INTRODUCTION TO TRADITIONAL TRIGONOMETRY
A Lesson Plan

You've seen the videos on circle-ometry ([1]), you've shared them with your students (or not—your curriculum won't allow radical deviations from it), and you are left with:

> **Yeah ... but I still have to do SOHCAHTOA and all that with my students.**
> **What should I do?**

Here's a lesson guide that cuts through all the clutter of the traditional introduction to trigonometry and teaches students to rely on their natural wits to solve problems and thus develop the natural confidence to solve problems.

Only one geometry prerequisite is needed.

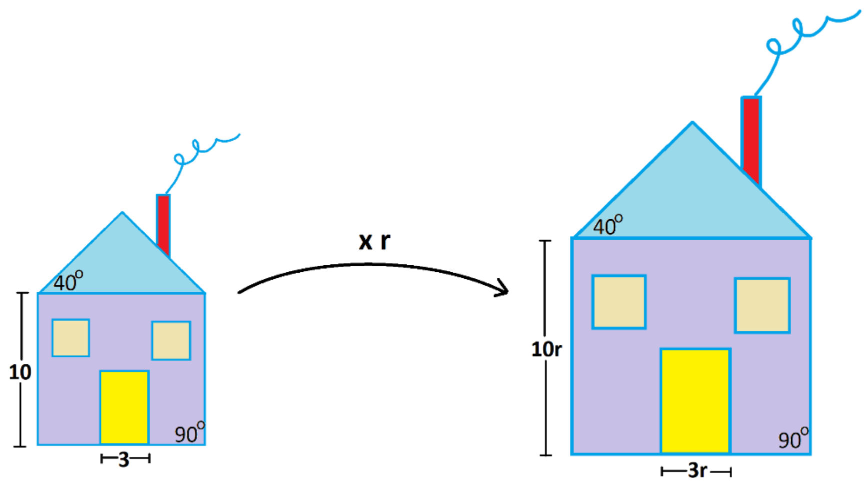
Here's the content of these notes.

[1] Videos 2 and 4 (and others) located here.

Prerequisite: Clarity on Scaling

Take a geometric figure to a photocopier and scale it by a factor r. What emerges is a scaled copy of that figure with

- all lengths changed by the factor r
- all angles unchanged in measure



This observation is usually couched within the notion of figures being *similar* (corresponding sides are scaled by a fixed factor and corresponding angles match exactly in measure).

It is a fundamental belief in geometry that **if two triangles have two corresponding angles that match in measure, then those triangles are scaled copies of each other.** (This is usually called the *AA principle.)*

Discuss this prerequisite, if needed, with your students in the language of your curriculum.

Natural Trigonometry

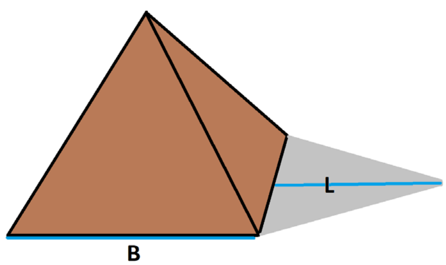
Lesson Start

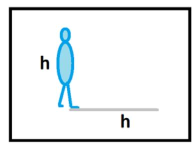
Here's a discussion item for your class. Draw diagrams on your board or projector, or share the first handout with your students.

Thales of Ionia (640-546 BCE) garnered tremendous fame as a geometer by deducing the height of the Great Pyramid of Egypt simply by measuring shadow lengths.

He waited for the day of the year that the Sun set directly behind the pyramid, and also waited for the exact time of day that his body cast a shadow the same length as his height. Then, at that very moment, he measured the length L of the shadow shown. As it is easy to measure the base width B of the Great Pyramid, Thales had all the information he needed to deduce its height.

Can you see how? Can you explain the mathematics behind his method?

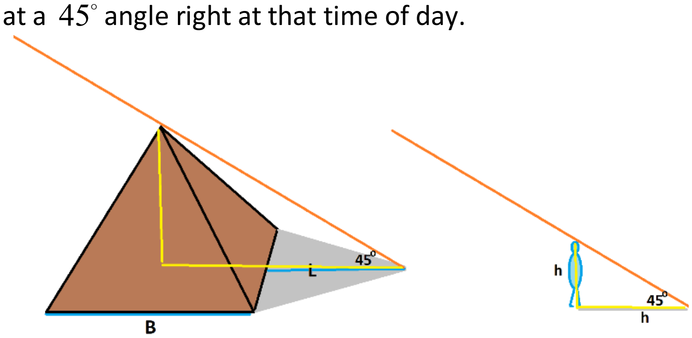




The base length of the pyramid is $B = 230$ m. Suppose the length of the shadow at his special time of the special day is $L = 32$ m. What is the height of the pyramid?

Help your students realise and articulate all the pieces here.

- The Suns rays when they reach Earth are essentially parallel.
- Thales' and his shadow form a right isosceles triangle. The rays from the Sun must be coming in at a 45° angle right at that time of day.



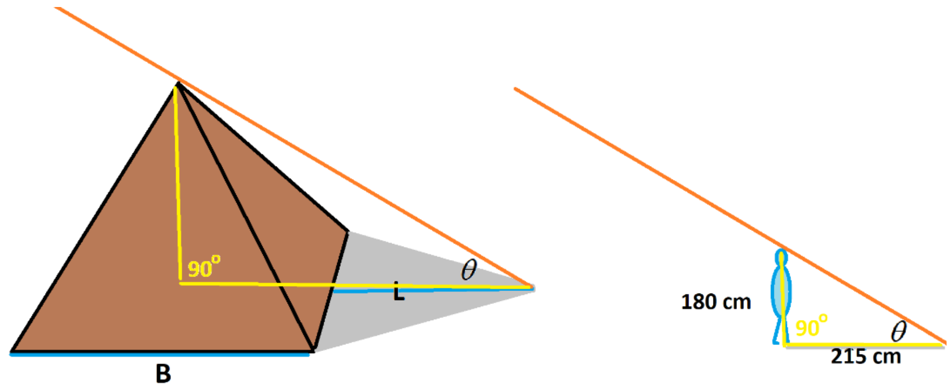
- Thus, the height of the Great Pyramid and the length $\frac{B}{2} + L$ form a right isosceles triangle too. These two side lengths must be of the same measure.
- The height of the Great Pyramid is $\frac{B}{2} + L$. Since he can measure B and measure L he can determine this quantity.

Pushing this Idea Further

Ask:

Did Thales really need to wait for the exact time of day when his shadow length matched his height?

Suppose Thales was 180 cm tall and he conducted this exercise at a time of day when his shadow was 215 cm long. Could he still have determined the height of the Great Pyramid?



Conduct a discussion (whole class, small groups, pairs – as you see fit) and help students realise

- Because the Sun's rays are essentially parallel, we have two triangles with two matching angles. We have similar triangles.
- Consequently, matching sides are scaled by the same factor.
- Because Thales' can measure B and measure L he knows what the scale factor is by comparing $\frac{B}{2}+L$ with the number 215.
- The height of the pyramid is the number 180 cm adjusted by this scale factor.

Now add

The base length of the pyramid is $B = 230$ m.

Given that you do already know the height of the pyramid from the previous exercise (147 m), what is the length L of the shadow in this scenario?

Help students come to the answer $L \approx 32$ m.

We have $\frac{B}{2}+L = 115 + L$ meters, and so $115 + L \;=\; 2.15r$ and $147 = 1.80r$ for some scale factor r. (We've written all measurements in terms of meters.) Thus $r = \frac{147}{1.80}$ and

$$L = 2.15 \times \frac{147}{1.80} - 115 \approx 32 \text{ meters.}$$

How tall is a flagpole?

ACTIVITY

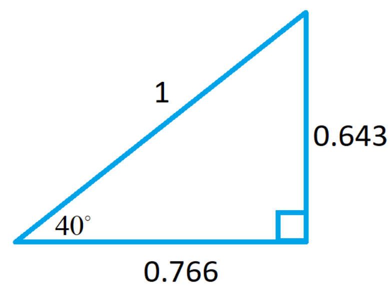
On a sunny day head out to a plaza or a field with flagpole equipped with a tape measure and a friend. Have your friend stand tall. Measure the length of her or his shadow. Measure the length of the shadow of the pole.

