Introduction to Turbomachines

1. Define turbo machines. Briefly classify Turbomachines (1a, 06,Dec18/19, 1a, 08,Dec17/Jan18)
2. Define turbo machines. Briefly classify on the basis of work transfer (1a, 04,June/July14,)
3. Define turbomachine . Explain with neat sketch construction and working of turbomachine (1a, 06, June/July 18,1a,06,Dec15/Jan16, 1a,4, Dec13/Jan14)
4. Enumerate the difference between a turbomachine and a positive displacement pump (1b, 06, June/July 18,1a,08, Dec 18/jan19, 1a, 05, June/july 17,1a, 05, Dec16/Jan17,1a,06, June/July16, 1a, 06, June/July13)
5. Define with appropriate expressions i) flow mcoefficient ii) head coefficient iii) power coefficient iv) specific speed (1a, 08, June/July 18 15ME53,1b, 05,June/July 17)
6. Define specific speed of pumps . Derive an expression for specific speed of a pump 1b,08, Dec16/Jan17,1b, 06, Dec14/Jan15)
7. Define specific speed of a turbine. Obtain an expression for the same in terms of P shaft power speed and head (1c,06, June/July13)
8. Define specific speed and specific power (1c, 04, June/July14)
9. Define specific speed of a pump and a turbine . Explain the significance of specific speed (1b, 06, Dec13/Jan14)
10. Define specific speed of pump . Show that specific speed of pump is given by
$N_s = \frac{N\sqrt{Q}}{H^{\frac{3}{4}}}$ (1b,06, Dec18/19, 1b, 06, Dec17/Jan 18)
11. What are Unit quantities.? Derive the expressions to each of them (1b, 06, June/July16)
12. With usual notations , derive expressions for unit Discharge coefficient, Head coefficient, and Power coefficient using Dimensional analysis (1c, 06, Dec15/Jan16)
13. Deducing an expression , expalain the significance of second law of thermodynamics applied to a turbo machine (1a,06, Dec12)
14. Explain the significance of first and second law of thermodynamics applied to a turbomachine (1a, 06, Dec14/Jan15)

15. Define the following efficiencies of power obsorbine macines i) Total to total efficiency ii) static to static efficiency (1b, 06, June/July14)

Turbo Machine: It is a device in which energy transfer takes place between a flowing fluid and a rotating element due to dynamic action and results in change of pressure and momentum of fluid

Example: Turbine, centrifugal compressors, centrifugal pumps:

Principle components of turbomachines:

1. Rotor which carries a series of blades, rotating in the steams of fluid flow
2. A stationary element (fixed blade) which usually acts as a guide way for the proper control of proper direction during energy conversion process
3. An input shaft

Classification of Turbo machine:

i) Classification Based on Direction of Energy Conversion.

The device in which the kinetic, potential or intermolecular energy held by the fluid is converted in the form of mechanical energy of a rotating member is known as a *turbine* .

The machines, on the other hand, where the mechanical energy from moving parts is transferred to a fluid to increase its stored energy by increasing either its pressure or velocity are known as *pumps, compressors, fans or blowers* .

ii) Classification based on basic working principle

Ie Impulse and Reaction turbine----------

The machine for which the change in static head in the rotor is zero is known *impulse machine.* In these machines, the energy transfer in the rotor takes place only by the change in dynamic head of the fluid

In reaction turbine energy transfer in the rotor takes place by change in static and dynamic head of the fluid

iii) Classification based on the direction of fluid flow:

- **Axial** in which fluid enters and leaves parallel to the axis of rotor
- **Radial** in which fluid enters and leaves along the direction perpendicular to the axis of shaft
- **Tangential** in which fluid flow is tangent to the shaft
- **Mixed flow**: in which fluid entry is axial , exit is radial or vice versa

Difference between positive displacement machine and turbo machine:

Sl No	Positive Displacement machine	Turbomachine
1	Energy transfer takes place due to static action and thermodynamic between rotor and static fluid	Energy transfer takes place between rotor and fluid due to dynamic action and thermodynamics between rotor and flowing fluid
2	Reciprocating in nature	Rotary in nature
3	Fluid flow is Unsteady	Fluid flow is Steady
4	Fluid containment is positive	Fluid containment is not positive
5	Low speed machine	High speed machine
6	Complex in design	Simple in design
7	Balancing of parts is difficult	Balancing of parts is easy
8	There is no problem of surging and cavitation	There is problem of surging and cavitiaon
9	Conversion efficiency is high	Conversion efficiency is low
10	Volumetric efficiency low	Volumetric efficiency is high

Dimension Analysis

Force(N)/resistnace	mass x acceleration	kg m/sec^2	MLT^{-2}
work/Energy/Torque	Force x displacement	Nm	$MLT^{-2}xL = ML^2T^{-2}$
Pressure/Change in Pressure	Force/Area	N/m^2	$MLT^{-2}/L^2=ML^{-1}T^{-2}$
Power	workdone /sec	Nm/s	$ML^2T^{-2}xT=ML^2T^{-3}$
Velocity	distance/sec	m/s	LT^{-1}
Density	mass/Volume	kg/m^3	ML^{-3}
Absolute viscosity		Ns/m^2	$MLT^{-2}xT/L^2=ML^{-1}T^{-1}$
Kinematic viscosity	Absolute Viscosity/Density	m^2/s	$ML^{-1}T^{-1}/ ML^{-3}=L^2T^{-1}$
Surface tension		N/m	$MLT^{-2}/L =MT^{-2}$
Discharge		m^3/s	L^3T^{-1}
Energy per Unit mass	gH	m^2/s	$ML^2T^{-2}/M=L^2T^{-2}$
Surface roughness		m	L
Length/Diameter/Height		m	L
Angular speed, speed of rotor		rad/sec, rpm	T^{-1}
Efficiency/pressure ratio		No dimension	Dimensionless number

Performance of a turbomachine depends upon the following

Discharge Q, speed or rpm N, size of the rotor D, energy per unit mass gH, Power P, density ρ, dynamic viscosity μ, Using dimensional analysis find the π terms

$f(Q, N, D, gH, P, \rho, \mu) = 0$

no of variables n =7

no of fundamental variables m = 3

no of π terms = n-m=7-3 =4

Let us select repeated variables , ρ (fluid property) N (dynamic property) D (Geometrical Property)

$\Pi_1 = \rho^{a1}, N^{b1}, D^{c1}, Q$

$\Pi_2 = \rho^{a2}, N^{b2}, D^{c2}, gH$

$\Pi_3 = \rho^{a3}, N^{b3}, D^{c3}, P$

$\Pi_4 = \rho^{a4}, N^{b4}, D^{c4}, \mu$

$Q=L^3T^{-1}$, $N=T^{-1}$, $D=L$, $\rho=ML^{-3}$, $gH= L^2T^{-2}$, $\mu=ML^{-1}T^{-1}$, $P=ML^2T^{-3}$

Π_1

$M^0L^0T^0 = (ML^{-3})^{a1}\ (T^{-1})^{b1}\ L^{c1}\ L^3T^{-1}$

M ----- $0=a_1$

T------ $0= -b_1 -1$ ie $b_1 = -1$

L------- $0= -3a_1+ C_1 + 3$ ie $0= -3x(0) + c_1+3$ ie $C_1= -3$

$\Pi_1= \rho^0, N^{-1}, D^{-3}, Q$

$=\frac{Q}{ND^3}$ **Flow coefficient**

Π_2

$M^0L^0T^0 = (ML^{-3})^{a2}\ (T^{-1})^{b2}\ L^{c2}\ L^2T^{-2}$

M ----- $0=a_2$

T------ $0= -b_2 -2$ ie $b_2 = -2$

L------- $0= -3a_2+ C_2 + 2$ ie $0= -3x(0) + c_2+2$ ie $C_2= -2$

$\Pi_2= \rho^0, N^{-2}, D^{-2}, gH$; **$\Pi_2$= $=\frac{gH}{N^2D^2}$ Head coefficient**

Π_3

$M^0L^0T^0 = (ML^{-3})^{a3}\ (T^{-1})^{b3}\ L^{c3}\ ML^2T^{-3}$

M ----- $0=a_3+1$ ie $a_3=-1$

T------ $0= -b_3 -3$ ie $b_3 = -3$

L------- $0= -3a_3+ C_3 + 2$ ie $0= -3x(-1) + c_3+2$ ie $C_3= -5$

$\Pi_3= \rho^{-1}, N^{-3}, D^{-5}, P$**;** **$\Pi_3=$** $=\boldsymbol{\frac{P}{\rho^1 N^3 D^5}}$ **Power coefficient**

Π_4

$M^0L^0T^0 = (ML^{-3})^{a4}\ (T^{-1})^{b4}\ L^{c4}\ ML^{-1}T^{-1}$

M ----- $0=a_4+1$ ie $a_4=-1$

T------ $0= -b_4 -1$ ie $b_4 = -1$

L------- $0= -3a_4+ C_4 - 1$ ie $0= -3x(-1) + c_4 -1$ ie $C_4= -2$

$\Pi_4= \rho^{-1}, N^{-1}, D^{-2}, \mu$; **$\Pi_4=$** $=\boldsymbol{\frac{\mu}{\rho^1 N^1 D^2}}$

<u>Significance of π terms</u>

$\Pi_1 =\frac{Q}{ND^3}$ is called as flow coefficient / capacity coefficient

It is defined as the volume flow rate of the fluid through a turbomachine of unit diameter of runner operating at unit speed ie flow coefficient = Q when N=1 and H=1

From above π_1 term for a pump of certain diameter running at various speeds the discharge is proportional to the speed of the pump. This is called as First fan Law

$\Pi_2 =$ $=\frac{gH}{N^2D^2}$ is called as Head coefficient

Since U is directly proportional to DN, N^2D^2 can be replaced in π term as U^2

Hence $\pi_2 = \frac{H}{\frac{U^2}{g}}$

From the above expression, for a given impeller, head varies as the square of the tangential speed of the rotor. This is called second fan law

$\Pi_3=\frac{P}{\rho^1 N^3 D^5}$ Power coefficient

From the above expression for the same runner of turbomachine and same fluid Power developed by the turbomachine is directly proportional to the cube power of speed. This is called 3rd fan law

Specific speed for a pump: It is defined as the speed of the geometrically similar turbomachine (pump) which discharges $1m^3/s$ under unit head

$N=N_s$ when $Q=1m^3/s$ and $H=1m$

$\Pi_2 = \frac{gH}{N^2D^2}$ is called as Head coefficient

From the above expression $H\alpha D^2 N^2$

$D\alpha \frac{\sqrt{H}}{N}$ -------------1

From flow coefficient $\frac{Q}{ND^3}$; Ie $Q\alpha ND^3$

Substituting 1 in above eqution

$Q \alpha N \left(\frac{\sqrt{H}}{N}\right)^3$; $Q\alpha \frac{H^{3/2}}{N^2}$; $Q =\frac{kH^{3/2}}{N^2}$

From the definition of specific speed $N=N_s$ when Q=1 and H=1

Hence $1= \frac{k}{N_s^2}$ ie $k =N_s^2$; Hence $Q =\frac{N_s^2}{N^2} H^{3/2}$; $N_s= \frac{N\sqrt{Q}}{H^{3/4}}$

Specific speed for a Turbine: It is defined as the speed of the geometrically similar turbomachine (turbine) which develops unit power under unit head

$N=N_s$ when $Q=1m^3/s$ and $H=1m$

$\Pi_2 = \frac{gH}{N^2D^2}$ is called as Head coefficient

From the above expression $H\alpha D^2 N^2$

$D\alpha \frac{\sqrt{H}}{N}$ -------------1

From Power coefficient $\frac{P}{\rho^1 N^3 D^5}$

Ie $P\alpha N^3 D^5$ for same fluid

Substituting 1 in above equation

$P \alpha N^3 \left(\frac{\sqrt{H}}{N}\right)^5$; $P\alpha \frac{H^{5/2}}{N^2}$; $P =\frac{kH^{5/2}}{N^2}$

From the definition of specific speed $N=N_s$ when P=1 and H=1

Hence $1=\frac{k}{N_s^2}$ ie $k = N_s^2$

Hence $P = \frac{N_s^2}{N^2} H^{5/2}$; $N_s = \frac{N\sqrt{P}}{H^{5/4}}$ where P is in kW

Unit quantities: (Applied to same machine)

Unit discharge Q_u: is defined as the discharge of a pump under unit Head

$Q = AV$; $Q \alpha \sqrt{H}$ for a given pump as $V=\sqrt{2gH}$; $Q = k\sqrt{H}$ –eqn 1

From definition of unit discharge $Q= Q_u$ when $H=1$

$Q_u = k x 1$; $k= Q_u$

Substituting k in eqn 1 ; $Q=Q_u \sqrt{H}$; Therefore $Q_u=\frac{Q}{\sqrt{H}}$

Unit Speed N_U

Unit Speed is defined as a speed of the turbomachine working under unit head

From flow coefficient $gH \alpha N^2D^2$

For the given turbomachine $H \alpha N^2$; $N \alpha \sqrt{H}$; $N = k\sqrt{H} - eqn\ 1$

From definition of unit speed $N=N_u$ when $H=1$

$N_u = k x 1$; $k= N_u$

Substituting k in eqn 1 ; $N=N_u\sqrt{H}$; $N_u = \frac{N}{\sqrt{H}}$

Unit Power P_U

Unit Power defined as the power of turbomachine working under unit head

$P=\omega QH$

$P\alpha \sqrt{H}$ H as $Q\alpha\sqrt{H}$ for a given pump

$P\alpha H^{3/2}$; $P=k H^{3/2} - eqn\ 1$

From the definition of unit power $P=P_u$ when $H=1m$

$P_u = k x 1$; $k= P_u$

Substituting k in equation 1 ; $P= P_u H^{3/2}$; $P_u= \frac{P}{H^{3/2}}$

Reynolds Number: is defined as a ratio of inertia force to viscous force

Reynold number= $\frac{Inertia\ force}{Viscous\ force}$ =$\frac{\rho VD}{\mu}$

In a pipe flow if R_e<2000------Laminar flow

If 2000<R_e<3000 -------Transition

If $R_e > 3000$ ----------turbulent flow

In turbomachine Reynold mumber is not such an important parameter since machine losses are not determined by viscous force alone because various other losses such as losses due to shock at entry, turbulence, impact, friction , leakage and roughness

Most of the turbomachines use relatively low viscous fluid like air steam, water and lighoils. Therefore, the flow in a turbomachine is turbulent in nature

According to Moodys friction factor depends only on relative roughness and not on Reynold number which becomes constant for turbulent flow

For Hydraulic turbine, prototype will have low relative roughness due to its large size, even though model has a smooth surface. Due to this dissimilarties of surface roughness the model similarity loss must ve corrected for Reynolds number dependency. Moody has suffested an equation to determine efficiencies from experiment on a geometrically similar model

The equation I s η_p= 1- (1-η_m)$\left(\frac{D_m}{D_p}\right)^2$

1. An output of 10kW was recorded on a turbine, 0.5m diameter, revolving at a speed of 800rpm, under a head of 20m. What is the diameter and output of another turbine which works under a head of 180m at a speed of 200rpm when their efficiencies are same. Find the specific speed and name the turbine can be used.(1c, 10, June/July 17)

   Solution
   $P_1 = 10kW$; D_1=0.5m; N_1=800rpm; H_1=20m
   D_1=?; P_2 =? N_2=200rpm; H_2=180m

   $\eta_1 = \eta_2$; N_s =?

   $\frac{gH_1}{N_1^2 D_1^2} = \frac{gH_2}{N_2^2 D_2^2}$; $D_2^2 = \frac{H_2}{H_1}\left(\frac{N_1}{N_2}\right)^2 D_1^2$; $D_2^2 = \frac{180}{20}\left(\frac{800}{200}\right)^2 0.5^2$; $D_2^2 = 36$; $D_2 = 6m$

   $\frac{P_1}{\rho\, N_1^3 D_1^5} = \frac{P_2}{\rho\, N_1^3 D_1^5}$; $P_2 = P_1\left(\frac{N_2}{N_1}\right)^3\left(\frac{D_2}{D_1}\right)^5$; $P_2 = 10 * \left(\frac{200}{800}\right)^3 * \left(\frac{6}{0.5}\right)^5$; $P_2 = 38880kW$

$N_s = \frac{N_1\sqrt{P_1}}{H_1^{\frac{5}{4}}}$; $N_s = \frac{800\sqrt{10}}{20^{\frac{5}{4}}}$; N_s =59.81

2. Tests on a turbine runner 1.25m in diameter at 30m head gave the following results, power developed =736kW, speed is 180rpm and discharge 2.70m^3/sec. Find the diameter speed and discharge of a runner to operate at 45m head and give 1472kW at the same efficiency . What is the specific speed of both the turbines? (1c, 08, Dec18/19,1b,08, Dec18?jan19 15ME53,1c, 08 Dec16/17,1c, 10, Dec13/Jan14)

D_1=1.25m; H_1=30m; $P_1 = 736kW$ N_1=180rpm $Q_1 = 2.70$ m^3/s

D_2=? ; N_2 =? Q_2 =? H_2= 45m ; P_2=1472kW;

Discharge:

$$\eta_1 = \eta_2$$

$$\frac{P_1}{\omega Q_1 H_1} = \frac{P_2}{\omega Q_2 H_2}$$

$$\frac{736}{2.7*30} = \frac{1472}{Q_2*45}; Q_2 = 3.6m^3/s;$$

Speed

$$N_s = \frac{N_1\sqrt{P_1}}{H_1^{\frac{5}{4}}} = \frac{N_2\sqrt{P_2}}{H_2^{\frac{5}{4}}}; \frac{180\sqrt{736}}{30^{\frac{5}{4}}} = \frac{N_2\sqrt{1472}}{45^{\frac{5}{4}}}; \quad N_2 = 211.28rpm$$

Diameter

$$\frac{Q_1}{N_1 D_1^3} = \frac{Q_2}{N_2 D_2^3}; \frac{2.7}{180*1.25^3} = \frac{3.6}{211.28*D_2^3}; \ D_2^3 = 2.219 \quad D_2 = 1.303m$$

Specific Speed

$$N_s = \frac{N_1\sqrt{P_1}}{H_1^{\frac{5}{4}}} = \frac{N_2\sqrt{P_2}}{H_2^{\frac{5}{4}}}; \quad N_s = \frac{180\sqrt{736}}{30^{\frac{5}{4}}} = 69.55$$

Type of turbine:

3. A model of turbine 1m in diameter acting under a head of 2m runs at 150rpm. Estimate the scale ratio of the prototype develops 20MW under a head of 225m with specific speed of 100 (1d, 06, June/July14)

D_m=1m; H_m=2m; N_m=150rpm;scale ratio = $\frac{D_m}{D_p}$ =?

