Introduction to Management Science
(8th Edition, Bernard W. Taylor III)

Chapter 12
Network Flow Models

Chapter Topics

- The Shortest Route Problem
- The Minimal Spanning Tree Problem
- The Maximal Flow Problem

Overview

- A network is an arrangement of paths connected at various points through which one or more items move from one point to another.
- The network is drawn as a diagram providing a picture of the system thus enabling visual interpretation and enhanced understanding.
- A large number of real-life systems can be modeled as networks which are relatively easy to conceive and construct.

Network Components (1 of 2)

- Network diagrams consist of nodes and branches.
- *Nodes* (circles), represent junction points, or locations.
- *Branches* (lines), connect nodes and represent flow.

Network Components (2 of 2)

- Four nodes, four branches in figure.
- "Atlanta", node 1, termed *origin*, any of others *destination.*
- Branches identified by beginning and ending node numbers.
- Value assigned to each branch (distance, time, cost, etc.).

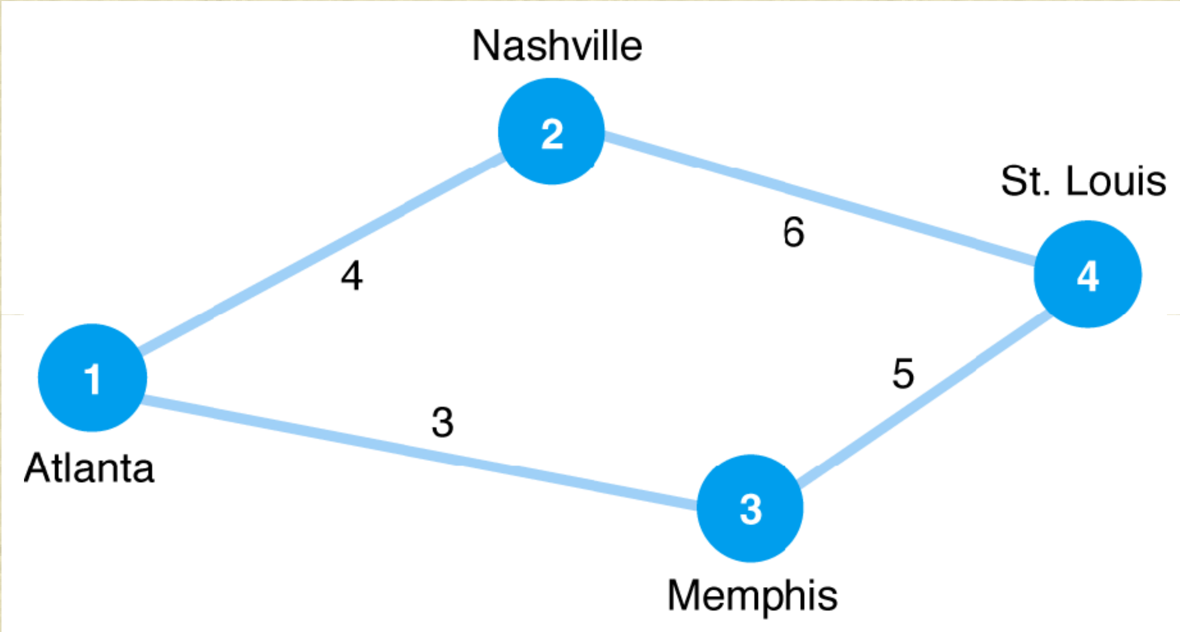


Figure 12.1
Network of Railroad Routes

The Shortest Route Problem Definition and Example Problem Data (1 of 2)

- Problem: Determine the shortest routes from the origin to all destinations.

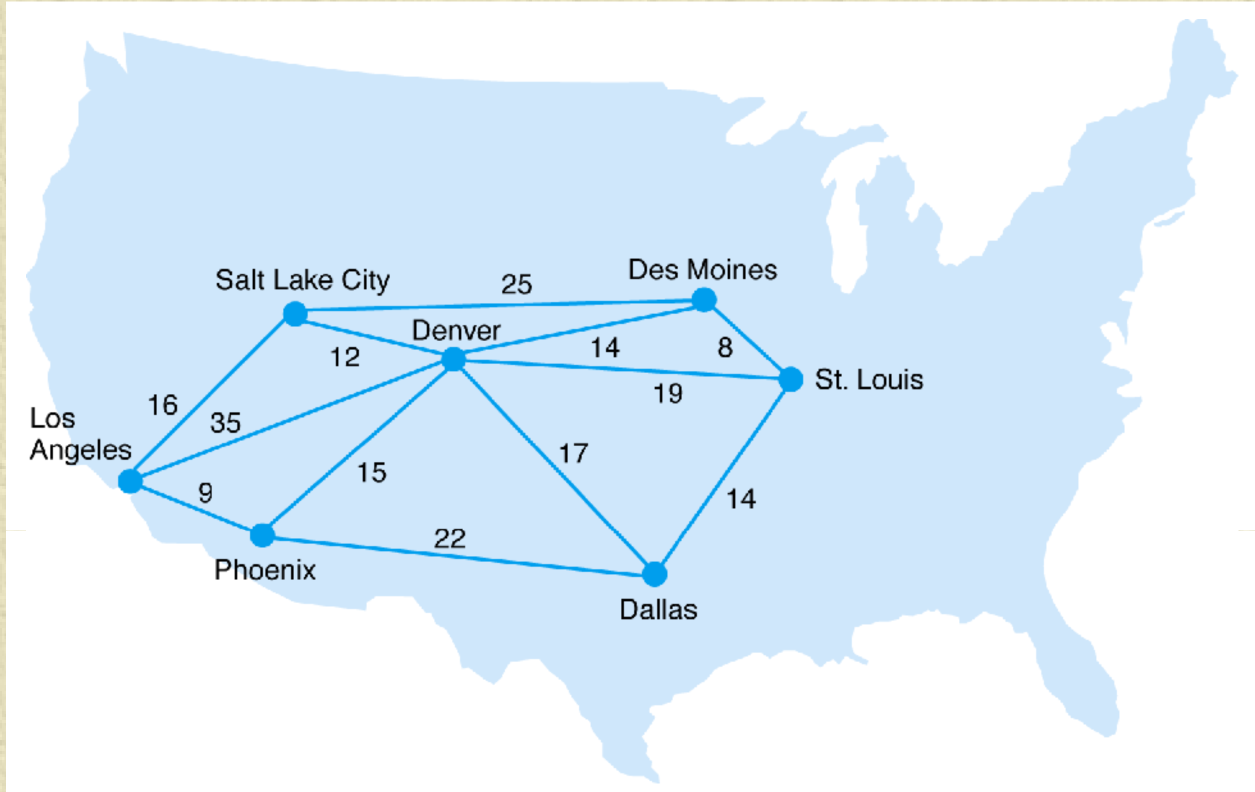


Figure 12.2
Shipping Routes from Los Angeles

The Shortest Route Problem
Definition and Example Problem Data (2 of 2)

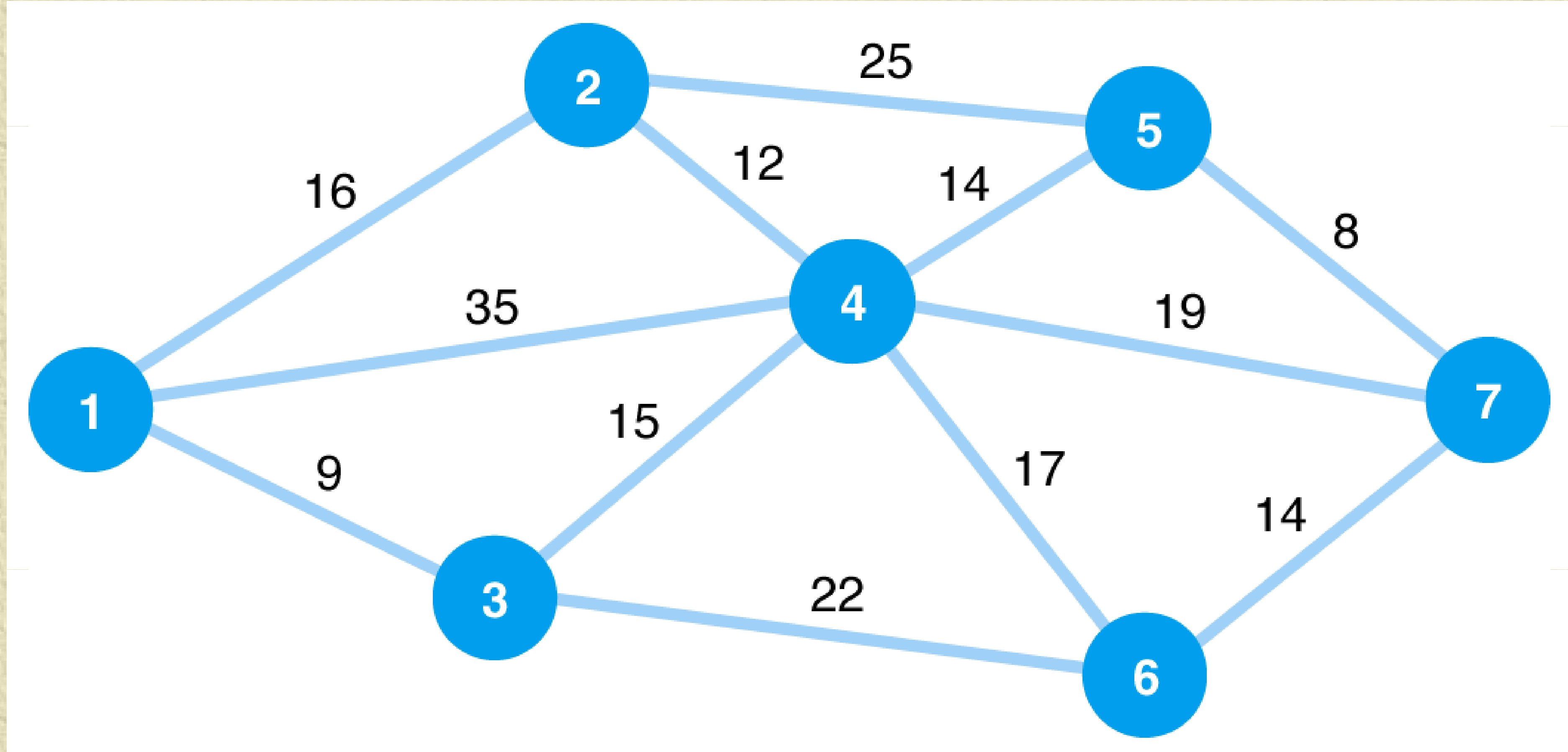
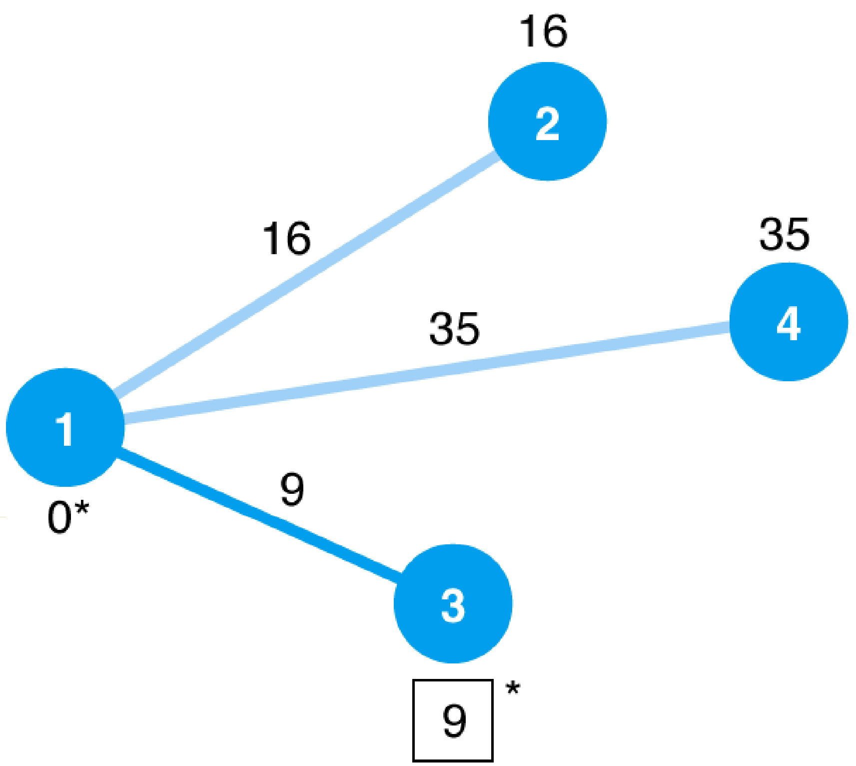


Figure 12.3
Network of Shipping Routes

The Shortest Route Problem Solution Approach (1 of 8)

- Determine the initial shortest route from the origin (node 1) to the closest node (3).

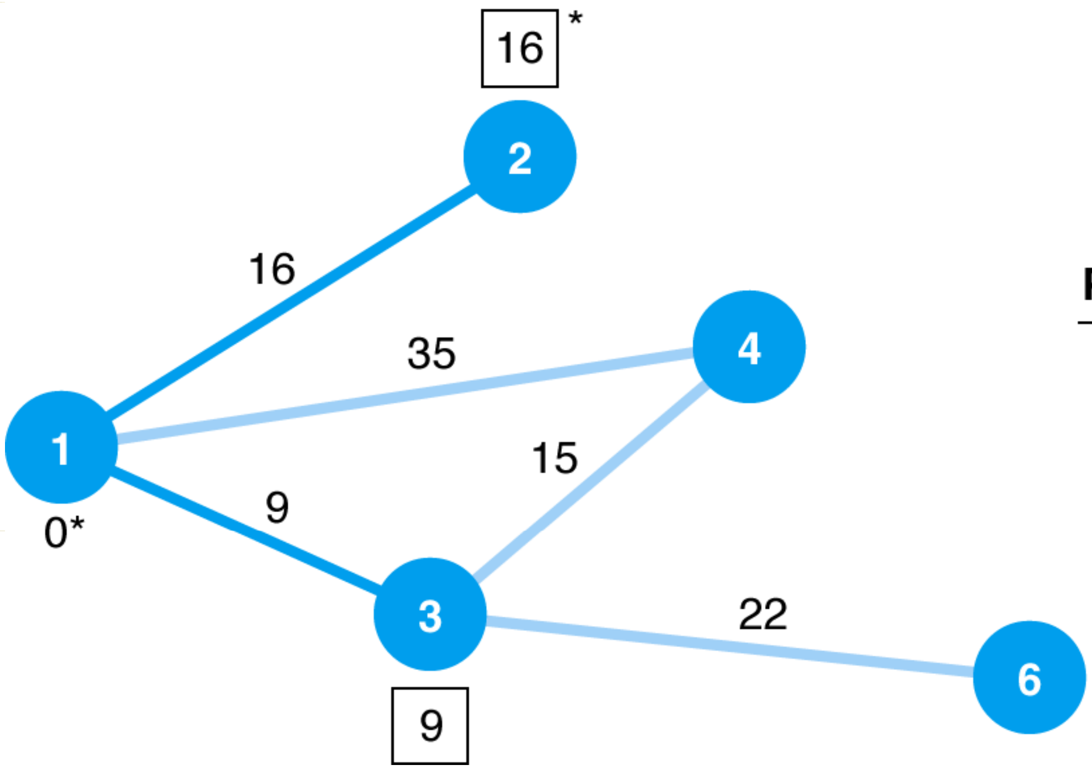


Permanent set	Branch	Time
{1}	1–2	16
	1–4	35
	1–3	9 *

Figure 12.4
Network with Node 1 in the Permanent Set

The Shortest Route Problem
Solution Approach (2 of 8)

- Determine all nodes directly connected to the permanent set.

