Introduction to Laser Doppler Velocimetry

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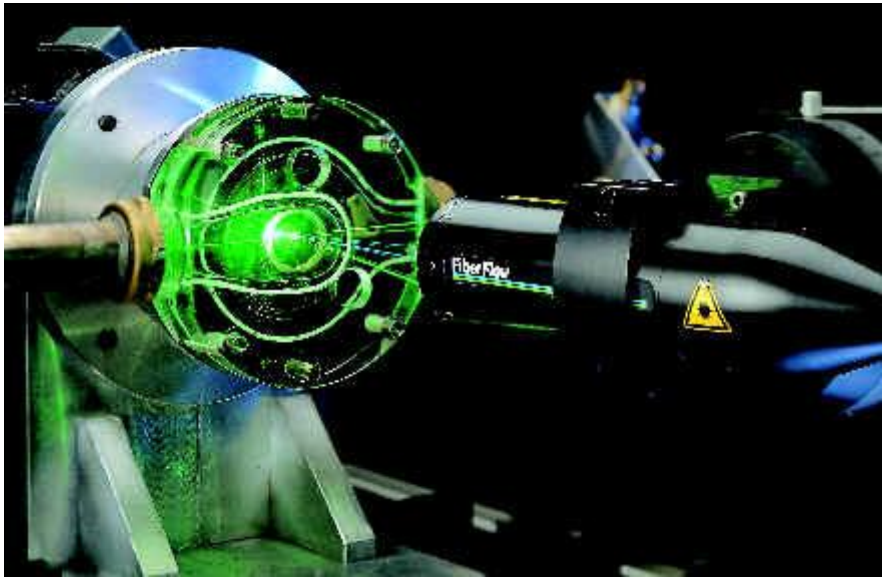
Laser Doppler Anemometry (LDA)

- **Single-point optical velocimetry method**

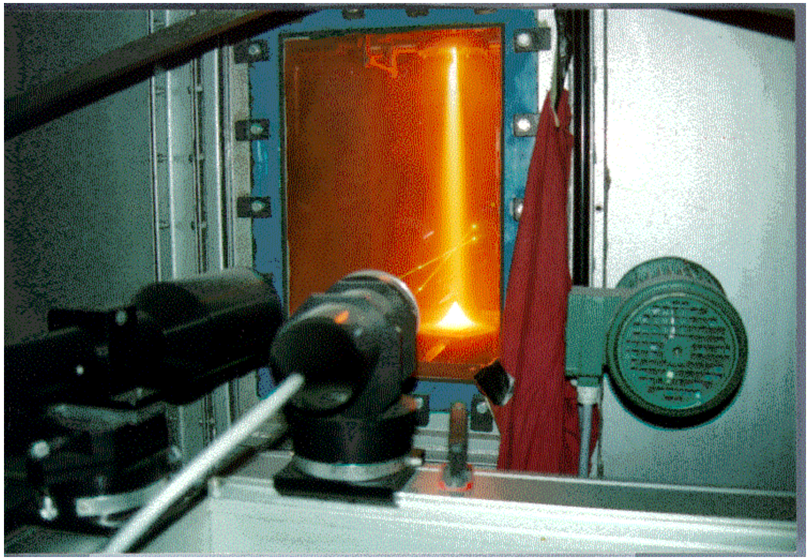


Study of the flow between rotating impeller blades of a pump

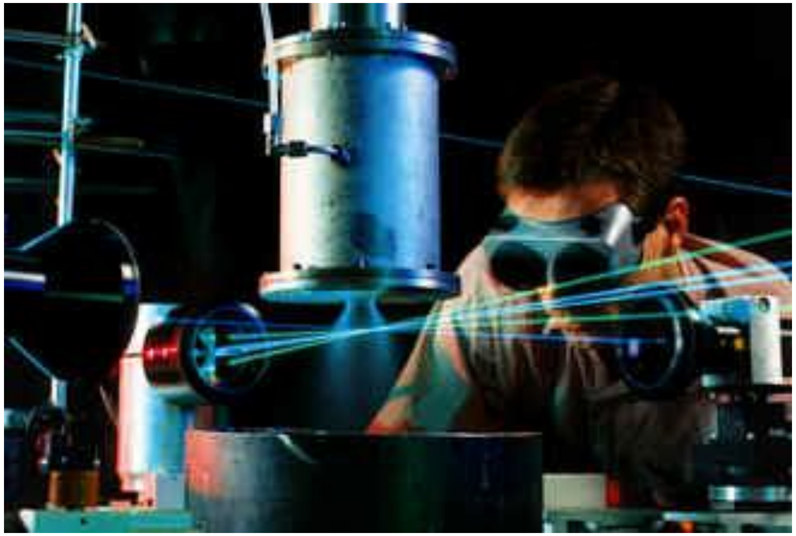
3-D LDA Measurements on a 1:5 Mercedes-Benz E-class model car in wind tunnel

Phase Doppler Anemometry (PDA)

- **Single point particle sizing/velocimetry method**



Drop Size and Velocity measurements in an atomized Stream of Moleten Metal

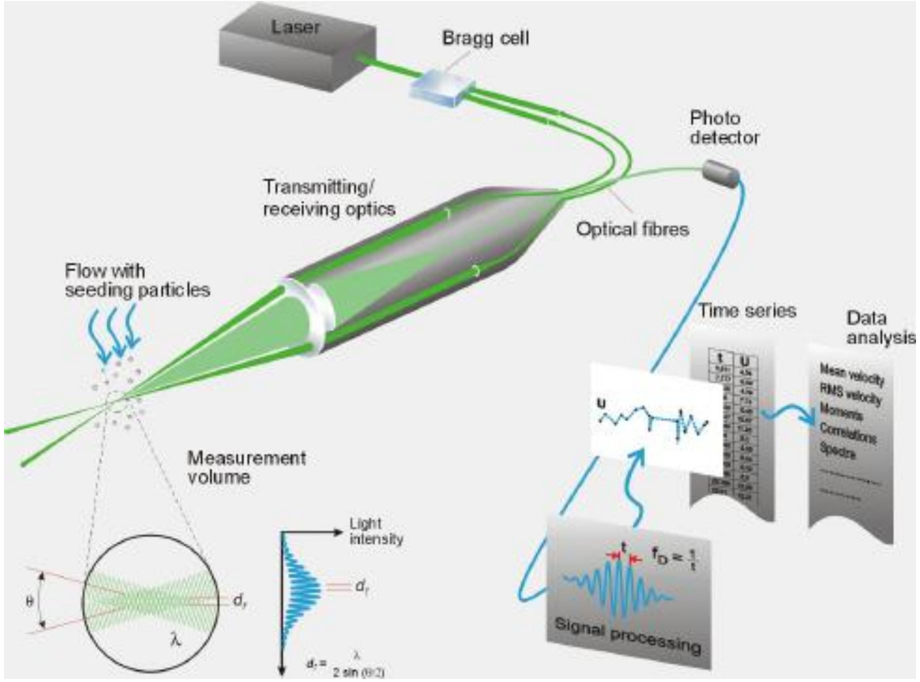


Droplet Size Distributions Measured in a Kerosene Spray Produced by a Fuel Injector

Laser Doppler Anemometry

- **LDA**
  - A high resolution - single point technique for velocity measurements in turbulent flows

A Back Scatter LDA System for One Velocity Component Measurement (Dantec Dynamics)



