac{\Delta x_{i}^{kl} \Delta x_{j}^{kl}}{r_{0}^{2}} \right)$$

• The strain field is a measure of geometric deformation of the atomic lattice

- The local atomic strain is calculated by comparing the local deviation of the lattice from a reference configuration.
- Usually, the reference configuration is taken to be the undeformed lattice.
- In the atomistic simulations, the information about the position of every atom is readily available, either in the current or in the reference configuration and thus calculation of the virial strain is relatively straightforward.
- Unlike the virial stress, the atomic strain is valid instantaneously in space and time. However, the expression is only strictly applicable away from surfaces and interfaces.

Jonathan Zimmerman, Continuum and atomistic modeling of dislocation nucleation at crystal surface ledges. PhD Thesis, Stanford University, 1999. © 2005 Markus J. Buehler, CEE/MIT

O Stress versus strain from atomistics...



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Computation and numerical issues

Typical simulation procedure

- Pre-processing (define geometry, build crystal etc.)
- 2. Energy relaxation (minimization)
- 3. Annealing (equilibration at specific temperature)
- 4. "Actual" calculation; e.g. apply loading to crack
- 5. Analysis







(Buehler et al., to appear 2006)

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C TOP500 List for November 2004



http://www.top500.org







Concept:

Divide the workload

No immediate long range interaction (only via dynamics)

- Each CPU is responsible for part of the problem
- Atoms can move into other CPUs (migration)
- Need to know topology or the geometric environment on other CPUs (green region)





- The strength of MD is not its predictive power (time scale limitations...)
- Rather use it in a differential way
- Hypothesis: MD only gives relative differential information
- Consequence: No quantitative number but only slope and thus additional integration needed to make information useful, use <u>model systems</u>





- Use MD methods to perform virtual experiments
- Computational microscope
- As long as valid, ideal method to gain fundamental understanding about behavior of materials
- Have intrinsic length scale given by the atomic scale (distance)
- Handles stress singularities intrinsically
- Ideal for deformation under high strain rate etc., not accessible by other methods (FE, DDD..)

Experimental verification of intersonic cracking

- Mike Marder's group at Univ. of Texas verified the phenomenon of intersonic cracking in a hyperelastic stiffening material (PRL, 2004)
- Agreement and confirmation of our theoretical predictions

Cracks in Rubber Propagate Faster than the Speed of Sound

Since the classical work by Griffith, Inglis, and Irwin on the physics of cracking, one of the most fundamental questions associated with crack dynamics is the maximum speed that cracks can propagate. Depending on the type of loading (e.g., tensile, shear, or antiplane shear), there is a unique maximum speed cracks can achieve. For tensile-loaded cracks, theory predicts that this limiting speed is the Rayleigh wave speed, the speed of elastic waves on a surface. Recent theoretical work, including atomistic simulations, has challenged this classical view. Now, P.J. Petersan and co-workers from the University of Texas at Austin have shown experimentally that tensileloaded cracks in rubber can actually propagate faster than the Rayleigh wave speed and even break the sound barrier.

As reported in the July issue of *Physical Review Letters* (105504), Petersan and colleagues identified the intersonic crack speed by the observation of shock fronts near the crack tip by high-speed photogra-

MRS BULLETIN/OCTOBER 2004





Multiple-exposure photograph of a crack propagating in a rubber sample ($\lambda_x = 1.2, \lambda_y = 2.4$); speed of the crack, ~56 m/s (Petersan *et al.*).

nuch up by high speed photogra





Some example applications



How "stuff" deforms?







Ductile versus brittle materials











- Large-scale atomistic models with up to 70,000,000 atoms
- Simple model potential (next slide)



Strain field close to cracks



v/c_r=1



$$\sigma_{ij}(\Theta, v) = \frac{K_I(t, v)}{\sqrt{2\pi r}} \Sigma_{ij}(\Theta, v) + \sigma_{ij}^{(1)} + O(1)$$

(e.g. Freund, 1990)

Result: Reasonable agreement

(Buehler, Gao, Huang; Theor. Appl. Fracture Mechanics, 2004)



Stress field close to cracks





Shear stress near a crack at a bimaterial interface (Buehler *et al.*, 2006)



Hoop stress near a moving crack (Buehler and Gao, Nature, to appear)



Increase in computing power: Parallelization





5 µm



A simulation with 1,000,000,000 particles



LJ potential... (simple interaction but VERY complex behavior!!)

(Abraham et a., 2002, Buehler, Hartmaier et al., 2004)



LJ in 2D... model system for brittle material





Why 30% versus 73%?

Does hyperelasticity play the governing role?

Attempts of explanation:

- Yoffe (linear elasticity, 1951)
- Gao (purely hyperelastic, 1996)
- Marder (lattice models, 1992-2000)
- Abraham (lattice vibration, 1994)

Critical speed for onset of surface roughening=instability speed

Dynamical crack tip instabilities





⁽Buehler and Gao, Nature, 2005 (to appear))

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Supersonic interface cracking





• Shear dominated loading

(Buehler et al., JCIE, 2005)

• Bimaterial interface (upper part: stiff, lower part: soft)

Cross-scale interactions: Brittle fracture



In brittle fracture, the macroscopic behavior of the materials depends on its underlying atomic interaction across several hierarchies of scales

Concurrent multi-scale simulations



Concurrent integration of various scales and paradigms





• Concurrent FE-atomistic-ReaxFF scheme in a crack problem (crack tip treated by ReaxFF) and an interface problem (interface treated by ReaxFF).