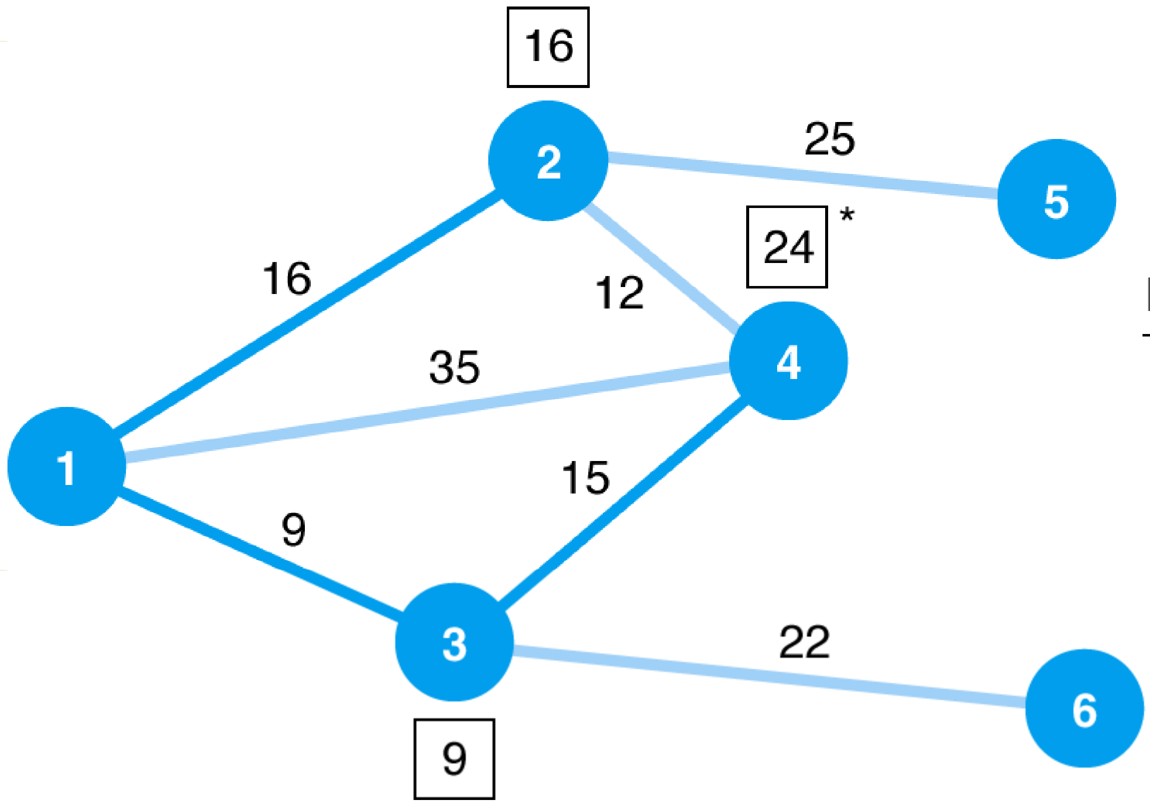


Permanent set	Branch	Time
{1, 3}	1–2	16*
	1–4	35
	3–4	24
	3–6	31

Figure 12.5
Network with Nodes 1 and 3 in the Permanent Set

The Shortest Route Problem
Solution Approach (3 of 8)

- Redefine the permanent set.

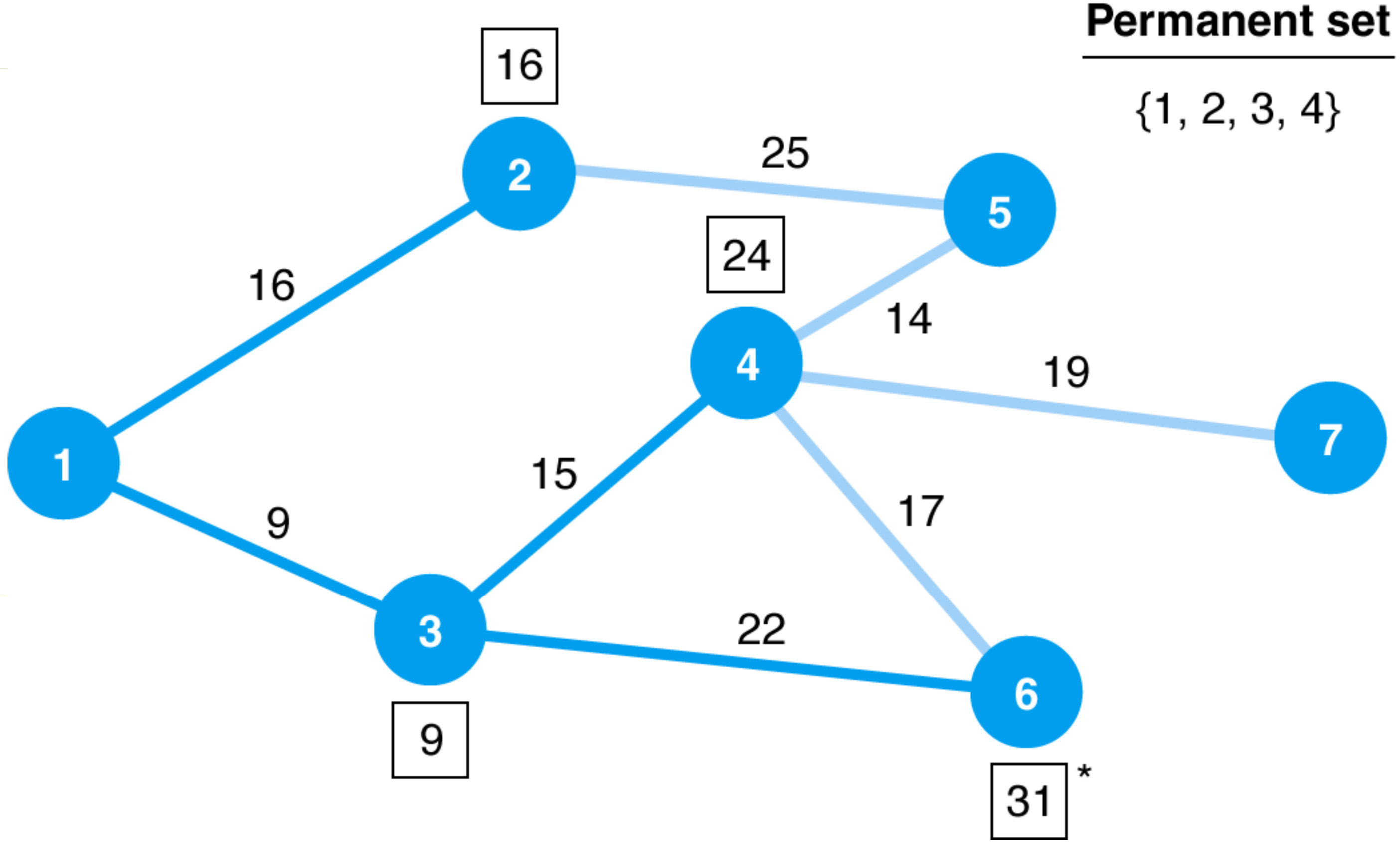


Permanent set	Branch	Time
{1, 2, 3}	1–4	35
	2–4	28
	2–5	41
	3–4	24 *
	3–6	31

Figure 12.6
Network with Nodes 1, 2, and 3 in the Permanent Set

The Shortest Route Problem Solution Approach (4 of 8)

- Continue

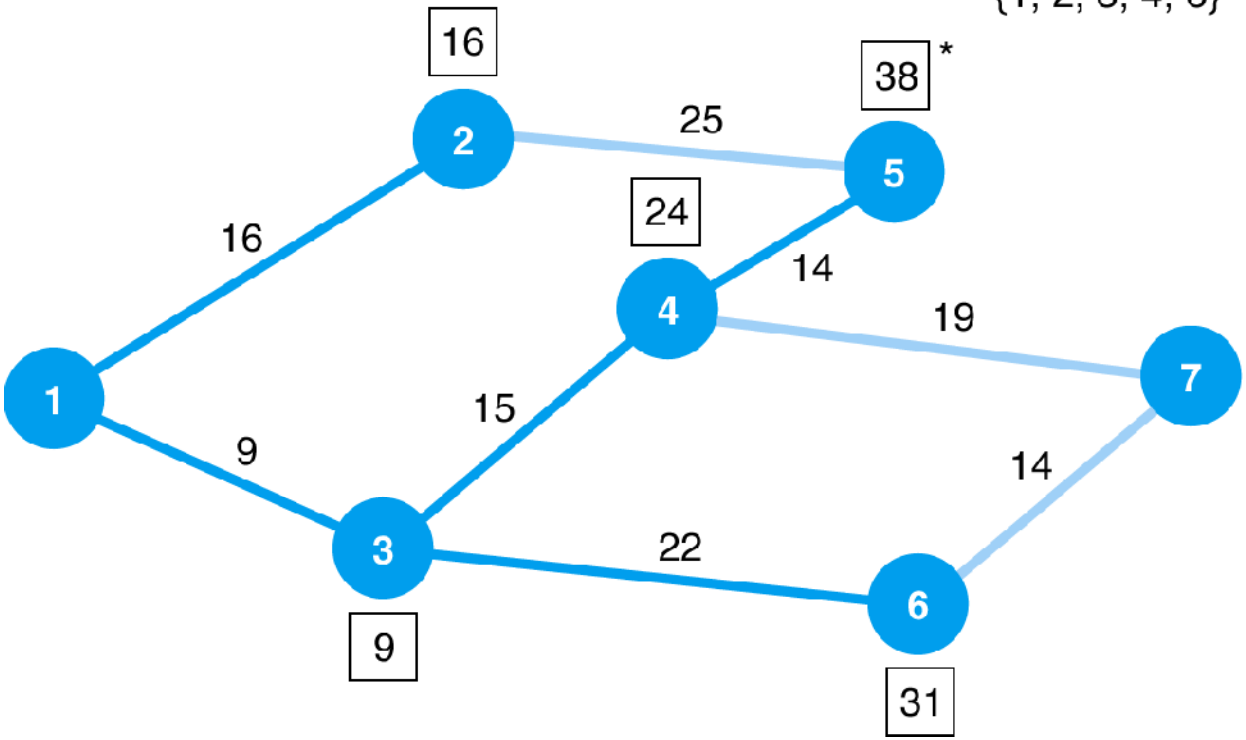


Permanent set	Branch	Time
{1, 2, 3, 4}	2–5	41
	3–6	31 *
	4–5	38
	4–7	43
	4–6	41

Figure 12.7
Network with Nodes 1, 2, 3, and 4 in the Permanent Set

The Shortest Route Problem
Solution Approach (5 of 8)

- Continue

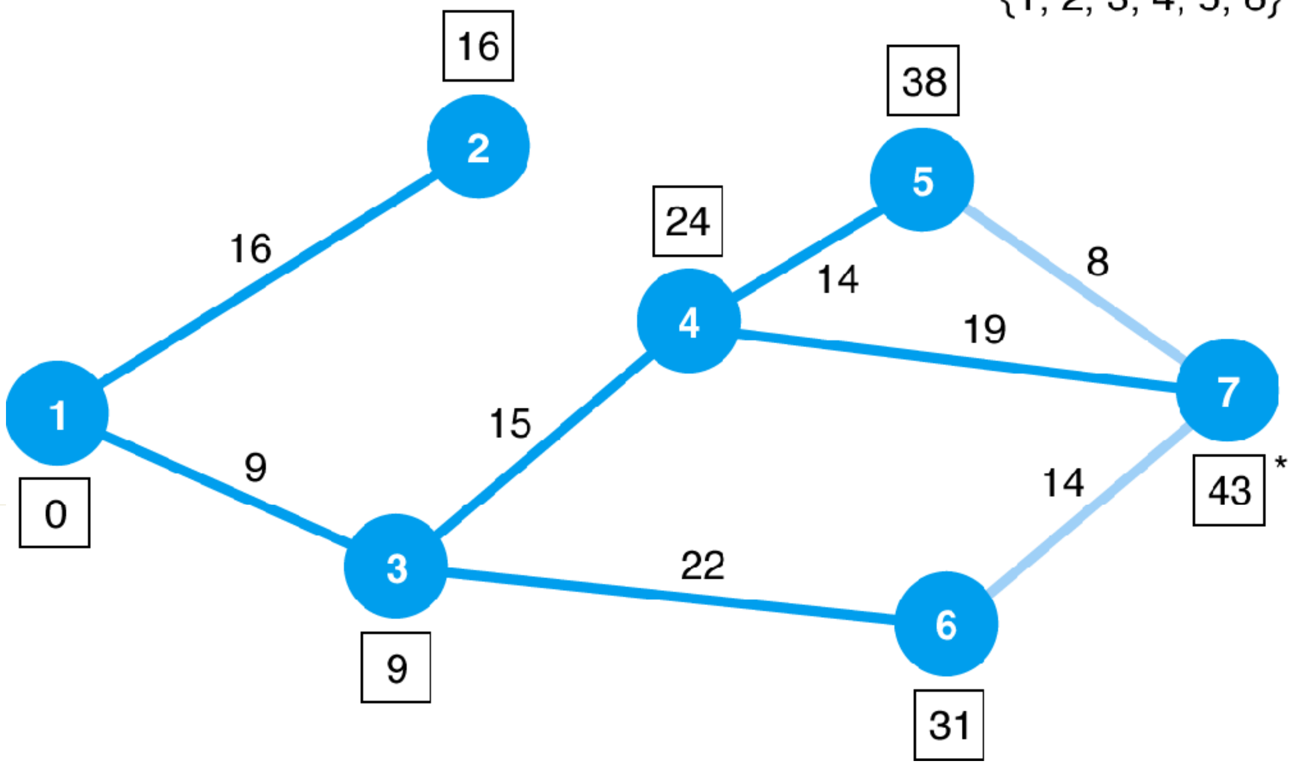


Permanent set	Branch	Time
{1, 2, 3, 4, 6}	2–5	41
	4–5	38 *
	4–7	43
	6–7	45

Figure 12.8
Network with Nodes 1, 2, 3, 4, and 6 in the Permanent Set

The Shortest Route Problem Solution Approach (6 of 8)

