

Introduction to Process Control

Lecture 1, 2016/2017

Control & System Eng. Dept. , 4th year

Subject: Process Control.

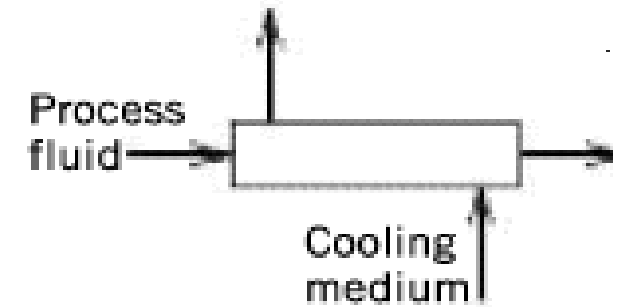
Dr. Safanah M. Raafat

REPRESENTATIVE PROCESS CONTROL PROBLEMS

- ***Process:*** *The conversion of feed materials to products using chemical and physical operations.*
- *In practice, the term process tends to be used for both the processing operation and the processing equipment.*
- Note: this definition applies to three types of common processes:
 - continuous,
 - batch,
 - and semi-batch

Continuous Processes

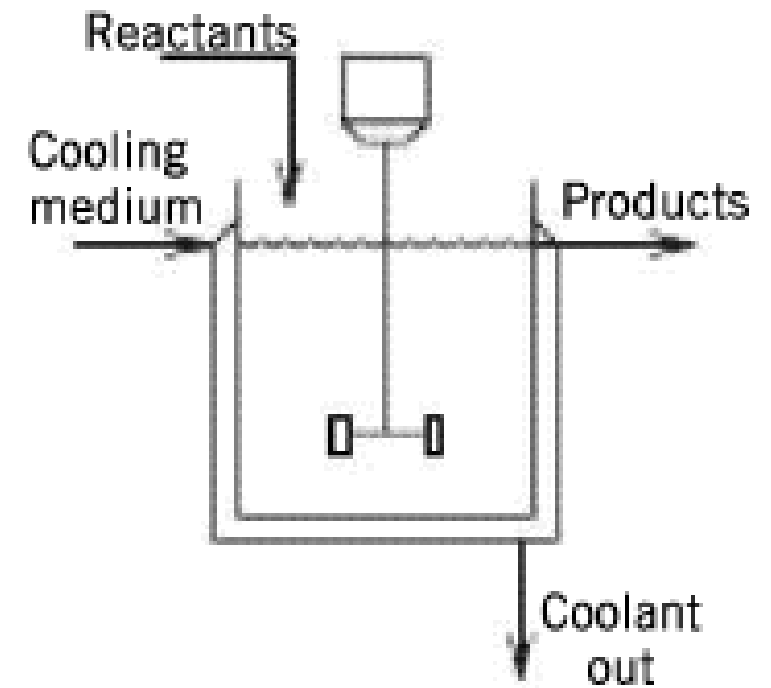
- ***Tubular heat exchanger.*** A process fluid on the tube side is cooled by cooling water on the shell side.
- Typically, the exit temperature of the process fluid is controlled by manipulating the cooling water flow rate. Variations in the inlet temperatures and the process fluid flow rate affect the heat exchanger operation. Consequently, these variables are considered to be disturbance variables.



(a) Heat exchanger

Continuous Processes-cont.

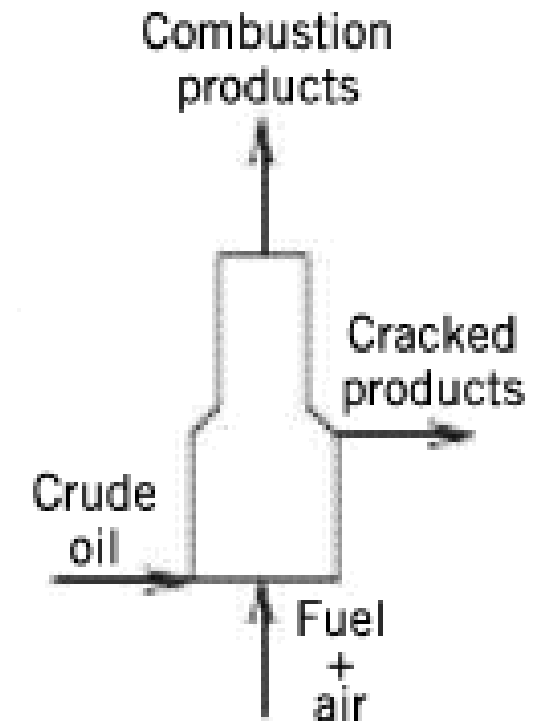
- ***Continuous stirred-tank reactor (CSTR).***
- If the reaction is highly exothermic, it is necessary to control the reactor temperature by manipulating the flow rate of coolant in a jacket or cooling coil. The feed conditions (composition, flow rate, and temperature) can be manipulated variables or disturbance variables.



(b) Chemical reactor

Continuous Processes-cont.

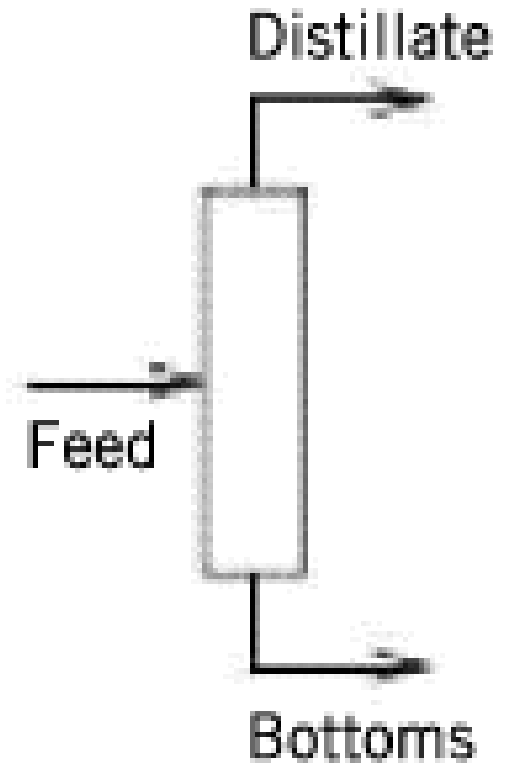
- ***Thermal cracking furnace.***
- Crude oil is broken down ("cracked") into a number of lighter petroleum fractions by the heat transferred from a burning fuel/air mixture. The furnace temperature and amount of excess air in the flue gas can be controlled by manipulating the fuel flow rate and the fuel/air ratio. The crude oil composition and the heating quality of the fuel are common disturbance variables.



(c) Cracking furnace

Continuous Processes-cont.

- ***Multicomponent distillation column.***
- Many different control objectives can be formulated for distillation columns.
- For example, the distillate composition can be controlled by adjusting the reflux flow rate or the distillate flow rate. If the composition cannot be measured on-line, a tray temperature near the top of the column can be controlled instead. If the feed stream is supplied by an upstream process, the feed conditions will be disturbance variables.



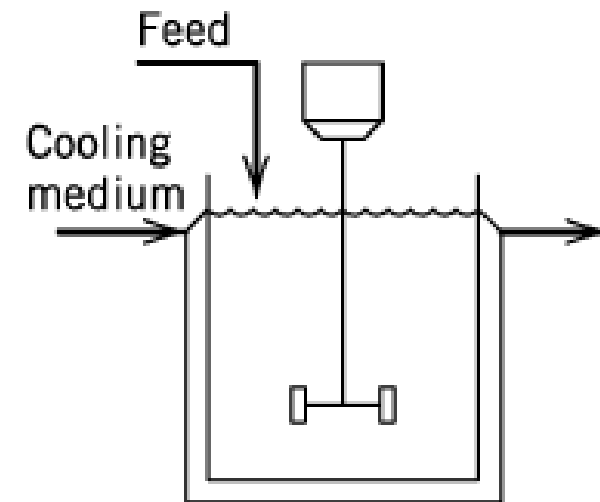
(d) Distillation column

- For each of these four examples, the process control problem has been characterized by identifying three important types of process variables.
- ***Controlled variables (CVs)***: The process variables that are controlled. The desired value of a controlled variable is referred to as its *set point*.
- ***Manipulated variables (MVs)***: The process variables that can be adjusted in order to keep the controlled variables at or near their set points. Typically, the manipulated variables are flow rates.
- ***Disturbance variables (DVs)***: Process variables that affect the controlled variables but cannot be manipulated. Disturbances generally are related to changes in the operating environment of the process:
 - for example, its feed conditions or ambient temperature. Some disturbance variables can be measured on-line, but many cannot such as the crude oil composition for Process (c), a thermal cracking furnace.

The specification of CVs, MVs, and DVs is a critical step in developing a control system.

Batch and Semi-Batch Processes

- Batch and semi-batch processes provide needed flexibility for multiproduct plants, especially when products change frequently and production quantities are small.
- ***Batch or semi-batch reactor.***
- An initial charge of reactants is brought up to reaction conditions, and the reactions are allowed to proceed for a specified period of time or until a specified conversion is obtained.
- Typically, the reactor temperature is controlled by manipulating a coolant flow rate.
- Batch and semi-batch reactors are used routinely in specialty chemical plants, polymerization plants, and in pharmaceutical and other bioprocessing facilities

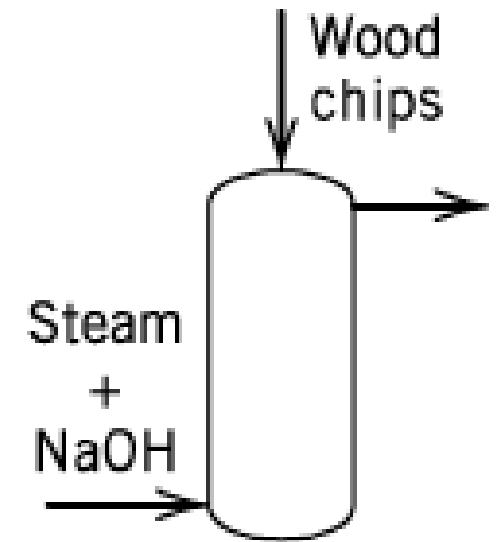


(e) Semi-batch reactor

Batch and Semi-Batch Processes-cont.

Batch digester in a pulp mill.

- Both continuous and semi-batch digesters (شبه دفعة هاضم) are used in paper manufacturing to break down wood chips in order to extract the cellulosic fibers. The end point of the chemical reaction is indicated by the kappa number, a measure of lignin content. It is controlled to a desired value by adjusting the digester temperature, pressure, and/or cycle time.



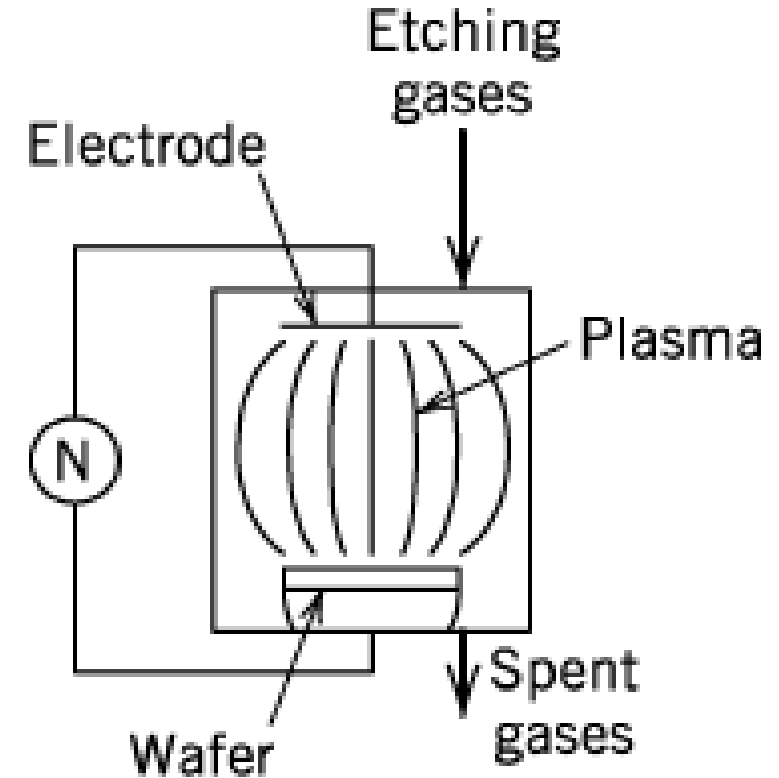
(f) Wood chip digester

- (lignin content : a complex organic polymer deposited in the cell walls of many plants)

Batch and Semi-Batch Processes-cont.

Plasma etcher in semiconductor processing

- A single wafer (رقاقة) containing hundreds of printed circuits is subjected to a mixture of etching gases (غازات النقش)
-) under conditions suitable to establish and maintain a plasma (a high voltage applied at high temperature and extremely low pressure). The unwanted material on a layer of a microelectronics circuit is selectively removed by chemical reactions. The temperature, pressure, and flow rates of etching gases to the reactor are controlled by adjusting electrical heaters and control valves.

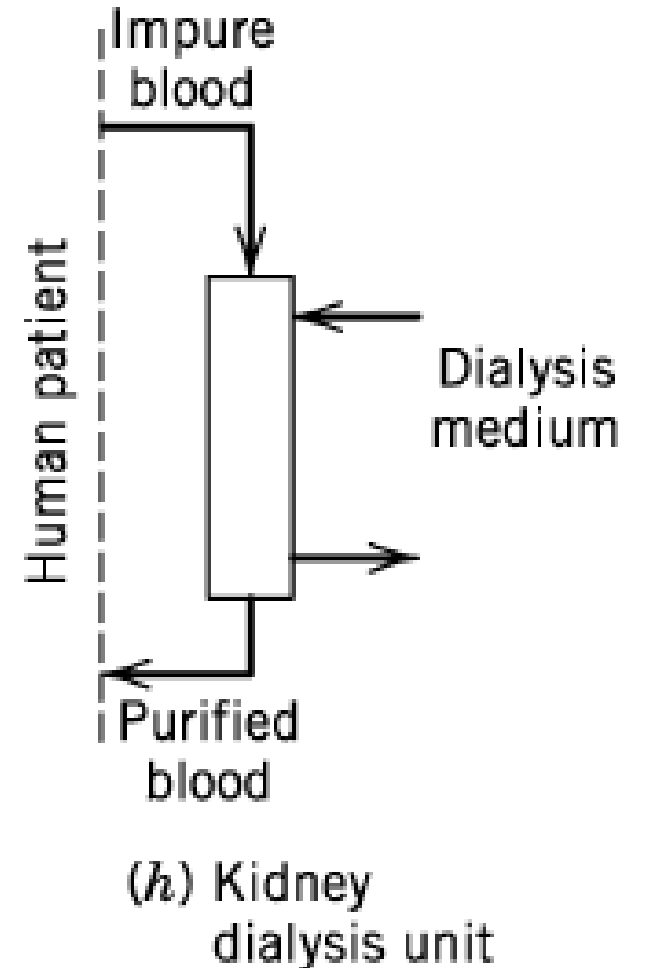


(g) Plasma etcher

Batch and Semi-Batch Processes-cont.

Kidney dialysis unit (وحدة غسيل الكلى).

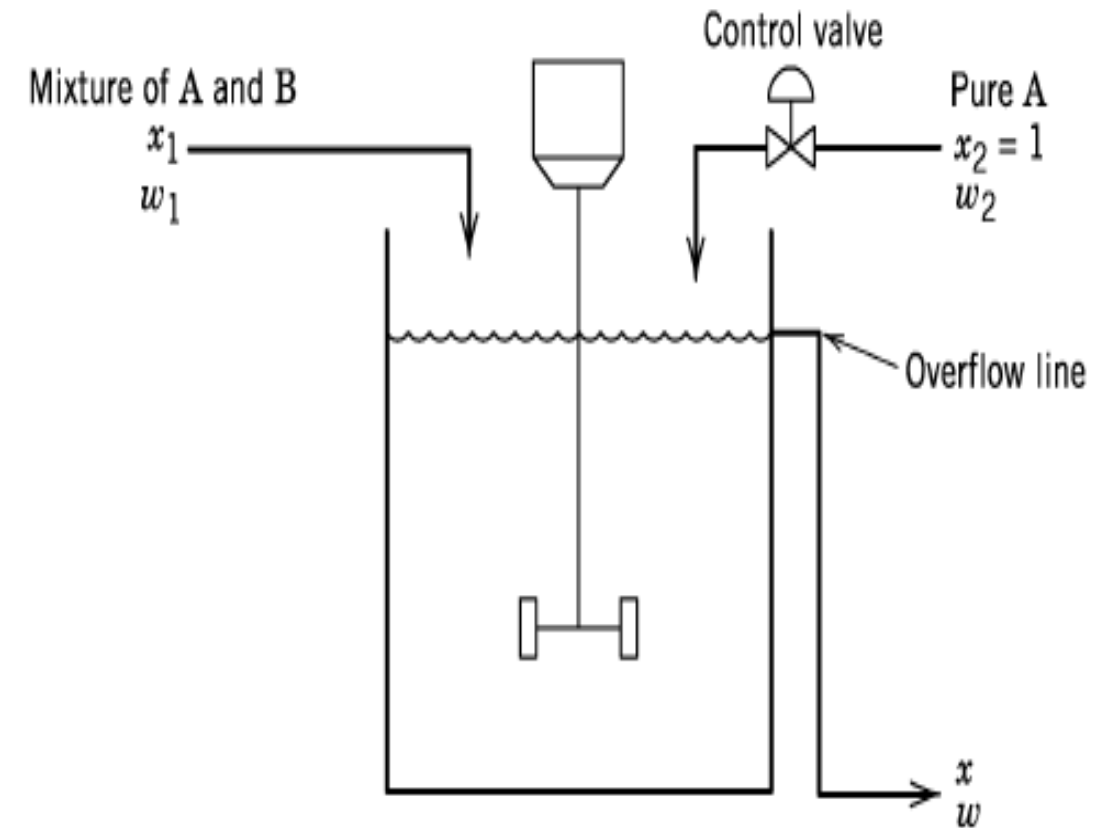
- This medical equipment is used to remove waste products from the blood of human patients whose own kidneys are failing or have failed. The blood flow rate is maintained by a pump, and "ambient conditions," such as temperature in the unit, are controlled by adjusting a flow rate. The dialysis is continued long enough to reduce waste concentrations to acceptable levels.



ILLUSTRATIVE EXAMPLE- A BLENDING PROCESS

(A continuous, stirred-tank blending system)

- The control objective is to blend the two inlet streams to produce an outlet stream that has the desired composition. Stream 1 is a mixture of two chemical species, A and B.
- We assume that its mass flow rate w_1 is constant, but the mass fraction of A, x_1 , varies with time.
- Stream 2 consists of pure A and thus $x_2 = 1$.
- The mass flow rate of Stream 2, w_2 , can be manipulated using a control valve.
- The mass fraction of A in the exit stream is denoted by x and the desired value (set point) by X_{sp} .
- Thus for this control problem, the controlled variable is x , the manipulated variable is w_2 , and the disturbance variable is x_1 .



Design Question. If the nominal value of x_1 is \bar{x}_1 , what nominal flow rate \bar{w}_2 is required to produce the desired outlet concentration, x_{sp} ?

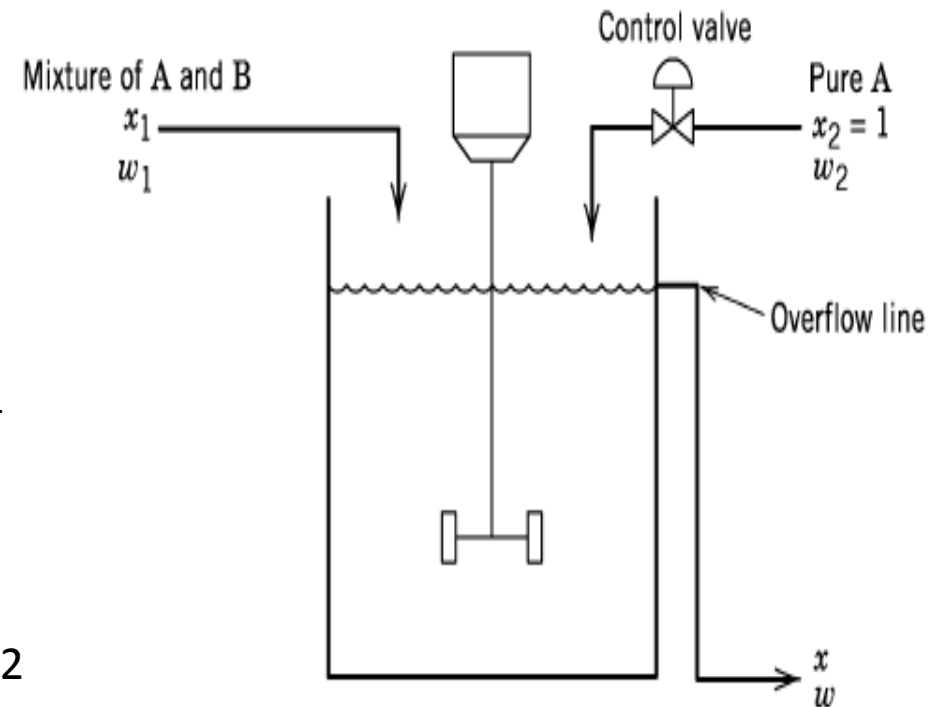
To answer this question, we consider the steady-state material balances:

Overall balance:

$$0 = \bar{w}_1 + \bar{w}_2 - \bar{w} \quad 1$$

Component A balance:

$$0 = \bar{w}_1 \bar{x}_1 + \bar{w}_2 \bar{x}_2 - \bar{w} \bar{x} \quad 2$$



According to the process description, $\dot{\bar{x}}_2 = 1$ and $\bar{x} = x_{sp}$.

$$\bar{w}_2 = \bar{w}_1 \frac{x_{sp} - \bar{x}_1}{1 - x_{sp}} \quad 3$$

Equation 3 is the design equation for the blending system. If our assumptions are correct and if $x_1 = \bar{x}_1$, then this value of w_2 will produce the desired result, $x = x_{sp}$. But what happens if conditions change?

Control Question. Suppose that inlet concentration x_1 varies with time. How can we ensure that the outlet composition x remains at or near its desired value, x_{sp} ?

As a specific example, assume that x_1 increases to a constant value that is larger than its nominal value, \bar{x}_1 . It is clear that the outlet composition will also increase due to the increase in inlet composition. Consequently, at this new steady state, $x > x_{sp}$.

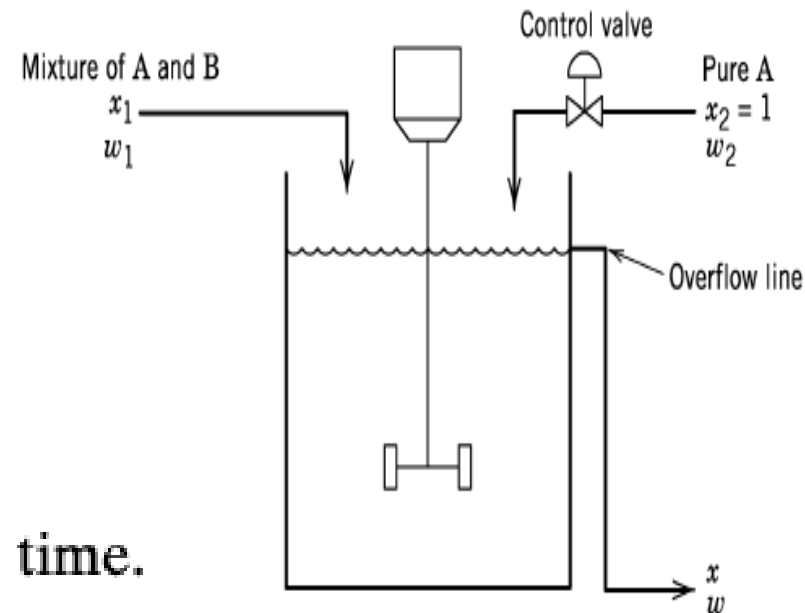
Strategies for reducing the effects of x_1 on x

Method 1. *Measure x and adjust w_2 .* It is reasonable to measure controlled variable x and then adjust w_2 accordingly. For example, if x is too high, w_2 should be reduced; if x is too low, w_2 should be increased. This control strategy could be implemented by a person (*manual control*). However, it would normally be more convenient and economical to automate this simple task (*automatic control*).

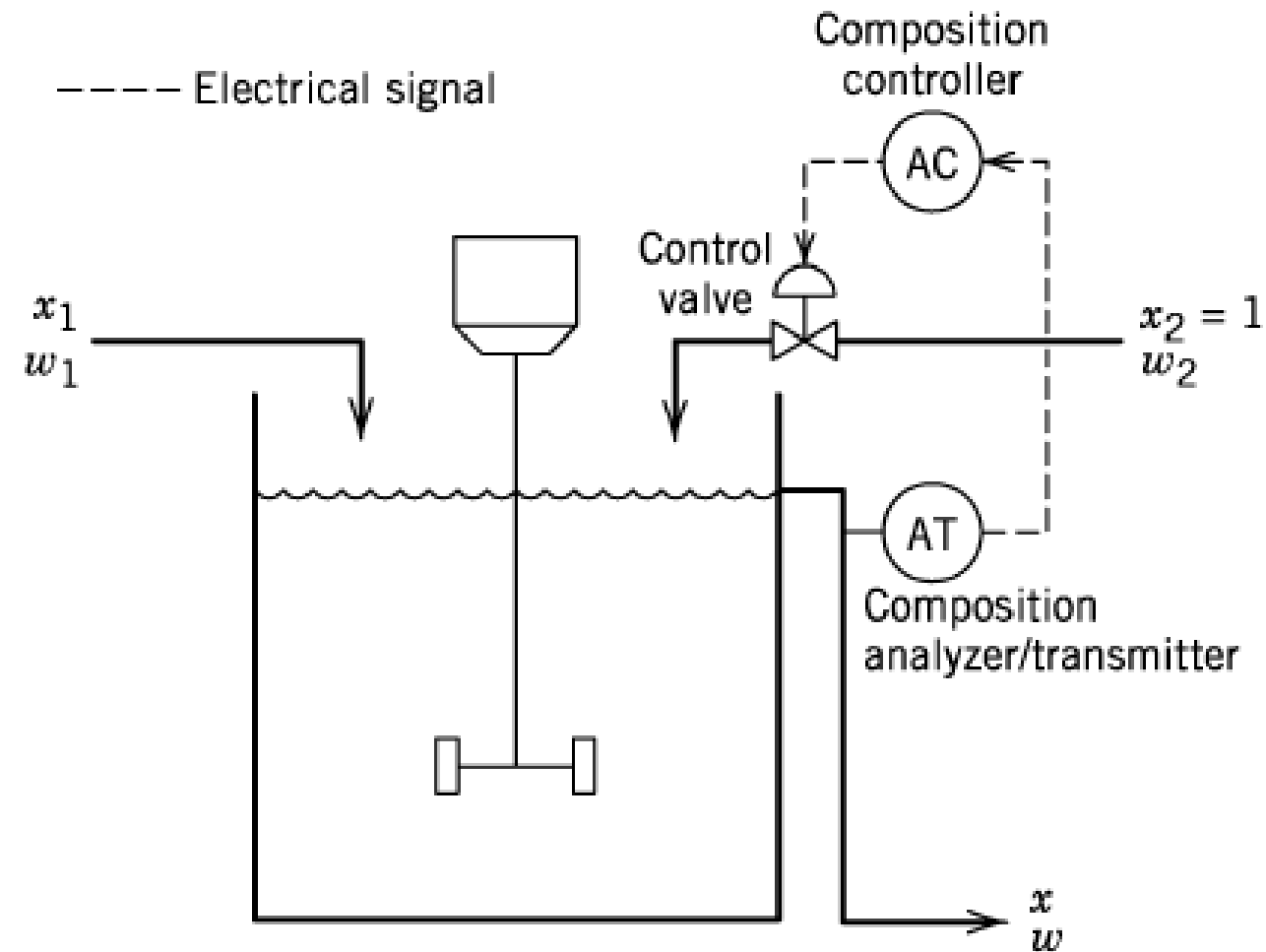
$$w_2(t) = \bar{w}_2 + K_c[x_{sp} - x(t)]$$

4

where K_c is a constant called the *controller gain*. The symbols, $w_2(t)$ and $x(t)$, indicate that w_2 and x change with time.



