Introduction to Shell Structures

Assoc. Prof. Adrian Dogariu

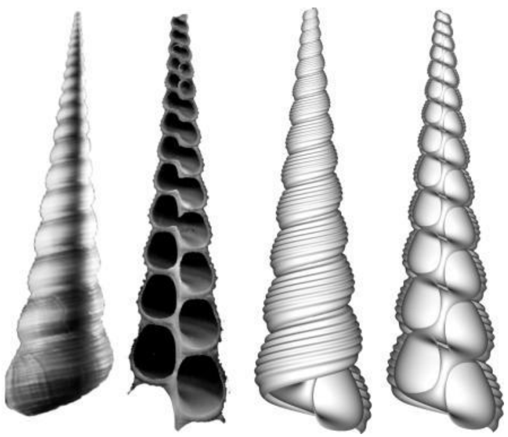
General

- Metalic shells

Introduction to Design of Shell Structures

General

- Natural shells

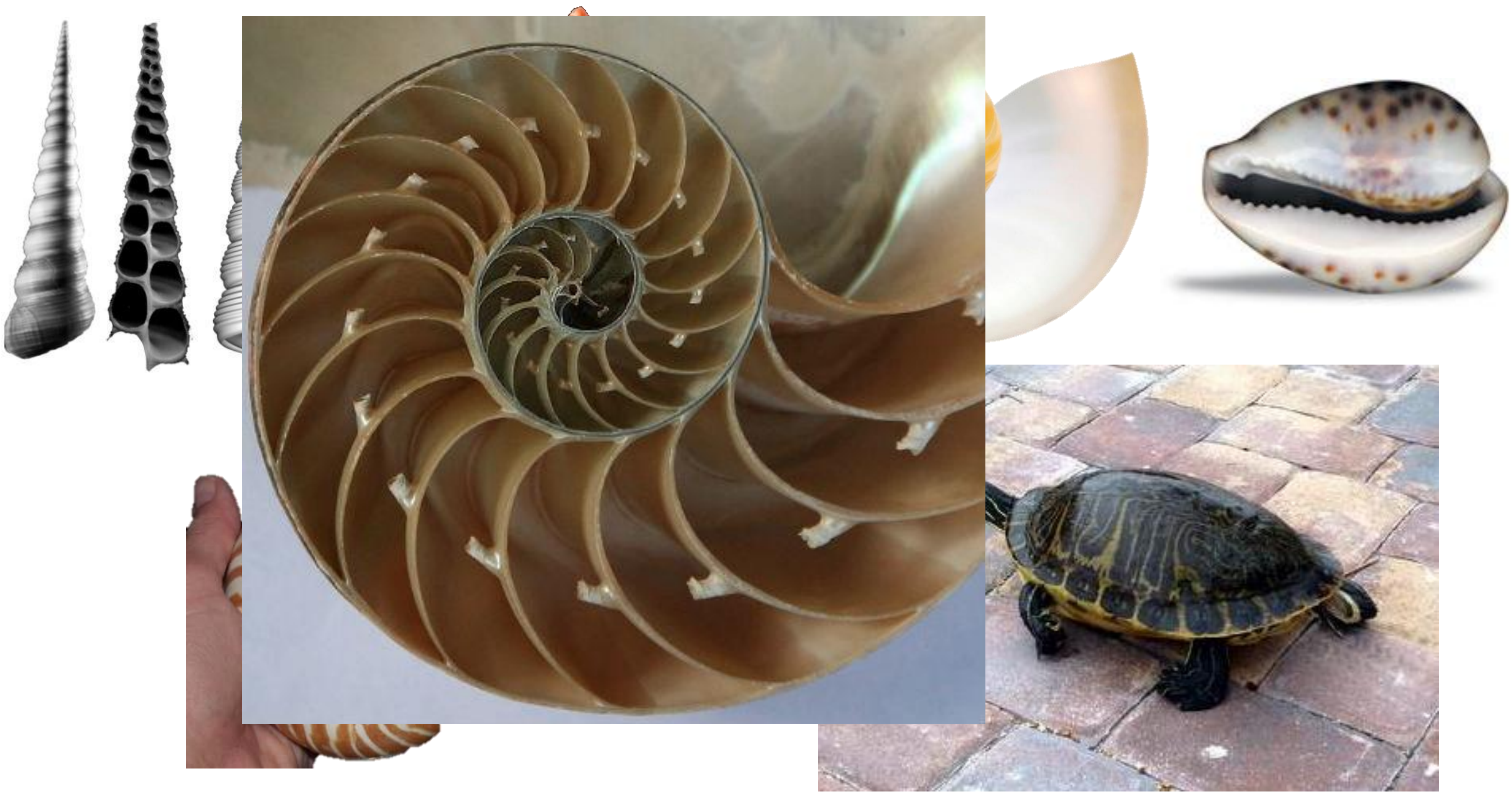





Introduction to Design of Shell Structures

General

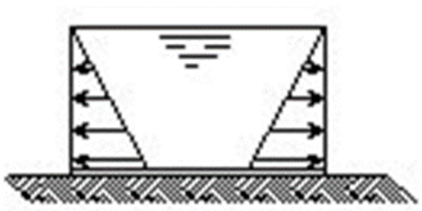
- Natural shells

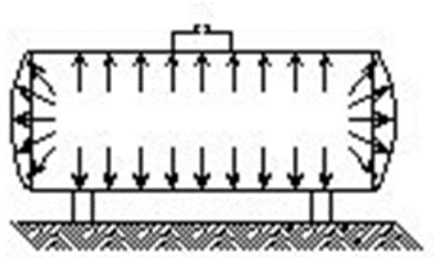


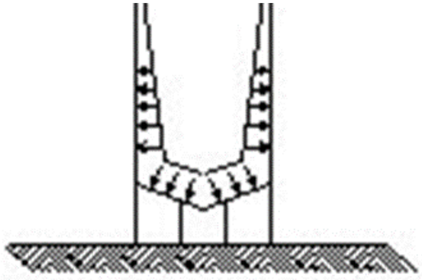
Introduction to Design of Shell Structures

General

- **Definition:**
  - **A shell is a thin structure composed of curved sheets of material, so that the curvature plays an important role in the structural behavior, realizing a spatial form**
- **Motivation:**
  - **A shell is the most efficient way of using the material, and can be very useful in case o storage of fluids and solids (uniform loads)**







Difficulties

- The curved form may lead to different failure modes and often unexpected behavior occurs
- The analytical formulas are very complex and complicated in comparison with all the other structural forms

- Shell structures are very attractive light weight structures which are especially suited to building as well as industrial applications.

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Range of application

- The shell structure is typically found
  - in nature
  - as well as in classical architecture.

- There are two principal uses of shells in civil engineering:
  - industrial structures:
    - silos, tanks, cooling towers, reactor vessels etc.
  - aesthetic and architectural special structures

Main documents

- Eurocode on strength and stability of Steel Shell Structures – EN1993 Part 1.6 (2007)
- Generic normative standard on shells for chimneys, towers, masts, silos, tanks, pipelines
- Buckling of Steel Shells European Design Recommendations 5th Edition (ECCS – 2008)

Introduction to Design of Shell Structures

General

- **Built structural shells**

Introduction to Design of Shell Structures

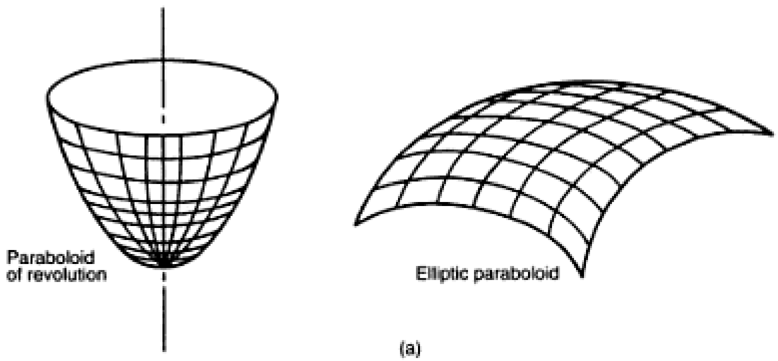
General

- Built structural shells

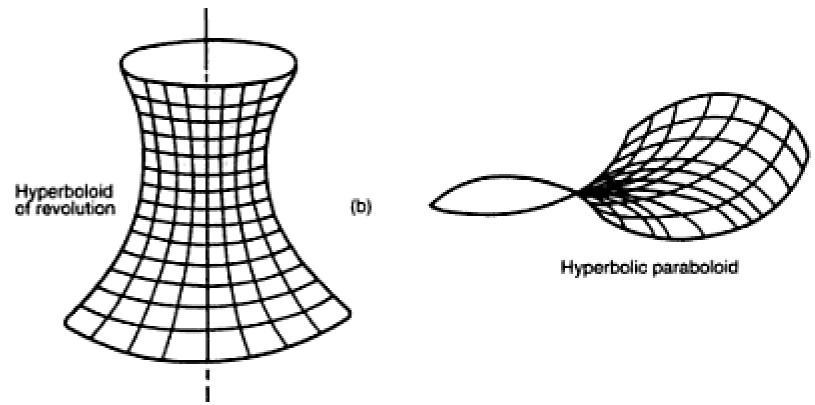
Reinforced concrete
Steel
Aluminium alloys
Plastics
Glass
Timber

Introduction to Design of Shell Structures

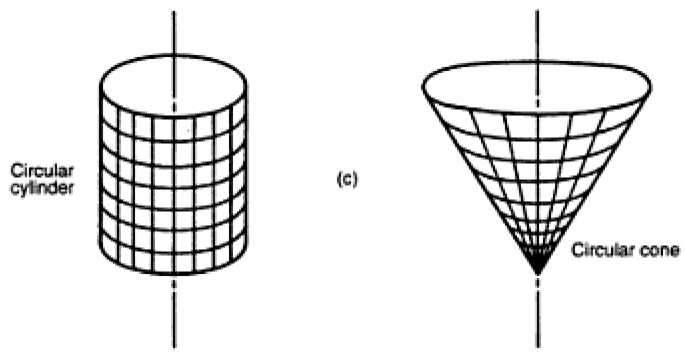
Structural typologies



Eliptic paraboloid



Hyperbolic paraboloid



Circular cylinder/cone

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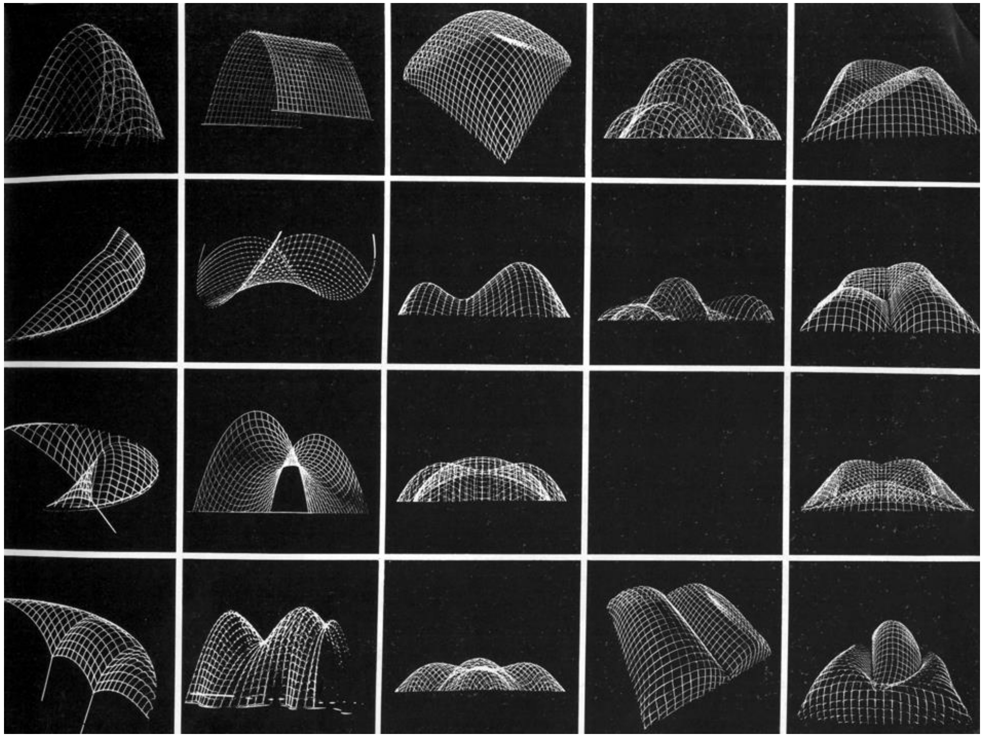
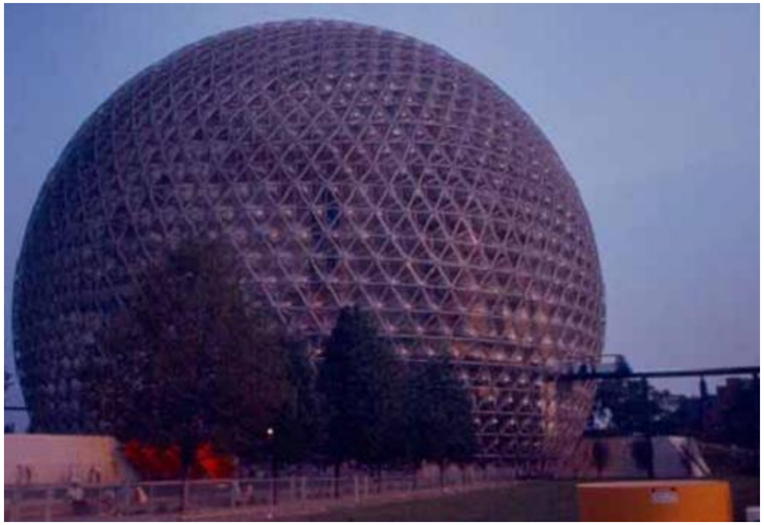
Structural typologies

Shells are the most difficult form of structure to analyse and the form with the most complex behaviour. As a result, all but the simplest conditions must be analysed using computers.



Examples – Steel reticulated dome



US pavilion Expo 67 Montreal

Architect: Buckminster Fuller & Shoji Sadao

The 250ft diameter by 200ft high dome roughly presents a three-quarter sphere, while geodesic domes before 1967 were hemispherical. The dome consists of steel pipes and 1,900 acrylic panels. To keep the indoor temperature acceptable, the design included mobile triangular panels that would move over the inner surface following the sun. Although brilliant on paper, this feature was too advanced for its time and never worked.

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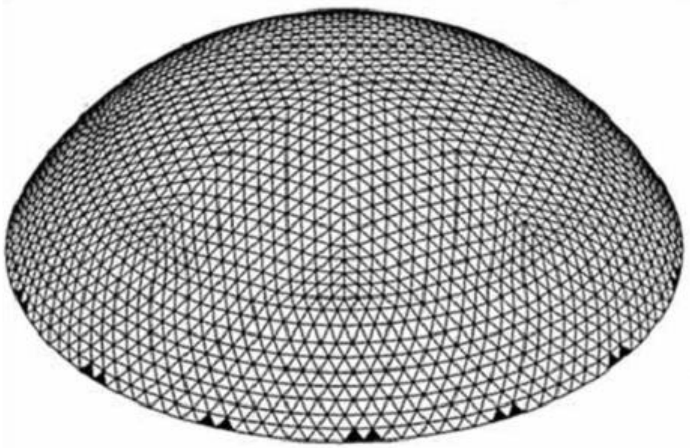
Examples – Aluminium alloy reticulated dome

Spruce Goose dome, Long Beach, USA

Architect: R. Duell and Associates

Engineer/builder: Temcor



A - Aluminum cover plate with silicone seal

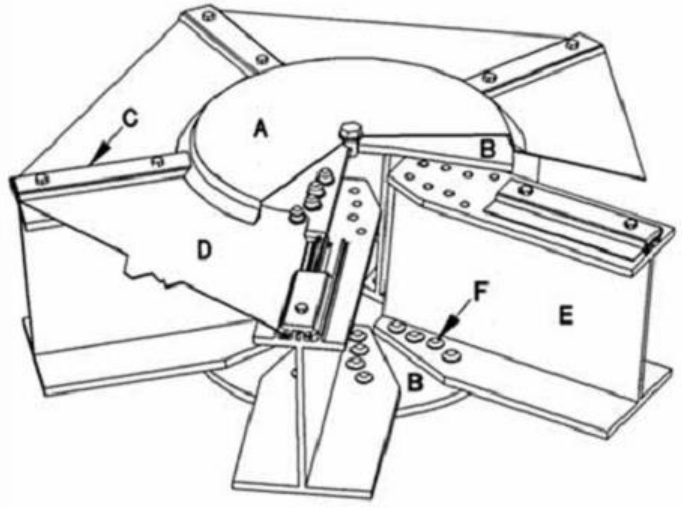
B - Aluminum gusset plates, bolted to struts

C - Aluminum batten secure silicone gaskets

D - Triangular aluminum panels

E - Wide-flange aluminum struts

F - Stainless steel bolts



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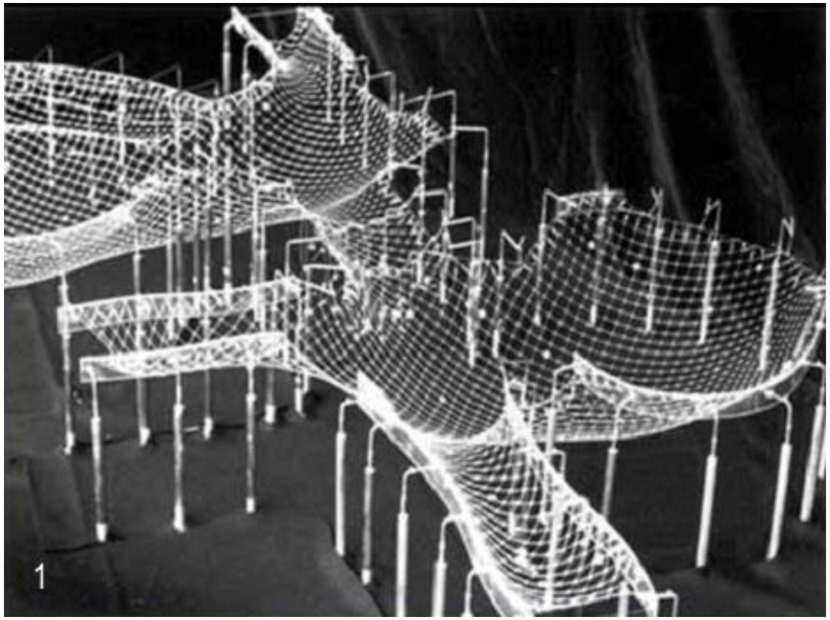
Examples – Timber-steel free form grid shell

Multi-hall, Mainz, Germany

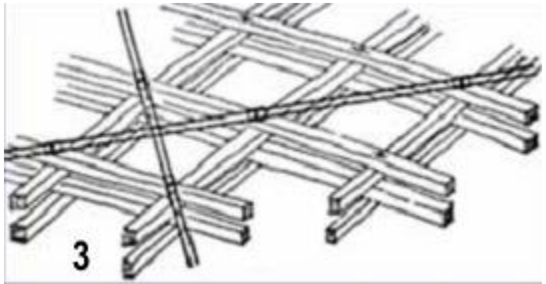
Architect: Mutschler, Frei Otto, consultant

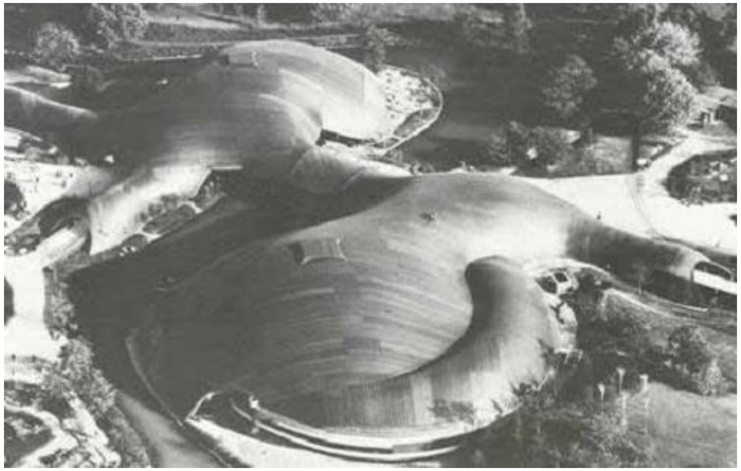
Engineer: Ove Arup



The multi-purpose dome for the 1975 garden show spans max. 60m with 50x50mm twin wood slats of 50cm squares that deformed into rhomboids.

1 - Form-finding model
2 - Interior
3 - Mesh detail (steel bands resist shear)





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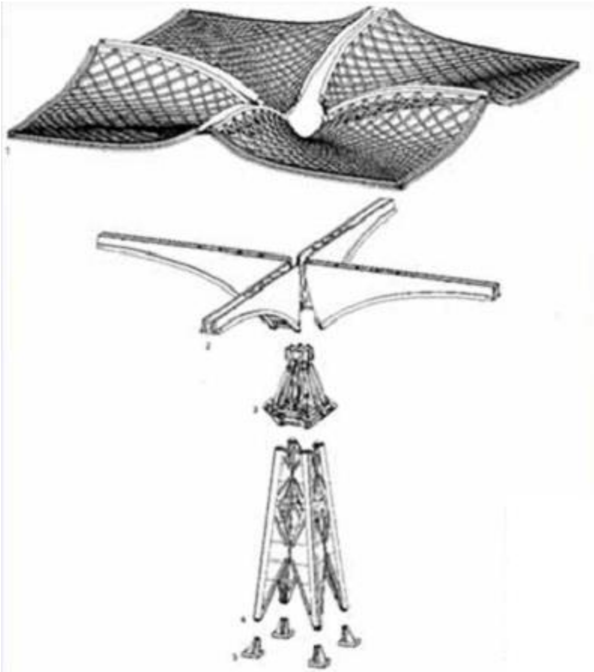
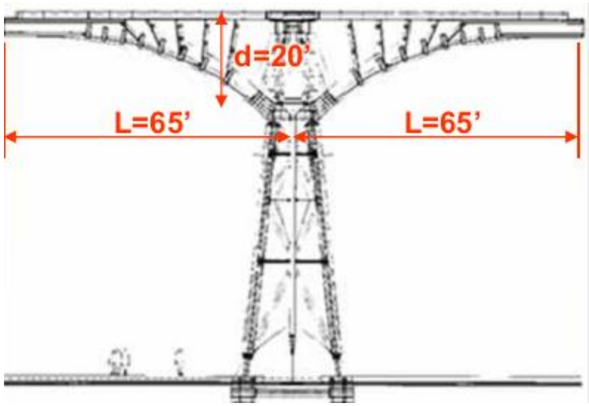
Examples – Timber-steel free form grid shell



Architect: Thomas Herzog
Engineer: Julius Natterer

Wood grid shell with PTFE membrane
The theme pavilion advanced the philosophy:

- Wood is the only renewable material
- Requires the least energy for production
- Use of wood maintains healthy forests



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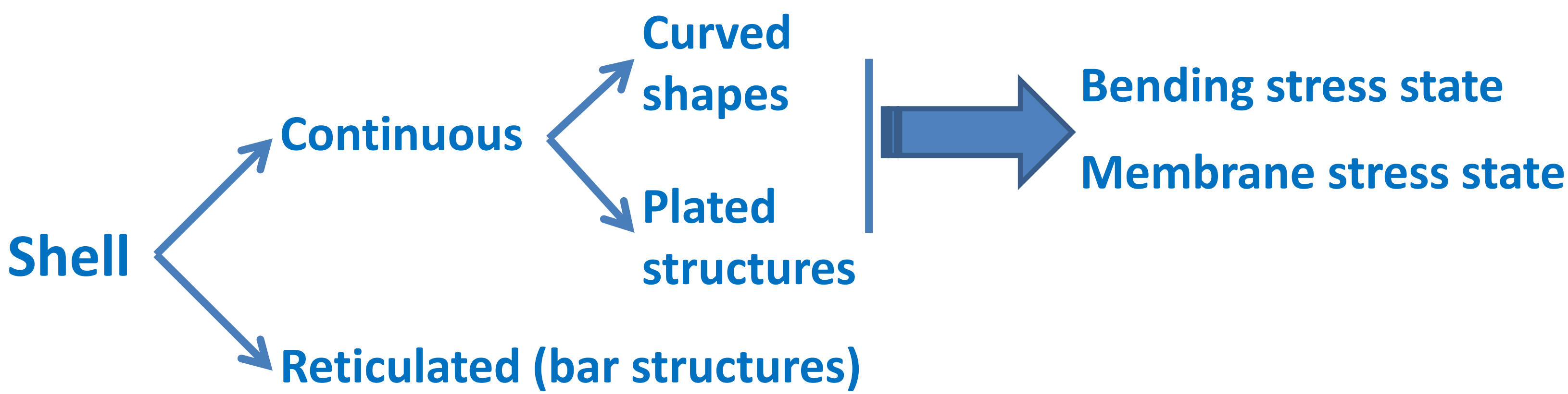
Shell Analysis



- Continuous (or reticulated) shells
  - Linear behaviour
  - Non-linear behaviour
    - Elastic
    - Elastic-plastic

Shell Design

- Resistance
- Stability
  - Highly sensitive to imperfections
    - Buckling is a process by which a structure cannot withstand loads with its original shape, so that it changes this shape in order to find a new equilibrium configuration. This is an undesired process (from the point of view of the engineer), and occurs for a well–defined value of the load.
    - The consequences of buckling are basically geometric:
      - The are large displacements in the structure
      - There may also be consequences for the material, in the sense that deflections may induce plasticity in the walls of the structure

Local buckling of a tank

Global buckling of a wind turbine tower

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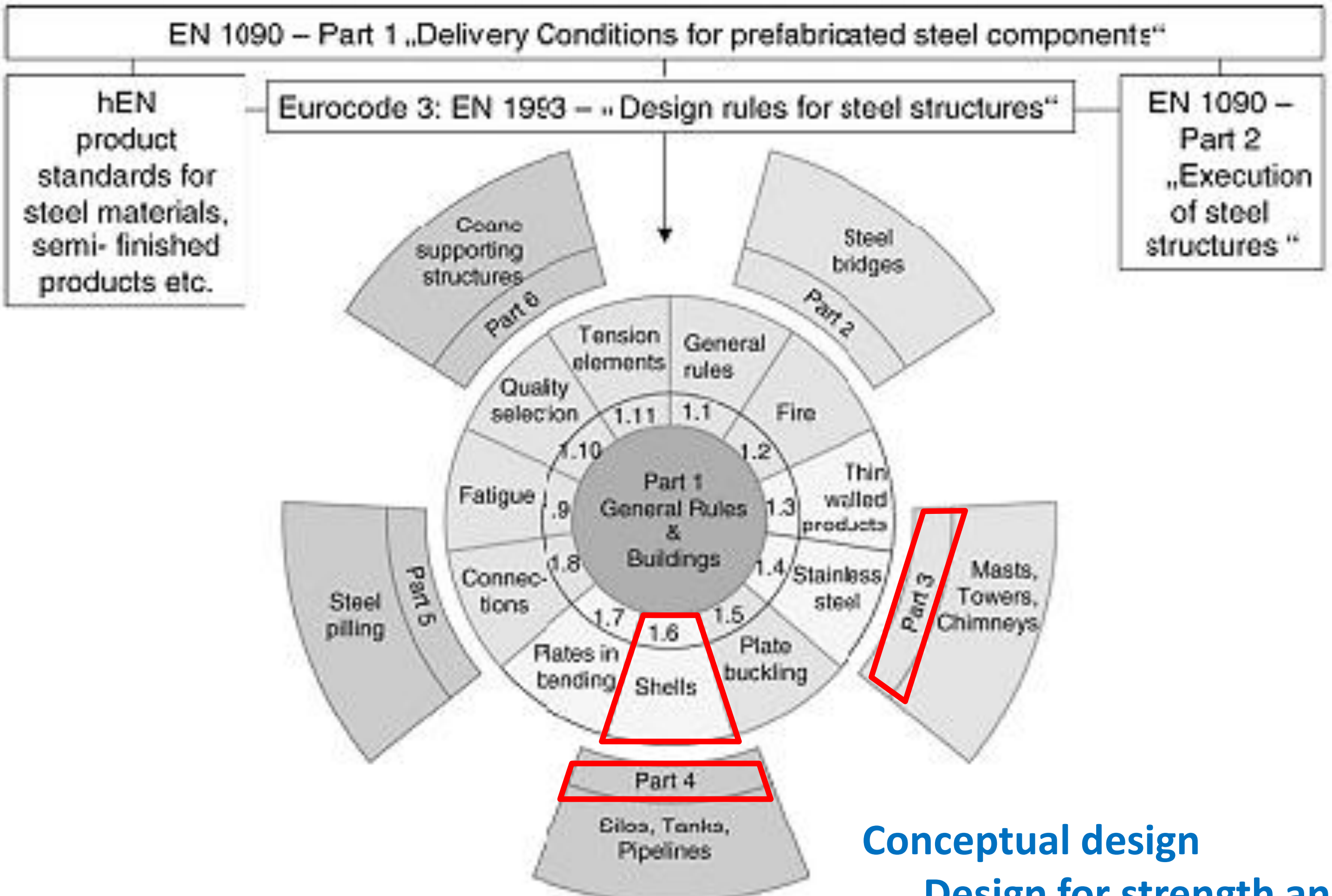
Steel Shell Design: Codification