Do you have all the information you need to find the height of the pole? (If not, measure something in addition!)

Let's be a little more abstract and general in our discussions.

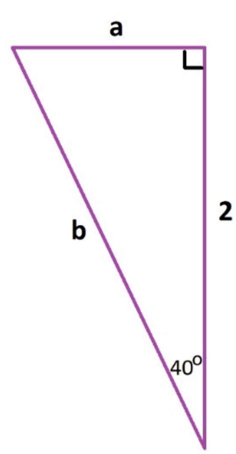
Here's a question to guide a class discussion.

Here are the side lengths of a right triangle possessing an angle of 40°.



(These numbers were found by drawing such a right triangle with a hypotenuse 1 and measuring the two side-lengths by hand. Measurements were conducted to three decimal places.)

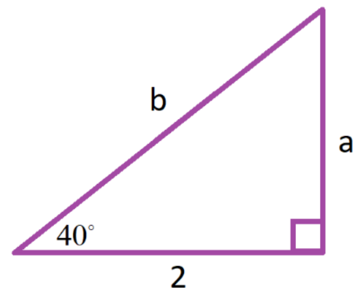
Can you now determine the lengths a and b in this figure?



Help your students come to realise that there is enough information here to deduce what the scale factor between the two figures must be.

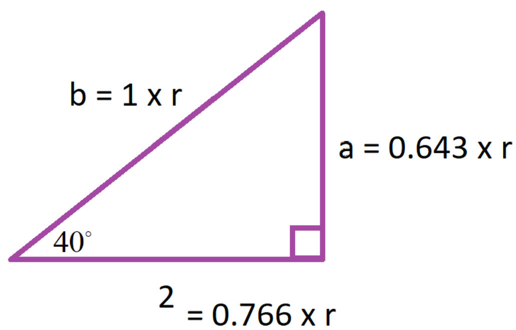
Go through the following specific pointers:

1. It could help to redraw the purple triangle so that its orientation is less befuddling.



2. This is a scaled copy of the given basic right triangle. It is helpful to write in the scaled-values of the basic right triangle.

   If we scale the basic triangle with side lengths 1, 0.643 and 0.766 by a factor r, we obtain a triangle with side length r, $0.643r$, and $0.766r$.



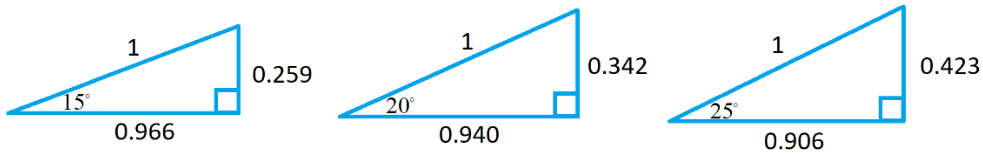
3. We can now see what the value of r must be (it's $\dfrac{2}{0.766}$) and hence deduce that

   $a = 0.643 \times \dfrac{2}{0.766} = 1.679$ and $b = 1 \times \dfrac{2}{0.766} = 2.611$.

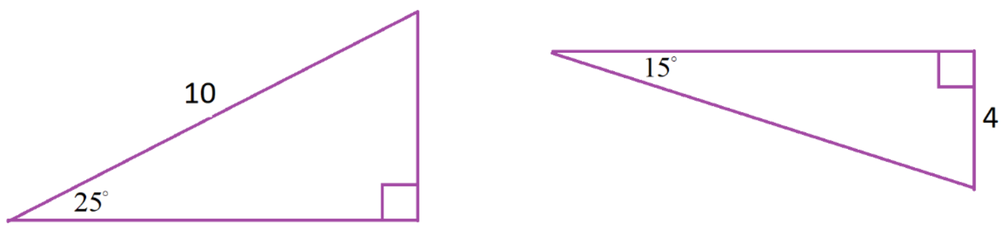
Offer your students the following practice problems.

Some Practice

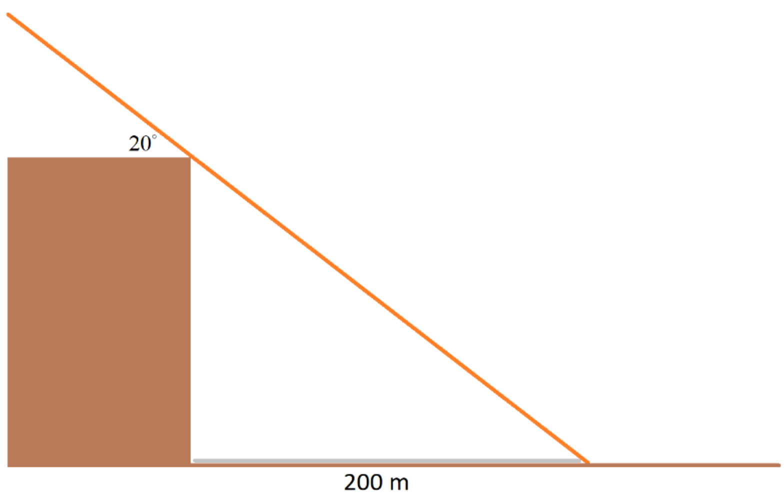
Here the side lengths of three right triangles each with hypotenuse 1. Use them for the following two questions.



Question 1: Find the two missing side lengths in each of these right triangles.

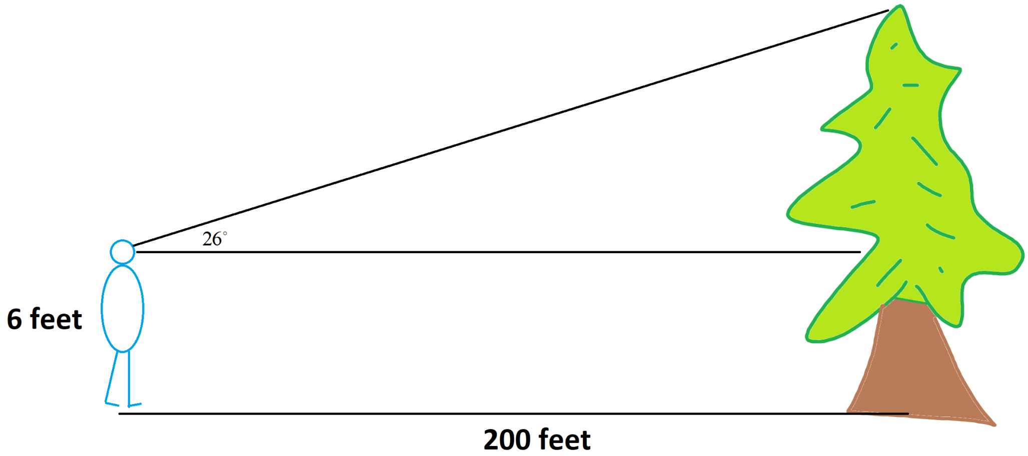


Question 2: A cliff casts a shadow 200 meters long when the rays of Sun hit the surface of the Earth at an angle of 20°. How high is the cliff?



(Scientists use essentially this method to determine the depth of craters on the Moon by looking at the lengths of shadows on surface photographs and knowing the angle of the Sun at the time the photographs were taken.)

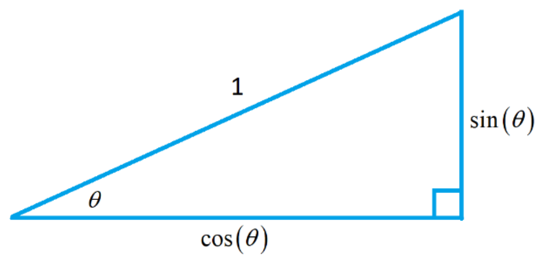
Question 3: Estimate the height of the tree.