P_p=20MW=20000kW; H_P= 225m; $N_s = 100$

$N_s = \frac{N_m\sqrt{P_m}}{H_m^{\frac{5}{4}}}$; $100 = \frac{150*\sqrt{P_m}}{2^{\frac{5}{4}}}$; $P_m = 2.514kW$

$N_s = \frac{N_P\sqrt{P_P}}{H_P^{\frac{5}{4}}}$; $100 = \frac{N_P\sqrt{20000}}{225^{\frac{5}{4}}}$; $N_P = 616.188rpm$

$\frac{P_m}{\rho_m N_m^3 D_m^5} = \frac{P_P}{\rho_P N_p^3 D_P^5}$; ; $\left(\frac{D_m}{D_P}\right)^5 = \frac{2{,}514}{20000}\left(\frac{150}{616.188}\right)^3$; $\left(\frac{D_m}{D_P}\right)^5 = 1.813 * 10^{-6}$; $\frac{D_m}{D_P} = \frac{1}{14.07}$

Scale ratio is 1: 14.07

4. A windmill model of 1:10 scale develops 2kW under a head of 6m at 500rpm. A prototype work under a head of 40m. Assuming that the efficiencies of model and prototype remains same Determine the power developed , speed of the prototype and its specific speed (1c, 08, June/July18)

$\frac{D_m}{D_P} = \frac{1}{10}$; $P_m = 2kW$; H_m=6m; N_m=500rpm;

$H_P = 40m$; $\eta_m = \eta_P$; P_P =?; N_P=?; N_s=?;

<u>Speed of the prototype</u>

$\frac{gH_m}{N_m^2 D_m^2} = \frac{gH_p}{N_P^2 D_P^2}$; $N_P^2 = N_m^2 * \frac{H_P}{H_m} * \left(\frac{D_m}{D_P}\right)^2$; $N_P^2 = 500^2 * \frac{40}{6} * \left(\frac{1}{10}\right)^2$ $N_P^2 = 16666.66$

N_P=129.1rpm

<u>Power developed by prototype</u>

$\frac{P_m}{\rho_m N_m^3 D_m^5} = \frac{P_P}{\rho_P N_p^3 D_P^5}$; $P_P = P_m * \left(\frac{N_P}{N_m}\right)^3 * \left(\frac{D_P}{D_m}\right)^5$; $P_P = 2 * \left(\frac{129.1}{500}\right)^3 * 10^5$;

$P_P = 3442.9kW$

<u>Specific speed</u>

$N_s = \frac{N_m\sqrt{P_m}}{H_m^{\frac{5}{4}}}$; $N_s = \frac{500\sqrt{2}}{6^{\frac{5}{4}}}$ $N_s = 75.30$

5. A turbine model of 1:10 develops 2.0kW under a head of 6m at 500rpm. Find the power developed by the prototype under a head of 40m. Also find the speed of prototype and its specific speed. Assume the turbine efficiencies to remain same (1c, 06, Dec17/Jan18)

   Solution is same as above problem

6. A one fourth scale turbine model is tested under a head of 10meters. The prototype is required to work under a head of 30meters and to run at 425rpm. Estimate the speed of the model if it develops 125kW and uses 1.1m^3/s of water at this speed . Also

calculate the power output of the prototype and suggest the type of turbine (1c, 08, Dec14/Jan15)

$\frac{D_m}{D_P} = \frac{1}{4}; H_m = 10m; H_P = 30m$ N_P=425rpm $P_m = 125kW$; Q_m=1.1m^3/s; N_m=?;

P_P =?;

Speed of the model

$\frac{gH_m}{N_m^2 D_m^2} = \frac{gH_p}{N_P^2 D_P^2}$; $N_m^2 = N_P^2 * \frac{H_m}{H_P} * \left(\frac{D_P}{D_m}\right)^2$; $N_m^2 = 425^2 * \frac{10}{30} *(4)^2$

N_m=981.49rpm

Power developed by Prototype

$\frac{P_m}{\rho_m N_m^3 D_m^5} = \frac{P_P}{\rho_P N_p^3 D_P^5}$; $P_P = P_m * \left(\frac{N_P}{N_m}\right)^3 * \left(\frac{D_P}{D_m}\right)^5$; $P_P = 125 * \left(\frac{425}{981.5}\right)^3 * 4^5$;

$P_P = 10392.16kW$

Specific speed

$N_s = \frac{N_m\sqrt{P_m}}{H_m^{\frac{5}{4}}}$; $N_s = \frac{981.5\sqrt{125}}{10^{\frac{5}{4}}}$ $N_s = 617.08$

Type of turbine suggested:

7. The quantity of water available for a hydroelectric power station is 260m^3/sec. The head developed is 1.73m. If the speed of the turbines is 50rpm and efficiency 82.5%, find the number of turbines . Assume specific speed to be 760.(1c, 06, Dec25/Jan16, ,)*

Q=260cumecs =260m^3/s; H=1.73m; N=50rpm; η_o=82.5% number of turbine=?

$\eta_o = \frac{P}{\omega QH}$; $0.85 = \frac{P_T}{9810*260*173}$ ie $P_T = 3640343W$=$3640.343kW$

$N_{s\ each} = \frac{N\sqrt{P_{each}}}{H^{5/4}}$; $760 = \frac{50\sqrt{P_{each}}}{1.73^{5/4}}$ $\sqrt{P_{each}} = 30.15$; $P_{each} = 909.49kW$

No of turbine required = $\frac{P_T}{P_{each}}$; n $= \frac{3640.343}{909.49} = 4$

8. A single stage centrifugal pump with impellor diameter of 30cm rotates at 2000rpm and with 3m^3 of water per second to a height of 30m with an efficiency of 75%. Find a) the number of stages and b) diameter of each impeller of a similar multistage pump to lift 5m^3 of water per sec to a height of 200m, when rotating at 1500rpm

A single stage centrifugal pump with impellor diameter of 30cm rotates at 2000rpm and with 3m^3 of water per second to a height of 30m with an efficiency of 75%.

D_1=30cm; N_1=2000rpm; Q_1 =3m^3/s ; H_1 =30m;

In multistage: find the number of stage required if similar single stage pumps (as above) are used to lift 5m^3/s to a height o 200m when rotating at 1500rpm

Q_2=5m^3/s; N_2=2000rpm H_T =200m

$\frac{N_1\sqrt{Q_1}}{{H_1}^{3/4}}=\frac{N_2\sqrt{Q_2}}{{H_2}^{3/4}}$; $\frac{2000\sqrt{3}}{30^{3/4}}=\frac{1500\sqrt{5}}{{H_2}^{3/4}}$; H_2=28.73m

<u>No of stage required =</u>

$H_T = nH_2$; 200 =n x 28.73; n=6.96 ie 7

i) <u>Diameter of the impellor</u>

$\frac{gH_1}{N_1^2D_1^2}=\frac{gH_2}{N_2^2D_2^2}$; $\frac{30}{2000^2x300^2}=\frac{28.73}{1500^2xD_2^2}$; $D_2 = 39m$

9. A quantity of water available for hydel station is 310cumecs under a head of 1.8m. Assuming speed of each turbine is 60rpm and efficiency of 85% find the no of turbines required and power produced by each turbine. Each turbine has a specific speed of 800(metric)

- A quantity of water available for hydel station is 310cumecs under a head of 1.8m. ie Q=310cumecs =310m^3/s; H=1.8m
- Assuming speed of each turbine is 60rpm and efficiency of 85% N=60rpm; η_o=0.85
- find the no of turbines required and power produced by each turbine. Each turbine has a specific speed of 800
  ie no of turbines=? P_{each}=?; N_{seach} =800rpm (metric)

$\eta_o=\frac{P}{\omega QH}$; 0.85=$\frac{P_T}{9810x310x1.8}$ ie $P_T = 4652883\ W$ =4652.9kW

$N_{s\ each}=\frac{N\sqrt{P_{each}}}{H^{5/4}}$;800= $\frac{60\sqrt{P_{each}}}{1.8^{5/4}}$ ie $P_{each} = 772.8\ Metric\ HP$ since specific in turbine is metric power is in HP

P_{each} =772.5x 0.7355 kW; P_{each} = = 566.25kW

No of turbine required = $\frac{P_T}{P_{each}}$ No of turbine required = $\frac{4652.9}{566.25} = 8.2$

Hence 9 turbine required

9. From the performance curves of the turbine it is seen that a turbine of 1m diameter acting under a head of 1m develops a speed of 25 rpm. What diameter should be prototyped if it is developed 1000kW working under a head of 200m with a specific speed of 150 (SI units)
10. A model of a centrifugal pump absorbs 5kW at a speed of 1500rpm, pumping water against a head of 6m. The large prototype pump is required to pump water to a head of 30m. The scale ratio of diameter is 4. Assume same efficiency and similarities, find

(a) the speed (b) power of prototype and (c) the ratio of discharge of prototype and model (1b, 08, June/July 18, 15ME53)

A model of a centrifugal pump absorbs 5kW at a speed of 1500rpm, pumping water against a head of 6m. ie P_m =5kW; N_m =1500rpm; H_m=6m

The large prototype pump is required to pump water to a head of 30m ie H_P=30m

The scale ratio of diameter is 4. Ie $\frac{D_P}{D_m} = 4$

Speed of the model

$\frac{gH_m}{N_m^2 D_m^2} = \frac{gH_p}{N_P^2 D_P^2}$; $N_P^2 = \frac{H_P}{H_m} x \left(\frac{D_m}{D_p}\right)^2 x N_m^2$; $N_P^2 = \frac{30}{6} x \left(\frac{1}{4}\right)^2 x 1500^2$; N_P=838.5 rpm

Power of Prototype

$\frac{P_m}{\rho_m N_m^3 D_m^5} = \frac{P_P}{\rho_P N_P^3 D_P^5}$; $\rho_m = \rho_P$ (same fluid); $P_P = P_m \left(\frac{D_p}{D_m}\right)^5 \left(\frac{N_P}{N_m}\right)^3$

$P_P = 5(4)^5 \left(\frac{838.5}{1500}\right)^3$; P_P=894.34kW

Ratio of discharge of prototype and model

$\frac{Q_m}{N_m D_m^3} = \frac{Q_P}{N_P D_P^3}$; $\frac{Q_P}{Q_m} = \frac{N_P}{N_m} x \left(\frac{D_p}{D_m}\right)^3$; $\frac{Q_P}{Q_m} = \frac{838.5}{1500} x(4)^3$; $\frac{Q_P}{Q_m} = 35.76$

11. Two geometrically similar pumps are running at same speed of 1000rpm. One pump has an impeller diameter of 0.3m and lifts water at the rate of 20litres /sec against a head of 15m Determine the head and impeller diameter of other pump to deliver half the discharge (1b, 08, June/July 13)

$N_1 = N_2 = 1000rpm; D_1 = 0.3m; Q_1 = 20lit/s = 20 * 10^{-3}m^3/s; H_1 = 15m; H_2 = ?\ D_2 =?\ Q_2 = \frac{Q_1}{2}$

$\frac{Q_1}{N_1 D_1^3} = \frac{Q_2}{N_2 D_2^3}$; $\frac{Q_1}{D_1^3} = \frac{Q_2}{D_2^3}$ as $N_1 = N_2 = 1000rpm$

$\frac{Q_1}{D_1^3} = \frac{\frac{Q_1}{2}}{D_2^3}$; $\frac{1}{D_1^3} = \frac{1}{2D_2^3}$; $\frac{1}{0.3^3} = \frac{1}{2D_2^3}$; $D_2^3 = \frac{0.3^3}{2}$; $D_2 = 0.238m$

$\frac{gH_1}{N_1^2 D_1^2} = \frac{gH_2}{N_2^2 D_2^2}$; $H_2 = H_1 \left(\frac{D_1}{D_2}\right)^2 \left(\frac{N_2}{N_1}\right)^2$; $H_2 = 15 * \left(\frac{0.3}{0.238}\right)^2 * \left(\frac{1000}{1000}\right)^2$; $H_2 = 9.48m$

12. A model of Francis turbine of 1:5 scale ratio is tested under a head of 1.5m. It develops 3kW at 360rpm. Determine the speed and power developed under a head of 6m. Find its specific speed

- A model of Francis turbine of 1:5 scale ratio is tested under a head of 1.5m ie $\frac{D_m}{D_p}=\frac{1}{5}$ and H_m=1.5m
- It develops 3kW at 360rpm ie P_{m1}=3kW and N_{m1}=360rpm
- Determine the speed and power developed under a head of 6m. Find its specific speed ie N_P=? P_P=? H_P=6m

Solution

$\frac{gH_m}{N_m^2 D_m^2}=\frac{gH_p}{N_P^2 D_P^2}$; $N_P^2=\frac{H_p}{H_m}x\left(\frac{D_m}{D_P}\right)^2 xN_m^2$; $N_P^2=\frac{6}{1.5}x\left(\frac{1}{5}\right)^2 x\ 360^2$; $N_P^2=20736$

N_P=144rpm

$N_s=\frac{N_m\sqrt{P_m}}{H_m^{5/4}}$; $N_s=\frac{360\sqrt{3}}{1.5^{5/4}}$=375.62

$N_s=\frac{N_P\sqrt{P_P}}{H_P^{5/4}}$; $375.62=\frac{144\sqrt{P_P}}{6^{5/4}}$; $P_P=600kW$

13. A Pelton wheel produces 10000kW while working under a head of 400m and running at a speed of 300rpm. Assuming an overall efficiency of 82% , find the unit quantities, During the off season, the head over the turbine reduces to 350m. Find the corresponding speed, discharge and power for the same efficiency

$P=10000x10^3W$; H=400m; N=300rpm; η_o=0.82

$\eta_o=\frac{P}{\omega QH}$; $0.82=\frac{10000x10^3}{9810xQx400}$; Q=3.1078$m^3/s$

Unit Discharge

$Q_u=\frac{Q}{\sqrt{H}}$; $Q_u=\frac{3.1078}{\sqrt{400}}$; Q_u=0.1553m^3/s

Unit speed:

$N_u=\frac{N}{\sqrt{H}}$; $N_u=\frac{300}{\sqrt{400}}$; N_u =15rpm

Unit Power

$P_u=\frac{P}{H^{3/2}}$; $P_u=\frac{10000x10^3}{400^{3/2}}$; P_u=1250W

If head reduces to 350m

$\frac{Q_1}{\sqrt{H_1}}=\frac{Q_2}{\sqrt{H_2}}$; $\frac{3.1078}{\sqrt{400}}=\frac{Q_2}{\sqrt{350}}$; Q_2=2.907m^3/s

$\frac{N_1}{\sqrt{H_1}}=\frac{N_2}{\sqrt{H_2}}$; $\frac{300}{\sqrt{400}}=\frac{N_2}{\sqrt{350}}$; N_2=280.624rpm

$\frac{P_1}{{H_1}^{3/2}} = \frac{P_2}{{H_2}^{3/2}}$; $\frac{10000x10^3}{400^{3/2}} = \frac{P_2}{350^{3/2}}$; P_2=8184571.29W

14. The following data were obtained from the main characteristics of a Kaplan turbine of runner diameter 1m P_u=30.695, Q_u =108.6, N_u =63.6. Estimate a) the runner diameter b) the discharge c) the speed of a similar runner working under a head of 30m and developing 2000kW. Also, d) determine the specific speed of the runner (1c, June/July16)

$D_1 = 1m; P_u = 30.695; Q_u = 108.6;\ N_u = 63.6;$

$D_2 =?;\ Q_2 =?;\ N_2 =?;\ H_2 = 30m;\ P_2 = 2000kW;\ N_s =?$

$N_s = \frac{N\sqrt{P}}{H^{5/4}}; N_s = \frac{N\sqrt{P}}{\sqrt{H}H^{3/4}}; N_s = \frac{N}{\sqrt{H}} * \left(\frac{P}{H^{\frac{3}{2}}}\right)^{\frac{1}{2}};\ N_s = N_u\sqrt{P_u};$

$N_s = 63.6\sqrt{30.695}; N_s = 352.36$

$N_s = \frac{N_2\sqrt{P_2}}{H_2^{5/4}};\quad 352.36 = \frac{N_2\sqrt{2000}}{30^{5/4}};\ N_2 = 553.18rpm$

$\frac{gH_1}{N_1^2D_1^2} = \frac{gH_2}{N_2^2D_2^2}$; $\left(\frac{\sqrt{H_1}}{N_1}\right)^2 * \frac{1}{D_1^2} = \frac{H_2}{N_2^2D_2^2}$; $\left(\frac{1}{N_{u1}}\right)^2 * \frac{1}{D_1^2} = \frac{H_2}{N_2^2D_2^2}$;

$\frac{1}{63.6^2} * \frac{1}{1^2} = \frac{30}{553.18^2 * D_2^2}; D_2^2 = 0.3965; D_2 = 0.629m$

$\frac{Q_1}{N_1D_1^3} = \frac{Q_2}{N_2D_2^3}$; $\frac{Q_1}{N_1D_1D_1^2} = \frac{Q_2}{N_2D_2D_2^2}$; $\frac{Q_1}{U_1D_1^2} = \frac{Q_2}{U_2D_2^2}$; $\frac{Q_1}{V_1D_1^2} = \frac{Q_2}{V_2D_2^2}$; $\frac{Q_1}{\sqrt{H_1}D_1^2} = \frac{Q_2}{\sqrt{H_2}D_2^2}$

$\frac{Q_{u1}}{D_1^2} = \frac{Q_2}{\sqrt{H_2}D_2^2}$; $\frac{108.6}{1^2} = \frac{Q_2}{\sqrt{30} * 0.629^2}$; $Q_2 = 235.33m^3/s$

15. A model of Kaplan turbine having scale ratio 1:12 tested under a head of 3m. The prototype of Kaplan turbine is designed to produce a power of 8000kW under a head of 8m running at a speed of 150rpm with a overall efficiency of 85%. Find the speed, flow, power and specific speed of the model

- A model of Kaplan turbine having scale ratio 1:12 tested under a head of 3m
  Ie $\frac{D_m}{D_p} = \frac{1}{12}\ and\quad H_m = 3m$
- The prototype of Kaplan turbine is designed to produce a power of 8000kW under a head of 8m running at a speed of 150rpm with a overall efficiency of 85%.
  Ie $P_P = 8000kW\ , H_P = 8m\ N_P = 150rpm$ η_o =85%
- Find the speed, flow, power and specific speed of the model
  Ie N_m =? Q_m =? Q_P =? N_s =?