Conceptual design

Design for strength and buckling

Detailing

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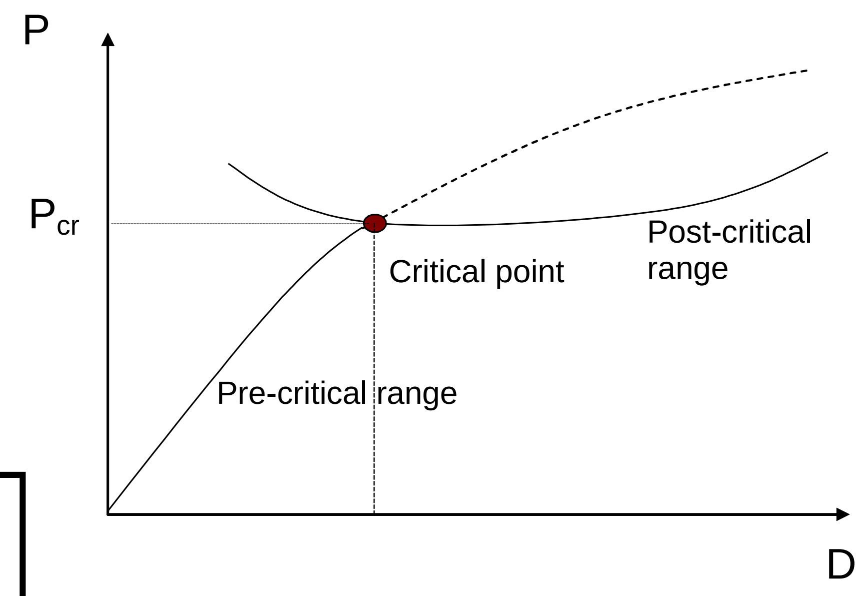
Behavioural phenomenology of shells

- **Behavior of a given structure (slender!) can be controlled by design if the three characteristic ranges of load-deformation curve are correctly defined**
  - **Pre-critical range**
  - **Critical point (or range)**
  - **Post-critical range**



$P \in (0, P_{cr}]$	➔ **Structural stability**
$P > P_{cr}$	➔ **Structural instability**

Buckling: { **elastic**, **plastic**, **dynamic**

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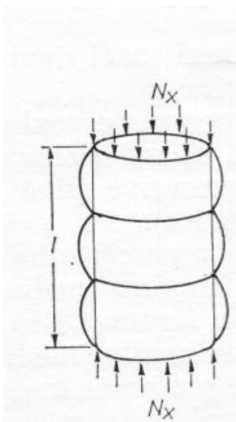
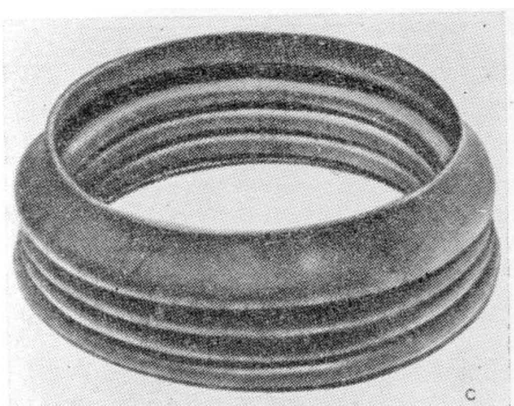
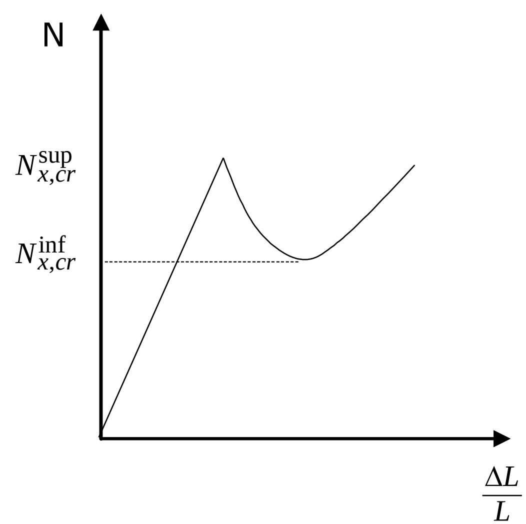
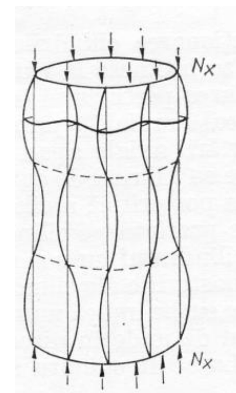
Behavioural phenomenology of shells

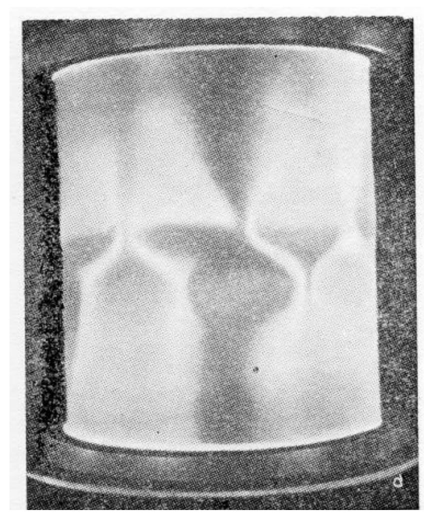
- **Instability phenomenon e.g. bifurcation instability of cylinders**











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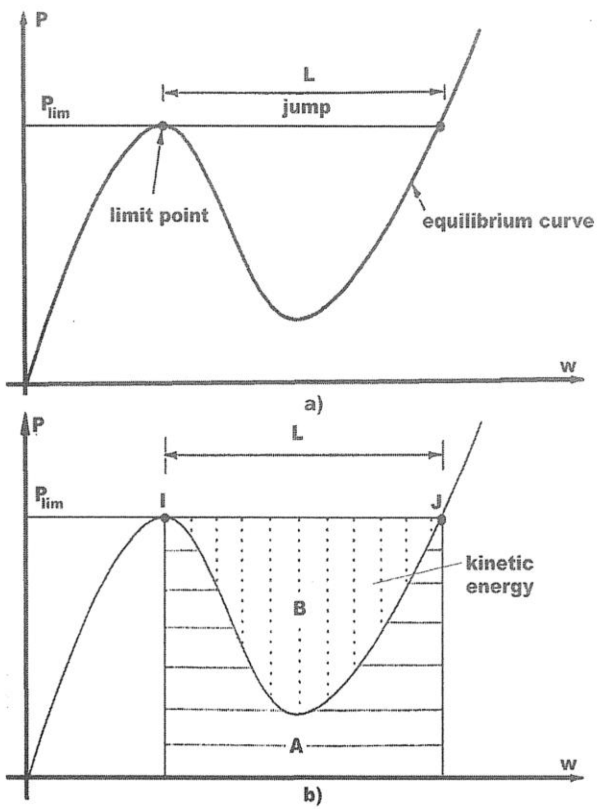
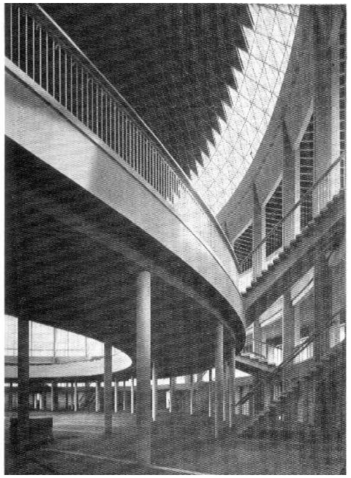
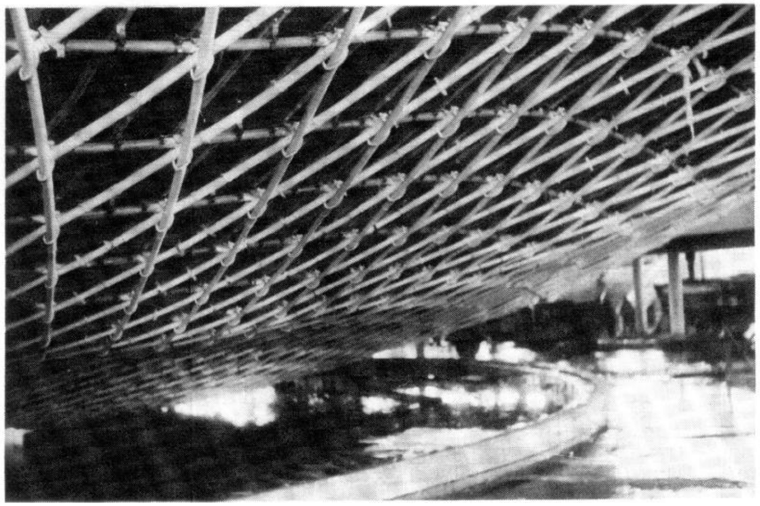
Behavioural phenomenology of shells

- **Instability phenomenon – Jump of Equilibrium or Snap Through Instability**
  - **Affects shallow arches and shells, reticulated shells**



EREN Exhibition hall, Bucharest, 1963





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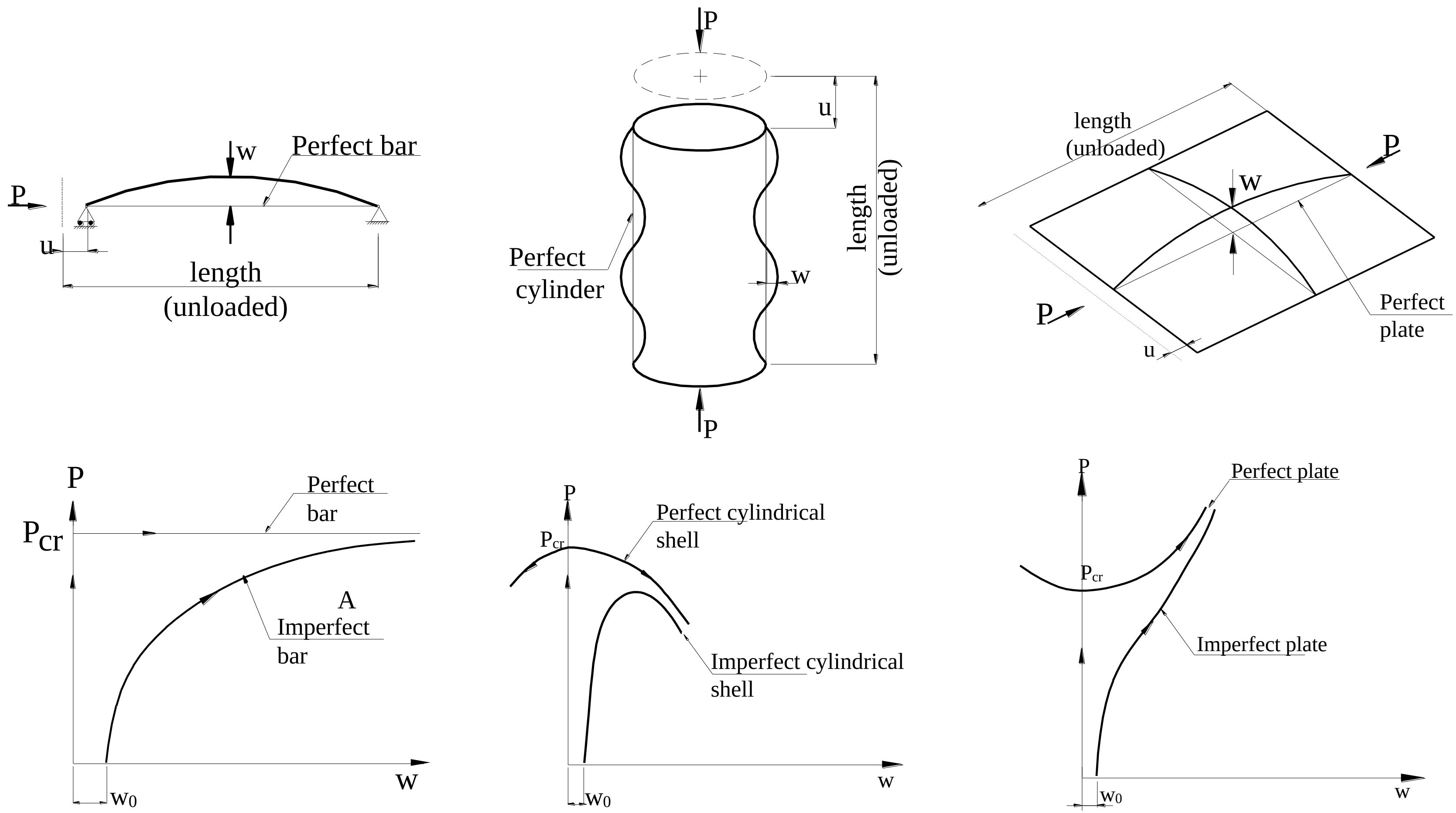
Critical and post-critical behaviour of elastic structures



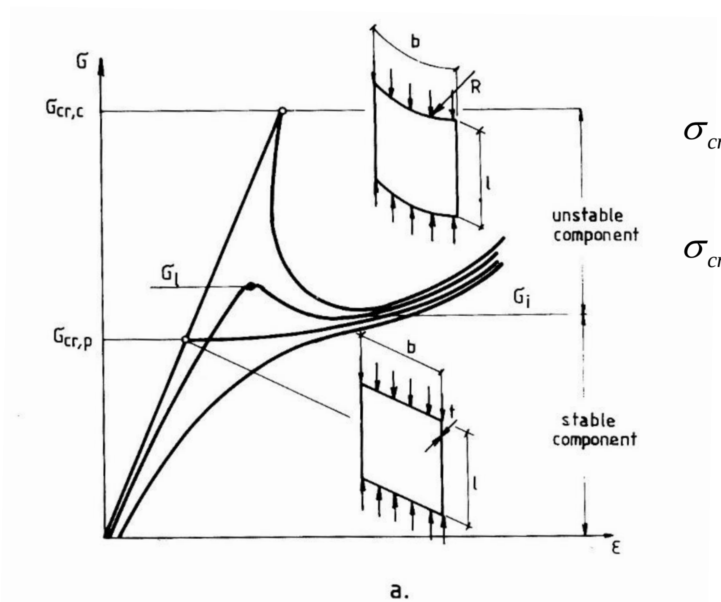
Columns
indifferent post-critical path

Cylinders
unstable post-critical path

Plates
stable post-critical path

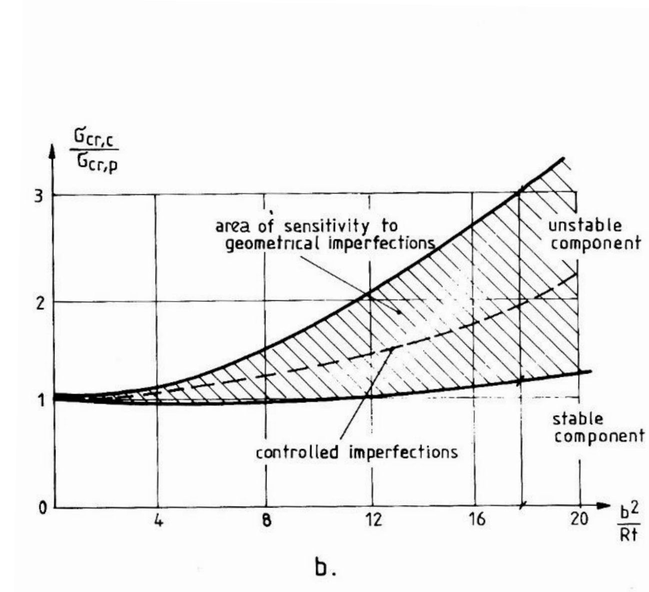
Favourable and unfavourable effects of spatiality

- **Curvature effect in axial compression**



$$\sigma_{cr,p} = \frac{\pi^2 E}{3(1-\mu^2)}\left(\frac{t}{b}\right)^2$$

$$\sigma_{cr,c} = \underbrace{\frac{\pi^2 E}{3(1-\mu^2)}\left(\frac{t}{b}\right)^2}_{\text{Stable component}} + \underbrace{\frac{E}{4\pi^2}\left(\frac{b}{r}\right)^2}_{\text{Unstable component}}$$



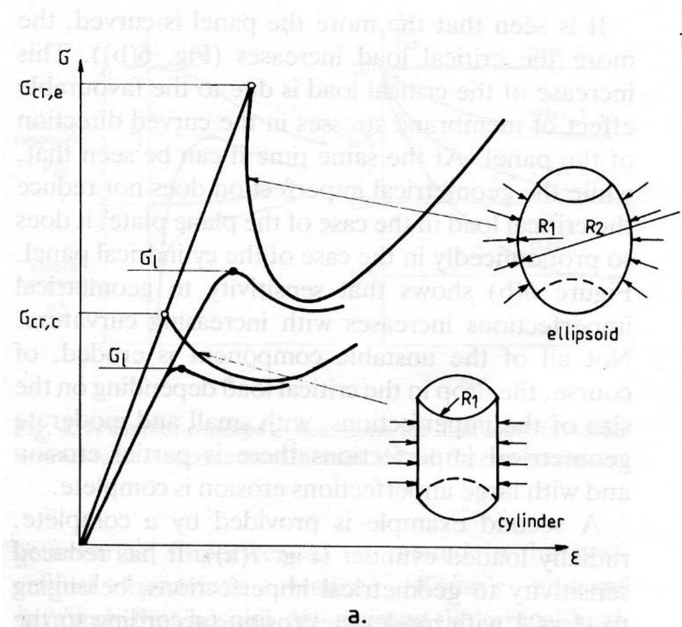
increase in critical load

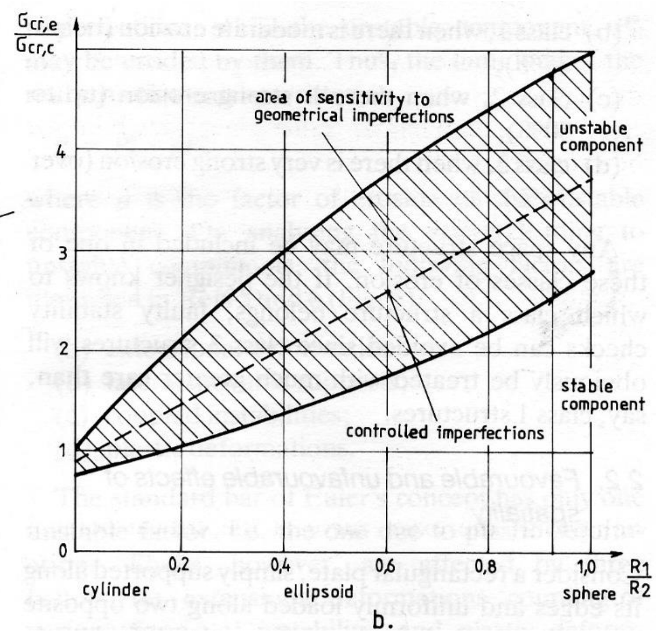
increase in sensitivity to geometrical imperfections

Introduction to Design of Shell Structures

Favourable and unfavourable effects of spatiality

- **Curvature effect in axi-symetrical compression**





increase in critical load

increase in sensitivity to geometrical imperfections

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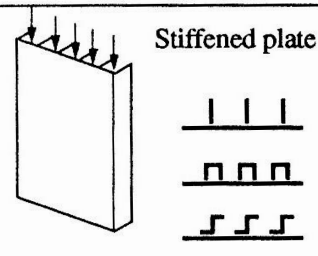
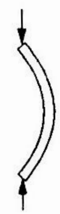
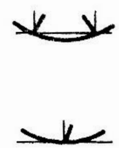
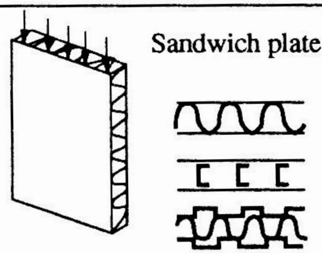
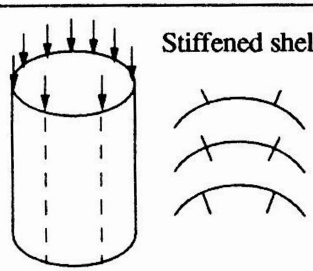
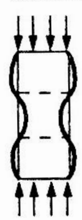
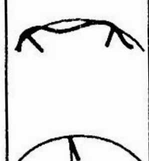
Coupled instabilities for plate and shell elements

	Structure	Instability modes		Class of interaction
		First	Second	
1	Stiffened plate 	Overall 	Local 	S-VS
2	Sandwich plate 	Overall 	Local 	S
3	Stiffened shell 	Overall 	Local 	S-VS

W – weak interaction

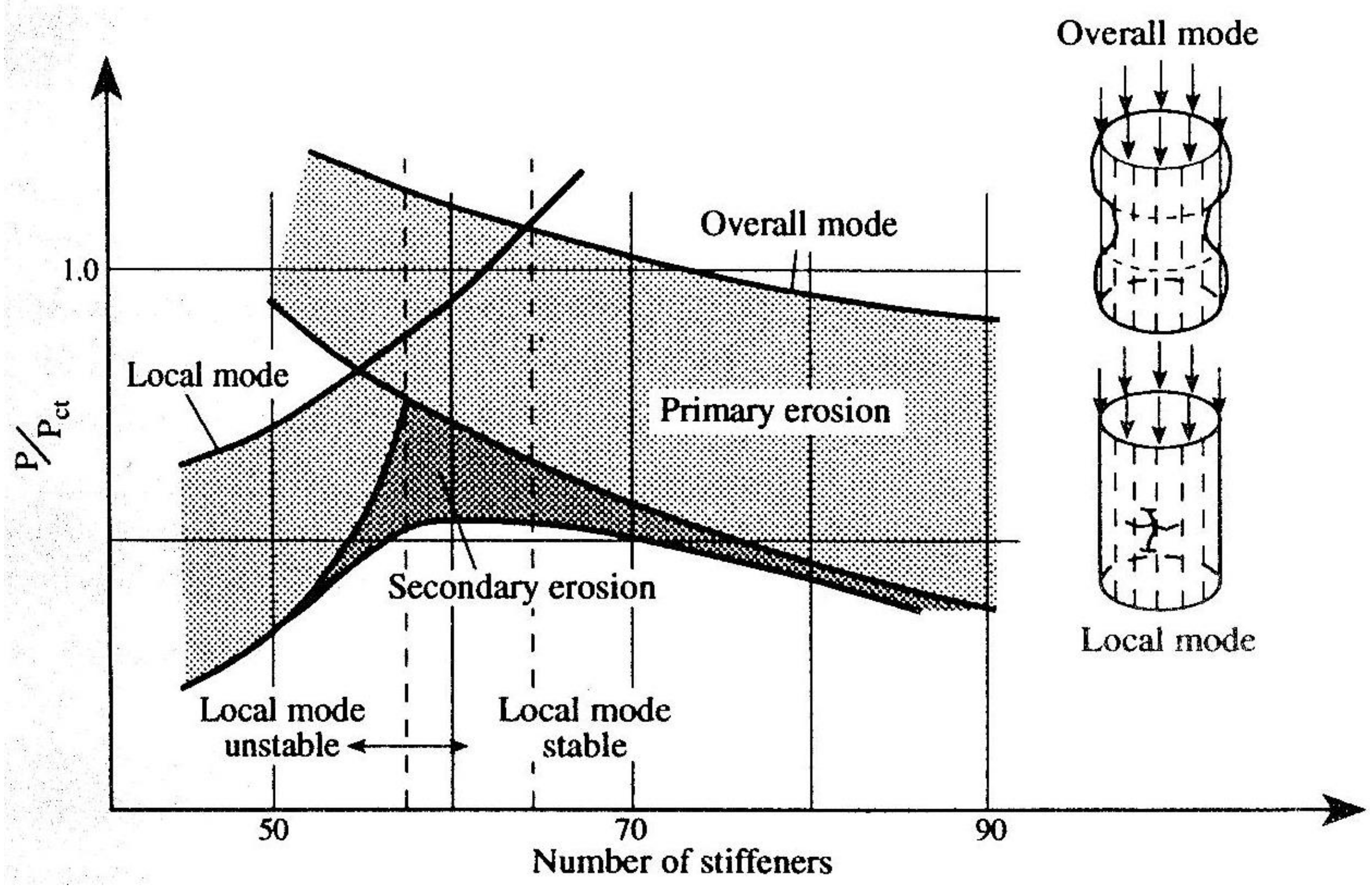
M – moderate interaction

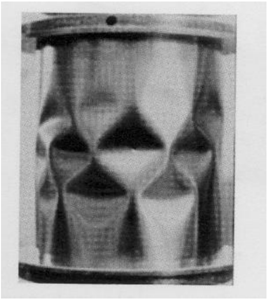
S – strong interaction

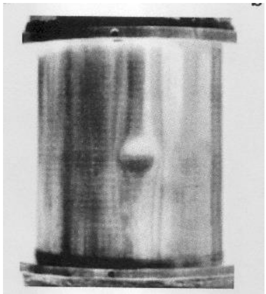
VS – very strong interaction

Coupled instabilities for plate and shell elements

- **Erosion of Theoretical Critical Buckling Load**







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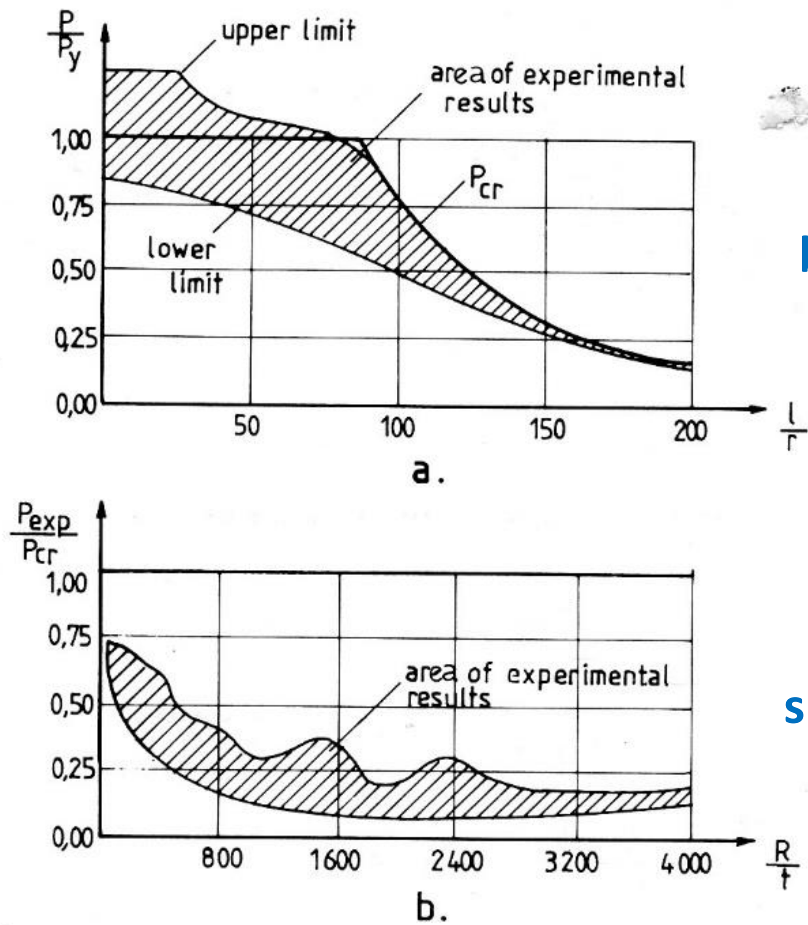
Instability phenomena: Influence of imperfections

- **Agreement of theoretical and experimental values**

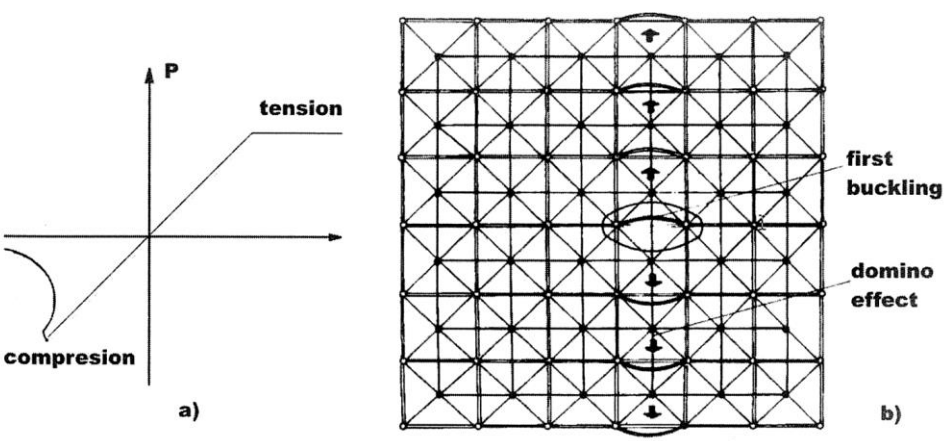


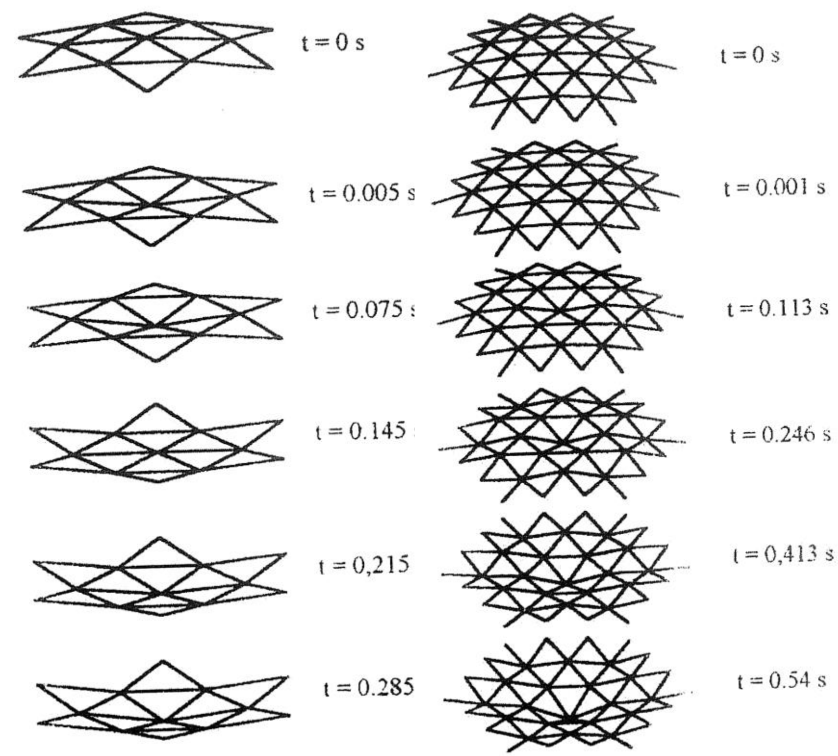
bars

shells

Instability phenomena: basic types and models

- Dynamic propagation of instability or progressive instability
  - Domino effect
    (double layer grids)
  - Instability propagation
    (single layer reticulated shells)