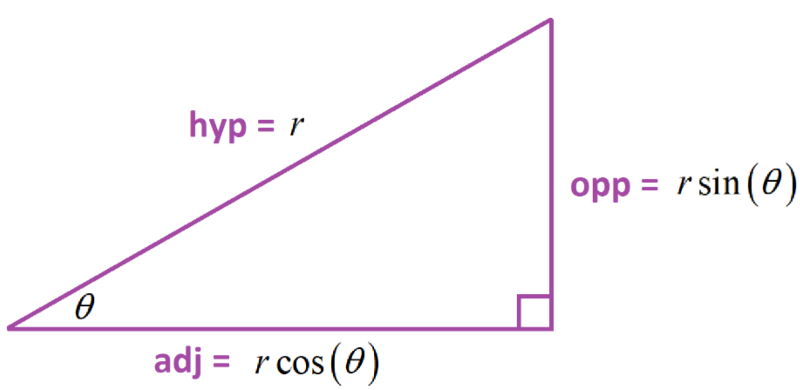
Continuing the Discussion

Share the following with your students.

> Humankind since the time of Thales has realised that knowing the side lengths of basic right triangles can be of immense practical use in matters of architecture, engineering, geography, navigation, and such. Just by measuring angles and knowing only one length it is possible to deduce additional lengths- be they heights of pyramids or trees, or distances to objects, or depths of craters.
>
> Thus it became a study of great interest to know the side-lengths of basic right triangles. For historical reasons, folk worked with right triangles of hypotenuse 1.
>
> If a right triangle contains an angle θ, then the side lengths of the triangle are called *sine* and *cosine* and are denoted $\sin(\theta)$ and $\cos(\theta)$. They are the sides shown in this diagram.



> These are strange names and the story that led us to them is fascinating. Watch this video up to the 13:43 mark to see this history (and afterwards you will have no trouble remembering the diagram above: "sine" is height and cosine is "overness.")
>
> Your calculator has been programmed to give you these sine and cosine values for any angle θ. (Oh how scholars of ancient times yearned for the means to know these values so readily!)
>
> The key thing to remember is that
>
> > **Any right triangle with the same angle θ is a scaled copy of this basic right triangle and so must be of the following form for some value r.**



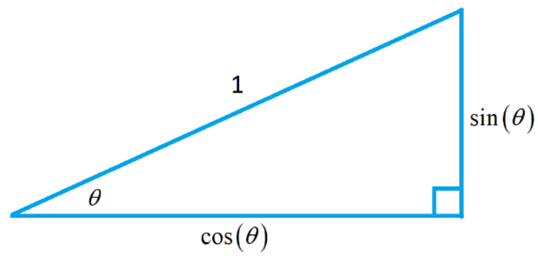
To be explicit:

The hypotenuse (hyp) will have a value r.

The side opposite the given angle θ (opp) will be $r\sin(\theta)$.

The side adjacent to the given angle θ different from the hypotenuse (adj) will be $r\cos(\theta)$.

Better yet, just keep the image of a scaled copy of the basic triangle in one's mind.



For example, a calculator gives

$$\sin\left(23^\circ\right) = 0.391$$
$$\cos\left(23^\circ\right) = 0.921$$

and so a right triangle with possessing an angle of 23° and a hypotenuse of length $r = 100$ has sides of lengths

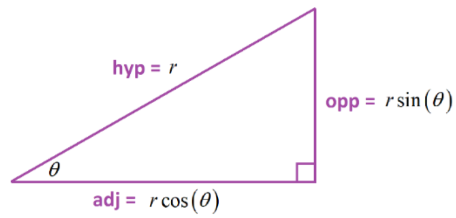
$$opp = r\sin\left(23^\circ\right) = 39.1$$
$$adj = r\cos\left(23^\circ\right) = 92.1.$$

Some more practice problems.

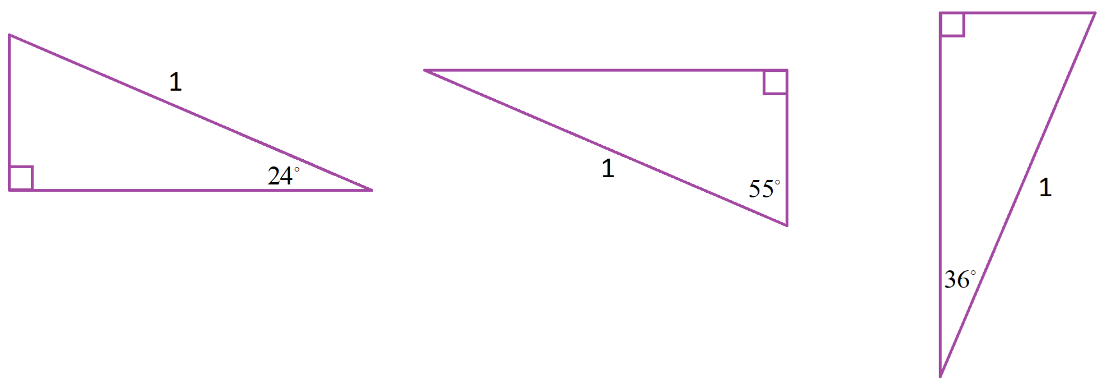
More Practice

For a right triangle possessing an angle θ we have

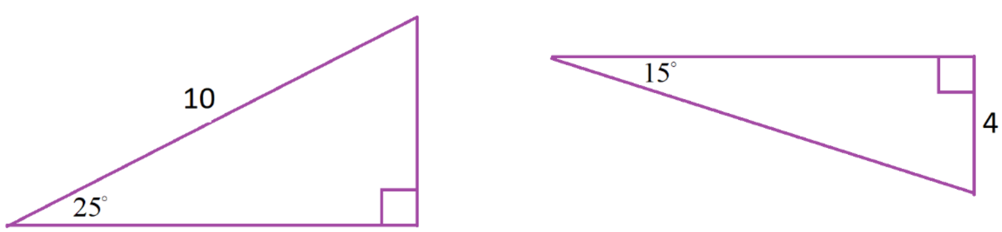
$$hyp = r$$
$$opp = r\sin(\theta)$$
$$adj = r\cos(\theta)$$



Question 1: According to your calculator, to three decimal places, what are the two missing side lengths in each of these triangles?



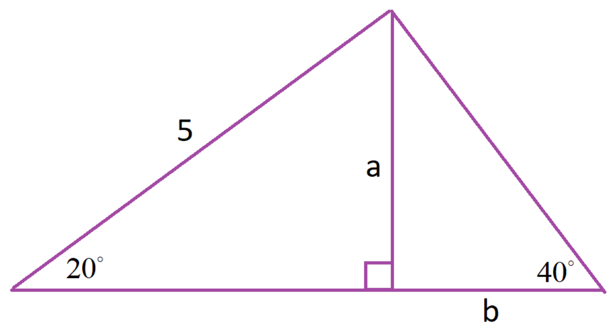
Question 2: Find the two missing side lengths in each of these right triangles.



Here are some more challenging practice problems. You might like to work with one of the problems in the set with the class as a whole first.

Here's a problem from the next practice set.

Find the values of lengths a *and* b *in this figure. Round your answers to three decimal places.*



Any first thoughts on how to get started?

Help your students reason as follows.