- Continue



Permanent set	Branch	Time
{1, 2, 3, 4, 5, 6}	4–7	43 *
	6–7	45
	5–7	46

Figure 12.9
Network with Nodes 1, 2, 3, 4, 5, and 6 in the Permanent Set

The Shortest Route Problem
Solution Approach (7 of 8)

- Optimal Solution

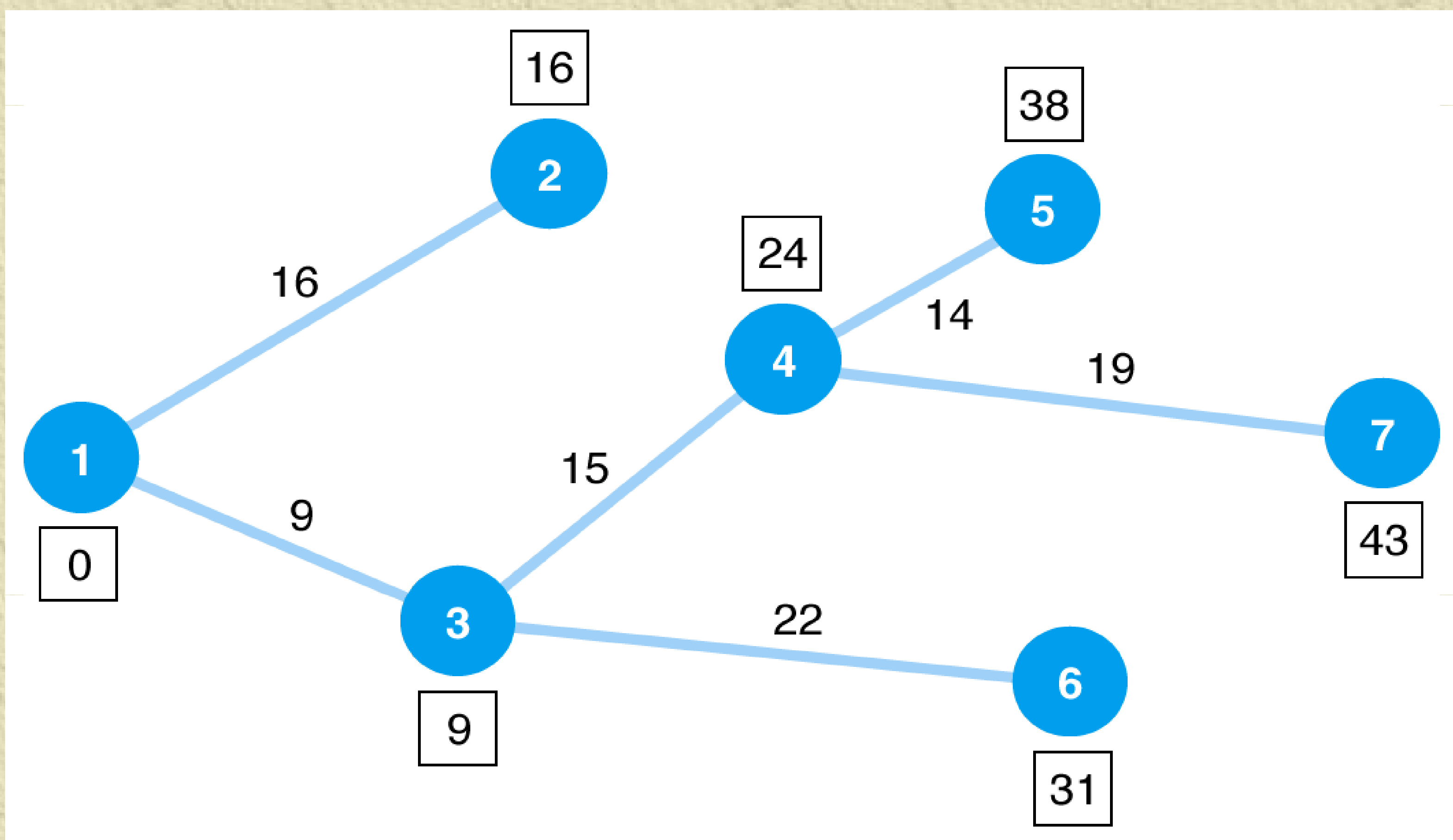


Figure 12.10
Network with Optimal Routes from Los Angeles to All Destinations

The Shortest Route Problem Solution Approach (8 of 8)

- Solution Summary

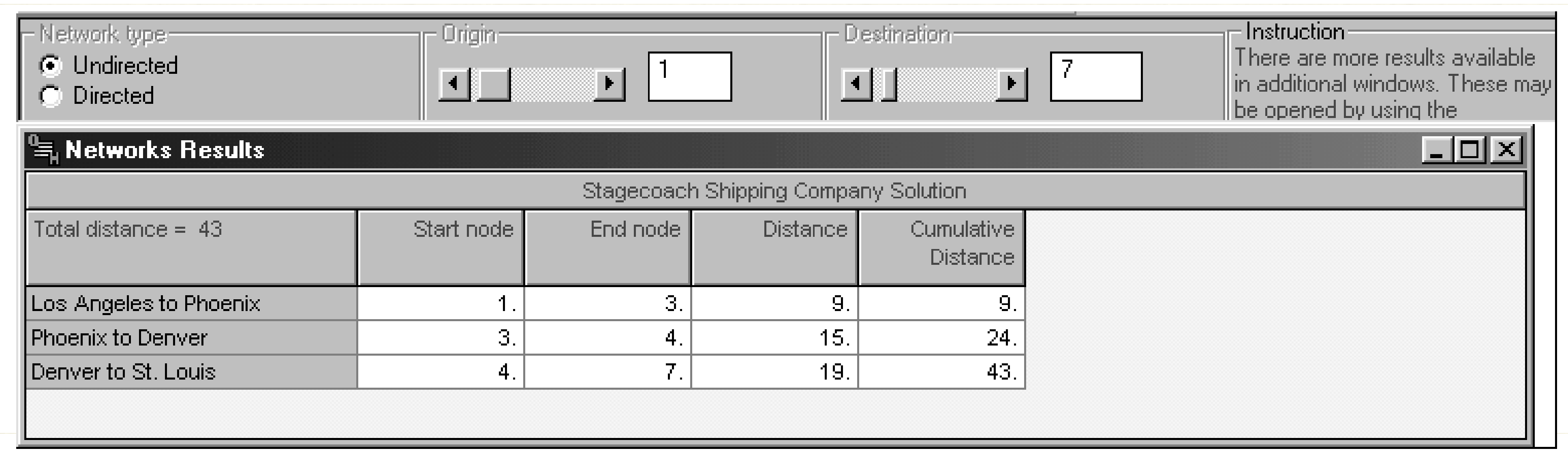
From Los Angeles to:	Route	Total Hours
Salt Lake City (node 2)	1–2	16
Phoenix (node 3)	1–3	9
Denver (node 4)	1–3–4	24
Des Moines (node 5)	1–3–4–5	38
Dallas (node 6)	1–3–6	31
St. Louis (node 7)	1–3–4–7	43

Table 7.1
Shortest Travel Time from Origin to Each Destination

The Shortest Route Problem Solution Method Summary

- Select the node with the shortest direct route from the origin.
- Establish a *permanent set* with the origin node and the node that was selected in step 1.
- Determine all nodes directly connected to the permanent set nodes.
- Select the node with the shortest route (branch) from the group of nodes directly connected to the permanent set nodes.
- Repeat steps 3 and 4 until all nodes have joined the permanent set.

The Shortest Route Problem
Computer Solution with QM for Windows (1 of 2)



Total distance = 43	Start node	End node	Distance	Cumulative Distance
Los Angeles to Phoenix	1.	3.	9.	9.
Phoenix to Denver	3.	4.	15.	24.
Denver to St. Louis	4.	7.	19.	43.

Exhibit 12.1

The Shortest Route Problem
Computer Solution with QM for Windows (2 of 2)

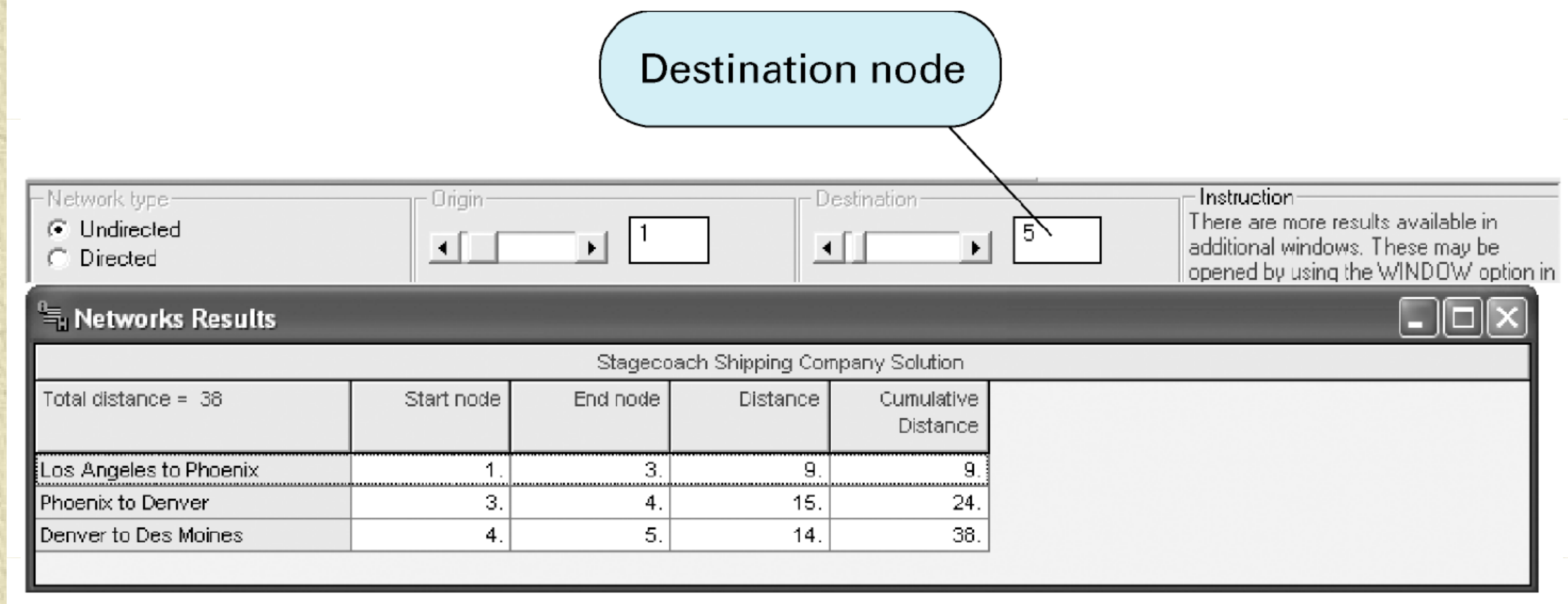


Exhibit 12.2

The Shortest Route Problem Computer Solution with Excel (1 of 4)

Formulation as a 0 - 1 integer linear programming problem.

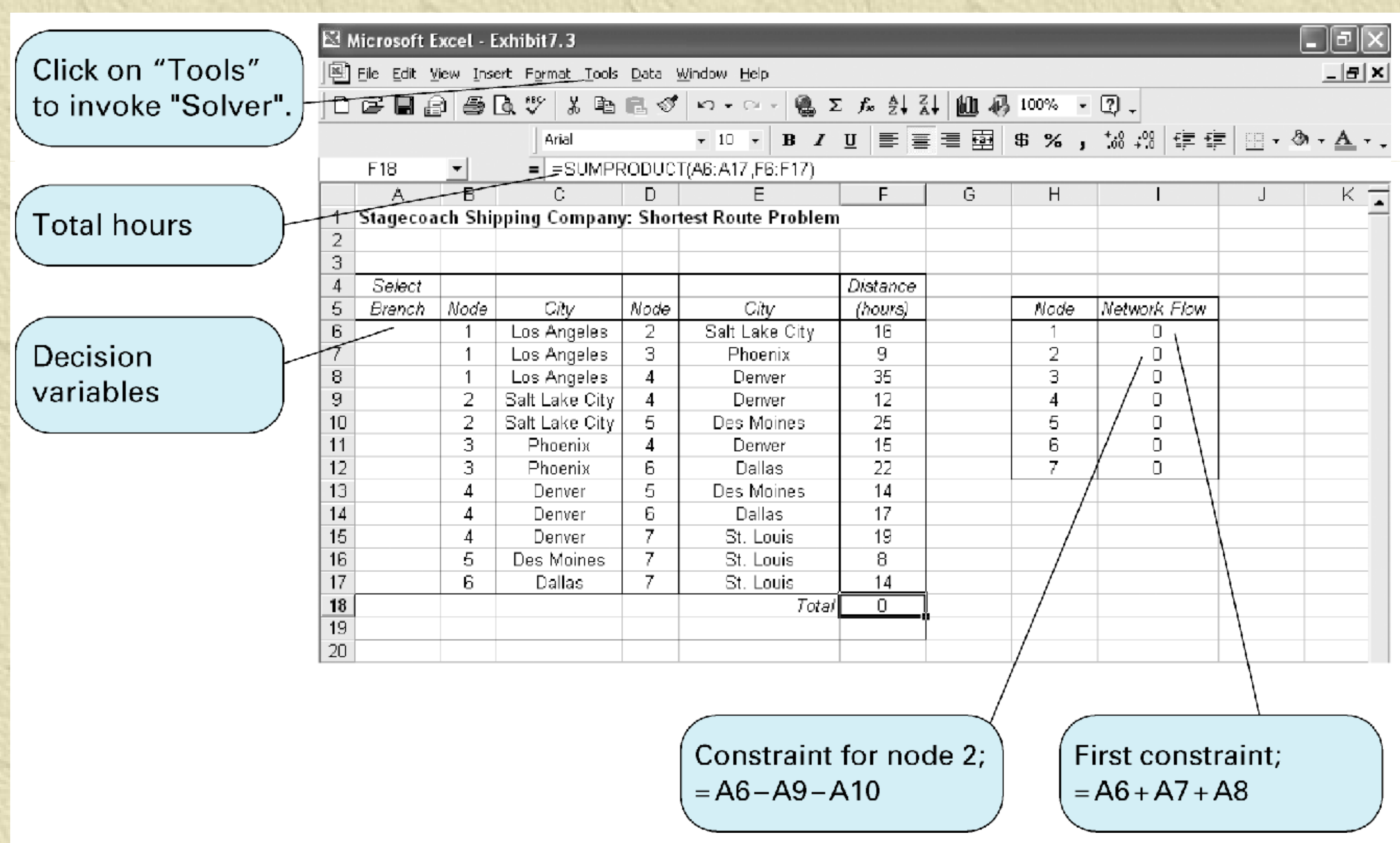
x_{ij} = 0 if branch i-j is not selected as part of the shortest route and 1 if it is selected.