$\frac{gH_m}{N_m^2D_m^2} = \frac{gH_p}{N_P^2D_P^2}$; $N_m^2 = \frac{H_m}{H_P}x\left(\frac{D_P}{D_m}\right)^2 xN_P^2$; $N_m^2 = \frac{3}{8}x(12)^2x150^2$ N_m^2 =12144999.15 ie N_m=1102.27rpm

$\eta_P = \frac{P_P}{\omega Q_P H_p}$; 0.85=$\frac{8000x10^3}{9810\ Q_P 8}$; $Q_P = 119.92$ m^3/s

$\frac{Q_p}{N_p D_P^3} = \frac{Q_m}{N_m D_m^3}$; $Q_m = \frac{N_m}{N_P} x \left(\frac{D_m}{D_P}\right)^3$ x Q_p; $Q_m = \frac{1102.27}{150} x \left(\frac{1}{12}\right)^3$x119.92

$Q_m = 0.509\text{m}^3\text{/s}$

$\frac{P_P}{P_m} = \frac{\eta_P\ \omega Q_P H_P}{\eta_m\ \omega Q_m H_m}$; $\frac{8000}{P_m} = \frac{119.92x8}{0.509x3}$; $P_m = 12.73kW$

16. A model of a turbine built to a scale of 1:4 is tested under a head of 10m. The prototype has to work under a head of 50m at 450rpm (a) what speed should the model run be if it develops 60kW using 0.9cumecs at this speed. (b) what power will be obtained from the prototype assuming that its efficiency is 3% better than that of model

- scale=$\frac{1}{4} = \frac{D_m}{D_P}$
- Model: $H_m = 10m$; $N_m =?$ $P_m = 60kW$; $Q_m = 0.9m^3/s$
- Prototype $H_P = 50m$; $N_P = 450\ rpm$ $P_P = ?$
- what power will be obtained from the prototype assuming that its efficiency is 3% better than that of model P_P=? if η_P=1.03 η_M

$$\frac{gH_m}{N_m^2 D_m^2} = \frac{gH_p}{N_P^2 D_P^2}$$

<u>Speed of the model</u>

$N_m^2 = \frac{H_m}{H_P} x \left(\frac{D_P}{D_m}\right)^2 x N_P^2$; $N_m^2 = \frac{10}{50} x(4)^2 x 450^2$; N_m=805 rpm

<u>Power of Prototype</u>

$\eta = \frac{\omega QH}{P}$; P =ηωQH

Hence $\frac{\eta_P\ Q_P H_P}{\eta_m\ Q_m H_m} = \frac{P_P}{P_m}$

But $\frac{Q_p}{N_p D_P^3} = \frac{Q_m}{N_m D_m^3}$; $\frac{Q_p}{Q_m} = \left(\frac{D_P}{D_m}\right)^3 x \frac{N_p}{N_m}$

Hence,

$\frac{\eta_P}{\eta_m} x \left(\frac{D_P}{D_m}\right)^3 x \frac{N_p}{N_m} x \frac{H_P}{H_m} = \frac{P_P}{P_m}$

1.03 $x(4)^3 x \frac{450}{805} x \frac{50}{10} = \frac{P_P}{60}$

P_P =11055kW

17. A Francis turbine model of 1:5 scaleThe data for model is P=4kW, N=3500rpm, H=2m and for prototype , H=6m Assume that the overall efficiency is 70%, Calculate i) speed of the prototype ii) Power of prototype Use Moodys equation (1c, 10, Dec 12)

$\frac{D_m}{D_P} = \frac{1}{5}$; $P_m = 4kW$; $N_m = 3500rpm; H_m = 2m; H_P = 6m; \eta_m = 0.7; N_P = ?\ P_P = ?$

$\frac{gH_m}{N_m^2 D_m^2} = \frac{gH_p}{N_P^2 D_P^2}; N_P^2 = N_m^2 * \frac{H_m}{H_p} * \left(\frac{D_m}{D_P}\right)^2; N_P^2 = 3500^2 * \frac{2}{6} * \left(\frac{1}{5}\right)^2; N_P^2$ =163170;

$N_P = 403.94rpm$

η_p= 1- (1-η_m)$\left(\frac{D_m}{D_p}\right)^{0.2}$; η_p= 1- (1-0.7) $\left(\frac{1}{5}\right)^{0.2} = 0.782$

$\eta_m = \frac{P_m}{\omega Q_m H_m} - eqn\ 1; \eta_P = \frac{P_P}{\omega Q_P H_P} - eqn\ 2$

$eqn\ 2/\ eqn\ 1;\ \frac{\eta_P}{\eta_m} = \frac{P_P}{P_m} * \frac{Q_m}{Q_P} * \frac{H_m}{H_P};\ \frac{\eta_P}{\eta_m} = \frac{P_P}{P_m} * \frac{D_m^2\sqrt{H_m}}{D_P^2\sqrt{H_P}} * \frac{H_m}{H_P};\ \ \frac{\eta_P}{\eta_m} = \frac{P_P}{P_m} * \left(\frac{D_m}{D_P}\right)^2 * \left(\frac{H_m}{H_P}\right)^{3/2}$

$\frac{0.782}{0.7} = \frac{P_P}{4} * \left(\frac{1}{5}\right)^2 * \left(\frac{2}{6}\right)^{3/2};\ \ P_P = 4 * (5)^2 * \left(\frac{6}{2}\right)^{3/2} * \frac{0.782}{0.7}; P_P$ =580.48kW

18. A small scale model of hydraulic turbine runs at a speed of 350rpm, under a head of 20m and produces 8kW as output Find : a) Unit Discharge b) Unit speed and c) Unit Power assuming total to total efficiency of a turbine as 0.79 find the output power of the actual turbine which is 12 times the model size, assuming the model and prototype efficiencies are related by Moodys formula

Model : H_m=20m; N_m=350rpm; P_m=8kW; η_{tt}=0.79; $\frac{D_p}{D_M}$ =12

$\eta_{tt} = \frac{P_m}{\omega Q_m H_m}$; 0.79=$\frac{8x10^3}{9810xQ_m x20}$; Q_m=0.0516m^3/s

Unit Discharge

$Q_u = \frac{Q}{\sqrt{H}} = \frac{0.056}{\sqrt{20}}$; Q_u=0.0115m^3/s

Unit speed:

$N_u = \frac{N}{\sqrt{H}}$; $N_u = \frac{350}{\sqrt{20}}$ =78.262rpm

Unit Power

$P_u = \frac{P}{H^{3/2}}$; $P_u = \frac{8x10^3}{20^{3/2}}$ =89.442W

η_p= 1- (1-η_m)$\left(\frac{D_m}{D_p}\right)^{0.2}$; η_p= 1- (1-0.79) $\left(\frac{1}{12}\right)^{0.2}$ =0.8722

For model; $\eta_m = \frac{P_m}{\omega Q_m H_m}$

For Prototype; $\eta_P = \frac{P_P}{\omega Q_P H_p}$

$$\frac{\eta_P}{\eta_m} = \frac{P_P}{P_m}\frac{\omega Q_m H_m}{\omega Q_P H_P} - eqn1$$

Discharge Q=AV

$Q \alpha D^2 \sqrt{H}$ *as* $A \alpha D^2$ *and* $V \alpha \sqrt{H}$

Hence $\frac{Q_m}{Q_P} = \frac{D_m^2\sqrt{H_m}}{D_p^2\sqrt{H_p}}$

Substituting $\frac{Q_m}{Q_P}$ *in eqn 1*

$\frac{\eta_P}{\eta_m} = \frac{P_P}{P_m}\frac{D_m^2\sqrt{H_m}}{D_p^2\sqrt{H_p}}\frac{H_m}{H_P}$; $\frac{\eta_P}{\eta_m} = \frac{P_P}{P_m} * \left(\frac{D_m}{D_p}\right)^2 * \left(\frac{H_m}{H_P}\right)^{3/2}$;

Assuming $D \propto H$; $\frac{D_m}{D_p} = \frac{H_m}{H_P}$; $\frac{1}{12} = \frac{H_m}{H_P}$

$\frac{0.8722}{0.79} = \frac{P_P}{8} * \left(\frac{1}{12}\right)^2 * \left(\frac{1}{12}\right)^{3/2}$; $P_P = 8 * \frac{0.8722}{0.79} * (12)^2 * (12)^{3/2}$; P_P =52870.49kW

19. A centrifugal pump is required to handle water at a capacity 6.75m^3/s, head of 125m and a speed of 350rpm. In designing a model of this pump the laboratory conditions impose a maximum capacity of 0.127m^3/s and a power consumption of 220kW model and prototype efficiencies are assumed same, find the speed of model and scale ratio

20. An axial flow pump with a rotor diameter 30cm handles water at the rate of 2.7m^3/min, while operating at 1500rpm. The corresponding energy input is 125J/kg. The total to total efficiency is 75%. If a second geometrically similar pump with a diameter of 20cm operates at 3000rpm, what is its flow rate? What is the change in total pressure

The total to total efficiency being 75%.

Ie D_1=30cm; N_1=1500rpm; Q_1 =2.7m^3/s ;

energy input is 125J/kg ie gH_1 =125J/kg ; The total to total efficiency is 75%.

If a second geometrically similar pump with a diameter of 20cm operates at 3000rpm,

ie D_2=20cm; N_1=3000rpm

what are a) its flow rate b) power input c) change in total pressure

Q_2 =? ; P_2 =? change in total pressure =?

$\frac{Q_1}{N_1 D_1^3} = \frac{Q_2}{N_2 D_2^3}$; $\frac{2.7}{1500x30^3} = \frac{Q_2}{3000x20^3}$; Q_2=1.6m^3/min

$\frac{gH_1}{N_1^2 D_1^2} = \frac{gH_2}{N_2^2 D_2^2}$; $\frac{125}{1500^2 x0.30^2} = \frac{gH_2}{3000^2 x0.20^2}$; gH_2 =222.22 J/kg = $\left(\frac{E}{m}\right)_2 = \Delta h_{o2}$

P=$m * \frac{E}{m}$

m_2= ρQ_2

Power input = $\rho Q_2\, gH_2$

=1000x 1.6 x222.22=58.32x10^3 W

Change in pressure

$\eta_{tt} = \frac{\Delta h_{os}}{\Delta h_o}$; $0.75 = \frac{\Delta h_{os}}{222.22}$

$\Delta h_{os} = 0.75 * 222.22$; $\Delta h_{os} = 166.65 J/kg$

$\Delta h_{os} = \frac{\Delta p_o}{\rho}$; $166.65 = \frac{\Delta p_o}{1000}$; $\Delta p_o = 166.65 * 1000 N/m^2$; $\Delta p_o = 1.66\ bar$

Module 1 : Thermodynamics of fluid flow in Turbomachines

Important point:

Enthalpy $h = C_pT$ where C_p is the specific heat in kJ/kgK , T is in temperature in K and h is in kJ/kg

If C_p is in J/kgK then h is in J/kg

$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$ if the process is isentropic

$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$ if the process is Polytropic

$\frac{T_{01}}{T_1} = \left(\frac{P_{01}}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$ since static to stagnation is always isentropic

<u>Efficiency of compressor or power absorbing machine</u>

$$\eta = \frac{isentropic\ enthalpy\ increase}{Actual\ enthlpy\ drop}$$

$$\eta = \frac{h_{02s} - h_{01}}{h_{02} - h_{01}};\ \eta = \frac{T_{02s} - T_{01}}{T_{02} - T_{01}}$$

<u>Efficiency of Turbine or power generating machine</u>

$$\eta = \frac{Actual\ enthalpy\ drop}{isentropic\ enthlpy\ drop}$$

$$\eta = \frac{h_{01} - h_{02}}{h_{01} - h_{02s}};\ \eta = \frac{T_{01} - T_{02}}{T_{01} - T_{02s}}$$

Static and stagnation states:

Static state: various properties such as pressure, temperature and volume may be determined at any given fluid particle

Static properties are those properties which are measured with instruments or devices which are at rest relative to the fluid . For example static temperature of any fluid particle moving with a given speed, the measuring thermometer or thermocouple should theoretically move with the same speed as the fluid particle itself while the measurements is being made

Example : measurements made by instrument fitted at the wall of the conduit in which fluid is flowing is static properties (because fluid particles at the wall has zero velocity, measuring instrument fitted has zero velocity hence relative velocity between the fluid and measuring instrument is zero)

Stagnation state is defined as the terminal state of fictitious, isentropic and work free thermodynamic process, during which the macroscopic kinetic and potential energies of the fluid particle are reduced to zero in steady flow . Measurement made by the instrument in which sensing element is fixed at the centre of conduit represents stagnation property of fluid (because Instrument has zero velocity and fluid at the centre of conduit is having stream velocity of the fluid)

Stagnation state , as defined above, does not represent the existing state of a fluid at any point;

From Ist law of thermodynamics applied to static to stagnation

$$\dot{Q} + \dot{m}\left(h + \frac{V^2}{2} + Zg\right) = \dot{W} + \dot{m}\left(h_0 + \frac{V_0^2}{2} + Z_0 g\right)$$

Note that h, V, Z are static enthalpy, velocity, elevation at given point and h_0 is the stagnation enthalpy at same point, $\dot{Q}$ is the rate of heat transfer and $\dot{W}$ is the rate of work done

Also note that without suffix is the static properties at the given point and properties with suffix 0 represents stagnation properties

As stagnation point is the terminal state of fictitious, isentropic and work free; $\dot{Q} = 0$; $\dot{W} = 0$; $V_o = 0$; $Z_0 = 0$

$$\dot{m}\left(h + \frac{V^2}{2} + Zg\right) = \dot{m}\,(h_0); \qquad h_0 = h + \frac{V^2}{2} + Zg;$$

$$C_p T_0 = C_p T + \frac{V^2}{2} + Zg; \qquad T_0 = T + \frac{V^2}{2C_p} + \frac{Zg}{C_p}$$

If PE is neglected, $stagntion\ enthalpy\ h_0 = h + \frac{V^2}{2}$;

$C_p T_0 = C_p T + \frac{V^2}{2}$; Hence stagnation temperature, $T_0 = T + \frac{V^2}{2C_p}$

Stagnation pressure

Bernoulli's equation between static and stagnation prperties

$\frac{p}{\omega}+\frac{V^2}{2g}+Z=\frac{p_0}{\omega}+\frac{V_0^2}{2g}+Z_0;\ \ \frac{p_0}{\omega}=\frac{p}{\omega}+\frac{V^2}{2g}+Z$ since at stagnation point velocity and potential energy become zero

$\frac{p_0}{\rho g}=\frac{p}{\rho g}+\frac{V^2}{2g}+Z\ ;p_0=p+\frac{\rho V^2}{2}+\ Z\,\rho g$

If potential energy is neglected , ; $p_0=p+\frac{\rho V^2}{2}$

Note that

Static to stagnation property (relation between Temperature and pressure)

$\frac{T_0}{T}=\left(\frac{p_0}{p}\right)^{\frac{\gamma-1}{\gamma}}$ since static to stagnation process is isentropic Hence $\gamma$is used

Efficiency of turbomachine:

In turbomachines , losses occur in turbomachine is due to a) bearing friction, windage etc which is referred as Mechanical losses and b) Unsteady flow, friction between the blade and fluid losses referred to hydraulic losses

In Power generating machine;

Referred to Mechanical losses

$\eta_{mech}=\frac{Power\ developed\ at\ the\ shaft\ of\ runner}{Power\ developed\ at\ the\ runner}$

Referred to Hydraulic losses

$\eta_{blade}=\frac{Power\ developed\ at\ the\ runner}{Fluid\ Power\ available\ at\ the\ inlet\ of\ turbine}$

Overall efficiency:

$\eta_0=\frac{Power\ developed\ at\ the\ shaft\ of\ runner}{Fluid\ Power\ available\ at\ the\ inlet\ of\ turbine}=\eta_{mech}\eta_{blade}$

In Power absorbing machine;

Referred to Mechanical losses

$\eta_{mech}=\frac{Power\ availabe\ at\ the\ impeller}{Input\ Power\ to\ the\ shaft}$

Referred to Hydraulic losses

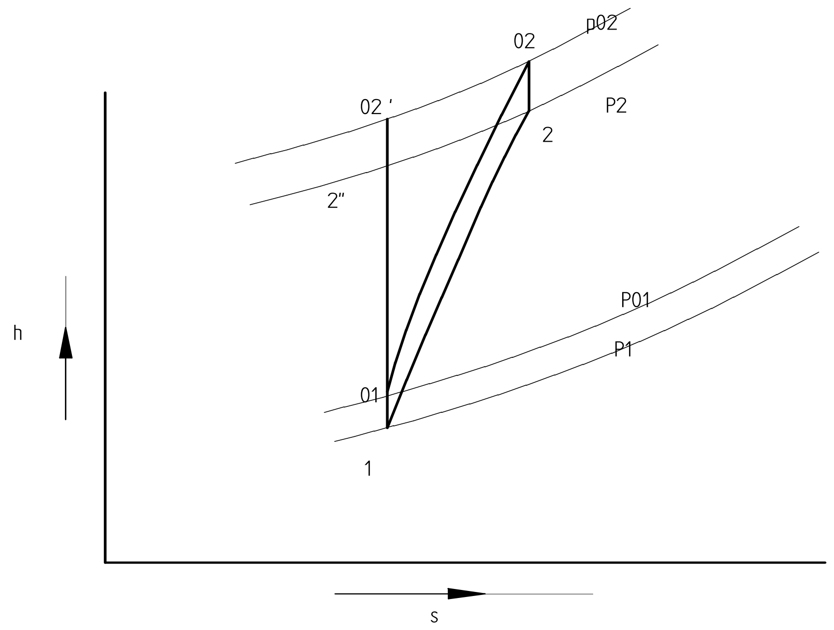
$$\eta_{manometric} = \frac{Fluid\ Power\ develped\ at\ outlet\ of\ the\ turbomachine}{Fluid\ Power\ available\ at\ the\ impeller}$$

Overall efficiency :

$$\eta_0 = \frac{Fluid\ Power\ develped\ at\ outlet\ of\ the\ turbomachine}{Input\ Power\ to\ the\ shaft} = \eta_{mech}\eta_{manometric}$$

Various efficiencies based on static and stagnation properties

Power absorbing machine:



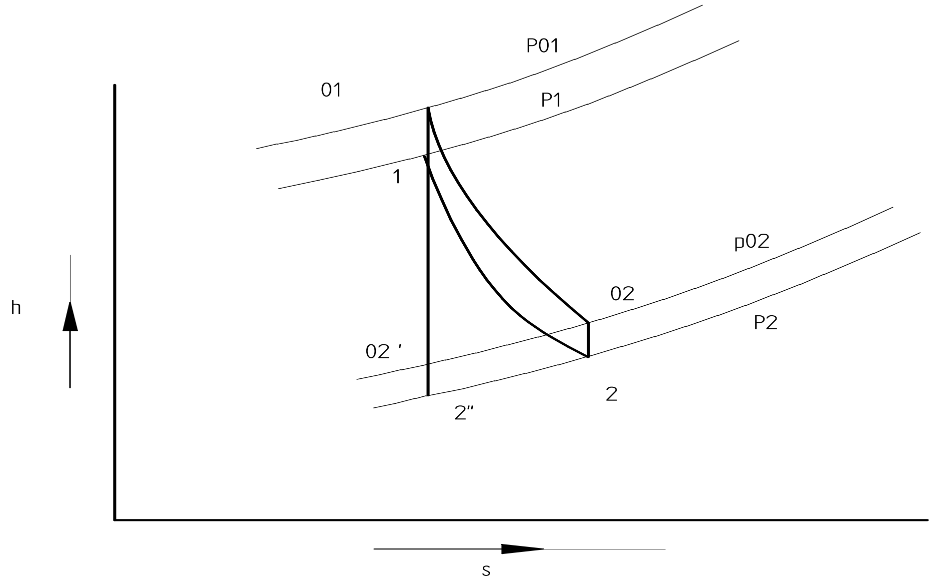
i) $W_{t\text{-}t} = h_{02s} - h_{01}$; $\eta_{t-t} = \frac{h_{02s} - h_{01}}{h_{02} - h_{01}}$

ii) $W_{t\text{-}s} = h_{2s} - h_{01}$; $\eta_{t-s} = \frac{h_{2s} - h_{01}}{h_{02} - h_{01}}$

iii) $W_{s\text{-}t} = h_{02s} - h_1$; $\eta_{s-t} = \frac{h_{02s} - h_1}{h_{02} - h_{01}}$

iv) $W_{s\text{-}s} = h_{2s} - h_1$; $\eta_{s-s} = \frac{h_{2s} - h_1}{h_{02} - h_{01}}$

Power generating machine



i) $W_{t\text{-}t} = h_{01} - h_{02s}$; $\eta_{t-t} = \frac{h_{01}-h_{02}}{h_{01}-h_{02s}}$

ii) $W_{t\text{-}s} = h_{01} - h_{2s}$; $\eta_{t-s} = \frac{h_{01}-h_{02}}{h_{01}-h_{2s}}$

iii) $W_{s\text{-}t} = h_1 - h_{02s}$; $\eta_{s-t} = \frac{h_{01}-h_{02}}{h_1-h_{02s}}$

iv) $W_{s\text{-}s} = h_{2s} - h_1$; $\eta_{s-s} = \frac{h_{01}-h_{02}}{h_1-h_{2'}}$

Application of First Law and Second Law of thermodynamics to Turbomachines

The fluid flow in any turbomachine is slightly varies with time (Steady flow) but unsteady flow near blade tips at entry and exit of cascades. But overall fluid flow is steady

Hence applying First law of thermodynamics for steady flow

$$\dot{Q} + \dot{m}\left(h_1 + \frac{V_1^2}{2} + Z_1 g\right) = P + \dot{m}\left(h_2 + \frac{V_2^2}{2} + Z_2 g\right)$$

But stagnation enthalpy: $h_0 = h + \frac{V^2}{2} + Zg$

Hence $\dot{Q} + \dot{m}\, h_{01} = P + \dot{m}\, h_{02}$

$$\dot{Q} - P = \dot{m}(h_{02} - h_{01})$$

$$\frac{\dot{Q}}{\dot{m}} - \frac{\dot{P}}{\dot{m}} = h_{02} - h_{01}$$

$$q - w = \Delta h_0$$

q =0 as turbomachine is ideally assumed as adiabatic

Hence, $-w = \Delta h_0$

Hence energy transfer as work per unit mass flow is therefore numerically equal to change in stagnation enthalpy of the fluid between the turbomachine inlet and outlet

In power generating turbomachine, w is positive so that $\Delta h_0 = h_{02} - h_{01}$ is negative

In power absorbing turbomachine, w is negative so that $\Delta h_0 = h_{02} - h_{01}$ is positive

For incompressible fluid, internal energy changes are negligible, and density is constant

H= u+pv

$\Delta h = (u_2 - u_1) + (p_2 v_2 - p_1 v_1)$

$u_2 - u_1 = 0$ as For incompressible fluid, internal energy changes are negligible

$\Delta h = \Delta pv; \quad v = \frac{1}{\rho}; \ \Delta h = \Delta \frac{p}{\rho}$

$\Delta h_0 = \Delta h + \Delta KE + \Delta P$

Application of II law of thermodynamics to turbomachine

From 2 law of thermaodynamics

$T_0 ds_0 = dh_0 - v_0 dp_0 - A$

From Ist law of thermodynamics, $dh_0 = -\delta w$

$T_0 ds_0 = -\delta w - v_0 dp_0 - 1$

In ideal turbo machine , $T_0 \, ds_0 = 0;$ $work\ done\ is\ \delta w_i$

Hence equation A becomes $0 = -\delta w_i - v_0 dp_0; \qquad \delta w_i = -v_0 dp_0$

Hence, substituting $\delta w_i = -v_0 dp_0$, $eq\ 1\ becomes$

$T_0 ds_0 = -\delta w + \delta w_i; \qquad \delta w_i - \delta w = T_0 ds_0$

Efficiencies of the compression process

i) Total to total efficiency

It is defined as the ratio of ideal work to the actual work between the stagnation states.

$\eta_{tt} = \frac{Ideal\ work\ between\ the\ stagnation\ states}{Actual\ work}$

$\eta_{tt}=\frac{h'_{02}-h_{01}}{h_{02}-h_{01}}=\frac{C_p\,(T'_{02}-T_{01})}{C_p\,(T_{02}-T_{01})}$; $\eta_{tt}=\frac{T'_{02}-T_{01}}{T_{02}-T_{01}}$; $\eta_{tt}=\frac{T_{01}\left(\frac{T'_{02}}{T_{01}}-1\right)}{T_{02}-T_{01}}$ -------------1

But $\frac{T'_{02}}{T_{01}}=\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}}$; $\frac{T'_{02}}{T_{01}}=(p_{ro})^{\gamma-1}$; $\eta_{tt}=\frac{T_{01}(p_{ro}^{\gamma-1}-1)}{T_{02}-T_{01}}$

Actual Power required

Actual Power required, P $=mC_p(T_{02}-T_{01})$; $P=mC_p\left(\frac{T'_{02}-T_{01}}{\eta_{tt}}\right)$; $P=\frac{mC_p}{\eta_{tt}}T_{01}\left(\frac{T'_{02}}{T_{01}}-1\right)$

$$P=\frac{mC_p}{\eta_{tt}}T_{01}\left(p_{ro}^{\gamma-1}-1\right)$$

If Mechanical Efficiency is given, $P=\frac{mC_p}{\eta_{tt}\quad\eta_{mech}}T_{01}\left(p_{ro}^{\gamma-1}-1\right)$

Efficiencies of the Expansion process

Turbine

i) Total to total efficiency

It is defined as the ratio of $Actual\ work$ to the $Ideal\ work$ between the stagnation states.

$\eta_{tt}=\frac{Actual\ work}{Ideal\ work\ between\ the\ stagnation\ states}$; $\eta_{tt}=\frac{h_{01}-h_{02}}{h_{01}-h'_{02}}$; $\eta_{tt}=\frac{C_p\,(T_{01}-T_{02})}{C_p\,(T_{01}-T'_{02})}$

$\eta_{tt}=\frac{T_{01}-T_{02}}{T_{01}-T_{02s}}$; $\eta_{tt}=\frac{T_{01}-T_{02}}{T_{01}\left(1-\frac{T_{02s}}{T_{01}}\right)}$; $T_{01}-T_{02}=\eta_{tt}T_{01}\left(1-\frac{T_{02s}}{T_{01}}\right)$

But $\frac{T_{02s}}{T_{01}}=\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}}$ $\frac{T_{02s}}{T_{01}}=\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}}$; $\frac{T_{02s}}{T_{01}}=\left(\frac{p_{01}}{p_{02}}\right)^{-\left(\frac{\gamma-1}{\gamma}\right)}$;

$\frac{T_{02s}}{T_{01}}=p_{ro}^{-\left(\frac{\gamma-1}{\gamma}\right)}$ where p_{ro} is the total pressure ratio $\frac{p_{01}}{p_{02}}$

$\eta_{tt}=\frac{T_{01}-T_{02}}{T_{01}\left(1-p_{ro}^{-\left(\frac{\gamma-1}{\gamma}\right)}\right)}$ where p_{ro} is the total pressure ratio $\frac{p_{01}}{p_{02}}$

Actual Power required

Actual Power required , $P=\dot{m}C_p(T_{01}-T_{02})$

$$P=\eta_{tt}mC_p(T_{01}-T_{02s})$$

$$=\eta_{tt}mC_pT_{01}\left(1-\frac{T_{02s}}{T_{01}}\right)$$

$$=\eta_{tt}mC_pT_{01}\left(1-p_{ro}^{-\left(\frac{\gamma-1}{\gamma}\right)}\right)$$

If Mechanical Efficiency is given

P= $m\,\eta_{tt}\eta_{mech}C_pT_{01}\left(1-p_{ro}^{-\left(\frac{\gamma-1}{\gamma}\right)}\right)$

1. A stream of combustion gases at the point of entry to a turbine has a static temperature of 1050K , static pressure of 600kPa and a velocity of 150m/s. For the gases C_p =1.004kJ/kgK and $\gamma = 1.41$. Find total temperature and total pressure of the gases. Also find the difference between their static and total enthalpies. (2b. 08, Dec/Jan 19, 15ME19)

   Solution:

   $T_1 = 1050K; P_1 = 600kPa; V_1 = 150m/s$

   $T_0 = T + \frac{V^2}{2C_p} + \frac{Zg}{C_p}$; since elevation is not given $T_0 = T + \frac{V^2}{2C_p}$

   $T_{01} = T_1 + \frac{V_1^2}{2C_p};$

   $T_{01} = 1050 + \frac{150^2}{2*1004};\quad T_{01} = 1061.21K$

   $p_0 = p + \frac{\rho V^2}{2} + Z\rho g$; since elevation is not given $p_0 = p + \frac{\rho V^2}{2}$

   $P_{01} = P_1 + \frac{\rho V^2}{2};$

   $\rho = \frac{p}{RT};\ \rho = \frac{600}{0.287*1050};\ \rho = 1.991kg/m^3$

   $P_{01} = (600*1000) + \frac{1.991*150^2}{2};\quad P_{01} = 622398.75Pa = 6.223bar$

   Static enthalpy $h_1 = C_pT_1; h_1 = 1.004*1050;\ h_1 = 1051.2kJ/kgK$

   Stagnation enthalpy $h_{01} = C_pT_{01}; h_{01} = 1.004*1061.21;\ h_{01} = 1065.45kJ/kgK$

   Difference between static enthalpy and total enthalpy = $h_{01} - h_1 = 1065.45 - 1051.2$

   Difference between static enthalpy and total enthalpy = $14.25kJ/kgK$

2. Air enters a compressor at a static pressure of 15 bar, a static temperature of 15°C and a flow velocity of 50m/s, At the exit the static pressure is 30 bar , the static temperature is 100°C and the flow velocity is 100m/s . The outlet is 1m above the inlet Evaluate i) the isentropic change in enthalpy ii) The actual change in enthalpy . Take C_p for air as 1005J/kgK. Also draw the relevant T-S diagram (2b. 10 June/July 13)

   $T_1 = 15^oC = 288K; P_1 = 15\ bar = 15*10^5; V_1 = 50m/s\ ; Z_1 = 0$

   $T_2 = 100^oC = 373K; P_2 = 30\ bar = 30*10^5; V_2 = 100m/s\ ; Z_2 = 1\text{m}$

   Isentropic enthalpy drop

   $\frac{T_{2s}}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}};\qquad \frac{T_{2s}}{288} = \left(\frac{30}{15}\right)^{0.286};\quad T_{2s} = 351.14K$

   $h_{2s} - h_1 = C_p(T_{2s} - T_1);\quad h_{2s} - h_1 = 1.005(351.14 - 288);\ h_{2s} - h_1 = 63.45kJ/kg$

   $h_1 = C_pT_1;\qquad h_1 = 1005*288 = 289440J/kg$

   $h_{2s} = C_pT_{2s};\qquad h_{2s} = 1005*351.14 = 352895.7J/kg$

$h_{01} = h_1 + \frac{V_1^2}{2} + Z_1 g$; $h_{01} = C_p T_1 + \frac{V_1^2}{2} + Z_1 g$; $h_{01} = 289440 + \frac{50^2}{2} + 0$;

$h_{01} = 290690 J/kgK$

$h_{02s} = h_{2s} + \frac{V_2^2}{2} + Z_2 g$; $\qquad$ $h_{02s} = C_p T_2 + \frac{V_2^2}{2} + Z_2 g$;

$h_{02s} = 352895.7 + \frac{100^2}{2} + (1 * 9.81)$; $\qquad$ $h_{02s} = 357904.81 J/kgK$

Isentropic enthalpy drop

$h_{02s} - h_{01} = 357904.81 - 290690$; $\qquad$ $h_{02s} - h_{01} = 67214.81 J/kg$

Actual enthalpy drop

$h_{01} = h_1 + \frac{V_1^2}{2} + Z_1 g$; $\qquad$ $h_{01} = 289440 + \frac{50^2}{2} + 0$; $\quad$ $h_{01} = 290690 J/kgK$

$h_{02} = h_2 + \frac{V_2^2}{2} + Z_2 g$; $h_{01} = C_p T_2 + \frac{V_2^2}{2} + Z_2 g$; $h_{02} = (1005 * 373) + \frac{100^2}{2} + (1 * 9.81)$;

$h_{02} = 379874.81 J/kgK$

Change in total enthalpy $= h_{02} - h_{01} = 379874.81 - 290690 = 89184.84 J/kg$=89.184kJ/kg

Efficiency of the compressor

$\eta_{tt} = \frac{h_{02s} - h_{01}}{h_{02} - h_{01}}$; $\qquad$ $\eta_0 = \frac{67214.81}{89184.8}$; $\quad$ $\eta_{tt} = 0.75365$

3. Air enters a compressor at a static pressure of 1.5 bar, a static temperature of 15°C and a flow velocity of 50m/s, At the exit the static pressure is 3 bar , the static temperature is 100°C and the flow velocity is 100m/s . The outlet is 1m above the inlet Evaluate i) the isentropic change in enthalpy ii) The actual change in enthalpy iii) Efficiency of the compressor (2c. 10 June/July 17) (2b. O8 June/July 18, 15ME53)

$p_1 = 1.5\ bar$; $T_1 = 15°\text{C}$;$V_1 = 50 m/s$

$p_2 = 3\ bar$; $T_2 = 100°\text{C}$;$V_2 = 1000 m/s$

$\Delta h_{o'} = ?$; $\Delta h_o = ?$; $\eta_o = ?$

Isentropic enthalpy drop

$\frac{T_{2s}}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$; $\qquad$ $\frac{T_{2s}}{288} = \left(\frac{3}{1.5}\right)^{0.286}$; $\quad$ $T_{2s} = 351.14K$

$h_{2s} - h_1 = C_p(T_{2'} - T_1)$; $\quad$ $h_{2s} - h_1 = 1.005(351.14 - 288)$; $h_{2'} - h_1 = 63.45 kJ/kg$

$$h_{02s} - h_{01} = h_{2s} - h_1 + \frac{V_2^2 - V_1^2}{2} + (Z_2 - Z_1)g$$

$$h_{02s} - h_{01} = 63.45x10^3 + \frac{100^2 - 50^2}{2} + 1x9.81$$

$$h_{02s} - h_{01} = 67.20x10^3 J/kg$$

Actual enthalpy drop

$$h_{02} - h_{01} = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + (Z_2 - Z_1)g$$

$$h_{02} - h_{01} = C_p(T_2 - T_1) + \frac{V_2^2 - V_1^2}{2} + (Z_2 - Z_1)g$$

$$h_{02} - h_{01} = 1005(100 - 15) + \frac{100^2 - 50^2}{2} + 1x9.81$$

$$h_{02} - h_{01} = 89184.81 J/kg$$

$$\eta_0 = \frac{h_{02\prime} - h_{01}}{h_{02} - h_{01}}; \ \eta_0 = \frac{67.20x10^3}{89184.8}$$

$$\eta_0 = 0.7536$$

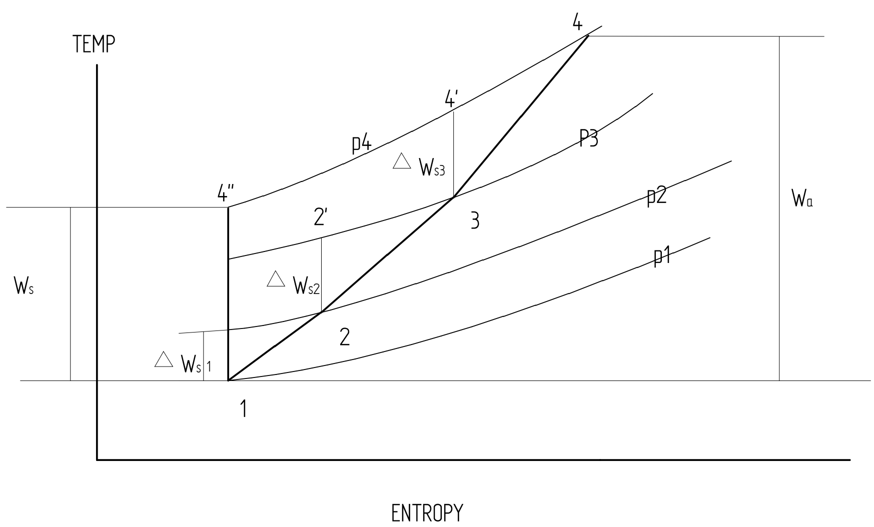
Effect of preheat in multistage compression or prove that preheat factor is always less than 1

The overall isentropic efficiency is useful as it indicates the overall performance of a turbomachine. But it is not always indicate the true efficiency from hydrodynamic point of view which is measure of fluid losses within the machine.

A compressor stage with a finite pressure stage is called as finite stage

In a multistage compressor, in each stage efficiency depends on inlet temperature of fluid to the stage and pressure ratio in each stage

Thus in multistage compressor for the same efficiency, each succeeding stage is suffered by the inefficiency of preceding stage which is handling the fluid at higher temperature



Here , a three stage compressor is considered between the inlet pressure p_1 and delivery pressure p_4

Assume that stage pressure ratio for all stage is same $\frac{p_2}{p_1} = \frac{p_3}{p_2} = \frac{p_4}{p_3}$

stage efficiency for all the stages are same $\eta_{s1} = \eta_{s2} = \eta_{s3} = \eta_s$

Let $14''$ and 14 are the total isentropic and actual compression process respectively.