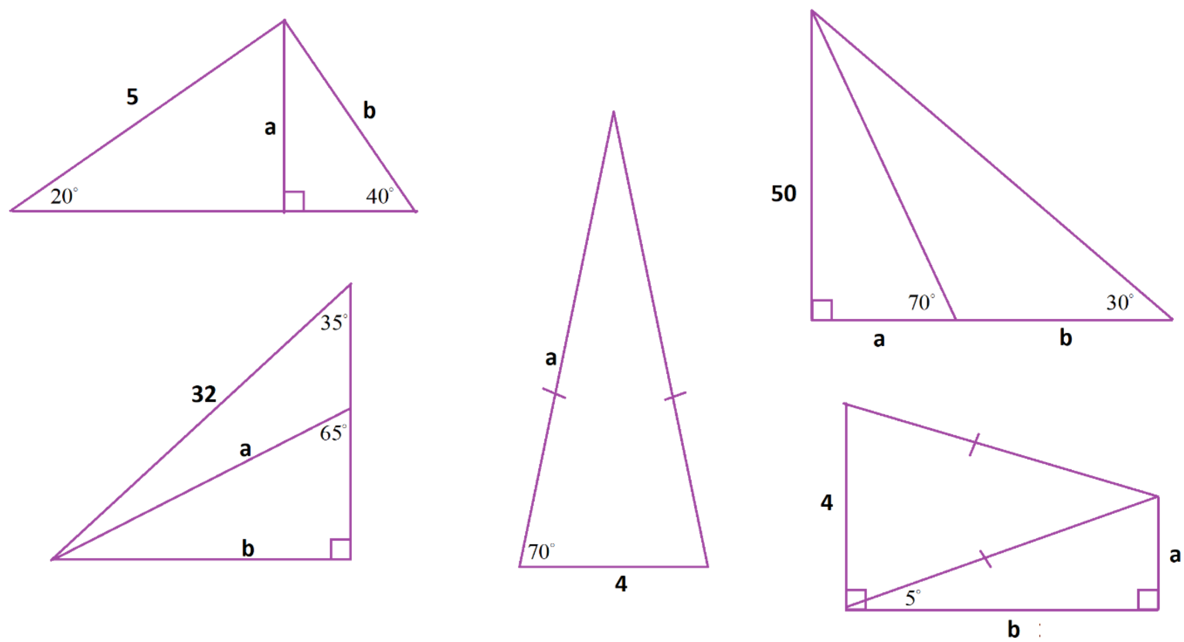
- In the right triangle with angle 20°, we have hypotenuse 5 and a is the side opposite angle 20°. Thus $a = 5\sin\left(20^\circ\right) = 1.710$, to three decimal places.
- In the right triangle with angle 40°, we don't know the value of the hypotenuse r, but we do know that $a = r\sin\left(40^\circ\right)$.

  So $1.710 = r\sin\left(40^\circ\right)$ and this gives $r = 2.660$.
- Then it follows that $b = r\cos\left(40^\circ\right) = 1.710 \times 0.766 = 1.310$

Answers may vary in the latter decimal place(s) depending on whether rounding is conducted with intermediate values along the way or left as a final step.

Yet More Practice

Find the lengths a and b indicated. Write your answers to three decimal places. Be sure to use your wits for these!



Now your students are ready for any standard textbook question on this topic, just by exercising their wits and being agents of their own learning and growth of understanding.

What follows is a sampler of trickier textbook challenges. Don't first explain the jargon that might be foreign to your students. Use these moments as opportunities for students to exercise their agency.

Also, there has been no mention of the tangent function as of yet. This is deliberate.

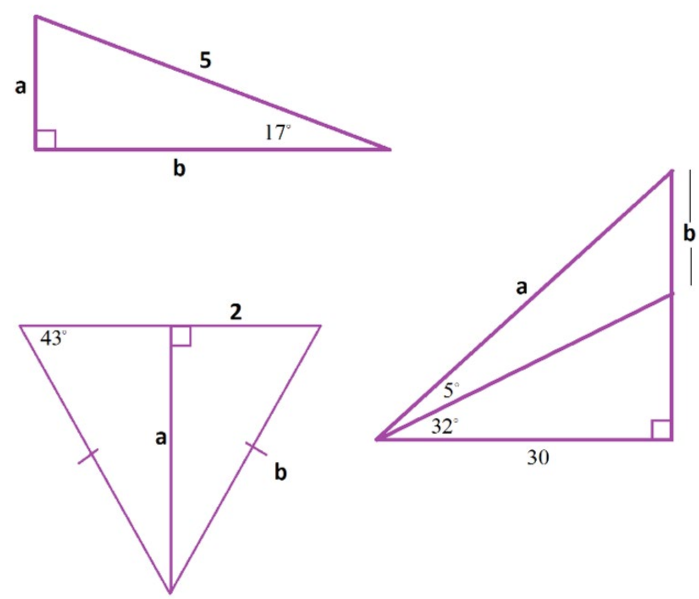
Still Yet More Practice Material

Here are some more problems to try. Let's assume all answers are to be to three-decimal places.

There might be words in some of these questions you have never seen before. Just do your best to use common sense to make good educated guesses as to their meaning. Usually drawing a picture of the situation at hand will reveal what must be meant. (One can, of course, always just google any unusual terms.)

Some of these problems are tricky. That is okay! Sit with them for a spell, try to make some progress and then let them be and go to some other tasks while you wait for a flash of insight to come to your brain.

Question 1: Find the missing side lengths.



Question 2: As observed at the top of a lighthouse 150 feet about seal level the angle of depression of a ship sailing directly towards the lighthouse changes from 30° to 40°. How far did the ship travel during this period of observation?

Question 3: I am looking out of my tenth-floor apartment window to a building directly opposite my building. The building I see is 50 meters away. Using my protractor I estimate the angle of elevation to the top of this second building to be 25° and the angle of depression down base of the building as 36°. How tall is that building?

Question 4: (From the 10th-grade Tamil Nadu State Curriculum, India.)
There are two temples, one on each back of a river just opposite one another. One temple is 40 meters high. As observed from the top of this temple, the angles of depression of the top and the foot of the other temple are 12° and 21°, respectively.

How wide is the river?

How tall is the other temple?

Question 5: At 6 pm the Sun has an angle of elevation of 10.6°. At this time, when I stand tall, my shadow is 32 feet long. To the nearest inch, how tall am I?

You calculator also has the means "undo" sine and cosine values. For instance, if you are wondering which angle θ in a right triangle has sine value 0.8 look for the $\sin^{-1}$ button on your calculator. (The superscript -1 indicates "undoing.") Then type in $\sin^{-1}(0.8)$ to get $53..1^\circ$.

Quick Practice 6: Find $\sin(1^\circ)$, $\sin^{-1}(0.5)$, $\cos\left(\frac{1^\circ}{2}\right)$, $\cos^{-1}\left(\frac{1}{2}\right)$.

Question 7: Why does your calculator give an error message for $\sin^{-1}(1.01)$? Should your calculator also give an error message for $\cos^{-1}(1.01)$? (Does it?)

Question 8: What are the three angles in a triangle with side lengths 3, ,4, and 5?

Question 9: Prodipta stands at the edge of a cliff on one side of a river, 20 meters wide. The cliff is 30 meters tall. What is the angle of depression of his line of site from the top of the cliff to the river bank directly opposite him?

Question 10: What is the angle of elevation of the graph of the line $y = 2x + 5$?

Question 11: A straight section of road has a 10% grade.

a) What is the angle of elevation of that section of road?
b) If I drive 50 meters upwards on that section of road, how high have I ascended?

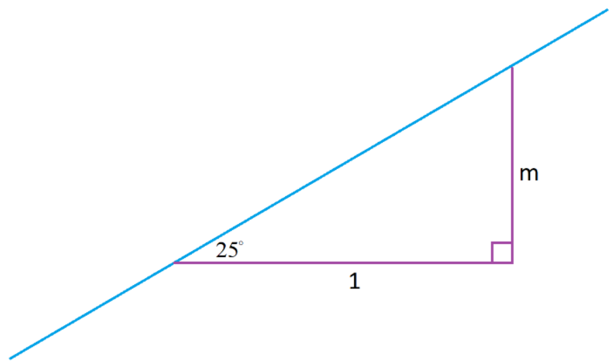
WHERE'S THE TANGENT FUNCTION?

The tangent function is not actually needed in any of the standard trigonometry work. But it is useful as a shortcut to repeated work.

Perhaps have the following discussion with your students.

Consider the following problem: *What is the slope of a line with angle of elevation* 25° *?*

To answer this, recall that the slope m of a line is "the rise needed to get back onto the line if one takes a unit step horizontally off of the line." We thus have this picture.