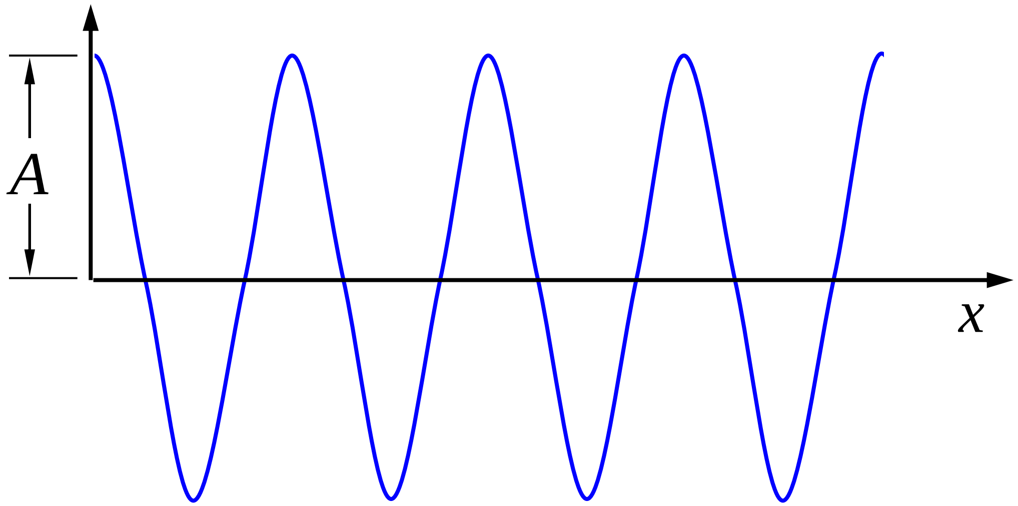
  - Basics
    - Seed flow with small tracer particles
    - Illuminate flow with one or more coherent, polarized laser beams to form a MV
    - Receive scattered light from particles passing through MV and interfere with additional light sources
    - Measurement of the resultant light intensity frequency is related to particle velocity

LDA in a nutshell

- **Benefits**
  - Essentially non-intrusive
  - Hostile environments
  - Very accurate
  - No calibration
  - High data rates
  - Good spatial & temporal resolution

- **Limitations**
  - Expensive equipment
  - Flow must be seeded with particles if none naturally exist
  - Single point measurement technique
  - Can be difficult to collect data very near walls

Review of Wave Characteristics



- **General wave propagation**

$$\psi(x,t) = A\cos\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{\tau}\right)\right] \qquad \psi(x,t) = \mathrm{Re}\left\{Ae^{i[kx-\omega t+\varepsilon]}\right\}$$

A = Amplitude
k = wavenumber
x = spatial coordinate
t = time
ω = angular frequency
ε = phase

$$k = \frac{2\pi}{\lambda} \qquad \tau = \frac{\lambda}{c} \Rightarrow \omega = \frac{2\pi}{\tau} = \frac{2\pi c}{\lambda}$$

Electromagnetic waves: coherence

- **Light is emitted in "wavetrains"**
  - Short duration, Δt
  - Corresponding phase shift, $\varepsilon(t)$; where ε may vary on scale $t > \Delta t$

$$\mathbf{E} = \mathbf{E}_o \exp\left[i\left(kx - \omega t + \varepsilon(t)\right)\right]$$

- **Light is *coherent* when the phase remains constant for a sufficiently long time**
  - Typical duration (Δt_c) and equivalent propagation length (Δl_c) over which some sources remain coherent are:

Source	λ_{nom} (nm)	Δl_c
White light	550	8 μm
Mercury Arc	546	0.3 mm
Kr^{86} discharge lamp	606	0.3 m
Stabilized He-Ne laser	633	$\leq$ 400 m

  - Interferometry is only practical with coherent light sources

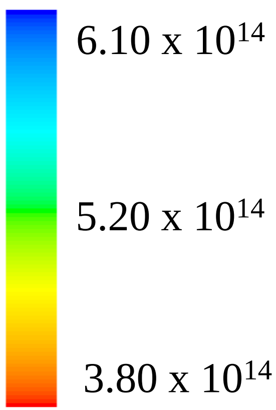
Electromagnetic waves: irradiance

- **Instantaneous power density given by Poynting vector**
  - Units of Energy/(Area-Time)

$$\mathbf{S} = c^2 \varepsilon_o \mathbf{E} \times \mathbf{B} \qquad S = c\varepsilon_o E^2$$

- **More useful: average over times longer than light freq.**

Frequency Range

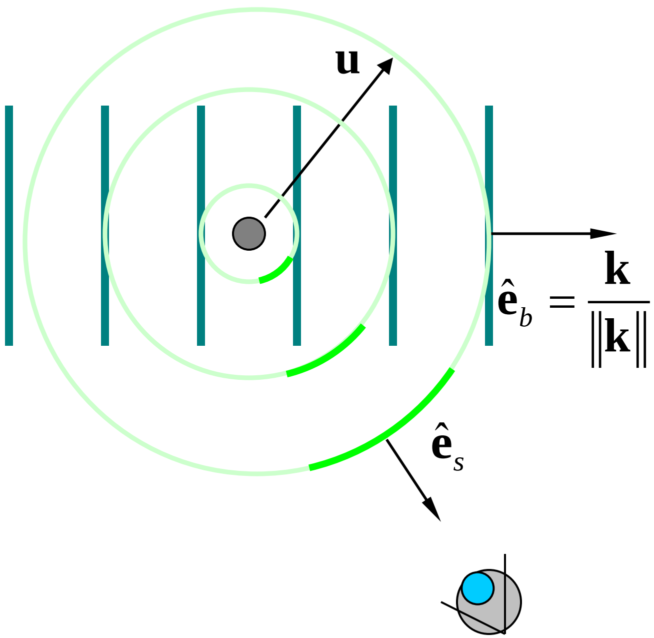


$$\langle f \rangle_T = \frac{1}{T} \int_{t-T/2}^{t+T/2} f(t')\,dt'$$

$$I = \langle S \rangle_T = c\varepsilon_o \langle E^2 \rangle_T = \frac{c\varepsilon_o}{2} \mathbf{E} \cdot \mathbf{E}^* = \frac{c\varepsilon_o}{2} E_0^2$$

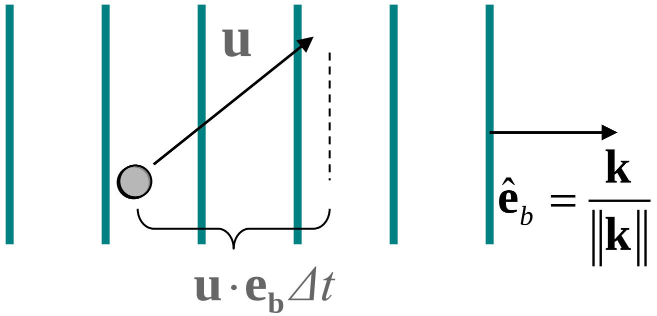
LDA: Doppler effect frequency shift

- **Overall Doppler shift due two separate changes**
  - The particle 'sees' a shift in incident light frequency due to particle motion
  - Scattered light from particle to stationary detector is shifted due to particle motion



LDA: Doppler shift, effect I

- **Frequency Observed by Particle**
  - The first shift can itself be split into two effects
    - (a) the number of wavefronts the particle passes in a time Δt, as though the waves were stationary…

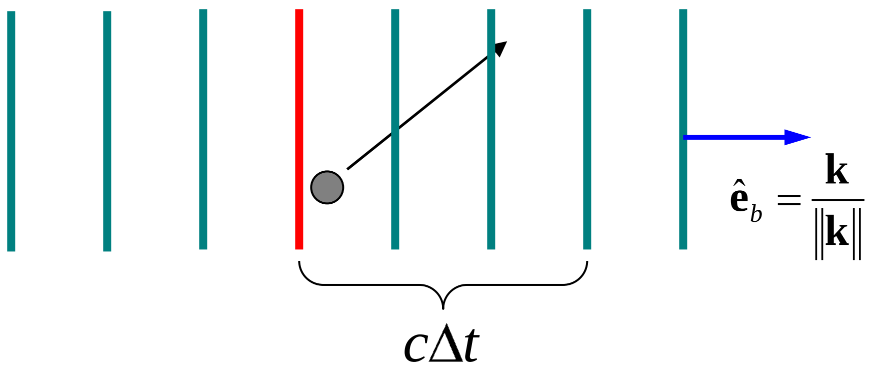


Number of wavefronts particle passes during Δt due to particle velocity:

$$\frac{\mathbf{u} \cdot \hat{\mathbf{e}}_b \Delta t}{\lambda}$$

LDA: Doppler shift, effect I

- **Frequency Observed by Particle**
  - The first shift can itself be split into two effects
    - (b) the number of wavefronts passing a stationary particle position over the same duration, Δt…