η_o is the overall efficiency, W_a and W_s are the total actual and total isentropic work absorbed

$\eta_o = \frac{W_s}{W_a}$; $\quad W_a = \frac{W_s}{\eta_o} \; - eqn\ 1$

For stage 1 , stage efficiency

$\eta_s = \frac{W_{s1}}{W_{a1}}$; $\quad W_{a1} = \frac{W_{s1}}{\eta_s}$

Similarly for stage 2, $W_{a2} = \frac{W_{s2}}{\eta_s}$; For stage 3, $W_{a3} = \frac{W_{s3}}{\eta_s}$

Total Work absorbed $W_a = W_{a1} + W_{a2} + W_{a3}$

Hence, $W_a = \frac{W_{s1}+W_{s2}+W_{s3}+\cdots\ldots}{\eta_s}$; $\quad W_a = \frac{\sum_{i=0}^{i=k} W_{s1}}{\eta_s} - eqn\ \ 2$

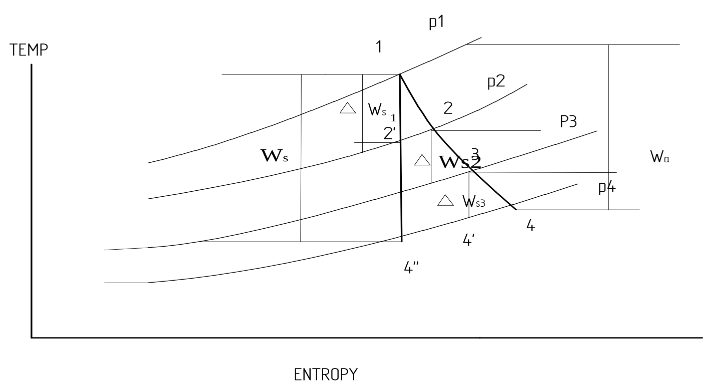
$Eqn\ 1 = Eqn\ 2;\quad \frac{W_s}{\eta_o} = \frac{\sum_{i=0}^{i=k} W_{si}}{\eta_s};\quad \eta_o = \eta_s \frac{W_s}{\sum_{i=0}^{i=k} W_{s1}}$

As the constant pressure lines are diverging in nature towards the right hand side of temperature entropy diagram, the isentropic work per stage increases as the temperature difference increases for the same pressure ratio and stage efficiency, therefore ,

$\frac{W_s}{\sum_{i=0}^{i=k} \Delta W_{s1}} < 1\ whch\ is\ called\ as\ pre - heat\ factor;\quad \eta_o = \eta_s$ x pre heat factor

<u>Effect of Reheat in multistage compression or prove that Rreheat factor is always greater than 1</u>

Thus in multistage turbine for the same efficiency, each succeeding stage is suffered by the inefficiency of preceding stage which is handling the fluid at higher temperature



Here a three stage Turbine is considered between the inlet pressure p_1 and delivery pressure p_4

Assume that stage pressure ratio for all stage is same $\frac{p_1}{p_2} = \frac{p_2}{p_3} = \frac{p_3}{p_4}$

and stage efficiency for all the stages are same $\eta_{s1} = \eta_{s2} = \eta_{s3} = \eta_s$

Let $14''$ and 14 are the total isentropic and actual compression process respectively.

η_o is the overall efficiency

W_a and W_s are the total actual and total isentropic work absorbed

$\eta_o = \frac{W_a}{W_s}; \quad W_a = \eta_o W_s - -eqn\ 1$

For stage 1 , stage efficiency; $\eta_s = \frac{W_{a1}}{W_{s1}}; \quad W_{a1} = \eta_s W_{s1}$

Similarly for stage 2, $W_{a2} = \eta_s W_{s2};$ For stage 3, $W_{a3} = \eta_s W_{s3}$

$W_a = W_{a1} + W_{a2} + W_{a3}; \quad W_a = \eta_s(W_{s1} + W_{s2} + W_{s3} + \cdots \dots.); \quad W_a = \eta_s \sum_{i=0}^{i=k} W_{si} - eqn\ 2$

$\eta_o W_s = \eta_s \sum_{i=0}^{i=k} W_{si}; \quad \eta_o = \eta_s \frac{\sum_{i=0}^{i=k} \Delta W_{s1}}{W_s}$

As the constant pressure lines are diverging in nature towards the right hand side of temperature entropy diagram, the isentropic work per stage increases as the temperature difference increases for the same pressure ratio and stage efficiency, therefore ,

$\frac{\sum_{i=0}^{i=k} W_{s1}}{W_s} > 1\ whch\ is\ called\ as\ Re - heat\ factor;$

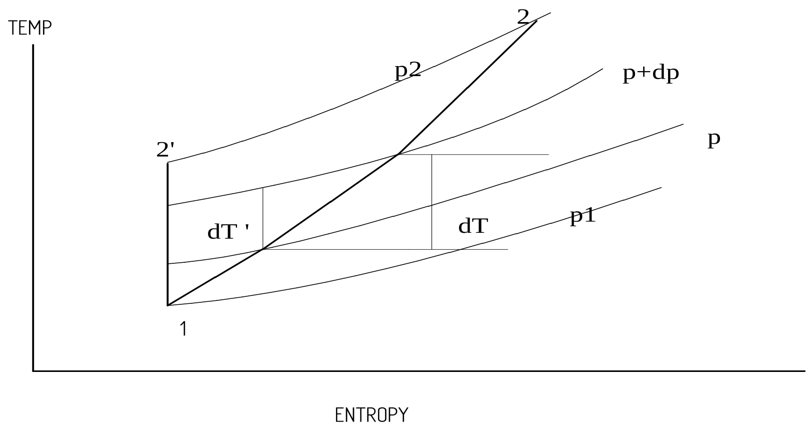
$\eta_o = \eta_s$ x Re heat factor= $\eta_0 = RF\ x\eta_s$

Infinitesimal stage efficiency or Polytropic efficiency in compression Process(compressor)

A finite compressor stage can be viewed as it made up of infinitesimal number of small stages. Each of these small stages has an efficiency, η_p, is called polytropic or infinitesimal stage efficiency

Consider a single stage compressor having stage efficiency η_s operates between p_1 and p_2 divided into infinitesimal stages.

Considering one intermediate stage operating between pressures p and p+dp and temperatures T and T+dT (efficiency of such stage is called Polytropic efficiency)



$\eta_p = \frac{Isentripic\ temp\ rise}{Actual\ temperature\ rise} = \frac{dT_s}{dT}$; $\quad dT = \frac{dT_s}{\eta_P}$; $\quad dT = \frac{T'-T}{\eta_P}$; $dT = \frac{T\left(\frac{T'}{T}-1\right)}{\eta_P}$

$\frac{dT}{T} = \frac{T\left(\left(\frac{p+dp}{P}\right)^{\frac{\gamma-1}{\gamma}}-1\right)}{\eta_P}$; $\quad \frac{dT}{T} = \frac{\left(\left(1+\frac{dp}{P}\right)^{\frac{\gamma-1}{\gamma}}-1\right)}{\eta_P}$

Using series of expansion $(1+x)^n = 1 + nx + \frac{n(n-1)}{2} \pm --$; Neglecting higher order $1 + nx$

$\frac{dT}{T} = \frac{1}{\eta_P}\left(1 + \frac{\gamma-1}{\gamma}\left(\frac{dp}{P}\right) - 1\right)$; $\quad \frac{dT}{T} = \frac{1}{\eta_P}\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{dp}{P}\right)$

Integrating above equation pressure from p_1 to p_2 and temperature from T_1 to T_2

$ln\frac{T_2}{T_1} = \frac{1}{\eta_P}\left(\frac{\gamma-1}{\gamma}\right) ln\frac{p_2}{p_1}$; $\quad \eta_P = \frac{\left(\frac{\gamma-1}{\gamma}\right)ln\frac{p_2}{p_1}}{ln\frac{T_2}{T_1}}$

Also, $ln\frac{T_2}{T_1} = \frac{1}{\eta_P}\left(\frac{\gamma-1}{\gamma}\right) ln\frac{p_2}{p_1}$; $\quad ln\frac{T_2}{T_1} = ln\left(\frac{p_2}{p_1}\right)^{\frac{1}{\eta_P}\left(\frac{\gamma-1}{\gamma}\right)}$; $\quad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{\eta_P}\left(\frac{\gamma-1}{\gamma}\right)}$-----------------1

Assuming the irreversible adiabatic compression process 1-2 as equivalent process with an index of compression n

$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\left(\frac{n-1}{n}\right)}$-----------------2

Comparing 1 and 2

$\frac{1}{\eta_P}\left(\frac{\gamma-1}{\gamma}\right) = \left(\frac{n-1}{n}\right)$; $\quad \eta_P = \left(\frac{\gamma-1}{\gamma}\right)\left(\frac{n}{n-1}\right)$

Hence there are two formulae to find polytropic efficiency

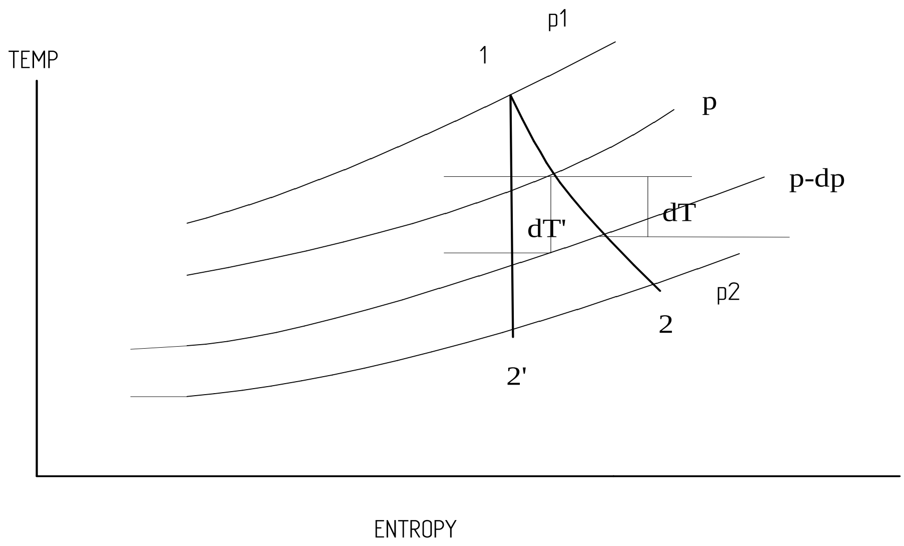
1. $\eta_P = \frac{\left(\frac{\gamma-1}{\gamma}\right)ln\frac{p_2}{p_1}}{ln\frac{T_2}{T_1}}$ 2. $\eta_P = \left(\frac{\gamma-1}{\gamma}\right)\left(\frac{n}{n-1}\right)$

Infinitesimal stage efficiency or Polytropic efficiency in expansion process (Turbine)

A finite turbine stage can be viewed as it made up of infinitesimal number of small stages. Each of these small stages has an efficiency, η_p, is called polytropic or infinitesimal stage efficiency

Consider a single stage turbine having stage efficiency η_s operates between p_1 and p_2 divided into infinitesimal stages.

Considering one intermediate stage operating between pressures p and p+dp and temperatures T and T+dp (efficiency of such stage is called Polytropic efficiency)



$\eta_p = \frac{Actual\ temperature\ rise}{Isentripic\ temp\ rise} = \frac{dT}{dT_s}$; $dT = \eta_P\, dT_s$; $dT = \eta_P (T - T_s)$; $dT = \eta_P T \left(1 - \frac{T_s}{T}\right)$

$\frac{dT}{T} = \eta_P \left(1 - \left(\frac{p-dp}{p}\right)^{\frac{\gamma-1}{\gamma}}\right)$; $\frac{dT}{T} = \eta_P \left(1 - \left(1 - \frac{dp}{P}\right)^{\frac{\gamma-1}{\gamma}}\right)$;

Using series of expansion $(1-x)^n = 1 - nx + \frac{n(n-1)}{2!} x^2 - \frac{n(n-1)(n-2)}{3!}$

Neglecting higher order $1 - nx$; $\frac{dT}{T} = \eta_P \left(1 - \left(1 - \frac{\gamma-1}{\gamma}\left(\frac{dp}{P}\right)\right)\right)$; $\frac{dT}{T} = \eta_P \left(\frac{\gamma-1}{\gamma}\right)\left(\frac{dp}{P}\right)$

Integrating above equation pressure from p_1 to p_2 and temperature from T_1 to T_2

$ln\frac{T_2}{T_1} = \eta_P \left(\frac{\gamma-1}{\gamma}\right) ln\frac{p_2}{p_1}$; $\eta_P = \frac{ln\frac{T_2}{T_1}}{\left(\frac{\gamma-1}{\gamma}\right) ln\frac{p_2}{p_1}}$;

$ln\frac{T_2}{T_1} = \eta_P \left(\frac{\gamma-1}{\gamma}\right) ln\frac{p_2}{p_1}$; $ln\frac{T_2}{T_1} = ln\left(\frac{p_2}{p_1}\right)^{\eta_P\left(\frac{\gamma-1}{\gamma}\right)}$; $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\eta_P\left(\frac{\gamma-1}{\gamma}\right)}$-----------------1

Assuming the irreversible adiabatic compression process 1-2 as equivalent process with an index of compression n

$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\left(\frac{n-1}{n}\right)}$-----------------2

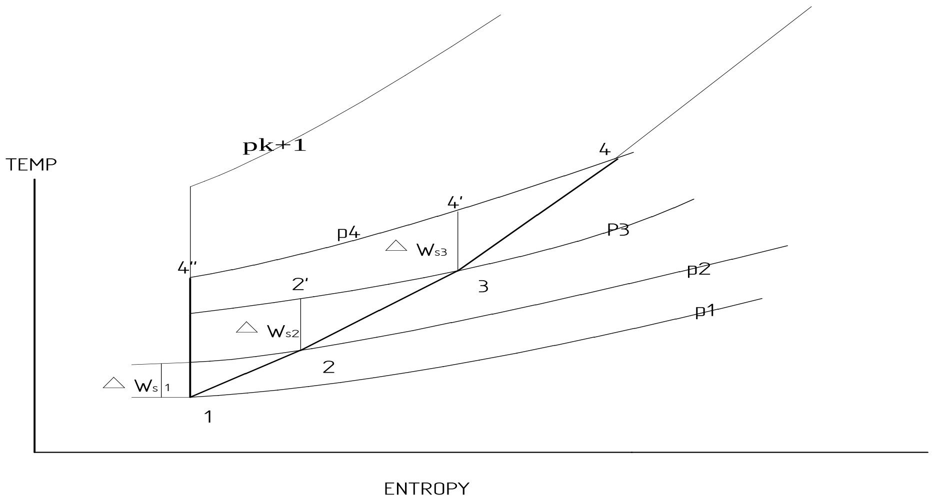
Comparing 1 and 2

$\eta_P \left(\frac{\gamma-1}{\gamma}\right) = \left(\frac{n-1}{n}\right)$; $\quad \eta_P = \left(\frac{\gamma}{\gamma-1}\right)\left(\frac{n-1}{n}\right)$ this is another formulae for polytropic efficiency

There are two formulae to determine polytropic efficiency for turbine

1. $\eta_P = \frac{ln\frac{T_2}{T_1}}{\left(\frac{\gamma-1}{\gamma}\right) ln\frac{p_2}{p_1}}$; 2. $\eta_P = \frac{ln\frac{T_2}{T_1}}{\left(\frac{\gamma-1}{\gamma}\right) ln\frac{p_2}{p_1}}$

Multistage compressors (Equal Pressure ratio case)



Consider multistage compression of k stages between the pressures p_1 and p_{k+1} with overall pressure ratio $\frac{p_{k+1}}{p_1}$ and having equal stage efficiencies η_{st} or η_p then the pressure ratio in each stage is given by

$$p_r = \frac{p_2}{p_1} = \frac{p_3}{p_2} = \frac{p_4}{p_3} = \cdots \ldots \ldots = \frac{p_{k+1}}{p_k}$$

$$p_{ro} = \frac{p_{k+1}}{p_1} = \frac{p_2}{p_1}\, x\, \frac{p_3}{p_2}\, x\, \frac{p_4}{p_3}\, x \ldots \ldots \ldots . x\, \frac{p_{k+1}}{p_k}$$

$$p_{ro} = \frac{p_{k+1}}{p_1} = p_r\, x\, p_r\, x\, p_r\, x \ldots \ldots \ldots . x\, p_r$$

$$p_{ro} = \frac{p_{k+1}}{p_1} = p_r^k$$

Stage Efficiency

Efficiency of stage in multi stage compressor is called stage efficiency

a. Compressor

$$\eta_{st} = \frac{T_2' - T_1}{T_2 - T_1}; \quad \eta_{st} = \frac{T_1\left(\frac{T_2'}{T_1} - 1\right)}{T_1\left(\frac{T_2}{T_1} - 1\right)}; \quad \eta_{st} = \frac{\left(\frac{p_2}{p_1}\right)^{\left(\frac{\gamma-1}{\gamma}\right)} - 1}{\left(\frac{p_2}{p_1}\right)^{\left(\frac{n-1}{n}\right)} - 1}$$

But , $\left(\frac{n-1}{n}\right) = \frac{1}{\eta_P}\left(\frac{\gamma-1}{\gamma}\right)$; Hence $\eta_{st} = \frac{\left(\frac{p_2}{p_1}\right)^{\left(\frac{\gamma-1}{\gamma}\right)} - 1}{\left(\frac{p_2}{p_1}\right)^{\frac{1}{\eta_P}\left(\frac{\gamma-1}{\gamma}\right)} - 1}$

$$\eta_{st} = \frac{p_r^{\left(\frac{\gamma-1}{\gamma}\right)} - 1}{p_r^{\frac{1}{\eta_P}\left(\frac{\gamma-1}{\gamma}\right)} - 1}$$

Overall efficiency of multistage compressor

For multistage compressor the stage efficiency η_{st} is replaced by the overall efficiency η_o and the stage pressure ratio p_r by the overall pressure ratio p_{ro} then the above equation becomes

$$\eta_0 = \frac{p_{r0}^{\left(\frac{\gamma-1}{\gamma}\right)} - 1}{p_{r0}^{\frac{1}{\eta_P}\left(\frac{\gamma-1}{\gamma}\right)} - 1}; \quad \eta_0 = \frac{p_r^{k\left(\frac{\gamma-1}{\gamma}\right)} - 1}{p_r^{\frac{k}{\eta_P}\left(\frac{\gamma-1}{\gamma}\right)} - 1} \text{ as } p_{r0} = p_r^{\,k}$$

Multi stage turbine (Equal Pressure ratio case)

For multistage turbine the stage efficiency η_{st} is replaced by the overall efficiency η_o and the stage pressure ratio p_r by the overall pressure ratio p_{ro} then the above equation becomes

$$\eta_0 = \frac{1 - (p_{r0})^{-\eta_P\left(\frac{\gamma-1}{\gamma}\right)}}{1 - (p_{r0})^{-\left(\frac{\gamma-1}{\gamma}\right)}} \text{Multistage turbine}$$