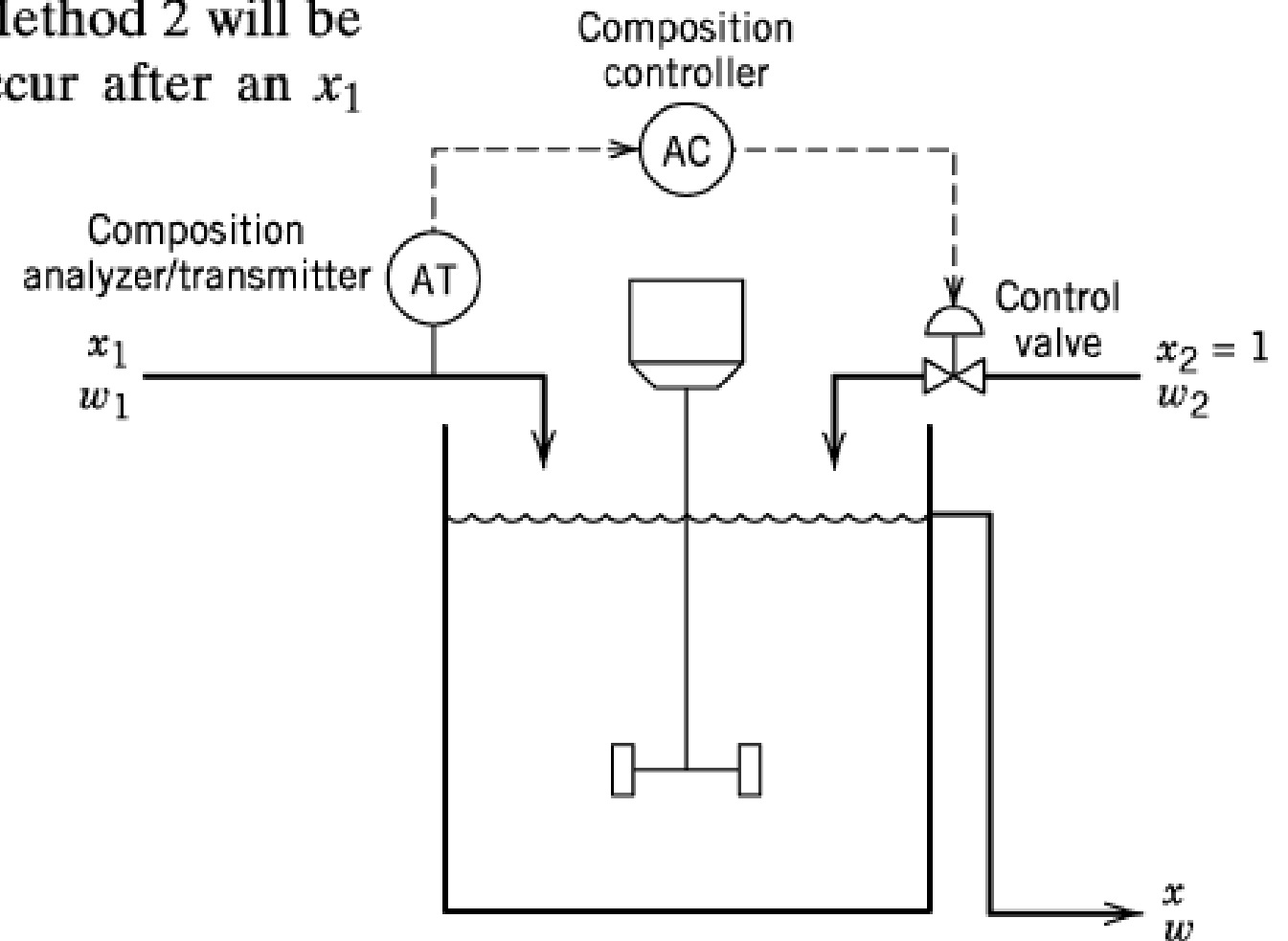
Blending system and Control Method 1.



Method 2. *Measure x_1 , adjust w_2 .* As an alternative to Method 1, we could measure disturbance variable x_1 and adjust w_2 accordingly. Thus, if $x_1 > \bar{x}_1$, we would decrease w_2 so that $w_2 < \bar{w}_2$. If $x_1 < \bar{x}_1$, we would increase w_2 . A control law based on Method 2 can be derived from Eq. 1-3 by replacing \bar{x}_1 with $x_1(t)$ and \bar{w}_2 with $w_2(t)$:

$$w_2(t) = \bar{w}_1 \frac{x_{sp} - x_1(t)}{1 - x_{sp}} \quad 5$$

Because Eq. 3 is valid only for steady-state conditions, it is not clear just how effective Method 2 will be during the transient conditions that occur after an x_1 disturbance.



Blending system and Control Method 2.

Method 3. Measure x_1 and x , adjust w_2 . This approach is a combination of Methods 1 and 2.

Method 4. Use a larger tank. If a larger tank is used, fluctuations in x_1 will tend to be damped out as a result of the larger volume of liquid. However, increasing tank size is an expensive solution due to the increased capital cost.

CLASSIFICATION OF PROCESS CONTROL STRATEGIES

- Method 1 is an example of a *feedback control* strategy.
- The distinguishing feature of feedback control is that the controlled variable is measured, and that the measurement is used to adjust the manipulated variable. For feedback control, the disturbance variable is *not* measured.
- It is important to make a distinction between *negative feedback* and *positive feedback*.
- negative feedback refers to the desirable situation in which the corrective action taken by the controller forces the controlled variable toward the set point.

CLASSIFICATION OF PROCESS CONTROL STRATEGIES-cont.

- when positive feedback occurs, the controller makes things worse by forcing the controlled variable farther away from the set point.
- For example, in the blending control problem, positive feedback takes place if $K_c < 0$, because w_2 will increase when x increases.
- An important advantage of feedback control is that corrective action occurs regardless of the source of the disturbance.
- Another important advantage is that feedback control reduces the sensitivity of the controlled variable to unmeasured disturbances and process changes.
- However, feedback control does have a fundamental limitation: no corrective action is taken until after the disturbance has upset the process, that is, until after the controlled variable deviates from the set point.

CLASSIFICATION OF PROCESS CONTROL STRATEGIES

- Method 2 is an example of a *feedforward control strategy*.
- The distinguishing feature of feedforward control is that the disturbance variable is measured, but the controlled variable is not. The important advantage of feedforward control is that corrective action is taken *before* the controlled variable deviates from the set point.
- Ideally, the corrective action will cancel the effects of the disturbance so that the controlled variable is not affected by the disturbance.

CLASSIFICATION OF PROCESS CONTROL STRATEGIES

- Feedforward control has three significant disadvantages:
- (i) the disturbance variable must be measured (or accurately estimated),
- (ii) no corrective action is taken for unmeasured disturbances, and
- (iii) a process model is required.
- For example, the feedforward control strategy for the blending system (Method 2) does not take any corrective action for unmeasured w_1 disturbances.

CLASSIFICATION OF PROCESS CONTROL STRATEGIES

- A more practical approach is to use a combined feedforward-feedback control system, in which feedback control provides corrective action for unmeasured disturbances, while feedforward control reacts to eliminate measured disturbances before the controlled variable is upset.
- This approach is illustrated by Method 3, a combined feedforward-feedback control strategy because both x and x_1 are measured.
- Finally, Method 4 consists of a process design change and thus is not really a control strategy.

Reference:

- Process Dynamics and Control, by Dale E. Seborg, Thomas F. Edgar, Duncan A. Mellichamp, Francis J. Doyle III, John Wiley & Sons, Inc.2011.

Thank you

Introduction to Process Control

Lecture 3, 2016/2017

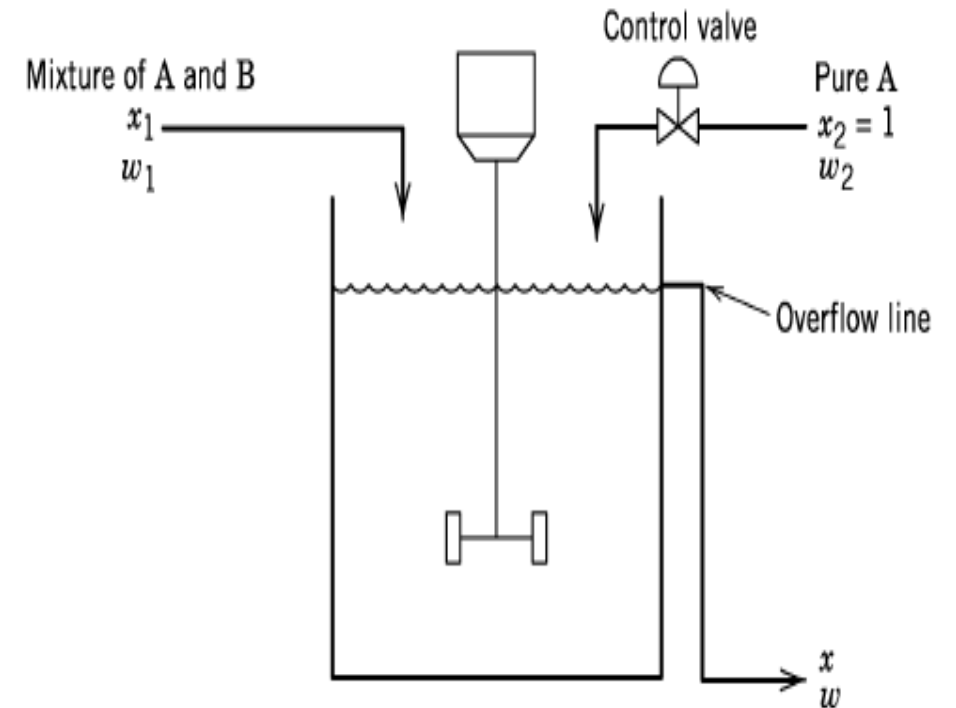
Control & System Eng. Dept. , 4th year

Subject: Process Control.

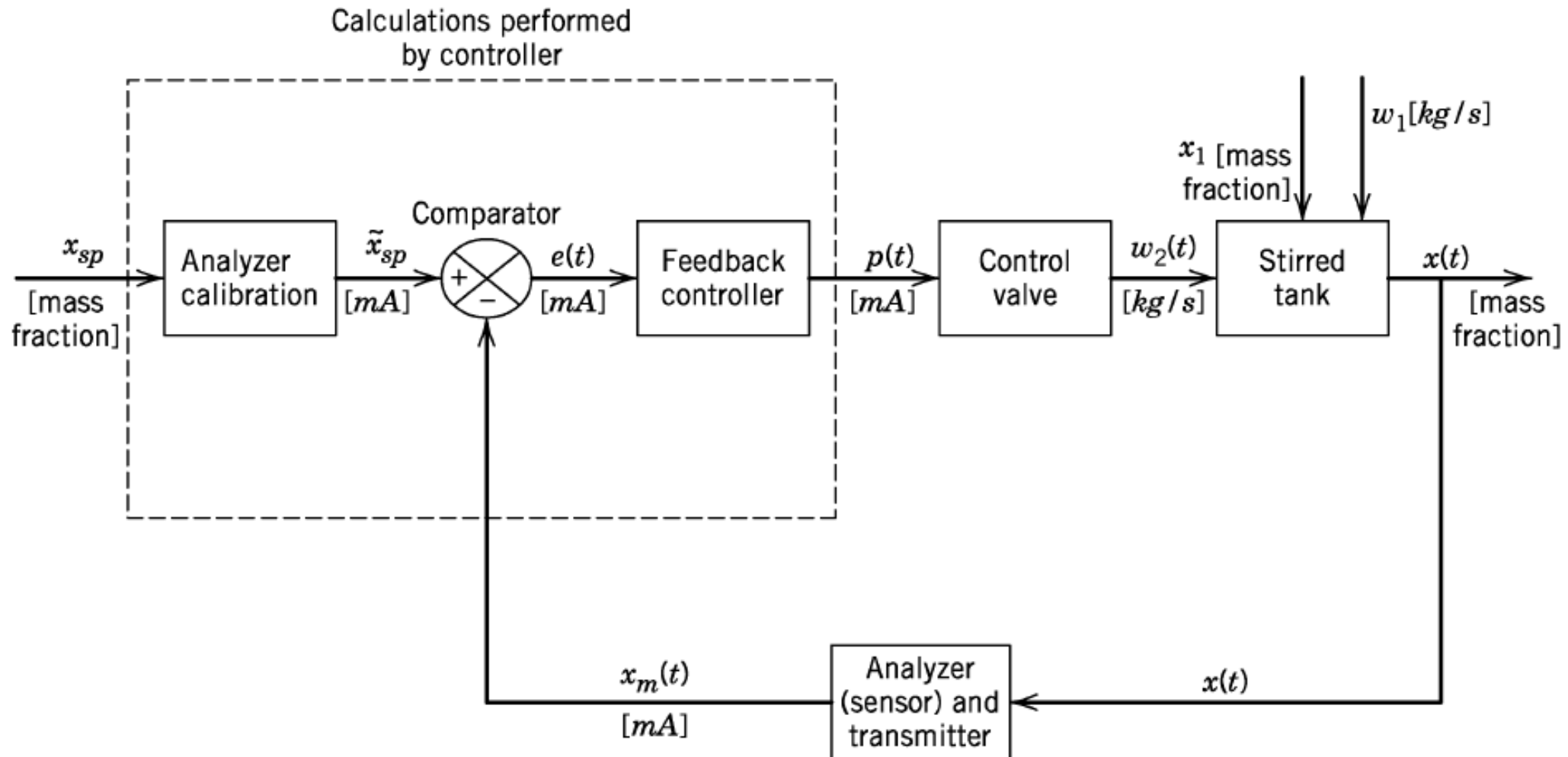
Dr. Safanah M. Raafat

Process Control Diagrams

- consider the equipment that is used to implement control strategies.
- For the stirred-tank mixing system under feedback control.
- the exit concentration x is controlled and the flow rate W_2 of pure species A is adjusted using proportional control.



Block diagram for composition feedback control system



composition feedback control system

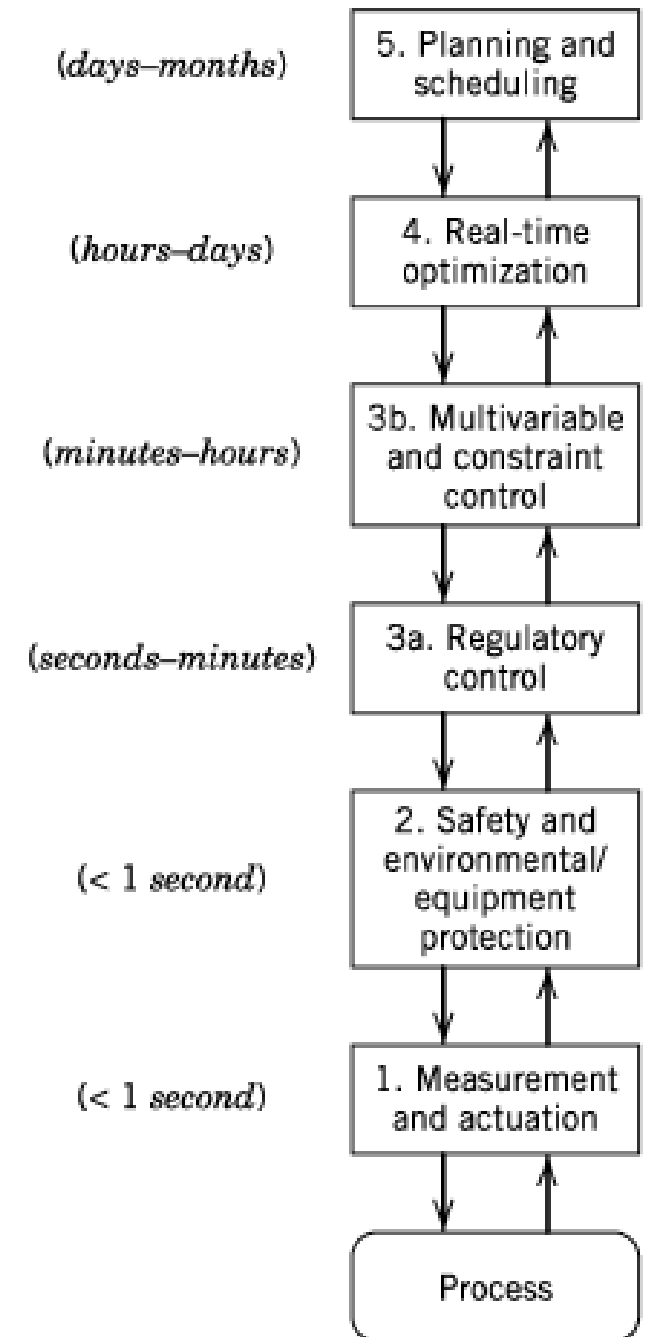
- Operation of the concentration control system can be summarized for the key hardware components as follows:
- **1. Analyzer and transmitter:** The tank exit concentration is measured by means of an instrument that generates a corresponding milliamper (mA)-level signal. This time-varying signal is then sent to the controller.
- **2. Feedback controller:** The controller performs three distinct calculations.
 - **First**, it converts the actual set point X_{sp} into an equivalent internal signal X_{sp} .
 - **Second**, it calculates an error signal $e(t)$ by subtracting the measured value $X_m(t)$ from the set point isp' that is, $e(t) = X_{sp} - X_m(t)$.
 - **Third**, controller output $p(t)$ is calculated from the proportional control law.

composition feedback control system

- ***Control valve:*** The controller output $p(t)$ in this case is a DC current signal that is sent to the control valve to adjust the valve stem position, which in turn affects flow rate $w_2(t)$.
- Because many control valves are pneumatic, i.e., are operated by air pressure, the controller output signal may have to be converted to an equivalent air pressure signal capable of adjusting the valve position.
- The block labeled "control valve" has $p(t)$ as its input signal and $w_2(t)$ as its output signal, which illustrates that the signals on a block diagram can represent either a physical variable such as $w_2(t)$ or an instrument signal such as $p(t)$.

THE HIERARCHY OF PROCESS CONTROL ACTIVITIES

- *Measurement and Actuation (Level 1)*
- Measurement devices (**sensors and transmitters**) and **actuation equipment** (for example, control valves) are used to measure process variables and implement the calculated control actions.
- These devices are interfaced to the control system, usually digital control equipment such as a digital computer.



THE HIERARCHY OF PROCESS CONTROL ACTIVITIES

- *Safety and Environmental/Equipment Protection (Level2)*
- One layer includes process control functions, such as alarm management during abnormal situations, and *safety instrumented systems* for emergency shutdowns.
- The safety equipment (including sensors and control valves) operates independently of the regular instrumentation used for regulatory control in Level3a.
- Sensor validation techniques can be employed to confirm that the sensors are functioning properly.

THE HIERARCHY OF PROCESS CONTROL ACTIVITIES

- *Regulatory Control (Level 3a)*
 - *regulatory control*, is achieved by applying standard feedback and feedforward control techniques.
- *Multivariable and Constraint Control (Level 3b)*
 - Many difficult process control problems have two distinguishing characteristics:
 - (i) significant interactions occur among key process variables,
 - and (ii) inequality constraints exist for manipulated and controlled variables.
- The ability to operate a process close to a limiting constraint is an important objective for advanced process control.

THE HIERARCHY OF PROCESS CONTROL ACTIVITIES

- *Real-time Optimization (Level 4)*
- The optimum operating conditions for a plant are determined as part of the process design. But during plant operations, the optimum conditions can change frequently owing to changes in equipment availability, process disturbances, and economic conditions (for example, raw material costs and product prices).
- Consequently, it can be very profitable to recalculate the optimum operating conditions on a regular basis.

THE HIERARCHY OF PROCESS CONTROL ACTIVITIES

- *Planning and Scheduling (Level 5)*
- For continuous processes, the production rates of all products and intermediates must be planned and coordinated, based on equipment constraints, storage capacity, sales projections, and the operation of other plants, sometimes on a global basis.
- planning and scheduling activities pose difficult optimization problems that are based on both engineering considerations and business projections.

Theoretical Models of Chemical Processes

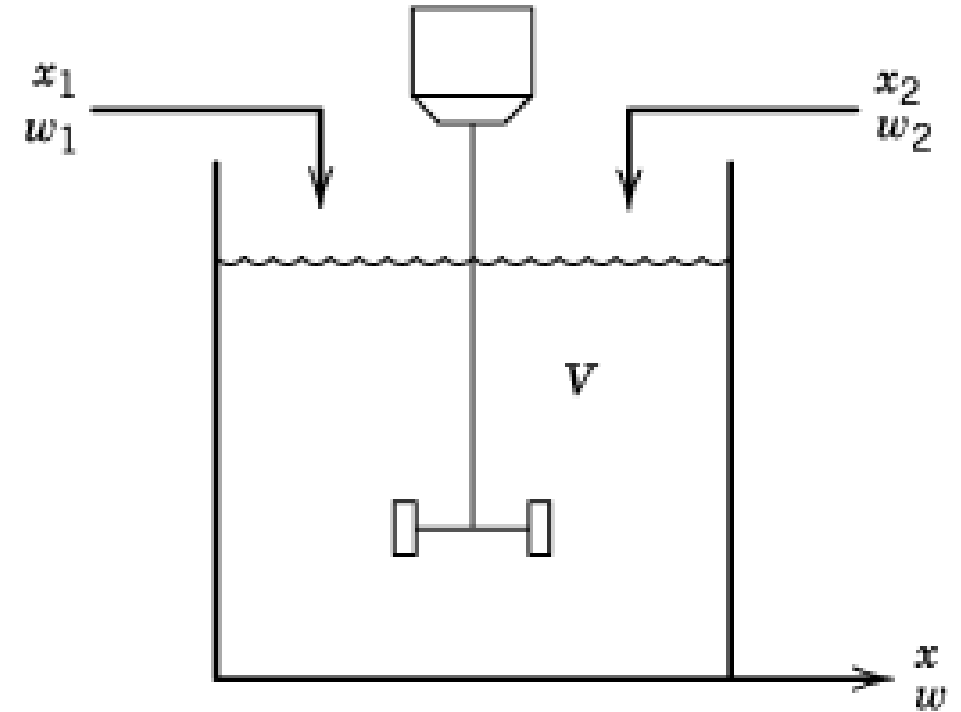
- Dynamic models play a central role in the subject of process dynamics and control. The models can be used to:
 - 1. *Improve understanding of the process.*
 - 2. *Train plant operating personnel.*
 - 3. *Develop a control strategy for a new process.*
 - 4. *Optimize process operating conditions.*

Theoretical Models of Chemical Processes

- Models can be classified based on how they are obtained:
- (a) *Theoretical models* are developed using the principles of chemistry, physics, and biology.
- (b) *Empirical models* are obtained by fitting experimental data.
- (c) *Semi-empirical models* are a combination of the models in categories (a) and (b);
- the numerical values of one or more of the parameters in a theoretical model are calculated from experimental data.

An Illustrative Example: A Blending Process

- consider the isothermal stirred-tank blending system
- the overflow line has been omitted and inlet stream 2 is not necessarily pure A (that is, $x_2 \neq 1$).
- Now the volume of liquid in the tank V can vary with time,
- and the exit flow rate is not necessarily equal to the sum of the inlet flow rates.
- An unsteady-state mass balance for the blending system has the form



$$\left\{ \begin{array}{l} \text{rate of accumulation} \\ \text{of mass in the tank} \end{array} \right\} = \left\{ \begin{array}{l} \text{rate of} \\ \text{mass in} \end{array} \right\} - \left\{ \begin{array}{l} \text{rate of} \\ \text{mass out} \end{array} \right\}$$

- the rate of mass accumulation is simply $d(V\rho)/dt$,

$$\frac{d(V\rho)}{dt} = w_1 + w_2 - w$$

- where w_1 , w_2 , and w are mass flow rates.
- We assume that the blending tank is perfectly mixed.
- This assumption has two important implications:
 - (i) there are no concentration gradients in the tank contents and
 - (ii) the composition of the exit stream is equal to the tank composition.
- The perfect mixing assumption is valid for low-viscosity liquids that receive an adequate degree of agitation.
- In contrast, the assumption is less likely to be valid for high-viscosity liquids such as polymers or molten metals.

- For the perfect mixing assumption, the rate of accumulation of component A is $d(V\rho x)/dt$, where x is the mass fraction of A.
- The unsteady-state component balance is

$$\frac{d(V\rho x)}{dt} = w_1x_1 + w_2x_2 - wx$$

- *Question : how to find the steady state model ?*
- a steady-state model is a special case of an unsteady-state model that can be derived by setting accumulation terms equal to zero.

Conservation Laws

- Theoretical models of chemical processes are based on conservation laws such as the conservation of mass and energy.
- consider important conservation laws and use them to develop dynamic models for representative processes.
- ***Conservation of Mass***

$$\left\{ \begin{array}{c} \text{rate of mass} \\ \text{accumulation} \end{array} \right\} = \left\{ \begin{array}{c} \text{rate of} \\ \text{mass in} \end{array} \right\} - \left\{ \begin{array}{c} \text{rate of} \\ \text{mass out} \end{array} \right\}$$

Conservation Laws

- *Conservation of Component i*

$$\left\{ \begin{array}{c} \text{rate of component } i \\ \text{accumulation} \end{array} \right\} = \left\{ \begin{array}{c} \text{rate of component } i \\ \text{in} \end{array} \right\} - \left\{ \begin{array}{c} \text{rate of component } i \\ \text{out} \end{array} \right\} + \left\{ \begin{array}{c} \text{rate of component } i \\ \text{produced} \end{array} \right\}$$

- Represents the rate of generation (or consumption) of component i as a result of chemical reactions.
- Conservation equations can also be written in terms of molar quantities, atomic species, and molecular species .