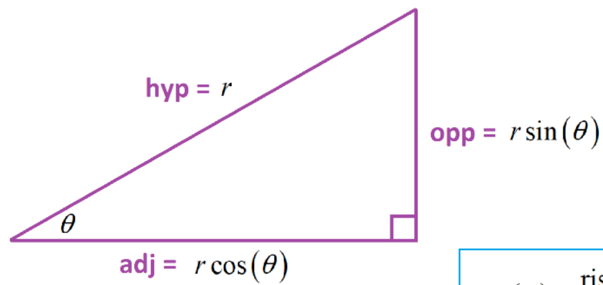
We see a right triangle. If we label its hypotenuse r, then we have

$$m = r\sin\left(25^\circ\right)$$
$$1 = r\cos\left(25^\circ\right)$$

Solving, we get $m = \dfrac{\sin\left(25^\circ\right)}{\cos\left(25^\circ\right)} \approx 0.466$.

In many practical applications it is important to know the slopes of lines and line segments, and so people have focused on giving the ratio "sine over cosine" its own name. It is called tangent.

We set $\tan(\theta) = \dfrac{\sin(\theta)}{\cos(\theta)}$.



$$\boxed{\tan(\theta)=\frac{\text{rise}}{\text{run}}=\frac{\text{opp}}{\text{adj}}=\frac{\cancel{r}\sin(\theta)}{\cancel{r}\cos(\theta)}}$$

Calculators have a $\tan$ button, as well as a $\tan^{-1}$ button to "undo" a tangent value.

For example,

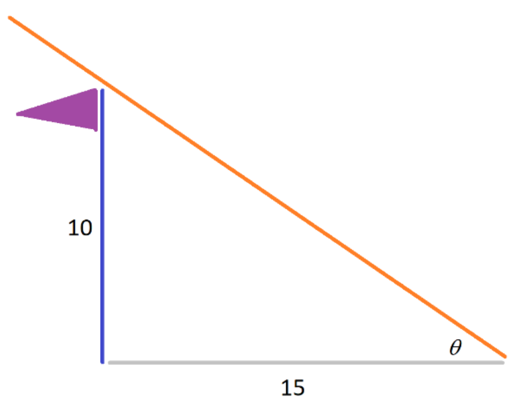
$\tan\left(25^{\circ}\right)\approx 0.466$ and a line with angle of elevation 25° has slope 0.466.

$\tan^{-1}(2)\approx 63^{\circ}$ and a line of slope 2 has angle of elevation 63°.

Practice Example: *A vertical flagpole 10 meters tall casts a shadow 15 meters long. To the nearest degree, what is the angle of elevation of the Sun at that moment?*

Answer without use of tangent: Let θ be the angle of elevation.

Drawing a diagram we see a natural right triangle to consider.



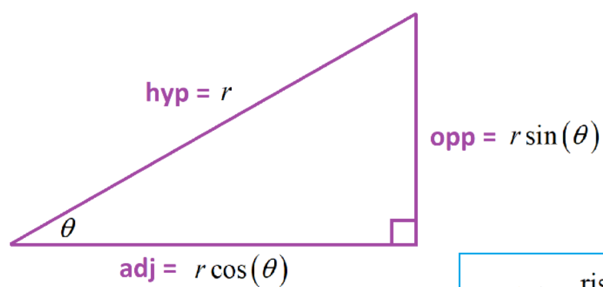
Natural Trigonometry

By the Pythagorean theorem, hypotenuse of this triangle is $\sqrt{325}$. From $10 = \sqrt{325}\sin(\theta)$ we see $\sin(\theta) = \dfrac{10}{\sqrt{325}}$, and so $\theta = \sin^{-1}\left(\dfrac{10}{\sqrt{325}}\right) \approx 33^\circ$. (We obtain this same answer if we work with $15 = \sqrt{325}\cos(\theta)$ instead.)

Answer using tangent: We have $\tan(\theta) = \text{slope} = \dfrac{\text{rise}}{\text{run}} = \dfrac{10}{15}$ and so $\theta = \tan^{-1}\left(\dfrac{10}{15}\right) \approx 33^\circ$.

Indeed, we see that having slope values, that is tangent values, already at hand can simplify work.

Final Practice Exercise



$$\boxed{\tan(\theta) = \frac{\text{rise}}{\text{run}} = \frac{\text{opp}}{\text{adj}} = \frac{\cancel{r}\sin(\theta)}{\cancel{r}\cos(\theta)}}$$

Which of the 11 questions in the previous problem set can be solved more swiftly making use of the tangent function?

If you are curious about some more history behind the story of trigonometry (how the "tangent" operation got this strange name), perhaps look at [this]() video and its follow-on video [here]().

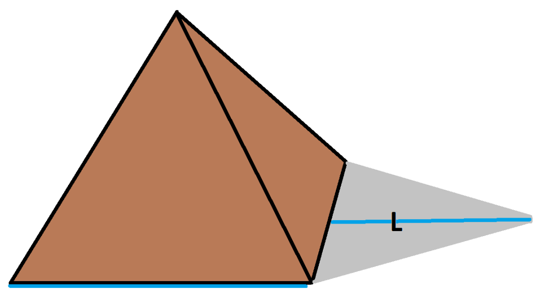
STUDENT HANDOUTS

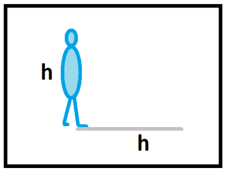
CLASS DISCUSSION ITEM 1

Thales of Ionia (640-546 BCE) garnered tremendous fame as a geometer by deducing the height of the Great Pyramid of Egypt simply by measuring shadow lengths.

He waited for the day of the year that the Sun set directly behind the pyramid, and also waited for the exact time of day that his body cast a shadow the same length as his height. Then, at that very moment, he measured the length L of the shadow shown. As it is easy to measure the base width B of the Great Pyramid, Thales had all the information he needed to deduce its height.

Can you see how? Can you explain the mathematics behind his method?



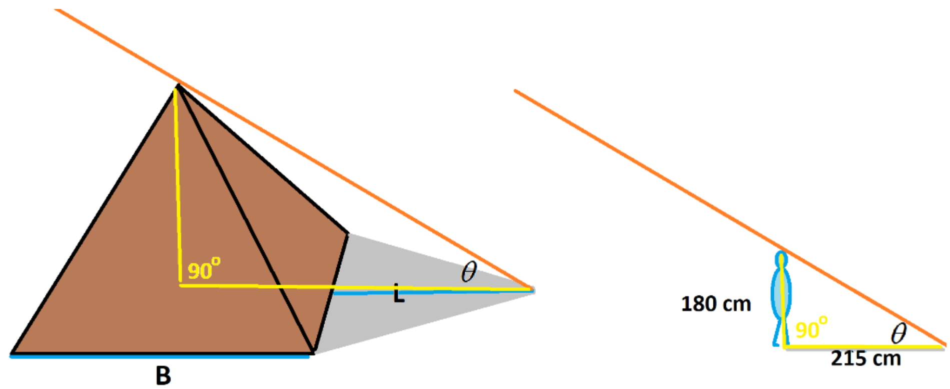


The base length of the pyramid is $B = 230$ m. Suppose the length of the shadow at his special time of the special day is $L = 32$ m. What is the height of the pyramid?

CLASS DISCUSSION ITEM 2

Did Thales really need to wait for the exact time of day when his shadow length matched his height?

Suppose Thales was 180 cm tall and he conducted this exercise at a time of day when his shadow was 215 cm long. Could he still have determined the height of the Great Pyramid?



The base length of the pyramid is $B = 230$ m.

Given that you do already know the height of the pyramid from the previous exercise (147 m), what is the length L of the shadow in this scenario?

How tall is a flagpole?

ACTIVITY

On a sunny day head out to a plaza or a field with flagpole equipped with a tape measure and a friend. Have your friend stand tall. Measure the length of her or his shadow. Measure the length of the shadow of the pole.

Do you have all the information you need to find the height of the pole? (If not, measure something in addition!)