Consider multistage expansion of k stages between the pressures p_1 and p_{k+1} with overall pressure ratio $\frac{p_1}{p_{k+1}}$ and having equal stage efficiencies η_{st} or η_p then the pressure ratio in each stage is given by

$$p_r = \frac{p_1}{p_2} = \frac{p_2}{p_3} = \frac{p_3}{p_4} = \cdots \ldots \ldots = \frac{p_k}{p_{k+1}}$$

$$p_{ro} = \frac{p_1}{p_{k+1}} = \frac{p_1}{p_2} \, x \, \frac{p_2}{p_3} \, x \, \frac{p_3}{p_4} \, x \ldots \ldots \ldots . x \, \frac{p_k}{p_{k+1}}$$

$$p_{ro} = \frac{p_1}{p_{k+1}} = p_r \, x \, p_r \, x \, p_r \, x \ldots \ldots \ldots . x \, p_r$$

$$p_{ro} = \frac{p_1}{p_{k+1}} = p_r^k$$

Stage Efficiency (Expansion Process – Turbine)

Stage efficiency

$$\eta_{st}=\frac{T_1-T_2}{T_1-T_2'};\qquad \eta_{st}=\frac{T_1\left(1-\frac{T_2}{T_1}\right)}{T_1\left(1-\frac{T_2'}{T_1}\right)};\qquad \eta_{st}=\frac{1-\left(\frac{p_2}{p_1}\right)^{\left(\frac{n-1}{n}\right)}}{1-\left(\frac{p_2}{p_1}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}};$$

But

$$\eta_P\left(\frac{\gamma-1}{\gamma}\right)=\left(\frac{n-1}{n}\right)$$

$$\eta_{st}=\frac{1-\left(\frac{p_2}{p_1}\right)^{\eta_P\left(\frac{\gamma-1}{\gamma}\right)}}{1-\left(\frac{p_2}{p_1}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}};\qquad \eta_{st}=\frac{1-\left(\frac{p_1}{p_2}\right)^{-\eta_P\left(\frac{\gamma-1}{\gamma}\right)}}{1-\left(\frac{p_1}{p_2}\right)^{-\left(\frac{\gamma-1}{\gamma}\right)}};\qquad \eta_{st}=\frac{1-(p_r)^{-\eta_P\left(\frac{\gamma-1}{\gamma}\right)}}{1-(p_r)^{-\left(\frac{\gamma-1}{\gamma}\right)}}\text{ where } p_r=\frac{p_1}{p_2}$$

Overall efficiency of multistage Turbine

For multistage expansion the stage efficiency η_{st} is replaced by the overall efficiency η_o and the stage pressure ratio p_r by the overall pressure ratio p_{ro} then the above equation becomes

$$\eta_0=\frac{1-(p_{ro})^{-\eta_P\left(\frac{\gamma-1}{\gamma}\right)}}{1-(p_{ro})^{-\left(\frac{\gamma-1}{\gamma}\right)}};\qquad \eta_0=\frac{1-(p_r)^{-K\eta_P\left(\frac{\gamma-1}{\gamma}\right)}}{1-(p_r)^{-K\left(\frac{\gamma-1}{\gamma}\right)}}$$

Overall efficiency for a finite number compressor stages in terms of stage efficiency for a Compressor

T_i is the initial temperature at which the fluid enters the turbine, K is the number of stages having equal pressure ratio, p_r is the pressure ratio in each stage, then the actual temperature rise in each stage can be calculated as follows

For First stage

$$\eta_s=\frac{T_{i+1}'-T_i}{T_{i+1}-T_i};\qquad \eta_s=\frac{T_i\left(\frac{T_{i+1}'}{T_i}-1\right)}{T_{i+1}-T_i};\qquad \eta_s=\frac{T_i\left(\left(\frac{p_{i+1}}{p_i}\right)^{\frac{\gamma-1}{\gamma}}-1\right)}{T_{i+1}-T_i};$$

$$T_{i+1}-T_i=\frac{T_i\left(\left(\frac{p_{i+1}}{p_i}\right)^{\frac{\gamma-1}{\gamma}}-1\right)}{\eta_s};\qquad \Delta T_i=\frac{T_i}{\eta_s}\left(\left(\frac{p_{i+1}}{p_i}\right)^{\frac{\gamma-1}{\gamma}}-1\right);\quad \Delta T_i=\frac{T_i}{\eta_s}\left((p_r)^{\frac{\gamma-1}{\gamma}}-1\right)$$

$$\Delta T_i=AT_i\text{ where }A=\frac{1}{\eta_s}\left((p_r)^{\frac{\gamma-1}{\gamma}}-1\right)$$

$$\textit{For First stage}\quad \Delta T_1=AT_1\,;$$

$$T_2-T_1=AT_1;\qquad T_2=T_1+AT_1;\qquad\qquad T_2=T_1(1+A)$$

$For\ Second\ stage\ \Delta T_2 = AT_2;$

Substituting T_2 in terms of T_1 , $\Delta T_2 = AT_1(1+A)$

$T_3 - T_2 = AT_2;\ T_3 = T_2 + AT_2;\ \ T_3 = T_2(1+A); T_3 = T_1(1+A)(1+A); T_3 = T_1(1+A)^2$

$For\ Third\ \ stage\ \Delta T_3 = AT_3;$

Substituting T_2 in terms of T_1; $\Delta T_3 = AT_1(1+A)^2$

For Kth stage

$\Delta T_4 = AT_1(1+A)^{K-1}$

$$\sum_{i=1}^{i=K} \Delta T_i = \Delta T_1 + \Delta T_2 + \Delta T_3 + \cdots \dots \dots . + \Delta T_k$$

$\sum_{i=1}^{i=K} \Delta T_i = AT_1 + T_1\, A(1+A) + \ AT_1(1+A)^2 + AT_1(1+A)^3 + \cdots \dots \dots .. + AT_1(1+A)^{K-1};$

$\sum_{i=1}^{i=K} \Delta T_i = AT_1(1 + (1+A) + (1+A)^2 + (1+A)^3 + \cdots \dots \dots \dots + (1+A)^{K-1});;$

$\sum_{i=1}^{i=K} \Delta T_i = T_1[(1+A)^K - 1]$

$$\Delta T_o = T_1\left[\left(1 + \frac{(p_r)^{\frac{\gamma-1}{\gamma}} - 1}{\eta_s}\right)^K - 1\right]$$

$$\eta_o = \frac{T'_{k+1} - T_1}{T_K - T_1}; \qquad \eta_o = \frac{T_1\left(\frac{T'_{k+1}}{T_1} - 1\right)}{\Delta T_o}; \quad \eta_o = \frac{T_1\left[(p_{r0})^{\frac{\gamma-1}{\gamma}} - 1\right]}{T_1\left[\left(1 + \frac{(p_r)^{\frac{\gamma-1}{\gamma}} - 1}{\eta_s}\right)^K - 1\right]}\ ;$$

$$\eta_o = \frac{\left[(p_{r0})^{\frac{\gamma-1}{\gamma}} - 1\right]}{\left[\left(1 + \frac{(p_r)^{\frac{\gamma-1}{\gamma}} - 1}{\eta_s}\right)^K - 1\right]}; \qquad \eta_o = \frac{\left[(p_r)^{K\frac{\gamma-1}{\gamma}} - 1\right]}{\left[\left(1 + \frac{(p_r)^{\frac{\gamma-1}{\gamma}} - 1}{\eta_s}\right)^K - 1\right]} \quad \text{as } p_{r0} = (p_r)^K$$

Let $S = 1 + (1+A) + (1+A)^2 + (1+A)^3 + \cdots \dots \dots \dots + (1+A)^{K-1}$

$S = 1 + (1+A)(1 + (1+A) + (1+A)^2 + (1+A)^3 + \cdots \dots \dots \dots + (1+A)^{K-2})$

$S = 1 + (1+A)(S - (1+A)^{K-1})$

$S - 1 = (1+A)S - (1+A)(1+A)^{K-1}$

$S - 1 = S + AS - (1+A)^K$

$(1+A)^K - 1 = \ AS$

$$\Delta T_o = \sum_{i=1}^{i=K} \Delta T_i = AT_1 S$$

$$= T_1[(1+A)^K - 1]$$

Overall efficiency for a finite number of turbine stages in terms of stage efficiency for a Turbine

T_i is the initial temperature at which the fluid enters the compressor, K is the number of stages having equal pressure ratio, p_r is the pressure ratio in each stage, then the actual temperature rise in each stage can be calculated as follows

For First stage

$$\eta_s = \frac{T_i - T_{i+1}}{T_i - T'_{i+1}}; \quad \eta_s = \frac{T_i - T_{i+1}}{T_i\left(1 - \frac{T'_{i+1}}{T_i}\right)}; \qquad \eta_s = \frac{T_i - T_{i+1}}{T_i\left(1 - \left(\frac{p_{i+1}}{p_i}\right)^{\frac{\gamma-1}{\gamma}}\right)}; \ T_i - T_{i+1} = \eta_s T_i\left(1 - \left(\frac{p_{i+1}}{p_i}\right)^{\frac{\gamma-1}{\gamma}}\right)$$

$$\Delta T_i = \eta_s T_i\left(1 - \left(\frac{p_{i+1}}{p_i}\right)^{\frac{\gamma-1}{\gamma}}\right); \quad \Delta T_i = \eta_s T_i\left(1 - \left(\frac{p_{i+1}}{p_i}\right)^{\frac{\gamma-1}{\gamma}}\right); \ \Delta T_i = \eta_s T_i\left(1 - \left(\frac{p_i}{p_{i+1}}\right)^{-\frac{\gamma-1}{\gamma}}\right)$$

$$\Delta T_i = \eta_s T_i\left(1 - (p_r)^{-\frac{\gamma-1}{\gamma}}\right)$$

$\Delta T_i = AT_i$ where $A = \eta_s\left(1 - (p_r)^{-\frac{\gamma-1}{\gamma}}\right)$

For First stage

$\Delta T_1 = AT_1;$

$T_1 - T_2 = AT_1; \ T_2 = T_1 - AT_1; \ T_2 = T_1(1-A)$

For Second stage, $\Delta T_2 = AT_2$

Substituting T_2 in terms of T_1, $\Delta T_2 = AT_1(1-A)$

$T_2 - T_3 = AT_2; \ T_3 = T_2 - AT_2; \ T_3 = T_2(1-A); \ T_3 = T_2(1-A); T_3 = T_1(1-A)(1-A)$

$T_3 = T_1(1-A)^2$

For third stage, $\Delta T_3 = AT_3;$

Substituting T_2 in terms of T_1, $\Delta T_3 = AT_1(1-A)^2$

Similarly for 4th stage, $\Delta T_4 = AT_1(1-A)^3$

For Kth stage, $\Delta T_4 = AT_1(1-A)^{K-1}$

$$\sum_{i=1}^{i=K} \Delta T_i = \Delta T_1 + \Delta T_2 + \Delta T_3 + \cdots \dots \dots . + \Delta T_k$$

$$\sum_{i=1}^{i=K} \Delta T_i = AT_1 + T_1\, A(1-A) + AT_1(1-A)^2 + AT_1(1-A)^3 + \cdots \dots \dots . . + AT_1(1-A)^{K-1};$$

$$\sum_{i=1}^{i=K} \Delta T_i = AT_1(1 + (1-A) + (1-A)^2 + (1-A)^3 + \cdots \dots \dots \dots + (1-A)^{K-1});$$

$$= T_1[1-(1-A)^K]$$

$$\Delta T_o = T_1[1-(1-A)^K]$$

$$\eta_o = \frac{T_1 - T_K}{T_1 - T'_{k+1}};\ \eta_o = \frac{\Delta T_o}{T_1\left(1 - \frac{T'_{k+1}}{T_1}\right)};\ \eta_o == \frac{T_1[1-(1-A)^K]}{T_1\left(1-\frac{T'_{k+1}}{T_1}\right)};\ \eta_o = \frac{[1-(1-A)^K]}{1-\left(\frac{p'_{k+1}}{p_1}\right)^{\frac{\gamma-1}{\gamma}}};\ \eta_o = \frac{[1-(1-A)^K]}{1-\left(\frac{p_1}{p'_{k+1}}\right)^{-\frac{\gamma-1}{\gamma}}}$$

$$\eta_o = \frac{\left[1-\left(1-\eta_s\left(1-(p_r)^{-\frac{\gamma-1}{\gamma}}\right)\right)^K\right]}{1-(p_{ro})^{-\frac{\gamma-1}{\gamma}}};\quad \eta_0 = \frac{\left[1-\left(1-\eta_s\left(1-(p_r)^{-\frac{\gamma-1}{\gamma}}\right)\right)^K\right]}{1-(p_r)^{-K\frac{\gamma-1}{\gamma}}}$$

Let $S = 1 + (1-A) + (1-A)^2 + (1-A)^3 + \cdots \dots \dots \dots + (1-A)^{K-1}$

$$S = 1 + (1-A)(1 + (1-A) + (1-A)^2 + (1-A)^3 + \cdots \dots \dots \dots + (1-A)^{K-2})$$

$$S = 1 + (1-A)(S - (1-A)^{K-1})$$

$$S - 1 = (1-A)S - (1-A)(1-A)^{K-1}$$

$$S - 1 = S - AS - (1-A)^K$$

$$AS = 1 - (1-A)^K$$

Mutli Stage Compressor (Constant temperature rise for compressor)

For constant stage work in a multistage compressor, the temperature rise in each stage is same, but the temperature at entry of each stage will be different. For given values of overall pressure ratio p_{ro} and polytropic efficiency η_P, the total temperature rise per stage is given by

$$\Delta T_{st} = \frac{\Delta T_0}{K};\quad \Delta T_0 = T_{0k+1} - T_{01};\quad \Delta T_0 = T_{01}\left(\frac{T_{0k+1}}{T_{01}} - 1\right);\ \Delta T_0 = T_{01}\left(\left(\frac{p_{k+1}}{p_1}\right)^{\frac{1}{\eta_P}\left(\frac{\gamma-1}{\gamma}\right)} - 1\right)$$

$$\Delta T_0 = T_{01}\left((p_{ro})^{\frac{1}{\eta_P}\left(\frac{\gamma-1}{\gamma}\right)} - 1\right);\ \Delta T_{st} = \frac{T_{01}\left((p_{ro})^{\frac{1}{\eta_P}\left(\frac{\gamma-1}{\gamma}\right)} - 1\right)}{K}$$

Knowing the temperature rise in each stage, the pressure ratio and hence the efficiency for each stages can now be calculated.

For i[th] stage, it is given by

$$\Delta T_i = T_{i+1} - T_i$$

$$= T_i\left(\frac{T_{i+1}}{T_i} - 1\right)$$

$$= T_i\left((p_{ri})^{\frac{1}{\eta_P}\left(\frac{\gamma-1}{\gamma}\right)} - 1\right)$$

$$\frac{\Delta T_i}{T_i} = (p_{ri})^{\frac{1}{\eta_P}\left(\frac{\gamma-1}{\gamma}\right)} - 1$$

$$1 + \frac{\Delta T_i}{T_i} = (p_{ri})^{\frac{1}{\eta_P}\left(\frac{\gamma-1}{\gamma}\right)}$$

Hence the pressure rise in each stage is

$$p_{ri} = \left(1 + \frac{\Delta T_i}{T_i}\right)^{\eta_P\left(\frac{\gamma}{\gamma-1}\right)}$$

From the above equation it can be seen that the pressure ratio in each stage decreases as T_i increases (as ΔT_i is constant for all stages

Hence stage efficiency is not constant and it varies for each stage, Hence

It can be calculated for each stage as

$$\eta_{sti} = \frac{p_{ri}^{\left(\frac{\gamma-1}{\gamma}\right)} - 1}{p_{ri}^{\frac{1}{\eta_P}\left(\frac{\gamma-1}{\gamma}\right)} - 1}$$

Constant temperature rise for Expansion stages (Turbine stage)

For constant stage work in a multistage turbine, the temperature rise in each stage is same, but the temperature at entry of each stage will be different. For given values of overall pressure ratio p_{ro} and polytropic efficiency η_P, the total temperature rise per stage is given by

$$\Delta T_{st} = \frac{\Delta T_0}{K};$$

$$\Delta T_0 = T_{01} - T_{0k+1}; \quad \Delta T_0 = T_{01}\left(1 - \frac{T_{0k+1}}{T_{01}}\right); \ \Delta T_0 = T_{01}\left(1 - \left(\frac{p_{k+1}}{p_1}\right)^{\eta_P\left(\frac{\gamma-1}{\gamma}\right)}\right)$$

$$\Delta T_0 = T_{01}\left(1 - \left(\frac{p_1}{p_{k+1}}\right)^{-\eta_P\left(\frac{\gamma-1}{\gamma}\right)}\right); \qquad \Delta T_0 = T_{01}\left(1 - (p_{ro})^{-\eta_P\left(\frac{\gamma-1}{\gamma}\right)}\right)$$

$$\Delta T_{st} = \frac{T_{01}\left(1 - (p_{ro})^{-\eta_P\left(\frac{\gamma-1}{\gamma}\right)}\right)}{K}$$

Knowing the temperature rise in each stage, the pressure ratio and hence the efficiency for each stages can now be calculated.