Number of wavefronts that pass a stationary particle during Δt due to the wavefront velocity: $\frac{c\Delta t}{\lambda}$

LDA: Doppler shift, effect I

- **The net effect due to a moving observer w/ a stationary source is then the difference:**

Number of wavefronts that pass a moving particle during Δt due to combined velocity (same as using relative velocity in particle frame):

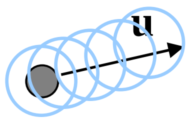
$$\frac{c\Delta t}{\lambda} - \frac{\mathbf{u}\cdot\hat{\mathbf{e}}_b \Delta t}{\lambda}$$

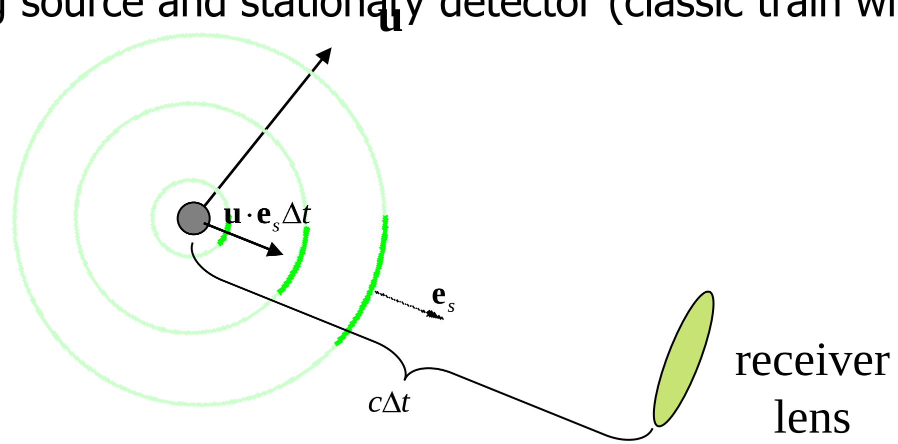
Net frequency observed by moving particle

$$f_p = \frac{\#\text{ of wavefronts}}{\Delta t} = \frac{c}{\lambda}\left(1 - \frac{\mathbf{u}\cdot\hat{\mathbf{e}}_b}{c}\right) = f_0\left(1 - \frac{\mathbf{u}\cdot\hat{\mathbf{e}}_b}{c}\right)$$

LDA: Doppler shift, effect II

- **An additional shift happens when the light gets scattered by the particle and is observed by the detector**
  - This is the case of a moving source and stationary detector (classic train whistle problem)





Distance a scattered wave front would travel during Δt in the direction of detector, if **u** were 0:

$$c\Delta t$$

Due to source motion, the distance is changed by an amount:

$$\mathbf{u} \cdot \hat{\mathbf{e}}_s \Delta t$$

Therefore, the effective scattered wavelength is:

$$\lambda_s = \frac{\text{net distance traveled by wave}}{\text{number of waves emitted}} = \frac{c\Delta t - \mathbf{u} \cdot \hat{\mathbf{e}}_s \Delta t}{f_p \Delta t} = \frac{c - \mathbf{u} \cdot \hat{\mathbf{e}}_s}{f_p}$$

LDA: Doppler shift, I & II combined

- **Combining the two effects gives:**

$$f_{obs} = \frac{c}{\lambda_s} = \frac{c f_p}{c - \mathbf{u}\cdot\hat{\mathbf{e}}_s} = \frac{f_p}{\left(1 - \frac{\mathbf{u}\cdot\hat{\mathbf{e}}_s}{c}\right)} = f_0 \frac{\left(1 - \frac{\mathbf{u}\cdot\hat{\mathbf{e}}_b}{c}\right)}{\left(1 - \frac{\mathbf{u}\cdot\hat{\mathbf{e}}_s}{c}\right)}$$

- **For u << c, we can approximate**

$$\begin{aligned} f_{obs} &= f_0\left(1 - \frac{\mathbf{u}\cdot\hat{\mathbf{e}}_b}{c}\right)\left(1 - \frac{\mathbf{u}\cdot\hat{\mathbf{e}}_s}{c}\right)^{-1} \\ &= f_0\left(1 - \frac{\mathbf{u}\cdot\hat{\mathbf{e}}_b}{c}\right)\left[1 + \frac{\mathbf{u}\cdot\hat{\mathbf{e}}_s}{c} - \left(\frac{\mathbf{u}\cdot\hat{\mathbf{e}}_s}{c}\right)^2 + \cdots\right] \\ &= f_0\left(1 + \frac{1}{c}\mathbf{u}\cdot(\hat{\mathbf{e}}_s - \hat{\mathbf{e}}_b) + \cdots\right) \\ &\cong f_0 + \frac{f_0}{c}\mathbf{u}\cdot(\hat{\mathbf{e}}_s - \hat{\mathbf{e}}_b) \end{aligned}$$

LDA: problem with single source/detector

- **Single beam frequency shift depends on:**
  - velocity magnitude
  - Velocity direction
  - observation angle

$$f_{obs} \cong f_0 + \frac{f_0}{c}\mathbf{u}\cdot\left(\hat{\mathbf{e}}_s - \hat{\mathbf{e}}_b\right)$$

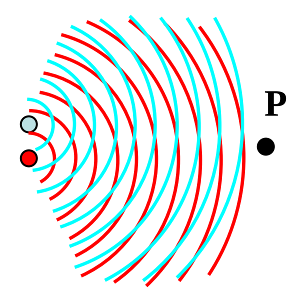
- **Additionally, base frequency is quite high…**
  - O[10^{14}] Hz, making direct detection quite difficult

- **Solution?**
  - Optical heterodyne
    - Use interference of two beams or two detectors to create a "beating" effect, like two slightly out of tune guitar strings, e.g.

$$\cos[\omega_1 t]\cos[\omega_2 t] = \frac{1}{2}\left(\cos[(\omega_1+\omega_2)t] + \cos[(\omega_1-\omega_2)t]\right)$$

  - Need to repeat for optical waves

$$\mathbf{E}_1 = \mathbf{E}_{o1}\cos\left(\mathbf{k}_1\cdot\mathbf{r} - \omega_1 t\right)$$
$$\mathbf{E}_2 = \mathbf{E}_{o2}\cos\left(\mathbf{k}_2\cdot\mathbf{r} - \omega_2 t\right)$$