Introduction to Design of Shell Structures

Models and Methods of Analysis

- Pre-Critical, Critical and Post-Critical Analysis

- Generic classification of structures in terms of characteristic instability types and sensitivity to imperfections
  - Linear, nonlinear, elastic, plastic models
    - Linear buckling analysis (eigen-buckling) – LBA
    - Geometrical nonlinear imperfection analysis – GNIA
    - Geometrical material nonlinear imperfection analysis – GMNIA
- Pre-critical solver methods (Newton – Raphson) or
- Post-critical solver methods (Arc-length); Designed load checking or load-deformation curve

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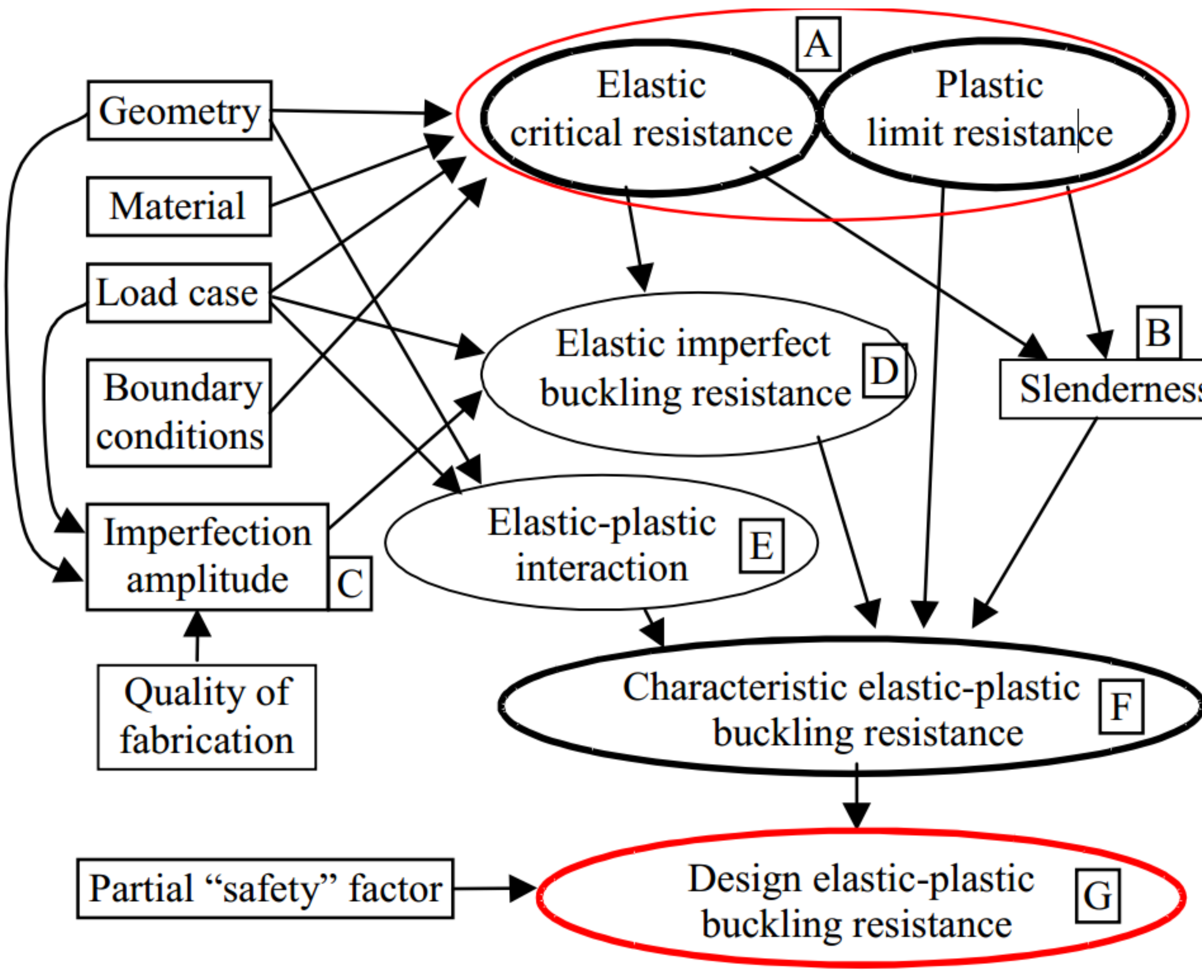
Models and Methods of Analysis

- Design flowchart for the Design of Shells according to EN 1993-1-6



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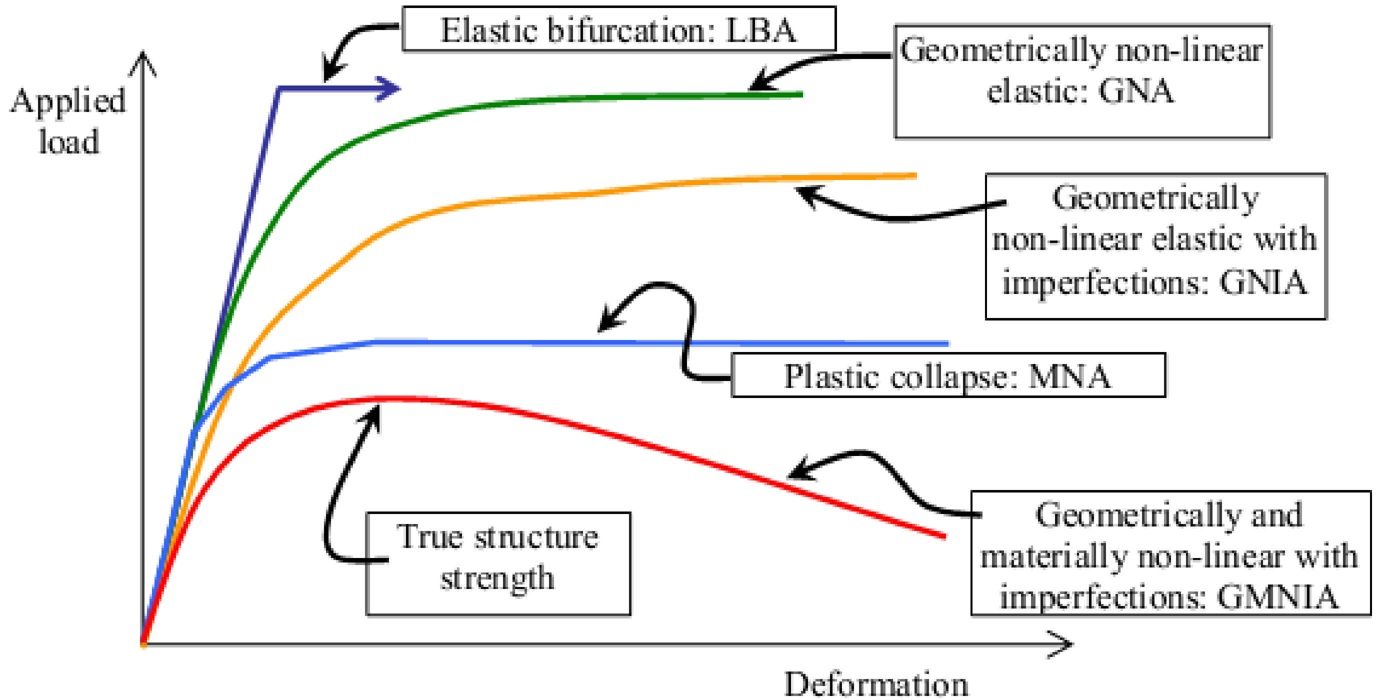
Methods of Analysis

- Methods of Analysis – Global Frame Analysis
  - Finite Elements Methods for Analysis and Design



Load-displacement curves found using different analyses of the same structure (Rotter, 2011)

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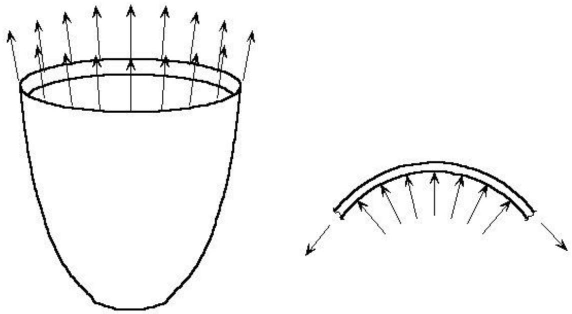
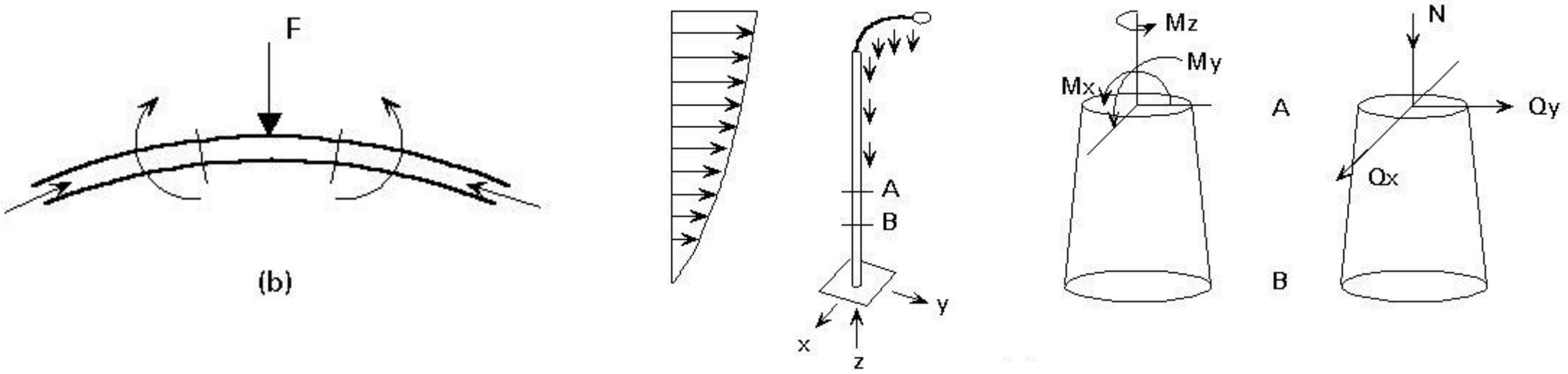
Methods of Analysis

- Basic modes for behaviour
  - a) Membrane
  - b) Bending shell
  - c) Shell like a member



a)



c)

- Basic Equations
  - Simplified Linear Shell Theory
    - The Love-Kirchhoff assumptions (simplified model)
      - The shell thickness is negligibly small in comparison with the least radius of curvature of the shell middle surface (shell is thin)
      - Strains and displacements that arise within the shell are small (products of deformations quantities that occur in the derivation of the theory may be neglected, ensuring that the system is described by a set of geometrically linear equations)
      - Straight lines that are normal to the middle surface prior to deformation remain straight and normal to the middle surface during deformation and experience no change in length (analogue to hypothesis for beams – plane sections before bending remain plane after bending)
      - The direct stress actin in the direction normal to the shell middle surface is negligible (not valid in the vicinity of concentrated transverse loads)

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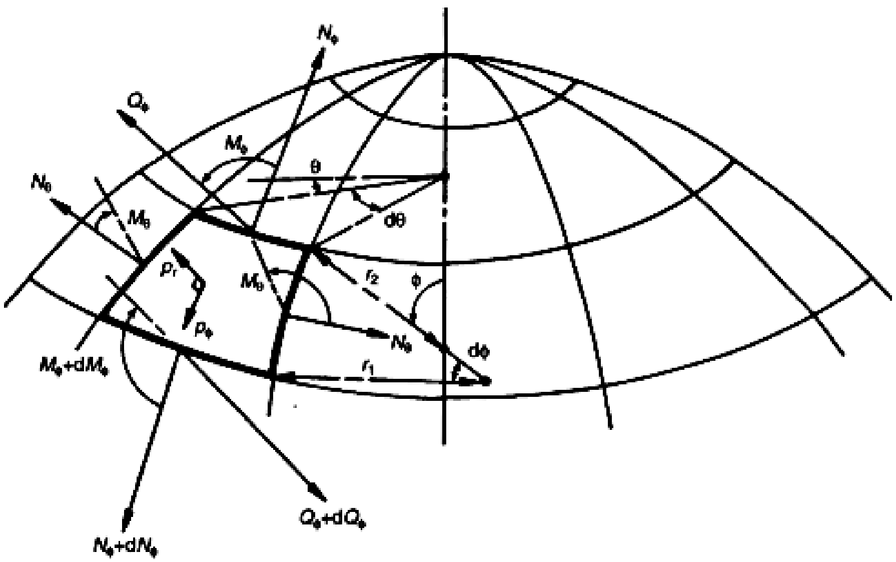
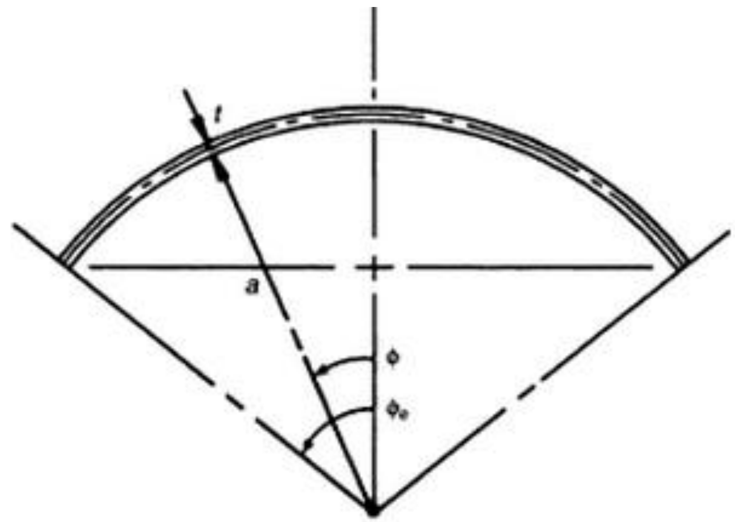
Methods of Analysis

- Model of an axi-symmetrical Loaded Shell



Model of an axisymetrically loaded shell



Geometrical parameter of the spherical shell

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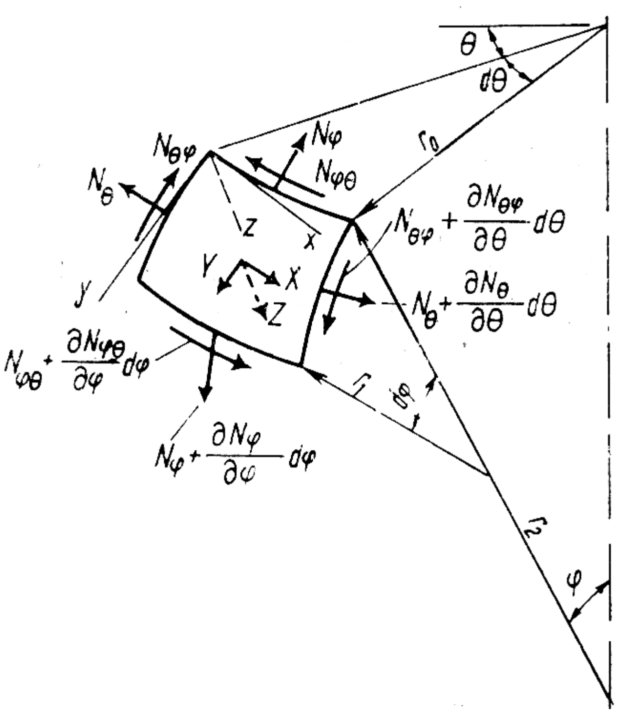
Methods of Analysis

- **General Rotation Shell**
  - **Membrane Theory: Equilibrium Equations for Unsymmetrical Actions**



$$\frac{\partial}{\partial\varphi}\left(N_\varphi r_0\right)+\frac{\partial N_{\theta\varphi}}{\partial\theta}r_1-N_\theta r_1\cos\varphi+Yr_1r_0=0$$

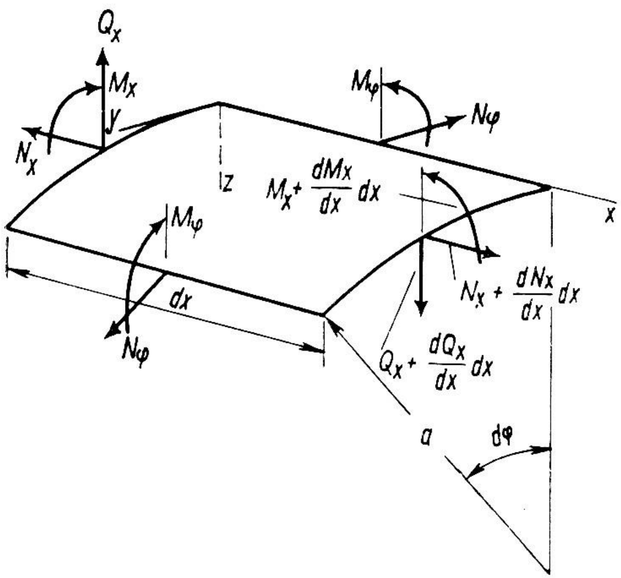
$$\frac{\partial}{\partial\varphi}\left(r_0N_{\varphi\theta}\right)+\frac{\partial N_\theta}{\partial\theta}r_1+N_{\theta\varphi}r_1\cos\varphi+Xr_0r_1=0$$

$$\frac{N_\varphi}{r_1}+\frac{N_\theta}{r_2}+z=0$$

Introduction to Design of Shell Structures

Methods of Analysis

- **Cylindrical Shells**
  - **Bending Theory Axisymmetric Loading**



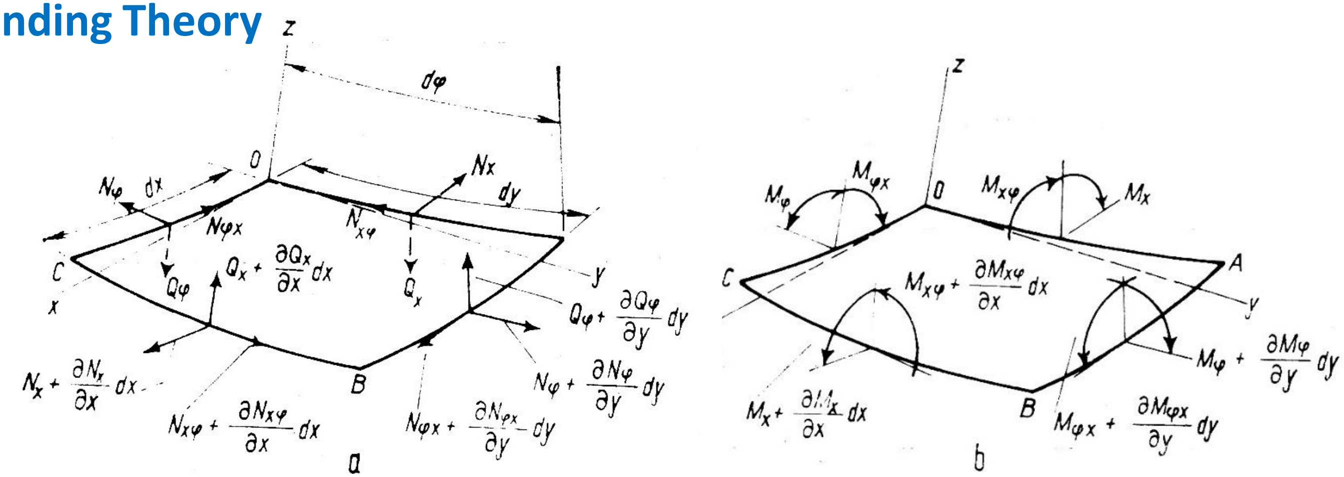
$$\begin{cases} \dfrac{dN_x}{dx} a \cdot dx \cdot d\varphi = 0 \\ \dfrac{dQ_x}{dx} a \cdot dx \cdot d\varphi + N_\varphi dx \cdot d\varphi + Z \cdot a \cdot dx \cdot d\varphi = 0 \\ \dfrac{dM_x}{dx} a \cdot dx \cdot d\varphi - Q_x \cdot a \cdot dx \cdot d\varphi = 0 \end{cases}$$

$$D\frac{d^4 w}{dx^4} + \frac{Eh}{a^2} w = Z \;; \qquad D = \frac{Eh^3}{12\left(1-\nu^3\right)}$$

Introduction to Design of Shell Structures

Methods of Analysis

- **Shells**
  - **General Bending Theory**



$$
\begin{cases}
a\dfrac{\partial N_x}{\partial x}+\dfrac{\partial N_{\varphi x}}{\partial \varphi}=0\\
\dfrac{\partial N_\varphi}{\partial \varphi}+a\dfrac{\partial N_{x\varphi}}{\partial x}-Q_\varphi=0\\
a\dfrac{\partial Q_x}{\partial x}+\dfrac{\partial Q_\varphi}{\partial \varphi}+N_\varphi+q\cdot a=0\\
a\dfrac{\partial M_{x\varphi}}{\partial x}-\dfrac{\partial M_\varphi}{\partial \varphi}+aQ_\varphi=0\\
\dfrac{\partial M_{\varphi x}}{\partial \varphi}+a\dfrac{\partial M_x}{\partial x}-aQ_x=0
\end{cases}
\rightarrow
\begin{cases}
a\dfrac{\partial N_x}{\partial x}+\dfrac{\partial N_{\varphi x}}{\partial \varphi}=0\\
\dfrac{\partial N_\varphi}{\partial \varphi}+a\dfrac{\partial N_{x\varphi}}{\partial x}+\dfrac{\partial M_{x\varphi}}{\partial x}-\dfrac{1}{a}\dfrac{\partial M_\varphi}{\partial \varphi}=0\\
N_\varphi+\dfrac{\partial^2 M_{\varphi x}}{\partial x\partial \varphi}+a\dfrac{\partial^2 M_x}{\partial x^2}-\dfrac{\partial^2 M_{x\varphi}}{\partial x\partial \varphi}+\dfrac{1}{a}\dfrac{\partial^2 M_\varphi}{\partial \varphi^2}+q\cdot a=0
\end{cases}
$$

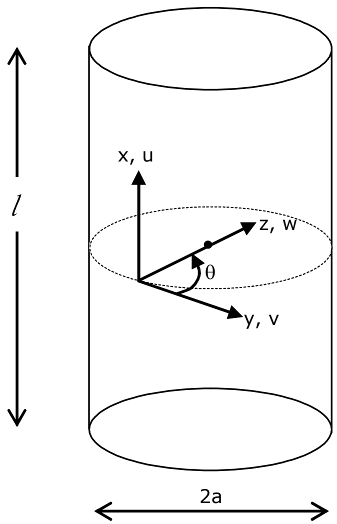
- Basic Equations

Aspect	Equation	Unknown
1. Equilibrium equations (static)	5	8
2. Deformability compatibility (geometric)	9	12
3. Physical aspect	6	---
TOTAL	20	20

Introduction to Design of Shell Structures

Methods of Analysis

- **Buckling of Cylindrical Shells in Compression**
  - **General Case**



Equilibrium equations for elastic buckling:

$$a\frac{\partial N_x}{\partial x}+\frac{\partial N_{yz}}{\partial \theta}=0$$

$$\frac{\partial N_y}{\partial \theta}+a\frac{\partial N_x}{\partial x}+aN_x\frac{\partial^2 v}{\partial x^2}+\frac{\partial M_{xy}}{\partial x}-\frac{\partial M_x}{a\partial \theta}=0$$

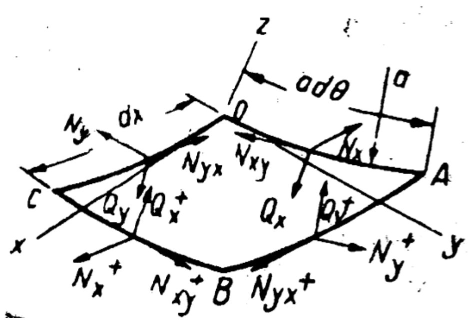
$$aN_x\frac{\partial^2 w}{\partial x^2}+N_y+a\frac{\partial^2 M_x}{\partial x^2}+\frac{\partial^2 M_{yx}}{\partial x\partial \theta}+\frac{\partial^2 M_y}{a\partial \theta^2}-\frac{\partial^2 M_{xy}}{\partial x\partial \theta}=0$$

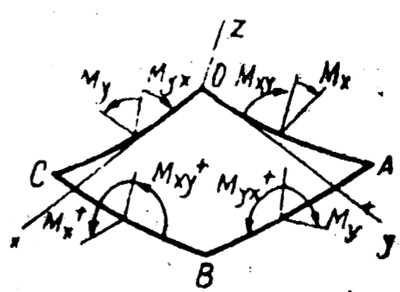
with solutions:

$$u=A\sin n\theta\cos\frac{m\pi x}{l}$$

$$v=B\cos n\theta\sin\frac{m\pi x}{l}$$

$$w=C\sin n\theta\sin\frac{m\pi x}{l}$$





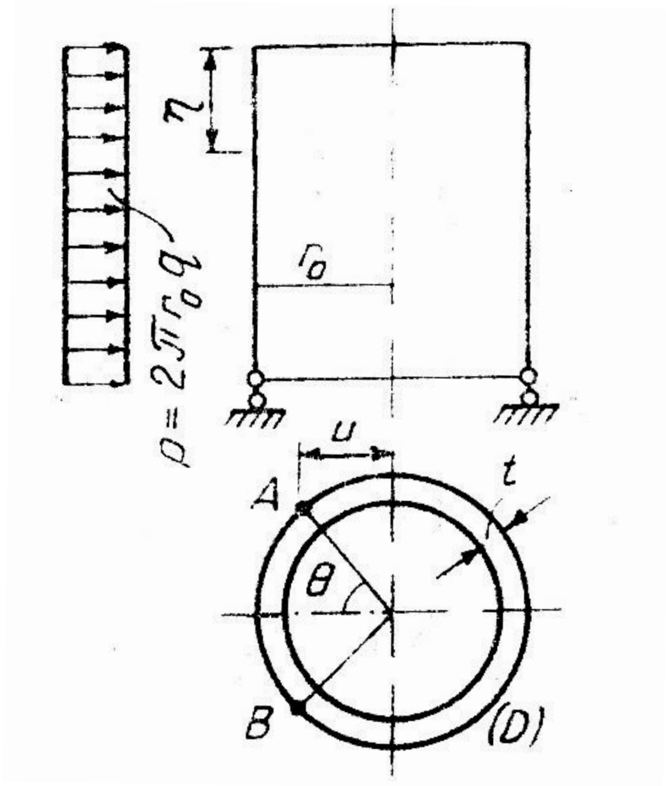
$$v=0$$

$$n=0\quad\Rightarrow\quad u,w=f(x)\quad axial-symetrical\ buckling$$

Introduction to Design of Shell Structures

Methods of Analysis

- Cylindrical Shells
  - Membrane Theory Application for Wind Action



$$N_{\varphi} = \frac{q\eta^2}{r_0}\cos\theta$$

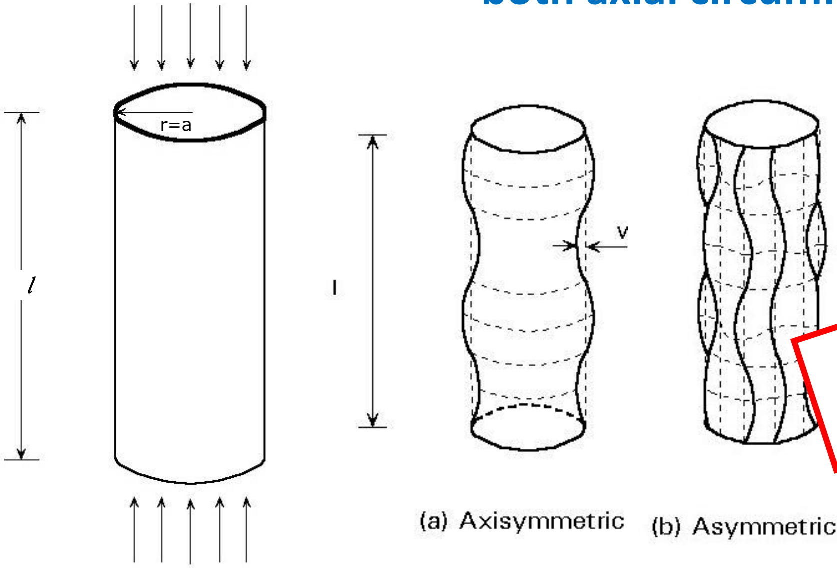
$$N_{\varphi\theta} = -2q\eta\sin\theta$$

$$N_{\theta} = -qr_0\cos\theta$$

Introduction to Design of Shell Structures

Methods of Analysis

- Simplified Design Formulae (Cylindrical shells)
  - Two possible approaches
    - Overall column buckling if l/r ratio is large
    - Shell buckling which involves the cross section deformation and can be, in general, either:
      - Axisymmetric, when the displacement are constant around circumferential section
      - Asymmetric (chessboard shape), when waves are formed in both axial circumferential directions