For i^{th} stage, it is given by

$$\Delta T_i = T_i - T_{i+1}; \quad \Delta T_i = T_i \left(1 - \frac{T_{i+1}}{T_i}\right); \quad \Delta T_i = T_i \left(1 - \left(\frac{p_{i+1}}{p_i}\right)^{\frac{n-1}{n}}\right); \quad \Delta T_i = T_i \left(1 - \left(\frac{p_i}{p_{i+1}}\right)^{-\frac{n-1}{n}}\right)$$

$$\Delta T_i = T_i \left(1 - \left(\frac{p_i}{p_{i+1}}\right)^{-\eta_P\left(\frac{\gamma-1}{\gamma}\right)}\right); \quad \frac{\Delta T_i}{T_i} = 1 - (p_{ri})^{-\eta_P\left(\frac{\gamma-1}{\gamma}\right)}$$

$$1 - \frac{\Delta T_i}{T_i} = (p_{ri})^{-\eta_P\left(\frac{\gamma-1}{\gamma}\right)}$$

Hence the pressure rise in each stage is

$$p_{ri} = \left(1 - \frac{\Delta T_i}{T_i}\right)^{-\frac{1}{\eta_P}\left(\frac{\gamma-1}{\gamma}\right)}$$

From the above equation it can be seen that the pressure ratio in each stage decreases as T_i increases (as ΔT_i is constant for all stages

Various formulae

Efficiency	Compressor	Turbine
Infinitesimal or Polytropic	$\eta_P = \dfrac{\left(\frac{\gamma-1}{\gamma}\right) ln\frac{p_2}{p_1}}{ln\frac{T_2}{T_1}}$ $\eta_P = \left(\frac{\gamma-1}{\gamma}\right)\left(\frac{n}{n-1}\right)$	$\eta_P = \dfrac{ln\frac{T_2}{T_1}}{\left(\frac{\gamma-1}{\gamma}\right) ln\frac{p_2}{p_1}}$ $\eta_P = \left(\frac{\gamma}{\gamma-1}\right)\left(\frac{n-1}{n}\right)$
Stage	$\eta_{st} = \dfrac{\left(\frac{p_2}{p_1}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}-1}{\left(\frac{p_2}{p_1}\right)^{\frac{1}{\eta_P}\left(\frac{\gamma-1}{\gamma}\right)}-1}$ $\eta_{st} = \dfrac{(p_r)^{\left(\frac{\gamma-1}{\gamma}\right)}-1}{(p_r)^{\frac{1}{\eta_P}\left(\frac{\gamma-1}{\gamma}\right)}-1}$	$\eta_{st} = \dfrac{1-\left(\frac{p_1}{p_2}\right)^{-\eta_P\left(\frac{\gamma-1}{\gamma}\right)}}{1-\left(\frac{p_1}{p_2}\right)^{-\left(\frac{\gamma-1}{\gamma}\right)}}$ $\eta_{st} = \dfrac{1-(p_r)^{-\eta_P\left(\frac{\gamma-1}{\gamma}\right)}}{1-(p_r)^{-\left(\frac{\gamma-1}{\gamma}\right)}}$
Overall	$\eta_{st} = \dfrac{\left(\frac{p_{k+1}}{p_1}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}-1}{\left(\frac{p_{k+1}}{p_1}\right)^{\frac{1}{\eta_P}\left(\frac{\gamma-1}{\gamma}\right)}-1}$ $\eta_{st} = \dfrac{(p_{r0})^{\left(\frac{\gamma-1}{\gamma}\right)}-1}{(p_{r0})^{\frac{1}{\eta_P}\left(\frac{\gamma-1}{\gamma}\right)}-1}$	$\eta_{st} = \dfrac{1-\left(\frac{p_1}{p_{k+1}}\right)^{-\eta_P\left(\frac{\gamma-1}{\gamma}\right)}}{1-\left(\frac{p_1}{p_{k+1}}\right)^{-\left(\frac{\gamma-1}{\gamma}\right)}}$ $\eta_{st} = \dfrac{1-(p_{r0})^{-\eta_P\left(\frac{\gamma-1}{\gamma}\right)}}{1-(p_{ro})^{-\left(\frac{\gamma-1}{\gamma}\right)}}$
Efficiency	**Compressor**	**Turbine**
Overall efficiency in terms of stage efficiency	$\eta_o = \dfrac{\left[(p_{r0})^{\frac{\gamma-1}{\gamma}}-1\right]}{\left[\left(1+\frac{(p_r)^{\frac{\gamma-1}{\gamma}}-1}{\eta_s}\right)^K-1\right]}$ $\eta_o = \dfrac{\left[(p_r)^{K\frac{\gamma-1}{\gamma}}-1\right]}{\left[\left(1+\frac{(p_r)^{\frac{\gamma-1}{\gamma}}-1}{\eta_s}\right)^K-1\right]}$ $p_{r0} = (p_r)^K$	$\eta_o = \dfrac{\left[1-\left(1-\eta_s\left(1-(p_r)^{-\frac{\gamma-1}{\gamma}}\right)\right)^K\right]}{1-(p_{r0})^{-\frac{\gamma-1}{\gamma}}}$ $\eta_0 = \dfrac{\left[1-\left(1-\eta_s\left(1-(p_r)^{-\frac{\gamma-1}{\gamma}}\right)\right)^K\right]}{1-(p_r)^{-K\frac{\gamma-1}{\gamma}}}$ $p_{r0} = (p_r)^K$

Numericals

4. A 16 stage axial flow compressor is to have a pressure ratio of 6.3 and tests have shown that a stage efficiency of 89.5% can be obtained. The intake conditions are 288K and 1 bar pressure Find i) Overall efficiency ii) Polytropic efficiency iii) Preheat factor(2c. 08 Dec/Jan 17)

K=16; $p_{r0} = 6.3$; $\eta_s = 0.895$; $T_1 = 288K$; $p_1 = 1\,bar$

η_0 =?; η_p =?;   PHF=?

$p_{r0} = (p_r)^K$; 6.3 =$(p_r)^{16}$; $p_r = 1.1219$

$$\eta_0 = \frac{\left[(p_{r0})^{\frac{\gamma-1}{\gamma}} - 1\right]}{\left[\left(1+\frac{(p_r)^{\frac{\gamma-1}{\gamma}}-1}{\eta_s}\right)^K - 1\right]} \quad \text{as } p_{r0} = (p_r)^K$$

$$\eta_0 = \frac{\left[(6.3)^{0.286}-1\right]}{\left[\left(1+\frac{(1.1219)^{0.286}-1}{0.895}\right)^K - 1\right]} = 0.8674$$

Also, Overall efficiency

$$\eta_0 = \frac{p_{r0}^{\left(\frac{\gamma-1}{\gamma}\right)}-1}{p_{r0}^{\frac{1}{\eta_P}\left(\frac{\gamma-1}{\gamma}\right)}-1}; \quad 0.8674 = \frac{6.3^{0.286}-1}{p_{r0}^{\frac{1}{\eta_P}0.286}-1}; \ \eta_p = 0.8967$$

$\eta_0 = \eta_s x PHF$; 0.8674=0.895xPHF ie PHF= 0.9691

5. In a three stage turbine the pressure ratio of each stage is 2 and the stage efficiency is75%. Calculate the overall efficiency and reheat factor (2c. 08, June/July 14)

$p_r = 2; \eta_s = 0.75;$

$$\eta_0 = \frac{\left[1-\left(1-\eta_s\left(1-(p_r)^{-\frac{\gamma-1}{\gamma}}\right)\right)^K\right]}{1-(p_r)^{-K\frac{\gamma-1}{\gamma}}}; \quad \eta_0 = \frac{\left[1-\left(1-0.75\left(1-(2)^{-0.286}\right)\right)^3\right]}{1-(2)^{-3x0.286}}; \ \eta_0 = \frac{0.3525}{0.4483}; \ \ \eta_0 = 0.7863$$

$\eta_0 = \eta_s x PHF$; $0.7863 = 0.75 * RHF$; $RHF = 1.048$

6. An air compressor has eight stages of equal pressure ratio 1:3.5. The flow rate through the compressor and its overall efficiency are 50kg/s and 82% respectively. If the conditions of air at the entry are 1 bar and 300K determine
   i) The state of air at compressor exit
   ii) Polytropic efficiency
   iii) Stage efficiency

$p_{ro} = 3.5$; $\dot{m} = 50kg/s; \eta_0 = 0.82; p_1 = 1\ bar; T_1 = 300K$

i) <u>The state of air at compressor exit</u>

$$\eta_0 = \frac{T_{09S}-T_{01}}{T_{09}-T_{01}}; \quad \eta_0 = \frac{T_{09S}-T_{01}}{T_{09}-T_{01}}; \quad \eta_0 = \frac{T_{01}\left(\frac{T_{09S}}{T_{01}}-1\right)}{T_{09}-T_{01}}$$

$$\eta_0 = \frac{T_{01}\left(p_{r0}^{k\left(\frac{\gamma-1}{\gamma}\right)}-1\right)}{T_{09}-T_{01}}; \quad 0.82 = \frac{300\left(3.5^{8(0.286)}-1\right)}{T_{09}-300}; \quad T_{09} = 6363.02K$$

ii) <u>Polytropic efficiency</u>

$$\eta_0 = \frac{p_r^{K\left(\frac{\gamma-1}{\gamma}\right)}-1}{p_r^{\frac{1}{\eta_P}K\left(\frac{\gamma-1}{\gamma}\right)}-1}; \qquad 0.82 = \frac{3.5^{(18*0.286)}-1}{3.5^{\left(\frac{18*0.286}{\eta_P}\right)}-1}; \qquad 3.5^{\left(\frac{18*0.286}{\eta_P}\right)} - 1 = 769.76 + 1;$$

$$\frac{18*0.286}{\eta_P} ln3.5 = ln770.76; \quad \eta_P = 0.970$$

iii) Stage efficiency

$$\eta_s = \frac{T_{02s}-T_{01}}{T_{02}-T_{01}}; \quad \eta_s = \frac{T_{01}\left(p_r^{\frac{\gamma-1}{\gamma}}-1\right)}{T_{01}\left(p_r^{\frac{n-1}{n}}-1\right)}; \quad \eta_s = \frac{\left(p_r^{\frac{\gamma-1}{\gamma}}-1\right)}{\left(p_r^{\frac{1}{\eta_P}\frac{\gamma-1}{\gamma}}-1\right)}$$

$$\eta_s = \frac{(3.5^{0.286}-1)}{\left(3.5^{\frac{0.286}{0.970}}-1\right)}; \quad \eta_s = \frac{0.431}{0.447}; \quad \eta_s = 0.9642$$

7. Air flows through an air turbine where its stagnation pressure is reduced in the ratio 5:1, the total to total efficiency is 80% . The air flow rate is 5kg/s If the total power output is 500kW, find i) inlet total temperature ii) actual exit temperature iii) actual exit static temperature if the flow velocity is 100m/s iv) total to static efficiency (2b. 10 June/July 16) (2c. 10 Dec17/Jan 18)

$p_{r0} = \frac{p_1}{p_{k+1}} = 5$; $\eta_{tt} = 80\%$; m=5kg/s; P=500kW ;T_{01} =?; T_{02} =?; T_2 =?

i) <u>inlet total temperature</u>

$$P = \dot{m}(h_{01} - h_{02}); \quad 500 = 5(h_{01} - h_{02}); \qquad (h_{01} - h_{02}) = 100kJ/kg$$

$$C_p(T_{01} - T_{02}) = 100; \qquad 1.005(T_{01} - T_{02}) = 100 \;\; ; \quad (T_{01} - T_{02}) = 99.52K$$

$$\eta_{tt} = \frac{T_{01}-T_{02}}{T_{01}\left(1-(p_{r0})^{-\frac{\gamma-1}{\gamma}}\right)} \; ; \; 0.8 = \frac{99.52}{T_{01}\left(1-(5)^{-0.286}\right)}; \quad T_{01} = 337.12K$$

ii) <u>Actual exit temperature</u>

$$(T_{01} - T_{02}) = 99.52K; \qquad (337.12 - T_{02}) = 99.52K; \; T_{02} = 237.6\text{K}$$

iii) **<u>Actual exit static temperature</u>**

$$T_{02} = T_2 + \frac{V_2^2}{2C_p}; \qquad 237.6 = T_2 + \frac{100^2}{2x1005}; \quad T_2 = 232.62K$$

v) **<u>Total to static efficiency</u>**

$$\eta_{t-s} = \frac{h_{01} - h_{02}}{h_{01} - h_{2s}}$$

$$\frac{T_{01}}{T_{02s}} = \left(\frac{p_{01}}{p_{02}}\right)^{\frac{\gamma-1}{\gamma}}; \qquad \frac{337.2}{T_{02s}} = (5)^{0.286}; \qquad T_{02s} = 212.80\text{K}$$

$$T_{02s} = T_{2s} + \frac{V_2^2}{2C_p}; \qquad 212.8 = T_{2s} + \frac{100^2}{2x1005}$$

$$T_{2s} = 207.82K$$

$$\eta_{t-s} = \frac{T_{01}-T_{02}}{T_{01}-T_{2s}}; \quad \eta_{t-s} = \frac{99.52}{337.12-207.82}; \quad \eta_{t-s} = 0.769$$

8. A gas turbine has 2 stages and develops 20MW power. The inlet temperature is 1450K. The overall pressure ratio is 7.5. Assume that pressure ratio of each stage is same and the expansion isentropic efficiency is 0.88. Claculate i) Pressure ratio at each stage i) Pressure ratio at each stage ii) Polytropic Efficiency iii) Mass flow rate iv) Stage efficnecy and power of each stage (2b. 10 Dec/Jan 12)
9. The output of three stage gas turbine is 30MW at the shaft coupling at an entry temperature of 1500K. The overall pressure ratio across the turbine is 11.0 and efficiency is 88%. If the pressure ratio of each stage is the same. Determine i) Pressure ratio of each stage ii) Polytropic effiency iii) The mass flow rate iv) The efficiency and power of each stage . Assume $\gamma_{air} =$ 1.4, C_p=1.005kJ/kgK, $\eta_{mech} = 91\%$ (2b. 10 Dec/Jan 19)

P=30MW =30000kW; $T_{01} = 1500K$ $p_{r0} = 11$; $\eta_0 = 88\%$; p_r =? ; η_p =?; $\dot{m}$ =? η_s =?

i) **<u>Pressure ratio in each stage</u>**

$$p_{r0} = (p_r)^K; \; 11 = (p_r)^3; \; p_r = 2.22$$

ii) **<u>Polytropic effieciency</u>**

$$\eta_0 = \frac{1-(p_{r0})^{-\eta_p\left(\frac{\gamma-1}{\gamma}\right)}}{1-(p_{r0})^{-\left(\frac{\gamma-1}{\gamma}\right)}};$$

$$0.88 = \frac{1-(11)^{-\eta_p(0.286)}}{1-(11)^{-(0.286)}}; 0.436 = 1-(11)^{-\eta_p(0.286)}; (11)^{-\eta_p(0.286)} = 0.564$$

$$-\eta_p(0.286)ln11 = ln0.564; \quad -\eta_p * 0.686 = -0.5727; \; \eta_p = 0.8348$$

$$Polytropic\ effeiciency = 83.48\%$$

iii) **<u>Total mass flow rate</u>**

$$P = \frac{P_s}{\eta_{mech}}; \; P = \frac{30000}{0.91}; \; P = 32967\text{kW}$$

$$\eta_0 = \frac{T_{01}-T_{04}}{T_{01}-T_{04s}}; \; \eta_0 = \frac{T_{01}-T_{04}}{T_{01}\left(1-\frac{T_{04s}}{T_{01}}\right)}; \; ; \; \eta_0 = \frac{T_{01}-T_{04}}{T_{01}\left(1-\left(\frac{p_4}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right)};$$

$$\eta_0 = \frac{T_{01}-T_{04}}{T_{01}\left(1-(p_{ro})^{-\left(\frac{\gamma-1}{\gamma}\right)}\right)}; \qquad 0.88 = \frac{T_{01}-T_{04}}{1500(1-(11)^{-0.286})}; T_{01}-T_{04} = 655.13\text{K}$$

$$P = \dot{m}C_P(T_4 - T_1); \; 32967 = \dot{m} * 1.005 * 655.13; \quad \dot{m} = 50.07kg/s$$

iv) **<u>The efficiency and power of each stage</u>**

$$\eta_{st} = \frac{1-(p_r)^{-\eta_P\left(\frac{\gamma-1}{\gamma}\right)}}{1-(p_r)^{-\left(\frac{\gamma-1}{\gamma}\right)}} \text{ where } p_r = \frac{p_1}{p_2}$$

$$\eta_{st} = \frac{1 - 2.33^{-(0.8348*0.286)}}{1 - 2.33^{-0.286}}$$

$$\eta_{st} = \frac{0.1828}{0.2148}; \qquad \eta_{st} = 0.851$$

$$\eta_{st} = \frac{W_a}{C_p(T_1 - T_{2s})}; \quad \eta_{st} = \frac{W_a}{C_p T_1\left(1 - \frac{T_{2s}}{T_1}\right)}; \quad \eta_{st} = \frac{W_a}{C_p T_1\left(1 - p_r^{-\left(\frac{\gamma-1}{\gamma}\right)}\right)}$$

$$0.851 = \frac{W_a}{1.005*1500(1 - 2.33^{-0.286})};$$

$$W_a = 275.67 kJ/kg$$

$$P_s = \dot{m} W_{sa}; P_s = 50.07 * 275.66; P_s = 13802.79 kW$$

10. A multi stage axial flow compressor, the air is taken at 1 bar and 15°C and compressed to a pressure of 6.4bar. The final true temperature is 300°C due to the compression process. Determine the overall compression efficiency and also the polytropic efficiency. Determine the number of stages required if the true temperature rise is limited to 13°K for each stage. Assume polytropic efficiency is equal to stage efficiency. (2c. 10 Dec/Jan 15)

$$p_1 = 1bar = 100kPa; T_1 = 15^oC = 288K; p_{k+1} = 6.4\ bar = 640kPa$$

$$T_{k+1} = 300^oC = 573K$$

$$\frac{T_{k+1s}}{T_1} = \left(\frac{p_{k+1}}{p_1}\right)^{\frac{\gamma-1}{\gamma}}; \frac{T_{k+1s}}{288} = \left(\frac{6.4}{1}\right)^{0.286}; T_{k+1s} = 489.73K$$

$$\eta_0 = \frac{T_{k+1s} - T_{01}}{T_{k+1} - T_{01}}; \eta_0 = \frac{489.73 - 288}{573 - 288}; \quad \eta_0 = 0.7078$$

$$\eta_0 = \frac{p_{r0}^{\left(\frac{\gamma-1}{\gamma}\right)} - 1}{p_{r0}^{\frac{1}{\eta_P}\left(\frac{\gamma-1}{\gamma}\right)} - 1}; 0.7078 = \frac{6.4^{0.286} - 1}{6.4^{\frac{0.286}{\eta_P}} - 1}; \quad 6.4^{\frac{0.286}{\eta_P}} = 1.9896; \ \frac{0.286}{\eta_P} ln6.4 = 1.9896$$

$$\eta_P = 0.7717$$

11. Air enters a compressor at a static pressure of 1.5 bar, a static temperature of 15°C and a flow velocity of 50m/s, At the exit the static pressure is 3 bar , the static temperature is 100°C and the flow velocity is 100m/s . The outlet is 1m above the inlet Evaluate i) the isentropic change in enthalpy ii) The actual change in enthalpy iii) Efficiency of the compressor (2c. 10 June/July 17) (2b. O8 June/July 18, 15ME53)
12. A 16 stage axial flow compressor is to have a pressure ratio of 6.3 with a stage efficiency of 89.5% can be obtained. The intake conditions are 15°C and 1 bar pressure Determine i) Expected Overall efficiency ii) Polytropic efficiency Take $\gamma = \frac{C_p}{C_v}$ =1.4 (2c. 08 June/July 18)

13. A 9 stage centrifugal compressor has overall stage pressure ratio 2.82. Air enters the compressor at 1 bar and 15°C. The efficiency of the compressor is 88%. Determine the following : i) Pressure ratio of each stage ii) Polytropic efficiency iii) Preheat factor (2b. 10 Dec/Jan 16