Optical Heterodyne

- **Repeat, but allow for different frequencies…**

$$\mathbf{E}_1 = \mathbf{E}_{01}\exp\left[i\left(k_1 x - \omega_1 t + \varepsilon_1\right)\right] = \mathbf{E}_{01}\exp\left[i\phi_1\right]$$

$$\mathbf{E}_2 = \mathbf{E}_{02}\exp\left[i\left(k_2 x - \omega_2 t + \varepsilon_2\right)\right] = \mathbf{E}_{02}\exp\left[i\phi_2\right]$$

$$I = \frac{c\varepsilon_o}{2}\left(\mathbf{E}_1 + \mathbf{E}_2\right)\cdot\left(\mathbf{E}_1^* + \mathbf{E}_2^*\right)$$

$$I = \frac{c\varepsilon_o}{2}\left[E_{o1}^2 + E_{o2}^2 + E_{01}\exp(i\phi_1)E_{02}\exp(-i\phi_2) + E_{01}\exp(-i\phi_1)E_{02}\exp(i\phi_2)\right]$$

$$I = \frac{c\varepsilon_o}{2}\left[E_{o1}^2 + E_{o2}^2 + 2E_{01}E_{02}\left\{\frac{\exp(i(\phi_1 - \phi_2)) + \exp(-i(\phi_1 - \phi_2))}{2}\right\}\right]$$

$$I = \frac{c\varepsilon_o}{2}\left[E_{o1}^2 + E_{o2}^2 + 2E_{01}E_{02}\cos(\phi_1 - \phi_2)\right]$$

$$I = \frac{c\varepsilon_o}{2}\left[E_{o1}^2 + E_{o2}^2 + 2E_{o1}E_{o2}\cos\left[(\mathbf{k}_1 - \mathbf{k}_2)\cdot\mathbf{r} - (\omega_1 - \omega_2)t + (\varepsilon_1 - \varepsilon_2)\right]\right]$$

$$= \frac{1}{2}\Big[\underbrace{I_{o1} + I_{o2}}_{I_{PED}} + \underbrace{2\sqrt{I_{o1}I_{o2}}}_{I_{AC}}\cos\left[(\mathbf{k}_1 - \mathbf{k}_2)\cdot\mathbf{r} - (\omega_1 - \omega_2)t + (\varepsilon_1 - \varepsilon_2)\right]\Big]$$

How do you get different scatter frequencies?

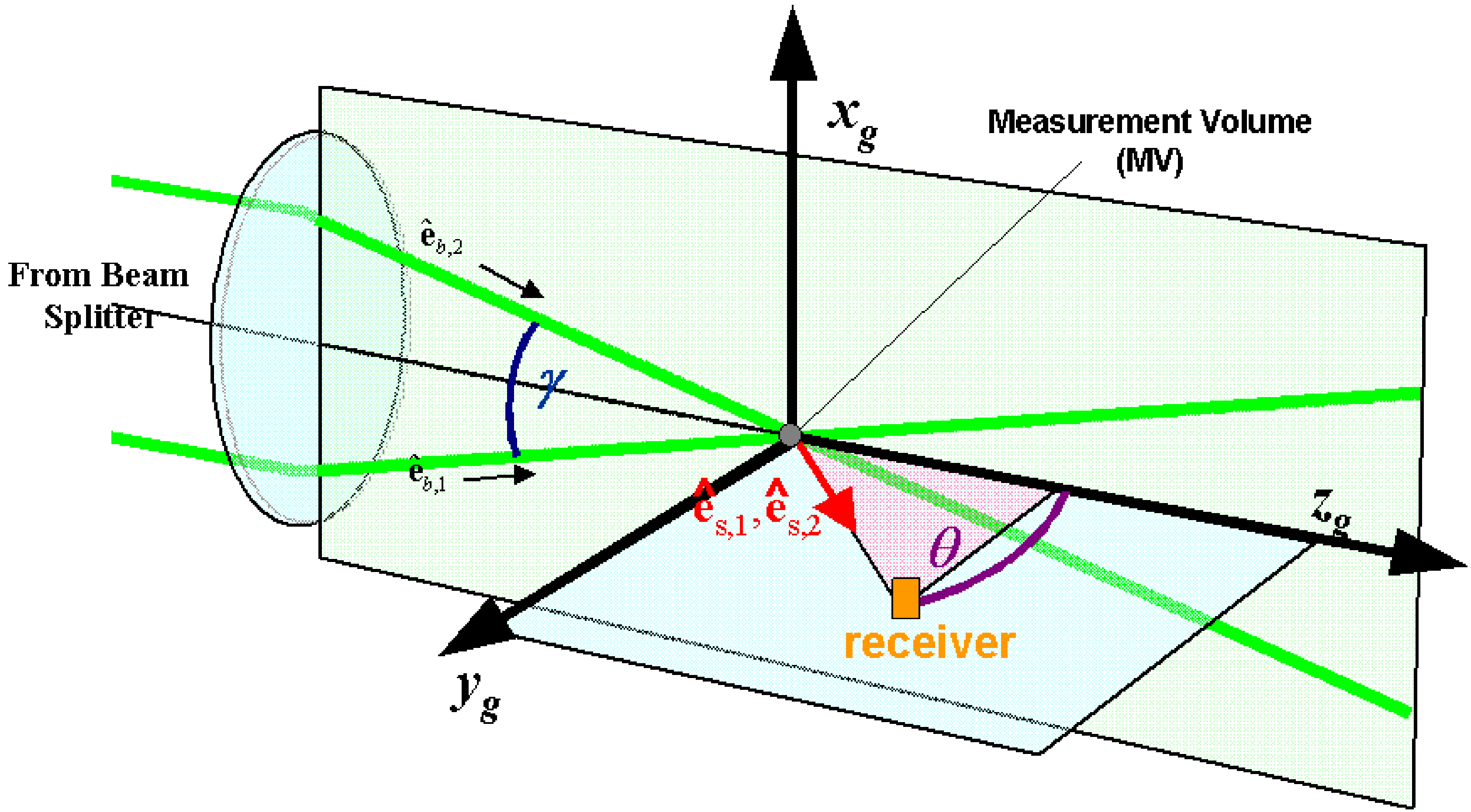
- **For a single beam**

$$f_s \cong f_0 + \frac{f_0}{c}\mathbf{u}\cdot\left(\hat{\mathbf{e}}_s - \hat{\mathbf{e}}_b\right)$$

  – Frequency depends on directions of e_s and e_b

- **Three common methods have been used**
  – Reference beam mode (single scatter and single beam)
  – Single-beam, dual scatter (two observation angles)
  – Dual beam (two incident beams, single observation location)

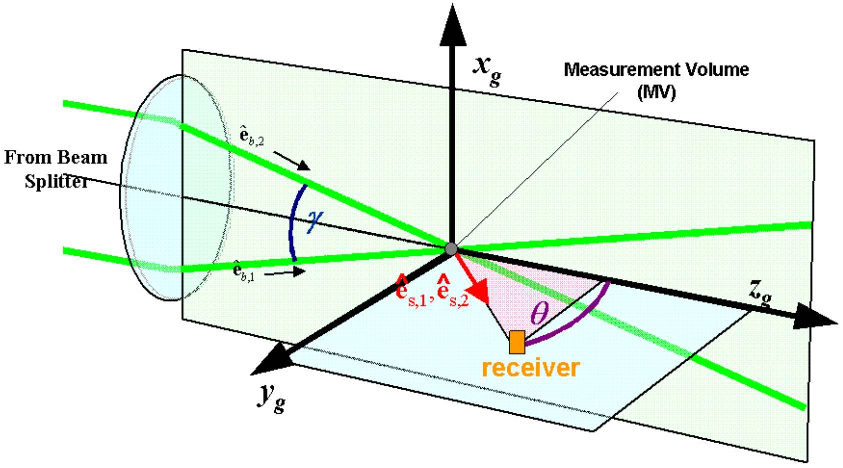
Dual beam method



Real MV formed by two beams
Beam crossing angle γ
Scattering angle θ

'Forward' Scatter Configuration

Dual beam method (cont)



$$f_{s,1} = f_0 + \frac{f_0}{c}\mathbf{u}\cdot\left(\hat{\mathbf{e}}_{s,1} - \hat{\mathbf{e}}_{b,1}\right)$$

$$f_{s,2} = f_0 + \frac{f_0}{c}\mathbf{u}\cdot\left(\hat{\mathbf{e}}_{s,2} - \hat{\mathbf{e}}_{b,2}\right)$$

$$\therefore \Delta f = \frac{f_0}{c}\mathbf{u}\cdot\left(\hat{\mathbf{e}}_{b,2} - \hat{\mathbf{e}}_{b,1}\right)$$