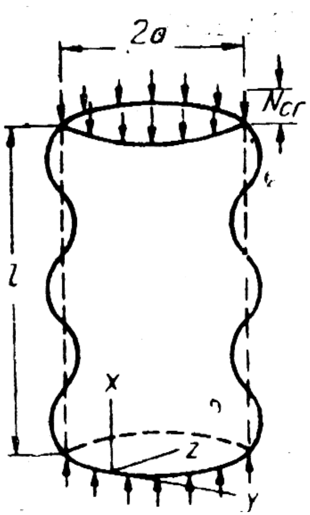
(a) Axisymmetric (b) Asymmetric

It can be shown theoretically that both modes correspond to the same buckling load

Introduction to Design of Shell Structures

Methods of Analysis

- **Simplified Design Formulae**
    - **Axial-symmetric buckling of cylindrical shell in compression**



$$D\frac{d^4 v}{dx^4} + N_x \frac{d^2 w}{dx^2} + Eh\frac{w}{a^2} = 0$$

$$D = \frac{Eh^3}{12\left(1-\mu^2\right)}$$

Radial displacement:

$$w = -A\sin\frac{m\pi x}{l}$$

Elastic critical axial stress $\left(\sigma_{cr} = \frac{N_{cr}}{h} \right)$

$$\sigma_{cr} = \frac{Eh}{a\sqrt{3\left(1-\nu^2\right)}} \quad ; \quad \frac{m\pi}{l} = \sqrt[4]{\frac{Eh}{a^2 D}}$$

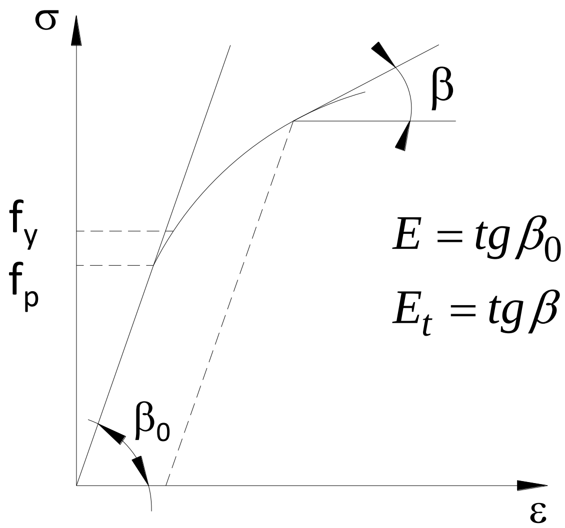
For ν=0.3 ➔ $\frac{l}{m} \approx 1.72\sqrt{ah}$

- **In case of axial-symmetrical buckling, the critical shear does not depend of cylinder length!**
- **If one of the cylinder ends is free ($w \neq 0$), σ_{cr} drops to 38% compared to simple supported case.**
- **Cylinder is highly sensitive to tangential displacements at the boundaries. If $v \neq 0$, critical stress drops to 50%!**

Introduction to Design of Shell Structures

Methods of Analysis

- Simplified Design Formulae
  - Axial-symmetric buckling of cylindrical shell in compression



Post-elastic critical buckling

$$\sigma_{cr} = \frac{h\sqrt{EE_t}}{a\sqrt{3\left(1-\nu^2\right)}}$$

$$\frac{l}{m} \approx 1.72\sqrt{ah}\sqrt{\frac{E_t}{E}}$$

Introduction to Design of Shell Structures

Methods of Analysis

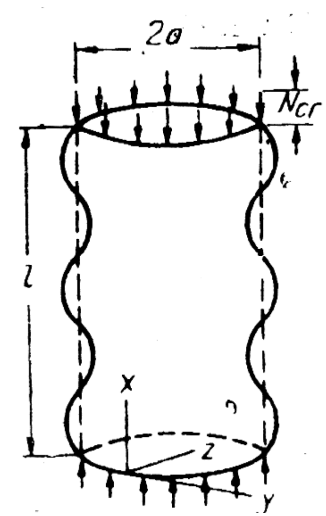
- Simplified Design Formulae
  - Axial-symmetric buckling of cylindrical shell in compression
  - Post-critical buckling: stable and unstable components

$$\sigma_{cr} = \frac{N_{cr}}{h} = D\left(\frac{m^2\pi^2}{hl^2} + \frac{E}{a^2 D}\frac{l^2}{m^2\pi^2}\right)$$

$$N_{cr} = \underbrace{D\left(\frac{m\pi^2}{l}\right)^2}_{\text{Stable component}} + \underbrace{\frac{Eh}{a^2}\left(\frac{h}{m\pi}\right)^2}_{\text{Unstable component}}$$

Stable component
(inextensional bending deformation – x direction)

Unstable component
(extensionally circumferential deformation – y direction



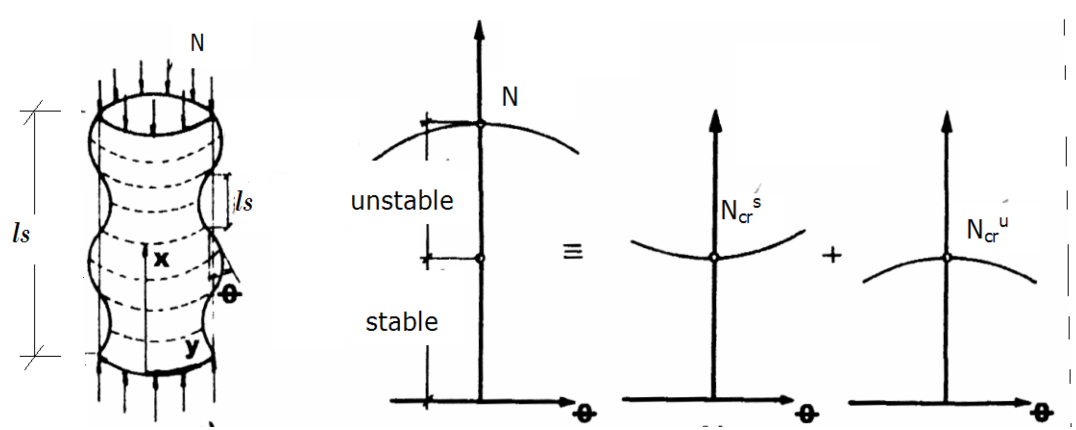
Introduction to Design of Shell Structures

Methods of Analysis

- **Simplified Design Formulae**
  - **Axial-symmetric buckling of cylindrical shell in compression**
  - **Post-critical buckling equation**

$$\frac{N}{N_{cr}} = 1 + \underbrace{\frac{1}{8}\theta_m^2}_{\text{Stable component}} - \underbrace{\frac{3}{8}\frac{Eh}{N_{cr}a^2}\left(\frac{l}{m\pi}\right)^2}_{\text{Unstable component}} \theta_m^2$$

Stable component **Unstable component**

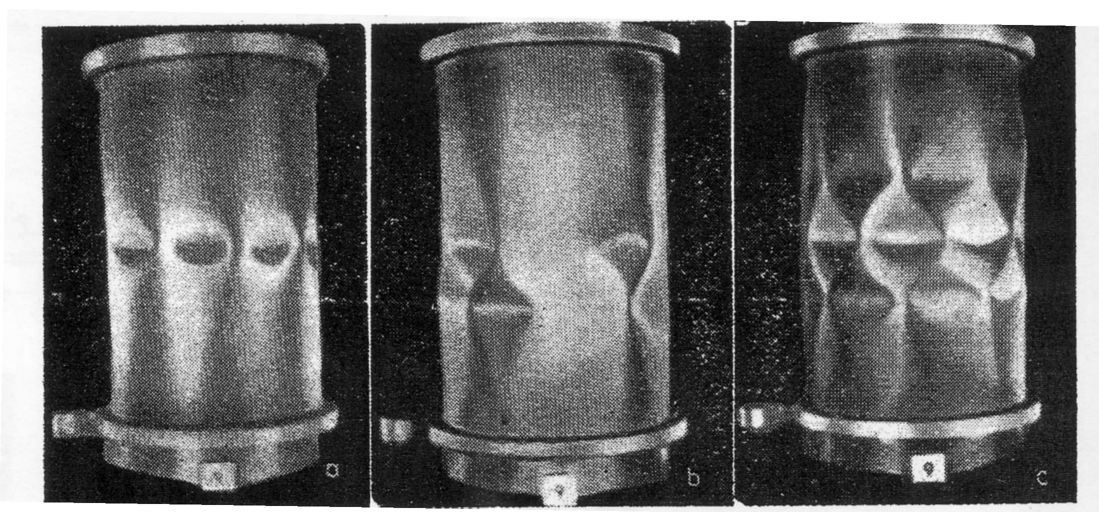


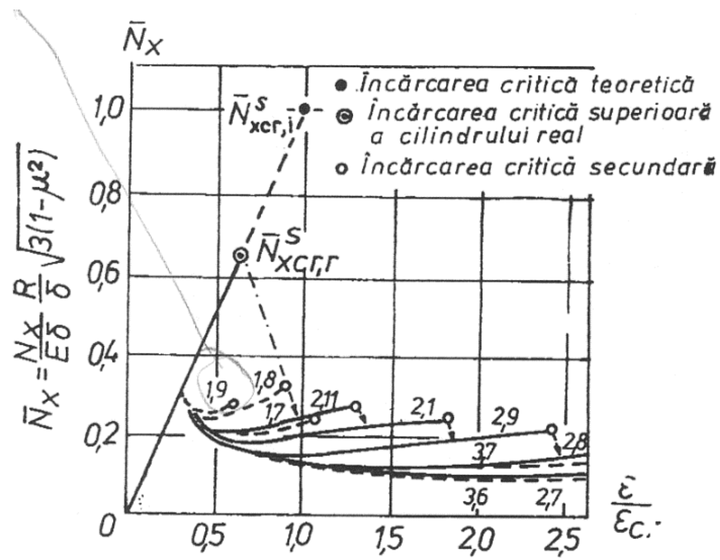
  - **The effect of circumferential extensional deformations increases the value of critical load, but change the type of instability from stable to unstable!**

Introduction to Design of Shell Structures

Methods of Analysis

- Simplified Design Formulae
  - Axial-symmetric buckling of cylindrical shell in compression
  - Examples: medium length cylinder
    - (i,j)   i = no. of longitudinal half-length waves;
      j = no. of circumferential half-length waves

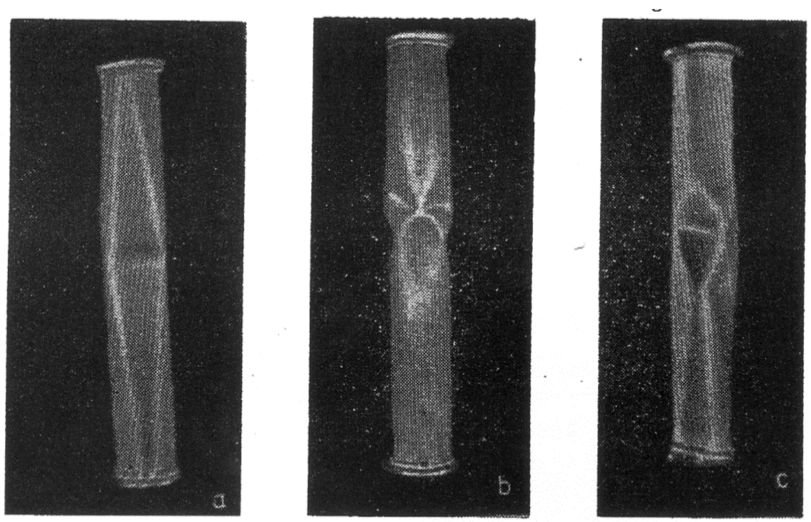


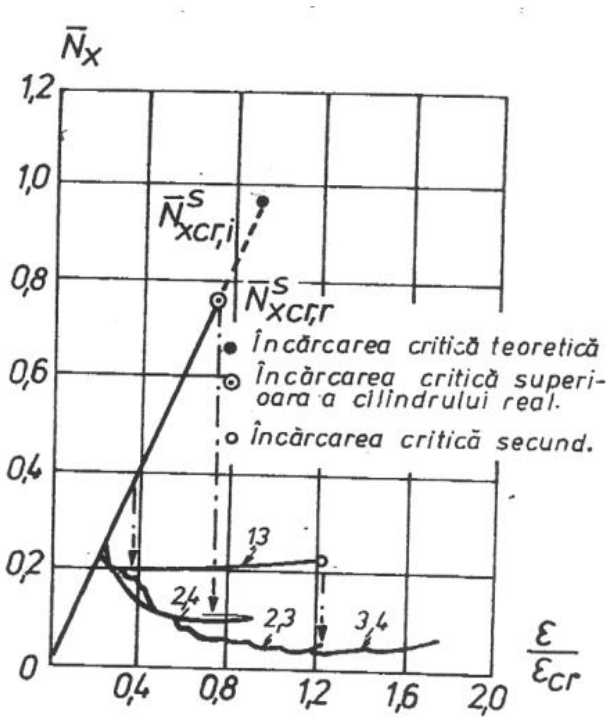


Introduction to Design of Shell Structures

Methods of Analysis

- Simplified Design Formulae
  - Axial-symmetric buckling of cylindrical shell in compression
  - Examples: long cylinder

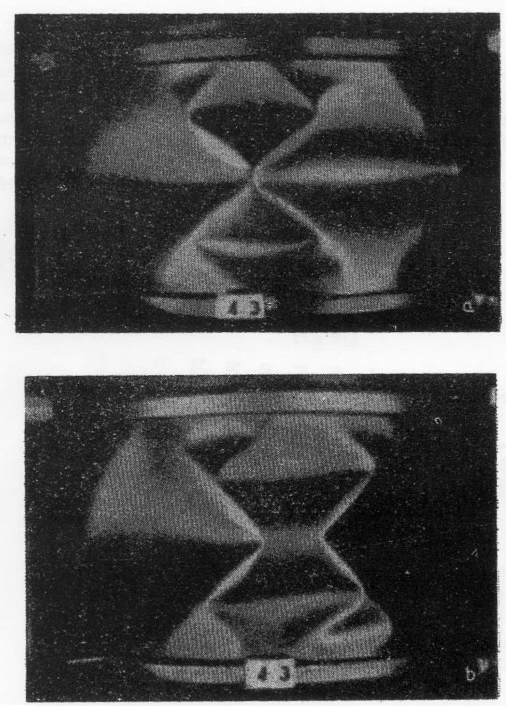


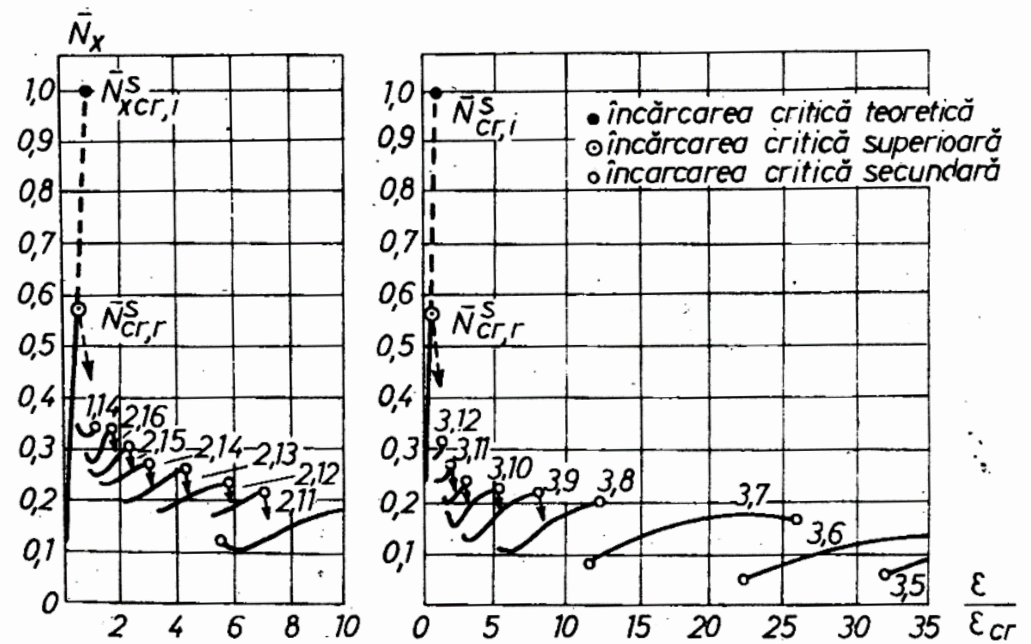


Introduction to Design of Shell Structures

Methods of Analysis

- Simplified Design Formulae
  - Axial-symmetric buckling of cylindrical shell in compression
  - Examples: short cylinder





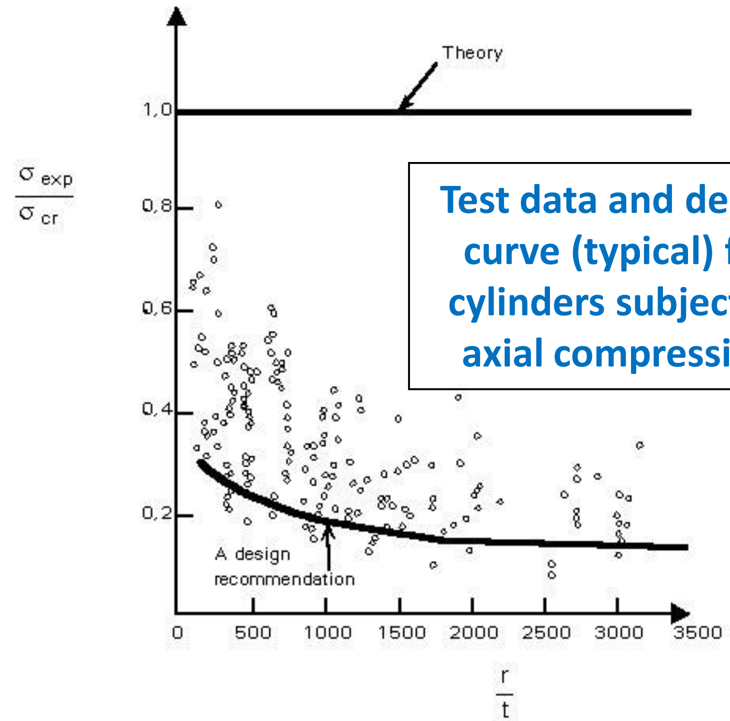
Introduction to Design of Shell Structures

Methods of Analysis

- Simplified Design Formulae
  - Axial-symmetric buckling of cylindrical shell in compression
  - Principle of ECCS approach (ECCS Recommendations 1998)
    - Real cylinders are highly sensitive to imperfections
    - “Knock-down” factor α is introduced to account for imperfections and for plastic effects

$$\sigma_d = \alpha \sigma_{cr}$$

- α depends on:
  - Shell geometry
  - Loading conditions
  - Initial imperfections
  - Material properties



Test data and design curve (typical) for cylinders subject to axial compression

Introduction to Design of Shell Structures

Methods of Analysis

- **Simplified Design Formulae**
  - **Axial-symmetric buckling of cylindrical shell in compression**
  - **Principle of ECCS approach**
    - **For unstiffened cylinders, is similar to the one for column in axial compression**

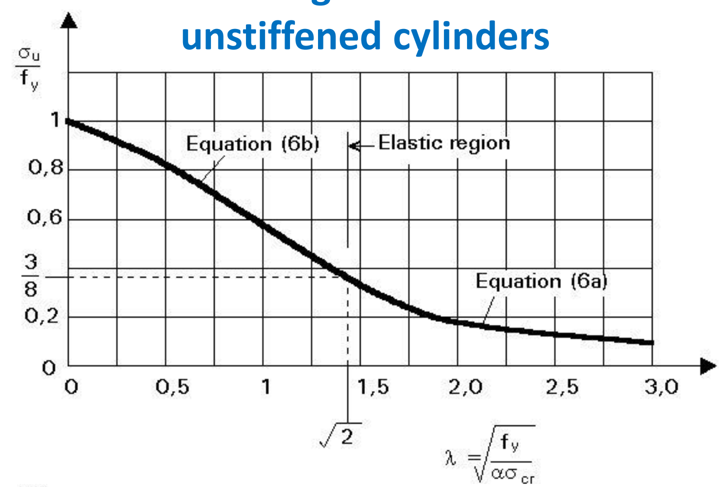
$$\lambda \geq \sqrt{2} \quad \Rightarrow \quad \alpha\sigma_{cr} \leq 0.5 f_y$$

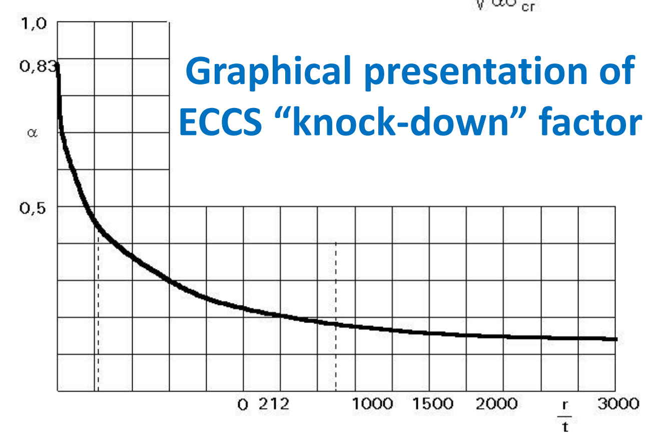
$$\frac{\sigma_n}{f_y} = \left(\frac{1}{\lambda^2}\right)\frac{1}{\gamma_{M1}}$$

$$\lambda \leq \sqrt{2} \quad \Rightarrow \quad \alpha\sigma_{cr} \geq 0.5 f_y$$

$$\frac{\sigma_n}{f_y} = 1 - 0.4123\lambda^{1.2}$$

ECCS design formulae for unstiffened cylinders





Graphical presentation of ECCS "knock-down" factor

Introduction to Design of Shell Structures

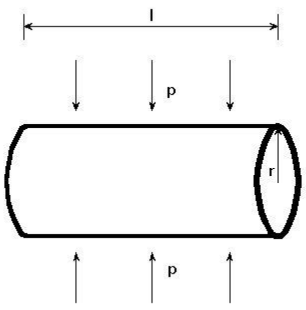
Methods of Analysis

- **Simplified Design Formulae**
  - **Buckling of cylindrical shells under external pressure**
  - **Membrane (hoop) stress in practical range**

$$\sigma_y = \frac{pa}{h}; \qquad \sigma_x = 0$$

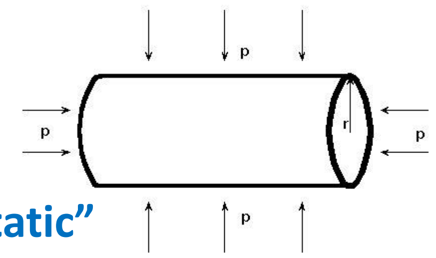
$$\sigma_y = \frac{pa}{h}; \qquad \sigma_x = 0.5\sigma_\theta$$

external uniform pressure



(a)

"hydrostatic" type pressure



(b)

  - **Von Misses formula for critical pressure, σ_{cr}**

$$\sigma_{cr} = \frac{Eh}{a}\left[\frac{1}{\left(n^2-1\right)\lambda_3^2} + \frac{h^2}{12a^2}\left(n^2 - 1 + \frac{2n^2-1-\nu}{\lambda_3}\right)\right]$$

$$\lambda_1 = \frac{\pi a}{l}; \quad \lambda_3 = 1 + \left(\frac{n}{\lambda_1}\right)^2$$

$$p_{cr} = \frac{Eh}{a}\left(\frac{1}{n^2\lambda_3^2} + \frac{h^2}{12a^2}n^2\lambda_3^2\right)\left[1 + \frac{1}{2}\left(\frac{\lambda_1}{2}\right)^2\right]^{-1}$$

n ➔ min p_{cr}

Simplified formulae for long cylinders

$$p_{cr} = \frac{Eh^3\left(n^2-1\right)}{12a^3\left(1-\nu^2\right)}$$

- **Simplified Design Formulae**
  - **Buckling of cylindrical shells under external pressure**
  - **Principle of ECCS approach**

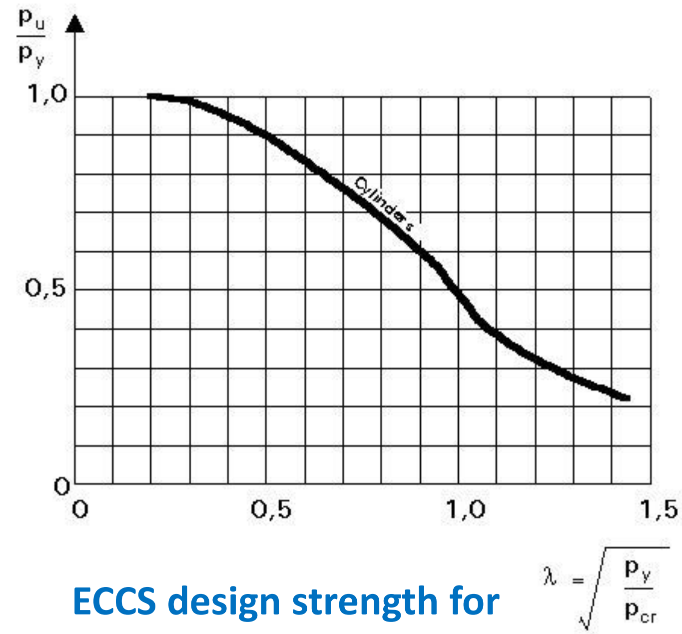
$$0 \le \lambda \le 1 \quad \frac{p_u}{p_y} = \frac{1}{1+\lambda^2}$$

$$\lambda > 1 \quad \frac{p_u}{p_y} = \frac{\alpha}{\lambda^2}; \quad \alpha = 0.5$$

$$\lambda = \sqrt{\left(\frac{p_y}{p_{cr}}\right)}$$

$$P_{cr} = E\left(\frac{h}{a}\right)\beta_{min}$$

for $l/a \ge 0.5$ ➔ $$\beta_{min} = \frac{0.855}{\left(1-\nu^2\right)}\frac{a}{l}\left(\frac{h}{a}\right)^{1.5}$$



ECCS design strength for unstiffened cylinder under uniform pressure

Introduction to Design of Shell Structures

Methods of Analysis

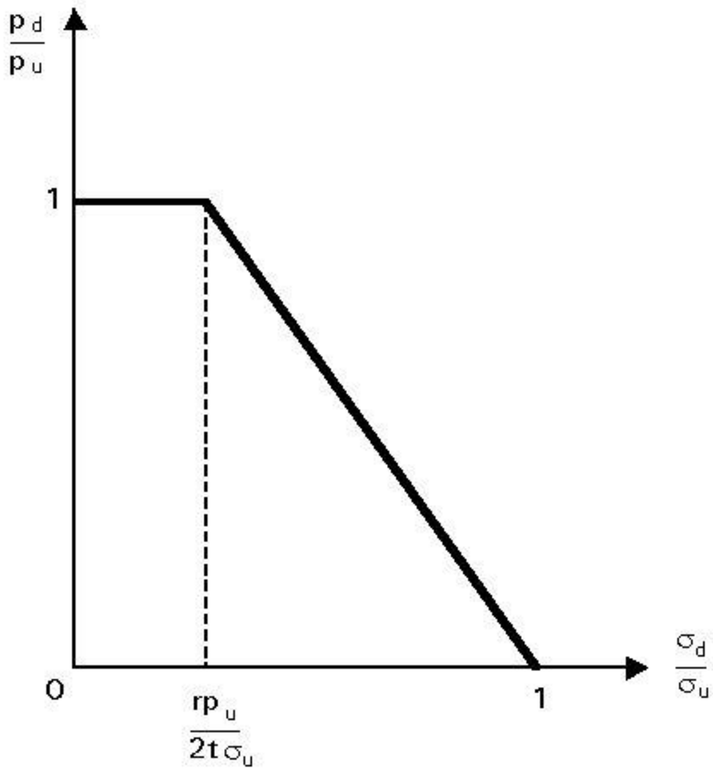
- Simplified Design Formulae
  - Buckling of cylindrical shells under external pressure
  - Principle of ECCS approach
    - Wind action is more complex than simply an external pressure
    - It is needed to check the cylinder stability separately for:
      - Wind radial pressure
      - Wind axial effects
      - Wind tangent effects
      - Interaction of the three
    - Approximately, wind critical pressure can be taken as 1.6 times critical external pressure (Maderspach, Gaunt, Sword)
    - ECCS Design Recommendations (No. 125/2008) offers also a solution