Conservation Laws

- ***Conservation of Energy***

- The general law of energy conservation is also called the First Law of Thermodynamics.

$$\left\{ \begin{array}{l} \text{rate of energy} \\ \text{accumulation} \end{array} \right\} = \left\{ \begin{array}{l} \text{rate of energy in} \\ \text{by convection} \end{array} \right\} - \left\{ \begin{array}{l} \text{rate of energy out} \\ \text{by convection} \end{array} \right\} + \left\{ \begin{array}{l} \text{net rate of heat addition} \\ \text{to the system from} \\ \text{the surroundings} \end{array} \right\} + \left\{ \begin{array}{l} \text{net rate of work} \\ \text{performed on the system} \\ \text{by the surroundings} \end{array} \right\}$$

Conservation Laws

- The total energy of a thermodynamic system, U_{tot} , is the sum of its internal energy, kinetic energy, and potential energy:

$$U_{\text{tot}} = U_{\text{int}} + U_{KE} + U_{PE}$$

- it is appropriate to make two assumptions:
 - 1. Changes in potential energy and kinetic energy can be neglected, because they are small in comparison with changes in internal energy.
 - 2. The net rate of work can be neglected, because it is small compared to the rates of heat transfer and convection.

Conservation Laws

- Therefore, the energy balance can be written as:

$$\frac{dU_{\text{int}}}{dt} = -\Delta(w\hat{H}) + Q$$

where U_{int} is the internal energy of the system, \hat{H} is the enthalpy per unit mass, w is the mass flow rate, and Q is the rate of heat transfer to the system. The Δ operator denotes the difference between outlet conditions and inlet conditions of the flowing streams. Consequently, the $-\Delta(w\hat{H})$ term represents the enthalpy of the inlet stream(s) minus the enthalpy of the outlet stream(s).

Reference:

- Process Dynamics and Control, by Dale E. Seborg, Thomas F. Edgar, Duncan A. Mellichamp, Francis J. Doyle III, John Wiley & Sons, Inc.2011.

Thank you

Process Control

4th year. Control Engineering

Dr. Safanah M. Raafat

2016/2017

Control & System Engineering Dept.,

University of Technology

Baghdad Iraq

2.4.2 Chemical Reactions

The rate of reaction per unit volume (mol/volume*time) is usually a function of the concentration of the reacting species. For example, consider the reaction $A + 2B \rightarrow C + 3D$. If the rate of the reaction of A is first-order in both A and B, we use the following expression:

$$r_A = -k C_A C_B \quad (2.45)$$

where

- r_A is the rate of reaction of A (mol A/volume*time)
- k is the reaction rate constant (constant for a given temperature)
- C_A is the concentration of A (mol A/volume)
- C_B is the concentration of B (mol B/volume)

Reaction rates are normally expressed in terms of generation of a species. The minus sign indicates that A is consumed in the reaction above. It is good practice to associate the units with all parameters in a model. For consistency in the units for r_A , we find that k has units of (vol/mol B* time). Notice that 2 mols of B react for each mol of A. Then we can write

$$\begin{aligned} r_B &= 2r_A = -2k C_A C_B \\ r_C &= -r_A = k C_A C_B \\ r_D &= -3r_A = 3k C_A C_B \end{aligned}$$

Usually, the reaction rate coefficient is a function of temperature. The most commonly used representation is the Arrhenius rate law

$$k(T) = A \exp(-E/RT) \quad (2.46)$$

where

- $k(T)$ = reaction rate constant, as a function of temperature
- A = frequency factor or preexponential factor (same units as k)
- E = activation energy (cal/gmol)
- R = ideal gas constant (1.987 cal/gmol K, or another set of consistent units)
- T = absolute temperature (deg K or deg R)

2.4.4 Heat Transfer

The rate of heat transfer through a vessel wall separating two fluids (a jacketed reactor, for example) can be described by

$$Q = UA\Delta T \quad (2.50)$$

where

Q = rate of heat transferred from the hot fluid to the cold fluid

U = overall heat transfer coefficient

A = area for heat transfer

ΔT = difference between hot and cold fluid temperatures

The heat transfer coefficient is often estimated from experimental data. At the design stage it can be estimated from correlations; it is a function of fluid properties and velocities.

2.4.5 Flow-through Valves

The flow-through valves are often described by the following relationship:

$$F = C_v f(x) \sqrt{\frac{\Delta P_v}{\text{s.g.}}} \quad (2.51)$$

where

F = volumetric flowrate

C_v = valve coefficient

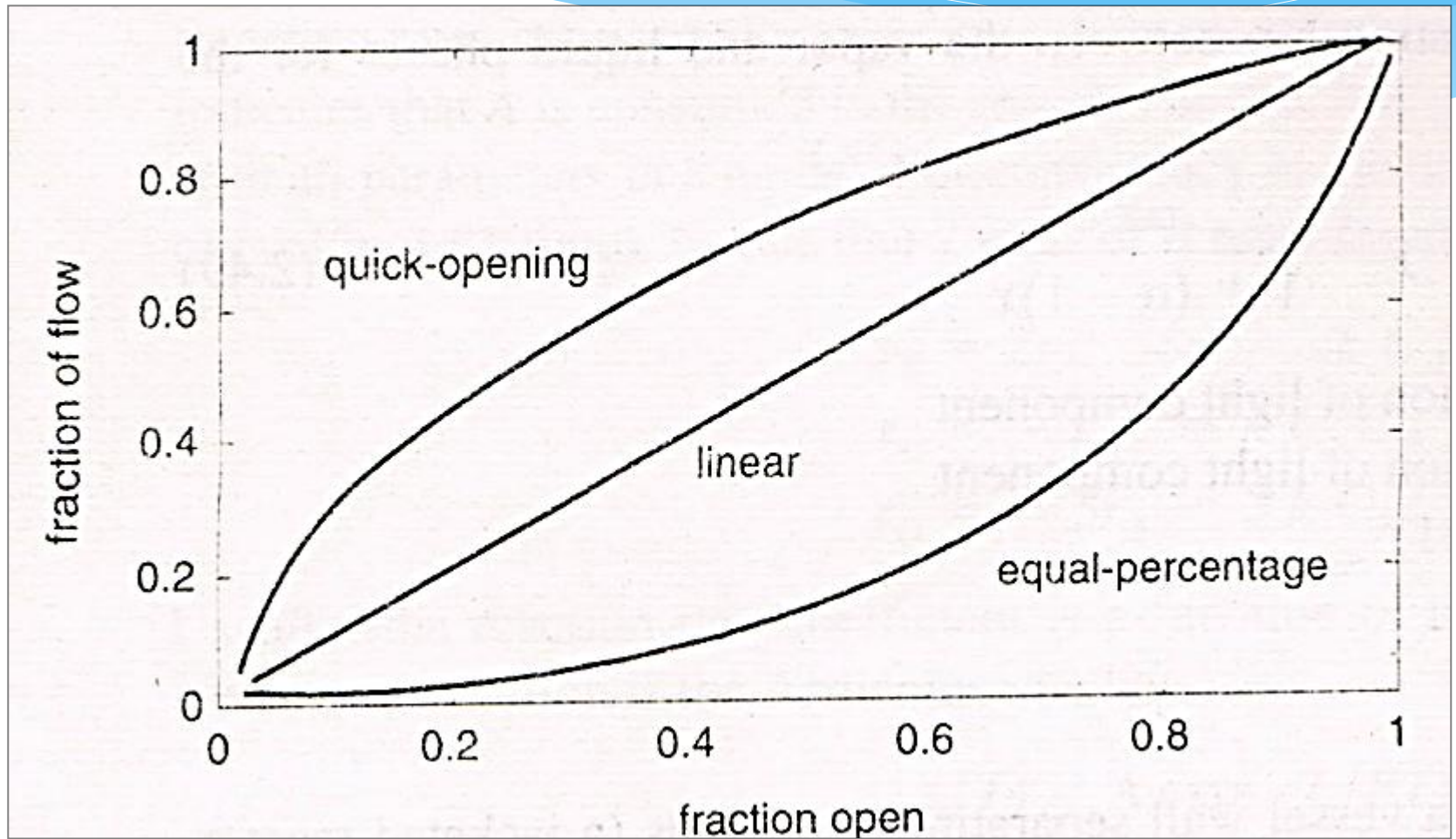
x = fraction of valve opening

ΔP_v = pressure drop across the valve

s.g. = specific gravity of the fluid

$f(x)$ = the flow characteristic (varies from 0 to 1, as a function of x)

Three common valve characteristics are (i) linear, (ii) equal-percentage, and (iii) quick-opening.



For a linear valve

$$f(x) = x$$

For an equal-percentage valve

$$f(x) = \alpha^{x-1}$$

For a quick-opening valve

$$f(x) = \sqrt{x}$$

Notice that for the **quick-opening** valve, the sensitivity of flow to valve position (fraction open) is high at low openings and low at high openings; the opposite is true for an **equal-percentage** valve. The sensitivity of a **linear** valve does not change as a function of valve position. The equal-percentage valve is commonly used in chemical processes, because of desirable characteristics when installed in piping systems where a significant piping pressure drop occurs at high flowrates. Knowledge of these characteristics will be important when developing feedback control systems.

DISTRIBUTED PARAMETER SYSTEMS

In this section we show how the balance equations can be used to develop a model for a distributed parameter system, that is, a system where the state variables change with respect to position and time.

Consider a tubular reactor where a chemical reaction changes the concentration of the fluid as it moves down the tube. Here we use a volume element ΔV and a time element Δt . The total moles of species A contained in the element ΔV is written $(\Delta V)C_A$. The amount of species A entering the volume is $FC_A|_V$ and the amount of species leaving the volume is $FC_A|_{V+\Delta V}$. The rate of A leaving by reaction (assuming a first-order reaction) is $(-k C_A)\Delta V$.

The balance equation is then:

$$(\Delta V)C_A|_{t+\Delta t} - (\Delta V)C_A|_t = \int_t^{t+\Delta t} [FC_A|_V - FC_A|_{V+\Delta V} - kC_A\Delta V] dt$$

Using the mean value theorem of integral calculus and dividing by Δt , we find:

$$\frac{\Delta V[C_A|_{t+\Delta t} - C_A|_t]}{\Delta t} = FC_A|_V - FC_A|_{V+\Delta V} - kC_A\Delta V \quad (2.64)$$

Dividing by ΔV and letting Δt and ΔV go to zero, we find:

$$\frac{\partial C_A}{\partial t} = -\frac{\partial FC_A}{\partial V} - kC_A \quad (2.65)$$

Normally, a tube with constant cross-sectional area is used, so $dV = Adz$ and $F = Av_z$, where v_z is the velocity in the z -direction. Then the equation can be written:

$$\frac{\partial C_A}{\partial t} = - \frac{\partial v_z C_A}{\partial z} - kC_A \quad (2.66)$$

Similarly, the overall material balance can be found as:

$$\frac{\partial \rho}{\partial t} = - \frac{\partial v_z \rho}{\partial z} \quad (2.67)$$

If the fluid is at a constant density (good assumption for a liquid), then we can write the species balance as

$$\frac{\partial C_A}{\partial t} = - v_z \frac{\partial C_A}{\partial z} - kC_A \quad (2.68)$$

To solve this problem, we must know the initial condition (concentration as a function of distance at the initial time) and one boundary condition. For example, the following boundary and initial conditions

$$\begin{aligned} C_A(z, t = 0) &= C_{A0}(z) \\ C_A(0, t) &= C_{Ain}(t) \end{aligned} \quad (2.69)$$

indicate that the concentration of A initially is known as a function of distance down the reactor, and that the inlet concentration as a function of time must be specified.

In deriving the tubular reactor equations we assumed that species A left a volume element only by convection (bulk flow). In addition, the molecules can leave by virtue of a concentration gradient. For example, the amount entering at V is

$$\left(FC_A + AD_{AZ} \frac{dC_A}{dz} \right) \bigg|_V \quad (2.70)$$

D_{AZ}

where is the diffusion coefficient.

DIMENSIONLESS MODELS

Models typically contain a large number of parameters and variables that may differ in value by several orders of magnitude. It is often desirable, at least for analysis purposes, to develop models composed of dimensionless parameters and variables. To illustrate the approach, consider a constant volume, isothermal CSTR modeled by a simple first-order reaction:

$$\frac{dC_A}{dt} = \frac{F}{V} (C_{Af} - C_A) - kC_A$$

It seems natural to work with a scaled concentration. Defining

$$x = C/C_{Af0}$$

where C_{Af0} is the nominal (steady-state) feed concentration of A, we find

$$\frac{dx}{dt} = \frac{F}{V} x_f - \left(\frac{F}{V} + k \right) x$$

where $x_f = C_{Af}/C_{Af0}$. It is also natural to choose a scaled time, $\tau = t/t^*$, where t^* is a scaling parameter to be determined. We can use the relationship $dt = t^*d\tau$ to write:

$$\frac{dx}{t^* d\tau} = \frac{F}{V} x_f - \left(\frac{F}{V} + k \right) x$$

A natural choice for t^* appears to be V/F (known as the residence time), so

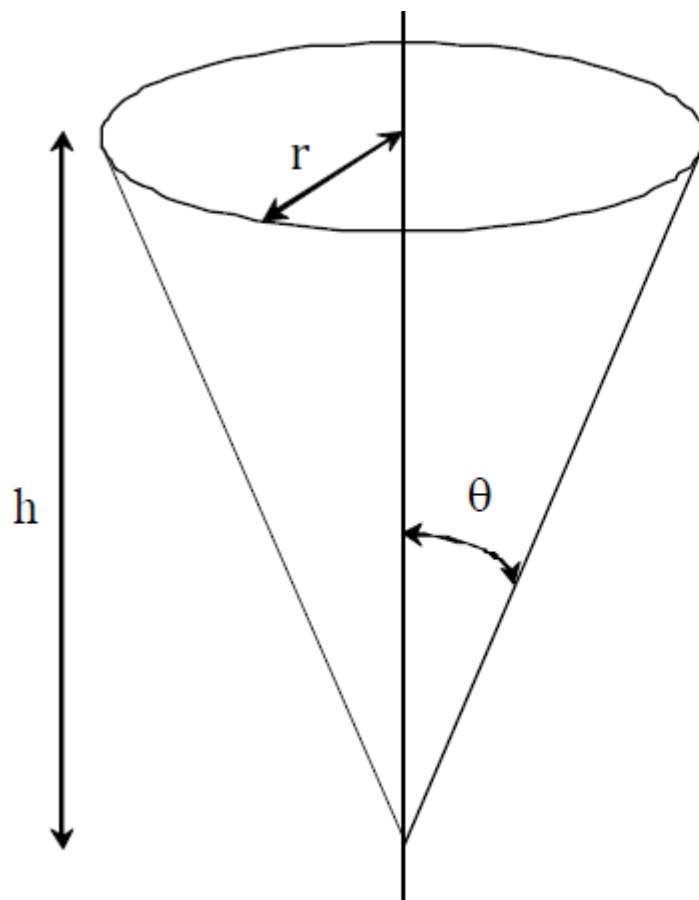
$$\frac{dx}{d\tau} = x_f - \left(1 + \frac{Vk}{F} \right) x$$

The term Vk/F is dimensionless and known as a Damkholer number in the reaction engineering literature. Assuming that the feed concentration is constant, $x_f = 1$, and letting $\alpha = Vk/F$, we can write:

$$\frac{dx}{d\tau} = 1 - x + \alpha x$$

which indicates that a single parameter, α , can be used to characterize the behavior of all first-order, isothermal chemical reactions. Similar results are obtained if the dimensionless state is chosen to be conversion

$$x = (C_A - C_{Af0})/C_{Af0}$$



Right Circular Cone

$$\tan \theta = \frac{r}{h}$$

$$V = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi h^3 \tan^2 \theta$$

$$\therefore \frac{dV}{dh} = (\pi \tan^2 \theta) h^2$$

references

- * B.Wayne Bequette. 'Process dynamics, modeling Analysis and Simulation', Prentice Hall International Series, 1998.
- * Colorado school of Mines CHEN403, lecture notes, 2008.



Thank you

Process Control

4th year. Control Engineering

Dr. Safanah M. Raafat

2016/2017

Control & System Engineering Dept.,

University of Technology

Baghdad Iraq

EXPLICIT SOLUTIONS TO DYNAMIC MODELS

Explicit solutions to nonlinear differential equations can rarely be obtained. The most common case where an analytical solution can be obtained is when a single differential equation has variables that are separable. This is a very limited class of problems.

Example: Nonlinear Tank Height

Consider a tank height problem where the outlet flow is a nonlinear function of tank height:

$$\frac{dh}{dt} = \frac{F_i}{A} - \frac{\beta}{A} \sqrt{h}$$

Here there is not an analytical solution because of the nonlinear height relationship and the forcing function. To illustrate a problem with an analytical solution, we will assume that there is no inlet flow to the tank:

$$\frac{dh}{dt} = - \frac{\beta}{A} \sqrt{h}$$

we can see that the variables are separable, so

$$\frac{dh}{\sqrt{h}} = - \frac{\beta}{A} dt$$

$$\int_{h_0}^h \frac{dh}{\sqrt{h}} = - \int_{t_0}^t \frac{\beta}{A} dt$$

which has the solution

$$2\sqrt{h} - 2\sqrt{h_0} = - \frac{\beta}{A} (t - t_0)$$

or

$$\sqrt{h} = \sqrt{h_0} - \frac{\beta}{2A} (t - t_0)$$

letting $t_0 = 0$, and squaring both sides, we obtain the solution

$$h(t) = \left[\sqrt{h_0} - \frac{\beta}{2A} t \right]^2$$

This analytical solution can be used, for example, to determine the time that it will take for the tank height to reach a certain level.

GENERAL FORM OF DYNAMIC MODELS

The dynamic models derived in this chapter consist of a set of first-order (meaning only first derivatives with respect to time), nonlinear, explicit, initial value ordinary differential equations. A representation of a set of first-order differential equations is

$$\begin{aligned}\dot{x}_1 &= f_1(x_1, \dots, x_n, u_1, \dots, u_m, p_1, \dots, p_r) \\ \dot{x}_2 &= f_2(x_1, \dots, x_n, u_1, \dots, u_m, p_1, \dots, p_r) \\ &\vdots \\ \dot{x}_n &= f_n(x_1, \dots, x_n, u_1, \dots, u_m, p_1, \dots, p_r)\end{aligned}$$

where x_i is a state variable, u_i is an input variable and p_i is a parameter. The notation \dot{x}_i is used to represent dx/dt . Notice that there are n equations, n state variables, m inputs, and r parameters.

Example:

Consider the following modelling equations

$$\frac{dV}{dt} = F_i - F$$

$$\frac{dC_A}{dt} = \frac{F_i}{V} (C_{Ai} - C_A) - k C_A C_B$$

$$\frac{dC_B}{dt} = \frac{F_i}{V} (C_{Bi} - C_B) - 2 k C_A C_B$$

$$\frac{dC_P}{dt} = -\frac{F_i}{V} C_P + k C_A C_B$$

There are four states (V, C_A, C_B, C_P),

Four inputs (F_i, F, C_{Ai}, C_{Bi})

Single parameter (k)

$$\begin{bmatrix} \dot{V} \\ \dot{C}_A \\ \dot{C}_B \\ \dot{C}_P \end{bmatrix} = \begin{bmatrix} F_I - F \\ \frac{F_I}{V} (C_{Ai} - C_A) - k C_A C_B \\ \frac{F_I}{V} (C_{Bi} - C_B) - 2 k C_A C_B \\ -\frac{F_I}{V} C_P + k C_A C_B \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} u_1 - u_2 \\ \frac{u_1}{x_1} (u_3 - x_2) - p_1 x_2 x_3 \\ \frac{u_1}{x_1} (u_4 - x_3) - 2 p_1 x_2 x_3 \\ -\frac{u_1}{x_1} x_4 + p_1 x_2 x_3 \end{bmatrix} = \begin{bmatrix} f_1(x, u, p) \\ f_2(x, u, p) \\ f_3(x, u, p) \\ f_4(x, u, p) \end{bmatrix}$$

Second Order Runge Kutta

Let m_1 represent the slope at the initial point and m_2 represent the slope (dx/dt) at the midpoint:

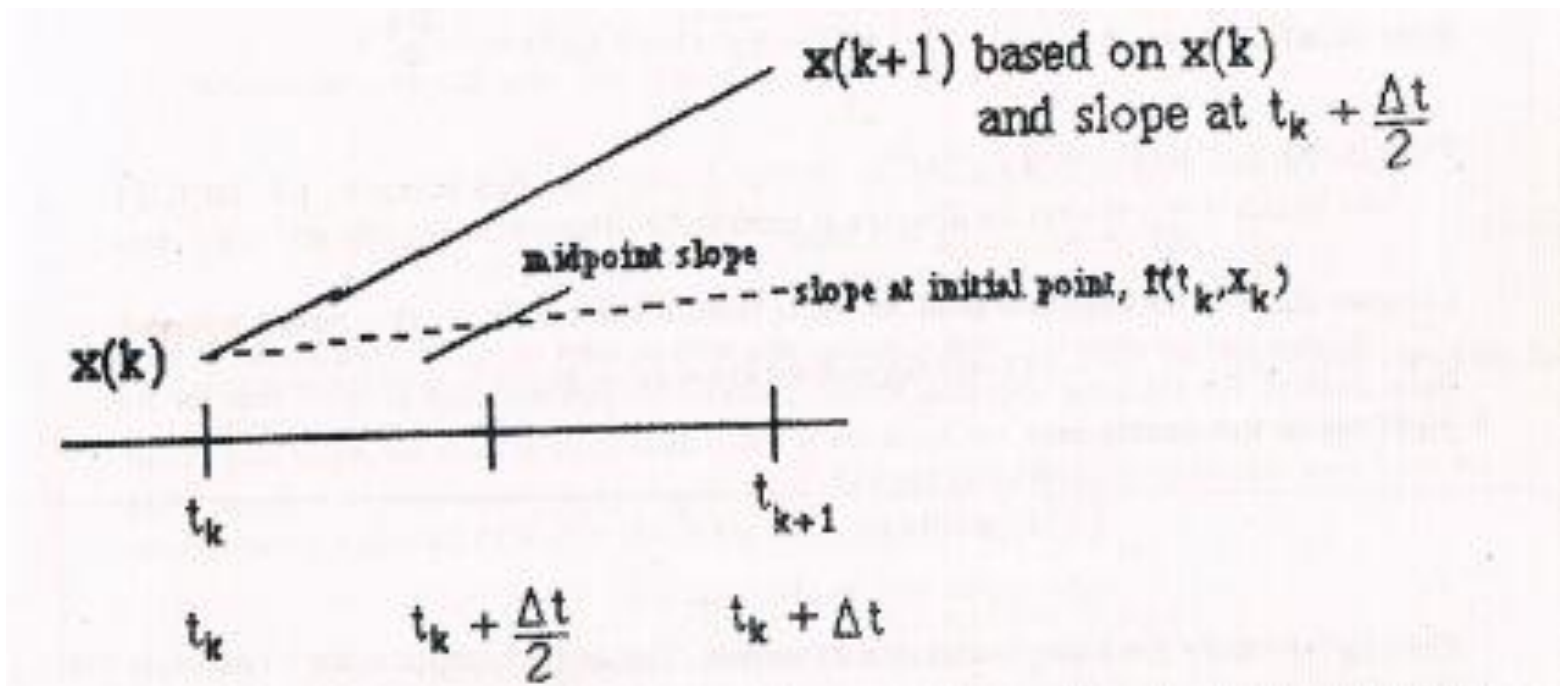
$$m_1 = f(x(k))$$

$$m_2 = f\left(x(k) + \frac{\Delta t}{2} m_1\right)$$

or

$$m_2 = f\left(x(k) + \frac{\Delta t}{2} f(x(k))\right)$$

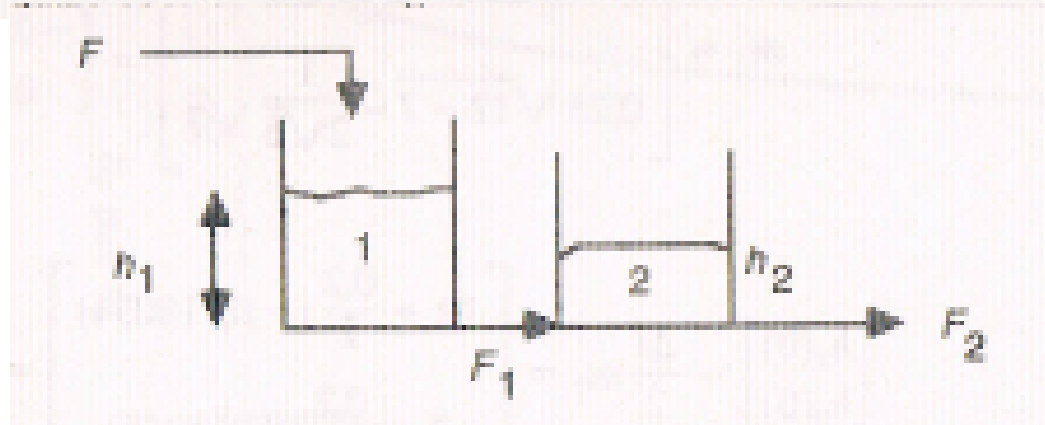
$$x(k + 1) = x(k) + m_2 \Delta t$$



example

Two-state Variable System, Second-order Runge-Kutta Method

Consider two interacting tanks in series, shown in Figure 4.6, with outlet flowrates that are a function of the square root of tank height. Notice that the flow from tank 1 is a function of $\sqrt{h_1 - h_2}$, while the flowrate out of tank 2 is a function of $\sqrt{h_2}$.



