# Lecture 1
## Introduction to Probability and Set Theory
Text: A Course in Probability by Weiss 1.2 $\sim$ 2.3

*STAT 225 Introduction to Probability Models*
January 13, 2014

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Notes



## Agenda

1. Introduction
2. Set Theory
3. Probability

Introduction
Set Theory
Probability



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## Motivation

Uncertainty/Randomness in our life

- Will it snow tomorrow?
- Can LeBron James and the Miami Heat win a third NBA title this year?
- If I flip a coin 100 times, what are the chances I get 50 heads and 50 tails?

We often want to assess how likely of such event occurs$\Rightarrow$ Probability is the right tool






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## Definitions

Probability theory is based on the paradigm of a random experiment, i.e. an action whose outcome cannot be predicted beforehand.

- Element: a single item (outcome), typically denoted by $\omega$
- Set: a collection of elements
- Sample space: the set of all possible outcomes for a random experiment and is denoted by $\Omega$
- Subset: a set itself which every element is contained in a large set.





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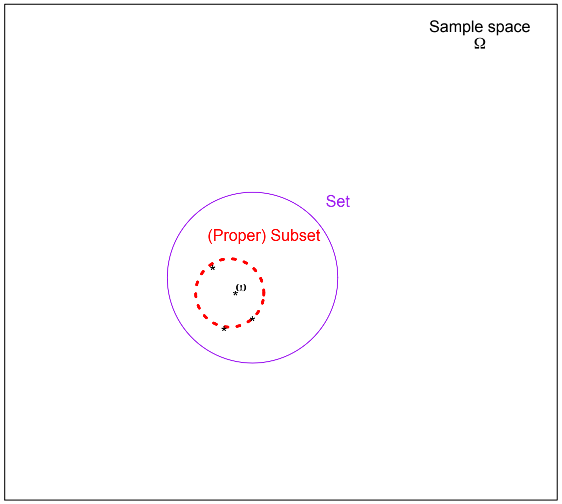
## Sample Space, Set, Subset



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## Example 1

We are interested in whether the price of the *S&P* 500 decreases, stays the same, or increases. If we were to examine the *S&P* 500 over one day, then $\Omega = \{$decrease, stays the same, increases$\}$. What would $\Omega$ be if we looked at 2 days?

**Solution.**



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## Definitions (cont'd)







- Empty (null) set: the opposite of the sample space. It is the set with 0 element and is written as $\emptyset$. $\Omega$ and $\emptyset$ are complements
- Complement: a set that contains all of the elements in the sample space that are not in the original set
- Event: any subset of the sample space. It can be one or more elements



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## Complement

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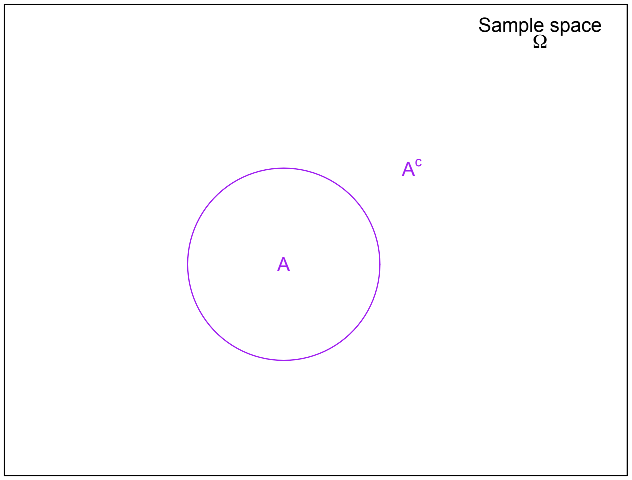




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## Example 2

Let us examine what happens in the flip of 3 fair coins. In this case $\Omega = \{(T, T, T), (T, T, H), (T, H, T), (H, T, T), (T, H, H), (H, T, H), (H, H, T), (H, H, H)\}$. Let $A$ be the event of exactly 2 tails. Let $B$ be the event that the first 2 tosses are tails. Let $C$ be the event that all 3 tosses are tails. Write out the possible outcomes for each of these 3 events

**Solution.**





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## Example 3

Start with a standard deck of 52 cards and remove all the hearts and all the spades, leaving 13 red and 13 black cards. List the cards in each of the following sets:

- $N =$ not a face card
- $R =$ neither red nor an ace
- $E =$ either black, even, or a Jack





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## Example 4

Suppose a fair six–sided die is rolled twice. Determine the number of possible outcomes

- For this experiment
- The sum of the two rolls is 5
- The two rolls are the same
- The sum of the two rolls is an even number

**Solution.**





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## Frequentist Interpretation of Probability

The probability of an event is the long–run proportion of times that the event occurs in independent repetitions of the random experiment. This is referred to as an empirical probability and can be written as

$$\mathbb{P}(event) = \frac{\text{number of times that event occurs}}{\text{number of random experiment}}$$





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## Equally Likely Framework

$$\mathbb{P}(\textit{event}) = \frac{\text{number of times that event occurs}}{\text{number of all possible outcomes}}$$

**Remark:**

- Any individual outcome of the sample space is equally likely as any other outcome in the sample space.
- In an equally likely framework, the probability of any event is the number of ways the event occurs divided by the number of total events possible.





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## Example 5

Find the probabilities associated with parts 2–4 of Example 4

**Solution.**





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## Probability Rules

1. Any probability must be between 0 and 1 inclusively
2. The sum of the probabilities for all the experimental outcomes must equal 1

If a probability model satisfies the two rules above, it is said to be legitimate





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## Example 6

An experiment with three outcomes has been repeated 50 times, and it was learned that outcome 1 occurred 20 times, outcome 2 occurred 13 times, and outcome 3 occurred 17 times. Assign probabilities to the outcomes. What method did you use?

**Solution.**

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## Example 7

A decision maker subjectively assigned the following probabilities to the four outcomes of an experiment:

$$\mathbb{P}(E_1) = 0.1 \quad \mathbb{P}(E_2) = 0.15 \quad \mathbb{P}(E_3) = 0.4 \quad \mathbb{P}(E_4) = 0.2$$

Are these probability assignments legitimate? Explain.

**Solution.**



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## Summary

In this lecture, we learned

- Set theory definitions: sample space, set, subset, element, empty set, complement, event
- The Frequentist Interpretation of Probability and the Equally Likely Framework
- Probability Rules



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## Homework 0

1. Read the Syllabus!
2. Visit the main course website at http://www.stat.purdue.edu/ cfurtner251/stat225http://www.stat.purdue.edu/ cfurtner/-stat225 and course website for section 081/091 http://www.stat.purdue.edu/ huang251/stat225.html





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