Introduction to Design of Shell Structures

Methods of Analysis

- Simplified Design Formulae
  - Buckling of cylindrical shells under compression and external pressure

$$\frac{\sigma_x}{\sigma_{x,cr}} + \frac{p}{p_{cr}}$$



Interaction curve

Introduction to Design of Shell Structures

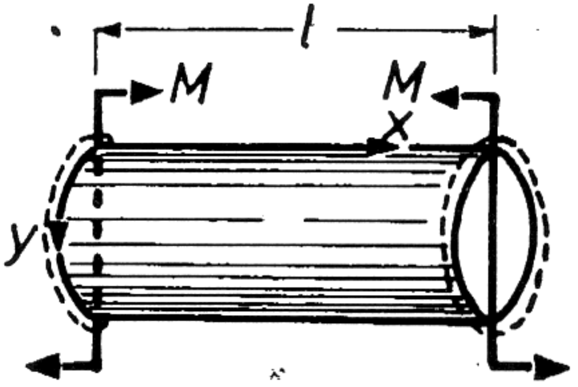
Methods of Analysis

- Simplified Design Formulae
  - Buckling of cylindrical shells in bending
    - (Flügge) ➔

$$\left(\sigma_{cr,x}\right)^{M} \approx 1.33\left(\sigma_{cr,x}\right)^{N_x}$$

    - Long cylinders (Brazier) ➔

$$M_{cr} = \frac{0.99}{\left(1-\nu^2\right)} E h^2 a$$



Introduction to Design of Shell Structures

Methods of Analysis

- Simplified Design Formulae
  - Buckling of cylindrical shells in torsion
    - (Swerin and Flügge) ➔ long cylinders

$$\tau_{cr} = \frac{E}{3\sqrt{2}\left(1-\nu^2\right)^{3/4}}\left(\frac{h}{a}\right)^{3/2}$$

    - (Donnel) ➔ short and medium long cylindrical shells
      - Fixed end

$$\tau_{cr} = \frac{E}{\left(1-\nu^2\right)}\left(\frac{h}{l}\right)^2\left\{4.6+\left[7.8+1.67\left(\sqrt{1-\nu^2}\,\frac{l}{2ah}\right)^{3/2}\right]^{1/2}\right\}$$

      - Simple Supported End

$$\tau_{cr} = \frac{E}{\left(1-\nu^2\right)}\left(\frac{h}{l}\right)^2\left\{2.8+\left[2.6+1.4\left(\sqrt{1-\nu^2}\,\frac{l^2}{2ah}\right)^{3/2}\right]^{1/2}\right\}$$

Introduction to Design of Shell Structures

Methods of Analysis

- Simplified Design Formulae
  - Cylindrical shells under interactive buckling

- Bending + torsion

$$\frac{\sigma}{\sigma_{cr}} + \left(\frac{\tau}{\tau_{cr}}\right)^2 = 1$$

- Bending + external pressure

$$\frac{M}{M_{cr}} + \frac{p}{p_{cr}} = 1 \quad or \quad \left(\frac{\sigma_x}{\sigma_{x,cr}}\right)^2 + \left(\frac{p}{0.9\,p_{cr}}\right)^2 = 1$$

- Compression + torsion

$$\frac{\sigma}{\sigma_{x,cr}} + \left(\frac{\tau}{\tau_{cr}}\right)^2 = 1$$

- Axial compression + external pressure + torsion

$$\frac{\sigma}{\sigma_{x,cr}} + \frac{p}{p_{cr}} + \left(\frac{\tau}{\tau_{cr}}\right)^2 = 1$$

- External pressure + torsion

$$\frac{p}{p_{cr}} + \left(\frac{\tau}{\tau_{cr}}\right)^2 = 1$$

- Axial compression + bending + torsion

$$\frac{\sigma_x}{\sigma_{x,cr}^N} + \frac{\sigma_x}{\sigma_{x,cr}^M} + \left(\frac{\tau}{\tau_{cr}}\right)^2 = 1$$

Design of Steel Structures: Strength and Stability of Shells

- **Basis of design and modelling**
  - Shells shall be designed in acc. with EN1990 and, in particular, to satisfy the following requirements:
    - Overall equilibrium
    - Equilibrium between actions and internal forces and moments
    - Limitation of cracks due to cyclic plastification
    - Limitation of cracks due to fatigue
  - Types of analysis:
    - Global analysis
    - Membrane theory analysis
    - Linear elastic shell analysis
    - Linear elastic bifurcation analysis
    - Geometrically nonlinear elastic analysis
    - Materially nonlinear analysis
    - Geometrically and materially nonlinear analysis
    - Geometrically nonlinear elastic analysis with imperfections included
    - Geometrically and materially nonlinear analysis with imperfections

Design of Steel Structures: Strength and Stability of Shells

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    - Equilibrium between actions and internal forces and moments
    - Limitation of cracks due to cyclic plastification
    - Limitation of cracks due to fatigue
  - Types of analysis:
    - Global analysis ← approximate treatments of certain parts of the structure
    - Membrane theory analysis
    - Linear elastic shell analysis
    - Linear elastic bifurcation analysis
    - Geometrically nonlinear elastic analysis
    - Materially nonlinear analysis
    - Geometrically and materially nonlinear analysis
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    - Materially nonlinear analysis
    - Geometrically and materially nonli
    - Geometrically nonlinear elastic analysis with imperfections included
    - Geometrically and materially nonlinear analysis with imperfections

Conditions of use:
- the boundary conditions are appropriate for transfer of the stresses in the shell into support reactions without causing bending effects;
- the shell geometry varies smoothly in shape (without discontinuities);
- the loads have a smooth distribution (without locally concentrated or point loads).

Design of Steel Structures: Strength and Stability of Shells

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    - Geometrically nonlinear elastic analysis with imperfections included
    - Geometrically and materially nonlinear analysis with imperfections

-linear elastic material law
- linear small deflection theory (undeformed geometry)

Introduction to Design of Shell Structures

Design of Steel Structures: Strength and Stability of Shells

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    - Geometrically and materially nonlinear analysis
    - Geometrically nonlinear elastic analysis with imperfections included
    - Geometrically and materially nonlinear analysis with imperfections

LBA
- linear elastic material law
- linear small deflection theory
- imperfections of all kinds are ignored
- the basis of the critical buckling resistance evaluation

Design of Steel Structures: Strength and Stability of Shells

- Basis of design and modelling
  - Shells shall be designed in acc. with EN1990 and, in particular, to satisfy the following requirements:
    - Overall equilibrium
    - Equilibrium between actions and internal forces and moments
    - Limitation of cracks due to cyclic plastification
    - Limitation of cracks due to fatigue
  - Types of analysis:
    - Global analysis
    - Membrane theory analysis
    - Linear elastic shell analysis
    - Linear elastic bifurcation analysis
    - Geometrically nonlinear elastic analysis
    - Materially nonlinear analysis
    - Geometrically and materially nonlinear analysis
    - Geometrically nonlinear elastic analysis with imperfections included
    - Geometrically and materially nonlinear analysis with imperfections

GNA
-change in the geometry of the structure
- the elastic buckling load of the perfect structure

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MNA
- gives the plastic limit load and the plastic strain increment Δε

Introduction to Design of Shell Structures

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GMNA
- gives the geometrically nonlinear plastic limit load and the plastic strain increment

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    - Geometrically and materially nonline
    - Geometrically nonlinear elastic
    - Geometrically and materially nonline

GNIA
- where compression or shear stresses dominate in the shell
- elastic buckling loads of the "real" imperfect structure

Introduction to Design of Shell Structures

Design of Steel Structures: Strength and Stability of Shells

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GMNIA
- gives the elasto-plastic buckling loads for the "real" imperfect structure

- elastic buckling loads of the "real" imperfect structure

Design of Steel Structures: Strength and Stability of Shells

Type of analysis	Shell theory	Material law	Shell geometry
Membrane theory of shells	membrane equilibrium	not applicable	perfect
Linear elastic shell analysis (LA)	linear bending and stretching	linear	perfect
Linear elastic bifurcation analysis (LBA)	linear bending and stretching	linear	perfect
Geometrically non-linear elastic analysis (GNA)	non-linear	linear	perfect
Materially non-linear analysis (MNA)	linear	non-linear	perfect
Geometrically and materially non-linear analysis (GMNA)	non-linear	non-linear	perfect
Geometrically non-linear elastic analysis with imperfections (GNIA)	non-linear	linear	imperfect
Geometrically and materially non-linear analysis with imperfections (GMNIA)	non-linear	non-linear	imperfect

Introduction to Design of Shell Structures

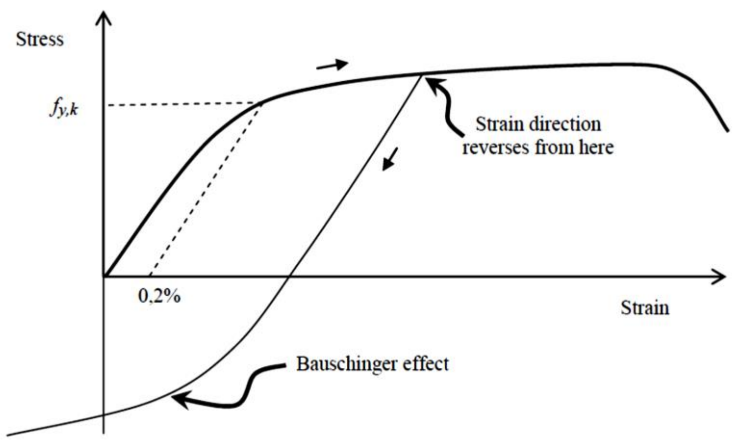
Design of Steel Structures: Strength and Stability of Shells

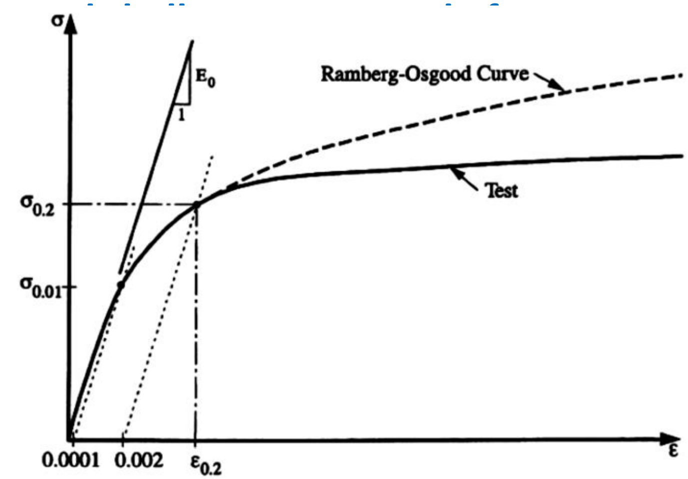
- Materials and geometry
  - The rules in EN 1993-1-6 are not limited to steel shell structures
  - The standard is valid for isotropic shells and shell segments made from any materials that may be represented as ideal elastic-plastic
  - For materials with no well defined yield point, 0.2% proof stress can be taken
  - The material properties apply to temperatures not exceeding 150 ºC (otherwise see EN 13084-7, 2005)
  - Where materials has a significant different stress strain curve, there are alternative ways of representation of the material behaviour
  - Bauschinger effect
  - For austenitic steels (and aluminium alloys) at higher plastic strains, Rasmussen (2003) curve is more appropriate than Ramberg-Osgood

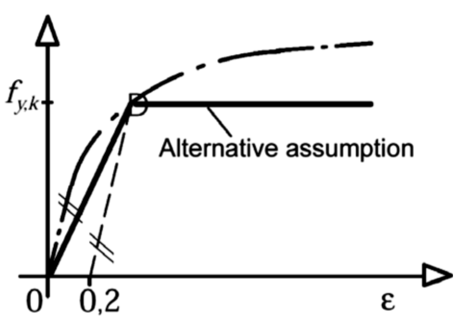
Introduction to Design of Shell Structures

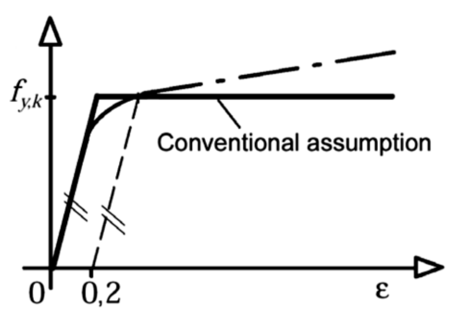
Design of Steel Structures: Strength and Stability of Shells

- **Materials and geometry**
  - The rules in EN 1993-1-6 are not limited to steel shell structures
  - ... erials
  - ... see EN
  - ... tive
  - ... (2003)
  - curve is more appropriate than Ramberg-Osgood







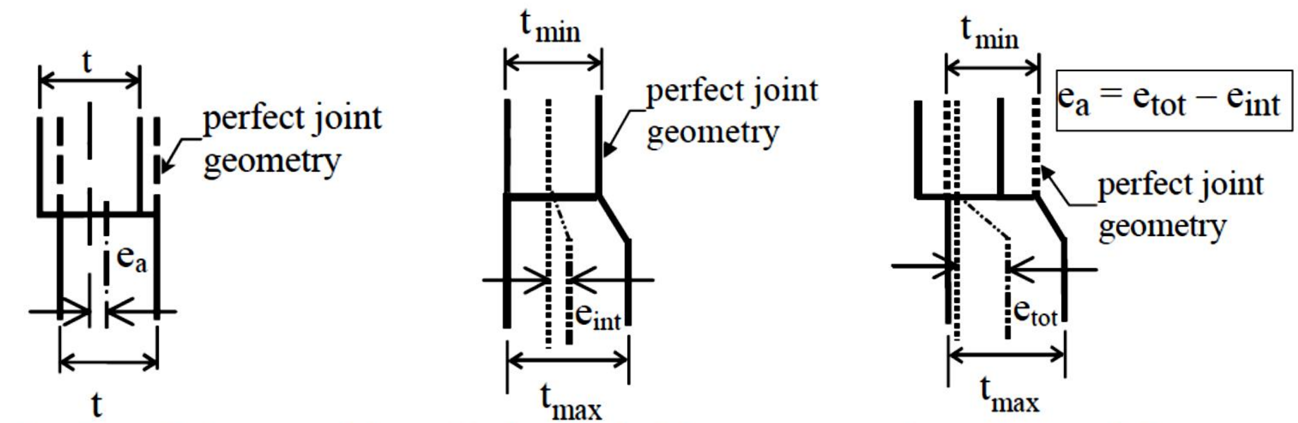


Design of Steel Structures: Strength and Stability of Shells

- Geometrical tolerances and imperfections
  - Relevant tolerances due to the requirements of serviceability:
    - out-of-roundness (deviation from circularity)
    - eccentricities (deviations from a continuous middle surface in the direction normal to the shell along junctions of plates)
    - local dimples (local normal deviations from the nominal middle surface)
  - Other forms of geometric imperfections:
    - deviations from nominal thickness
    - lack of evenness of supports
  - Material imperfections:
    - residual stresses caused by rolling, pressing, welding, straightening etc.
    - inhomogeneities and anisotropies
  - Wear and corrosion
  - Non-uniformities of loading
  - Residual stresses

Intro

ty of Shells



a) unintended eccentricity when there is no change of plate thickness

b) intended offset at a change of plate thickness without unintended eccentricity

c) total eccentricity (unintended plus intended) at change of plate thickness

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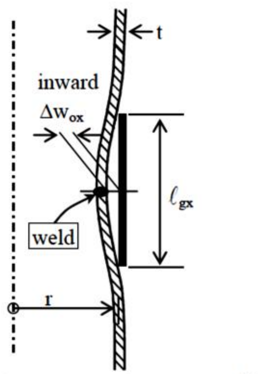
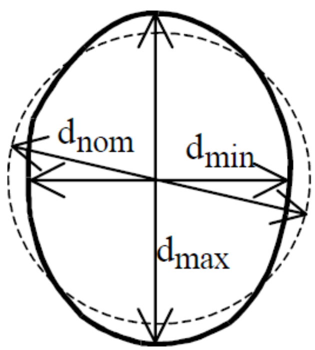
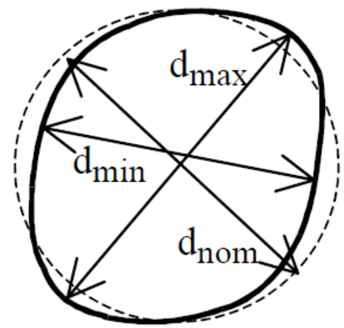
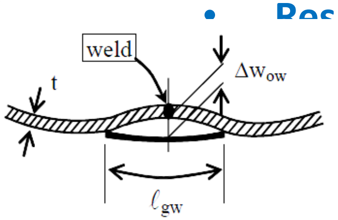


c) First measurement across a weld

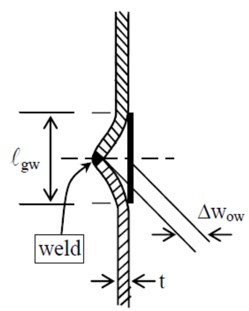


a) flattening

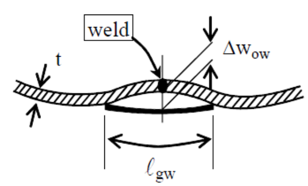




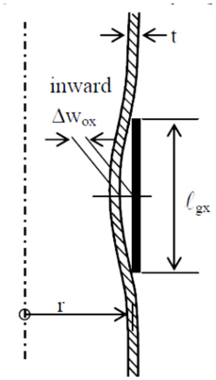
d) Second measurement on circumferential circle



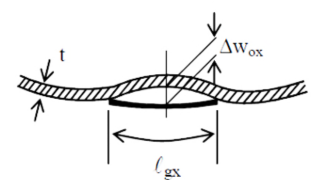
e) Second measurement across a weld with special gauge



f) Measurements on circumferential circle across weld



easurement on a meridian



b) First measurement on a circumferential circl

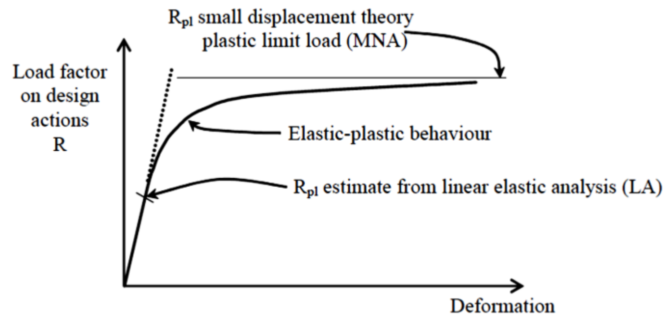
- Ultimate limit states in steel shells
  - LS1: Plastic limit
    - Identifies the strength of the structure when stability plays no significant role.
    - Covers two conditions:
      - tensile rupture or compressive yield through the full thickness
      - development of a plastic collapse mechanism involving bending
    - The plastic limit load is also relevant to a buckling strength assessment

$$\lambda = \sqrt{R_{pl} / R_{cr}}$$

R_{pl} - the plastic limit load
R_{cr} - the elastic critical load

    - The plastic limit load does not represent the real strength (even for stocky structures): strain hardening of material, stabilizing or destabilizing effects due to change in geometry

- **Ultimate limit states in steel shells**
  - **LS1: Plastic limit**
    - **Types of analysis:**
      - **MNA: often underestimates the strength very considerably**



      - **Membrane theory calculations:**
        - **If the stress state is entirely axisymmetric, it gives a close approximation to the true condition at plastic collapse**

$$n_{eq} = \sqrt{n_1^2 - n_1 n_2 + n_2^2} = \sqrt{n_x^2 - n_x n_\theta + n_\theta^2 + 3 n_{x\theta}^2}$$

        - **if the stresses are significantly unsymmetrical, this criterion often provides a very conservative estimate of the plastic limit load**

- Ultimate limit states in steel shells
  - LS1: Plastic limit
    - Types of analysis:
      - Linear elastic shell bending theory: This is commonly more conservative than membrane theory calculation (is based on the first yield on the surface)
      - Geometrically nonlinear calculation (GMNA): problems arise over whether the structure displays geometric hardening or geometric softening
    - The plastic limit load should be seen only as the ideal value of the plastic reference resistance

- Ultimate limit states in steel shells
  - LS2: Cyclic plasticity:
    - Repeated cycles of loading and unloading, eventually leading to local cracking by exhaustion of the energy absorption capacity of the material
    - Low cycle fatigue failure may be assumed to be prevented if the procedures set out in the standard are adopted
    - Methods of analysis:
      - expressions in Annex C
      - elastic analysis (LA or GNA)
      - MNA or GMNA and find plastic strains
  - LS4: Fatigue:
    - Repeated cycles of increasing and decreasing stress lead to the development of a fatigue crack
    - Methods of analysis:
      - expressions in Annex C (using stress concentration factors)
      - elastic analysis (LA or GNA), using stress concentration factors

Design of Steel Structures: Strength and Stability of Shells

- Ultimate limit states in steel shells
  - LS3: Buckling:
    - Caused by loss of stability under compressive membrane or shear membrane stresses in the shell wall, leading to inability to sustain any increase in the stress resultants, possibly causing catastrophic failure
    - Three approaches used in the assessment of buckling resistance:
      - GMNIA analysis
      - MNA/LBA analysis
      - Buckling stresses
    - The strength under LS3 depends strongly on the quality of construction
    - For this purpose, three fabrication quality classes are set out

Design of Steel Structures: Strength and Stability of Shells

- **Design concepts for the limit states design of shells**
  - **The limit state verification should be carried out using one of the following:**
    - **Stress design:**
      - primary
      - secondary
      - Local

In general, **primary** stress states control LS1, whereas **secondary** stress states affect LS2 and LS3 and **local** stresses govern LS4.

  - **Direct design by application of standard expressions:**
    - the limit states may be represented by standard expressions that have been derived from either membrane theory, plastic mechanism theory or linear elastic analysis
    - The membrane theory (Annex A) - primary stresses needed for assessing LS1 and LS3.
    - The plastic design (Annex B) - plastic limit loads for assessing LS1
    - The linear elastic analysis (Annex C) - stresses of the primary plus secondary stress type for assessing LS2 and LS4. An LS3 assessment may be based on the membrane part of these expressions.
  - **Design by global numerical analysis**

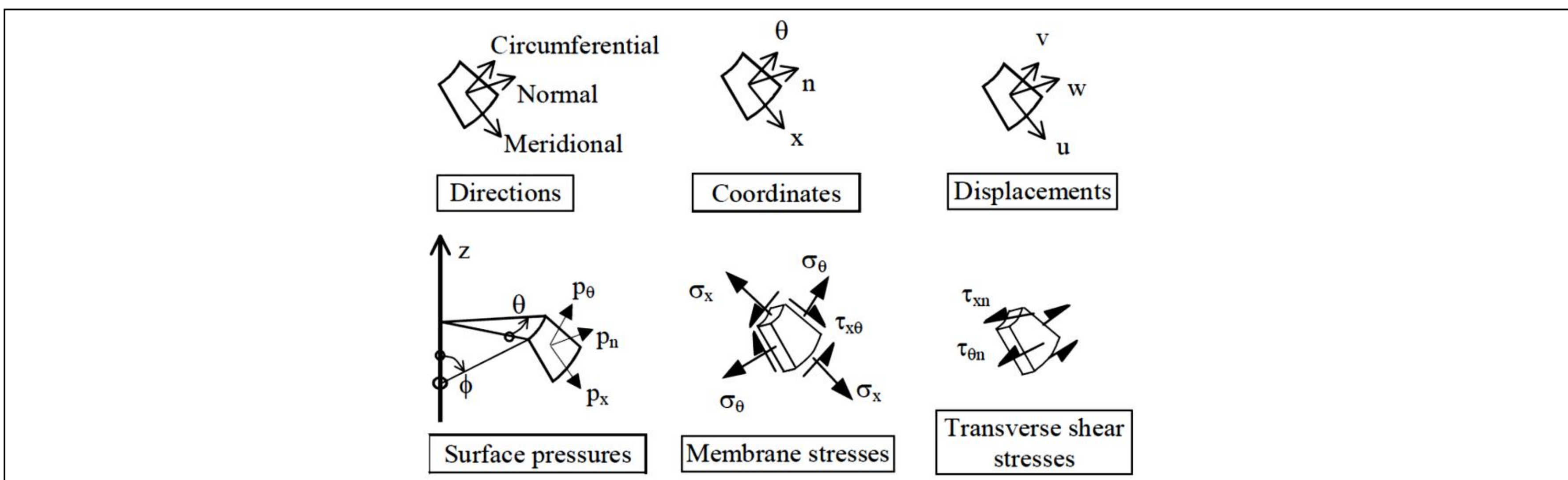
Introduction to Design of Shell Structures

Design of Steel Structures: Strength and Stability of Shells

- Stress resultants and stresses in shells
  - Stresses:
    - There are eight stress resultants in the shell
    - However, the shear stresses τ_{xn}, $\tau_{\theta n}$ due to the transverse shear forces q_{xn}, $q_{\theta n}$ are insignificant and they may usually be neglected in design
    - For most design purposes, the evaluation of the limit states may be made using only the six stress resultants in the shell wall n_x, n_θ, $n_{x\theta}$, m_x, m_θ, $m_{x\theta}$
    - Where the structure is axisymmetric and subject only to axisymmetric loading and support, only n_x, n_θ, m_x and m_θ need be used
  - Modelling:
    - Represention by its middle surface
    - Nominal radius of curvature, imperfections neglected (excepting LS3)
    - Eccentricities and steps if they induce significant effects
    - Eccentricity at junctions between shell segments
    - Stringers, corrugations, holes, depending on the conditions
  - Boundary conditions

Boundary condition code	Simple term	Description	Normal displacements	Vertical displacements	Meridional rotation
BC1r	Clamped	radially restrained meridionally restrained rotation restrained	w = 0	u = 0	$\beta\phi = 0$
BC1f		radially restrained meridionally restrained rotation free	w = 0	u = 0	$\beta\phi \neq 0$
BC2r		radially restrained meridionally free rotation restrained	w = 0	$u \neq 0$	$\beta\phi = 0$
BC2f	Pinned	radially restrained meridionally free rotation free	w = 0	$u \neq 0$	$\beta\phi \neq 0$
BC3	Free edge	radially free meridionally free rotation free	$w \neq 0$	$u \neq 0$	$\beta\phi \neq 0$

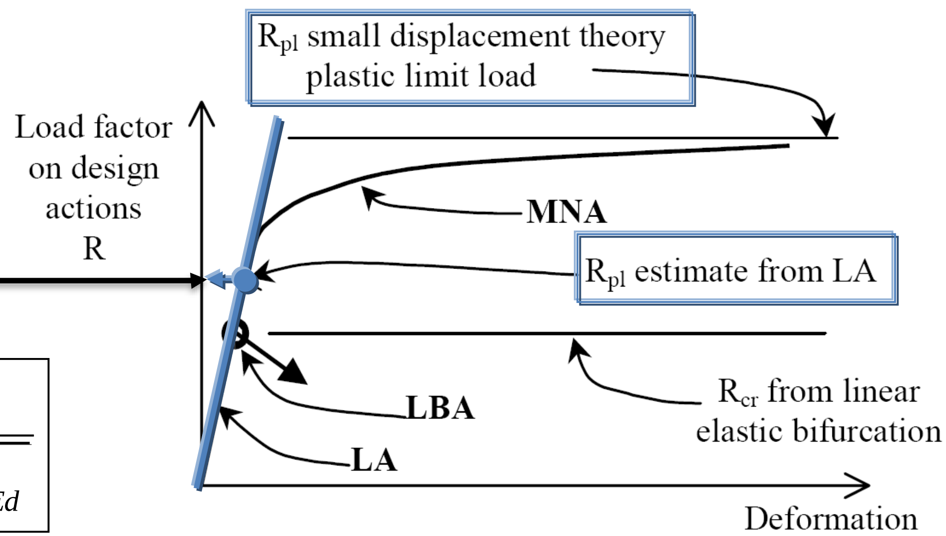
NOTE: The circumferential displacement v is closely linked to the displacement w normal to the surface so separate boundary conditions are not identified in paragraph (3) for these two parameters.



