Introduction to Quantum Optics
From the Semi-classical Approach to Quantized Light

Covering a number of important subjects in quantum optics, this textbook is an excellent introduction for advanced undergraduate and beginning graduate students, familiarizing readers with the basic concepts and formalism as well as the most recent advances.

The first part of the textbook covers the semi-classical approach where matter is quantized, but light is not. It describes significant phenomena in quantum optics, including the principles of lasers. The second part is devoted to the full quantum description of light and its interaction with matter, covering topics such as spontaneous emission, and classical and non-classical states of light. An overview of photon entanglement and applications to quantum information is also given. In the third part, nonlinear optics and laser cooling of atoms are presented, where the use of both approaches allows for a comprehensive description. Each chapter describes basic concepts in detail, and more specific concepts and phenomena are presented in 'complements'.

Gilbert Grynberg was a CNRS Senior Scientist at the Laboratoire Kastler Brossel at the Université Pierre et Marie Curie Paris 6, and a Professor at the Ecole Polytechnique. He was a pioneer in many domains, including atomic spectroscopy, nonlinear optics and laser-cooled atoms in optical lattices.

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CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore,
São Paulo, Delhi, Dubai, Tokyo, Mexico City

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521551120

Original edition: *Introduction aux Lasers et á l'Optique*, Ellipses 1997

Published with the assistance of the French Ministry of Culture–National Book Centre

Ouvrage publié avec le concours du Ministère français chargé de la culture – Centre National du Livre

First published 2010

Printed in the United Kingdom at the University Press, Cambridge

A catalogue record for this publication is available from the British Library

ISBN 978-0-521-55112-0 Hardback

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Cambridge University Press
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Foreword

Atomic, molecular and optical physics is a field which, during the last few decades, has known spectacular developments in various directions, like nonlinear optics, laser cooling and trapping, quantum degenerate gases, quantum information. Atom–photon interactions play an essential role in these developments. This book presents an introduction to *quantum optics* which, I am sure, will provide an invaluable help to the students, researchers and engineers who are beginning to work in these fields and who want to become familiar with the basic concepts underlying electromagnetic interactions.

Most books dealing with these subjects follow either a semi-classical approach, where the field is treated as a classical field interacting with quantum particles, or a full quantum approach where both systems are quantized. The first approach is often oversimplified and fails to describe correctly new situations that can now be investigated with the development of sophisticated experimental techniques. The second approach is often too difficult for beginners and lacks simple physical pictures, very useful for an initial understanding of a physical phenomenon. The advantage of this book is that it gives both approaches, starting with the first, illustrated by several simple examples, and introducing progressively the second, clearly showing why it is essential for the understanding of certain phenomena. The authors also clearly demonstrate, in the case of non-linear optics and laser cooling, how advantageous it may be to combine both approaches in the analysis of an experimental situation and how one can get from each point of view useful, complementary physical insights. I believe that this challenge to present and to illustrate both approaches in a single book has been taken up successfully. Whatever their ultimate interests, the readers of this work will be exposed to an important example of a broad and vibrant field of research and they will better understand the intellectual enrichment and the technical developments which result from it.

To write a book on such a broad topic, the authors must obviously possess wide knowledge of the field, they must have thought long and hard about the basic concepts and about the different levels of complexity with which one can approach the topics. They must also have a deep and concrete knowledge about experimental and technical details and the many problems which daily confront a laboratory researcher. Having worked extensively with them, I know the authors of this work fulfil these requirements. I have the highest admiration for their enthusiasm, their scientific rigour, their ability to give simple and precise physical explanations, and their quest to illuminate clearly the difficult points of the subject without oversimplification. Each of them has made many original contributions to the development of this important field of physics, and they and their younger collaborators for this book work at the cutting edge of modern quantum optics. In reading the book, I am therefore not surprised to find their many fine qualities reflected in the text. The general organisation of the main chapters and complementary sections allows reading on many different levels. When the authors discuss a new physical problem, they begin the analysis with the simplest possible model. A large variety of experiments and applications are presented with clear diagrams and explanations and with constant attention to highlighting the guiding principles, the orders of magnitude and the problems which remain open.

This work will allow a broad audience an easier access to a field of science which continues to see spectacular developments. I believe that science is not simply a matter of exploring new horizons. One must also make the new knowledge readily available and we have in this book, a beautiful example of such a pedagogical effort. I would like finally to evoke the memory of Gilbert Grynberg who participated with Alain Aspect and Claude Fabre in the writing of a preliminary, much less developed, French version of this book and who passed away in 2003. Gilbert was an outstanding physicist, a fine person, and had an exceptional talent for explaining in the clearest possible way the most difficult questions. I think that the present book is the best possible tribute to be paid to him.