$$\text{Minimize } Z = 16x_{12} + 9x_{13} + 35x_{14} + 12x_{24} + 25x_{25} + 15x_{34} + 22x_{36} + 14x_{45} + 17x_{46} + 19x_{47} + 8x_{57} + 14x_{67}$$

subject to:

$$x_{12} + x_{13} + x_{14} = 1$$
$$x_{12} - x_{24} - x_{25} = 0$$
$$x_{13} - x_{34} - x_{36} = 0$$
$$x_{14} + x_{24} + x_{34} - x_{45} - x_{46} - x_{47} = 0$$
$$x_{25} + x_{45} - x_{57} = 0$$
$$x_{36} + x_{46} - x_{67} = 0$$
$$x_{47} + x_{57} + x_{67} = 1 \qquad x_{ij} = 0 \text{ or } 1$$

The Shortest Route Problem
Computer Solution with Excel (2 of 4)



Stagecoach Shipping Company: Shortest Route Problem

Select Branch	Node	City	Node	City	Distance (hours)
	1	Los Angeles	2	Salt Lake City	16
	1	Los Angeles	3	Phoenix	9
	1	Los Angeles	4	Denver	35
	2	Salt Lake City	4	Denver	12
	2	Salt Lake City	5	Des Moines	25
	3	Phoenix	4	Denver	15
	3	Phoenix	6	Dallas	22
	4	Denver	5	Des Moines	14
	4	Denver	6	Dallas	17
	4	Denver	7	St. Louis	19
	5	Des Moines	7	St. Louis	8
	6	Dallas	7	St. Louis	14
				Total	0

Node	Network Flow
1	0
2	0
3	0
4	0
5	0
6	0
7	0

Exhibit 12.3

The Shortest Route Problem
Computer Solution with Excel (3 of 4)

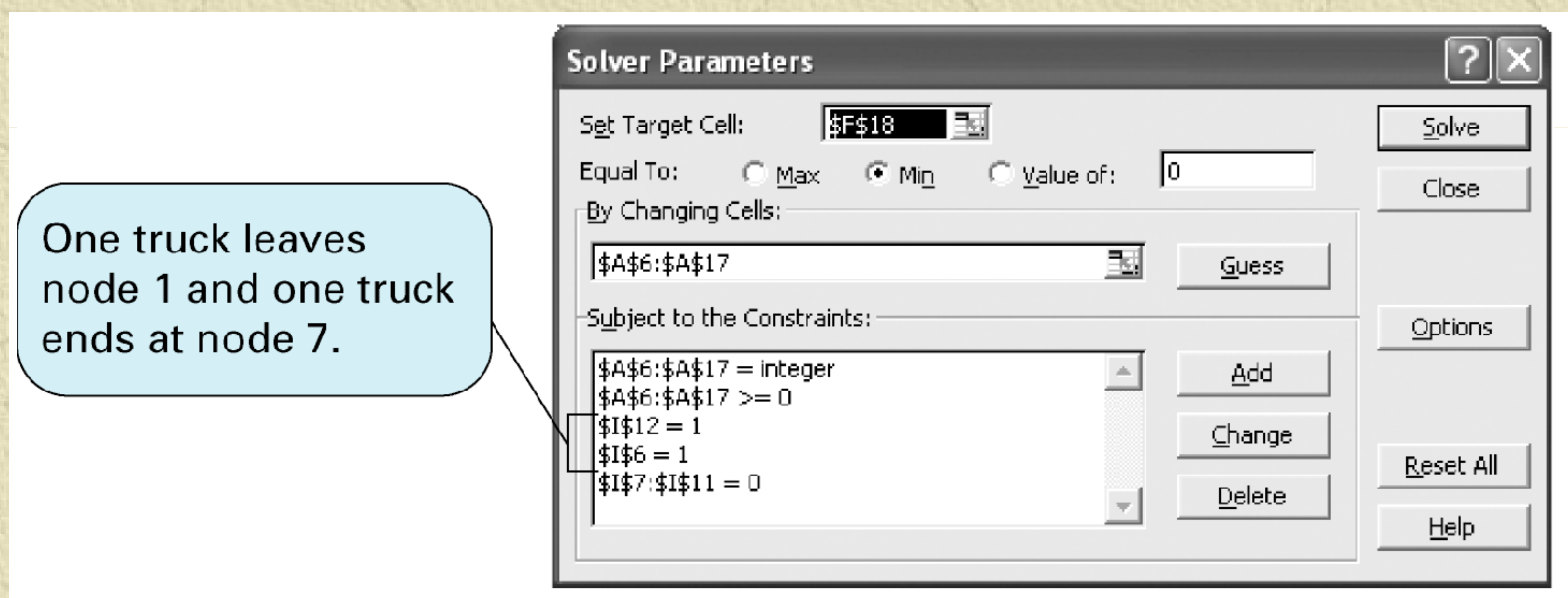


Exhibit 12.4

The Shortest Route Problem Computer Solution with Excel (4 of 4)

Microsoft Excel - Ch7-Stagecoach.xls

I9 = =A8+A9+A11-A13-A14-A15

Stagecoach Shipping Company: Shortest Route Problem

Select Branch	Node	City	Node	City	Distance (hours)
0	1	Los Angeles	2	Salt Lake City	16
1	1	Los Angeles	3	Phoenix	9
0	1	Los Angeles	4	Denver	35
0	2	Salt Lake City	4	Denver	12
0	2	Salt Lake City	5	Des Moines	25
1	3	Phoenix	4	Denver	15
0	3	Phoenix	6	Dallas	22
0	4	Denver	5	Des Moines	14
0	4	Denver	6	Dallas	17
1	4	Denver	7	St. Louis	19
0	5	Des Moines	7	St. Louis	8
0	6	Dallas	7	St. Louis	14
				Total	43

Node	Network Flow
1	1
2	0
3	0
4	0
5	0
6	0
7	1

Exhibit 12.5

The Minimal Spanning Tree Problem Definition and Example Problem Data

- Problem: Connect all nodes in a network so that the total branch lengths are minimized.

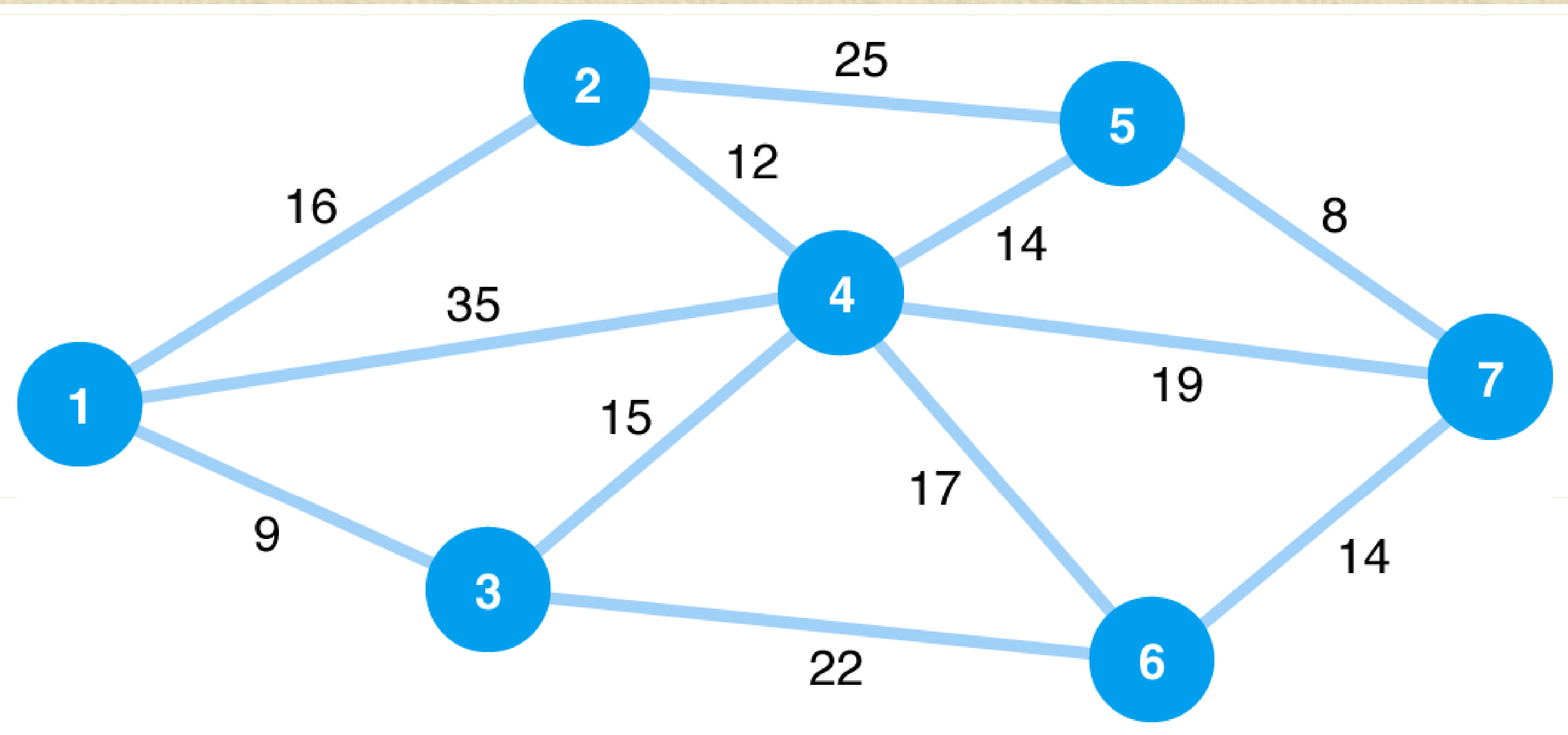


Figure 12.11
Network of Possible Cable TV Paths

The Minimal Spanning Tree Problem Solution Approach (1 of 6)

- Start with any node in the network and select the closest node to join the spanning tree.

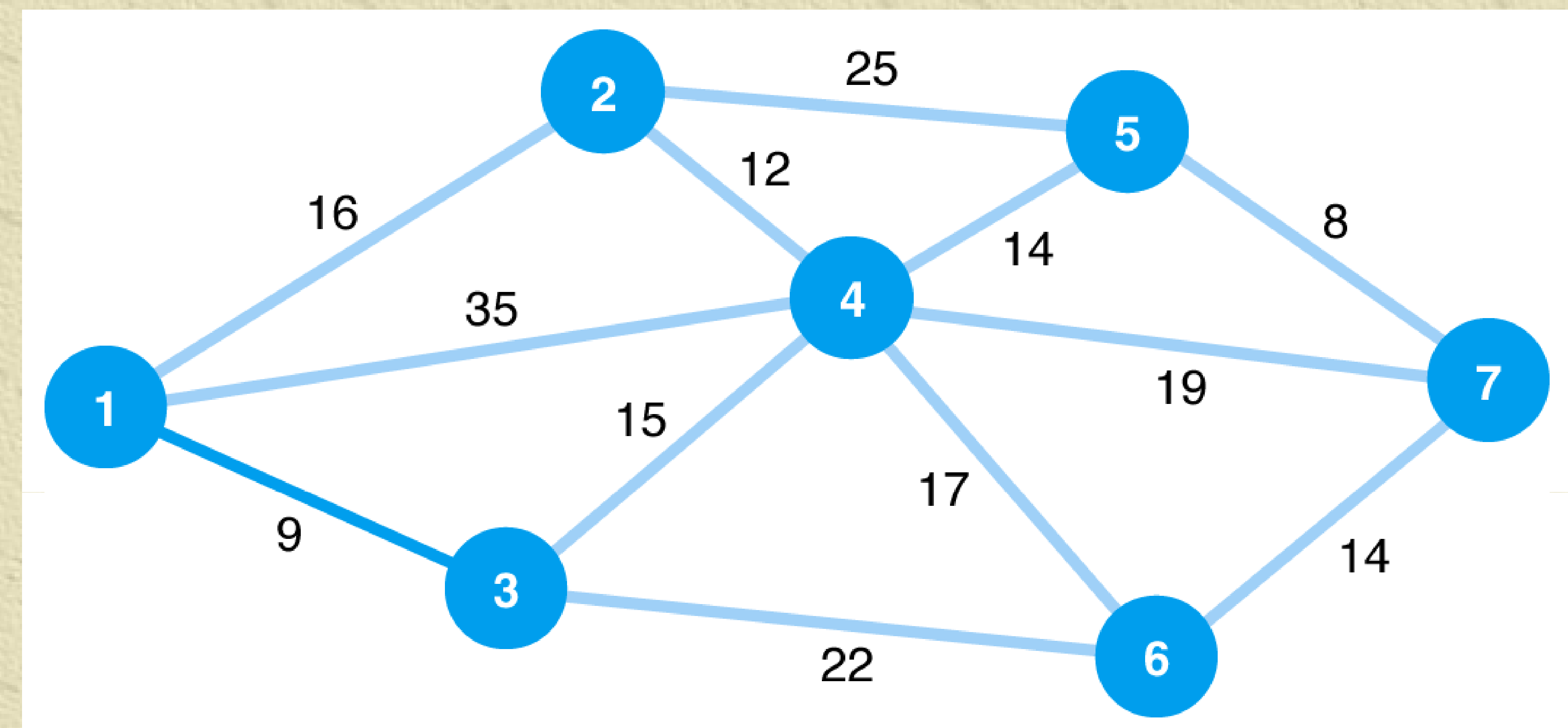


Figure 12.12
Spanning Tree with Nodes 1 and 3

The Minimal Spanning Tree Problem Solution Approach (2 of 6)

- Select the closest node not presently in the spanning area.