Design of Steel Structures: Strength and Stability of Shells

- **Plastic limit state (LS1)**
  - The plastic reference resistance R_{pl}
  - Where it is not possible to undertake a materially non-linear analysis (MNA), the plastic reference resistance R_{pl} may be conservatively estimated from linear shell analysis (LA) conducted using the design values of the applied combination of actions using the following procedure.
  - The three points where obtain maximum values for stresses

$$R_{pl} = \frac{t\, f_{y,k}}{\sqrt{n_{x,Ed}^2 - n_{x,Ed}\, n_{\theta,Ed} + n_{\theta,Ed}^2 + 3n_{x\theta,Ed}^2}}$$

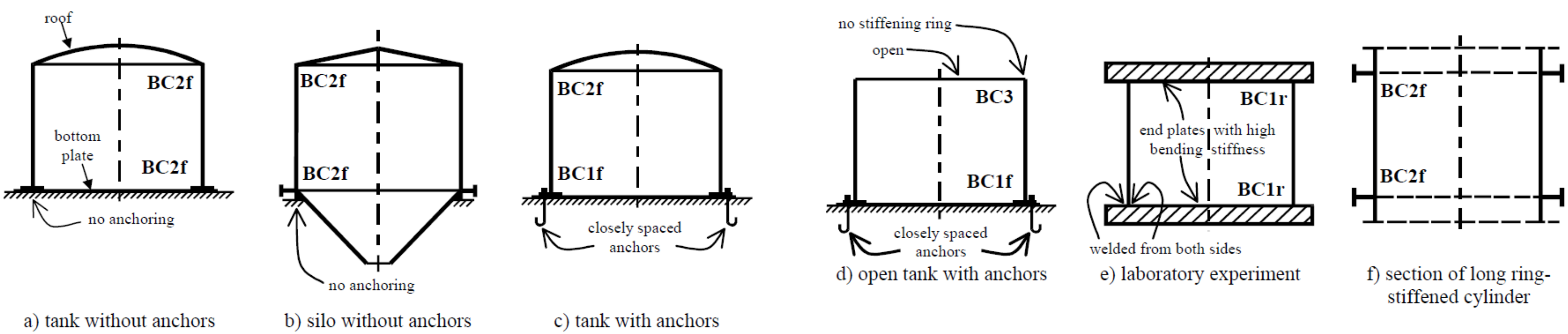


Introduction to Design of Shell Structures

Design of Steel Structures: Strength and Stability of Shells

- **Buckling limit state (LS3)**
  - **To find out the design buckling resistance**
  - **Defined as a load factor R applied to the design values of the combination of actions for the relevant load case**
  - **Different approaches have been proposed, difficult to generalise**
  - **In EN 1993-1-6, a considerable effort to produce general procedures applicable to all geometries, all loading conditions and all material conditions**

  - **Buckling-relevant boundary conditions**

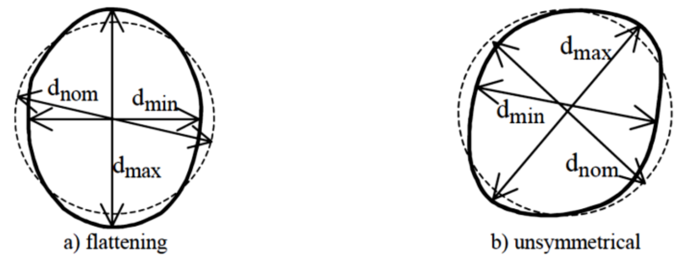


Introduction to Design of Shell Structures

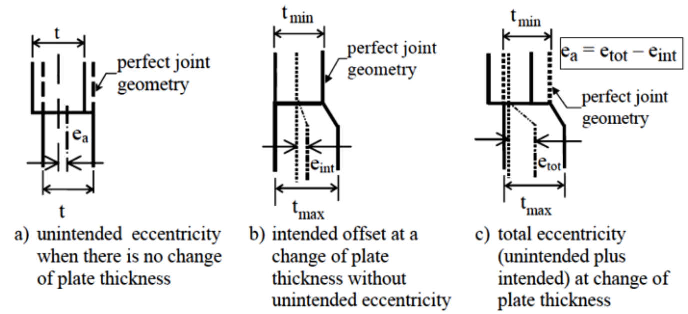
Design of Steel Structures: Strength and Stability of Shells

- **Buckling-relevant geometrical tolerances**

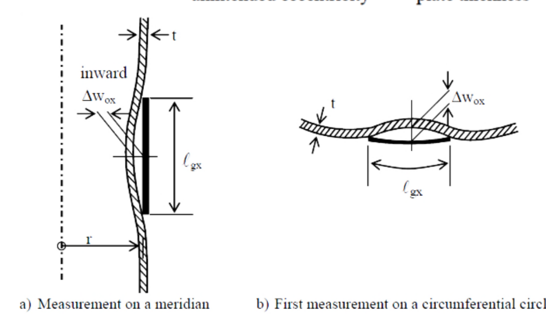
	Diameter range	$d \leq 0{,}50$m	$0{,}50\text{m} < d < 1{,}25$m	$1{,}25\text{m} \leq d$
Fabrication tolerance quality class	Description	Value of $U_{r,max}$		
Class A	Excellent	0,014	$0{,}007 + 0{,}0093(1{,}25-d)$	0,007
Class B	High	0,020	$0{,}010 + 0{,}0133(1{,}25-d)$	0,010
Class C	Normal	0,030	$0{,}015 + 0{,}0200(1{,}25-d)$	0,015



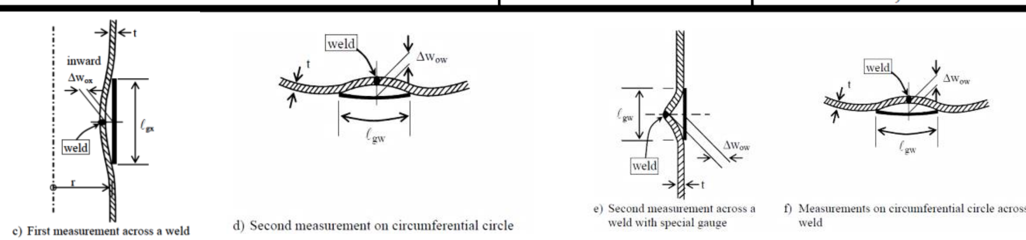
Fabrication tolerance quality class	Description	Maximum permitted accidental eccentricity
Class A	Excellent	$e_a \leq 2$ mm
Class B	High	$e_a \leq 3$ mm
Class C	Normal	$e_a \leq 4$ mm



Fabrication tolerance quality class	Description	Value of $U_{e,max}$
Class A	Excellent	0,14
Class B	High	0,20
Class C	Normal	0,30



Fabrication tolerance quality class	Description	Value of $U_{0,max}$
Class A	Excellent	0,006
Class B	High	0,010
Class C	Normal	0,016



- Stress design
  - Design values of stresses $\sigma_{x,Ed}$, $\sigma_{\theta,Ed}$ and $\tau_{x\theta,Ed}$: taken as the key values of compressive and shear membrane stresses obtained from linear shell analysis (LA).
  - Design resistance (buckling strength):

$$\sigma_{x,Rk} = \chi_x f_{y,k}, \quad \sigma_{\theta,Rk} = \chi_\theta f_{y,k}, \quad \tau_{x\theta,Rk} = \chi_t f_{y,k} / \sqrt{3}$$

$$\chi = 1 \quad \text{when} \quad \bar{\lambda} \leq \bar{\lambda}_0$$

$$\chi = 1 - \beta \left(\frac{\bar{\lambda} - \bar{\lambda}_0}{\bar{\lambda}_p - \bar{\lambda}_0} \right)^\eta \quad \text{when} \quad \bar{\lambda}_0 < \bar{\lambda} < \bar{\lambda}_p$$

$$\chi = \frac{\alpha}{\bar{\lambda}^2} \quad \text{when} \quad \bar{\lambda}_p \leq \bar{\lambda}$$

$$\bar{\lambda}_p = \sqrt{\frac{\alpha}{1 - \beta}}$$

λ_0 - squash limit relative slenderness

$$\bar{\lambda}_x = \sqrt{f_{y,k} / \sigma_{x,Rcr}}$$

$$\bar{\lambda}_\theta = \sqrt{f_{y,k} / \sigma_{\theta,Rcr}}$$

$$\bar{\lambda}_\tau = \sqrt{\left(f_{y,k} / \sqrt{3}\right) / \tau_{x\theta,Rcr}}$$

$$\sigma_{x,Rd} = \sigma_{x,Rk}/\gamma_{M1}, \quad \sigma_{\theta,Rd} = \sigma_{\theta,Rk}/\gamma_{M1}, \quad \tau_{x\theta,Rd} = \tau_{x\theta,Rk}/\gamma_{M1}$$

γ_{M1} = min 1,1

Design of Steel Structures: Strength and Stability of Shells

- Stress limitation (buckling strength verification)
  - The influence of bending stresses may be neglected provided they arise as a result of boundary compatibility effects.
  - In the case of bending stresses from local loads or from thermal gradients, special consideration should be given.
  - Following checks for the key values of single membrane stress components should be carried out:

$$\sigma_{x,Ed} \le \sigma_{x,Rd},\ \sigma_{\theta,Ed} \le \sigma_{\theta,Rd},\ \tau_{x\theta,Ed} \le \tau_{x\theta,Rd}$$

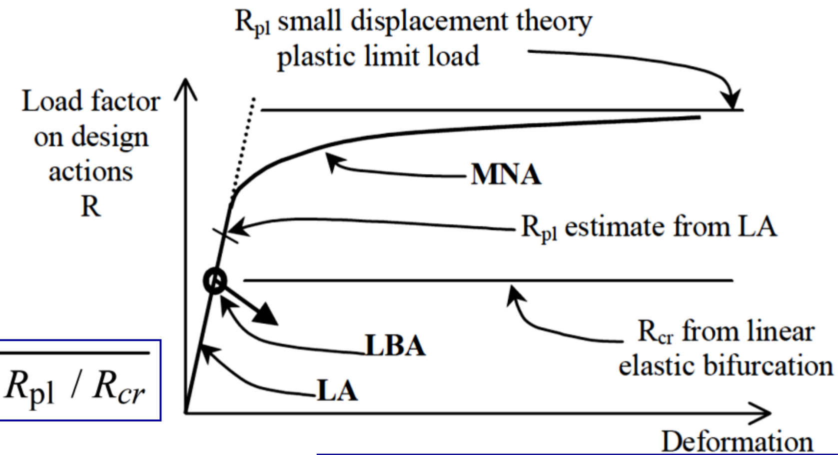
  - For more than one buckling-relevant membrane stress components, interaction check for the combined membrane stress state should be carried out:

$$\left(\frac{\sigma_{x,\mathrm{Ed}}}{\sigma_{x,\mathrm{Rd}}}\right)^{k_x} + \left(\frac{\sigma_{\theta,\mathrm{Ed}}}{\sigma_{\theta,\mathrm{Rd}}}\right)^{k_\theta} - k_i\left(\frac{\sigma_{x,\mathrm{Ed}}}{\sigma_{x,\mathrm{Rd}}}\right)\left(\frac{\sigma_{\theta,\mathrm{Ed}}}{\sigma_{\theta,\mathrm{Rd}}}\right) + \left(\frac{\tau_{x,\theta,\mathrm{Ed}}}{\tau_{x,\theta,\mathrm{Rd}}}\right)^{k_\tau} \le 1$$

Design of Steel Structures: Strength and Stability of Shells

- Buckling design
  - EN 1993-1-6 specifies three approaches that are approved for use in the assessment of buckling resistance:
    - Design by means of a global numerical MNA/LBA analysis
    - Design by means of a global numerical GMNIA analysis
    - Design by means of buckling stresses

- **Buckling design by global numerical MNA/LBA analysis**
  - **It is recommended for many applications**
  - **It has the same basis as the traditional stress design buckling approach**
  - **All relevant combinations of actions causing compressive membrane stresses or shear membrane stresses in the shell wall shall be taken into account**
  - **It involves the following steps, see left hand side figure**



$$\bar{\lambda}_{ov} = \sqrt{R_{pl} / R_{cr}}$$

$$\chi_{ov} = f\left(\bar{\lambda}_{ov}, \bar{\lambda}_{ov,0}, \alpha_{ov}, \beta_{ov}, \gamma_{ov}\right)$$

$$R_k = \chi_{ov} R_{pl}$$

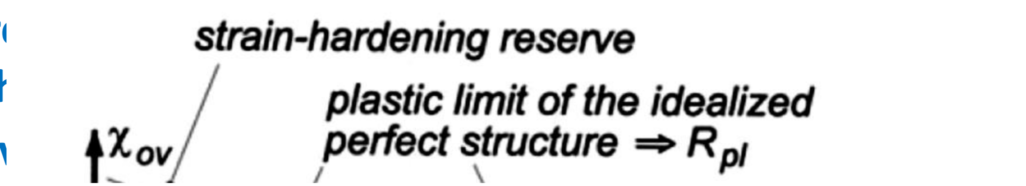
$$R_d = R_k / \gamma_{M1}$$

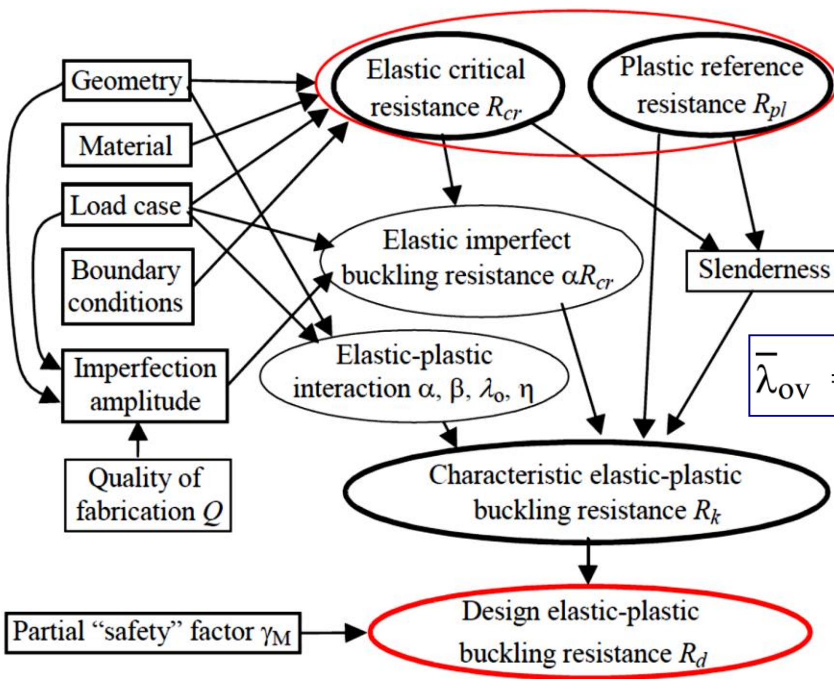
α_{ov} is the overall elastic imperfection factor,
β_{ov} is the plastic range factor,
η_{ov} is the interaction exponent and
$\lambda_{ov,0}$ is the squash limit relative slenderness

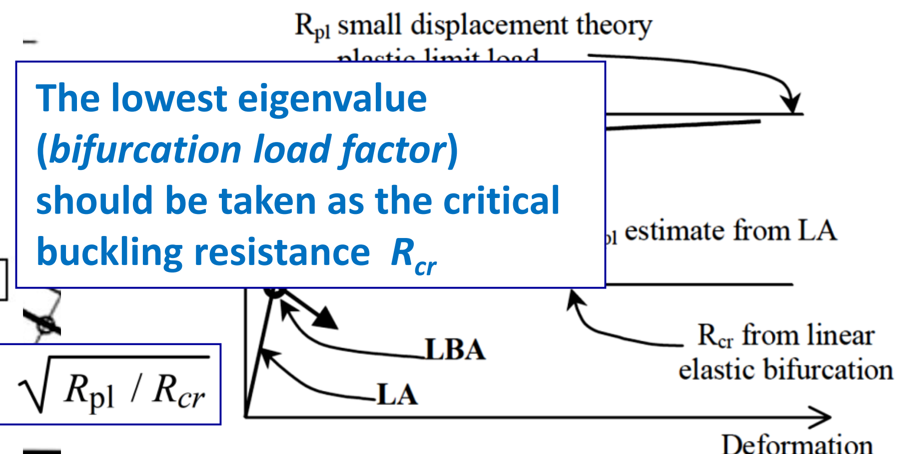
Introduction to Design of Shell Structures

Design of Steel Structures: Strength and Stability of Shells

- **Buckling design by global numerical MNA/LBA analysis**
  - It is recommended for many applications
  - It has the same basis as the traditional stress design buckling approach
  - All r… membrane stresses or sh… en into account
  - It inv…







The lowest eigenvalue (*bifurcation load factor*) should be taken as the critical buckling resistance R_{cr}

$$\bar{\lambda}_{ov} = \sqrt{R_{pl} / R_{cr}}$$

$$\bar{\lambda}_{ov} = \sqrt{R_{pl} / R_{cr}}$$

$$R_k = \chi_{ov} R_{pl}$$

$$R_d = R_k / \gamma_{M1}$$

$$\chi_{ov} = f\left(\bar{\lambda}_{ov}, \bar{\lambda}_{ov,0}, \alpha_{ov}, \beta_{ov}, \gamma_{ov}\right)$$

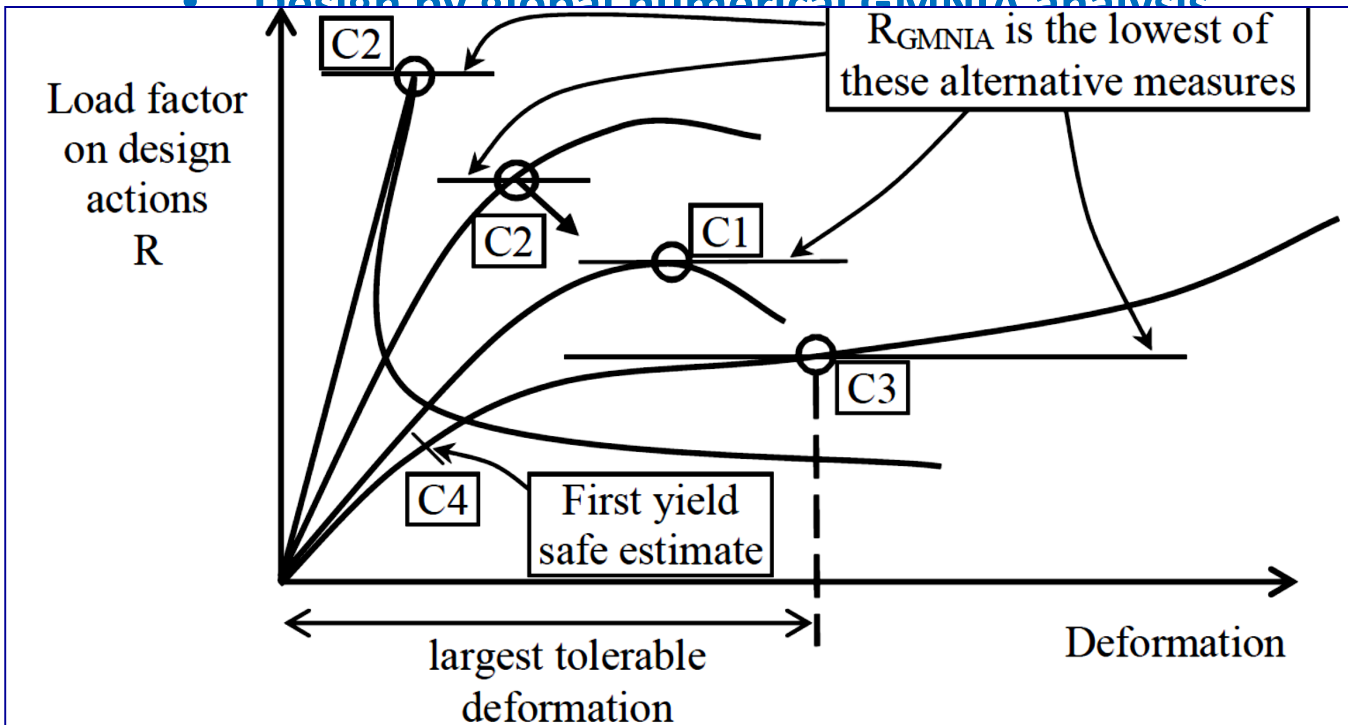
α_{ov} is the overall elastic imperfection factor,
β_{ov} is the plastic range factor,
η_{ov} is the interaction exponent and
$\lambda_{ov,0}$ is the squash limit relative slenderness

Design of Steel Structures: Strength and Stability of Shells

- Design by global numerical GMNIA analysis
  - Developed to exploit the full power of modern numerical analysis
  - Application is more complex than for frame or plated structures
  - Several sequence of analysis:
    - LA followed by a LBA to evaluate elastic critical buckling resistance
    - GMNA to identify the elastic-plastic buckling resistance of the perfect structure
    - GMNIA with different imperfection modes (the lowest value is selected)
    - Check the precision of the GMNIA by comparison with test or other relevant data
  - Methodology
    - Action combinations causing compressive membrane stresses or shear membrane stresses
    - R_k should be found from the imperfect elastic-plastic critical buckling resistance R_{GMNIA}, adjusted by the calibration factor k_{GMNIA}.
    - The design buckling resistance R_d should then be found using the partial factor γ_{M1}.

Design of Steel Structures: Strength and Stability of Shells

- Design by global numerical GMNIA analysis



C1: The maximum load factor on the load-deformation-curve (limit load);
C2: The bifurcation load factor, where this occurs during the loading path before reaching the limit point of the load-deformation-curve
C3: The largest tolerable deformation, where this occurs during the loading path before reaching the bifurcation load or the limit load

C4: The load factor at which the equivalent stress at the most highly stressed point on the shell surface reaches the design value of the yield stress

membrane stresses

- R_k should be found from the imperfect elastic-plastic critical buckling resistance R_{GMNIA}, adjusted by the calibration factor k_{GMNIA}.
- The design bu...kli... factor γ_{M1}.

$$R_k = k_{GMNIA} \; R_{GMNIA}$$

$$R_d = R_k/\gamma_{M1}$$

...ssessment of R_{GMNIA} may be obtained using a GNIA ... the lowest lo...

$$F_d \le R_d \quad F_d \qquad \text{or} \qquad R_d \ge 1$$

Design of Steel Structures: Strength and Stability of Shells

- GMNIA analysis
  - Allowances for imperfections:
    - geometric imperfections: pre-deformations, out of-roundness, irregularities at and near welds, thickness deviation, etc.
    - material imperfections: residual stresses, inhomogeneities, anis…
  - EN 1993-1-6 requires that imperfections are explicit… …lly, not just treated as small perturbations i…
  - They are introduced by mea… …e imperfections in the form of initial sh… …ular to the middle surface of the perfe…
  - …perfections with the most unfavorable effect should be …red (the most unfavorable effect on the buckling resistance RGMNIA of the shell); if practicable, they must reflect the constructional detailing and the boundary conditions

The designer should not forget equivalent geometric imperfections must also cover the effects of all other types of imperfections (see above)!

Design of Steel Structures: Strength and Stability of Shells

- GMNIA analysis
  - The analysis should be carried out for a sufficient number of different imperfection patterns, and the worst case (lowest value of R_{GMNIA}) should be identified.
  - The eigen-mode-affine pattern should be used (the critical buckling mode associated with the elastic critical buckling resistance R_{cr} based on an LBA analysis of the perfect shell)
  - The amplitude of the imperfection form - dependent on the fabrication tolerance quality class

Fabrication tolerance quality class	Description	Recommended value of Un1	Recommended value of Un2
Class A	Excellent	0,010	0,010
Class B	High	0,016	0,016
Class C	Normal	0,025	0,025

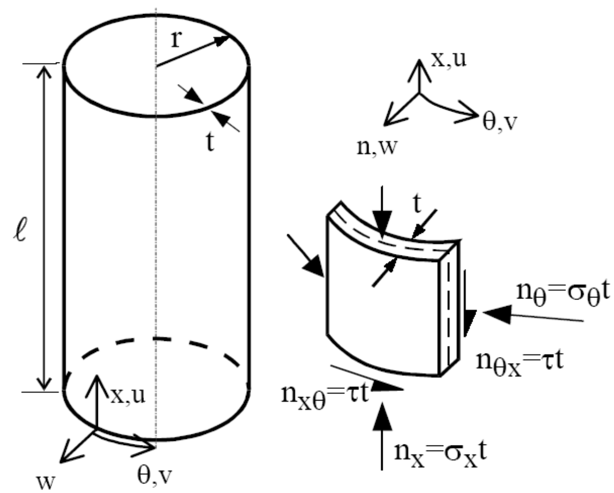
- **Imperfections**
  - **The maximum deviation of the geometry of the equivalent imperfection from the perfect shape $\Delta_{w0,eff} = \max(\Delta_{w0,eff,1}; \Delta_{w0,eff,2})$, where:**

$$\Delta w_{0,eff,1} = l_g U_{n1} \qquad l_{gx} = 4\sqrt{rt}$$

$$\Delta w_{0,eff,2} = n_i t U_{n2} \qquad l_{g,\theta} = 2,3\left(l^2 rt\right)^{0.25} \le r$$

l_g relevant gauge lengths
t local shell wall thickness
n_i multiplier to achieve an appropriate tolerance level
U_{n1}, U_{n2} dimple imperfection

- $n_i = 25$ — is a multiplier to achieve an appropriate tolerance level
- t — is the local shell wall thickness
- l_g — is all relevant gauge lengths (see Dimple tolerances)



Design of Steel Structures: Strength and Stability of Shells

- **GMNIA validation**
  - For each calculated value of the buckling resistance RGMNIA, the ratio of the imperfect to perfect resistance (R_{GMNIA} / R_{GMNA}) should be determined and compared with values of α found using the procedures of 8.5 and Annex D.
  - The reliability of the numerically determined critical buckling resistance R_{GMNIA} should be checked by one of the following methods:
    - by using the same program to calculate values R_{GMNIA}, check for other shell buckling cases for which characteristic buckling resistance values $R_{k,known,check}$ are known;
    - by comparison of calculated values ($R_{GMNIA,check}$) against test results ($R_{test,known,check}$).

$$k_{GMNIA} = \frac{R_{k,know,check}}{R_{GMNIA,check}} \qquad k_{GMNIA} = \frac{R_{test,know,check}}{R_{GMNIA,check}}$$

Design of Steel Structures: Strength and Stability of Shells

- GMNIA validation
  - Where a known characteristic value based on existing established theory is used to determine k_{GMNIA}, and the calculated value of k_{GMNIA} lies outside the range $0,8 < k_{GMNIA} < 1,2$, this procedure should not be used.
  - The characteristic buckling resistance should be obtained from:

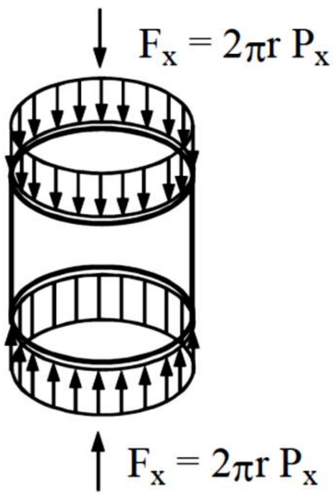
$$R_k = k_{GMNIA} R_{GMNIA}$$

  - R_{GMNIA} is the calculated imperfect elastic-plastic critical buckling resistance;
  - k_{GMNIA} is the calibration factor.

