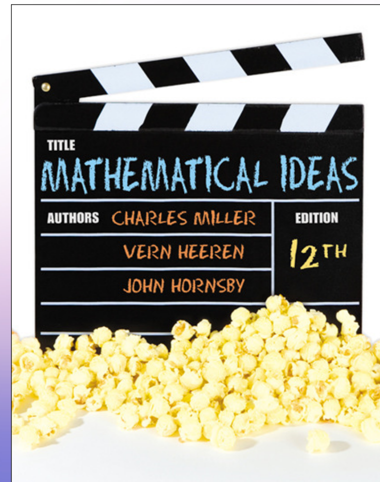
Chapter 3

Introduction to Logic



Chapter 3: Introduction to Logic

Section 3-1

Statements and Quantifiers

Statements and Qualifiers

- Statements
- Negations
- Symbols
- Quantifiers
- Sets of Numbers

Statements

A **statement** is defined as a declarative sentence that is either true or false, but not both simultaneously.

Compound Statements

A **compound statement** may be formed by combining two or more statements. The statements making up the compound statement are called the **component statements**. Various **connectives** such as *and*, *or*, *not*, and *if…then*, can be used in forming compound statements.

Example: Compound Statements

Decide whether each statement is compound.

a) If Amanda said it, then it must be true.
b) The gun was made by Smith and Wesson.

Solution

a) This statement is compound.
b) This is not compound since *and* is part of a manufacturer name and not a logical connective.

Negations

The sentence "Max has a valuable card" is a statement; the **negation** of this statement is "Max does not have a valuable card." The negation of a true statement is false and the negation of a false statement is true.

Inequality Symbols

Use the following inequality symbols for the next example.

Symbolism	Meaning
$a < b$	*a* is less than *b*
$a > b$	*a* is greater than *b*
$a \leq b$	*a* is less than or equal to *b*
$a \geq b$	*a* is greater than or equal to *b*

Example: Forming Negations

Give a negation of each inequality. Do *not* use a slash symbol.

a) $p < 3$

b) $3x - 2y \geq 12$

Solution

a) $p \geq 3$

b) $3x - 2y < 12$

Symbols

To simplify work with logic, we use symbols. Statements are represented with letters, such as *p*, *q*, or *r*, while several symbols for connectives are shown below.

Connective	Symbol	Type of Statement
and	$\wedge$	Conjunction
or	$\vee$	Disjunction
not	$\sim$	Negation

Example: Translating from Symbols to Words

Let *p* represent "It is raining," and let *q* represent "It is March." Write each symbolic statement in words.

a) $p \vee q$

b) $\sim (p \wedge q)$

Solution

a) It is raining or it is March.
b) It is not the case that it is raining and it is March.

Quantifiers

The words *all*, *each*, *every*, and *no*(*ne*) are called **universal quantifiers**, while words and phrases such as *some*, *there exists*, and (*for*) *at least one* are called **existential quantifiers**.

Quantifiers are used extensively in mathematics to indicate *how many* cases of a particular situation exist.

Negations of Quantified Statements

Statement	Negation
All do.	Some do not.
Some do.	None do.

Example: Forming Negations of Quantified Statements

Form the negation of each statement.

a) Some cats have fleas.
b) Some cats do not have fleas.
c) No cats have fleas.

Solution

a) No cats have fleas.
b) All cats have fleas.
c) Some cats have fleas.

Sets of Numbers

Natural (*counting*) {1, 2, 3, 4, …}
Whole numbers {0, 1, 2, 3, 4, …}
Integers {…,–3, –2, –1, 0, 1, 2, 3, …}
Rational numbers $\left\{ \frac{p}{q} \middle| p \text{ and } q \text{ are integers and } q \neq 0 \right\}$

May be written as a terminating decimal, like 0.25, or a repeating decimal like 0.333…
Irrational {x | x is not expressible as a quotient of integers} Decimal representations never terminate and never repeat.
Real numbers {x | x can be expressed as a decimal}

Example: Deciding Whether the Statements are True or False

Decide whether each of the following statements about sets of numbers is *true* or *false*.

a) Every integer is a natural number.
b) There exists a whole number that is not a natural number.

Solution

a) This is false, –1 is an integer and not a natural number.
b) This is true (0 is it).