Note that $(\hat{\mathbf{e}}_{b,1} - \hat{\mathbf{e}}_{b,2}) = 2\sin(\gamma/2)\ \hat{\mathbf{x}}_g$

so:

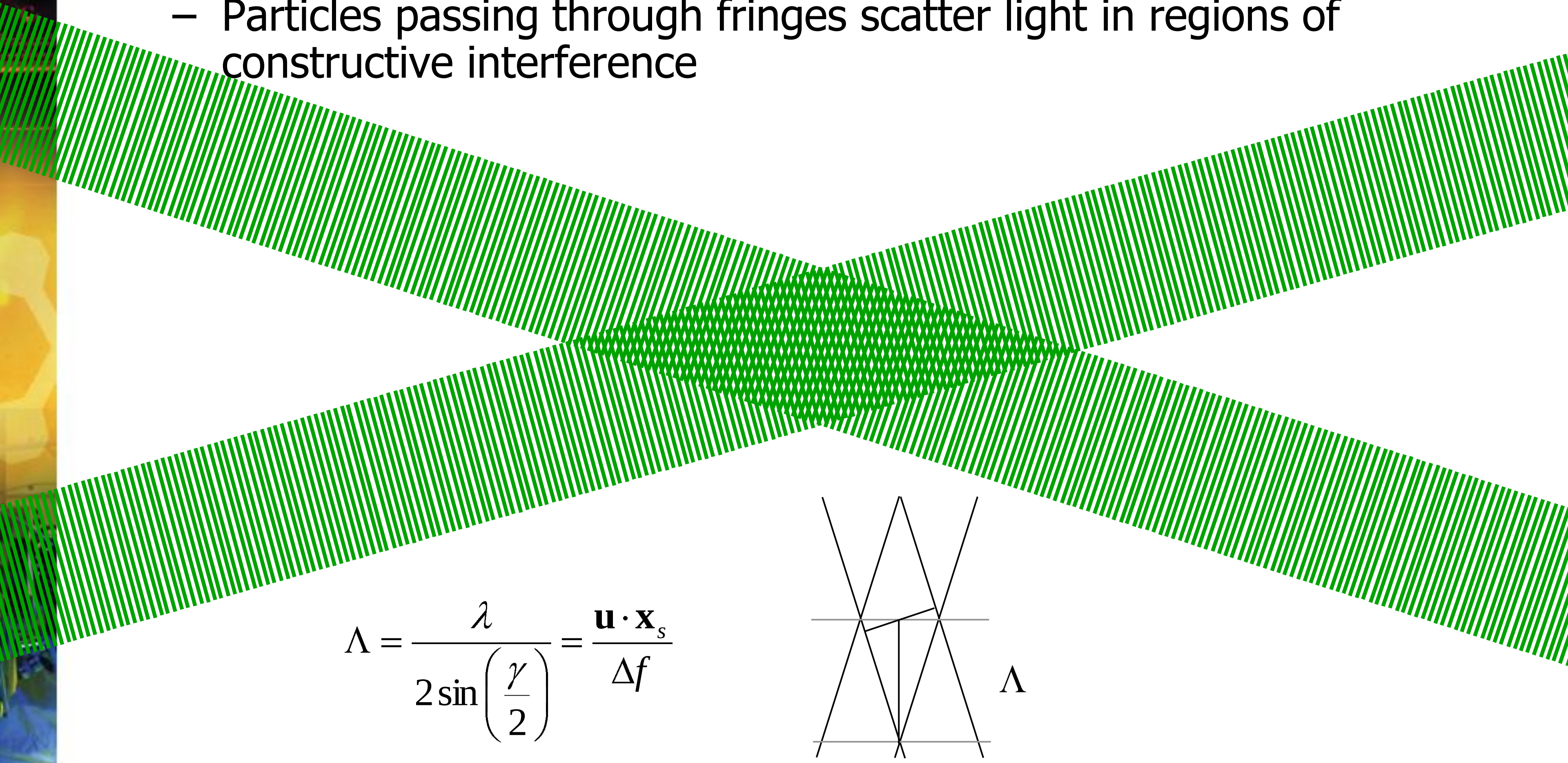
$$\mathbf{u}\cdot\mathbf{x}_g = \frac{\lambda}{2\sin(\gamma/2)}\Delta f_D$$

Measure the component of $\mathbf{u}$ *in the* $\hat{\mathbf{x}}_g$ *direction*

$$I(t) = \frac{1}{2}\left[I_{o1} + I_{o2} + 2\sqrt{I_{o1}I_{o2}}\cos\left[(\mathbf{k}_1 - \mathbf{k}_2)\cdot\mathbf{r} - \left(\frac{4\pi\sin(\gamma/2)}{\lambda}\mathbf{u}\cdot\mathbf{x_g}\right)t + (\varepsilon_1 - \varepsilon_2)\right]\right]$$

Fringe Interference description

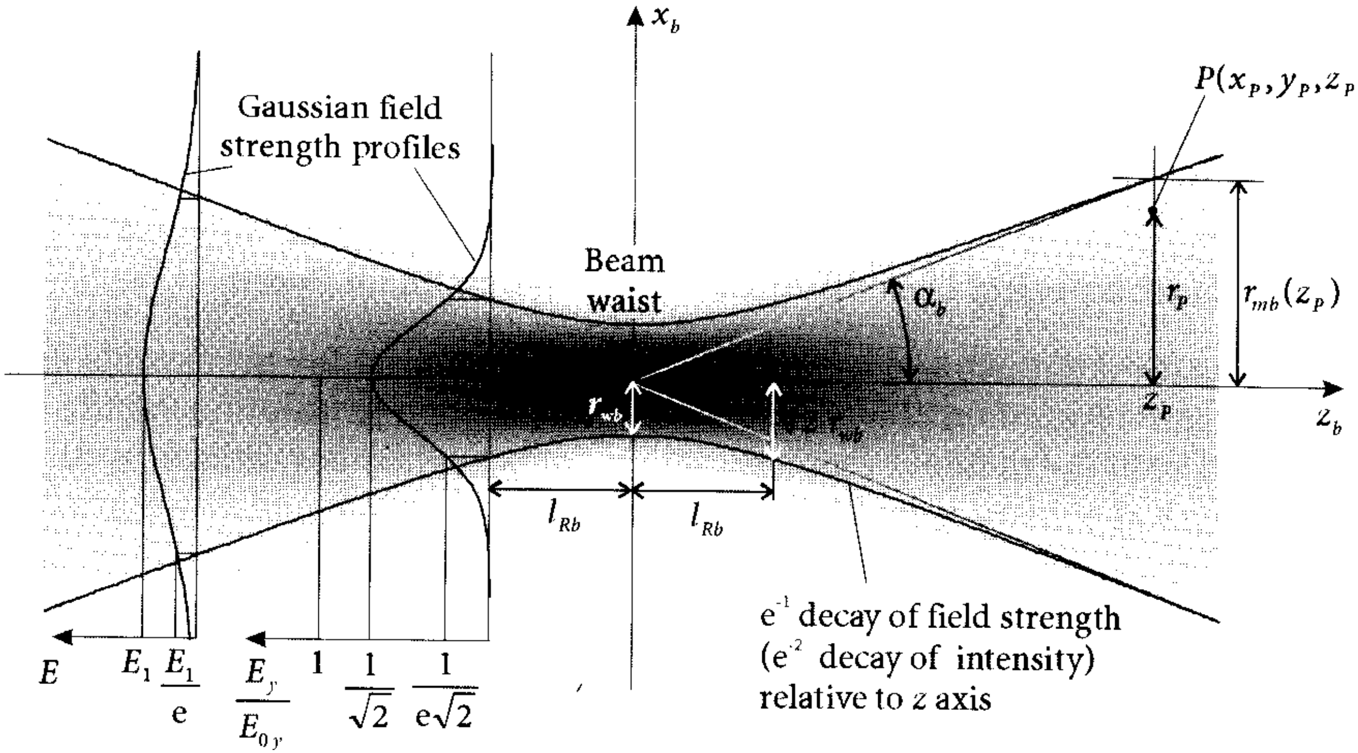
- **Interference "fringes" seen as standing waves**
  - Particles passing through fringes scatter light in regions of constructive interference



$$\Lambda = \frac{\lambda}{2\sin\left(\frac{\gamma}{2}\right)} = \frac{\mathbf{u}\cdot\mathbf{x}_s}{\Delta f}$$