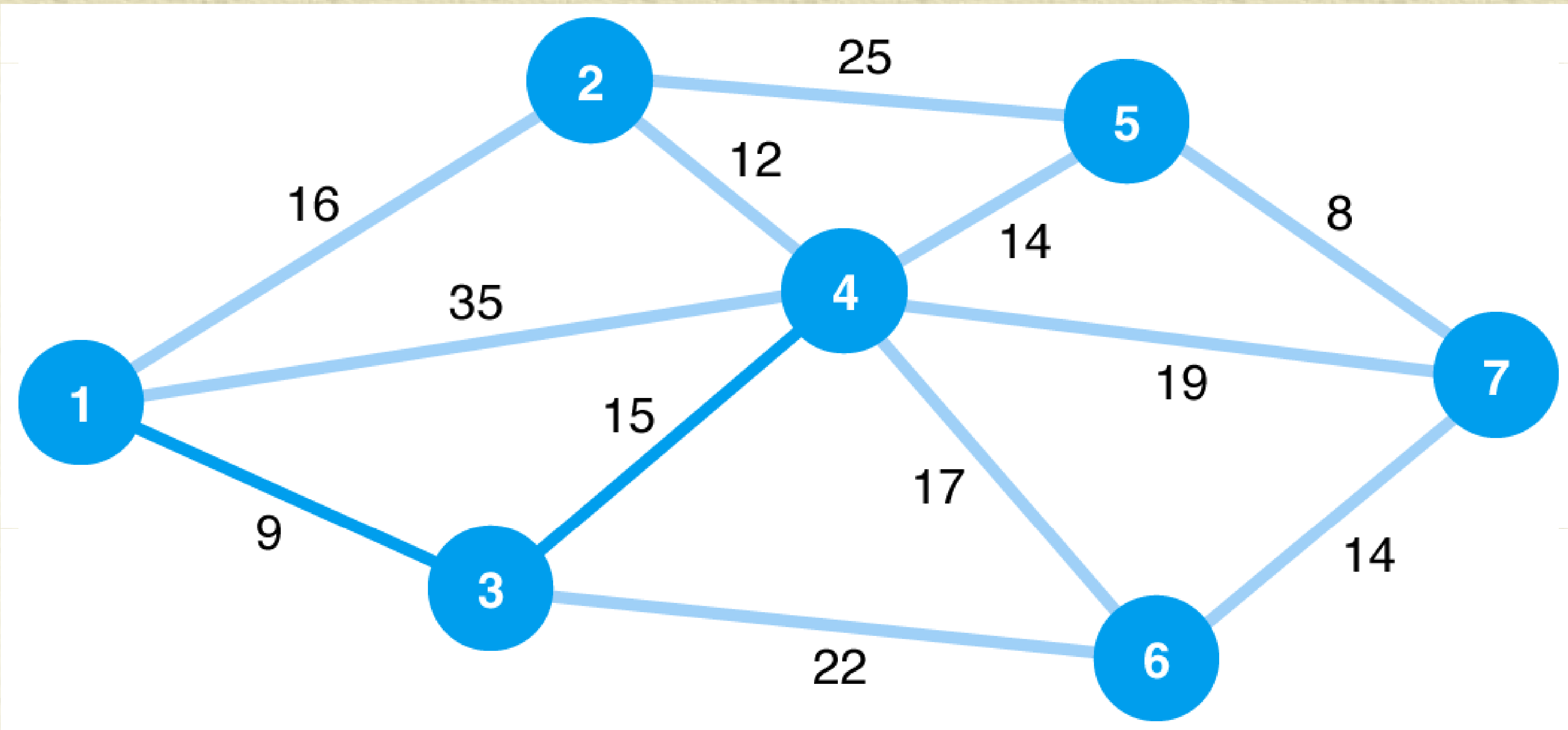


Figure 12.13
Spanning Tree with Nodes 1, 3, and 4

The Minimal Spanning Tree Problem Solution Approach (3 of 6)

- Continue

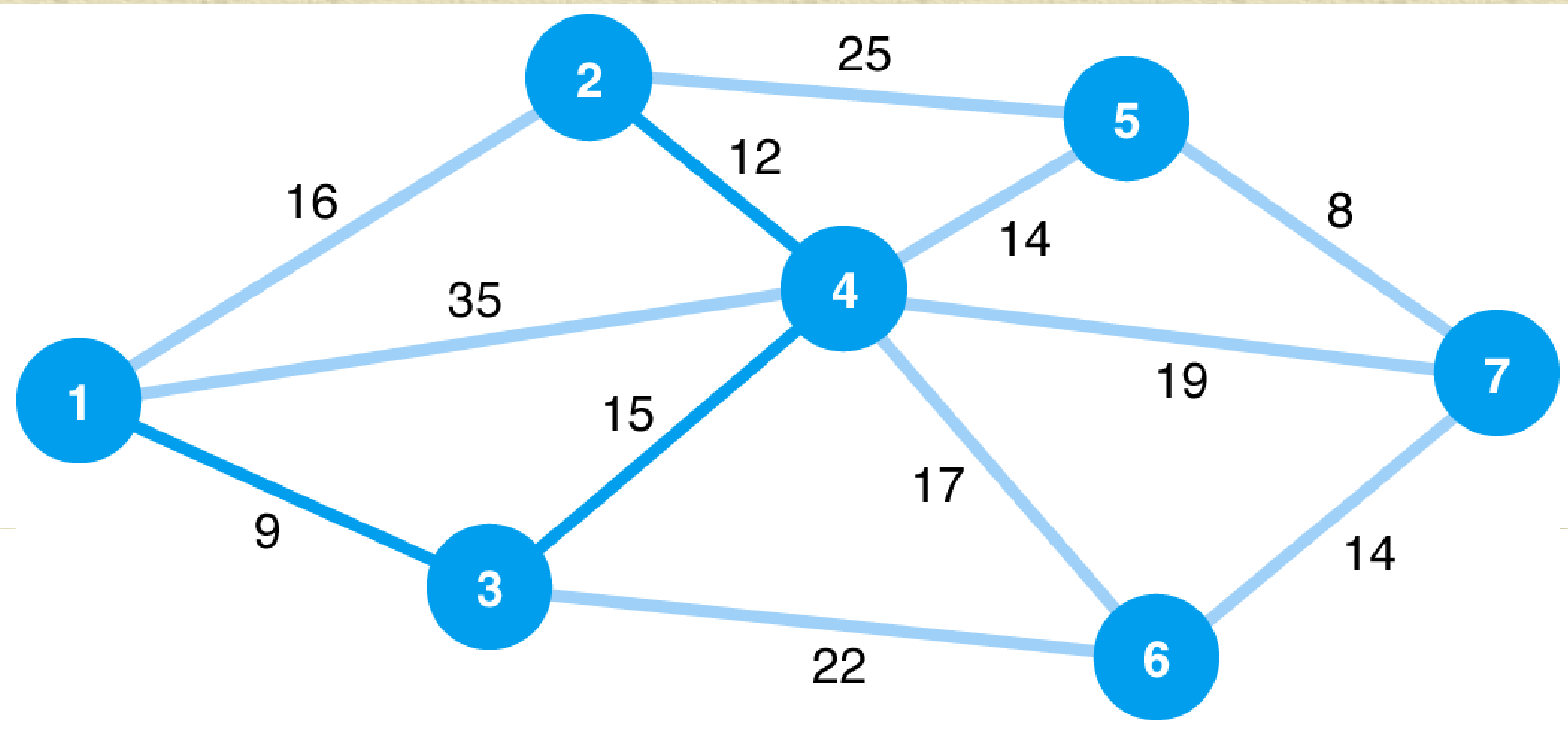


Figure 12.14
Spanning Tree with Nodes 1, 2, 3, and 4

The Minimal Spanning Tree Problem Solution Approach (4 of 6)

- Continue

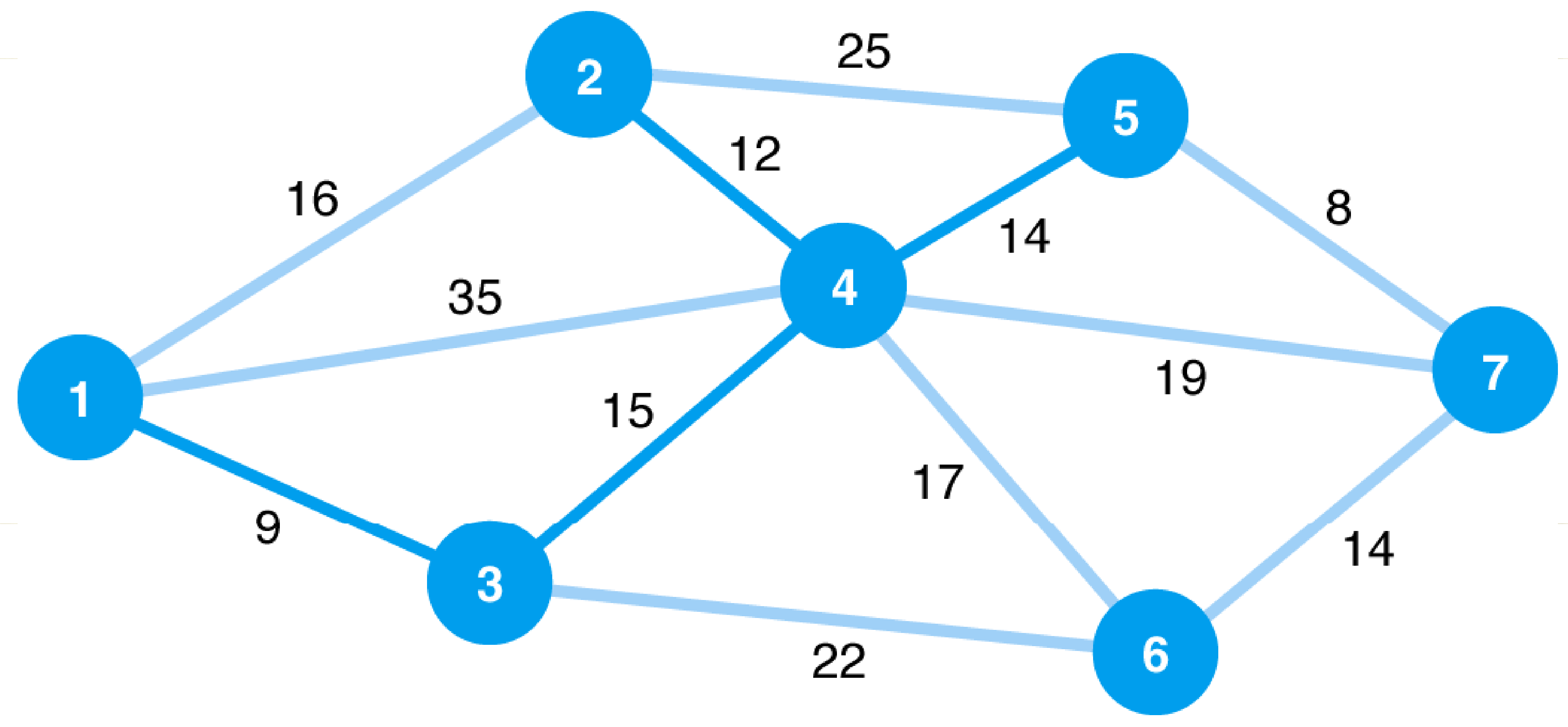


Figure 12.15
Spanning Tree with Nodes 1, 2, 3, 4, and 5

The Minimal Spanning Tree Problem Solution Approach (5 of 6)

- Continue

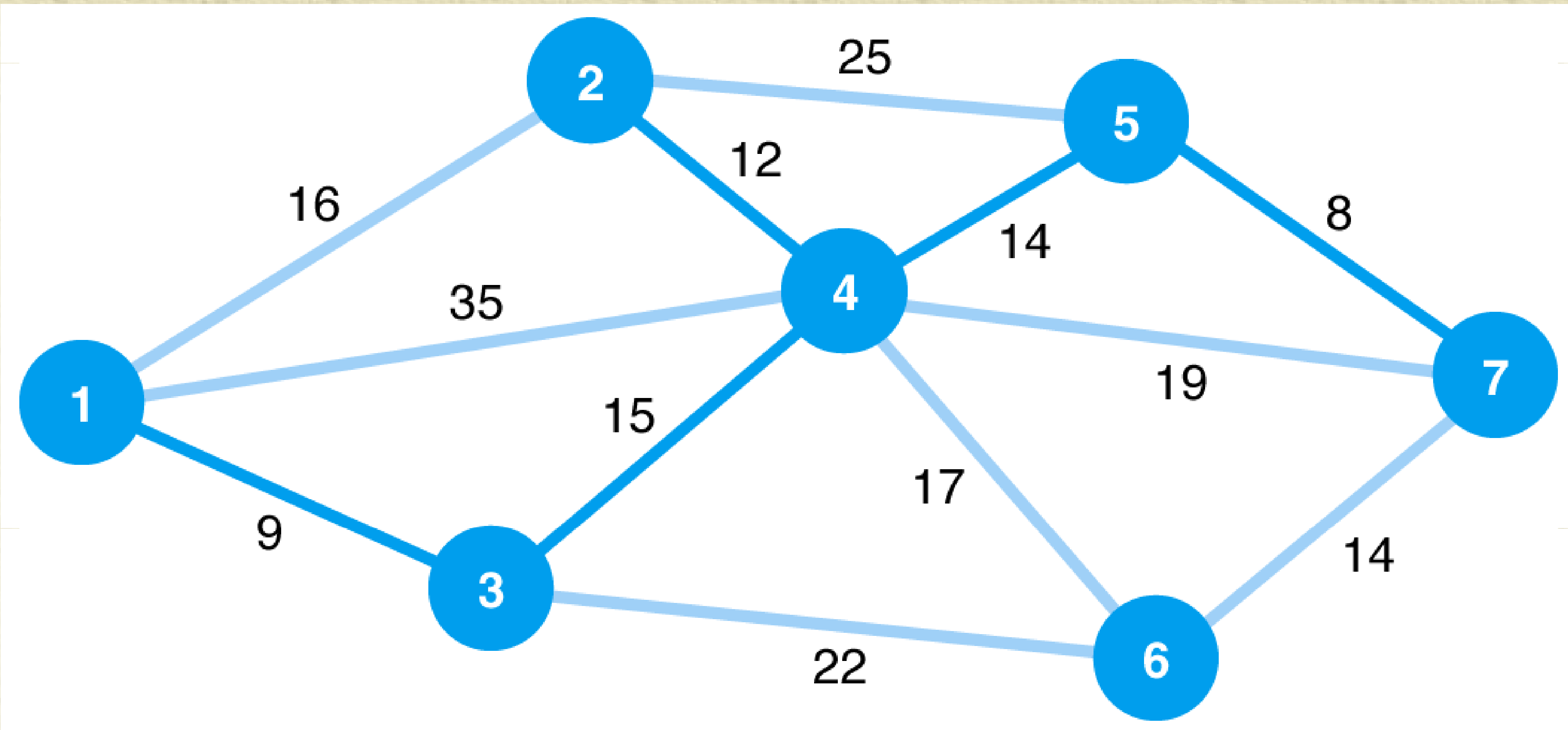


Figure 12.16
Spanning Tree with Nodes 1, 2, 3, 4, 5, and 7

The Minimal Spanning Tree Problem Solution Approach (6 of 6)