Claude Cohen-Tannoudji
Paris, September 2009

Preface

Since its invention in 1960, the laser has revolutionized both the study of optics and our understanding of the nature of light, prompting the emergence of a new field, *quantum optics*. Actually, it took decades until the words quantum optics took their current precise meaning, referring to phenomena which can be understood only by quantizing the electromagnetic field describing light. Surprisingly enough, such quantum optics phenomena hardly existed at the time that the laser was invented, and almost all optics effects could be fully understood by describing light as a classical electromagnetic field; the laser was no exception. As a matter of fact, to understand how a laser works, it suffices to use the *semi-classical description of matter–light interaction*, where the laser amplifying medium, made of atoms, molecules, ions or semi-conductors, is given a quantum mechanical treatment, but light itself is described by classical electromagnetic waves.

The first part of our book is devoted to presentation of the semi-classical approach and its use in describing various optical phenomena. It includes an elementary exposition of the physics of lasers, and some applications of this ubiquitous device. After recalling in **Chapter 1** some basic results of the quantum mechanical description of interaction induced transitions between the atomic energy levels, we use these results in **Chapter 2** to show how the interaction of a quantized atom with a classical electromagnetic wave leads to absorption or stimulated emission, and to derive the process of laser amplification that happens when a wave propagates in an inverted medium. **Chapter 3** gives an elementary exposition of the physics of laser sources and of the properties of laser light.

Although the quantum theory of light existed since its development by Dirac in the early 1930s, quantum optics theory in its modern sense started when Roy Glauber showed, in the early 1960s, how to apply it to classical optics devices such as the Michelson stellar interferometer or the Hanbury Brown and Twiss intensity interferometer. At that time it could have appeared to be an academic exercise without consequence, since the only known phenomenon that demanded quantization of light was spontaneous emission, and it was not clear whether quantum theory was at all useful for describing light freely propagating far from the source. Actually, Glauber developed a clear quantum formalism to describe optics phenomena, and introduced the important notion of quasi-classical states of light, a theoretical tool that allowed physicists to understand why all available sources of light, including lasers, delivered light whose properties could be totally understood in the framework of the semi-classical approach. But in doing so, he paved the way for the discovery of new phenomena which can be understood only if light is considered as a quantum system. It became possible to build sources delivering single photon wave packets, pairs of entangled photons, squeezed beams of light...

The second part of our book is devoted to the presentation of the quantum theory of light and its interaction with matter, and its use in describing many phenomena of modern quantum optics. We show in **Chapter 4** how it is possible to write the dynamical equations of a classical electromagnetic field, i.e. Maxwell equations, in a form allowing us to use the canonical quantization procedure to quantize the electromagnetic field, and obtain the notion of a photon. We then use our results, in **Chapter 5**, to describe some fully quantum effects observed in experiments with *single photons*, *squeezed light or pairs of entangled photons*. It is remarkable that many of these experiments, whose first goal was to demonstrate the highly counter-intuitive, non-classical properties of new types of light states, turned out to stimulate the emergence of a new field, *quantum information*, where one uses such properties to implement new ways of processing and transmitting data. In **Chapter 6**, we show how to use the quantum optics formalism to describe the interaction between light and atoms. We will then revisit in this new framework the phenomena of absorption and stimulated emission, already studied in Chapter 2. Moreover, we will now be able to give a *consistent treatment of spontaneous emission*.

Having introduced the full quantum optics formalism and reviewed some remarkable phenomena that could not have been discovered without such a formalism, we would not like to leave the reader with the impression that he/she can now forget the semi-classical approach. Both approaches are definitely useful. On the one hand, there is no reason to use the, usually more involved, fully quantum analysis, when the situation does not demand it. After all, nobody would use quantum mechanics to describe the motion of planets. Similarly, no experimentalist studying fusion plasmas with intense lasers would start using the quantum formalism of light. What is important then is to be able to recognize when the full quantum theory is necessary, and when one can content oneself with the semi-classical model. To help the reader to develop their intuition about this point, we present, in the third part of this book, two topics, non-linear optics in **Chapter 7**, laser cooling and trapping of atoms in **Chapter 8**, where it is convenient to 'juggle' between the two approaches, each being better adapted to one or the other particular phenomenon. As 'the cherry on the cake', we will give in Chapter 8 an elementary presentation of atomic Bose–Einstein condensates, and emphasize the analogy between such a system, where all atoms are described by the same matter wave, and a laser beam where all photons are described by the same mode of the electromagnetic field. When we started to write the first French version of this book, we had never dreamt of being able to finish it with a presentation on *atom lasers*.

This book is composed of **chapters**, in which we present the fundamental concepts and some applications to important quantum optics phenomena, and of **complements**, which present supplementary illustrations or applications of the theory presented in the main chapter. The choice of these examples is, of course, somewhat arbitrary. We present them as a snap-shot of the current state of a field which is rapidly evolving. Complements of another type are intended to give some supplementary details about a derivation or about concepts presented in the chapter.