Design of Steel Structures: Strength and Stability of Shells

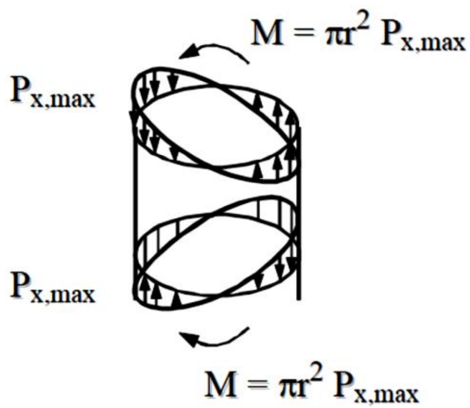
- **ANNEX A - Membrane theory stresses in shells**

Uniform axial load



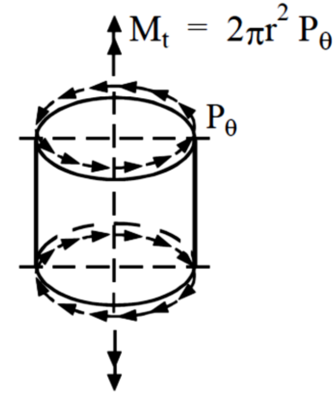
$$\sigma_x = -\frac{F_x}{2\pi r t}$$

Axial load from bending



$$\sigma_x = \pm\frac{M}{\pi r^2 t}$$

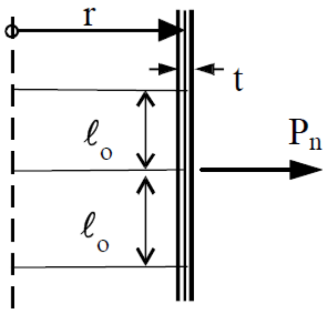
Uniform shear from torsion



$$\tau = \frac{M_t}{2\pi r^2 t}$$

- **ANNEX B - Additional expressions for plastic collapse resistances**

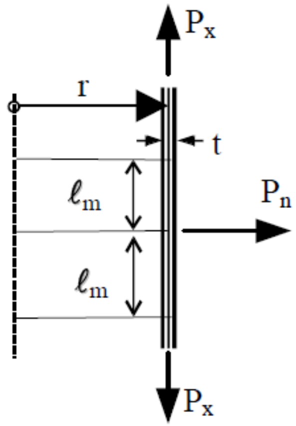
Cylinder:
Radial line load



$$\ell_o = 0{,}975\sqrt{rt}$$

$$\frac{P_{nR}}{2\ell_o} = f_y \frac{t}{r}$$

Cylinder:
Radial line load and axial load



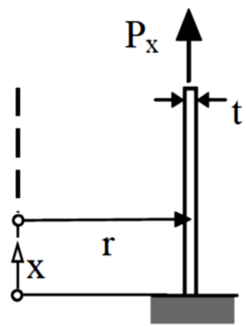
$$\ell_m = s_m \ell_o \quad \begin{cases} s_m = A + \sqrt{A^2 + 4(1 - s_x^2)} \\ A = +/- s_x - 1{,}50 \\ s_x = \dfrac{P_x}{f_y t} \\ \ell_o = 0{,}975\sqrt{rt} \end{cases}$$

$$\frac{P_{nR}}{2\ell_m} = f_y \frac{t}{r}$$

P_{nR} - plastic resistance (force per unit circumference)

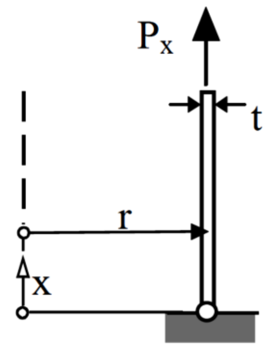
- **ANNEX C - Expressions for linear elastic membrane and bending stresses**

Cylinder, clamped: axial loading



$$\sigma_{MTx} = \frac{P_x}{t}$$

Cylinder, pinned: axial loading

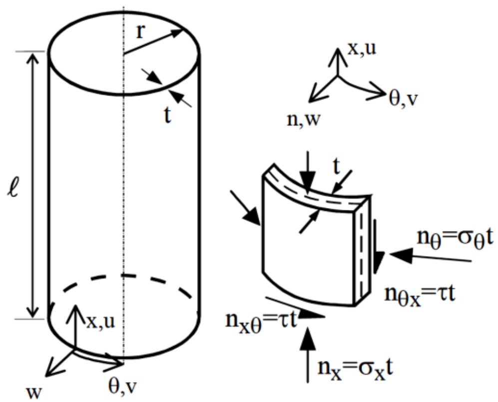


$$\sigma_{MTx} = \frac{P_x}{t}$$

Introduction to Design of Shell Structures

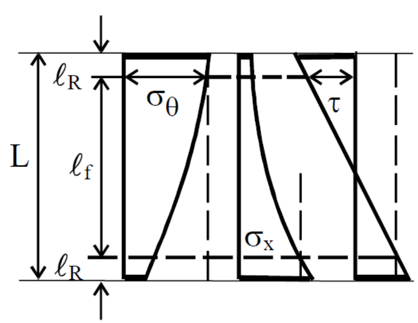
Design of Steel Structures: Strength and Stability of Shells

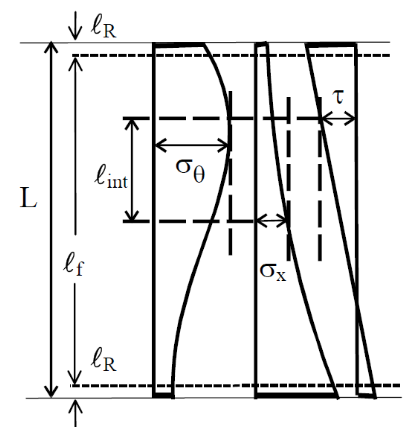
- ANNEX D - Expressions for buckling stress design
  - Unstiffened cylindrical shells of constant wall thickness
    - Meridional (axial) compression
      - Critical meridional buckling stresses
      - Meridional buckling parameters
    - Circumferential (hoop) compression
      - Critical circumferential buckling stresses
      - Circumferential buckling parameters
    - Shear
      - Critical shear buckling stresses
      - Shear buckling parameters



Cylinder geometry, membrane stress resistances and stress resultant resistances

- Combinations of meridional (axial) compression, circumferential (hoop) compression and shear





Introduction to Design of Shell Structures

Finite Element Application

- Principles, simplified and advanced models, concentration of stresses, stiffening

Hand calculation

Formulas for:

Bifurcation load
Plastic limit
Imperfection sensitivity
Elastic plastic interaction
Interaction between different stress components

Simple computer calculations

Computational evaluations:

Bifurcation load
Plastic limit
Interaction between different stress components

→ Formulas for:

Imperfection sensitivity
Elastic plastic interaction

→ LA
LBA
GNA
MNA
GMNA

Complete computer calculations

Computational evaluations:

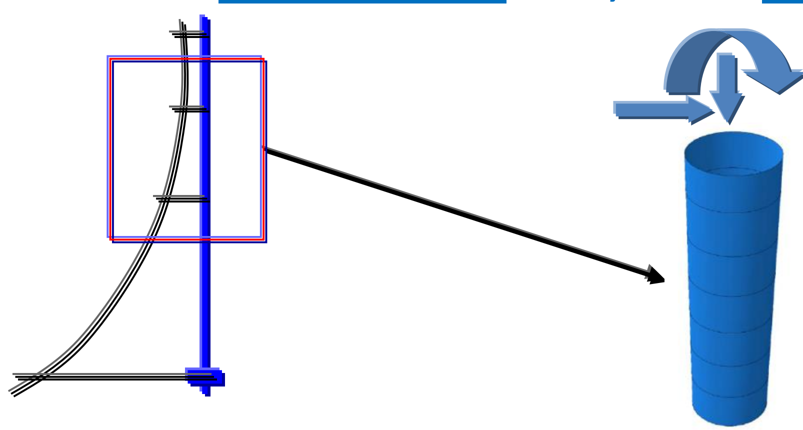
Bifurcation load
Plastic limit
Perfect structure strength
Imperfect structure strength

→ GNIA
GMNIA

Introduction to Design of Shell Structures

Finite Element Application

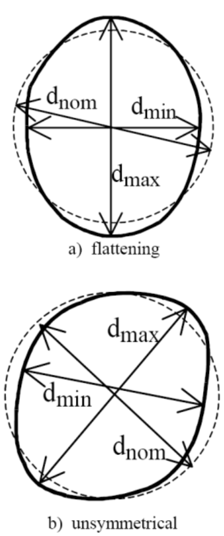
- Simplified FEM model
  - Global analysis
    - A simplified bar model using 1D beam finite elements, with or without the account of global imperfection, using an elastic material law.
    - Aim of analysis – find-out the internal forces needed for:
      - The global check of the tower capacity;
      - Refined local analysis of the relevant tower segments (between two consecutive flange that assure the appropriate boundary condition, radial restrained)
  - Local analysis of a tower segment
    - The 3D shell model should be build using 2D finite elements (plane), that can be: either <u>Homogeneous</u> shell, either <u>Membranes</u>

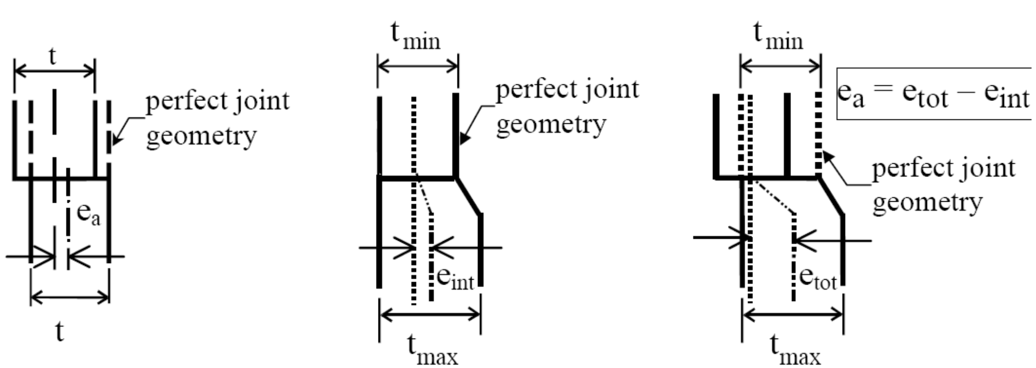


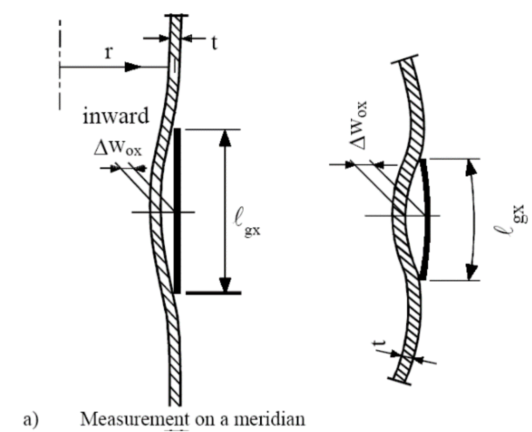
Introduction to Design of Shell Structures

Finite Element Application

- **Simplified FEM model**
  - **Local analysis of a tower segment**
    - **The 3D shell model should be build using 2D finite elements (plane), that can be: either Homogeneous shell, either Membranes**
    - **The geometry can be perfect / ideal or taking into account of the prescribed relevant geometrical imperfection:**
      - **out-of-roundness (deviation from circularity),**
      - **eccentricities (deviations from a continuous middle surface in the direction normal to the shell along junctions of plates),**
      - **local dimples (local normal deviations from the nominal middle surface).**







Introduction to Design of Shell Structures

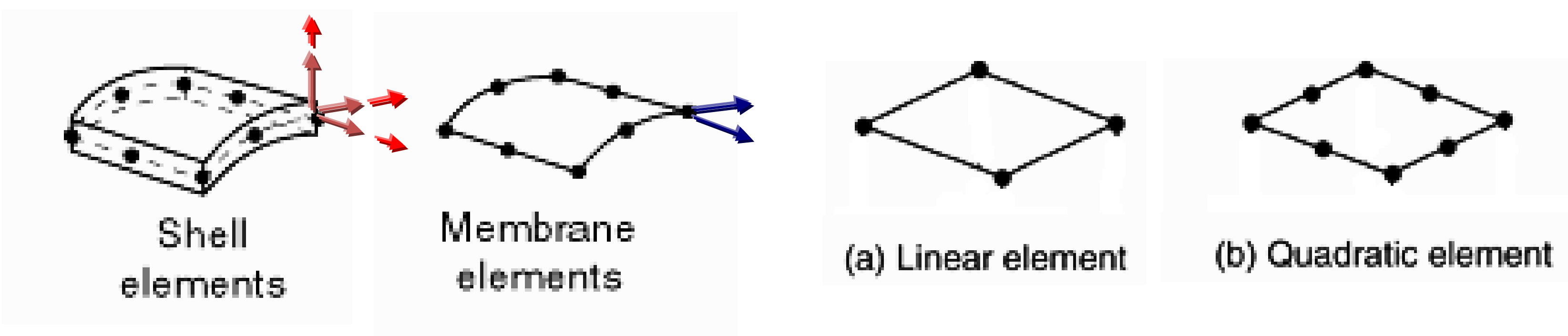
Finite Element Application

- Advanced FEM Model
  - Modelling the entire structure with 2D finite elements with a special attention of details (connection flanges, welds connection, etc);
  - Material behavior:
    - Elastic
    - Plastic
  - Geometry:
    - Ideal
    - Imperfect
  - Finite elements types:
    - Homogenous shell

Introduction to Design of Shell Structures

Finite Element Application

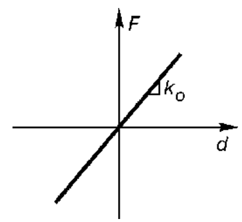
- Geometry and state of stress
  - Homogeneous shell with bending stiffness, sections consist of a shell thickness, material name, section Poisson's ratio.
  - Membranes represent thin surfaces in space that offer strength in the plane of the surface but have no bending stiffness. Membrane sections consist of a material name, membrane thickness, section Poisson's ratio.

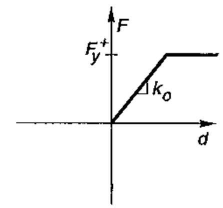


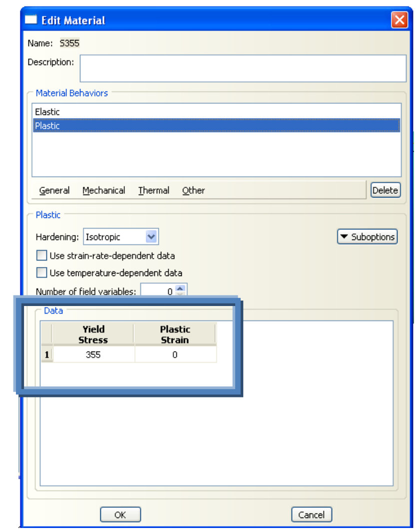
Introduction to Design of Shell Structures

Finite Element Application

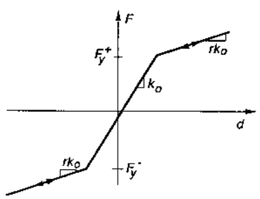
- Material behavior law
  - Elastic
  - Elastic – Plastic

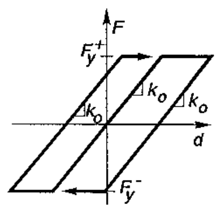


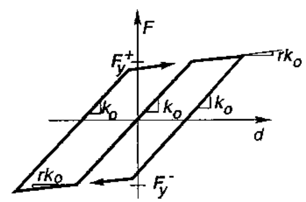




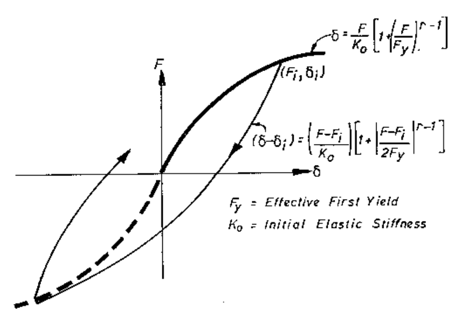
  - Linear
  - Multi-linear







  - Continuous
    - Powell,
    - Ramberg-Osgood



- Load cases
  - The load cases shall be determined from the combination of operational modes or other design situations, such as specific assembly, erection or maintenance conditions, with the external conditions.
  - All relevant load cases with a reasonable probability of occurrence shall be considered, together with the behavior of the control and protection system.
  - The design load cases used (IEC 61400-1:2005) to verify the structural integrity of a wind turbine shall be calculated by combining:
    - normal design situations and appropriate normal or extreme external conditions;
    - fault design situations and appropriate external conditions;
    - transportation, installation and maintenance design situations and appropriate external conditions.

Introduction to Design of Shell Structures

Finite Element Application

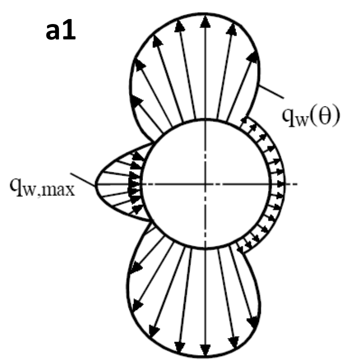
- Load cases and design situations
  - Load cases and assumptions for global analysis
    - IEC 61400-1:2005
      - Dead loads - Self-weight: tower head; tower body, installation etc.
      - Wind action (EN1991-1-4)
      - Seismic loads
      - Temperature
      - Ice
  - Design situations
    - Power production
    - Power production plus occurrence of fault or loss of electrical network connection
    - Start up
    - Normal shut down
    - Emergency shut down
    - Parked (standstill or idling)
    - Parked plus fault conditions
    - Transport, assembly, maintenance and repair

Introduction to Design of Shell Structures

Finite Element Application

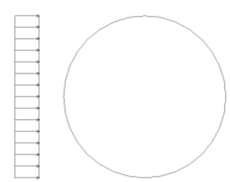
- Wind action using different load situation:
  - Global analysis to find-out the internal forces
    - Load of wind on tower (hub)
      - Simplified distribution (a2)
      - Surface distribution (a1)
    - Load from turbine’s machinery
      - Concentrated force and moment
  - Equivalent load for buckling verification
    - Axisymmetric pressure distribution

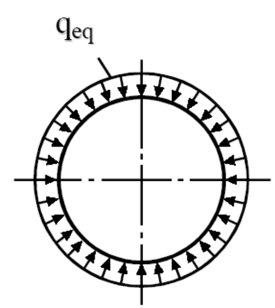
a1



a) wind pressure distribution around shell circumference

a2



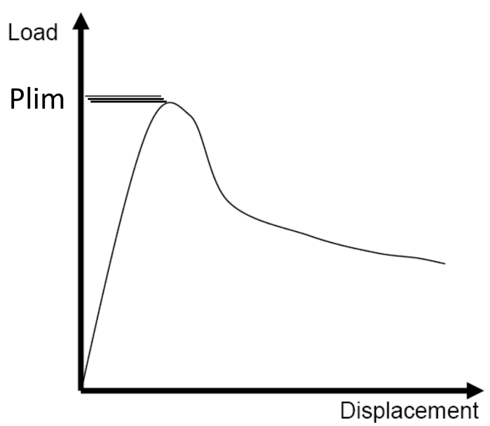


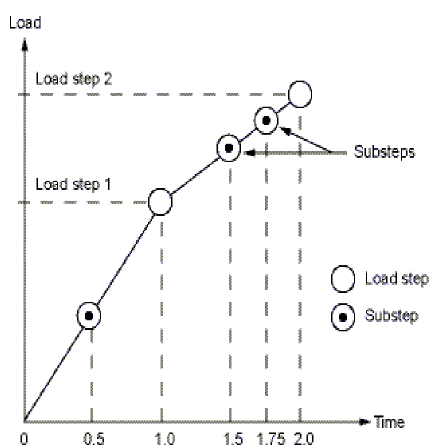
b) equivalent axisymmetric pressure distribution

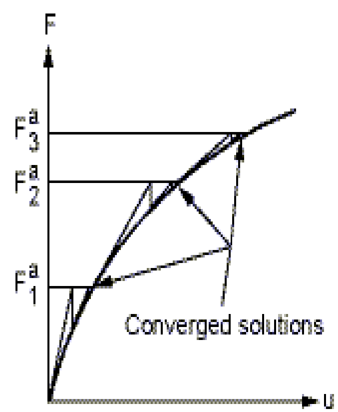
Introduction to Design of Shell Structures

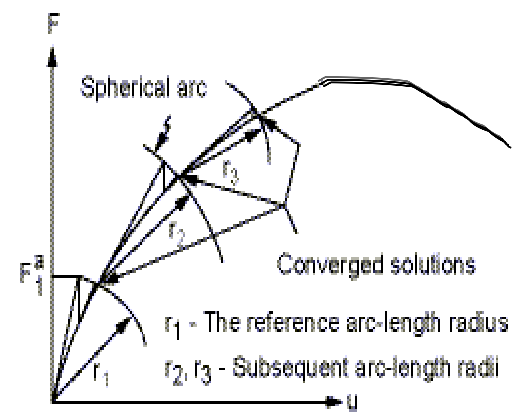
Finite Element Application

- Solver technique
  - Pre-critical analysis and point results (Newton-Raphson)
  - Post-critical analysis with deformation–to-failure (displacement control, arc-length, modal analysis)
    - The Static Riks step (based on arc-length solver) is able to find solution during unstable loading response, when Static General step (based on N-R solver) stops at limit load.







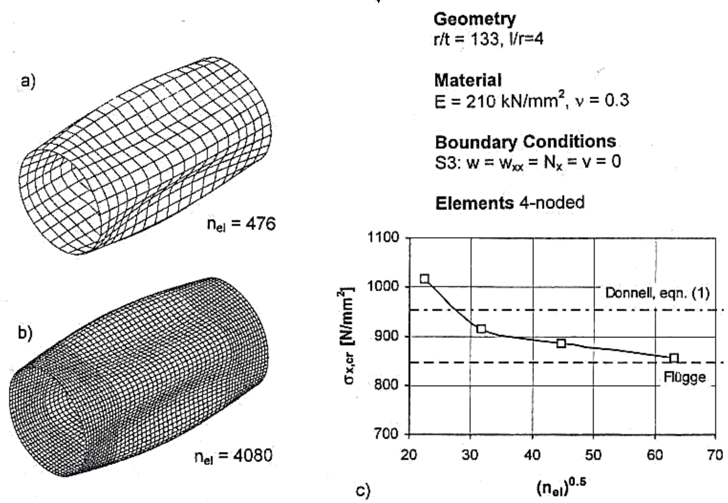


Introduction to Design of Shell Structures

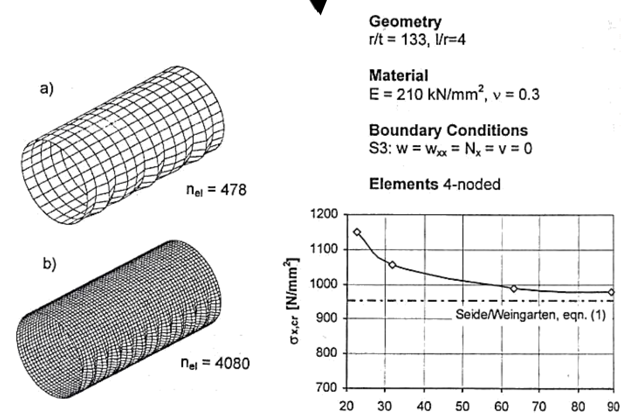
Finite Element Application

- **Results: Stability of steel shell structure General Report – Herbert Schmidt (JCSR 55(2000) 159-181)**

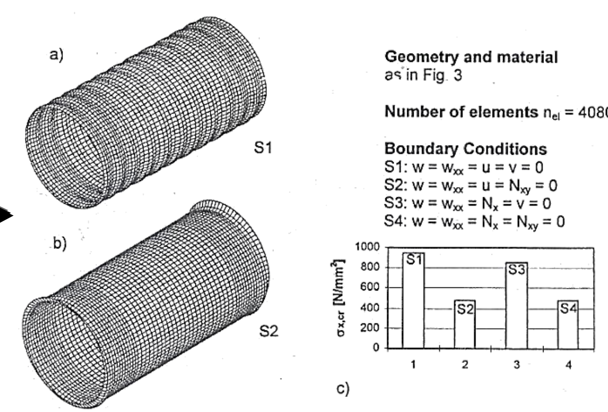
Eigenvalue results axial compression different discretization levels



Eigenvalue results pure bending different discretization levels



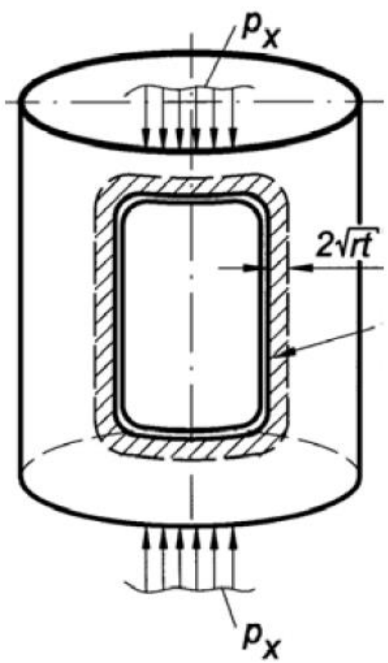
Eigenvalue results axial compression different boundary conditions

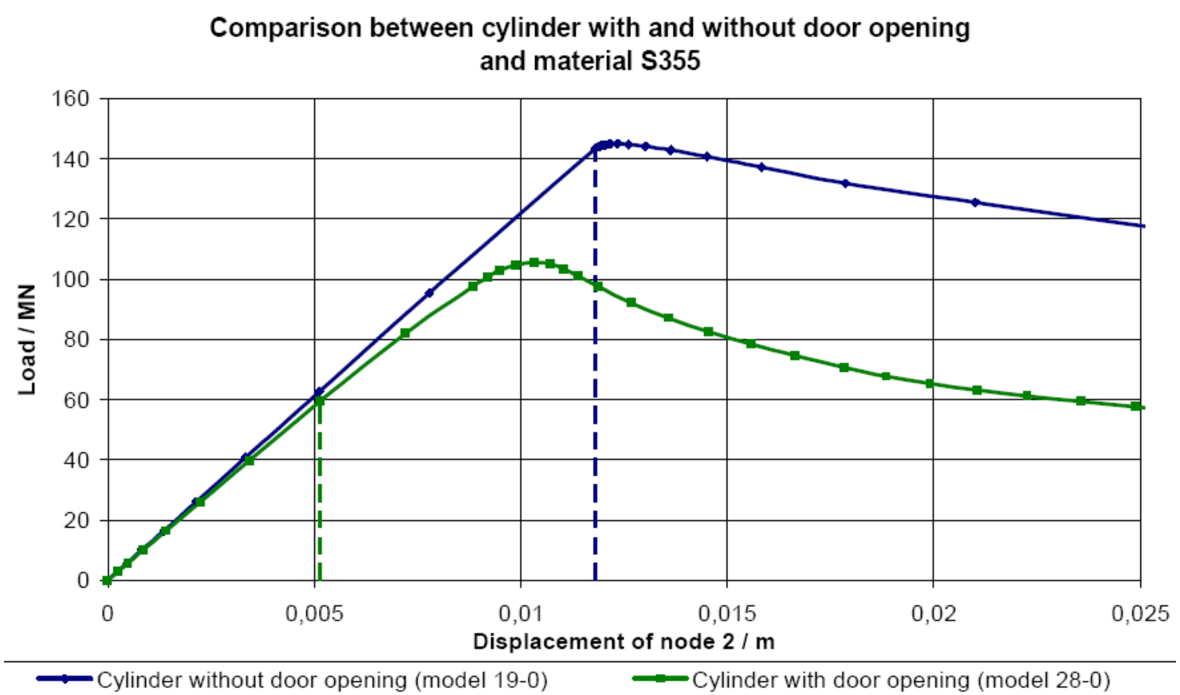


Introduction to Design of Shell Structures

Finite Element Application

- Stress concentrations
  - Modelling stress concentration (door opening)
  - Around door opening appear elevated values of the membrane stresses. This stresses that develop within a band of width of 2 √rt adjacent to a restrained edge need not be consider in buckling calculations

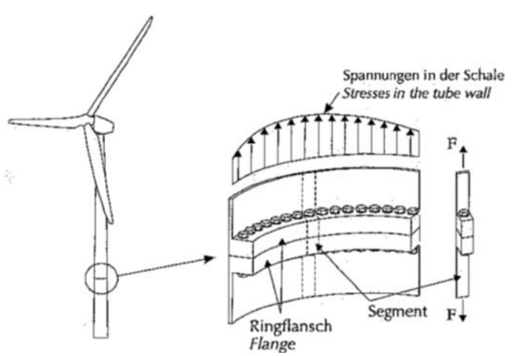


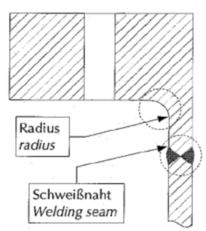


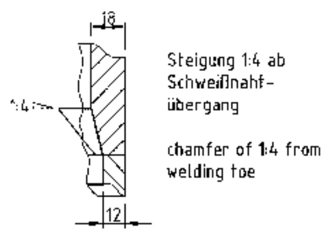
Introduction to Design of Shell Structures

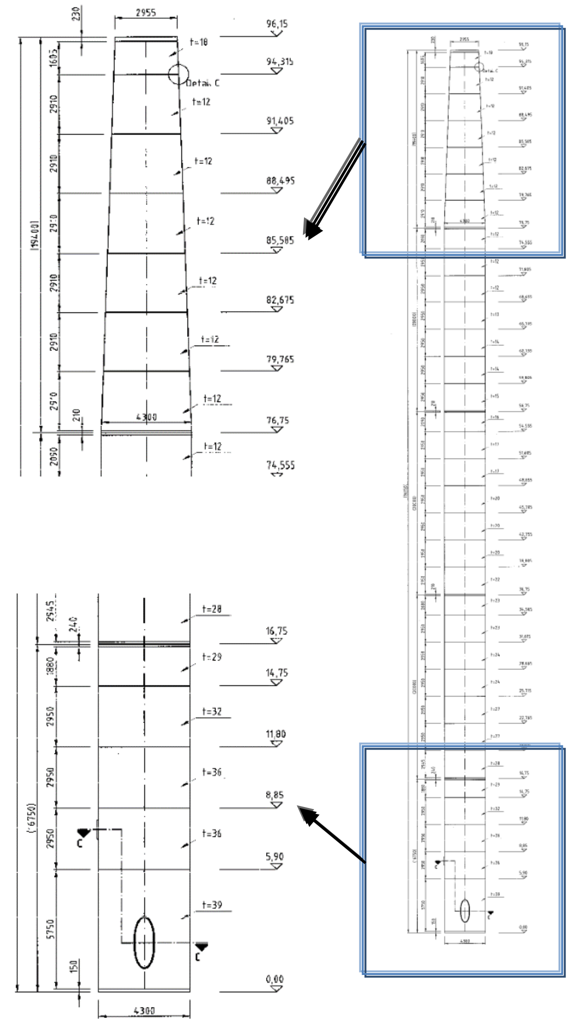
Finite Element Application and Case Study

- **Study Case**
  - General geometrical data
    - Total height: 96.15 m
  - Shape: cylinder (conic last segment) stepwise variable wall thickness
    - Base diameter: 4300 mm (t=39 mm)
    - Top diameter: 2955 mm (t=12 mm)
  - Divided in 5 segments of ~ 20m length
  - Detailing
    - Flange bolted connection (M48 - M27) between segments and welded connection between shells of different thickness