- Optimal Solution

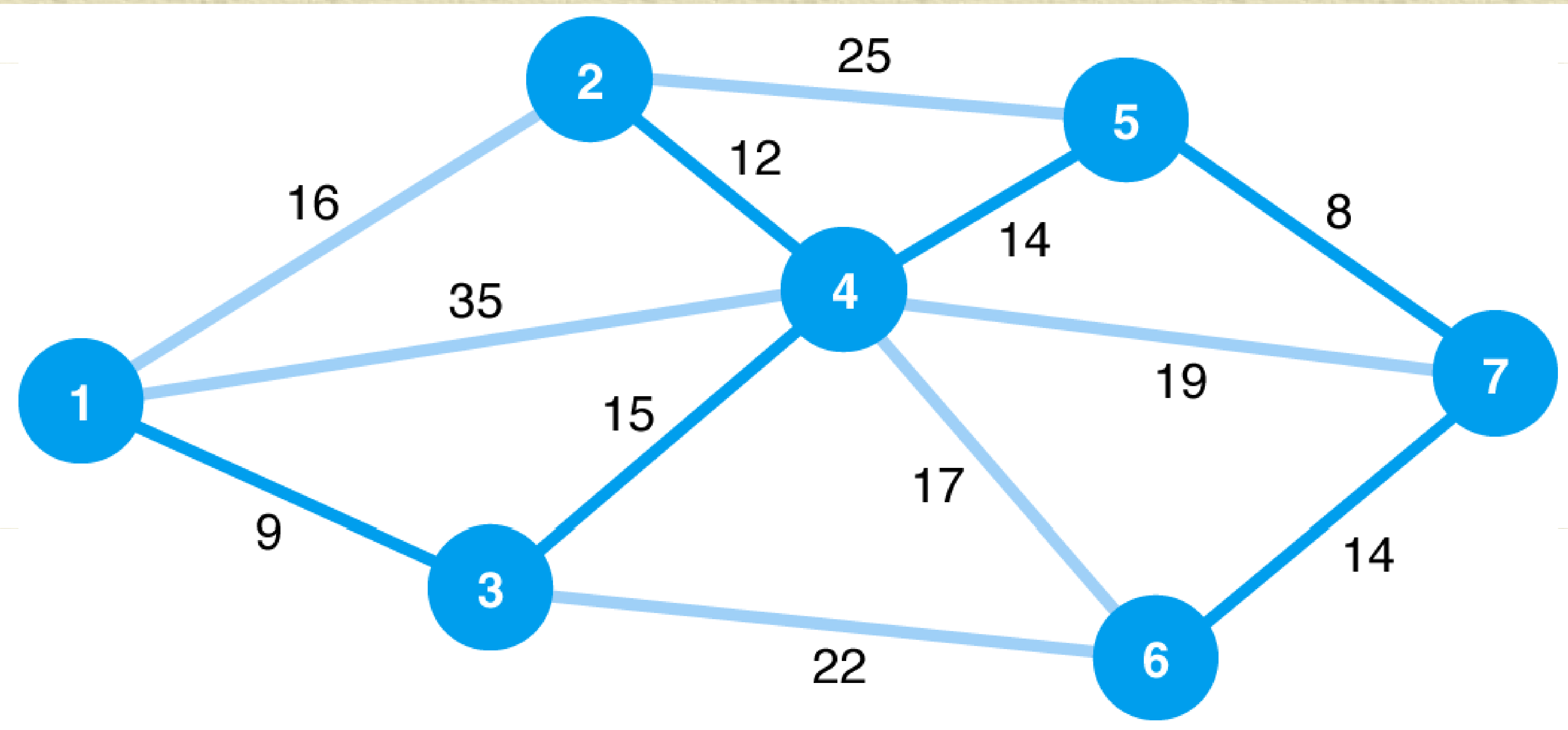


Figure 12.17
Minimal Spanning Tree for Cable TV Network

The Minimal Spanning Tree Problem Solution Method Summary

- Select any starting node (conventionally, node 1).
- Select the node closest to the starting node to join the spanning tree.
- Select the closest node not presently in the spanning tree.
- Repeat step 3 until all nodes have joined the spanning tree.

The Minimal Spanning Tree Problem Computer Solution with QM for Windows

Starting node for iterations: 1

Instruction: There are more results available in additional windows. These may be opened by using the WINDOW option in the Main Menu.

Networks Results

Metro Cable Television Company Solution

Branch name	Start node	End node	Cost	Include	Cost
1	1.	2.	16.		
2	1.	3.	9.	Y	9.
3	1.	4.	35.		
4	2.	4.	12.	Y	12.
5	2.	5.	25.		
6	3.	4.	15.	Y	15.
7	3.	6.	22.		
8	4.	5.	14.	Y	14.
9	4.	6.	17.		
10	4.	7.	19.		
11	5.	7.	8.	Y	8.
12	6.	7.	14.	Y	14.
Total					72.

Exhibit 12.6

The Maximal Flow Problem
Definition and Example Problem Data

- Problem: Maximize the amount of flow of items from an origin to a destination.

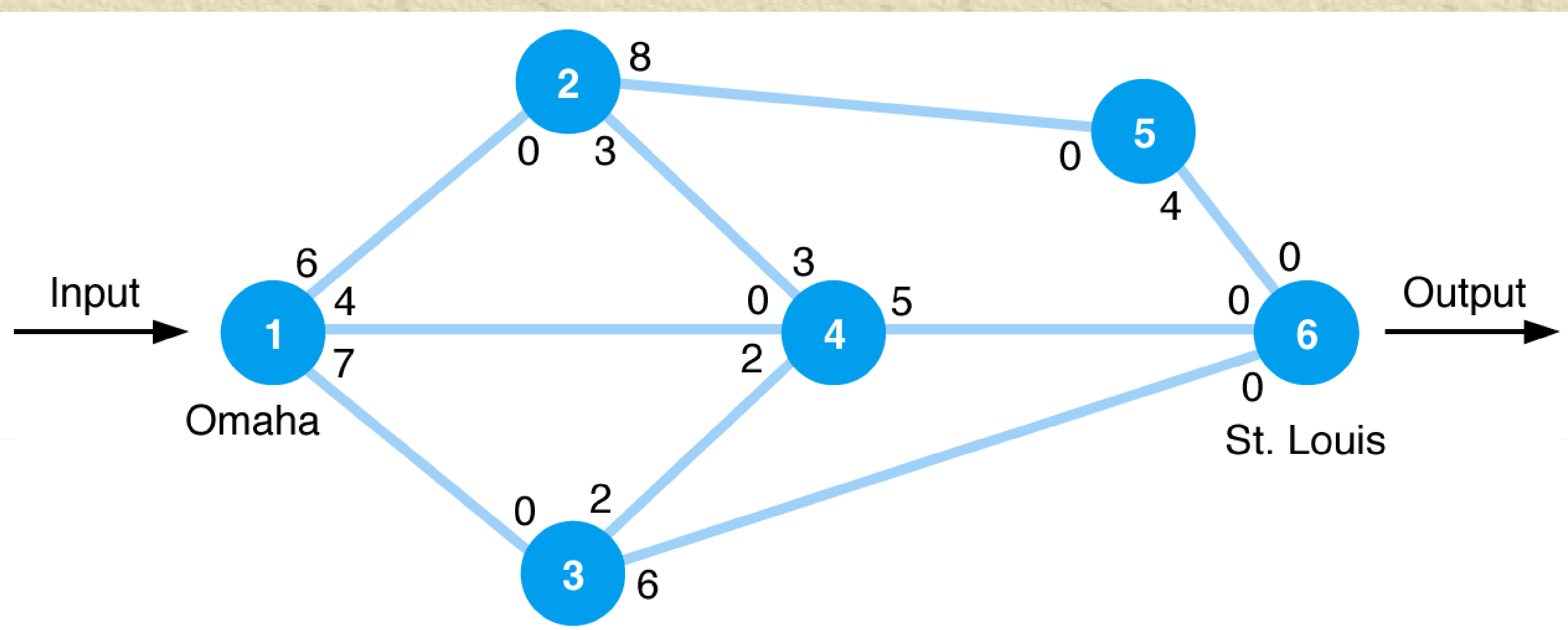


Figure 12.18
Network of Railway System

The Maximal Flow Problem
Solution Approach (1 of 5)

- Arbitrarily choose any path through the network from origin to destination and ship as much as possible.

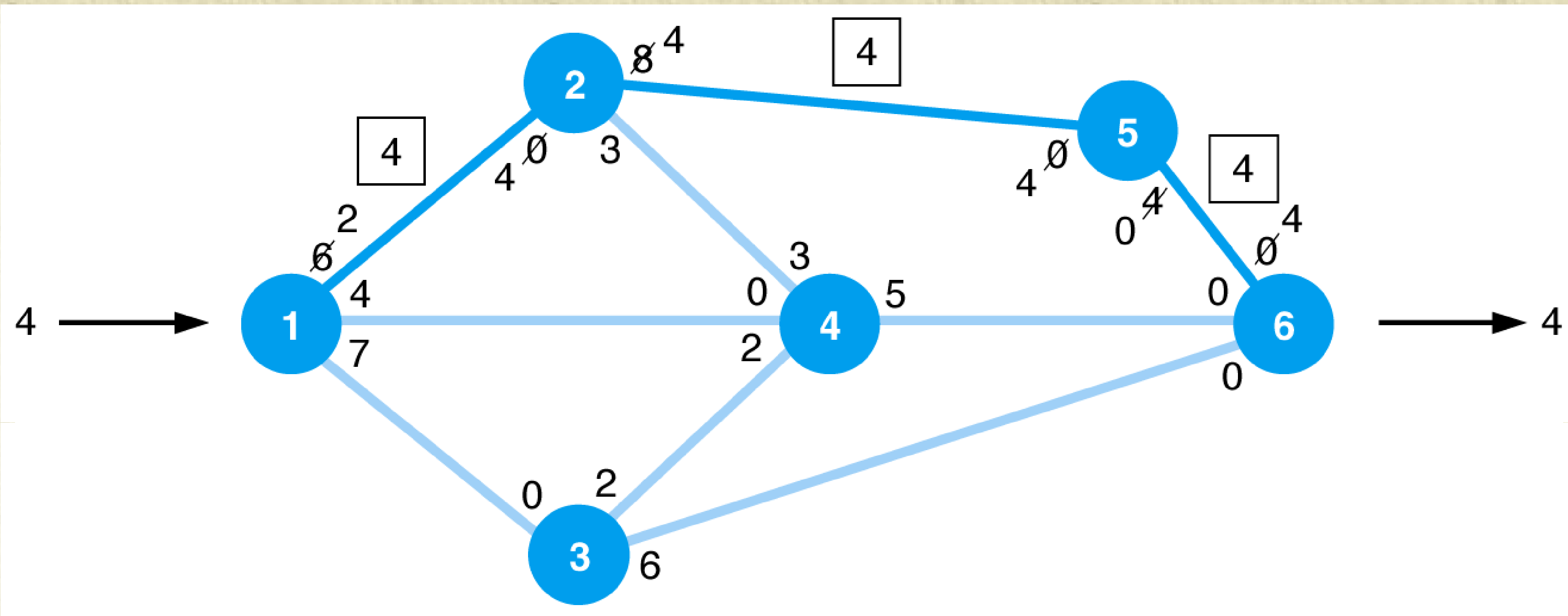


Figure 12.19
Maximal Flow for Path 1-2-5-6

The Maximal Flow Problem
Solution Approach (2 of 5)

- Re-compute branch flow in both directions and then select other feasible paths arbitrarily and determine maximum flow along the paths until flow is no longer possible.