  - Adequate explanation for particles smaller than individual fringes

Gaussian beam effects

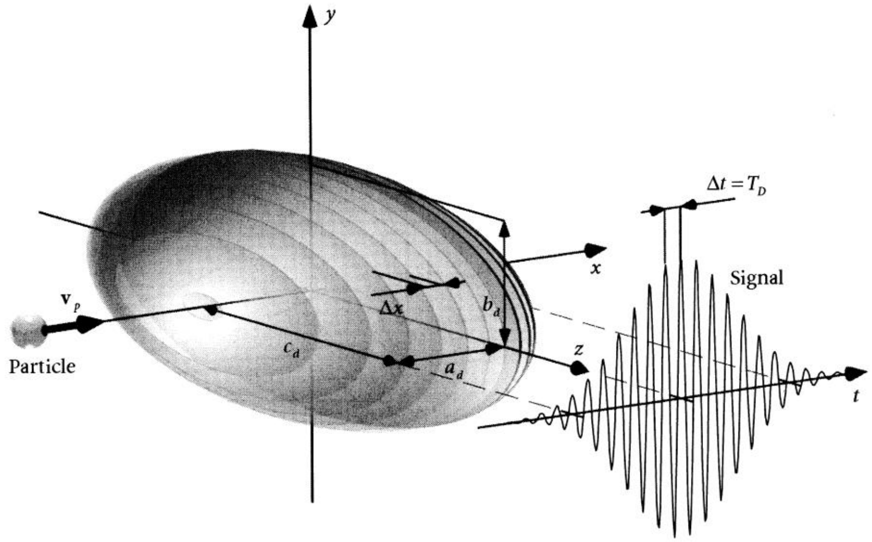


A single laser beam profile

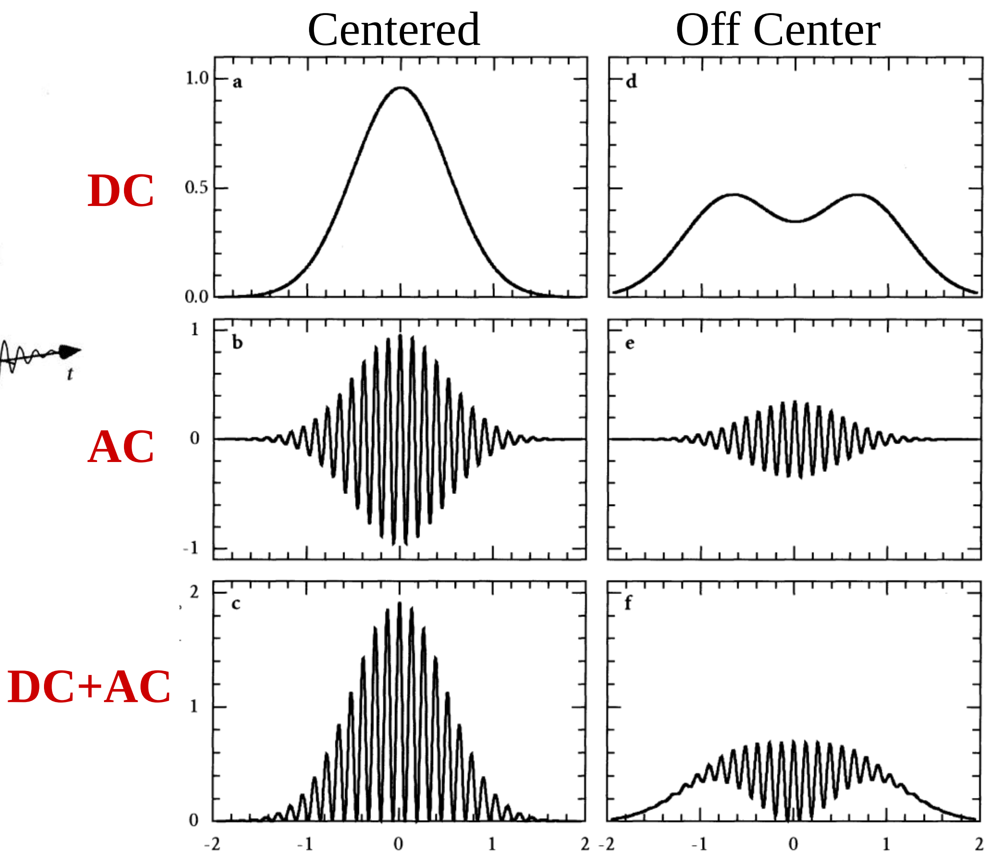
-Power distribution in MV will be Gaussian shaped
-In the MV, true plane waves occur only at the focal point
-Even for a perfect particle trajectory the strength of the Doppler 'burst' will vary with position

Figures from Albrecht et. al., 2003

Non-uniform beam effects



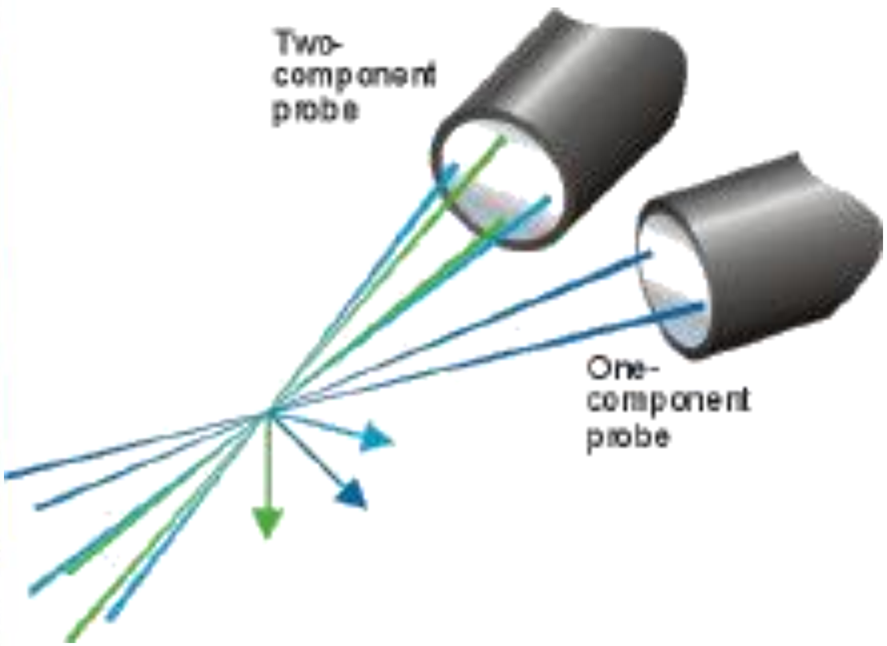
Particle Trajectory



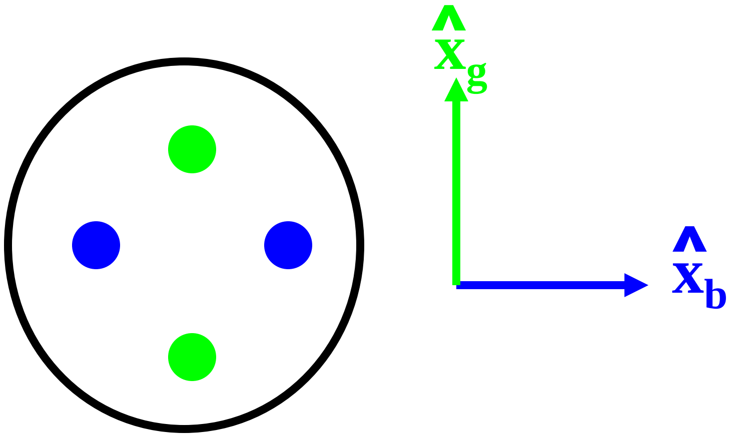
- *Off-center trajectory results in weakened signal visibility*
- *Pedestal (DC part of signal) is removed by a high pass filter after photomultiplier*