The following modeling equations describe this system

$$\begin{bmatrix} \frac{dh_1}{dt} \\ \frac{dh_2}{dt} \end{bmatrix} = \begin{bmatrix} f_1(h_1, h_2, F) \\ f_2(h_1, h_2, F) \end{bmatrix} = \begin{bmatrix} \frac{P}{A_1} - \frac{\beta_1}{A_1} \sqrt{h_1 - h_2} \\ \frac{\beta_1}{A_2} \sqrt{h_1 - h_2} - \frac{\beta_2}{A_2} \sqrt{h_2} \end{bmatrix}$$

For the following parameter values:

$$\beta_1 = 2.5 \frac{\text{ft}^{2.5}}{\text{min}} \quad \beta_2 = \frac{5}{\sqrt{6}} \frac{\text{ft}^{2.5}}{\text{min}} \quad A_1 = 5 \text{ ft}^2 \quad A_2 = 10 \text{ ft}^2$$

and the input: $F = 5 \text{ ft}^3/\text{min}$

the steady-state height values are:

$$h_{1s} = 10 \quad h_{2s} = 6$$

$$\begin{bmatrix} \frac{dh_1}{dt} \\ \frac{dh_2}{dt} \end{bmatrix} = \begin{bmatrix} f_1(h_1, h_2) \\ f_2(h_1, h_2) \end{bmatrix} = \begin{bmatrix} 1 - 0.5 \sqrt{h_1 - h_2} \\ 0.25 \sqrt{h_1 - h_2} - \frac{1}{2\sqrt{6}} \sqrt{h_2} \end{bmatrix}$$

Since this system is autonomous (no explicit dependence on time), we can leave t out of the arguments:

$$m_1 = f(h(k)) = \begin{bmatrix} f_1(h_1(k), h_2(k)) \\ f_2(h_1(k), h_2(k)) \end{bmatrix}$$

$$m_2 = \begin{bmatrix} f_1(h_1(k) + \frac{\Delta t}{2} m_{11}, h_2(k) + \frac{\Delta t}{2} m_{21}) \\ f_2(h_1(k) + \frac{\Delta t}{2} m_{11}, h_2(k) + \frac{\Delta t}{2} m_{21}) \end{bmatrix}$$

$$h(k+1) = \begin{bmatrix} h_1(k+1) \\ h_2(k+1) \end{bmatrix} = \begin{bmatrix} h_1(k) \\ h_2(k) \end{bmatrix} + m_2 \Delta t$$

Let the initial conditions be $h_1(0) = 12$ ft and $h_2(0) = 7$ ft. Also, let $\Delta t = 0.2$ minutes. For $k = 0$, we find

$$m_1 = \begin{bmatrix} 1 - 0.5 \sqrt{12} \\ 0.25 \sqrt{12 - 7} - \frac{1}{2\sqrt{6}} \sqrt{7} \end{bmatrix} = \begin{bmatrix} -0.118034 \\ 0.018955 \end{bmatrix}$$

$$\begin{bmatrix} h_1(0) \\ h_2(0) \end{bmatrix} + \frac{\Delta t}{2} m_1 = \begin{bmatrix} 12 + \frac{0.2}{2} (-0.118034) \\ 7 + \frac{0.2}{2} (0.018955) \end{bmatrix} = \begin{bmatrix} 11.988197 \\ 7.001896 \end{bmatrix}$$

$$h_1(0) + \frac{\Delta t}{2} m_{11} = 12 + \frac{0.2}{2} (-0.118034) = 11.988197 \text{ ft}$$

$$h_2(0) + \frac{\Delta t}{2} m_{21} = 7 + \frac{0.2}{2} (0.018955) = 7.001896 \text{ ft}$$

$$m_{12} = f_1\left(h_1(0) + \frac{\Delta t}{2} m_{11}, h_2(0) + \frac{\Delta t}{2} m_{12}\right) = f_1(11.988197, 7.001896)$$

$$= 1 - 0.5\sqrt{11.988197 - 7.001896} = -0.116501$$

$$m_{22} = f_2\left(h_1(0) + \frac{\Delta t}{2} m_{11}, h_2(0) + \frac{\Delta t}{2} m_{12}\right) = f_2(11.988197, 7.001896)$$

$$= 0.25\sqrt{11.988197 - 7.001896} - \frac{1}{2\sqrt{6}}\sqrt{7.001896} = 0.018116$$

$$h_1(1) = h_1(0) + m_{21}\Delta t = 12 + -0.116501(0.2) = 11.976700 \text{ ft}$$

$$h_2(1) = h_2(0) + m_{22}\Delta t = 7 + 0.018116(0.2) = 7.003623 \text{ ft}$$

Notice that when h_1 is greater than h_2 , the flow is from tank 1 to tank 2; while when h_1 is less than h_2 , the flow is from tank 2 to tank 1 (although this cannot occur at steady-state). Since we have assigned a positive value to F_1 when the flow is from tank 1 to tank 2, then a negative value of F_1 indicates the opposite flow. Care must be taken when solving this problem numerically, so that the square root of a negative number is not taken. For this purpose, the sign function is used

LINEARIZATION OF NONLINEAR MODELS

Most chemical process models are nonlinear, but they are often linearized to perform a stability analysis. Linear models are easier to understand (than nonlinear models) and are necessary for most control system design methods.

A general single variable nonlinear model is:

$$\frac{dx}{dt} = f(x)$$

The function of a single variable, $f(x)$, can be approximated by a truncated Taylor series approximation around the **steady-state operating point (x_s)**:

$$f(x) = f(x_s) + \left. \frac{\partial f}{\partial x} \right|_{x_s} (x - x_s) + \frac{1}{2} \left. \frac{\partial^2 f}{\partial x^2} \right|_{x_s} (x - x_s)^2 + \text{higher order terms}$$

Neglecting the quadratic and higher order terms, we obtain:

$$f(x) \approx f(x_s) + \left. \frac{\partial f}{\partial x} \right|_{x_s} (x - x_s)$$

Note that:

$$\frac{dx_s}{dt} = f(x_s) = 0$$


By definition of steady state

$$\frac{dx}{dt} = f(x) \approx \left. \frac{\partial f}{\partial x} \right|_{x_s} (x - x_s)$$

Where the
evaluated

notation $\left. \frac{\partial f}{\partial x} \right|_{x_s}$ is used to indicate the partial derivative of $f(x)$ with respect to x , at the steady-state. Since the derivative of a constant (x_s) is zero, we can write:

$$\frac{dx}{dt} = \frac{d(x - x_s)}{dt}$$



$$\frac{d(x - x_s)}{dt} \approx \left. \frac{\partial f}{\partial x} \right|_{x_s} (x - x_s)$$

The reason
state from
violation var
Or perturbat

for using the expression above is that we are often interested in deviations in a steady-state operating point. Sometimes the ' symbol is used to represent *deviations*, $x' = x - x_s$. We can see that a deviation variable represents the change (deviation) from a steady-state value.

$$\frac{dx'}{dt} = \left. \frac{\partial f}{\partial x} \right|_{x, u}$$



$$\frac{dx'}{dt} = a x'$$

One State Variable and One Input Variable

consider a function with one state variable and one input variable

$$\dot{x} = \frac{dx}{dt} = f(x, u)$$

a Taylor Series Expansion for $f(x, u)$:

$$\begin{aligned} \dot{x} = & f(x_s, u_s) + \left. \frac{\partial f}{\partial x} \right|_{x_s, u_s} (x - x_s) + \left. \frac{\partial f}{\partial u} \right|_{x_s, u_s} (u - u_s) \\ & + \frac{1}{2} \left. \frac{\partial^2 f}{\partial x^2} \right|_{x_s, u_s} (x - x_s)^2 + \left. \frac{\partial^2 f}{\partial x \partial u} \right|_{x_s, u_s} (x - x_s)(u - u_s) + \frac{1}{2} \left. \frac{\partial^2 f}{\partial u^2} \right|_{x_s, u_s} (u - u_s)^2 \\ & + \text{higher order terms} \end{aligned}$$

and truncating after the linear terms, we have:

$$\dot{x} = f(x_s, u_s) + \left. \frac{\partial f}{\partial x} \right|_{x_s, u_s} (x - x_s) + \left. \frac{\partial f}{\partial u} \right|_{x_s, u_s} (u - u_s)$$

and realizing that $f(x_s, u_s) = 0$ and $dx/dt = d(x - x_s)/dt$:

$$\frac{d(x - x_s)}{dt} \approx \left. \frac{\partial f}{\partial x} \right|_{x_s, u_s} (x - x_s) + \left. \frac{\partial f}{\partial u} \right|_{x_s, u_s} (u - u_s)$$

Using deviation variables, $x' = x - x_s$ and $u' = u - u_s$:

$$\frac{dx'}{dt} \approx \left. \frac{\partial f}{\partial x} \right|_{x_s, u_s} x' + \left. \frac{\partial f}{\partial u} \right|_{x_s, u_s} u'$$

which can be written:

$$\frac{dx'}{dt} = a x' + b u'$$

where $a = \partial f / \partial x|_{x_s}$ and $b = \partial f / \partial u|_{x_s, u_s}$

If there is a single output that is a function of the states and inputs, then:

$$y = g(x, u)$$

Again, performing a Taylor series expansion and truncating the quadratic and higher terms:

$$g(x,u) \approx g(x_s, u_s) + \left. \frac{\partial g}{\partial x} \right|_{x_s, u_s} (x - x_s) + \left. \frac{\partial g}{\partial u} \right|_{x_s, u_s} (u - u_s)$$

Since $g(x_s, u_s)$ is simply the steady-state value of the output (y_s), we can write:

$$y \approx g(x_s, u_s) + \left. \frac{\partial g}{\partial x} \right|_{x_s, u_s} (x - x_s) + \left. \frac{\partial g}{\partial u} \right|_{x_s, u_s} (u - u_s)$$

or

$$y - y_s = c (x - x_s) + d (u - u_s)$$

where $c = \left. \partial g / \partial x \right|_{x_s, u_s}$ and $d = \left. \partial g / \partial u \right|_{x_s, u_s}$

Using deviation notation:

$$y' = c x' + d u'$$

* Example:

Consider a Nonlinear Tank Height Problem

$$\frac{dh}{dt} = \frac{F}{A} - \frac{\beta}{A} \sqrt{h}$$

where h is the state variable, F is the input variable, β and A are parameters. The righthand side is:

$$f(h, F) = \frac{F}{A} - \frac{\beta}{A} \sqrt{h}$$

Using a truncated Taylor series expansion, we find:

$$f(h, F) \approx \left[\frac{F_s}{A} - \frac{\beta}{A} \sqrt{h_s} \right] + \frac{1}{A} [F - F_s] - \frac{\beta}{2A\sqrt{h_s}} [h - h_s]$$

The first term on the righthand side is zero, because the linearization is about a steady-state point. That is,

$$\left. \frac{dh}{dt} \right|_{h_s, F_s} = \frac{F_s}{A} - \frac{\beta}{A} \sqrt{h_s} = 0$$

We can now write:

$$\frac{d(h - h_s)}{dt} = -\frac{\beta}{2A\sqrt{h_s}} [h - h_s] + \frac{1}{A} [F - F_s]$$

and using deviation variable notation ($h' = h - h_s$ and $u' = F - F_s$), and dropping the =

$$\frac{dh'}{dt} = -\frac{\beta}{2A\sqrt{h_s}} h' + \frac{1}{A} F'$$

For convenience (simplicity in notation) we often drop the (') notation and assume that x and u are deviation variables ($x = h - h_s$, $u = F - F_s$) and write:

$$\frac{dx}{dt} = -\frac{\beta}{2A\sqrt{h_s}} x + \frac{1}{A} u$$

which is in the state-space form

$$\frac{dx}{dt} = a x + b u$$

references

- * B.Wayne Bequette. 'Process dynamics, modeling Analysis and Simulation', Prentice Hall International Series, 1998.



Thank you

Introduction to Process Control

2016/2017

Control & System Eng. Dept. , 4th year

Subject: Process Control.

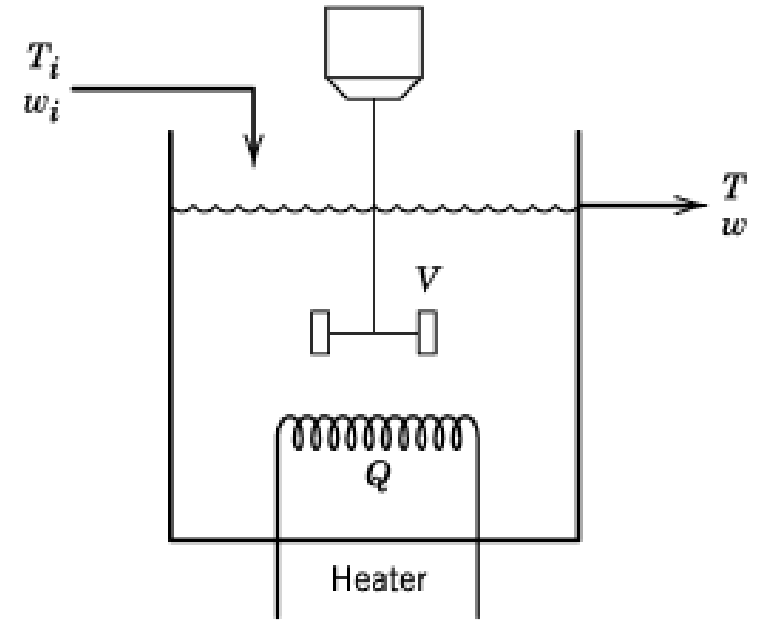
Dr. Safanah M. Raafat

Steam-Heated Stirred Tank

- Steam (or some other heating medium) can be condensed within a coil or jacket to heat liquid in a stirred tank, and the inlet steam pressure can be varied by adjusting a control valve.
- The condensation pressure P_s then fixes the steam temperature T_s through an appropriate thermodynamic relation or from tabular information such as the steam tables

$$T_s = f(P_s)$$

- Consider the stirred-tank heating system with constant holdup and a steam heating coil.
- We assume that the thermal capacitance of the liquid condensate is negligible compared to the thermal capacitances of the tank liquid and the wall of the heating coil.
- This assumption is reasonable when a steam trap is used to remove the condensate from the coil as it is produced.
- As a result of this assumption, the dynamic model consists of energy balances on the liquid and the heating coil wall:



$$mC \frac{dT}{dt} = wC(T_i - T) + h_p A_p (T_w - T) \quad (2-51)$$

$$m_w C_w \frac{dT_w}{dt} = h_s A_s (T_s - T_w) - h_p A_p (T_w - T) \quad (2-52)$$

- where the subscripts w , s , and p refer, respectively, to the wall of the heating coil and to its steam and process sides.
- Note that these energy balances are similar to **Eqs. 2-47 and 2-48** for the electrically heated example. **Copmpare**
- The dynamic model contains three output variables (T_s , T , and T_w) and three equations: an algebraic equation with T_s related to P_s (a specified function of time or a constant) and two differential equations. Thus,
- Eqs. 2-50 through 2-52 constitute an exactly specified model with three input variables: P_s , T_i , and w .
- Several important features are noted.

- 1. Usually $hsAs \gg hp-Ap$, because the resistance to heat transfer on the steam side of the coil is much lower than on the process side.
- **2.** The change from electrical heating to steam heating increases the complexity of the model (three equations instead of two) but does not increase the model order (number of first-order differential equations).
- **3.** As models become more complicated, the input and output variables may be coupled through certain parameters.
- For example, hp may be a function of w , or hs may vary with the steam condensation rate; sometimes algebraic equations cannot be solved explicitly for a key variable.
- In this situation, numerical solution techniques have to be used.
- Usually, implicit algebraic equations must be solved by iterative methods at each time step in the numerical integration.

Liquid Storage Systems

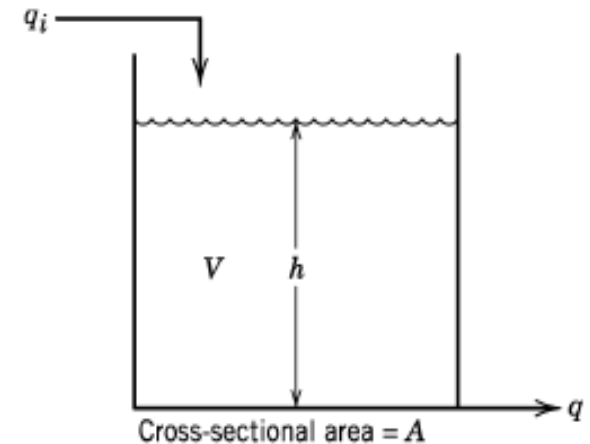
- A typical liquid storage process is shown in Fig. 2.5 where q_i and q are volumetric flow rates. A mass balance yields

$$\frac{d(\rho V)}{dt} = \rho q_i - \rho q \quad (2-53)$$

- Assume that liquid density ρ is constant and the tank is cylindrical with cross-sectional area, A .
- Then the volume of liquid in the tank can be expressed as $V = Ah$, where h is the liquid level (or *head*). Thus, (2-53) becomes

$$A \frac{dh}{dt} = q_i - q \quad (2-54)$$

- Note that Eq. 2-54 appears to be a *volume balance*.
- There are three important variations of the liquid storage process:
- **1.** The inlet or outlet flow rates might be constant;
- for example, exit flow rate q might be kept constant by a constant-speed, fixed-volume (metering) pump.
- An important consequence of this configuration is that the exit flow rate is then completely independent of liquid level over a wide range of conditions.
- Consequently, $q = \bar{q}$ where \bar{q}
- is the steady-state value.
- For this situation, the tank operates essentially as a flow *integrator*



- The tank exit line may function simply as a resistance to flow from the tank (distributed along the entire line), or it may contain a valve that provides significant resistance to flow at a single point.
- In the simplest case, the flow may be assumed to be linearly related to the driving force, the liquid level, in analogy to Ohm's law for electrical circuits ($E = IR$)

$$h = qR_v \quad (2-55)$$

- where R_v is the resistance of the line or valve.
- Rearranging (2-55) gives the following *flow-head equation*:

$$q = \frac{1}{R_v} h \quad (2-56)$$

- Substituting (2-56) into (2-54) gives a first-order differential equation:

$$A \frac{dh}{dt} = q_i - \frac{1}{R_v} h \quad (2-57)$$

- This model of the liquid storage system exhibits dynamic behavior similar to that of the stirred tank heating system of Eq. 2-36.
- **3.** A more realistic expression for flow rate q can be obtained when a fixed valve has been placed in the exit line and turbulent flow can be assumed.
- The driving force for flow through the valve is the pressure drop ΔP :

$$\Delta P = P - P_a \quad (2-58)$$

- where P is the pressure at the bottom of the tank and P_a is the pressure at the end of the exit line.
- We assume that P_a is the ambient pressure.
- If the valve is considered to be an orifice, a mechanical energy balance, or *Bernoulli equation*, can be used to derive the relation

$$q = C_v^* \sqrt{\frac{P - P_a}{\rho}} \quad (2-59)$$

- where C_v^* is a constant. The value of C_v^* depends on the particular valve and the valve setting (how much it is open).
- The pressure P at the bottom of the tank is related to liquid level h by a force balance

$$P = P_a + \frac{\rho g}{g_c} h \quad (2-60)$$

- where the acceleration of gravity g is constant.
- Substituting (2-59) and (2-60) into (2-54) yields the dynamic model

$$A \frac{dh}{dt} = q_i - C_v \sqrt{h} \quad (2-61)$$

where $C_v \triangleq C_v^* \sqrt{g/g_c}$. This model is nonlinear due to the square root term.

The Continuous Stirred-Tank Reactor (CSTR)

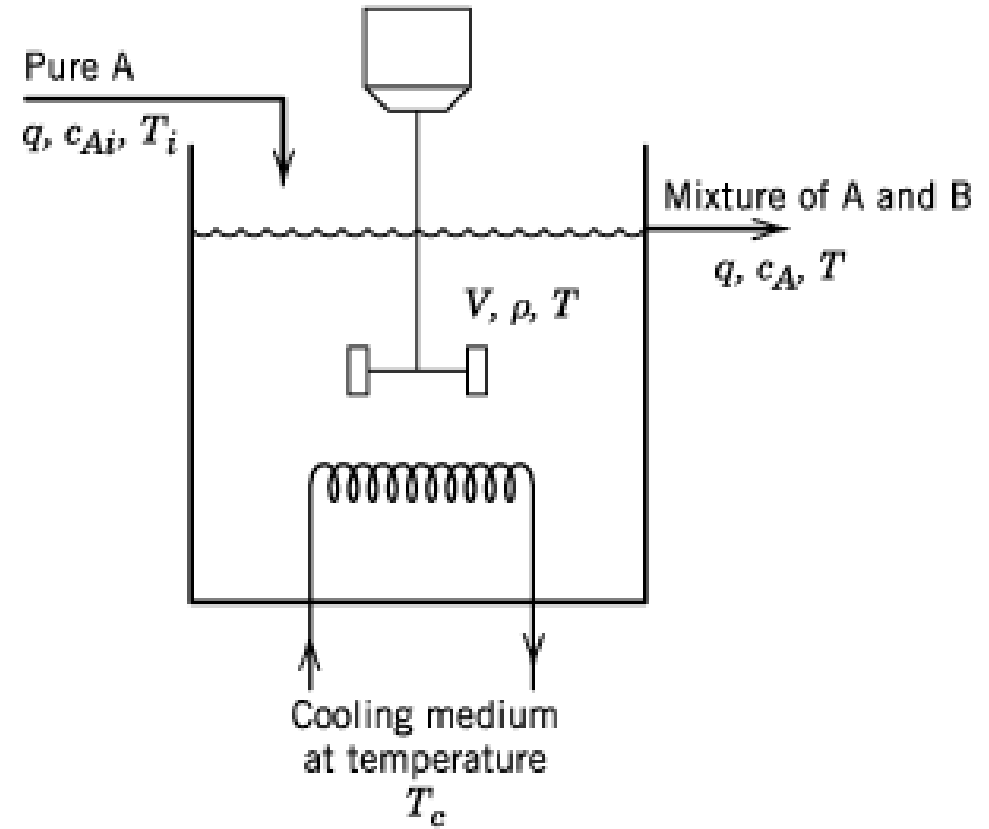
- Consider a simple liquid-phase, irreversible chemical reaction where chemical species A reacts to form species B. The reaction can be written as $A \rightarrow B$.
- We assume that the rate of reaction is first-order with respect to component A,

$$r = kc_A \quad (2-62)$$

- where r is the rate of reaction of A per unit volume, k is the reaction rate constant (with units of reciprocal time), and c_A is the molar concentration of species A.
- For single-phase reactions, the rate constant is typically a strong function of reaction temperature given by the Arrhenius relation,

$$k = k_0 \exp(-E/RT) \quad (2-63)$$

- where k_0 is the frequency factor,
- E is the activation energy,
- and R is the gas constant.
- The expressions in (2-62) and (2-63) are based on theoretical considerations, but model parameters k_0 and E are usually determined by fitting experimental data.
- Thus, these two equations can be considered to be *semi-empirical* relations.
- The inlet stream consists of pure component A with molar concentration, C_{Ai} .
- A cooling coil is used to maintain the reaction mixture at the desired operating temperature by removing heat that is released in the exothermic reaction.



- initial CSTR model development is based on three assumptions:
- **1.** The CSTR is perfectly mixed.
- **2.** The mass densities of the feed and product streams are equal and constant. They are denoted by ρ .
- **3.** The liquid volume V in the reactor is kept constant by an overflow line.

For these assumptions, the unsteady-state mass balance for the CSTR is:

$$\frac{d(\rho V)}{dt} = \rho q_i - \rho q \quad (2-64)$$

Because V and ρ are constant, (2-64) reduces to

$$q = q_i \quad (2-65)$$

Thus, even though the inlet and outlet flow rates may change due to upstream or downstream conditions,

Eq. 2-65 must be satisfied at all times. In Fig. 2.6, both flow rates are denoted by the symbol q .

For the stated assumptions, the unsteady-state component balances for species A (in molar units) is

$$V \frac{dc_A}{dt} = q(c_{Ai} - c_A) - Vkc_A \quad (2-66)$$

This balance is a special case of the general component balance in Eq. 2-7.

Next, we consider an unsteady-state energy balance for the CSTR. But first we make five additional assumptions:

4. The thermal capacitances of the coolant and the cooling coil wall are negligible compared to the thermal capacitance of the liquid in the tank.

5. All of the coolant is at a uniform temperature, T_c . (That is, the increase in coolant temperature as the coolant passes through the coil is neglected.)
6. The rate of heat transfer from the reactor contents to the coolant is given by

$$Q = UA(T_c - T) \quad (2-67)$$

where U is the overall heat transfer coefficient and A is the heat transfer area. Both of these model parameters are assumed to be constant.

7. The enthalpy change associated with the mixing of the feed and the liquid in the tank is negligible compared with the enthalpy change for the chemical reaction. In other words, the heat of mixing is negligible compared to the heat of reaction.

8. Shaft work and heat losses to the ambient can be neglected.

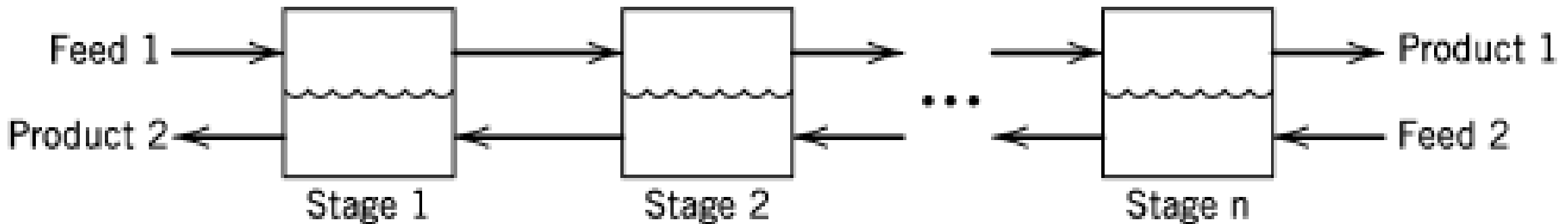
The following form of the CSTR energy balance is convenient for analysis and can be derived from Eqs. 2-62 and 2-63 and Assumptions 1–8 (Fogler, 2006; Russell and Denn, 1972),

$$\begin{aligned} V\rho C \frac{dT}{dt} = & wC(T_i - T) + (-\Delta H_R)Vkc_A \\ & + UA(T_c - T) \end{aligned} \quad (2-68)$$

where ΔH_R is the heat of reaction per mole of A that is reacted.