The prerequisite for using this book is to have followed an elementary course on both electromagnetism (Maxwell's equations) and quantum mechanics (Schrödinger formulation in the Dirac formalism of bras and kets, with application to the harmonic oscillator). The book is then self-consistent, and can be used for an advanced undergraduate, or for a first graduate course on quantum optics. Although we do not make use of the most advanced tools studied at graduate school, we make all efforts to provide the reader with solid derivations of the main results obtained in the chapters. For example, to quantize electromagnetic waves, first in free space, and then in interaction with charges, we do not use the Lagrangian formalism, but we introduce enough elements of the Hamiltonian formalism to be able to apply the canonical quantization rules. We are thus able to provide the reader with a solid derivation of the basic quantum optics formalism rather than bringing it in abruptly. On the other hand, when we want to present in a Complement a particularly important and interesting phenomenon, we do not hesitate to ask the reader to admit a result which results from more advanced courses.

We have done our best to merge the French teaching tradition of logical and deductive exposition with the more pragmatic approach that we use as researchers, and as advisors to Ph.D. and Masters students. We have taught the content of this book for many years to advanced undergraduate or beginning graduate students, and this text represents the results of our various teaching experiences.

Acknowledgements

In this book, we refer to a number of textbooks in which general elementary results of quantum mechanics are established, in particular the book by Jean-Louis Basdevant and Jean Dalibard,[1] which we indicate by the short-hand notation 'BD', and the one by Claude Cohen-Tannoudji, Bernard Diu and Franck Laloë,[2] which we denote by 'CDL'. On the more advanced side, we sometimes refer to more rigorous demonstrations, or to more advanced developments, that can be found in the two books written by Claude Cohen-Tannoudji, Jacques Dupont-Roc and Gilbert Grynberg, to which we refer under the short-hand notations 'CDG I' and 'CDG II', respectively.[3,4]

It is not possible to mention all those who have contributed to or influenced this work. We would first like to acknowledge, however, our principal inspiration, Claude Cohen-Tannoudji, whose lectures at the Collège de France we have had the good fortune to be able to follow for three decades. At the other end of the spectrum, we also owe a lot to our students at Ecole Polytechnique, Ecole Normale Supérieure, Institut d'Optique Graduate School, Université Pierre et Marie Curie, as well as the many graduate students we have advised towards Masters or Ph.D. work. By their sharp questioning, never content with a vague answer, they have forced us to improve our lectures year upon year. We cannot cite all of the colleagues with whom we have taught, and from whom we have borrowed many ideas and materials, but we cannot omit to mention the names of Manuel Joffre, Emmanuel Rosencher, Philippe Grangier, Michel Brune, Jean-François Roch, François Hache, David Guéry-Odelin, Jean-Louis Oudar, Hubert Flocard, Jean Dalibard, Jean-Louis Basdevant. In addition, Philippe Grangier was kind enough to write Complement 5E on quantum information.

Martine Maguer, Dominique Toustou, and all the team of Véronique Pellouin at the Centre Polymedia of Ecole Polytechnique have done an impressive and professional job in preparing the manuscript with its figures. We would like also to thank the Centre National du Livre, of the French Ministry of Culture, for its important financial support in the translation of our French text.

[1] J.-L. Basdevant and J. Dalibard, *Quantum Mechanics*, Springer (2002).
[2] C. Cohen-Tannoudji, B. Diu and F. Laloë, *Quantum Mechanics*, Wiley (1977).
[3] C. Cohen-Tannoudji, J. Dupont-Roc and G. Grynberg, *Photons and Atoms – Introduction to quantum electrodynamics*, Wiley (1989).
[4] C. Cohen-Tannoudji, J. Dupont-Roc and G. Grynberg, *Atom-photon Interactions: Basic processes and applications*, Wiley (1992).

Special acknowledgement

This book has three authors, who wrote the original French textbook on which it is based.[5] Sadly, as we had just started to prepare the English version, **Gilbert Grynberg** passed away, and for several years we were discouraged and not able to carry on working on the English version. Eventually, we realized that the best demonstration of all that we owe to our former friend and colleague was to resume this project. But we realized then that almost a decade after writing the French version, quantum optics had evolved tremendously, and we had also personally evolved in the ways in which we understood and taught the subject. The original French book, therefore, had not only to be translated but also widely revised and updated. In this long-term enterprise, we have been fortunate to have fantastic help from our younger colleagues (and former students) **Fabien Bretenaker** and **Antoine Browaeys**. For the past three years they have devoted innumerable hours to helping us complete the revised version, and without their help this would not have been possible. There is not a single chapter that has not been strongly influenced by their thorough criticisms, their strong suggestions, and their contributions to the rewriting of the text, not to speak of the double checking of equations. Moreover, they bring to this book the point of view of a new generation of physicists who have been taught quantum optics in its modern sense, in contrast to we who have seen it developing while we were already engaged in research. For their priceless contribution, we can only express to Fabien Bretenaker and Antoine Browaeys our immense gratitude. Gilbert would have been happy to have such wonderful collaborators.

Alain Aspect and Claude Fabre,
Palaiseau, Paris, July 2009.

[5] Gilbert Grynberg, Alain Aspect, Claude Fabre, *Introduction aux lasers et à l'Optique Quantique*, cours de l'Ecole Polytechnique, Ellipses, Paris (1997).