Figures from Albrecht et. al., 2003

Multi-component dual beam



Three independent directions



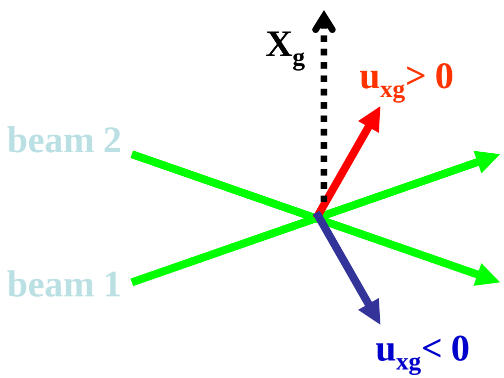
Two – Component Probe Looking Toward the Transmitter

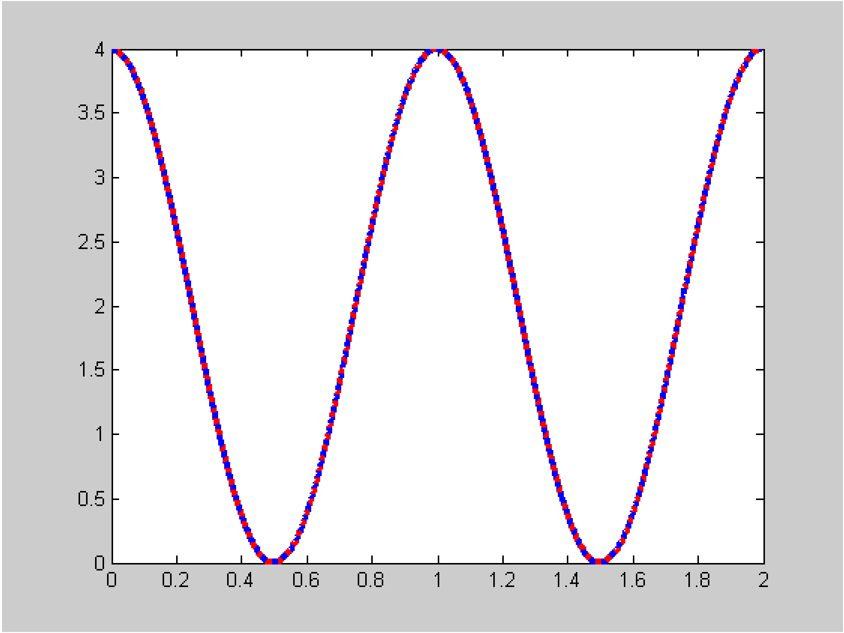
Sign ambiguity…

- **Change in sign of velocity has no effect on frequency**

$$I = \left[2I_o + 2I_o \cos\left((\mathbf{k}_1 - \mathbf{k}_2)\cdot\mathbf{r} - 2\pi\Delta f_D t + (\varepsilon_1 - \varepsilon_2)\right)\right]$$

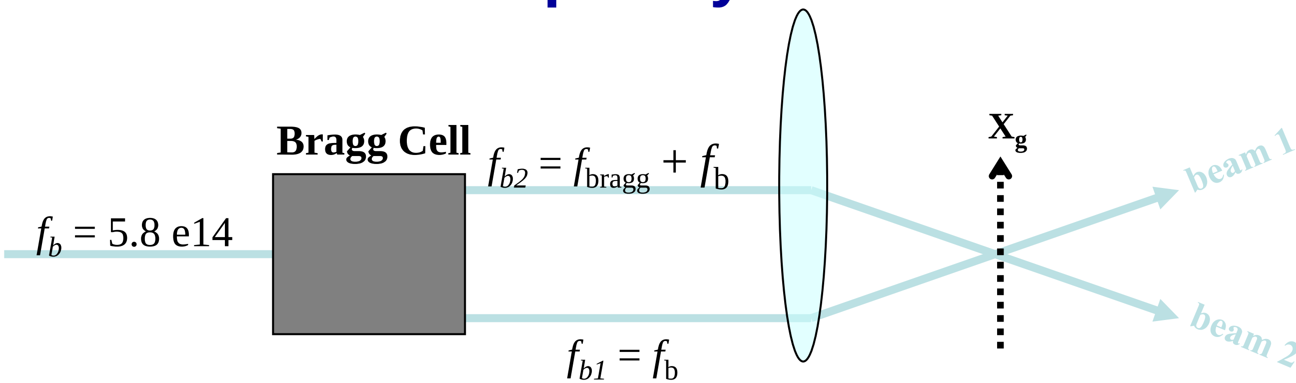
$$\mathbf{u}\cdot\mathbf{x}_s = \frac{\lambda}{2\sin(\kappa/2)}\Delta f_D$$





Velocity Ambiguity

- **Equal frequency beams**
  - No difference with velocity direction… cannot detect reversed flow
- **Solution: Introduce a frequency shift into 1 of the two beams**



$$f_{s,1} = f_b + \frac{f_b}{c}\mathbf{u}\cdot(\hat{\mathbf{e}}_{s,1} - \hat{\mathbf{e}}_{b,1})$$

$$f_{s,2} = (f_b + f_{bragg}) + \frac{f_b}{c}\mathbf{u}\cdot(\hat{\mathbf{e}}_{s,2} - \hat{\mathbf{e}}_{b,2})$$

$$\Delta f_D = f_{bragg} + \frac{f_b}{c}\mathbf{u}\cdot(\hat{\mathbf{e}}_{b,1} - \hat{\mathbf{e}}_{b,2}) = f_{bragg} + \Delta f_{D0}$$