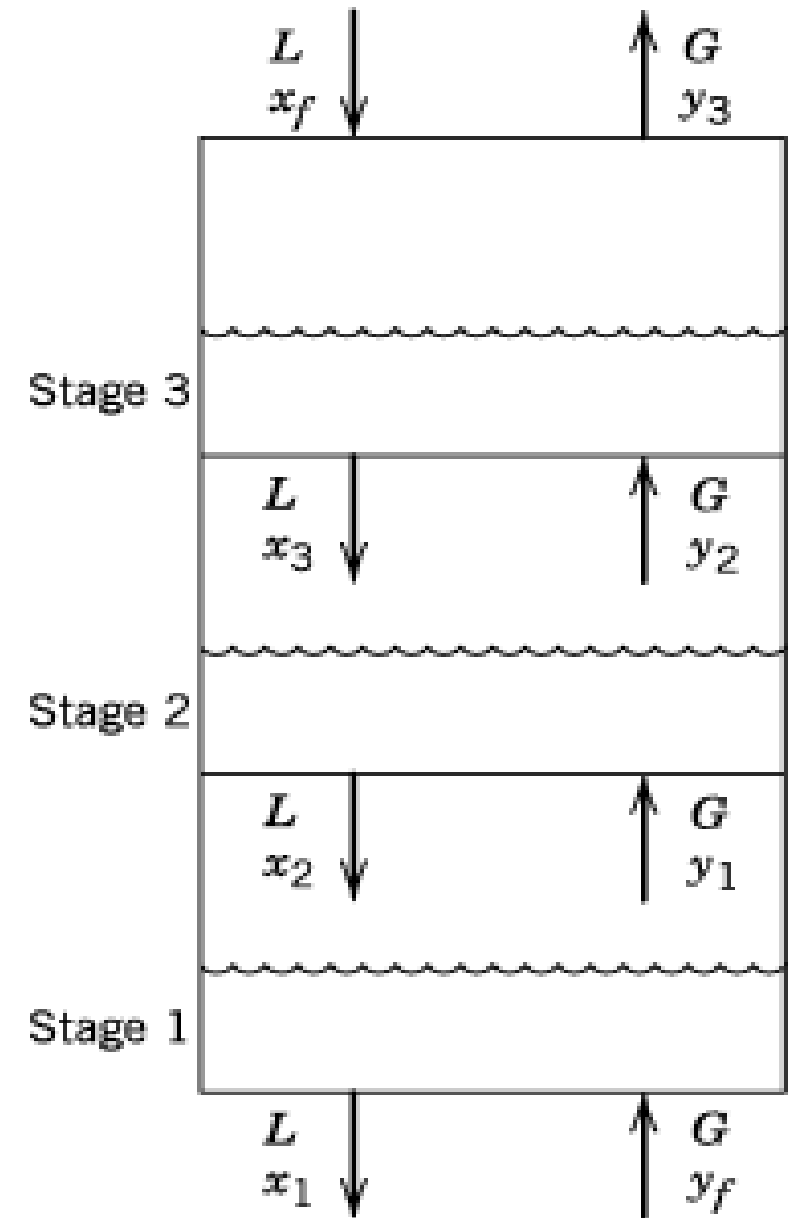
Staged Systems (a Three-Stage Absorber)

- Chemical processes, particularly separation processes, often consist of a sequence of stages. In each stage, materials are brought into intimate contact to obtain (or approach) equilibrium between the individual phases. The most important examples of staged processes include **distillation, absorption, and extraction**.
- The stages are usually arranged as a cascade with immiscible or partially miscible materials (the separate phases) flowing either cocurrently or countercurrently. Countercurrent contacting, shown in Fig. usually **permits the highest degree of separation to be attained in a fixed number of stages**



- The feeds to staged systems may be introduced at each end of the process, as in absorption units, or a single feed may be introduced at a middle stage, as is usually the case with distillation.
- The stages may be physically connected in either a vertical or horizontal configuration, depending on how the materials are transported, that is, whether pumps are used between stages, and so forth.
- we consider **a gas-liquid absorption process**, because its dynamics are somewhat simpler to develop than those of distillation and extraction processes. At the same time, it illustrates the characteristics of more complicated countercurrent staged processes.

- For the three-stage absorption unit shown in Fig. a gas phase is introduced at the bottom (**molar flow rate G**) and a single component is to be absorbed into a liquid phase introduced at the top (**molar flow rate L** , flowing countercurrently).
- A practical example of such a process is **the removal of sulfur dioxide (SO_2)** from combustion gas by use of a liquid absorbent. The gas passes up through the perforated (sieve) trays and contacts the liquid cascading down through them.



- series of weirs and down comers typically are used to retain a significant holdup of liquid on each stage while forcing the gas to flow upward through the perforations.
- Because of intimate mixing, we can assume that the component to be absorbed is in equilibrium between the gas and liquid streams leaving each stage i .
- a simple linear relation is often assumed.
- For stage i

$$y_i = ax_i + b \quad (2-70)$$

where y_i and x_i denote gas and liquid concentrations of the absorbed component. Assuming constant liquid

holdup H and perfect mixing on each stage, and neglecting the holdup of gas, the component material balance for any stage i is

$$H \frac{dx_i}{dt} = G(y_{i-1} - y_i) + L(x_{i+1} - x_i) \quad (2-71)$$

In Eq. 2-71 we also assume that molar liquid and gas flow rates L and G are unaffected by the absorption, because changes in concentration of the absorbed component are small, and L and G are approximately constant. Substituting Eq. 2-70 into Eq. 2-71 yields

$$H \frac{dx_i}{dt} = aGx_{i-1} - (L + aG)x_i + Lx_{i+1} \quad (2-72)$$

Why ?→ Dividing by L and substituting $\tau = H/L$ (the stage liquid residence time), $\mathfrak{s} = aG/L$ (the *stripping factor*), and $K = G/L$ (the gas-to-liquid ratio), the following model is obtained for the three-stage absorber:

$$\tau \frac{dx_1}{dt} = K(y_f - b) - (1 + \delta) x_1 + x_2 \quad (2-73)$$

$$\tau \frac{dx_2}{dt} = \delta x_1 - (1 + \delta) x_2 + x_3 \quad (2-74)$$

$$\tau \frac{dx_3}{dt} = \delta x_2 - (1 + \delta) x_3 + x_f \quad (2-75)$$

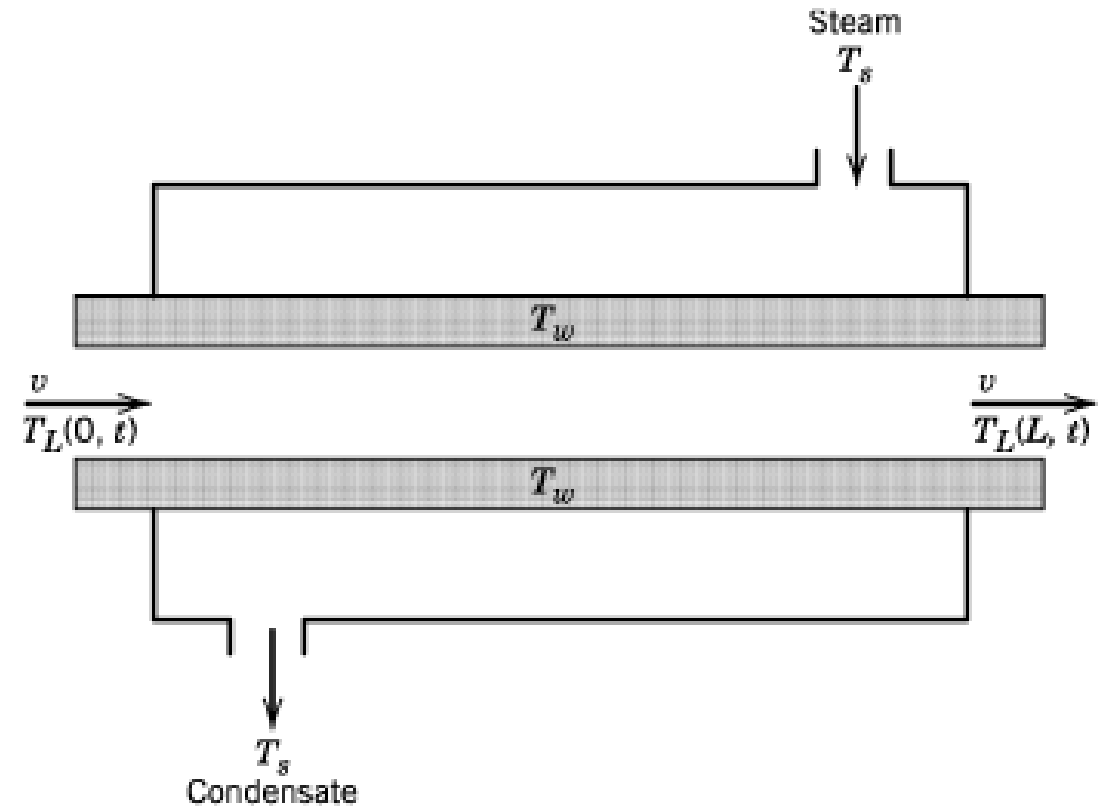
In the model of (2-73) to (2-75) note that the individual equations are linear but also coupled, meaning that each output variable— x_1 , x_2 , x_3 —appears in more than one equation. This feature can make it difficult to convert these three equations into a single higher-order equation in one of the outputs, as was done in Eq. 2-49.

Distributed Parameter Systems (the Double-Pipe Heat Exchanger)

- All of the process models discussed up to this point have been of the ***lumped parameter*** type, meaning that any dependent variable can be assumed to be a function only of time and not of spatial position.
- For the stirred tank systems discussed earlier, we assumed that any spatial variations of the temperature or concentration within the liquid could be neglected.
- Perfect mixing in each stage was also assumed for the absorber.
- Even when perfect mixing cannot be assumed, a lumped or average temperature may be taken as representative of the tank contents to simplify the process model.

- While lumped parameter models are normally used to describe processes, many important process units are inherently *distributed parameter*; that is, the output variables are functions of both time and position.
- Hence, their process models contain one or more partial differential equations. Pertinent examples include shell-and-tube heat exchangers, packed-bed reactors, packed columns, and long pipelines carrying compressible gases.
- In each of these cases, the output variables are a function of distance down the tube (pipe), height in the bed (column), or some other measure of location.
- In some cases, two or even three spatial variables may be considered; for example, concentration and temperature in a tubular reactor may depend on both axial and radial positions, as well as time.

- Figure shown illustrates a double-pipe heat exchanger where a fluid flowing through the inside tube with velocity v is heated by steam condensing in the outer tube.
- If the fluid is assumed to be in plug flow, the temperature of the liquid is expressed as $T_L(Z, t)$ where z denotes distance from the fluid inlet.
- The fluid heating process is truly distributed parameter; at any instant in time there is a temperature profile along the inside tube.
- The steam condensation, on the other hand, might justifiably be treated as a lumped process, because the steam temperature $T_s(t)$ can be assumed to be a function of the condensation pressure, itself presumably a function only of time and not a function of position.
- We also assume that the wall temperature $T_w(Z, t)$ is different from T_L and T_s due to the thermal capacitance and resistances.



- In developing a model for this process, assume that the liquid enters at temperature $T_L(0, t)$ -that is, at $z = 0$.
- Heat transfer coefficients (steam-to-wall h_s and wall-to liquid h_L) can be used to approximate the energy transfer processes.
- We neglect the effects of axial energy conduction, the resistance to heat transfer within the metal wall, and the thermal capacitance of the steam condensate.
- A distributed parameter model for the heat exchanger can be derived by applying Eq. 2-8 over a differential tube length Δz of the exchanger. In such a *shell* energy balance, the partial differential equation is obtained by taking the limit as $\Delta z \rightarrow 0$.
- Using the conservation law, Eq. 2-8, the following PDE results

$$\rho_L C_L S_L \frac{\partial T_L}{\partial t} = -\rho_L C_L S_L v \frac{\partial T_L}{\partial z} + h_L A_L (T_w - T_L) \quad (2-76)$$

where the following parameters are constant: ρ_L = liquid density, C_L = liquid heat capacity, S_L = cross-sectional area for liquid flow, h_L = liquid heat transfer coefficient, and A_L = wall heat transfer area of the liquid. This equation can be rearranged to yield

$$\frac{\partial T_L}{\partial t} = -v \frac{\partial T_L}{\partial z} + \frac{1}{\tau_{HL}} (T_w - T_L) \quad (2-77)$$

where $\tau_{HL} = \rho_L C_L S_L / h_L A_L$ has units of time and is called the characteristic time for heating of the liquid. An energy balance for the wall gives

$$\rho_w C_w S_w \frac{\partial T_w}{\partial t} = h_s A_s (T_s - T_w) - h_L A_L (T_w - T_L) \quad (2-78)$$

where the parameters associated with the wall are denoted by subscript w and the steam-side transport parameters are denoted by subscript s . Because T_w depends on T_L , it is also a function of time and position, $T_w(z, t)$. T_s is a function only of time, as noted above. Equation 2-78 can be rearranged as

$$\frac{\partial T_w}{\partial t} = \frac{1}{\tau_{sw}} (T_s - T_w) - \frac{1}{\tau_{wL}} (T_w - T_L) \quad (2-79)$$

where

$$\tau_{sw} = \frac{\rho_w C_w S_w}{h_s A_s} \quad \text{and} \quad \tau_{wL} = \frac{\rho_w C_w S_w}{h_L A_L} \quad (2-80)$$

are characteristic times for the thermal transport processes between the steam and the wall and the wall and the liquid, respectively.

- To be able to solve Eqs. 2-77 and 2-79, boundary conditions for both T_L and T_w at time $t = 0$ are required.
- Assume that the system initially is at steady state ($\partial T_L / \partial t = \partial T_w / \partial t = 0$; $T_s(0)$ is known).
- The steady state profile, $h(z, 0)$, can be obtained by integrating Eq. 2-77 with respect to z simultaneously with solving the steady-state version of Eq. 2-79, an algebraic expression.
- Note that the steady-state version of (2-77) is an ODE in z , with $T_L(0, 0)$ as the boundary condition.
- $T_w(Z, 0)$ is found algebraically from T_s and $T_L(z, 0)$.

With the initial and boundary conditions completely determined, the variations in $T_L(z, t)$ and $T_w(z, t)$ resulting from a change in the inputs, $T_s(t)$ or $T_L(0, t)$, can now be obtained by solving Eqs. 2-77 and 2-79 simultaneously using an analytical approach or a numerical procedure (Hanna and Sandall, 1995). Because analytical methods can be used only in special cases, we illustrate a numerical procedure here. A numerical approach invariably requires that either z , t , or both z and t be *discretized*. Here we use a finite difference approximation to convert the PDEs to ODEs. Although numerically less efficient than other techniques such as those based on weighted residuals (Chapra and Canale, 2010), finite difference methods yield more physical insight into both the method and the result of physical lumping.

To obtain ODE models with time as the independent variable, the z dependence is eliminated by discretization. In Fig. 2.12 the double-pipe heat exchanger has been redrawn with a set of grid lines to indicate points at which the liquid and wall temperatures will be evaluated. We now rewrite Eqs. 2-77 and 2-79 in terms of the liquid and wall temperatures $T_L(0)$, $T_L(1)$, \dots , $T_L(N)$ and $T_w(0)$, $T_w(1)$, \dots , $T_w(N)$. Utilizing the backward difference approximation for the derivative $\partial T_L / \partial z$ yields

$$\frac{\partial T_L}{\partial z} \approx \frac{T_L(j) - T_L(j-1)}{\Delta z} \quad (2-81)$$

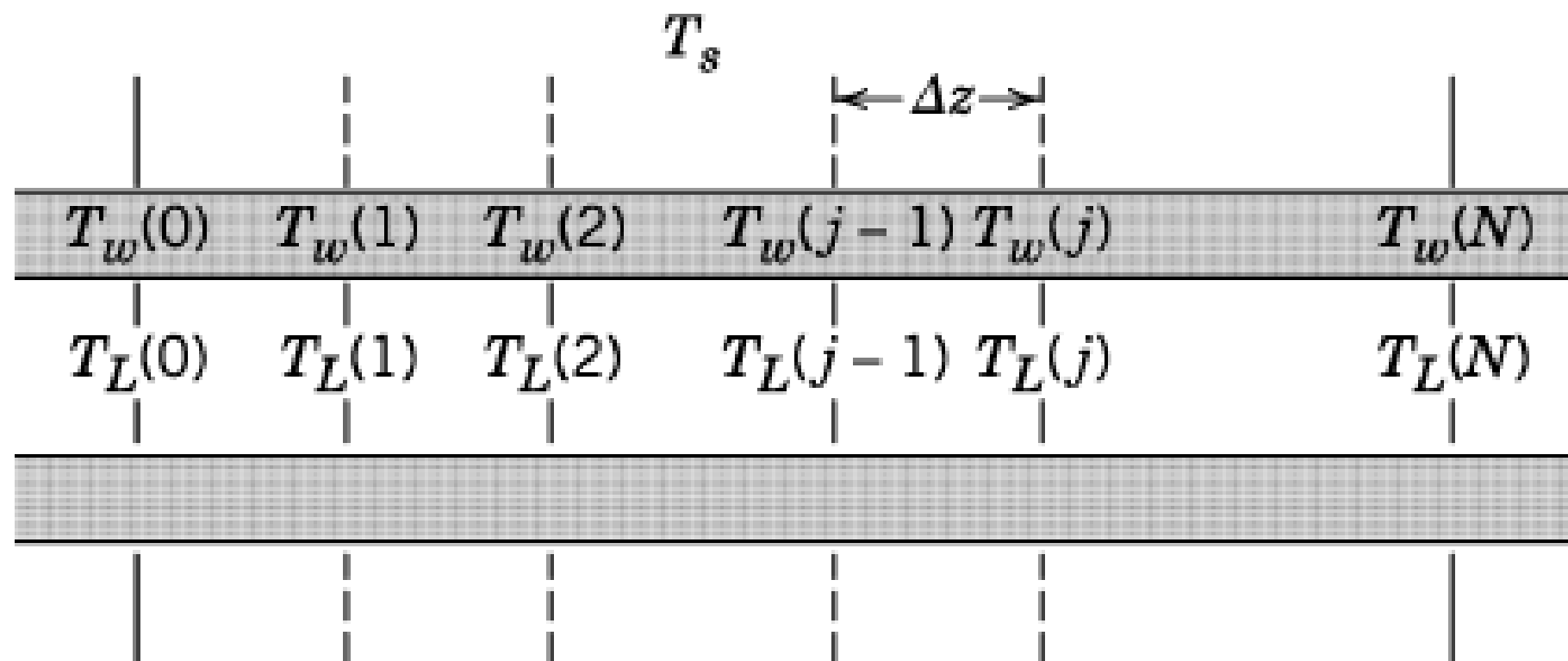


Figure 2.12 Finite-difference approximations for double-pipe heat exchanger.

where $T_L(j)$ is the liquid temperature at the j th node (discretization point). Substituting Eq. 2-81 into Eq. 2-77, the equation for the j th node is

$$\begin{aligned} \frac{dT_L(j)}{dt} = & -v \frac{T_L(j) - T_L(j-1)}{\Delta z} \\ & + \frac{1}{\tau_{HL}} [T_w(j) - T_L(j)] \quad (j = 1, \dots, N) \end{aligned} \quad (2-82)$$

The boundary condition at $z = 0$ becomes

$$T_L(0, t) = T_F(t) \quad (2-83)$$

where $T_F(t)$ is a specified forcing (input) function. Rearranging Eq. 2-82 yields

$$\begin{aligned}\frac{dT_L(j)}{dt} = & \frac{v}{\Delta z} T_L(j-1) - \left(\frac{v}{\Delta z} + \frac{1}{\tau_{HL}} \right) T_L(j) \\ & + \frac{1}{\tau_{HL}} T_w(j) \quad (j = 1, \dots, N)\end{aligned}\quad (2-84)$$

Similarly, for the wall equation,

$$\begin{aligned}\frac{dT_w(j)}{dt} = & - \left(\frac{1}{\tau_{sw}} + \frac{1}{\tau_{wL}} \right) T_w(j) + \frac{1}{\tau_{wL}} T_L(j) \\ & + \frac{1}{\tau_{sw}} T_s(j) \quad (j = 1, \dots, N)\end{aligned}\quad (2-85)$$

Note that Eqs. 2-84 and 2-85 represent $2N$ linear ordinary differential equations for N liquid and N wall temperatures. There are a number of anomalies associated with this simplified approach compared to the original PDEs. For example, it is clear that heat transfer from steam to wall to liquid is not accounted for at the zeroth node (the entrance), but is accounted for at all other nodes. Also, a detailed analysis of the discrete model will show that the steady-state relations between $T_L(j)$ and either input, T_s or T_F , are a function of the number of grid points and thus the grid spacing, Δz . The discrepancy can be minimized by making N large, that is, Δz small. The lowest-order model for this system that retains some distributed nature would be for $N = 2$. In this case, four equations result:

$$\frac{dT_{L1}}{dt} = \frac{v}{\Delta z} T_F(t) - \left(\frac{v}{\Delta z} + \frac{1}{\tau_{HL}} \right) T_{L1} + \frac{1}{\tau_{HL}} T_{w1} \quad (2-86)$$

$$\frac{dT_{L2}}{dt} = \frac{v}{\Delta z} T_{L1} - \left(\frac{v}{\Delta z} + \frac{1}{\tau_{HL}} \right) T_{L2} + \frac{1}{\tau_{HL}} T_{w2} \quad (2-87)$$

$$\frac{dT_{w1}}{dt} = - \left(\frac{1}{\tau_{sw}} + \frac{1}{\tau_{wL}} \right) T_{w1} + \frac{1}{\tau_{wL}} T_{L1} + \frac{1}{\tau_{sw}} T_s(t) \quad (2-88)$$

$$\frac{dT_{w2}}{dt} = - \left(\frac{1}{\tau_{sw}} + \frac{1}{\tau_{wL}} \right) T_{w2} + \frac{1}{\tau_{wL}} T_{L2} + \frac{1}{\tau_{sw}} T_s(t) \quad (2-89)$$

where the node number has been denoted by the second subscript on the output variables to simplify the notation. Equations 2-86 to 2-89 are coupled, linear, ordinary differential equations.

Process Control

4th year. Control Engineering

2nd Semester

Dr. Safanah M. Raafat

2016/2017

Control & System Engineering Dept.,

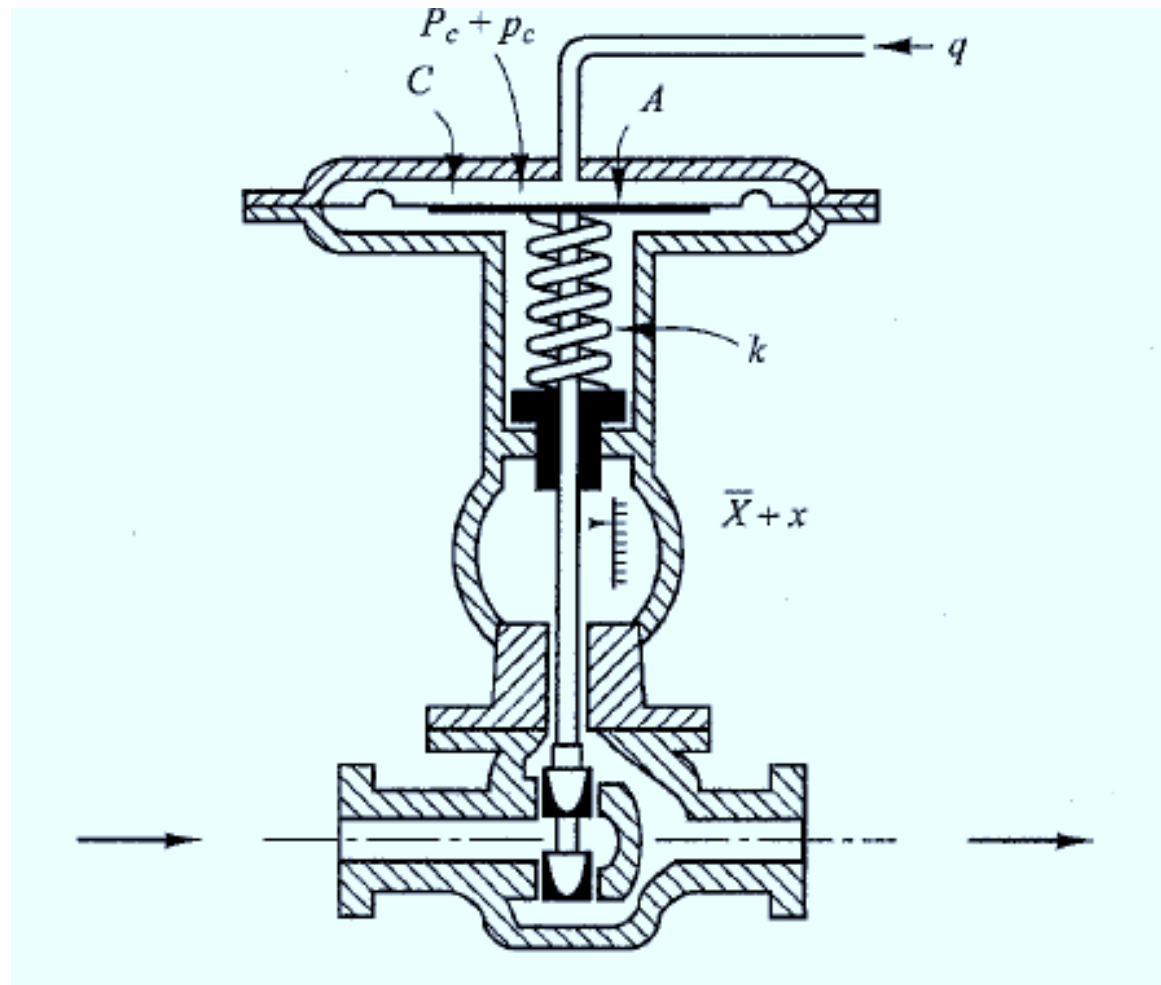
University of Technology

Baghdad Iraq

Pneumatic Actuating Valves

- One characteristic of pneumatic controls is that they almost exclusively employ pneumatic actuating valves.
- A pneumatic actuating valve can provide a large power output. (Since a pneumatic actuator requires a large power input to produce a large power output, it is necessary that a sufficient quantity of pressurized air be available.)
- In practical pneumatic actuating valves, the valve characteristics may not be linear; that is, the flow may not be directly proportional to the valve stem position, and also there may be other nonlinear effects, such as hysteresis.
- Consider the schematic diagram of a pneumatic actuating valve shown next, assume that the area of the diaphragm is A . Assume also that when the actuating error is zero the control pressure is equal to P_c and the valve displacement is equal to X .

Pneumatic Actuating Valves



Pneumatic Actuating Valves

- consider small variations in the variables and linearize the pneumatic actuating valve.
- define the small variation in the control pressure and the corresponding valve displacement to be p_c and x , respectively.
- Since a small change in the pneumatic pressure force applied to the diaphragm repositions the load, consisting of the spring, viscous friction, and mass, the force balance equation becomes

Pneumatic Actuating Valves

$$Ap_c = m\ddot{x} + b\dot{x} + kx$$

where m = mass of the valve and valve stem

b = viscous-friction coefficient

k = spring constant

If the force due to the mass and viscous friction are negligibly small, then this last equation can be simplified to

$$Ap_c = kx$$

The transfer function between x and p_c thus becomes

$$\frac{X(s)}{P_c(s)} = \frac{A}{k} = K_c$$

Pneumatic Actuating Valves

where $X(s) = \mathcal{L}[x]$ and $P_c(s) = \mathcal{L}[p_c]$. If q_i , the change in flow through the pneumatic actuating valve, is proportional to x , the change in the valve-stem displacement, then

$$\frac{Q_i(s)}{X(s)} = K_q$$

where $Q_i(s) = \mathcal{L}[q_i]$ and K_q is a constant. The transfer function between q_i and p_c becomes

$$\frac{Q_i(s)}{P_c(s)} = K_c K_q = K_v$$

where K_v is a constant.