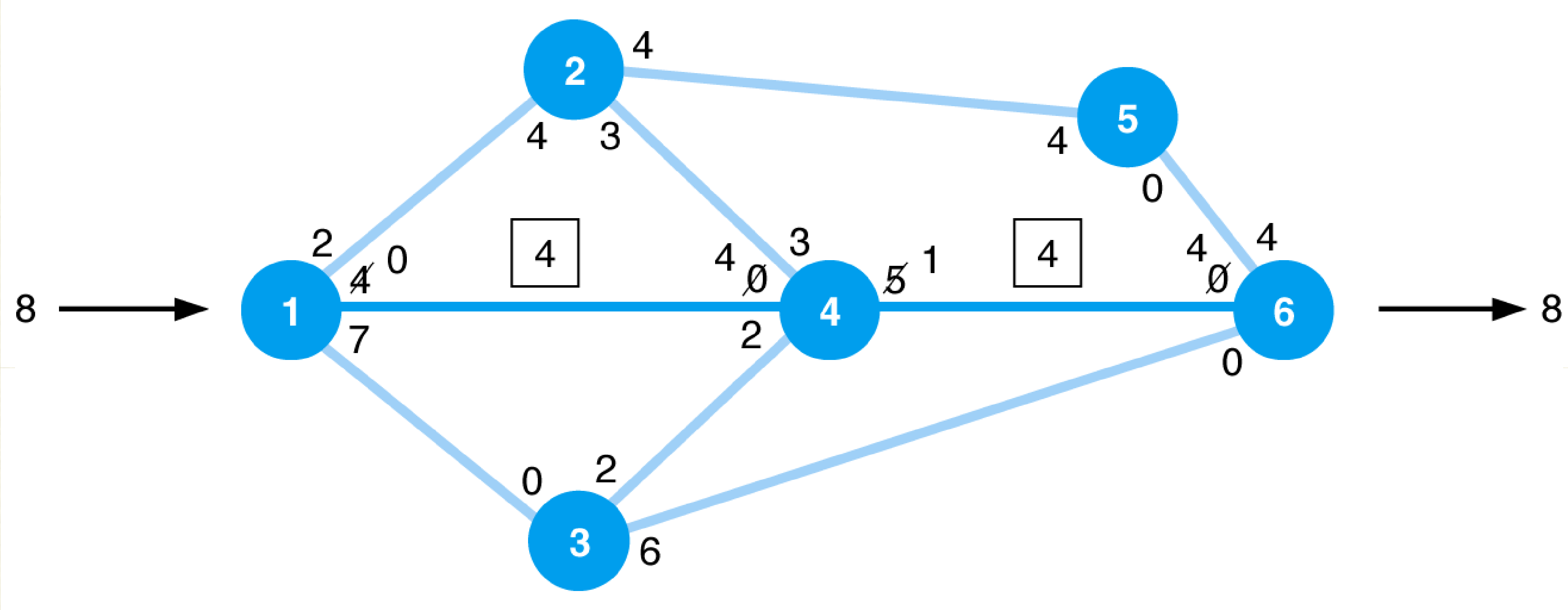


Figure 12.20
Maximal Flow for Path 1-4-6

The Maximal Flow Problem
Solution Approach (3 of 5)

- Continue

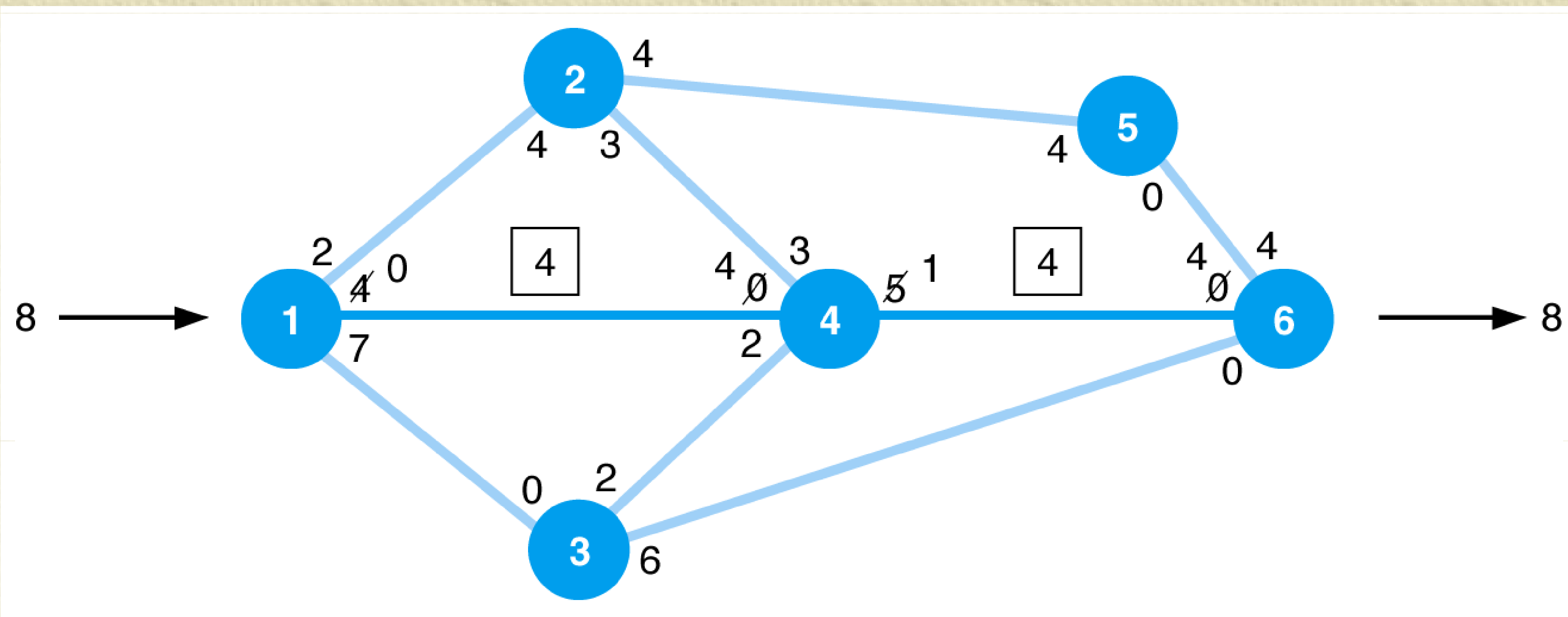


Figure 12.21
Maximal Flow for Path 1-3-6

The Maximal Flow Problem
Solution Approach (4 of 5)

- Continue

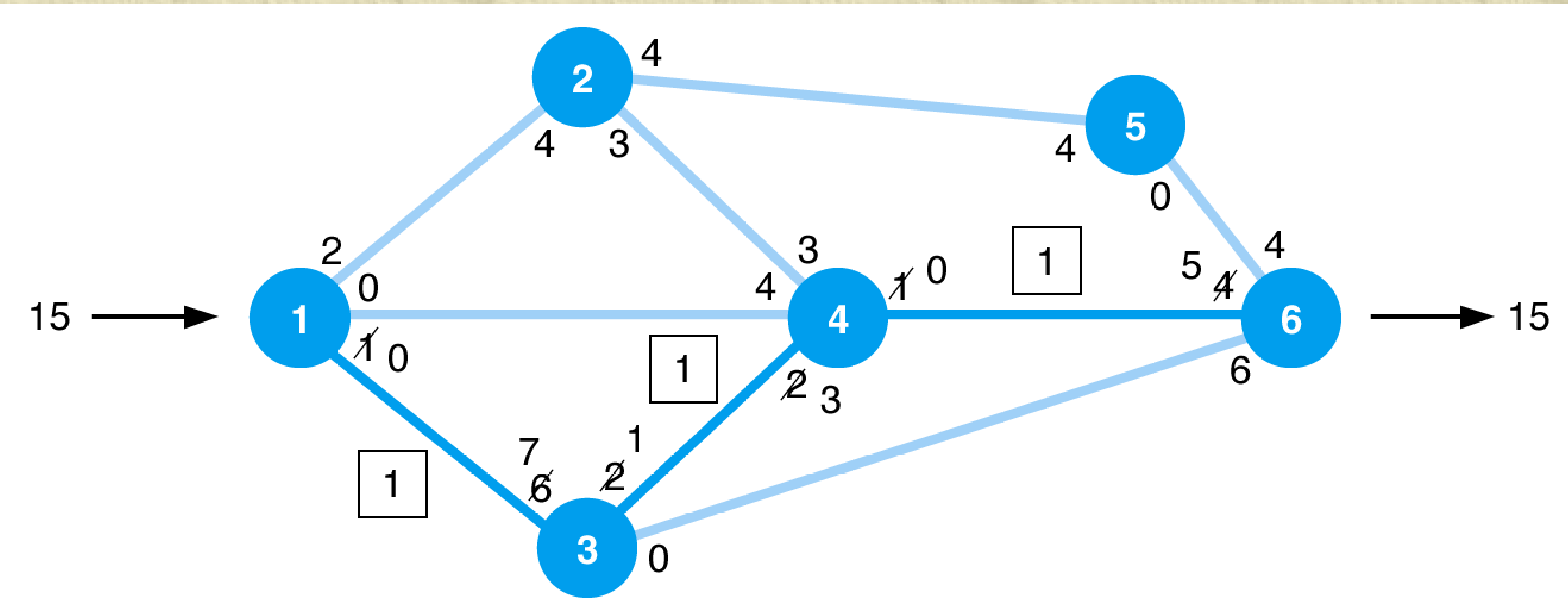


Figure 12.22
Maximal Flow for Path 1-3-4-6

The Maximal Flow Problem
Solution Approach (5 of 5)

- Optimal Solution

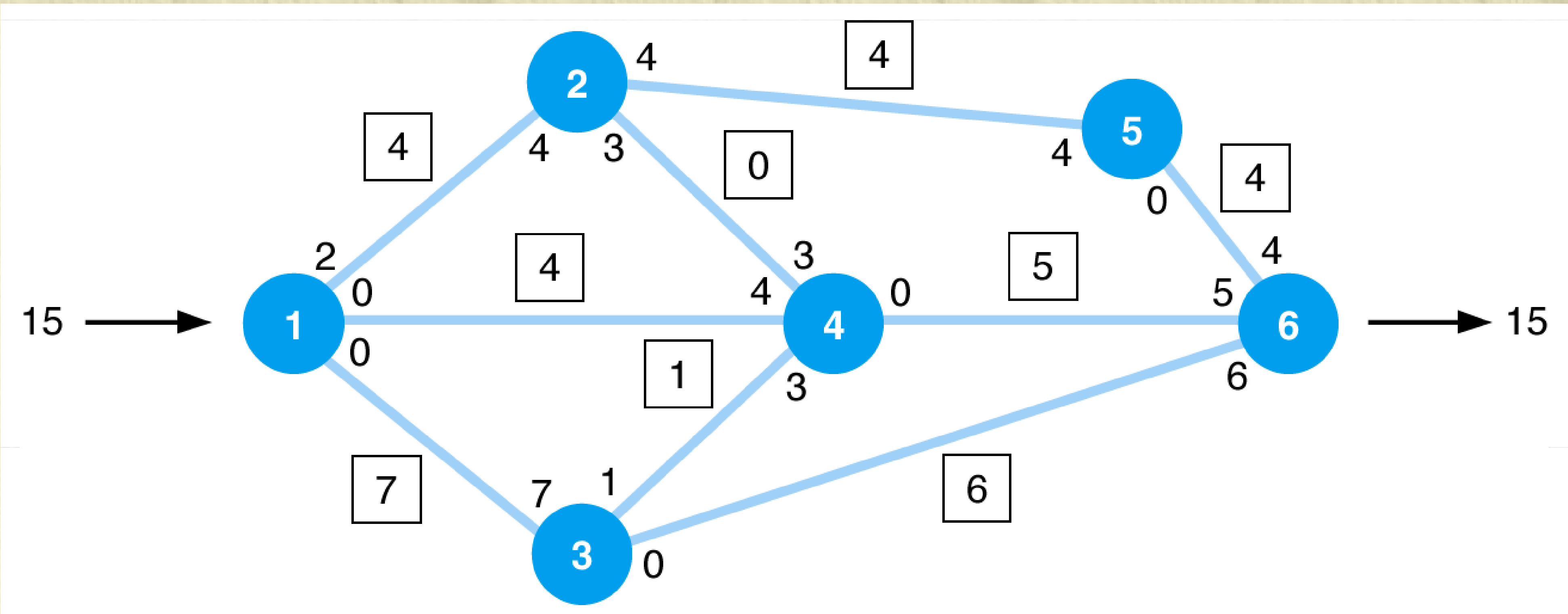


Figure 12.23
Maximal Flow for Railway Network

The Maximal Flow Problem Solution Method Summary

- Arbitrarily select any path in the network from origin to destination.
- Adjust the capacities at each node by subtracting the maximal flow for the path selected in step 1.
- Add the maximal flow along the path to the flow in the opposite direction at each node.
- Repeat steps 1, 2, and 3 until there are no more paths with available flow capacity.

The Maximal Flow Problem
Computer Solution with QM for Windows

Source: 1

Sink: 6

Instruction: There are more results available in additional windows. These may be opened by using the WINDOW option in the Main Menu.

Networks Results

Scott Tractor Company Solution

Branch name	Start node	End node	Capacity	Reverse capacity	Flow
Maximal Network Flow	15.				
1	1.	2.	6.	0.	5.
2	1.	3.	7.	0.	6.
3	1.	4.	4.	4.	4.
4	2.	4.	8.	3.	-4.
5	2.	5.	8.	0.	4.
6	3.	4.	2.	2.	0.
7	3.	6.	6.	0.	6.
8	4.	6.	5.	0.	5.
9	5.	6.	4.	0.	4.

Exhibit 12.7

The Maximal Flow Problem Computer Solution with Excel (1 of 4)

i_{ij} = flow along branch i-j and integer

Maximize $Z = x_{61}$

subject to:

$$x_{61} - x_{12} - x_{13} - x_{14} = 0$$
$$x_{12} - x_{24} - x_{25} = 0$$
$$x_{12} - x_{34} - x_{36} = 0$$
$$x_{14} + x_{24} + x_{25} - x_{46} = 0$$
$$x_{25} - x_{56} = 0$$
$$x_{36} + x_{46} + x_{56} - x_{61} = 0$$

$x_{12} \leq 6$ $\quad x_{24} \leq 3$ $\quad x_{34} \leq 2$

$x_{13} \leq 7$ $\quad x_{25} \leq 8$ $\quad x_{36} \leq 6$

$x_{14} \leq 4$ $\quad x_{46} \leq 5$ $\quad x_{56} \leq 4$

$x_{61} \leq 17$ $\quad x_{ij} \geq 0$

The Maximal Flow Problem
Computer Solution with Excel (2 of 4)

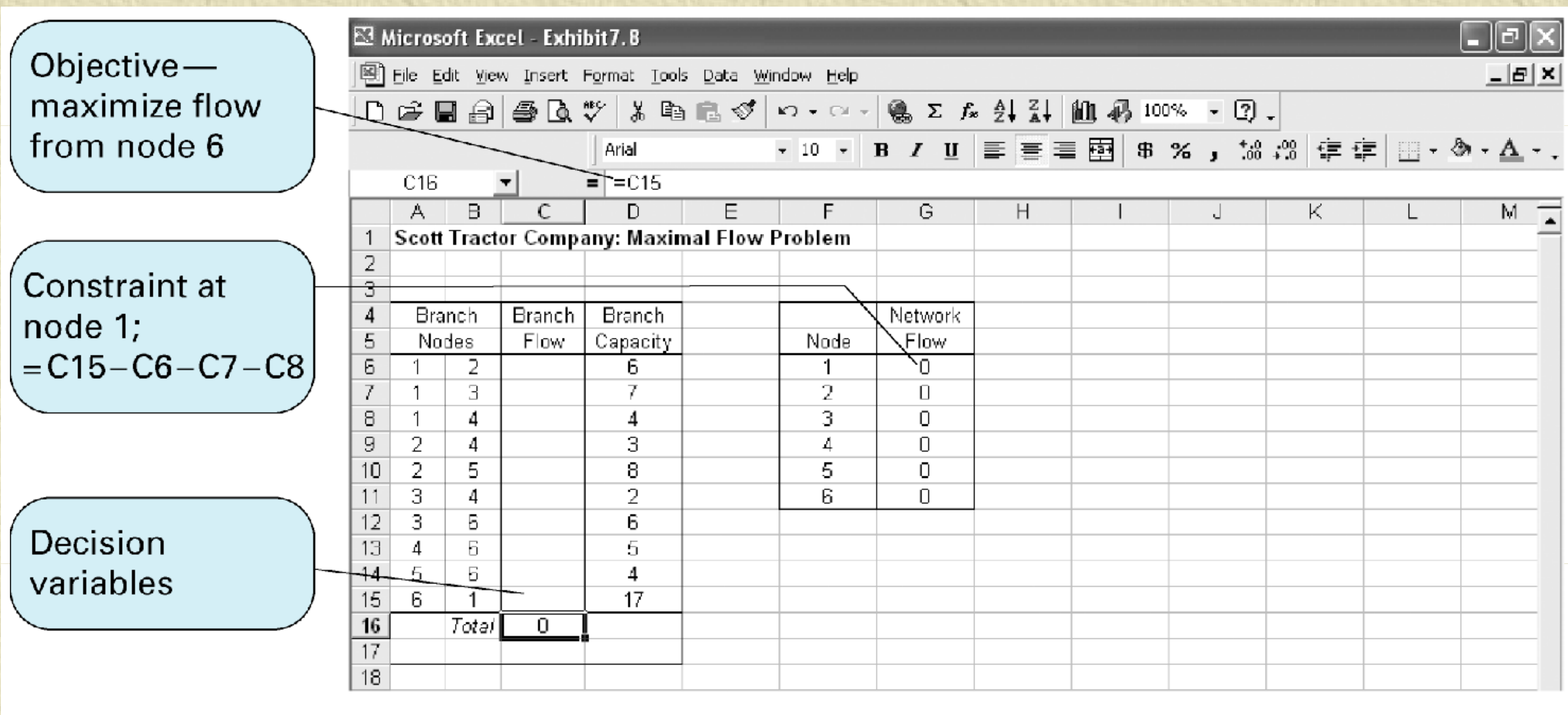


Exhibit 12.8

The Maximal Flow Problem
Computer Solution with Excel (3 of 4)