Some Practice

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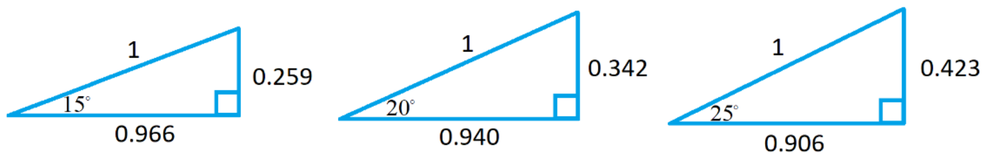

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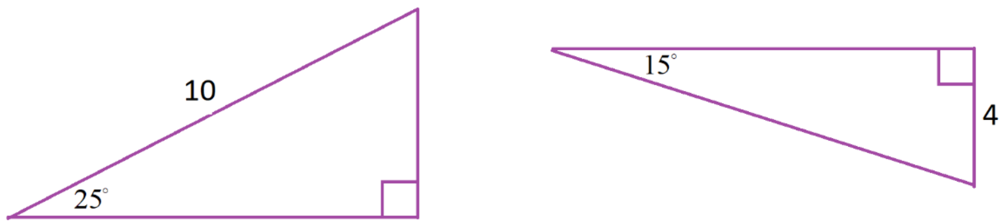
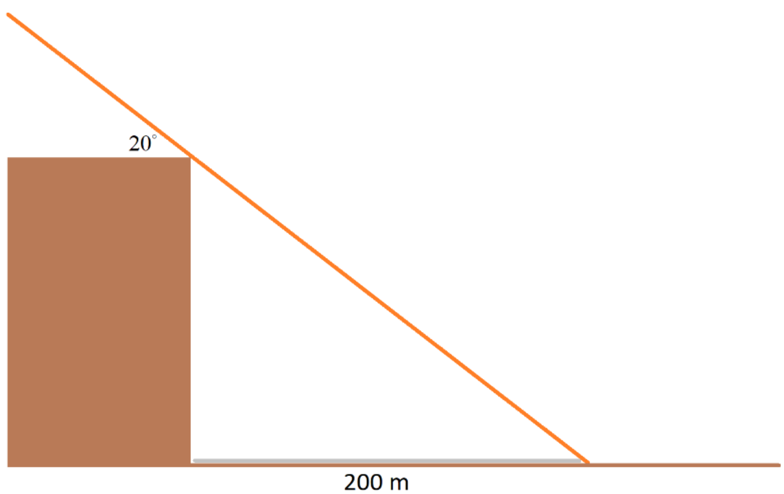
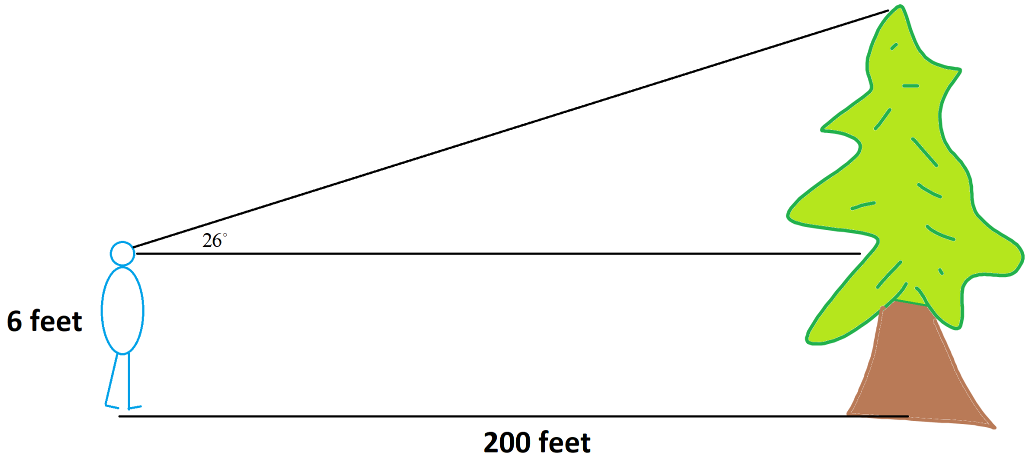


Question 2: A cliff casts a shadow 200 meters long when the rays of Sun hit the surface of the Earth at an angle of 20°. How high is the cliff?



(Scientists use essentially this method to determine the depth of craters on the Moon by looking at the lengths of shadows on surface photographs and knowing the angle of the Sun at the time the photographs were taken.)

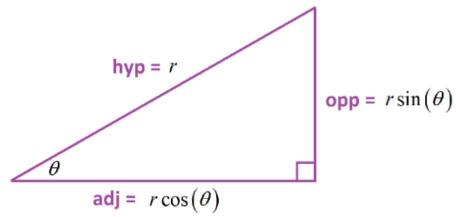
Question 3: Estimate the height of the tree.



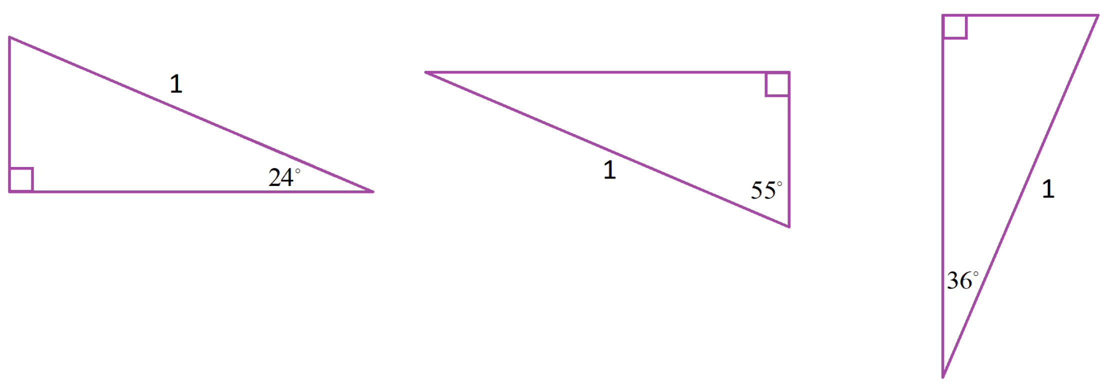
More Practice

For a right triangle possessing an angle θ we have

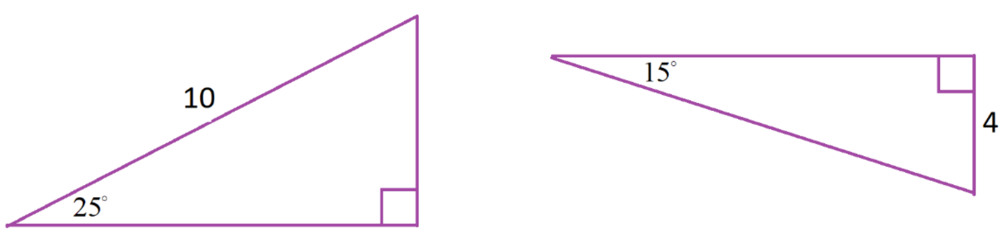
$$hyp = r$$
$$opp = r\sin(\theta)$$
$$adj = r\cos(\theta)$$



Question 1: According to your calculator, to three decimal places, what are the two missing side lengths in each of these triangles?

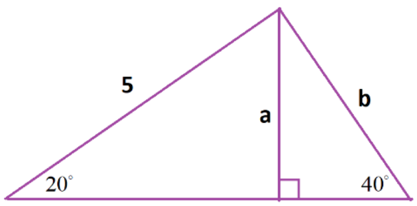


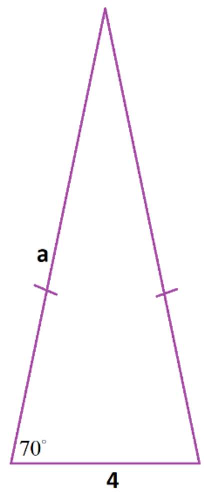
Question 2: Find the two missing side lengths in each of these right triangles.