Pneumatic Actuating Valves

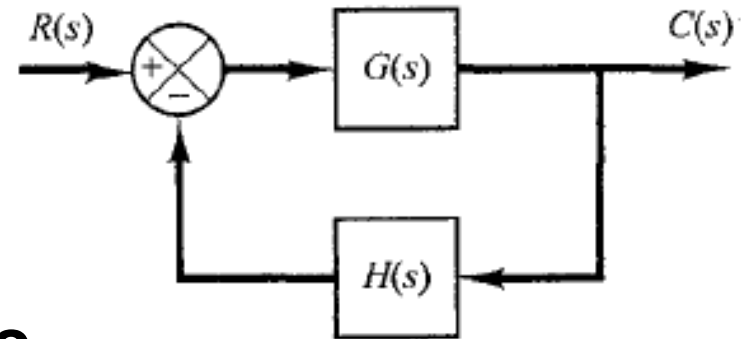
- The standard control pressure for this kind of a pneumatic actuating valve is between 3 and 15 psig.
- The valve-stem displacement is limited by the allowable stroke of the diaphragm and is only a few inches. If a longer stroke is needed, a piston-spring combination may be employed.
- In pneumatic actuating valves, the static-friction force must be limited to a low value so that excessive hysteresis does not result.
- Because of the compressibility of air, the control action may not be positive; that is, an error may exist in the valve-stem position.
- The use of a valve positioner results in improvements in the performance of a pneumatic actuating valve.

Basic Principle for Obtaining Derivative Control Action.

methods for obtaining derivative control action.

- **The basic principle for generating a desired control action is to insert the inverse of the desired transfer function in the feedback path.**
- For the system shown , the closed-loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$



If $|G(s)H(s)| \gg 1$, then $C(s)/R(s)$ can be modified to

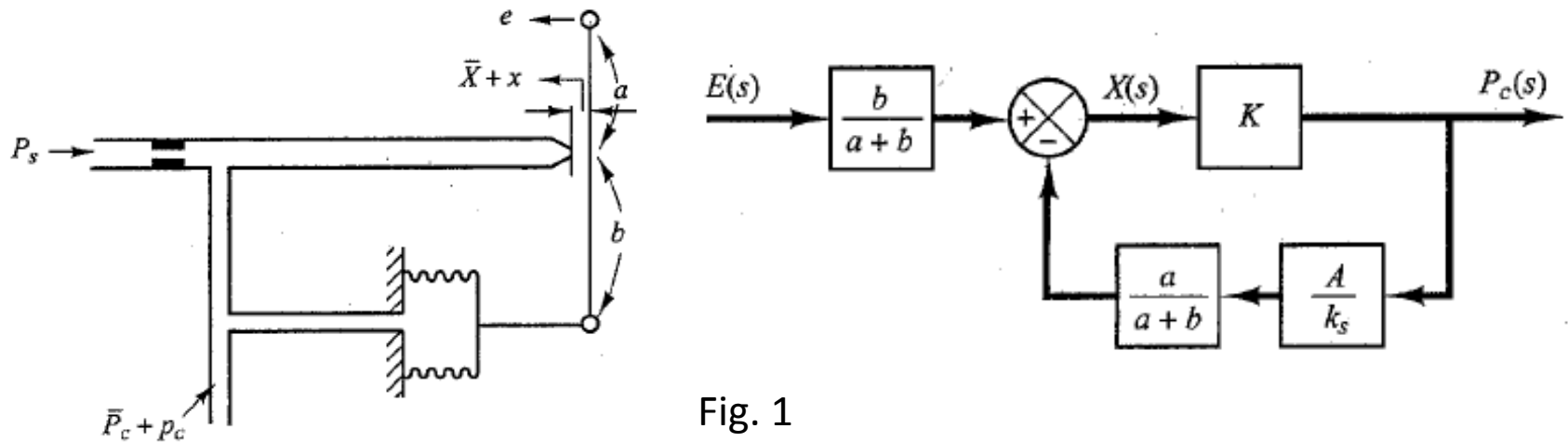
$$\frac{C(s)}{R(s)} = \frac{1}{H(s)}$$

Thus, if proportional-plus-derivative control action is desired, we insert an element having the transfer function $1/(Ts + 1)$ in the feedback path.

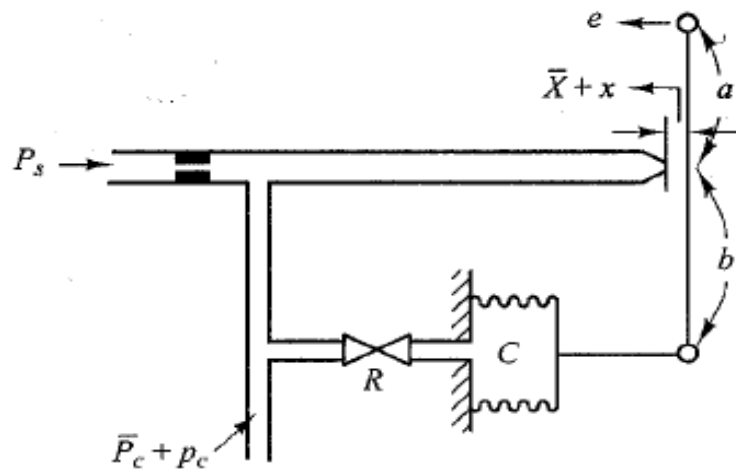


Explain Why ?

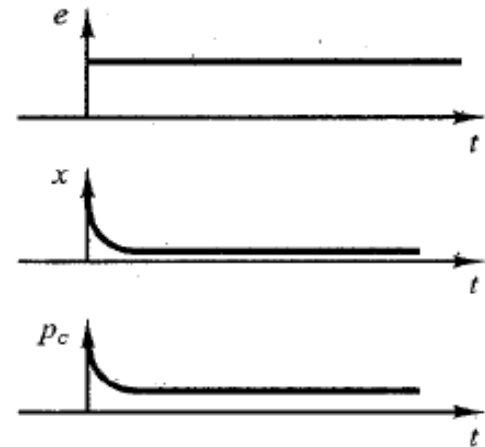
- Considering small changes in the variables, we can draw a block diagram of this controller as shown



- What is the type of this controller ?
- We shall now show that the addition of a restriction in the negative feedback path will modify the proportional controller to a proportional-plus-derivative controller, or a PD controller.

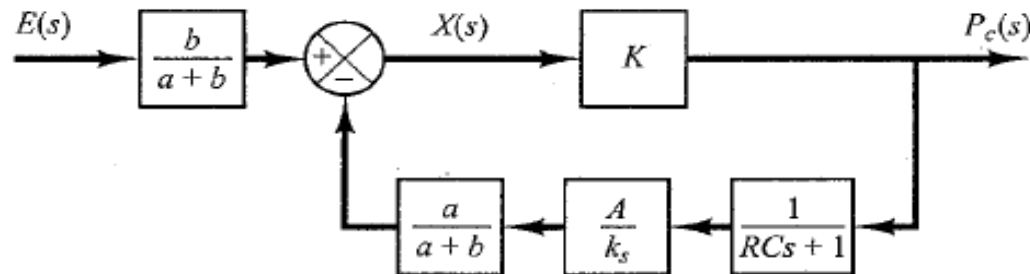


(a)



(b)

Fig. 2



(c)

Assuming again small changes in the actuating error, nozzle-flapper distance, and control pressure, we can summarize the operation of this controller as follows:

Let us first assume a small step change in e . Then the change in the control pressure p_c will be instantaneous. The restriction R will momentarily prevent the feedback bellows from sensing the pressure change p_c .

- Thus the feedback bellows will not respond momentarily, and the pneumatic actuating valve will feel the full effect of the movement of the flapper. As time goes on, the feedback bellows will expand. The change in the nozzle-flapper distance x and the change in the control pressure p_c can be plotted against time t , as shown in Figure (b).
- At steady state, the feedback bellows acts like an ordinary feedback mechanism. The curve p_c versus t clearly shows that this controller is of the proportional plus-derivative type. (why ?)
- A block diagram corresponding to this pneumatic controller is shown in Figure (c). In the block diagram, K is a constant, A is the area of the bellows, and k_s is the equivalent spring constant of the bellows.
- The transfer function between p_c and e can be obtained from the block diagram as follows:

$$\frac{P_c(s)}{E(s)} = \frac{\frac{b}{a+b} K}{1 + \frac{Ka}{a+b} \frac{A}{k_s} \frac{1}{RCs + 1}}$$

In such a controller the loop gain $|KaA/[(a + b)k_s(RCs + 1)]|$ is made much greater than unity. Thus the transfer function $P_c(s)/E(s)$ can be simplified to give

$$\frac{P_c(s)}{E(s)} = K_p(1 + T_d s)$$

where

$$K_p = \frac{bk_s}{aA}, \quad T_d = RC$$

Gived detailed derivative of the equation

Thus, delayed negative feedback, or the transfer function $1/(RCs + 1)$ in the feedback path, modifies the proportional controller to a proportional-plus-derivative controller.

Note that if the feedback valve is fully opened the control action becomes proportional.

If the feedback valve is fully closed, the control action becomes narrow-band proportional (on-off).

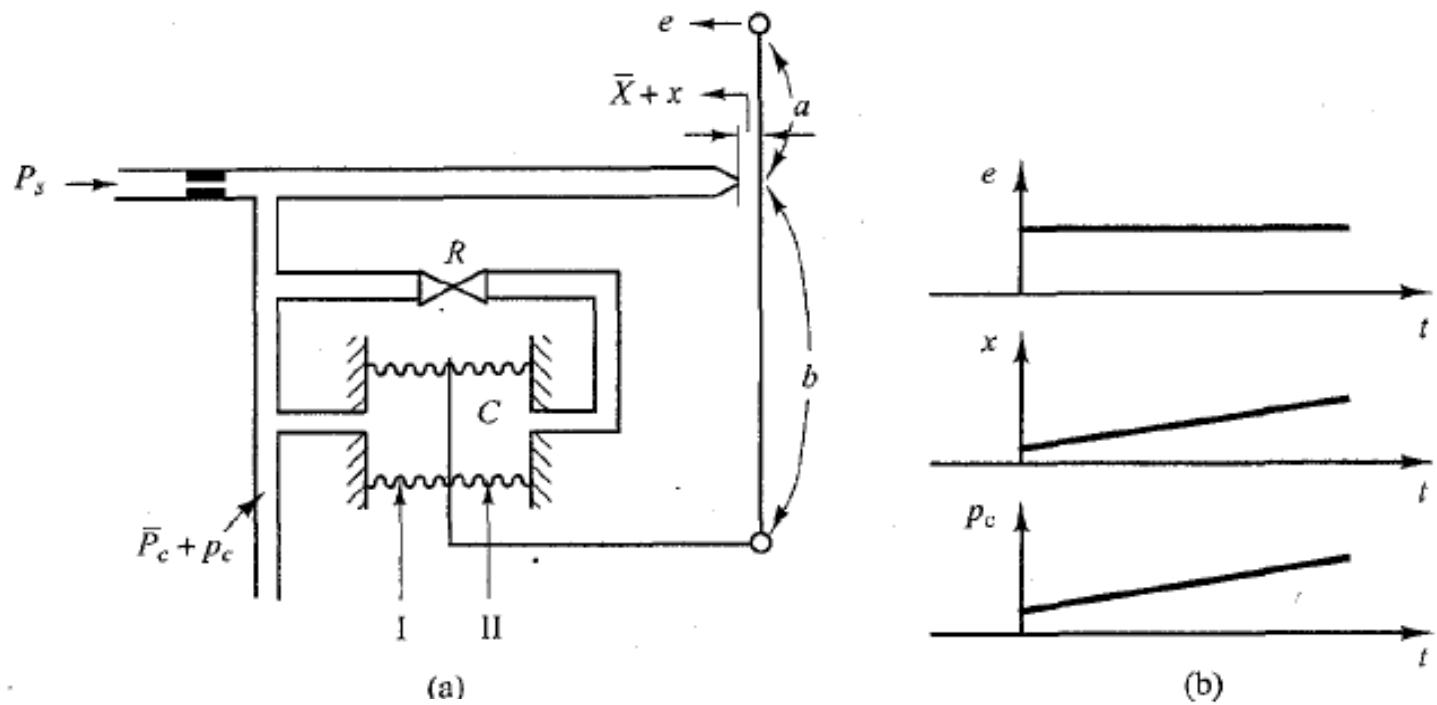
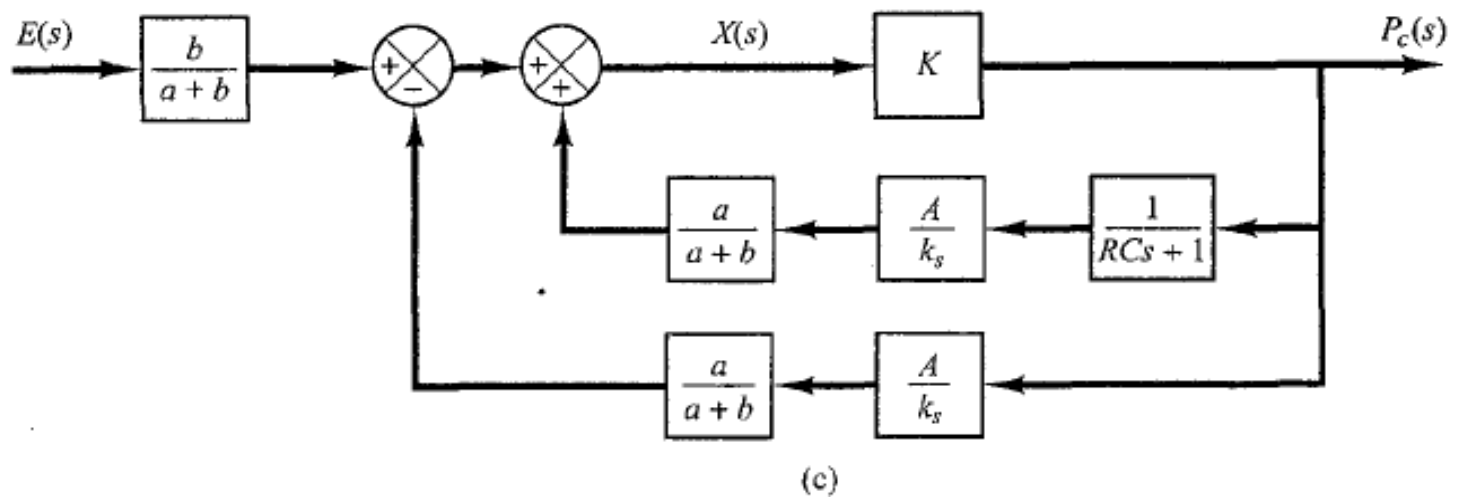


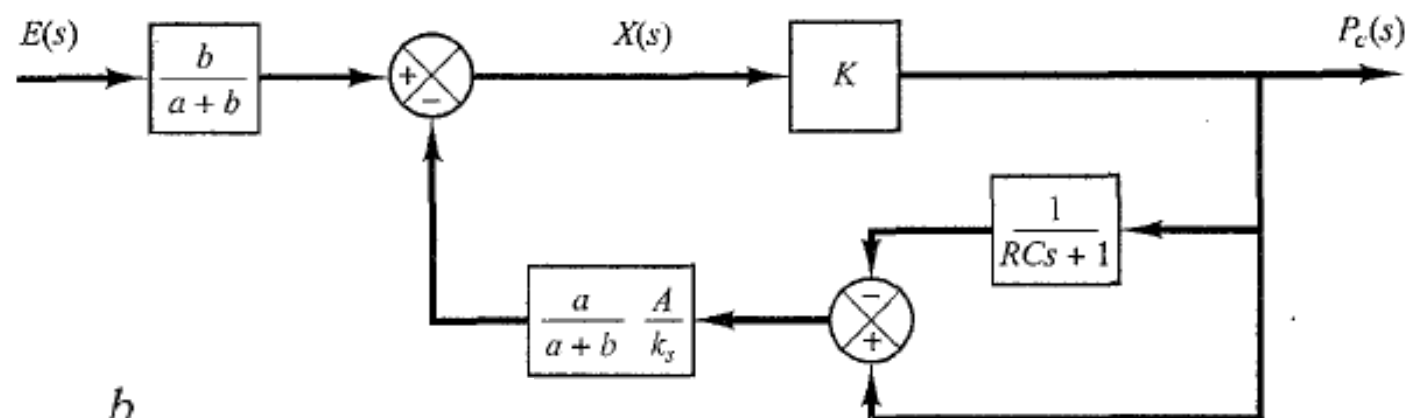
Fig. 3 Pneumatic Proportional plus integral controller



Obtaining Pneumatic Proportional-Plus-Integral Control Action. Consider the proportional controller shown in Figure 1 (a). Considering small changes in the variables, we can show that the addition of delayed positive feedback will modify this proportional controller to a proportional-plus-integral controller, or a PI controller.

Consider the pneumatic controller shown in Figure 3 (a). The operation of this controller is as follows: The bellows denoted by I is connected to the control pressure source without any restriction. The bellows denoted by II is connected to the control pressure source through a restriction. Let us assume a small step change in the actuating error. This will cause the back pressure in the nozzle to change instantaneously. Thus a change in the control pressure p_c also occurs instantaneously. Due to the restriction of the valve in the path to bellows II, there will be a pressure drop across the valve. As time goes on, air will flow across the valve in such a way that the change in pressure in bellows II attains the value p_c . Thus bellows II will expand or contract as time elapses in such a way as to move the flapper an additional amount in the direction of the original displacement e . This will cause the back pressure p_c in the nozzle to change continuously, as shown in Figure 3 (b).

Note that the integral control action in the controller takes the form of slowly canceling the feedback that the proportional control originally provided.



$$\frac{P_c(s)}{E(s)} = \frac{\frac{b}{a+b} K}{1 + \frac{Ka}{a+b} \frac{A}{k_s} \left(1 - \frac{1}{RCs+1}\right)}$$

where K is a constant, A is the area of the bellows, and k_s is the equivalent spring constant of the combined bellows. If $|KaARCs/[(a+b)k_s(RCs+1)]| \gg 1$, which is usually the case, the transfer function can be simplified to

$$\frac{P_c(s)}{E(s)} = K_p \left(1 + \frac{1}{T_i s}\right)$$

where

$$K_p = \frac{bk_s}{aA}, \quad T_i = RC$$

Obtaining Pneumatic Proportional-Plus-Integral-Plus-Derivative Control Action. A combination of the pneumatic controllers shown in Figures 2 (a) and 3 (a) yields a proportional-plus-integral-plus-derivative controller, or a PID controller. Figure 4 (a) shows a schematic diagram of such a controller. Figure 4 (b) shows a block diagram of this controller under the assumption of small variations in the variables.

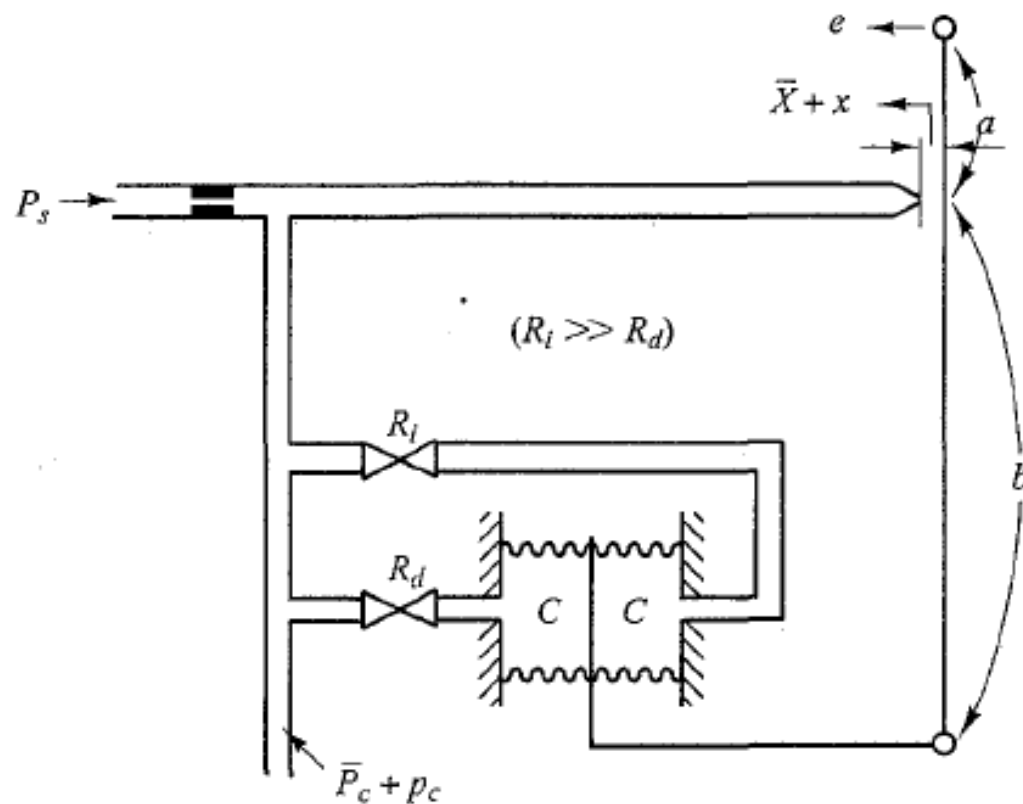


Fig. 4-a

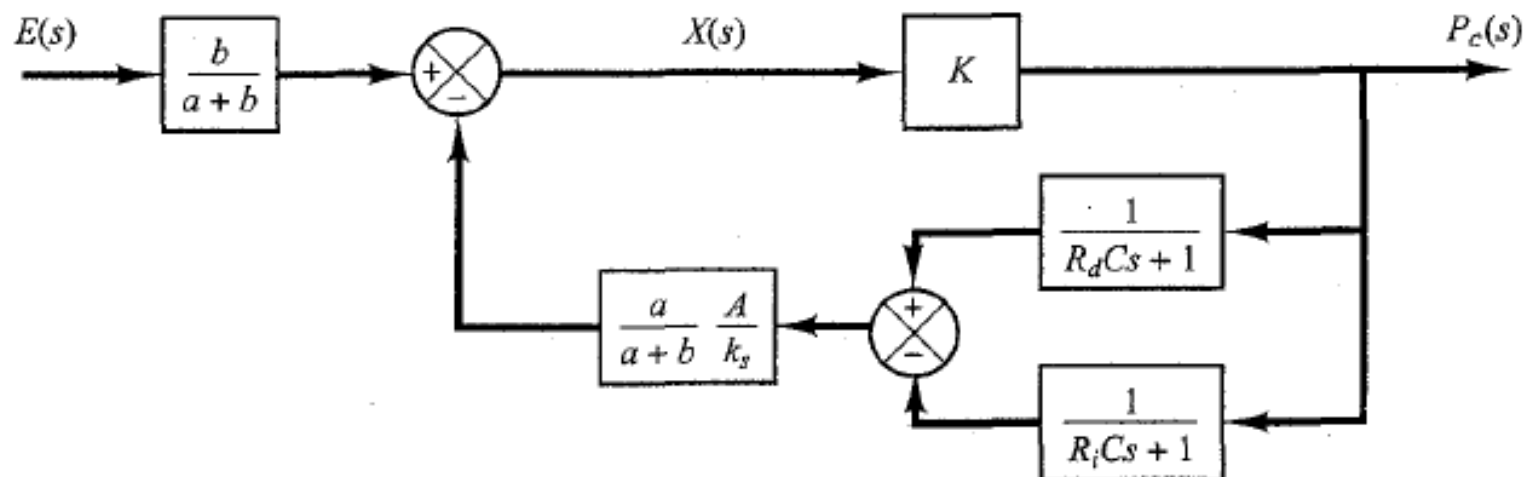


Fig. 4-b

The transfer function of this controller is

$$\frac{P_c(s)}{E(s)} = \frac{\frac{bK}{a+b}}{1 + \frac{Ka}{a+b} \frac{A}{k_s} \frac{(R_iC - R_dC)s}{(R_dCs + 1)(R_iCs + 1)}}$$

By defining

$$T_i = R_iC, \quad T_d = R_dC$$

and noting that under normal operation $|KaA(T_i - T_d)s / [(a + b)k_s(T_d s + 1)(T_i s + 1)]| \gg 1$ and $T_i \gg T_d$, we obtain

$$\begin{aligned} \frac{P_c(s)}{E(s)} &\doteq \frac{bk_s (T_d s + 1)(T_i s + 1)}{aA (T_i - T_d)s} \\ &\doteq \frac{bk_s}{aA} \frac{T_d T_i s^2 + T_i s + 1}{T_i s} \\ &= K_p \left(1 + \frac{1}{T_i s} + T_d s \right) \end{aligned}$$

where

$$K_p = \frac{bk_s}{aA}$$

HYDRAULIC SYSTEMS

- The widespread use of hydraulic circuitry in machine tool applications, aircraft control systems, and similar operations occurs because of such factors as positiveness, accuracy, flexibility, high horsepower-to-weight ratio, fast starting, stopping, and reversal with smoothness and precision, and simplicity of operations.
- The operating pressure in hydraulic systems is somewhere between 145 and 5000 lbf/in.² (between 1 and 35 MPa). In some special applications, the operating pressure may go up to 10,000 lbf/in.² (70 MPa).
- For the same power requirement, the weight and size of the hydraulic unit can be made smaller by increasing the supply pressure. **With high-pressure hydraulic systems, very large force can be obtained.**
- **Rapid-acting, accurate positioning of heavy loads is possible with hydraulic systems.** A combination of electronic and hydraulic systems is widely used because it combines the advantages of both electronic control and hydraulic power.

HYDRAULIC SYSTEMS

- Advantages and Disadvantages of Hydraulic Systems.
- Some of the advantages are the following:
 1. Hydraulic fluid acts as a lubricant, in addition to carrying away heat generated in the system to a convenient heat exchanger.
 2. Comparatively small sized hydraulic actuators can develop large forces or torques.
 3. Hydraulic actuators have a higher speed of response with fast starts, stops, and speed reversals.
 4. Hydraulic actuators can be operated under continuous, intermittent, reversing, and stalled conditions without damage.
 5. Availability of both linear and rotary actuators gives flexibility in design.
 6. Because of low leakages in hydraulic actuators, speed drop when loads are applied is small.