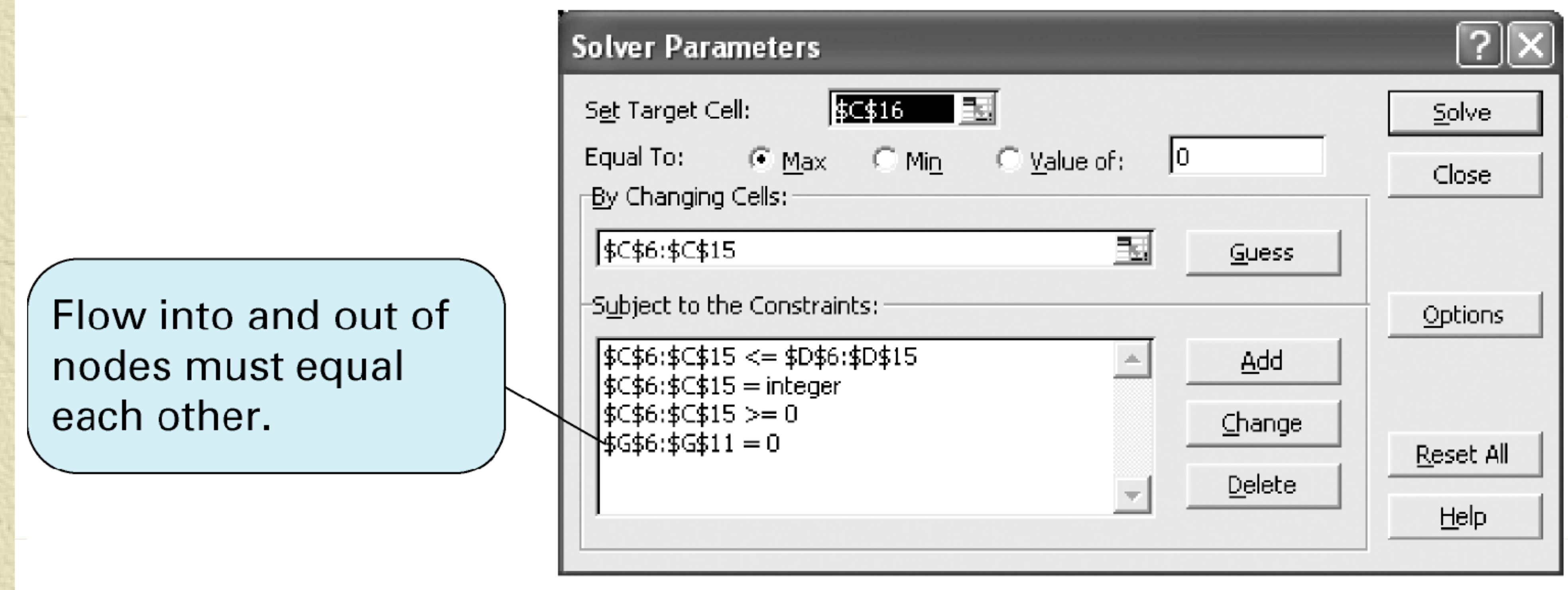


Exhibit 12.9

The Maximal Flow Problem
Computer Solution with Excel (4 of 4)

Microsoft Excel - Ch7-MaxFlow.xls

C16 = =C15

Scott Tractor Company: Maximal Flow Problem

Branch Nodes		Branch Flow	Branch Capacity
1	2	4	6
1	3	7	7
1	4	4	4
2	4	0	3
2	5	4	8
3	4	1	2
3	6	6	6
4	6	5	5
5	6	4	4
6	1	15	17
	Total	15	

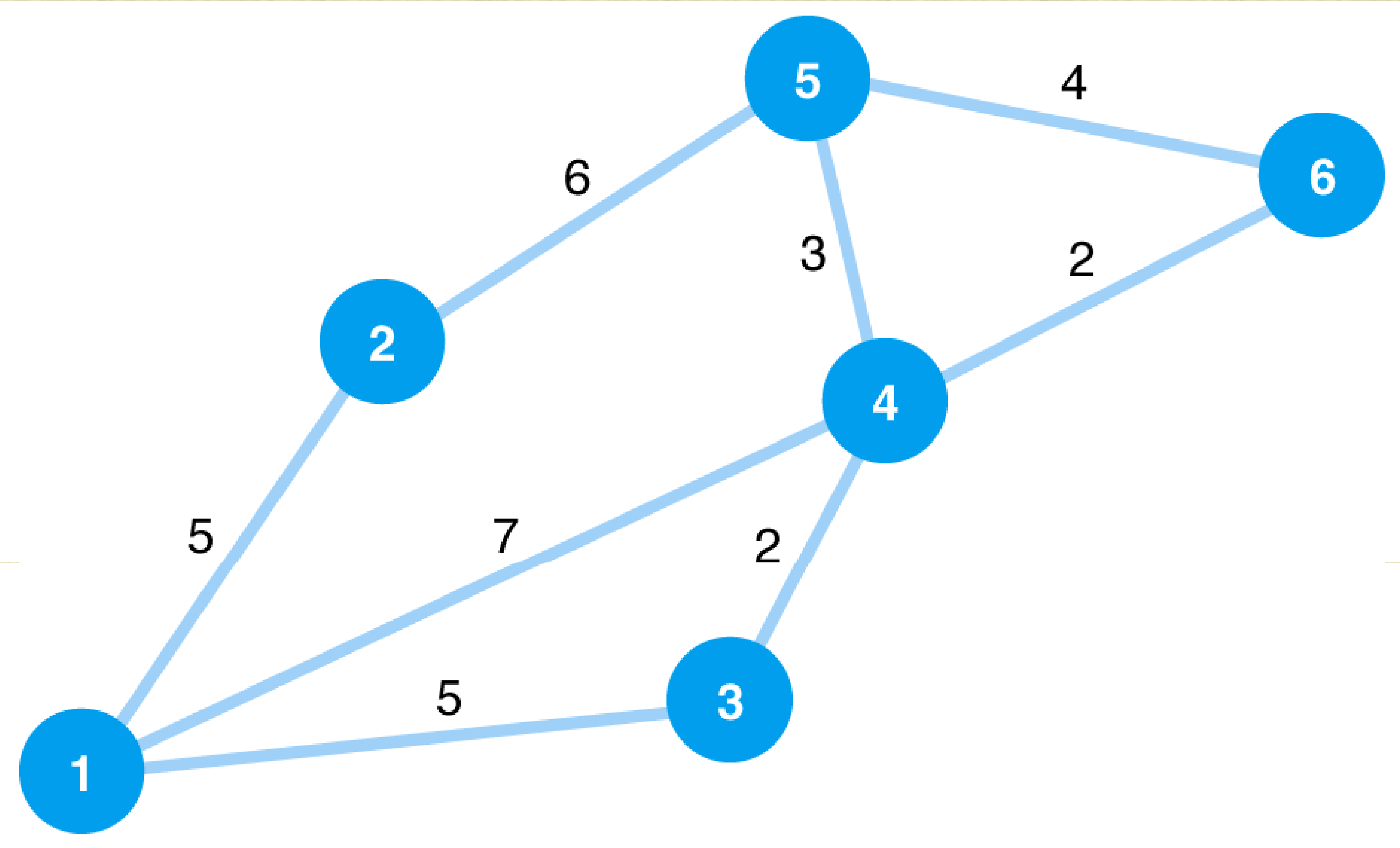
Node	Network Flow
1	0
2	0
3	0
4	0
5	0
6	0

Exhibit 12.10

The Maximal Flow Problem
Example Problem Statement and Data (1 of 2)

- Determine the shortest route from Atlanta (node 1) to each of the other five nodes (branches show travel time between nodes).
- Assume branches show distance (instead of travel time) between nodes, develop a minimal spanning tree.

The Maximal Flow Problem
Example Problem Statement and Data (2 of 2)

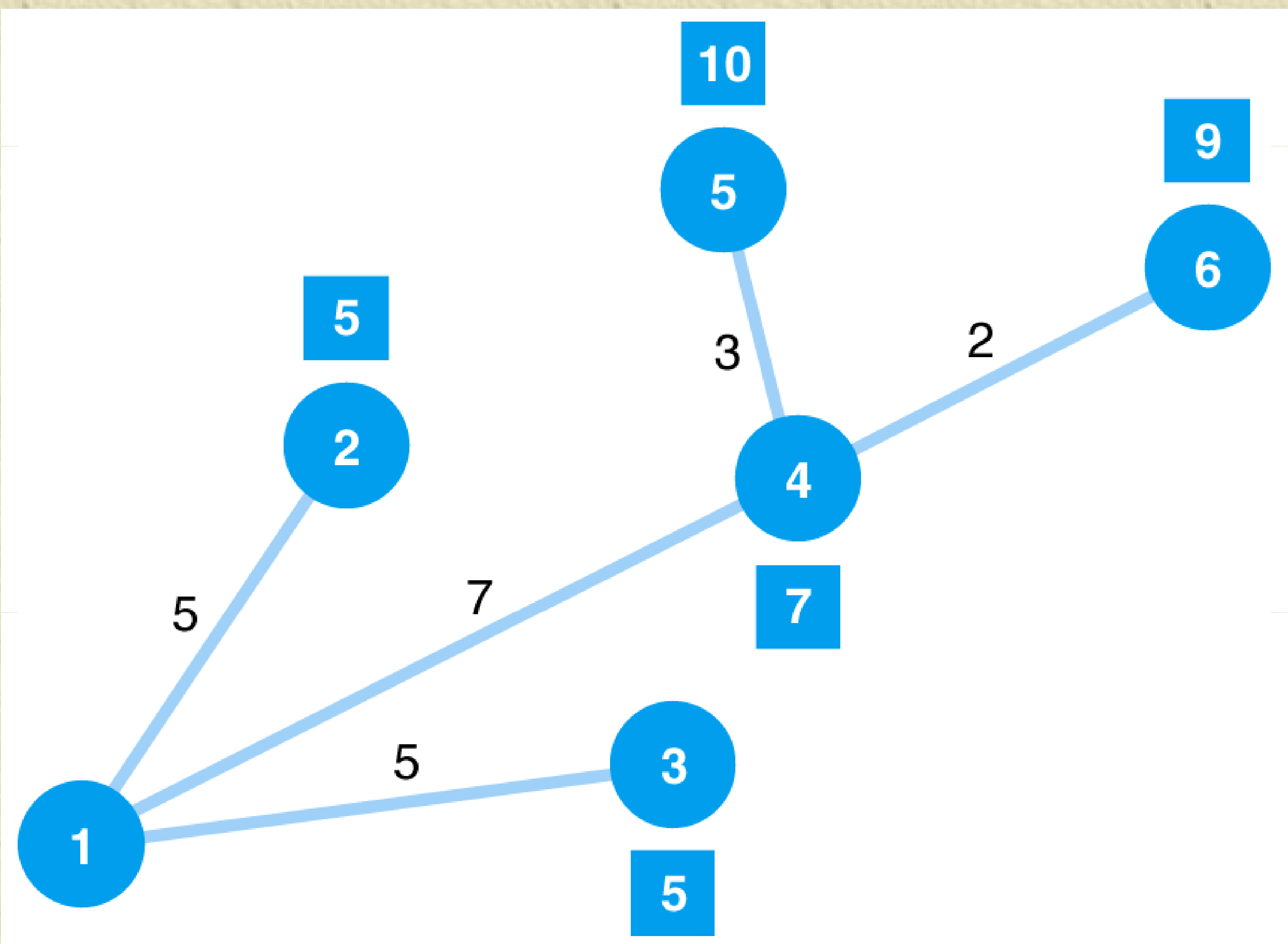


The Maximal Flow Problem
Example Problem, Shortest Route Solution (1 of 2)

Step 1 (part A): Determine the Shortest Route Solution

1.	Permanent Set	Branch	Time
	{1}	1-2	[5]
		1-3	5
		1-4	7
2.	{1,2}	1-3	[5]
		1-4	7
		2-5	11
3.	{1,2,3}	1-4	[7]
		2-5	11
		3-4	7
4.	{1,2,3,4}	4-5	10
		4-6	[9]
5.	{1,2,3,4,6}	4-5	[10]
		6-5	13
6.	{1,2,3,4,5,6}		

The Maximal Flow Problem
Example Problem, Shortest Route Solution (2 of 2)



The Maximal Flow Problem
Example Problem, Minimal Spanning Tree (1 of 2)

- The closest unconnected node to node 1 is node 2.
- The closest to 1 and 2 is node 3.
- The closest to 1, 2, and 3 is node 4.
- The closest to 1, 2, 3, and 4 is node 6.
- The closest to 1, 2, 3, 4 and 6 is 5.
- The shortest total distance is 17 miles.

The Maximal Flow Problem
Example Problem, Minimal Spanning Tree (2 of 2)