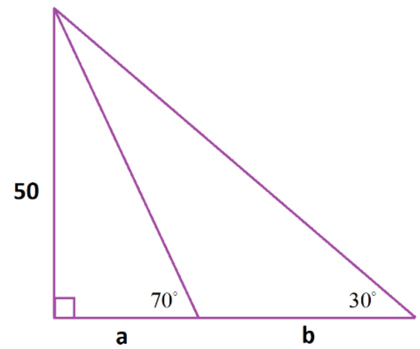


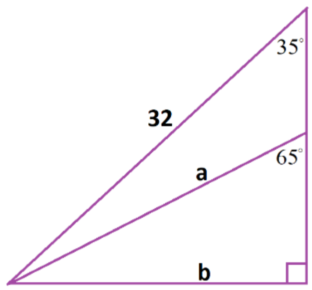
Yet More Practice

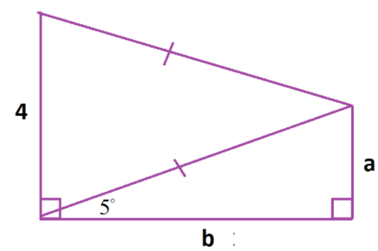
Find the lengths a and b indicated. Write your answers to three decimal places. Be sure to use your wits for these!











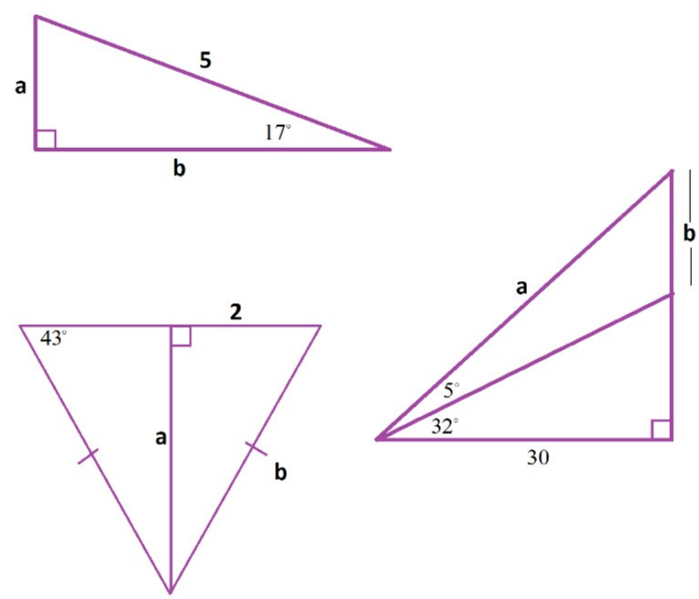
Still Yet More Practice Material

Here are some more problems to try. Let's assume all answers are to be to three-decimal places.

There might be words in some of these questions you have never seen before. Just do your best to use common sense to make good educated guesses as to their meaning. Usually drawing a picture of the situation at hand will reveal what must be meant. (One can, of course, always just google any unusual terms.)

Some of these problems are tricky. That is okay! Sit with them for a spell, try to make some progress and then let them be and go to some other tasks while you wait for a flash of insight to come to your brain.

Question 1: Find the missing side lengths.



Question 2: As observed at the top of a lighthouse 150 feet about seal level the angle of depression of a ship sailing directly towards the lighthouse changes from 30° to 40°. How far did the ship travel during this period of observation?

Question 3: I am looking out of my tenth-floor apartment window to a building directly opposite my building. The building I see is 50 meters away. Using my protractor I estimate the angle of elevation to the top of this second building to be 25° and the angle of depression down base of the building as 36°. How tall is that building?

Question 4: (From the 10^{th}-grade Tamil Nadu State Curriculum, India.)
There are two temples, one on each back of a river just opposite one another. One temple is 40 meters high. As observed from the top of this temple, the angles of depression of the top and the foot of the other temple are 12° and 21°, respectively.

How wide is the river?

How tall is the other temple?

Question 5: At 6 pm the Sun has an angle of elevation of 10.6°. At this time, when I stand tall, my shadow is 32 feet long. To the nearest inch, how tall am I?

You calculator also has the means "undo" sine and cosine values. For instance, if you are wondering which angle θ in a right triangle has sine value 0.8 look for the $\sin^{-1}$ button on your calculator. (The superscript -1 indicates "undoing.") Then type in $\sin^{-1}(0.8)$ to get $53..1^\circ$.

Quick Practice 6: Find $\sin(1^\circ)$, $\sin^{-1}(0.5)$, $\cos\left(\frac{1}{2}^\circ\right)$, $\cos^{-1}\left(\frac{1}{2}\right)$.

Question 7: Why does your calculator give an error message for $\sin^{-1}(1.01)$? Should your calculator also give an error message for $\cos^{-1}(1.01)$? (Does it?)

Question 8: What are the three angles in a triangle with side lengths 3, ,4, and 5?

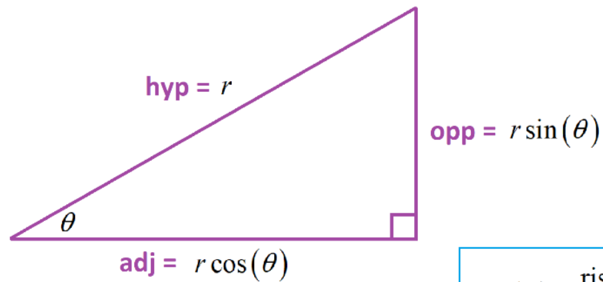
Question 9: Prodipta stands at the edge of a cliff on one side of a river, 20 meters wide. The cliff is 30 meters tall. What is the angle of depression of his line of site from the top of the cliff to the river bank directly opposite him?

Question 10: What is the angle of elevation of the graph of the line $y = 2x + 5$?

Question 11: A straight section of road has a 10% grade.

c) What is the angle of elevation of that section of road?
d) If I drive 50 meters upwards on that section of road, how high have I ascended?

Final Practice Exercise



$$\boxed{\tan(\theta) = \frac{\text{rise}}{\text{run}} = \frac{\text{opp}}{\text{adj}} = \frac{\cancel{r}\sin(\theta)}{\cancel{r}\cos(\theta)}}$$

Which of the 11 questions in the previous problem set can be solved more swiftly making use of the tangent function?

BRIEF SOLUTIONS

Some Practice

Question 1:

* The triangle with side of length **10.**
We have a scale factor of 10.
The remaining two sides are $0.423 \times 10 = 4.23$ and $0.906 \times 10 = 9.06$.

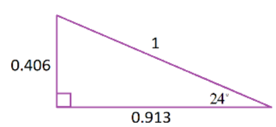
* The triangle with side of length **4.** **neate**
Scale factor is given by $4 = 0.259r$ and so $r = 15.444$. The remaining two sides are $0.966 \times 15.444 = 14.919$ and $1 \times 15.444 = 15.444$.

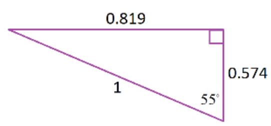
Question 2: We see a right triangle with angle 20°. Let r be the scale factor between this triangle and the basic one given. We have $200 = r \times 0.940$ and so $r = \dfrac{200}{\cos\left(20^\circ\right)} \approx 212.766$. Thus the height of the cliff is $r \times 0.342 \approx 73$ meters.

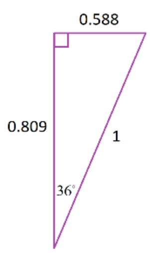
Question 3: We see a right triangle with angle approximately 25°. So $200 = r \times 0.906$ for some scale value r. We have $r \approx 220.751$. Thus the height of the tree is approximately $6 + 0.423 \times 220.751 \approx 102$ feet.

More Practice

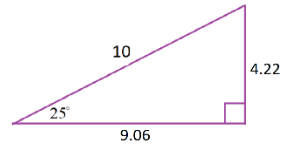
Question 1

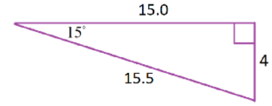




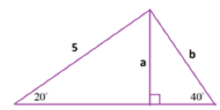


Question 2:

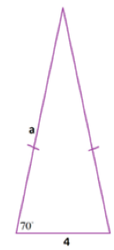




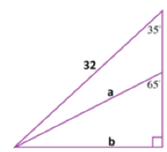
Yet More Practice



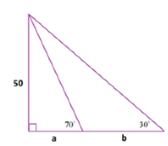
a = 1.710
b = 1.310



2 = a cos(70)
a = 5.848

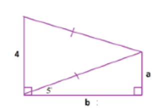


b = 32cos(35) = 26.213
26.213 = a x sin(65)
a = 28.923



50 = r x sin(70). So r = 53.209.
a = r x cos(70) = 18.199

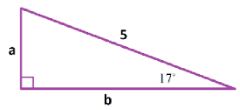
50 = R x sin(30). So R = 100.
a+b = R x cos(30) = 86.603
b = 86,603 - a = 68.404



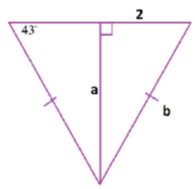
We see that a = 2.
Let r be the hypotenuse of the right triangle.
2 = r x sin(5) and so r = 22.947,
b = r x cos(5) = 22.860.