HYDRAULIC SYSTEMS

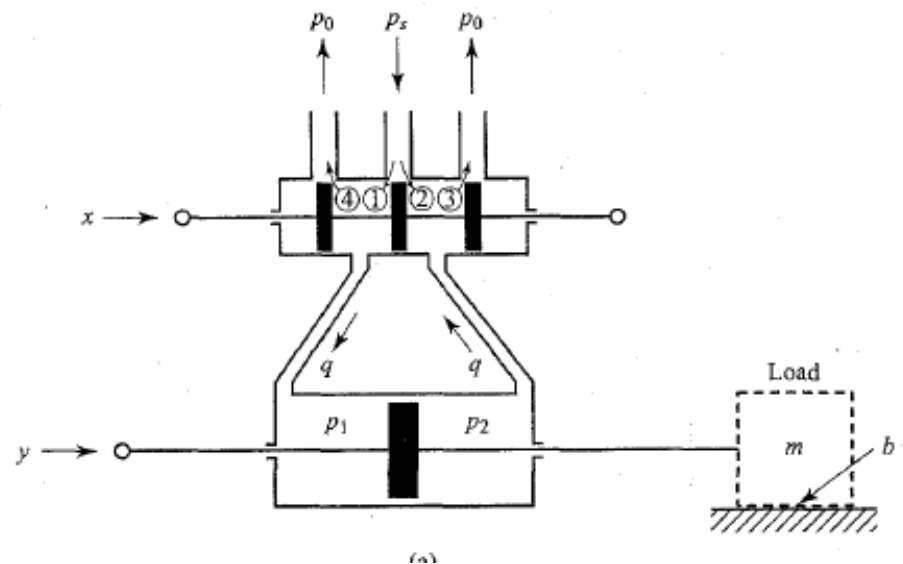
- On the other hand, several disadvantages tend to limit their use.
- 1. Hydraulic power is not readily available compared to electric power.
- 2. Cost of a hydraulic system may be higher than a comparable electrical system performing a similar function.
- 3. Fire and explosion hazards exist unless fire-resistant fluids are used.
- 4. Because it is difficult to maintain a hydraulic system that is free from leaks, the system tends to be messy.
- 5. Contaminated oil may cause failure in the proper functioning of a hydraulic system.
- 6. As a result of the nonlinear and other complex characteristics involved, the design of sophisticated hydraulic systems is quite involved.
- 7. Hydraulic circuits have generally poor damping characteristics. If a hydraulic circuit is not designed properly, some unstable phenomena may occur or disappear, depending on the operating condition.

Note that most hydraulic systems are nonlinear. Sometimes, however, it is possible to linearize nonlinear systems so as to reduce their complexity and permit solutions that are sufficiently accurate for most purposes.

Hydraulic Servo System

- Figure below shows a hydraulic servomotor.
- It is essentially a **pilot-valve-controlled hydraulic power amplifier and actuator**.
- The pilot valve is a balanced valve, in the sense that the pressure forces acting on it are all balanced.
- A very large power output can be controlled by a pilot valve, which can be positioned with very little power.

In practice, the ports shown **often made wider than the corresponding valves**. In such a case, **always leakage through the valves there is, which improves both the sensitivity and the linearity of the hydraulic servomotor**

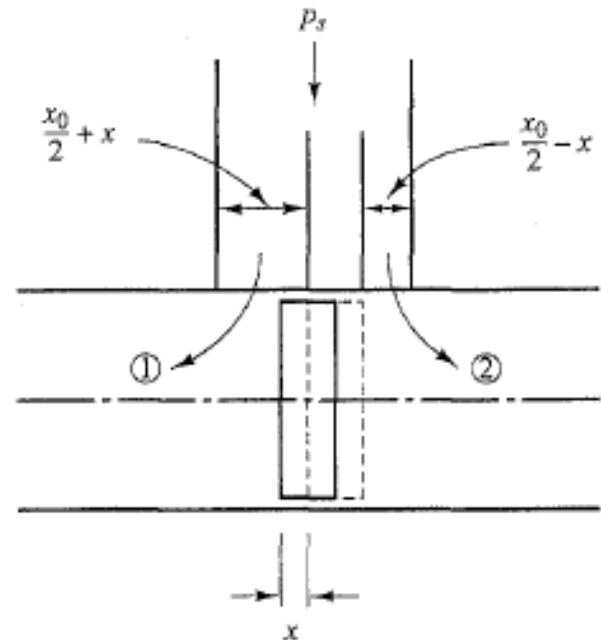


- Let us define the valve orifice areas of ports 1,2,3,4 as A_1 , **A_2** , A_3 , A_4 , respectively. Also, define the flow rates through ports 1,2,3,4 as q_1 , q_2 , q_3 , q_4 , respectively.
- valve is symmetrical, $A_1 = A_3$ and $A_2 = \mathbf{A_4}$. Assuming the displacement x to be small, we obtain

$$A_1 = A_3 = k \left(\frac{x_0}{2} + x \right)$$

$$A_2 = A_4 = k \left(\frac{x_0}{2} - x \right)$$

where k is a constant.



- Furthermore, we shall assume that the return pressure p_0 in the return line is small and thus can be neglected. Then flow rates through valve orifices are

$$q_1 = c_1 A_1 \sqrt{\frac{2g}{\gamma} (p_s - p_1)} = C_1 \sqrt{p_s - p_1} \left(\frac{x_0}{2} + x \right)$$

$$q_2 = c_2 A_2 \sqrt{\frac{2g}{\gamma} (p_s - p_2)} = C_2 \sqrt{p_s - p_2} \left(\frac{x_0}{2} - x \right)$$

$$q_3 = c_1 A_3 \sqrt{\frac{2g}{\gamma} (p_2 - p_0)} = C_1 \sqrt{p_2 - p_0} \left(\frac{x_0}{2} + x \right) = C_1 \sqrt{p_2} \left(\frac{x_0}{2} + x \right)$$

$$q_4 = c_2 A_4 \sqrt{\frac{2g}{\gamma} (p_1 - p_0)} = C_2 \sqrt{p_1 - p_0} \left(\frac{x_0}{2} - x \right) = C_2 \sqrt{p_1} \left(\frac{x_0}{2} - x \right)$$

where $C_1 = c_1 k \sqrt{2g/\gamma}$ and $C_2 = c_2 k \sqrt{2g/\gamma}$, and γ is the specific weight and is given by $\gamma = \rho g$, where ρ is mass density and g is the acceleration of gravity. The flow rate q to the left-hand side of the power piston is

$$q = q_1 - q_4 = C_1 \sqrt{p_s - p_1} \left(\frac{x_0}{2} + x \right) - C_2 \sqrt{p_1} \left(\frac{x_0}{2} - x \right)$$

The flow rate from the right-hand side of the power piston to the drain is the same as this q and is given by

$$q = q_3 - q_2 = C_1 \sqrt{p_2} \left(\frac{x_0}{2} + x \right) - C_2 \sqrt{p_s - p_2} \left(\frac{x_0}{2} - x \right)$$

In the present analysis we assume that the fluid is incompressible. Since the valve is symmetrical we have $q_1 = q_3$ and $q_2 = q_4$. By equating q_1 and q_3 , we obtain

$$p_s - p_1 = p_2$$

or

$$p_s = p_1 + p_2$$

If we define the pressure difference across the power piston as Δp or

$$\Delta p = p_1 - p_2$$

then

$$p_1 = \frac{p_s + \Delta p}{2}, \quad p_2 = \frac{p_s - \Delta p}{2}$$

For the symmetrical valve shown in Figure 1(a), the pressure in each side of the power piston is $(1/2)p_s$ when no load is applied, or $\Delta p = 0$. As the spool valve is displaced, the pressure in one line increases as the pressure in the other line decreases by the same amount.

In terms of p_s and Δp , we can rewrite the flow rate q

$$q = q_1 - q_4 = C_1 \sqrt{\frac{p_s - \Delta p}{2}} \left(\frac{x_0}{2} + x \right) - C_2 \sqrt{\frac{p_s + \Delta p}{2}} \left(\frac{x_0}{2} - x \right)$$

Noting that the supply pressure p_s is constant, the flow rate q can be written as a function of the valve displacement x and pressure difference Δp , or

$$q = C_1 \sqrt{\frac{p_s - \Delta p}{2}} \left(\frac{x_0}{2} + x \right) - C_2 \sqrt{\frac{p_s + \Delta p}{2}} \left(\frac{x_0}{2} - x \right) = f(x, \Delta p)$$

By applying the linearization technique to this case, the linearized equation about point $x = \bar{x}$, $\Delta p = \Delta \bar{p}$, $q = \bar{q}$ is

$$q - \bar{q} = a(x - \bar{x}) + b(\Delta p - \Delta \bar{p}) \quad *$$

where

$$\bar{q} = f(\bar{x}, \Delta\bar{p})$$

$$a = \left. \frac{\partial f}{\partial x} \right|_{x=\bar{x}, \Delta p=\Delta\bar{p}} = C_1 \sqrt{\frac{p_s - \Delta\bar{p}}{2}} + C_2 \sqrt{\frac{p_s + \Delta\bar{p}}{2}}$$

$$b = \left. \frac{\partial f}{\partial \Delta p} \right|_{x=\bar{x}, \Delta p=\Delta\bar{p}} = - \left[\frac{C_1}{2\sqrt{2} \sqrt{p_s - \Delta\bar{p}}} \left(\frac{x_0}{2} + \bar{x} \right) + \frac{C_2}{2\sqrt{2} \sqrt{p_s + \Delta\bar{p}}} \left(\frac{x_0}{2} - \bar{x} \right) \right] < 0$$

Coefficients a and b here are called *valve coefficients*. Equation (*) is a linearized mathematical model of the spool valve near an operating point $x = \bar{x}$, $\Delta p = \Delta\bar{p}$, $q = \bar{q}$. The values of valve coefficients a and b vary with the operating point. Note that $\partial f / \partial \Delta p$ is negative and so b is negative.

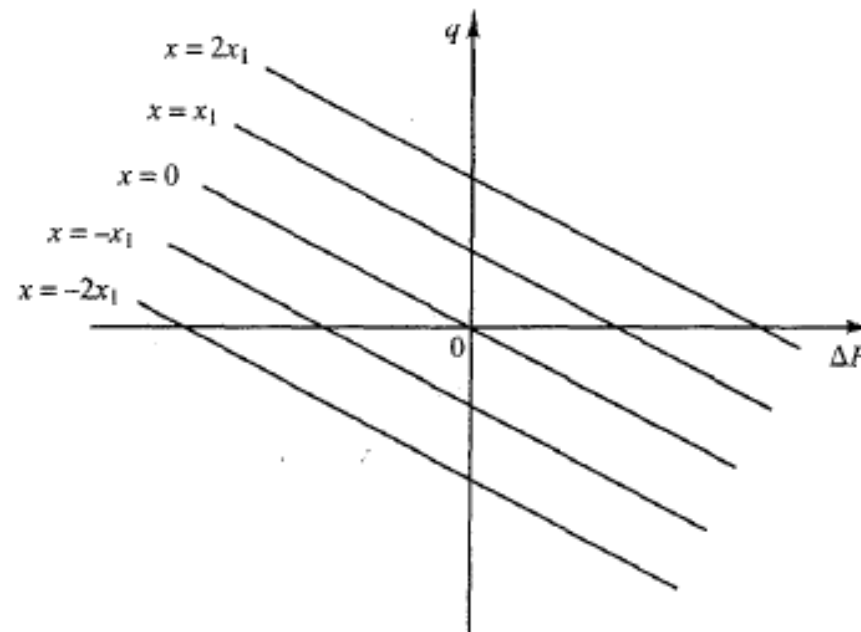
Since the normal operating point is the point where $\bar{x} = 0$, $\Delta\bar{p} = 0$, $\bar{q} = 0$, near the normal operating point, Equation * becomes

$$q = K_1 x - K_2 \Delta p$$

where

$$K_1 = (C_1 + C_2) \sqrt{\frac{p_s}{2}} > 0$$

$$K_2 = (C_1 + C_2) \frac{x_0}{4\sqrt{2} \sqrt{p_s}} > 0$$



$$A\rho \, dy = q \, dt$$

Notice that for a given flow rate q the larger the piston area A is, the lower will be the velocity dy/dt . Hence, if the piston area A is made smaller, the other variables remaining constant, the velocity dy/dt will become higher. Also, an increased flow rate q will cause an increased velocity of the power piston and will make the response time shorter.

$$\Delta P = \frac{1}{K_2} \left(K_1 x - A\rho \frac{dy}{dt} \right)$$

The force developed by the power piston is equal to the pressure difference ΔP times the piston area A or

$$\begin{aligned} \text{Force developed by the power piston} &= A \Delta P \\ &= \frac{A}{K_2} \left(K_1 x - A\rho \frac{dy}{dt} \right) \end{aligned}$$

For a given maximum force, if the pressure difference is sufficiently high, the piston area, or the volume of oil in the cylinder, can be made small. Consequently, to minimize the weight of the controller, we must make the supply pressure sufficiently high.

- Assume that the power piston moves a load consisting of a mass and viscous friction.
- Then the force developed by the power piston is applied to the load mass and friction, and we obtain

$$m\ddot{y} + b\dot{y} = \frac{A}{K_2}(K_1x - A\rho\dot{y})$$

- or

$$m\ddot{y} + \left(b + \frac{A^2\rho}{K_2}\right)\dot{y} = \frac{AK_1}{K_2}x$$

where m is the mass of the load and b is the viscous-friction coefficient.

- Assuming that the pilot valve displacement x is the input and the power piston displacement y is the output, we find that **the transfer function for the hydraulic servomotor** is, from Equation

$$\begin{aligned}\frac{Y(s)}{X(s)} &= \frac{1}{s \left[\left(\frac{mK_2}{AK_1} \right) s + \frac{bK_2}{AK_1} + \frac{A\rho}{K_1} \right]} \\ &= \frac{K}{s(Ts + 1)}\end{aligned}$$

where

$$K = \frac{1}{\frac{bK_2}{AK_1} + \frac{A\rho}{K_1}} \quad \text{and} \quad T = \frac{mK_2}{bK_2 + A^2\rho}$$

$nK_2/(bK_2 + A^2\rho)$ is negligibly small or the time constant T is negligible, the transfer function $Y(s)/X(s)$ can be simplified to give

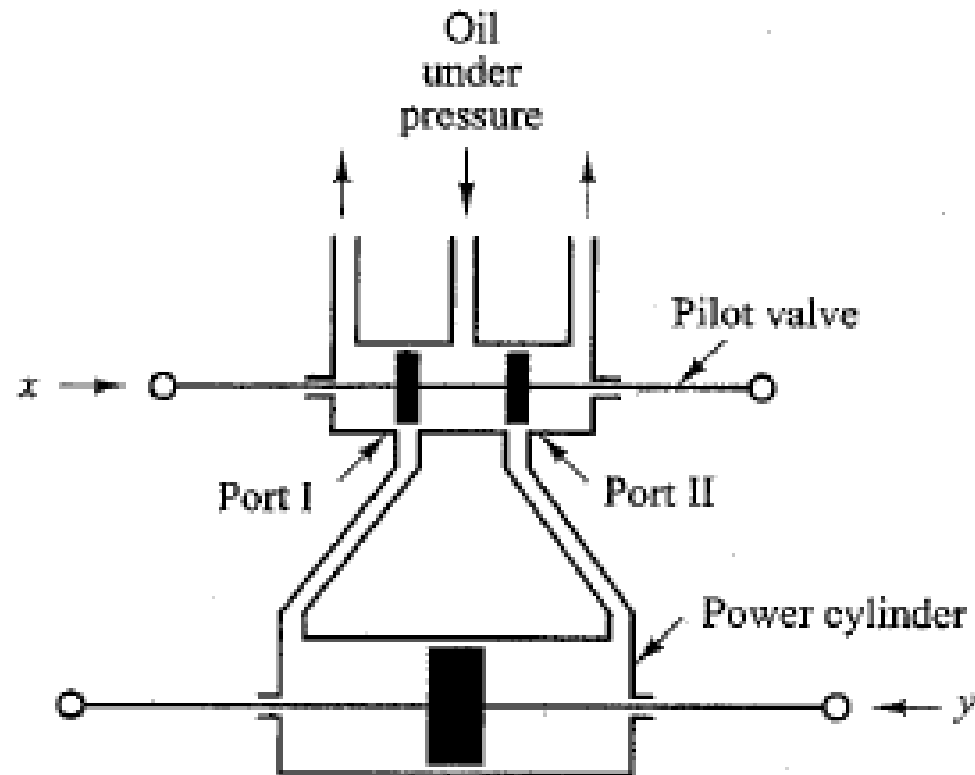
$$\frac{Y(s)}{X(s)} = \frac{K}{s}$$

It is noted that a more detailed analysis shows that if oil leakage, compressibility (including the effects of dissolved air), expansion of pipelines, and the likes are taken into consideration, the transfer function becomes

$$\frac{Y(s)}{X(s)} = \frac{K}{s(T_1s + 1)(T_2s + 1)}$$

where T_1 and T_2 are time constants. As a matter of fact, these time constants depend on the volume of oil in the operating circuit. The smaller the volume, the smaller the time constants.

Hydraulic Integral Controller.



Hydraulic
servomotor.

In the hydraulic servomotor shown the pilot valve (a four-way valve) has two lands on the spool. If the width of the land is smaller than the port in the valve sleeve, the valve is said to be *underlapped*. *Overlapped* valves have a land width greater than the port width. A *zero-lapped* valve has a land width that is identical to the port width. (If the pilot valve is a zero-lapped valve, analyses of hydraulic servomotors become simpler.)

In the present analysis, we assume that hydraulic fluid is incompressible and that the inertia force of the power piston and load is negligible compared to the hydraulic force at the power piston. We also assume that the pilot valve is a zero-lapped valve, and the oil flow rate is proportional to the pilot valve displacement.

Operation of this hydraulic servomotor is as follows. If input x moves the pilot valve to the right, port II is uncovered, and so high-pressure oil enters the right-hand side of the power piston. Since port I is connected, to the drain port, the oil in the left-hand side of the power piston is returned to the drain. The oil flowing into the power cylinder is at high pressure; the oil flowing out from the power cylinder into the drain is at low pressure. The resulting difference in pressure on both sides of the power piston will cause it to move to the left.

Note that the rate of flow of oil $q(\text{kg/sec})$ times dt (sec) is equal to the power piston displacement $dy(\text{m})$ times the piston area $A(\text{m}^2)$ times the density of oil $\rho(\text{kg/m}^3)$. Therefore,

$$A\rho dy = q dt$$

Because of the assumption that the oil flow rate q is proportional to the pilot valve displacement x , we have

$$q = K_1 x$$

where K_1 is a positive constant.

$$A\rho \frac{dy}{dt} = K_1 x$$

The Laplace transform of this last equation, assuming a zero initial condition, gives

$$A\rho sY(s) = K_1 X(s)$$

$$\frac{Y(s)}{X(s)} = \frac{K_1}{A\rho s} = \frac{K}{s}$$

where $K = K_1/(A\rho)$.

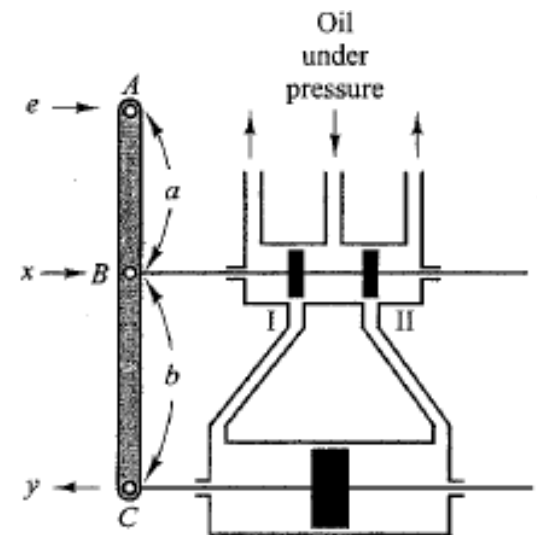
Hydraulic Proportional Controller.

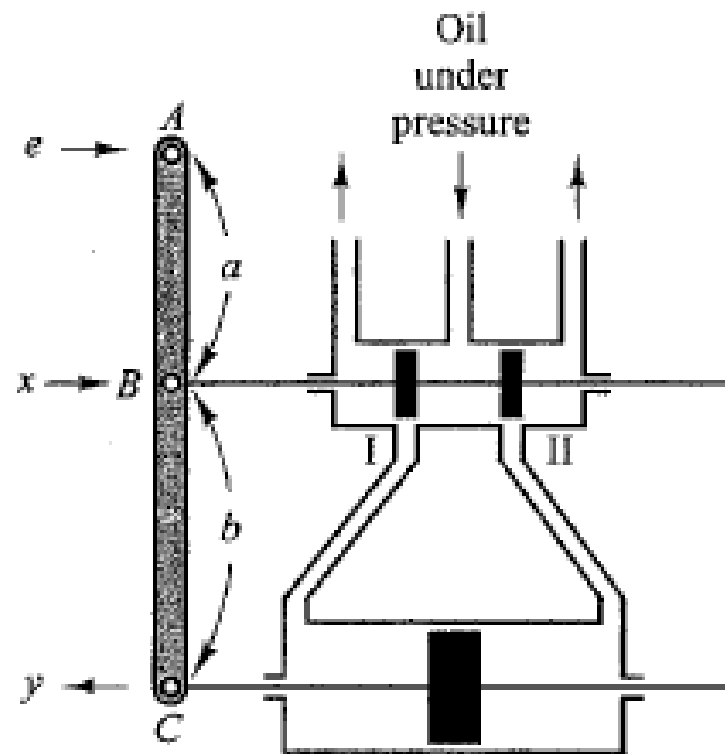
servomotor can be modified to a pro-

portional controller by means of a feedback link.

The left-hand side of the pilot valve is joined to the left-hand

side of the power piston by a link ABC . This link is a floating link rather than one moving about a fixed pivot.



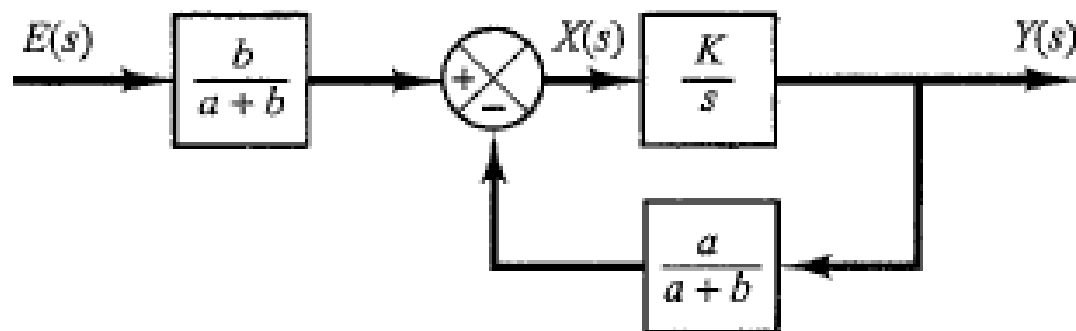


The controller here operates in the following way. If input e moves the pilot valve to the right, port II will be uncovered and high-pressure oil will flow through port II into the right-hand side of the power piston and force this piston to the left. The power piston, in moving to the left, will carry the feedback link ABC with it, thereby moving the pilot valve to the left. This action continues until the pilot piston again covers ports I and II.

$$\frac{Y(s)}{E(s)} = \frac{\frac{b}{a+b} \frac{K}{s}}{1 + \frac{K}{s} \frac{a}{a+b}}$$

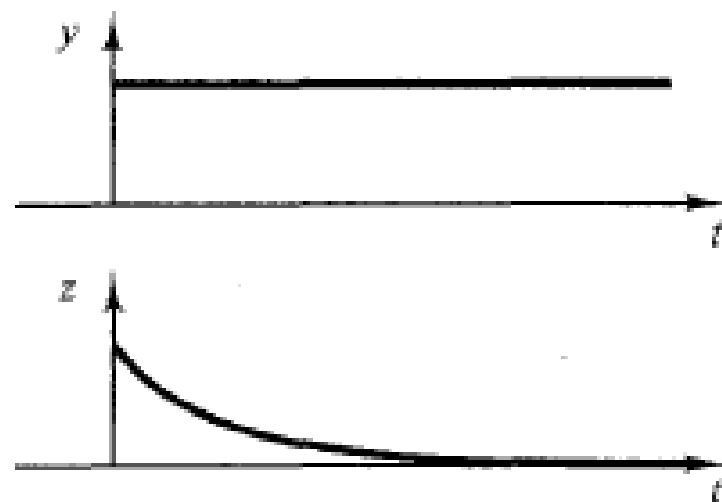
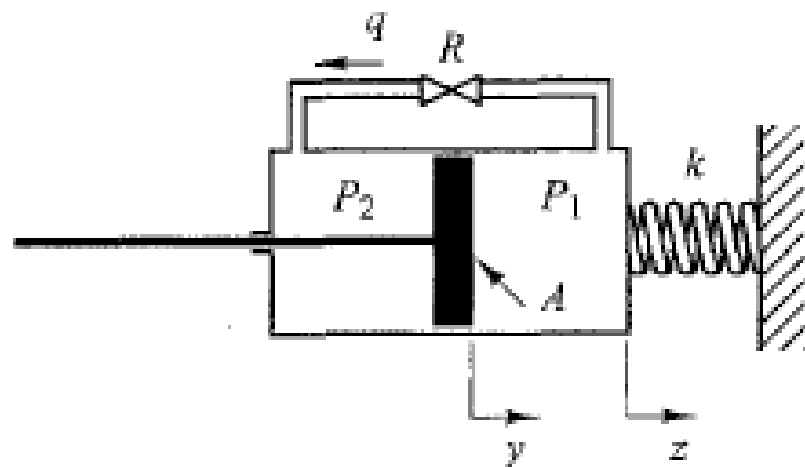
Noting that under the normal operating conditions we have $|Ka/[s(a + b)]| \gg 1$, this last equation can be simplified to

$$\frac{Y(s)}{E(s)} = \frac{b}{a} = K_p$$



Dashpots. damper)

differentiating element. Suppose that we introduce a step displacement to the piston position y . Then the displacement z becomes equal to y momentarily. Because of the spring force, however, the oil will flow through the resistance R and the cylinder will come back to the original position. The curves y versus t and z versus t are shown in



Let us derive the transfer function between the displacement z and displacement y . Define the pressures existing on the right and left sides of the piston as $P_1(\text{lb}_f/\text{in.}^2)$ and $P_2(\text{lb}_f/\text{in.}^2)$, respectively. Suppose that the inertia force involved is negligible. Then the force acting on the piston must balance the spring force. Thus

$$A(P_1 - P_2) = kz$$

where A = piston area, in.^2

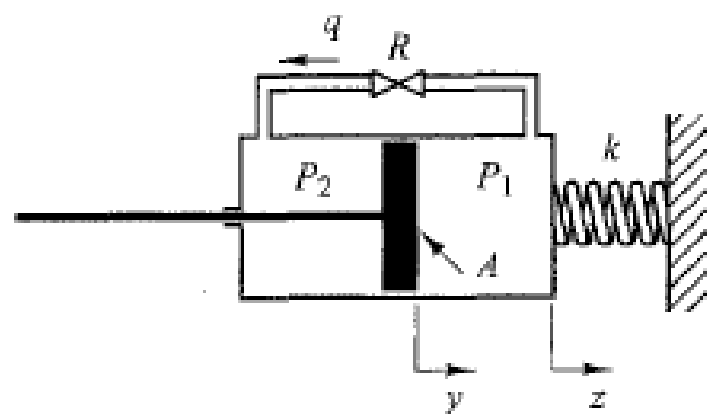
k = spring constant, $\text{lb}_f/\text{in.}$