• Highlighted transition regions as handshake domains between different scale and methods.



Example for code coupling: Concept of mixed Hamiltonian





 Schematic showing the coupling of reactive and nonreactive potentials

• The simpler nonreactive potential is trained to resemble the reactive potential for small deviations from the equilibrium configuration.

Agreement at small deformation: LINK



Mixed Hamiltonians





Cracking in Silicon: Model within CMDF

- To model cracking in Silicon efficiently, we developed a multi-paradigm scheme that combines the Tersoff potential and ReaxFF
- The ReaxFF region is moving with the crack tip (region determined based on local atomic strain)

New hybrid scheme within CMDF

(110) crack surface, 10 % strain

Reactive region is moving with crack tip



C

(a)

(c)

Oxidation versus brittle fracture Including complex chemistry





Crack dynamics in silicon without (subplots (a) and (c)) and with oxygen molecules present (subplots (b) and (d))

Subplots (a) and (b) show the results for 5 percent applied strain, whereas subplots (c) and (d) show the results

for 10 percent applied strain.

• The systems contain 13,000 atoms and $Lx \approx 160$ Å and $Ly \approx 310$ Å.





Diffusion of H on Pt



TAD low temperature 400 K (high sampling temperature 1300 K)





Diffusion of H on Pt









Summary and wrap-up

O we need atoms to describe how materials behave?



Atomic details needed for some applications and situations, including:

- Small-scale materials: Miniaturization as a new engineering frontier and potential (nanomaterials and small-scale structures)
 - □ Thin films, IC technology
 - Basis for modern technologies: Coatings
 - □ New metals, alloys, composites, including structural applications
- Interfaces between dissimilar materials (living systems and technologies, bio-chips or N/MEMS)

"Interfacial materials" (incl. nanomaterials)

- Quantum effects, confinement, size effects: Now important for engineers and exploited for technologies
- Thus: MD may play a critical role as engineering tool ("new" engineers trained in physics, chemistry, biology etc. and the intersections of various scientific disciplines)



Size effects in materials



This helps to define novel machine and materials design principles

C Linkage of experiment-theory-simulation

✓ Atomistic simulations is an increasingly important tool in materials science; it can be used to...

- Advance theory and discover new physical phenomena
- Augment and explain experiment

✓ With its **limitations** understood, MD simulation is an ideal tool to study small-scale dynamics materials phenomena; gain insight into mechanisms





Scales covered at CEE: From nano to macro to help understand the "world"









- At CEE, we use a holistic approach to understand the scientific concepts "how" the world works
- A key focus is the "system perspective" and integration of dissimilar hierarchies of materials, methods, and interactions of technologyhuman/society
- Genuine interest in multi-scale phenomena and their modeling, experimental investigation and understanding
- To develop deep understanding of scale problems we need different perspectives and views, including nano-view (atomistic), systems perspective, macroscale properties and many others
- This involves a variety of numerical, theoretical and experimental approaches across scales and disciplines, including atomistic and mesoscale simulations
- Helps to understand the similarities in behavior across disciplines and across the scales for development of new engineering concepts





Fall 2005

- Oct. 27, 1 PM, Room 1-134: Introduction to atomistic modeling techniques: Do we need atoms to describe how materials behave?
- Nov. 3, 1 PM, Room 1-134: Methods and techniques for modeling metals and their alloys and application to the mechanics of thin metal films
- Nov. 17, 1 PM, Room 1-134: Scale coupling techniques: From nano to macro
- Dec. 5, 1 PM, Room 1-150: Reactive versus nonreactive potentials: Towards unifying chemistry and mechanics in organic and inorganic systems

IAP 2006: From nano to macro: Introduction to atomistic modeling techniques and application in a case study of modeling fracture of copper (1.978 PDF)

- Jan. 9 (Monday): Introduction to classical molecular dynamics: Brittle versus ductile materials behavior
- Jan. 11 (Wednesday): Deformation of ductile ma terials like metals using billion-atom simulations with massively parallelized computing techniques
- Jan. 13 (Friday): Dynamic fracture of brittle materials: How nonlinear elasticity and geometric confinement governs crack dynamics
- Jan. 16 (Monday): Size effects in deformation of materials: Smaller is stronger
- Jan. 18 (Wednesday): Introduction to the problem set: Atomistic modeling of fracture of copper
- The IAP activity can be taken for credit. Both undergraduate and graduate level students are welcome to participate. Details will be posted on the IAP website (<u>http://web.mit.edu/iap/</u>).

Spring 2006

- TBD. Atomistic modeling of biological and natural materials: Mechanics of protein crystals and collagen
- TBD. Mechanical properties of carbon nanotubes: Scale effects and self-folding mechanisms
- TBD. Atomistic and multi-scale modeling in civil and environmental engineering: Current status and future development

http://web.mit.edu/mbuehler/www/Teaching/LS/





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- B. deCelis, A.S. Argon, and S. Yip. Molecular-dynamics simulation of crack tip processes in alpha-iron and copper. J. Appl. Phys., 54(9):4864–4878, 1983.
- See additional references & material on the website: <u>http://web.mit.edu/mbuehler/www/Teaching/LS/lecture-1-supp.htm</u>