Still Yet More Practice

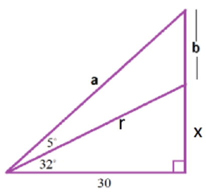
Question 1:



a =5sin(17) = 1.462
b = 5cos(17) = 4.782



2 = bcos(43), so b = 2.735.
a = bsin(43) = 1.865

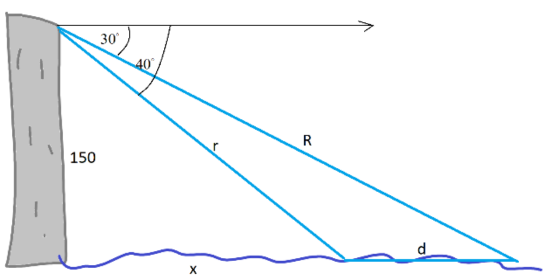


30 = a cos(37), so a = 37.564.
b+x = asin(37) = 22.607

30 = rcos(32), so r = 35.375.
x = rsin(32) = 18.746.

b = 22.607 - 18.746 = 3.861

Question 2:
Label a diagram as shown.



$150 = r\sin(40)$ and so $r = 233.359$.

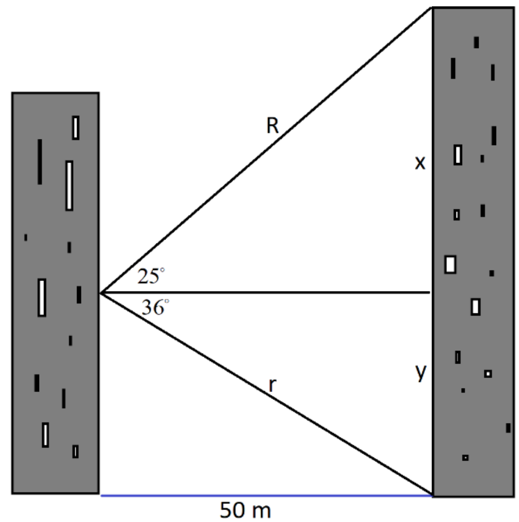
$150 = R\sin(30)$ and so $R = 300$.

$x = r\cos(40) = 178.763$.

$x + d = R\cos(30) = 259.808$.

So $d = 81.045$ feet.

Question 3:
Label a diagram as shown.

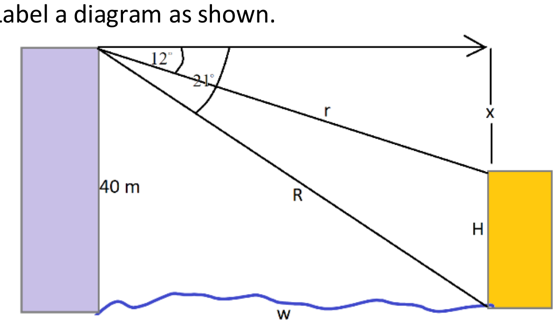


$50 = r\cos(36)$ and so $r = 61.803$.

$50 = R\cos(25)$ and so $R = 55.169$.

$x + y = r\sin(36) + R\sin(25) = 59.642$
meters.

Question 4:
Label a diagram as shown.



$40 = R\sin(21)$ and so $R = 111.617$.

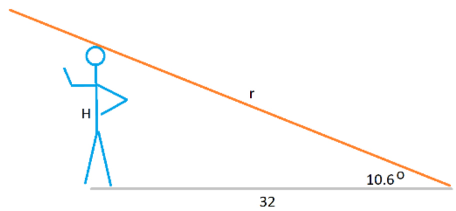
$w = R\cos(21) = 104.204$ meters.

$w = r\cos(12)$ and so $r = 106.532$.

$x = r\sin(12) = 22.149$.

$H + x = 40$.
$H = 40 - x = 17.851$ meters.

Question 5:
Label a diagram as shown.



$32 = r\cos(10.6)$ and so $r = 32.556$.

$H = r\sin(10.6) = 5.989$ feet.

Question 6:

$\sin(1^\circ) \approx 0.017$, $\sin^{-1}(0.5) = 30^\circ$.

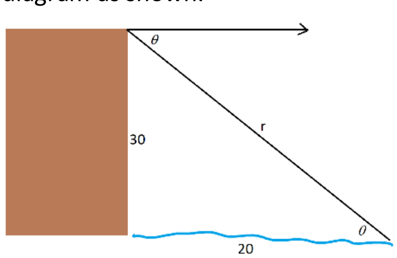
$\cos\left(\frac{1}{2}^\circ\right) \approx 0.99996$, $\cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$.

Question 7: The hypotenuse is the longest side of a right triangle. (Reason: If a right triangle has sides a and b, then its hypotenuse is $\sqrt{a^2+b^2}$. This is larger than $\sqrt{a^2+0^2} = a$ and $\sqrt{0^2+b^2} = b$.)

There is no right triangle of hypotenuse 1 with an opposite side or adjacent side of length 1.01.

Question 8: This is a right triangle, so one angle has measure 90°.
Let θ be the angle opposite the side of length 3.
Then $3 = 5\sin(\theta)$ and so $\sin(\theta) = 0.6$ giving $\theta = \sin^{-1}(0.6) \approx 36.870^\circ$. The other angle in the triangle is the complement to this: 53.130°.

Question 9:
Label a diagram as shown.

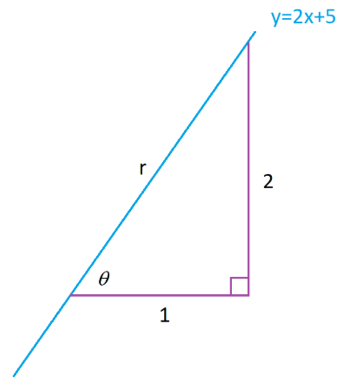


By the Pythagorean Theorem, $r = \sqrt{1300}$.

We have $30 = r\sin(\theta)$ and so

$\theta = \sin^{-1}\left(\frac{30}{\sqrt{1300}}\right) \approx 56.310^\circ$.

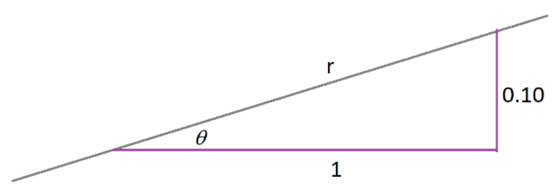
Question 10: We have a line of slope 2.
Label a diagram as shown.



Here $r = \sqrt{5}$ and $2 = \sqrt{5}\sin(\theta)$. Thus

$\theta = \sin^{-1}\left(\frac{2}{\sqrt{5}}\right) \approx 63.435^\circ$

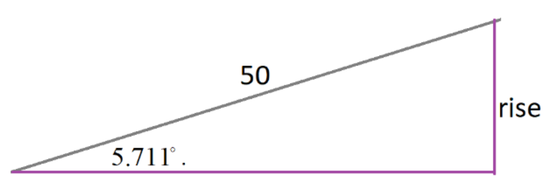
Question 11:
a) Label a diagram as shown.



We have $r = \sqrt{1.01}$ and

$\theta = \sin^{-1}\left(\frac{0.10}{\sqrt{1.01}}\right) \approx 5.711^\circ$.

b) $rise = 50\sin(5.711) = 4.975$ meters.



Final Practice Exercise

Elements of question 1, and questions 2, 3, 4 5, 8, 9, 10, and 11 of the previous problem set are more swiftly solved using the tangent operation.