The flow rate q is given by

$$q = \frac{P_1 - P_2}{R}$$

where q = flow rate through the restriction, lb/sec

R = resistance to flow at the restriction, $\text{lb}_f\text{-sec}/\text{in.}^2\text{-lb}$

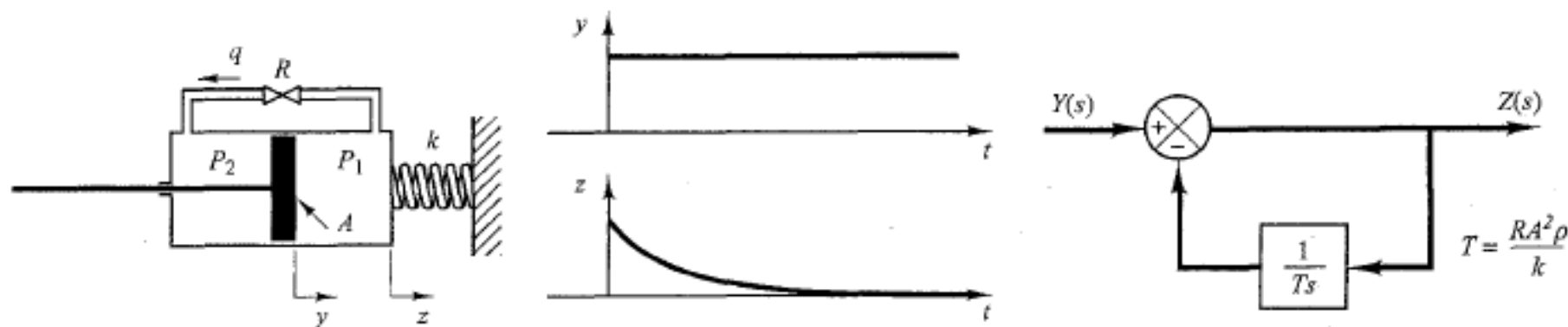


Since the flow through the restriction during dt seconds must equal the change in the mass of oil to the left of the piston during the same dt seconds, we obtain

$$q \, dt = A\rho(dy - dz)$$

where ρ = density, lb/in.³. (We assume that the fluid is incompressible or ρ = constant.) This last equation can be rewritten as

$$\frac{dy}{dt} - \frac{dz}{dt} = \frac{q}{A\rho} = \frac{P_1 - P_2}{RA\rho} = \frac{kz}{RA^2\rho}$$



or

$$\frac{dy}{dt} = \frac{dz}{dt} + \frac{kz}{RA^2\rho}$$

Taking the Laplace transforms of both sides of this last equation, assuming zero initial conditions, we obtain

$$sY(s) = sZ(s) + \frac{k}{RA^2\rho} Z(s)$$

The transfer function of this system thus becomes

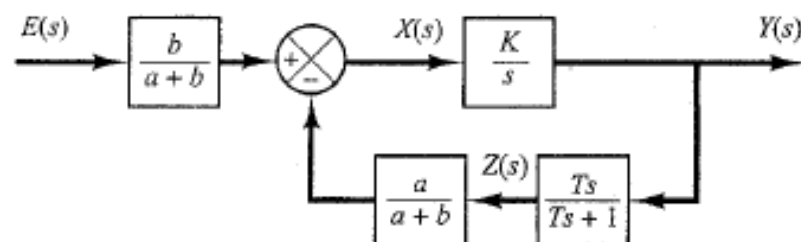
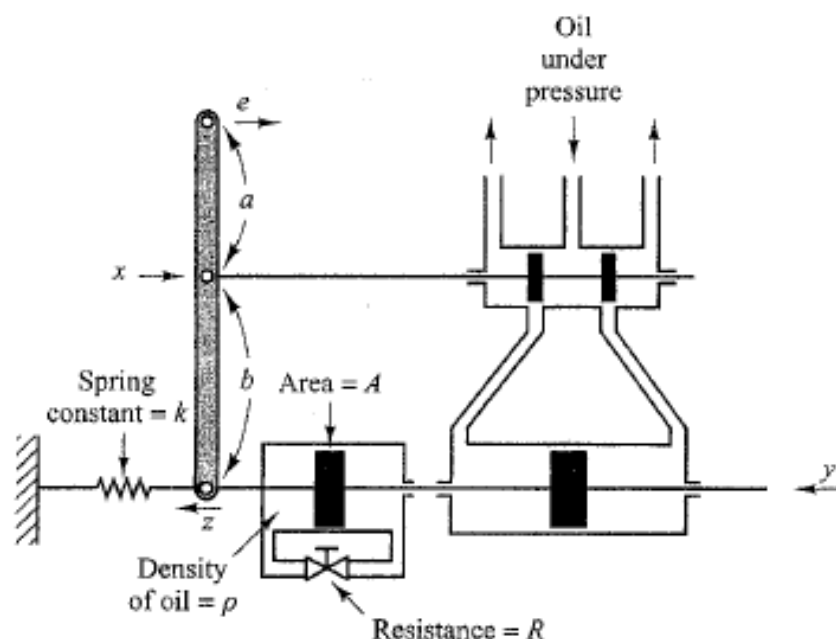
$$\frac{Z(s)}{Y(s)} = \frac{s}{s + \frac{k}{RA^2\rho}}$$

Let us define $RA^2\rho/k = T$. (Note that $RA^2\rho/k$ has the dimension of time.) Then

$$\frac{Z(s)}{Y(s)} = \frac{Ts}{Ts + 1} = \frac{1}{1 + \frac{1}{Ts}}$$

Obtaining Hydraulic Proportional-Plus-Integral Control Action.

$$\frac{Y(s)}{E(s)} = \frac{\frac{b}{a+b} \frac{K}{s}}{1 + \frac{Ka}{a+b} \frac{T}{Ts+1}}$$



In such a controller, under normal operation $|KaT/[(a + b)(Ts + 1)]| \gg 1$, with the result that

$$\frac{Y(s)}{E(s)} = K_p \left(1 + \frac{1}{T_i s} \right)$$

where

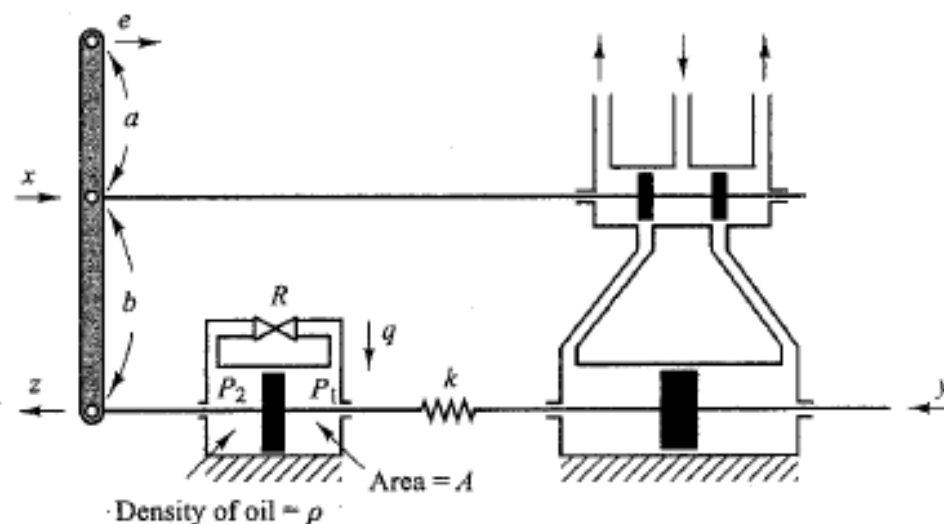
$$K_p = \frac{b}{a}, \quad T_i = T = \frac{RA^2\rho}{k}$$

Obtaining Hydraulic Proportional-Plus-Derivative Control Action.

$$k(y - z) = A(P_2 - P_1)$$

$$q = \frac{P_2 - P_1}{R}$$

$$q \, dt = \rho A \, dz$$

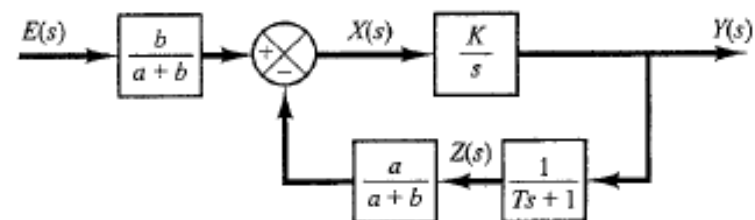
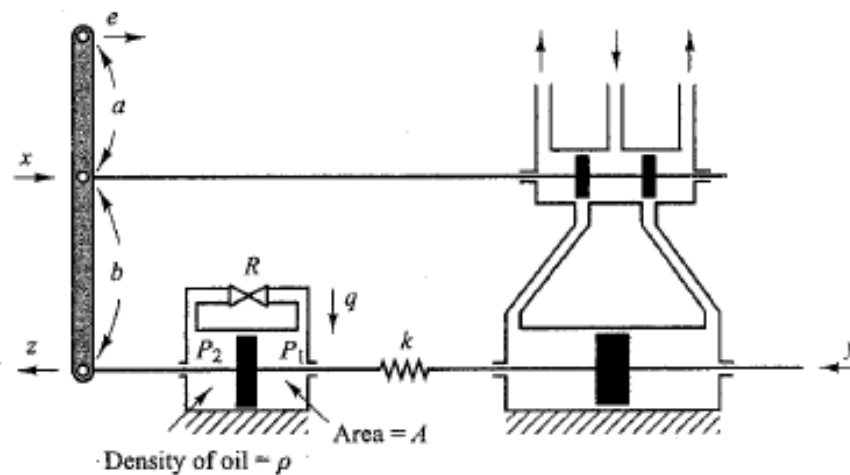


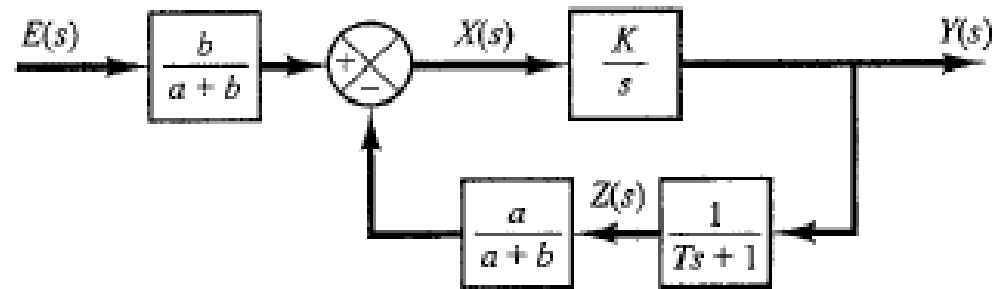
$$y = z + \frac{A}{k} q R = z + \frac{RA^2 \rho}{k} \frac{dz}{dt}$$

$$\frac{Z(s)}{Y(s)} = \frac{1}{Ts + 1}$$

where

$$T = \frac{RA^2 \rho}{k}$$





transfer function $Y(s)/E(s)$ can be obtained as

$$\frac{Y(s)}{E(s)} = \frac{\frac{b}{a+b} \frac{K}{s}}{1 + \frac{a}{a+b} \frac{K}{s} \frac{1}{Ts+1}}$$

Under normal operation we have $|aK/[(a+b)s(Ts+1)]| \gg 1$. Hence

$$\frac{Y(s)}{E(s)} = K_p(1 + Ts)$$

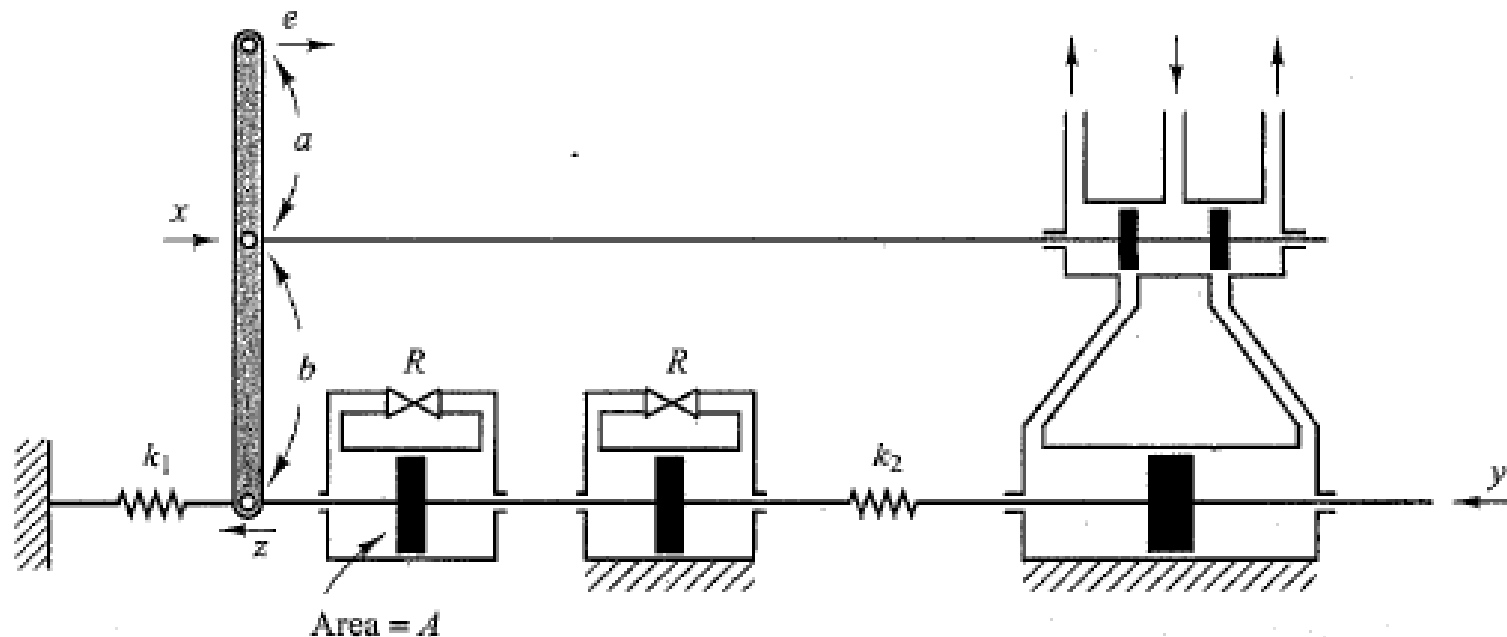
where

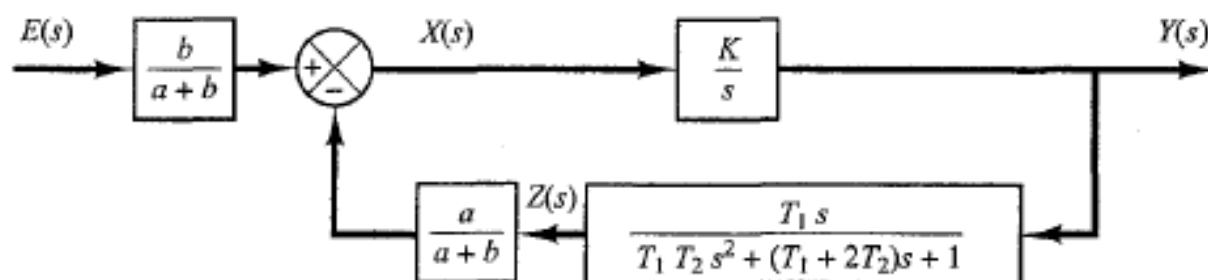
$$K_p = \frac{b}{a}, \quad T = \frac{RA^2\rho}{k}$$

If the two dashpots are identical, the transfer function $Z(s)/Y(s)$ can be obtained as follows:

$$\frac{Z(s)}{Y(s)} = \frac{T_1 s}{T_1 T_2 s^2 + (T_1 + 2T_2)s + 1}$$

(For the derivation of this transfer function, refer to Problem A-4-12.)





A block diagram for this system is shown in Figure 4.10. The transfer function $Y(s)/E(s)$ can be obtained as

$$\frac{Y(s)}{E(s)} = \frac{b}{a+b} \frac{\frac{K}{s}}{1 + \frac{a}{a+b} \frac{K}{s} \frac{T_1 s}{T_1 T_2 s^2 + (T_1 + 2T_2)s + 1}}$$

Under normal operation of the system we have

$$\left| \frac{a}{a+b} \frac{K}{s} \frac{T_1 s}{T_1 T_2 s^2 + (T_1 + 2T_2)s + 1} \right| \gg 1$$

Hence

$$\begin{aligned} \frac{Y(s)}{E(s)} &= \frac{b}{a} \frac{T_1 T_2 s^2 + (T_1 + 2T_2)s + 1}{T_1 s} \\ &= K_p + \frac{K_i}{s} + K_d s \end{aligned}$$

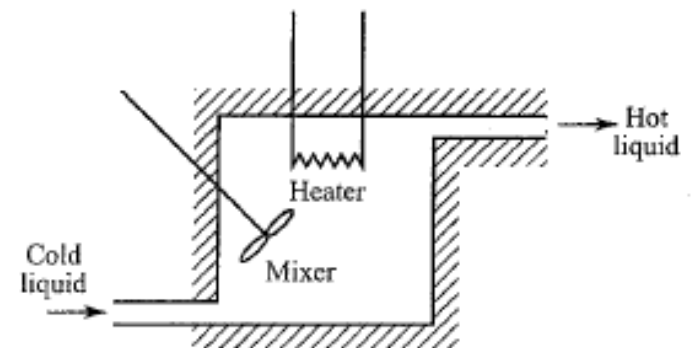
where

$$K_p = \frac{b}{a} \frac{T_1 + 2T_2}{T_1}, \quad K_i = \frac{b}{a} \frac{1}{T_1}, \quad K_d = \frac{b}{a} T_2$$

THERMAL SYSTEMS

Thermal systems are those that involve the transfer of heat from one substance to another. Thermal systems may be analyzed in terms of resistance and capacitance, although the thermal capacitance and thermal resistance may not be represented accurately as lumped parameters since they are usually distributed throughout the substance. For precise analysis, distributed-parameter models must be used. Here, however, to simplify the analysis we shall assume that a thermal system can be represented by a lumped-parameter model, that substances that are characterized by resistance to heat flow have negligible heat capacitance, and that substances that are characterized by heat capacitance have negligible resistance to heat flow.

There are three different ways heat can flow from one substance to another: conduction, convection, and radiation. Here we consider only conduction and convection. (Radiation heat transfer is appreciable only if the temperature of the emitter is very high compared to that of the receiver. Most thermal processes in process control systems do not involve radiation heat transfer.)



For conduction or convection heat transfer,

$$q = K \Delta\theta$$

where q = heat flow rate, kcal/sec

$\Delta\theta$ = temperature difference, °C

K = coefficient, kcal/sec °C

The coefficient K is given by

$$K = \frac{kA}{\Delta X}, \quad \text{for conduction}$$

$$= HA, \quad \text{for convection}$$

where k = thermal conductivity, kcal/m sec °C

A = area normal to heat flow, m²

ΔX = thickness of conductor, m

H = convection coefficient, kcal/m² sec °C

Thermal Resistance and Thermal Capacitance. The thermal resistance R for heat transfer between two substances may be defined as follows:

$$R = \frac{\text{change in temperature difference, } ^\circ\text{C}}{\text{change in heat flow rate, kcal/sec}}$$

The thermal resistance for conduction or convection heat transfer is given by

$$R = \frac{d(\Delta\theta)}{dq} = \frac{1}{K}$$

Since the thermal conductivity and convection coefficients are almost constant, the thermal resistance for either conduction or convection is constant.

The thermal capacitance C is defined by

$$C = \frac{\text{change in heat stored, kcal}}{\text{change in temperature, } ^\circ\text{C}}$$

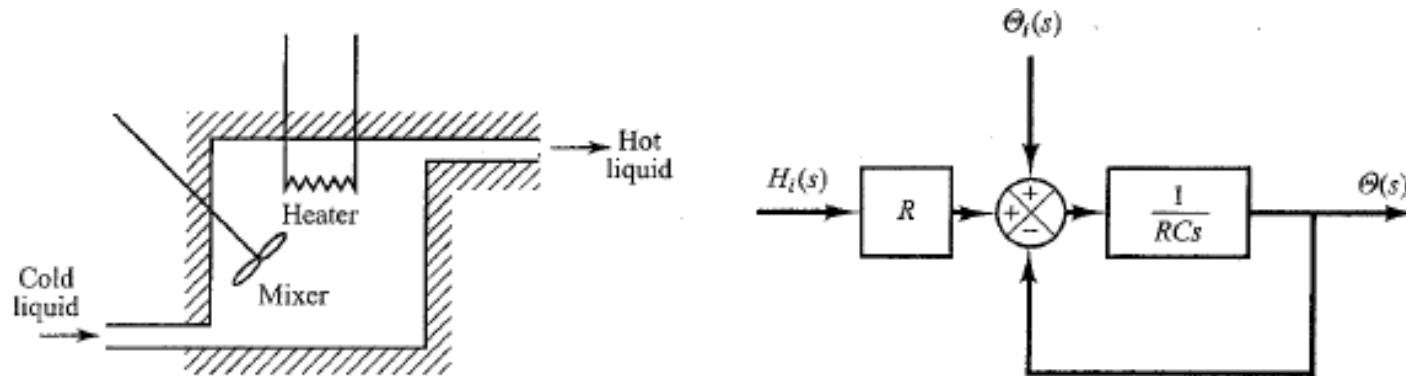
or

$$C = mc$$

where m = mass of substance considered, kg

c = specific heat of substance, kcal/kg $^\circ\text{C}$

Thermal System. Consider the system shown in Figure . It is assumed that the tank is insulated to eliminate heat loss to the surrounding air. It is also assumed that there is no heat storage in the insulation and that the liquid in the tank is perfectly mixed so that it is at a uniform temperature. Thus, a single temperature is used to describe the temperature of the liquid in the tank and of the outflowing liquid.



Let us define

$\bar{\theta}_i$ = steady-state temperature of inflowing liquid, °C

$\bar{\theta}_o$ = steady-state temperature of outflowing liquid, °C

G = steady-state liquid flow rate, kg/sec

M = mass of liquid in tank, kg

c = specific heat of liquid, kcal/kg °C

R = thermal resistance, °C sec/kcal

C = thermal capacitance, kcal/°C

\bar{H} = steady-state heat input rate, kcal/sec

Assume that the temperature of the inflowing liquid is kept constant and that the heat input rate to the system (heat supplied by the heater) is suddenly changed from \bar{H} to $\bar{H} + h_i$, where h_i represents a small change in the heat input rate. The heat outflow rate will then change gradually from \bar{H} to $\bar{H} + h_o$. The temperature of the outflowing liquid will also be changed from $\bar{\theta}_o$ to $\bar{\theta}_o + \theta$. For this case, h_o , C , and R are obtained, respectively, as

$$h_o = Gc\theta$$

$$C = Mc$$

$$R = \frac{\theta}{h_o} = \frac{1}{Gc}$$

The heat balance equation for this system is

$$Cd\theta = (h_i - h_o)dt$$

or

$$C \frac{d\theta}{dt} = h_i - h_o$$

which may be rewritten as

$$RC \frac{d\theta}{dt} + \theta = Rh_i$$

Note that the time constant of the system is equal to RC or M/G seconds. The transfer function relating θ and h_i is given by

$$\frac{\Theta(s)}{H_i(s)} = \frac{R}{RCs + 1}$$

where $\Theta(s) = \mathcal{L}[\theta(t)]$ and $H_i(s) = \mathcal{L}[h_i(t)]$.

In practice, the temperature of the inflowing liquid may fluctuate and may act as a load disturbance. (If a constant outflow temperature is desired, an automatic controller may be installed to adjust the heat inflow rate to compensate for the fluctuations in the temperature of the inflowing liquid.) If the temperature of the inflowing liquid is suddenly changed from $\bar{\theta}_i$ to $\bar{\theta}_i + \theta_i$ while the heat input rate H and the liquid flow rate G are kept constant, then the heat outflow rate will be changed from \bar{H} to $\bar{H} + h_o$, and the temperature of the outflowing liquid will be changed from $\bar{\theta}_o$ to $\bar{\theta}_o + \theta$. The heat balance equation for this case is

$$C d\theta = (Gc\theta_i - h_o) dt$$

or

$$C \frac{d\theta}{dt} = Gc\theta_i - h_o$$

which may be rewritten

$$RC \frac{d\theta}{dt} + \theta = \theta_i$$

The transfer function relating θ and θ_i is given by

$$\frac{\Theta(s)}{\Theta_i(s)} = \frac{1}{RCs + 1}$$

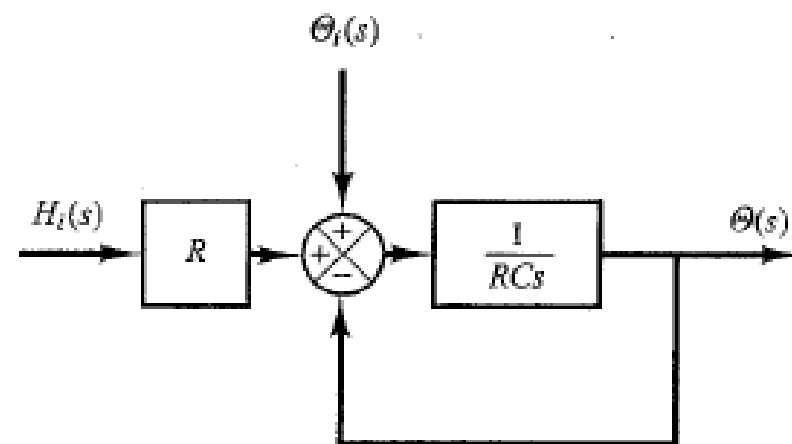
where $\Theta(s) = \mathcal{L}[\theta(t)]$ and $\Theta_i(s) = \mathcal{L}[\theta_i(t)]$.

If the present thermal system is subjected to changes in both the temperature of the inflowing liquid and the heat input rate, while the liquid flow rate is kept constant, the change θ in the temperature of the outflowing liquid can be given by the following equation:

$$RC \frac{d\theta}{dt} + \theta = \theta_i + Rh_i$$

A block diagram corresponding to this case is shown in Figure

Notice that the



References

- Ogata, K., Modern control engineering, 4th Ed.
, 2002.

Thank you