Hypothetical shift Without Bragg Cell

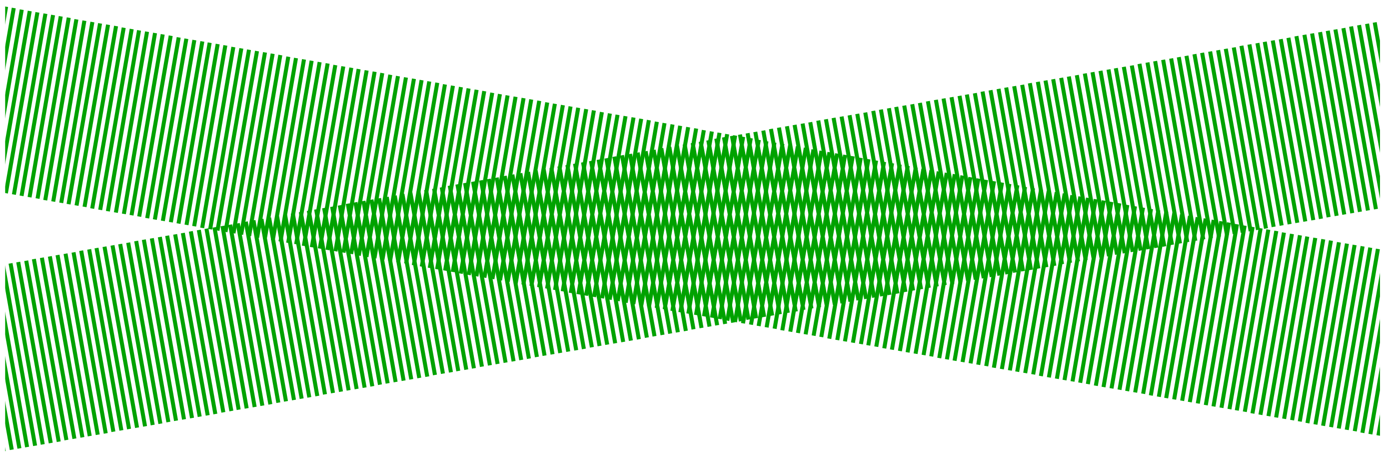
New Signal

$$I \propto 2E_{01}^2 \cos(-2\pi\{\overbrace{\Delta f_{D0} + f_{bragg}}\}t)$$

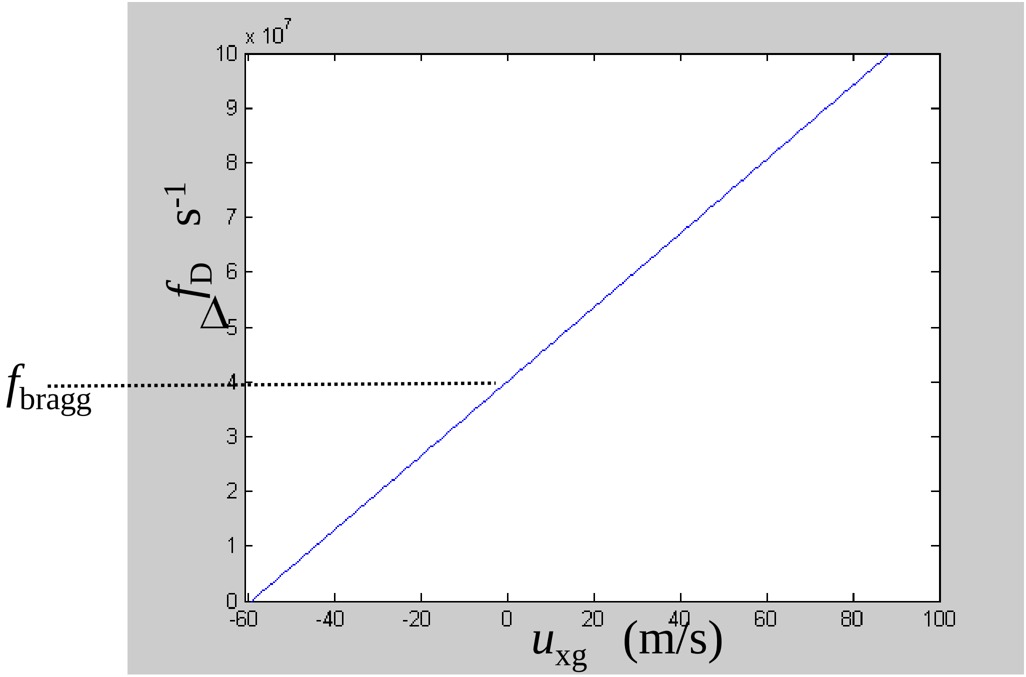
If $\boldsymbol{\Delta f_D < f_{bragg}}$ ***then*** $u < 0$

Frequency shift: Fringe description

- **Different frequency causes an apparent velocity in fringes**
  - Effect result of interference of two traveling waves as slightly different frequency



Directional ambiguity (cont)



$$|u_{xg}| = \frac{\lambda(\Delta f_{D0} - f_{bragg})}{2\sin(\gamma/2)}$$

$\lambda = 514$ nm, f_{bragg} = 40 MHz and $\gamma = 20°$

Upper limit on positive velocity limited only by time response of detector

Velocity bias sampling effects

- **LDA samples the flow based on**
  - Rate at which particles pass through the detection volume
  - Inherently a flux-weighted measurement
  - Simple number weighted means are biased for unsteady flows and need to be corrected
- **Consider:**
  - Uniform seeding density (# particles/volume)
  - Flow moves at steady speed of 5 units/sec for 4 seconds (giving 20 samples) would measure:

$$\frac{5*20}{20} = 5$$

  - Flow that moves at 8 units/sec for 2 sec (giving 16 samples), then 2 units/sec for 2 second (giving 4 samples) would give

$$\frac{16*8+4*2}{20} = 6.8$$

Laser Doppler Anemometry
Velocity Measurement Bias

$$\overline{U}_x = \sum_{i=1}^{N} U_{x,i}\tau_i \bigg/ \sum_{i=1}^{N}\tau_i$$

Mean Velocity

$$\overline{U_x^n} = \sum_{i=1}^{N} \left(U_{x,i} - \overline{U}_x\right)^n \tau_i \bigg/ \sum_{i=1}^{N}\tau_i$$

nth moment

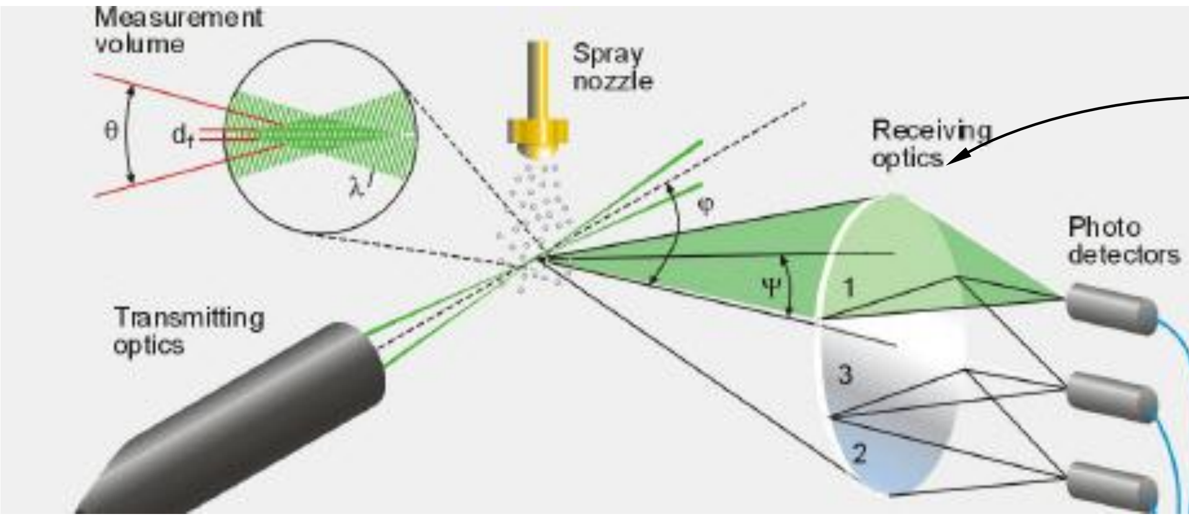
Bias Compensation Formulas

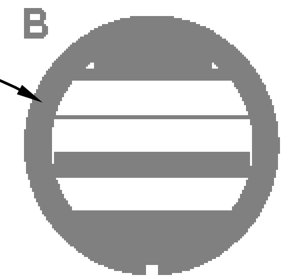
- ***The sampling rate of a volume of fluid containing particles increases with the velocity of that volume***
- ***Introduces a bias towards sampling higher velocity particles***

Phase Doppler Anemometry

The overall phase difference is proportional to particle diameter

$$\Delta\varepsilon = \frac{2\pi n_i D}{\lambda}\beta(\theta,\psi,\gamma,n_p,n_i)$$





Multiple Detector Implementation

The geometric factor, β

- Has closed form solution for p = 0 and 1 only
- Absolute value increases with ψ (elevation angle relative to 0°)
- Is independent of n_p for reflection