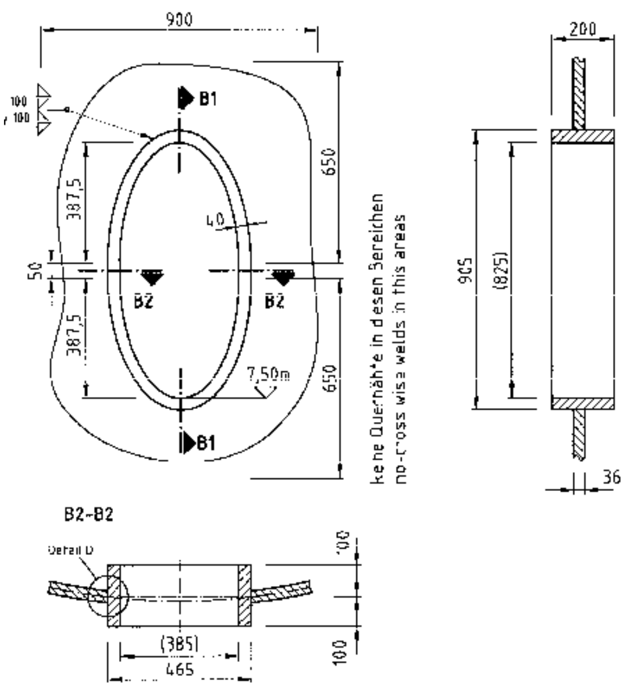


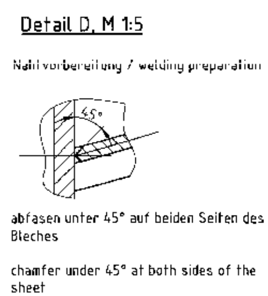


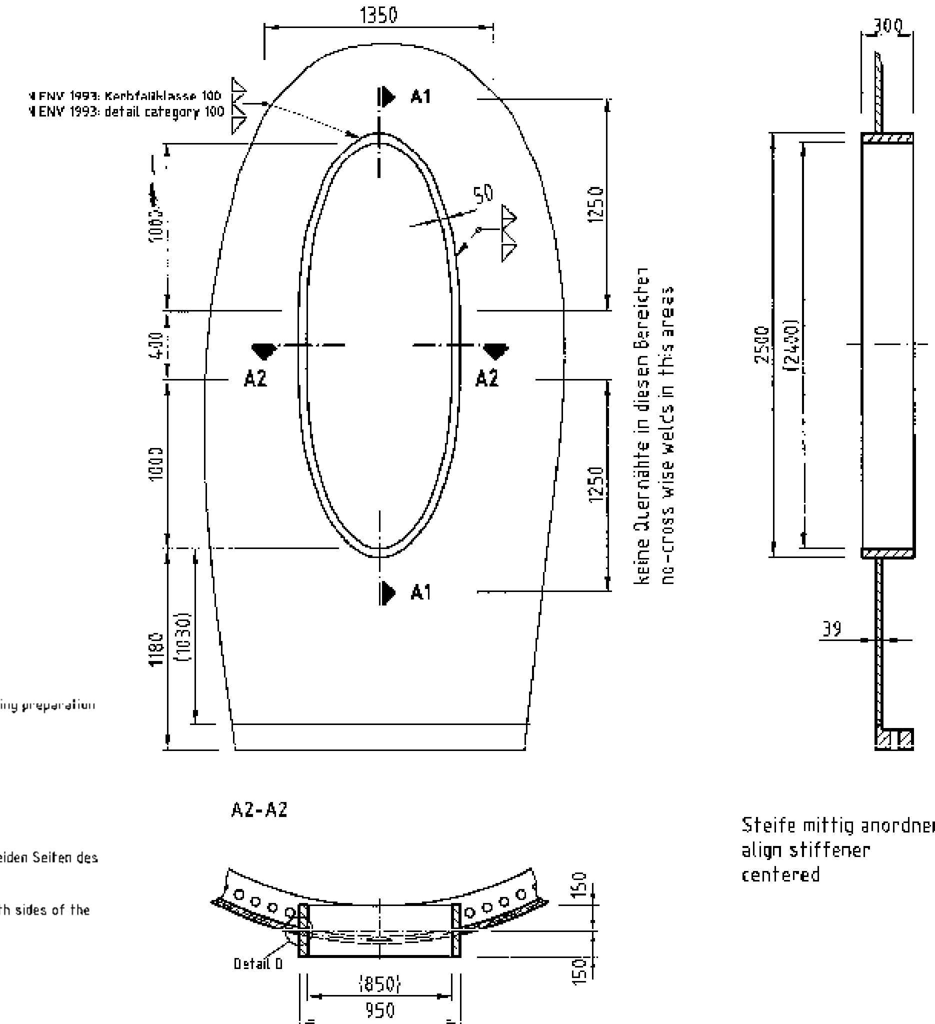
Introduction to Design of Shell Structures

Finite Element Application and Case Study

- **Study Case**
  - Door opening details
  - Ventilation opening







- Study Case
  - Numerical models
    - Simplified bar model
    - Refined local segment model
      - Relevant segments
      - Door opening segment
    - Entire model
  - Verification procedures
    - Analytical determination of moment capacity (LA) → M_{Rd}
    - Characteristic buckling resistance (LBA + MNA)
      - The plastic reference resistance → R_{pl}
      - The elastic critical buckling resistance → R_{cr}
        - The overall buckling reduction factor → $R_k = \chi_{ov} R_{pl}$
    - Characteristic buckling resistance (GMNIA)
      - Calibration factor → $R_k = k_{GMNIA} R_{GMNIA}$

- Study Case
  - Expressions for buckling stress design
  - Unstiffened cylindrical shells
    - Critical meridional buckling stresses

$$\omega = \frac{l}{r}\sqrt{\frac{r}{t}} = \frac{l}{\sqrt{rt}} = \frac{20000}{\sqrt{2150 \cdot 13{,}14}} = 119$$

$$\sigma_{x,Rcr} = 0{,}605 \cdot E \cdot c_x \cdot \frac{t}{r} = 0{,}605 \cdot 2{,}1 \cdot 10^5 \cdot 0{,}943 \cdot \frac{12}{2150} = 669 \frac{N}{mm^2}$$

$$\frac{r}{t} = \frac{2150}{13{,}14} = 164 \Rightarrow c_x = c_{x,N} = 1 + \frac{0{,}2}{c_{xb}}\left(1 - 2\omega\frac{t}{r}\right)$$

$$c_{x,N} = 1 + \frac{0{,}2}{1}\left(1 - 2 \cdot 119 \frac{12}{2150}\right) = 0{,}9434 \geq 0{,}6$$

- Study Case
  - Expressions for buckling stress design (hand calculation)
  - Unstiffened cylindrical shells
    - Meridional buckling parameters

$$\alpha_x = \frac{0,62}{1+1,91\left(\frac{\Delta w_k}{t}\right)^{1,44}} = \frac{0,62}{1+1,91\left(\frac{10,51}{13,14}\right)^{1,44}} = 0,25999$$

$$\Delta w_k = \frac{1}{Q}\sqrt{\frac{r}{t}}\cdot t = \frac{1}{16}\sqrt{\frac{2150}{13,14}}\cdot 13,14 = 10,51$$

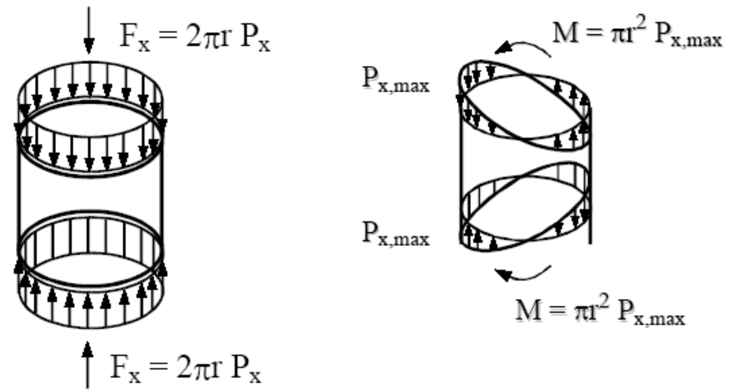
$$\begin{cases} \bar{\lambda}_{x0} = 0,20 \\ \beta = 0,60 \quad \rightarrow \bar{\lambda}_p = 0,806 \qquad \bar{\lambda}_x = 0,728 \leq 0,806 \\ \eta = 1,0 \end{cases}$$

$$\chi = 1 - 0,60\cdot\left(\frac{0,728-0,2}{0,806-0,2}\right) = 0,477 \Rightarrow \sigma_{x,Rk} = 0,477\cdot 355 = 169\frac{N}{mm^2}$$

- Study Case
  - Expressions for buckling stress design (hand calculation)
  - Unstiffened cylindrical shells
    - Meridional stresses

$$\sigma^{N}{}_{x,Ed} = -\frac{F_x}{2\pi r t} = \frac{1770 \cdot 10^3}{2\pi \cdot 2150 \cdot 12} = 10,92 \frac{N}{mm^2}$$

$$\sigma^{M}{}_{x,Ed} = \pm \frac{M}{\pi r^2 t} = \frac{17702 \cdot 10^6}{\pi \cdot 2150^2 \cdot 12} = 102 \frac{N}{mm^2}$$



$\sigma_x = -\frac{F_x}{2\pi r t}$ $\sigma_x = \pm \frac{M}{\pi r^2 t}$

- Study Case
  - Expressions for buckling stress design (hand calculation)
  - Unstiffened cylindrical shells
    - Critical circumferential buckling stresses

$$\omega = \frac{l}{r}\sqrt{\frac{r}{t}} = \frac{l}{\sqrt{rt}} = \frac{20000}{\sqrt{2150 \cdot 13,14}} = 119$$

$$\sigma_{\theta,Rcr} = 0,92 \cdot E \cdot \frac{c_\theta}{\omega} \cdot \frac{t}{r} = 0,92 \cdot 2,1 \cdot 10^5 \cdot \frac{1}{119} \cdot \frac{12}{2150} = 9,06 \frac{N}{mm^2}$$

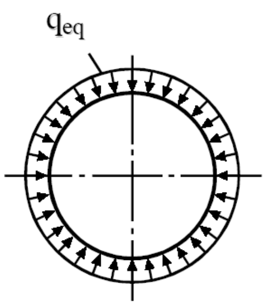
$$1,63 \cdot \frac{r}{t} = 267,32$$

- Study Case
  - Expressions for buckling stress design (hand calculation)
  - Unstiffened cylindrical shells
    - Circumferential buckling parameters

$$\alpha_\theta = 0,5 \qquad (Class\,C)$$

$$\begin{cases} \bar{\lambda}_{\theta 0} = 0,40 \\ \beta = 0,60 \quad \rightarrow \bar{\lambda}_p = 1,12 \qquad \bar{\lambda}_\theta = 6,26 \geq 1,12 \\ \eta = 1,0 \end{cases}$$

$$\chi = \frac{\alpha}{\bar{\lambda}^2} = \frac{0,5}{6,26^2} = 0,0128 \Rightarrow \sigma_{\theta,Rk} = 0,0128 \cdot 355 = 4,53 \frac{N}{mm^2}$$



$$\sigma_{\theta,Ed} = p_n \frac{r}{t} = 1,5 \cdot 1,102 \cdot 10^{-3} \frac{2150}{12} = 0,296 \frac{N}{mm^2}$$

Introduction to Design of Shell Structures

Finite Element Application and Case Study

- Study Case – Complete Model
  - Imperfection amplitude

$$l_{gx} = 4\sqrt{rt} = 4 \cdot \sqrt{4300 \cdot 12} = 908,63\,mm$$

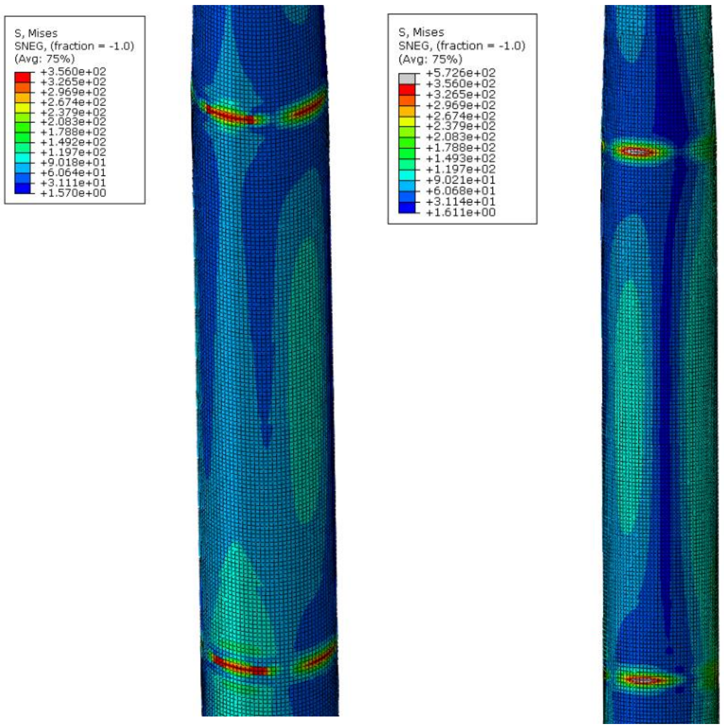
$$\Delta w_{0,eff,1} = l_g\, U_{n1}$$

Fabrication tolerance quality class	Description	Value of U_{n1}	Geometric tolerance normal to the shell surface
Class A	Excellent	*0,010*	*9,086*
Class B	High	0,016	*14,538*
Class C	Normal	*0,025*	*22,7158*

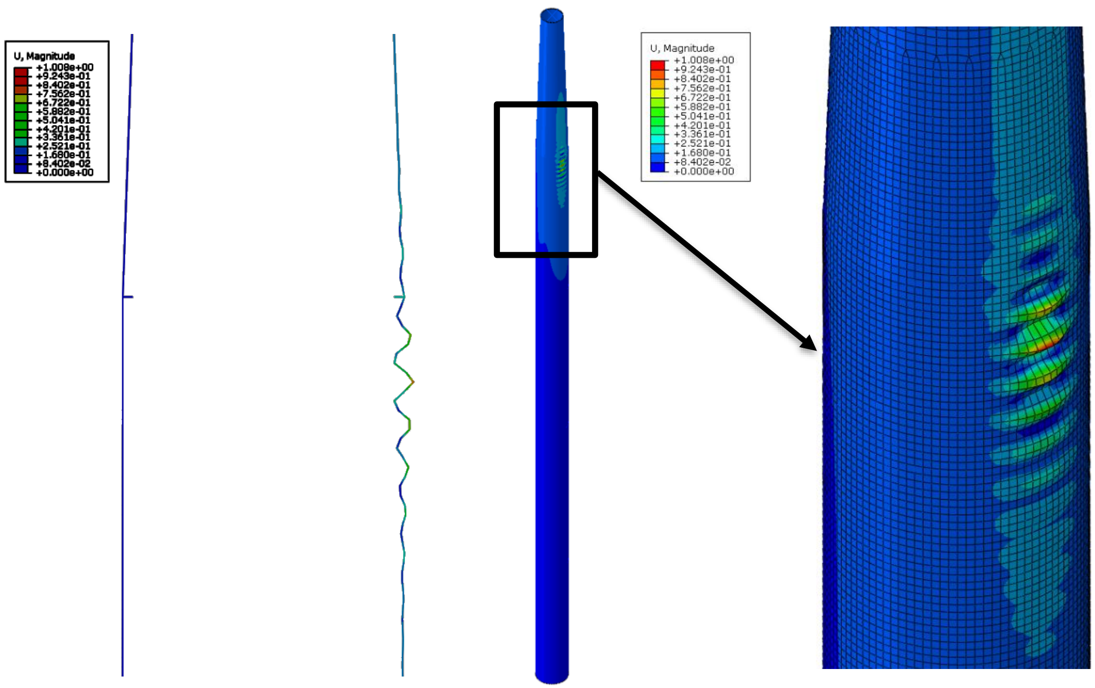
Introduction to Design of Shell Structures

Finite Element Application and Case Study

- Study Case – Complete Model
  - LA / MNA at reference load results
    - Local plastic zones around connecting flanges



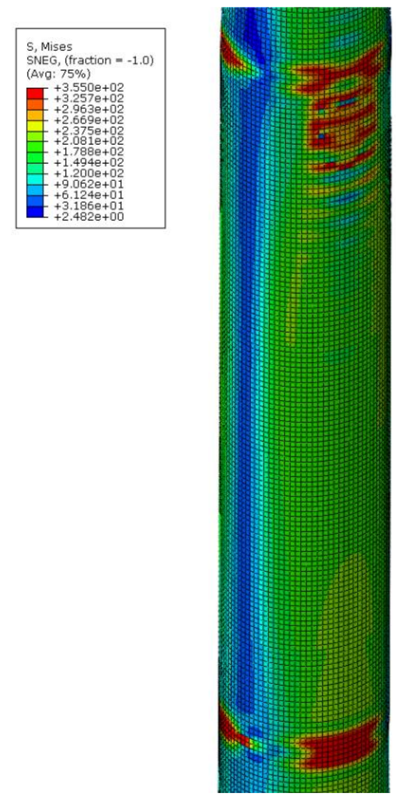
  - LBA results R_{cr} = 5,877

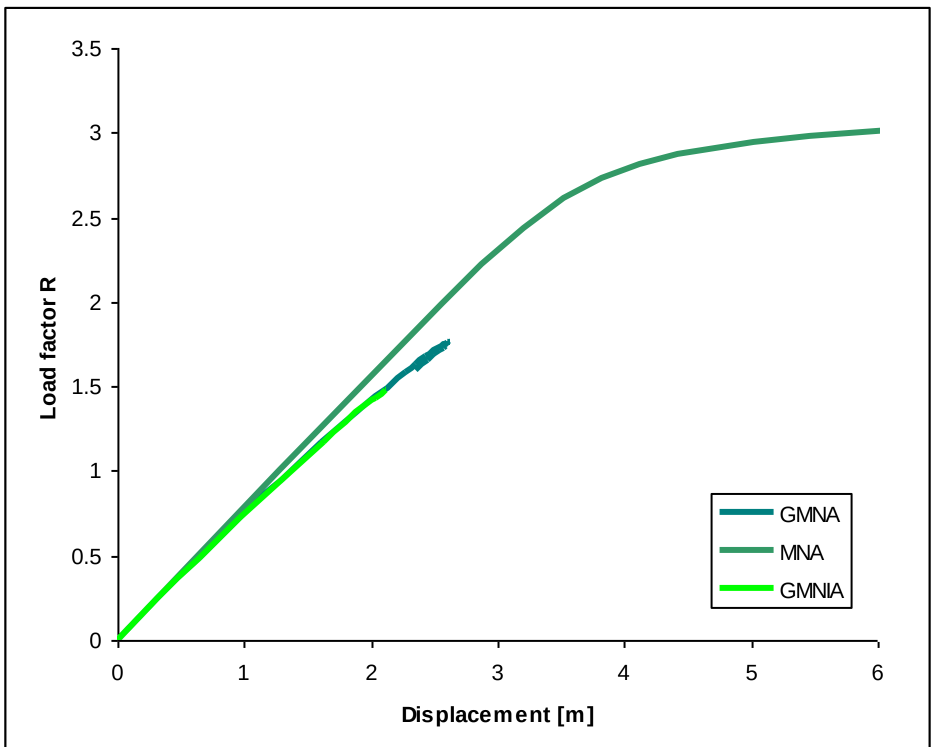


Bottom section	Axial force [kN]	Shear force [kN]	Bending moment [kNm]
LA	4323	1260	80088
MNA	4323	1260	80088

Finite Element Application and Case Study

- Study Case – Complete Model
  - MNA R_{pl} = 3,07
  - GMNA R_{pl} = 1,77
  - GMNIA R_{pl} = 1,49





Reactions at	Axial force [kN]	Shear force [kN]	Bending moment [kNm]
R = 1	4323	1260	80088

- Study Case – Complete Model
  - Description of the complete shell model
    - LBA - Linear elastic bifurcation analysis
      - The elastic critical buckling load factor R_{cr} = 5,8772
      - Imperfection shape
    - MNA – Material nonlinear analysis
      - The plastic reference load factor R_{pl} = 3,07
  - The overall relative slenderness λ_{ov} for the complete shell

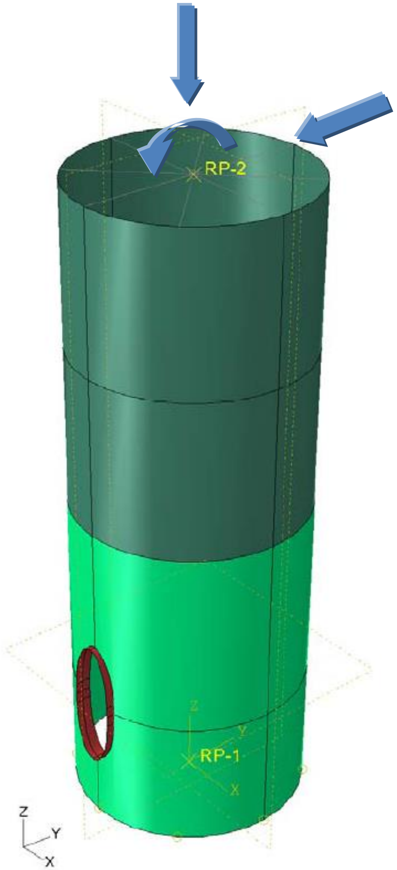
$$\overline{\lambda_{ov}} = \sqrt{\frac{3,07}{5,877}} = 0,522$$

  - The overall buckling reduction factor
  - Annex D EN1993-1-6 gives values for:
    - α_{ov} is the overall elastic imperfection factor
    - β_{ov} is the plastic range factor = 0,60
    - η_{ov} is the interaction exponent = 1,0

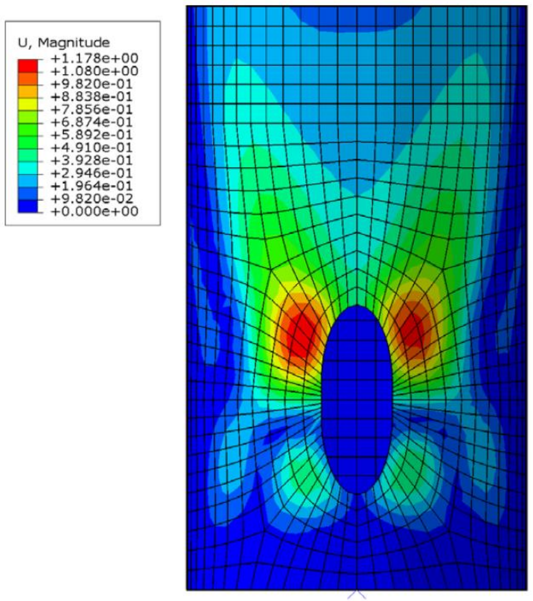
Finite Element Application and Case Study

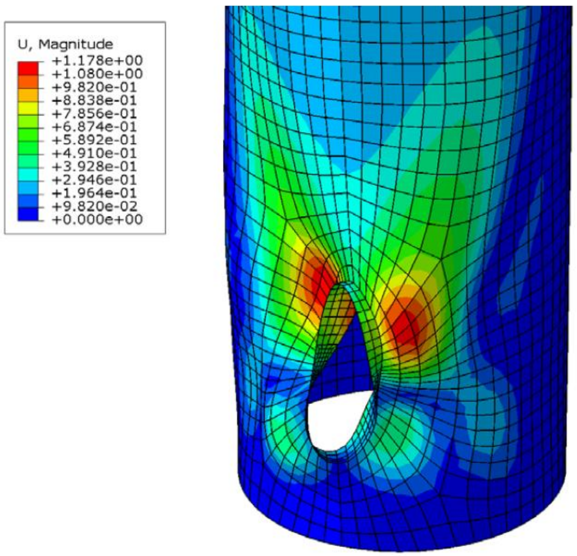
- **Study Case – Refined door opening segment**

- **Numerical model**



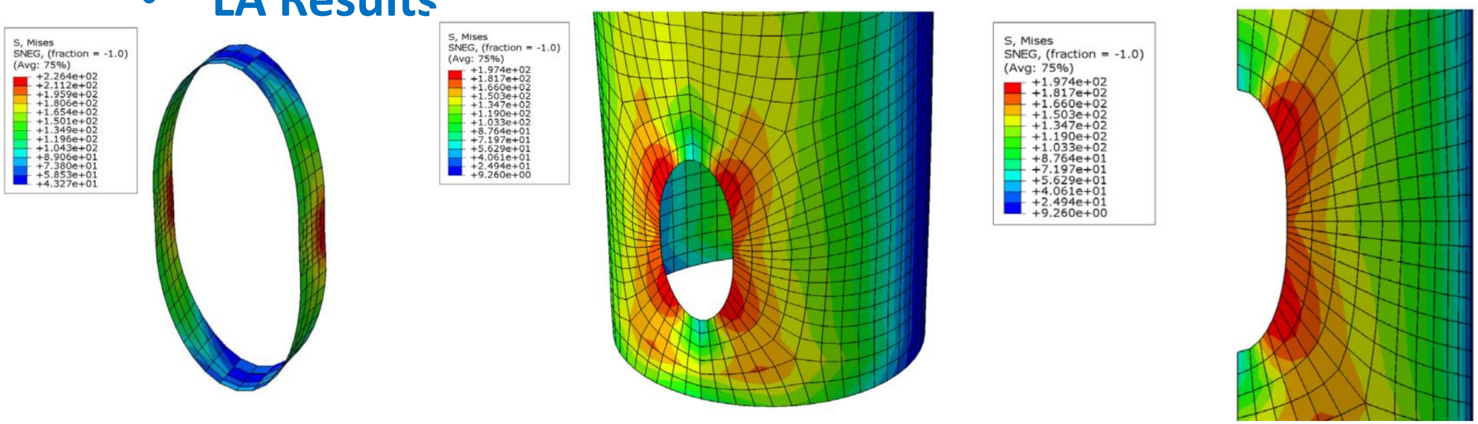
LBA results R_{cr} = 12.248



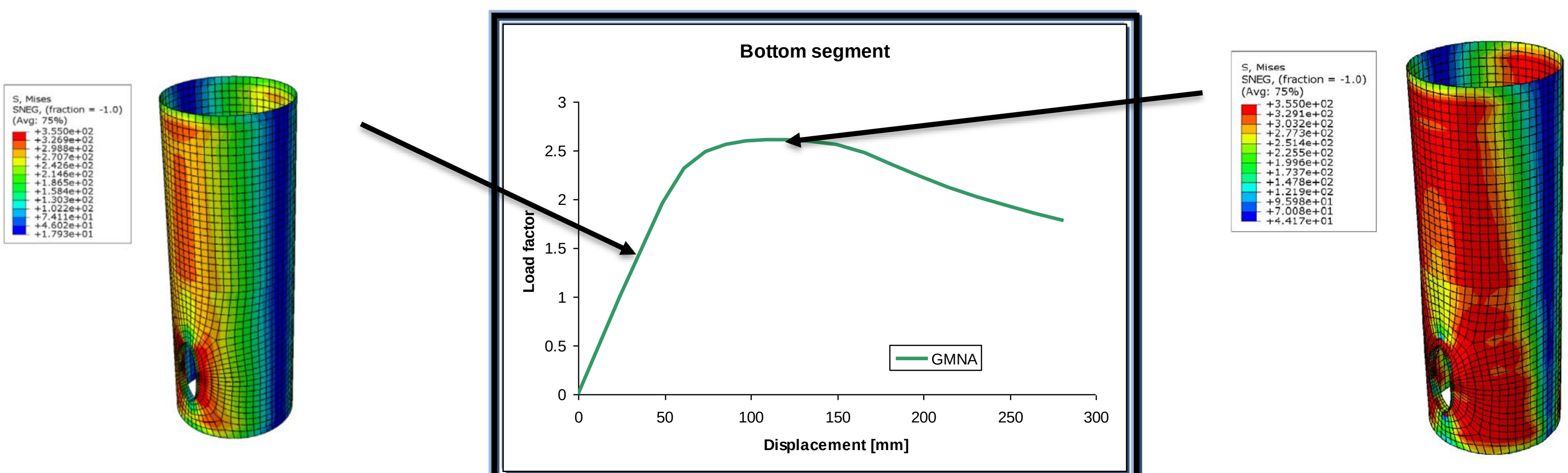


Section	Axial force [kN]	Shear force [kN]	Bending moment [kNm]
Upper section	4413	1231	73988
Bottom section	4413	1231	88514

- Study Case – Refined door opening segment
  - LA Results



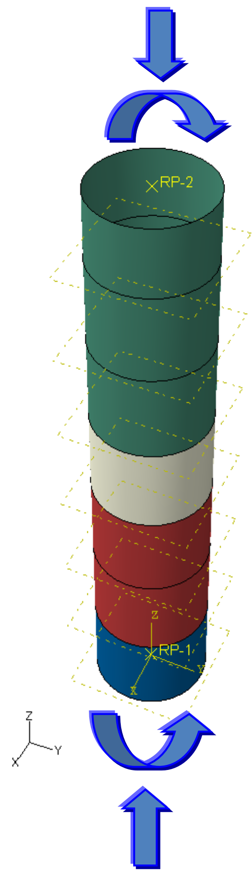
  - GMNA Results



Introduction to Design of Shell Structures

Finite Element Application and Case Study

- **Study Case – Refined door opening segment**
  - **Numerical model**



Section	Axial force [kN]	Shear force [kN]	Bending moment [kNm]
Upper section	1586	409	13055
Bottom section	2015	666	23897

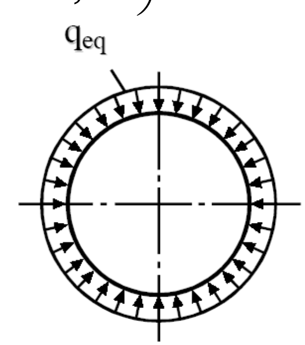
- LA R_{pl} = (126/355)=2,82

- Transformation of typical wind pressure load distribution

$$k_w = 0,46\left(1+0,1\sqrt{\frac{c_\theta}{\omega}\frac{r}{t}}\right) = 0,46\left(1+0,1\sqrt{\frac{1}{119}\cdot\frac{2150}{13,14}}\right) = 0,514$$

$$\omega = \frac{l}{r}\sqrt{\frac{r}{t}} = \frac{l}{\sqrt{rt}} = \frac{20000}{\sqrt{2150\cdot 13,14}} = 119$$

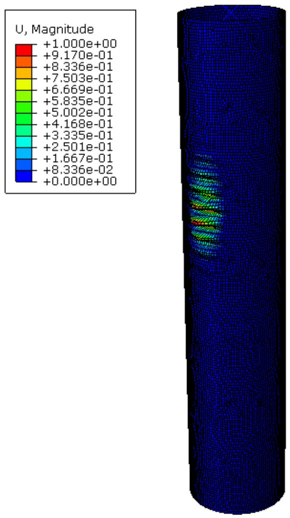
$$q_{eq} = k_w\, q_{\max} = 0,541\cdot 1430\cdot 1,5 = 1102\,\frac{N}{m^2}$$



Introduction to Design of Shell Structures

Finite Element Application and Case Study

- Study Case – Refined door opening segment
  - LBA Results
  - GMNIA results
    - Imperfection afine first buckling mode
    - Amplitude of imperfection 23mm and 17mm corresponding to normal and high tolerance



Section	R_{cr}
Without EWL	6,8006
With EWL	6,8003

Quality class	k_{GMNIA}
High *(~17 mm)*	1,79
Normal *(~23 mm)*	1,72