ENERGY TRANSFER IN TURBOMACHINES

1. With neat sketch derive an expression for Eulers turbine equation (3a. 10 June/July 17)
2. Derive an alternate form of Euler's turbine equation and explain the significance of each energy equations (3a. 10, June/July14)(3a,10, Dec17/Jan18) (3a,10, Dec18/Jan19,08scheme)
3. Show that the alternate form of Eulers Turbine equation can be expressed as

$$W = \frac{(V_1^2 - V_2^2) + (U_1^2 - U_2^2) - (V_{r1}^2 - V_{r2}^2)}{2}$$ (3a. 10 June/July 13)

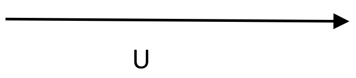
Draw the velocity triangles

4. Define degree of reaction (R) . Derive an expression relating utilization factor with degree of reaction (2b. 10, Dec16/Jan 17)
5. Define utilization factor for a turbine. Derive an expression relating utilization factor with degree of reaction for an axial flow turbine (3a. 10, Dec14/Jan 15) (3a. 10, June/July18) (3a. 08, June/July18, 15 scheme)
6. Why the discharge blade angles has considerable effect in the analysis of turbomachine. Give reasons (3a,04, Dec18/Jan 19,10scheme)
7. Draw the velocity triangles at inlet and outlet of an axial flow turbine when i) R is $-ve$,ii) R=0, iii) R=0.5 iv) R=1 v) $R > 1$. Discuss the energy in each case (3b,10, Dec18/Jan 19,10scheme)
8. Explain why turbine with reaction $R > 1$ and $R < 0$ are not in practical use (4a, 4, June/July18)

Energy Transfer in Turbo machines

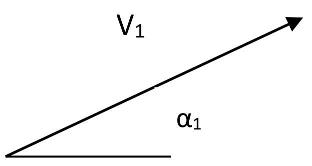
Rotor Speed- tangential speed –peripheral speed of the shaft -$U = \frac{\pi D N}{60}$

In velocity triangle is always horizontal



Velocity of fluid (steam, water, air,jet)----- Absolute velocity of fluid----- V

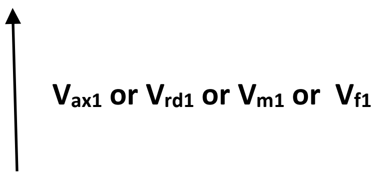
Fluid Angle at inlet , nozzle exit angle (Impulse turbine), exit angle of guide (fixed) blade α_1 with the direction of U



Absolute Velocity at is to be resolved into two components ---

1) along tangential direction and is called as tangential component velocity of fluid V_{u1} (whirl velocity V_{w1}) ---- along horizontal direction (along U) ie the image of V_1 along the direction of U

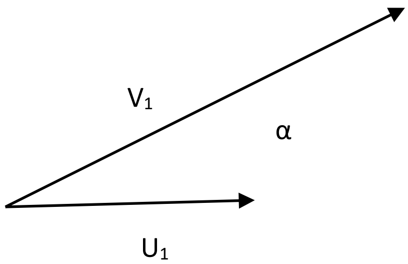
2) Along axial direction in axial turbomachine V_{ax1} (called as axial component), along radial direction in radial flow turbomachine V_{rd1} (called as radial component). Axial and radial direction represented in velocity triangle in Y direction



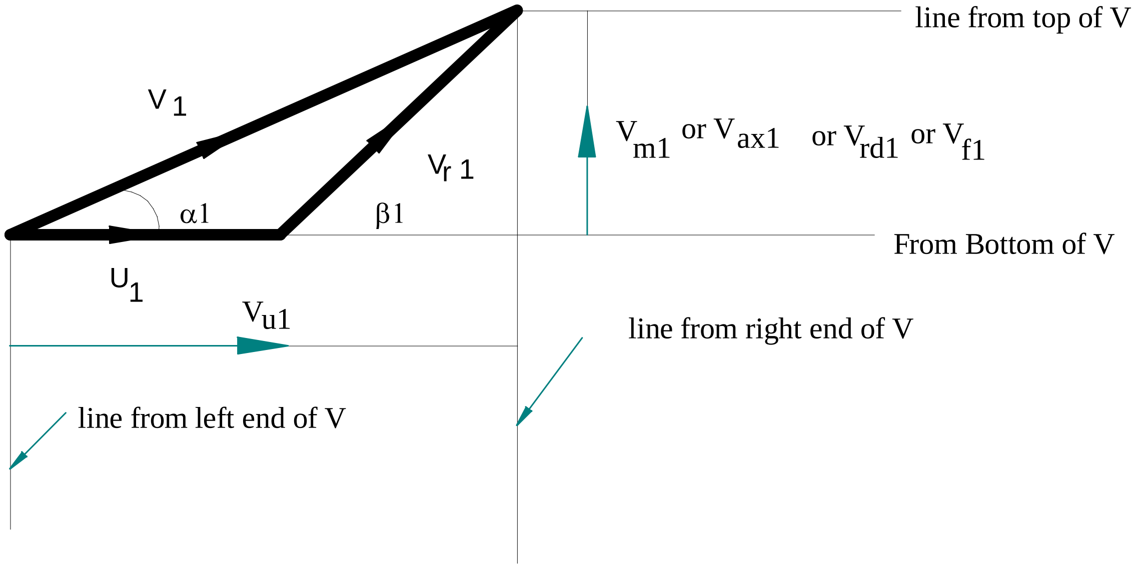
Axial component in axial flow turbomachine and radial component in radial flow turbine is called as velocity of flow

Symbol used in y direction is V_{ax1} or V_{rd1} or V_{m1} or V_{f1}

In drawing velocity triangle U_1 and V_1 should lead from common point



Vector difference between absolute velocity of the fluid and tangential speed of rotor is called as relative velocity and in velocity diagram this is the line connecting tip of U and V as given below and arrow opposes V and V_r follows U



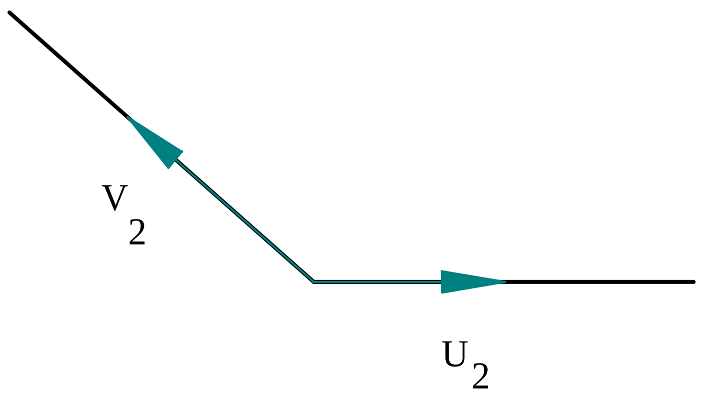
Above triangle is the general velocity triangle at inlet of the turbine

Direction of V_r is the moving vane angle (vane (blade)angle, runner vane (blade) angle, moving vane (blade) angle) and it is denoted by β

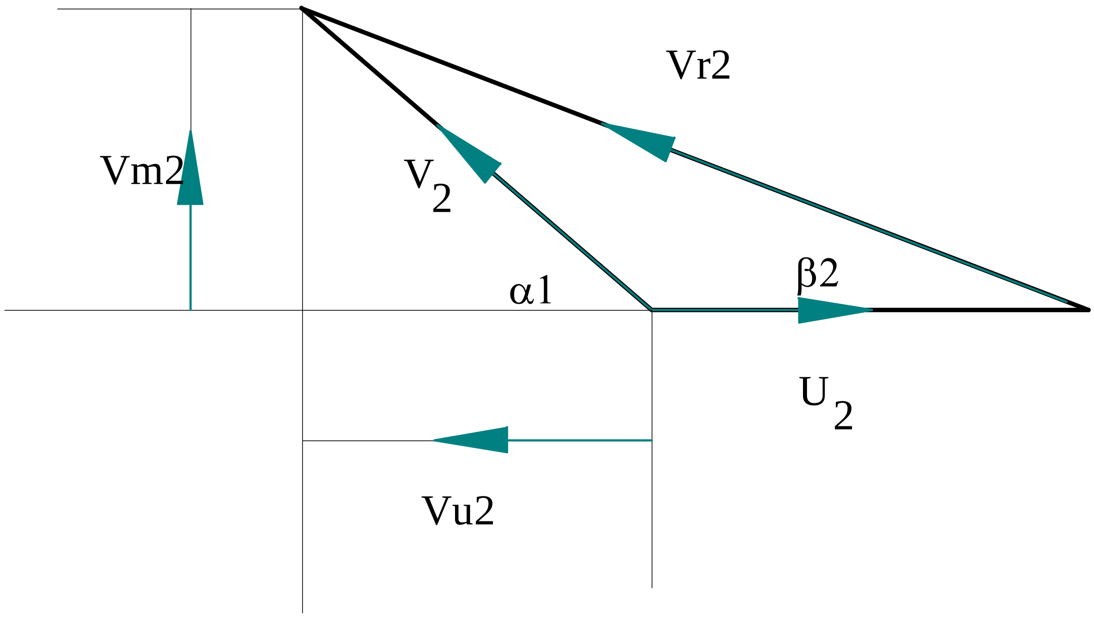
Hence α is always associated with V and β is always associated with V_r

$V_{u1} = V_1\ cos\alpha_1;\quad V_{m1} or\ V_{f1} = V_1\ sin\alpha_1;\ \ tan\ \beta_1 = \frac{V_{m1}}{V_{u1}-U_1};\quad V_{r1} = V_{m1} sin\beta_1$

General outlet triangle as given below



U_2 and V_2 are emerging from single point and line joining tip of V_2 and $U_2$is relative velocity at outlet



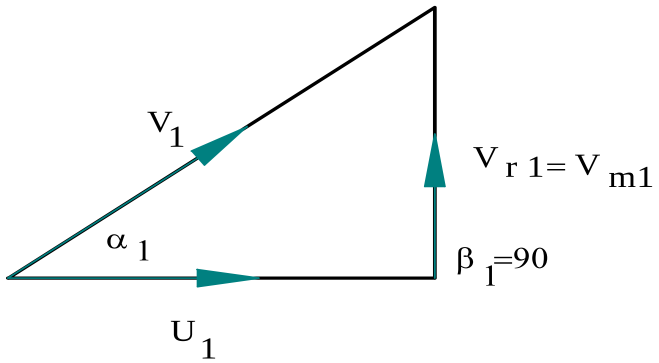
$\overleftarrow{V_{u2}} = V_{r2}\ cos\ \beta_2 - U_2$; V_{m2}or V_{f2} =$V_{r2}\ sin\ \beta_2$

Note down the difference between inlet velocity triangle for turbine (general) :

1) Direction of V_1 is towards right in the inlet velocity triangle where as Direction of V_2 in the outlet velocity triangle is towards right
2) Direction of V_1 is towards right in the inlet velocity triangle where as Direction of V_2in the outlet velocity triangle is towards right

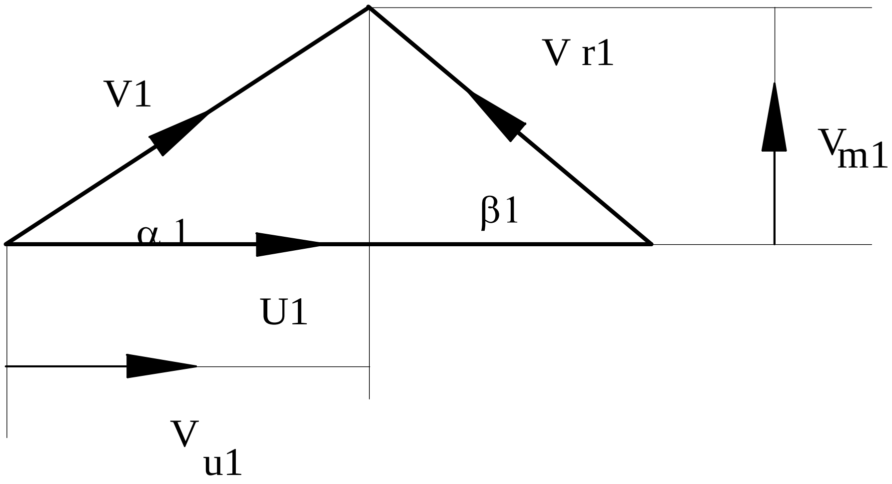
Inlet triangle for given condition:

If the vane angle at inlet is Axial/radial ie β_1 =90°



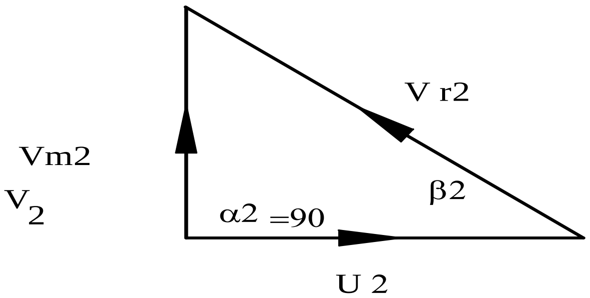
$\overrightarrow{V_{u1}} = U_1$; $V_{r1} = V_{m1}$; $tan\ \alpha_1 = \frac{V_{m1}}{u_1}$

<u>If U_1 is greater than V_{u1}</u>



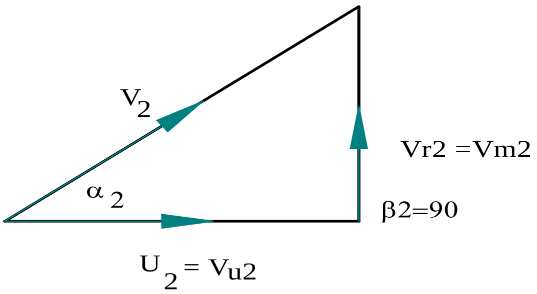
Outlet velocity triangle for turbine

If fluid exit is Axial/Radial or utilization factor is maximum α_2=90 (whirl velocity at outlet or tangential component at outlet =0)

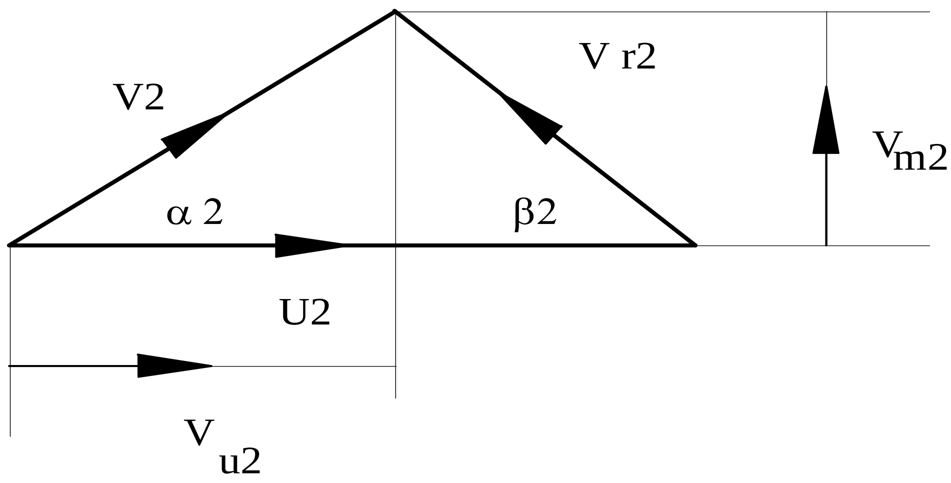


$\overrightarrow{V_{u2}}$ =0 ; $V_2 = V_{m2}$; $tan\, \beta_2 = \frac{V_{m2}}{U_2}$

If blade angle at outlet =90^0 ie β_2=90^{0+}



If U_2 is greater than $\overrightarrow{V_{u2}}$ or tangential component of absolute velocity (Whirl velocity) at outlet is in the opposite direction of tangential component at inlet



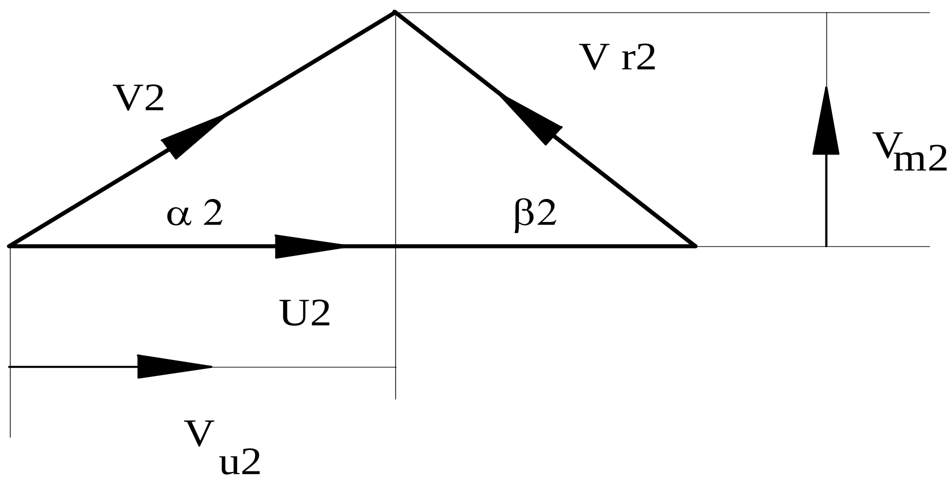
General velocity triangle for power absorbing turbomachine:

Direction of $\overrightarrow{V_{u1}}$ and $\overrightarrow{V_{u2}}$ are in the same direction

U_1 is greater than $\overrightarrow{V_{u1}}$

U_2 is greater than $\overrightarrow{V_{u2}}$

Outlet velocity triangle:



Here $\overrightarrow{V_{u2}}$ is greater than $\overrightarrow{V_{u1}}$

Eulers turbine equation:

Force= Rate change of momentum =mass (kg/s) x change in velocity

Force along tangential direction= mass (kg/s) x change in velocity along tangential direction

$F_u = \frac{\dot{m}}{g_c}\left(\overrightarrow{V_{u1}} - \overrightarrow{V_{u2}}\right)$ (tangential thrust (force)

Force along a axial/radial direction= mass (kg/s) x change in velocity along axial/radial direction (Axial/radial thrust)

$F_a = \frac{\dot{m}}{g_c}(V_{m1} - V_{m2})$

$Torque = force * radius$

$Torque\ along\ tangential\ direction = T = \frac{\dot{m}}{g_c}(\overrightarrow{V_{u1}}r_1 - \overrightarrow{V_{u2}}r_2)$

$Power = Torque * angular\ velocity$

$E = T * \omega$

$E = \frac{\dot{m}}{g_c}\left(\overrightarrow{V_{u1}}r_1 - \overrightarrow{V_{u2}}r_2\right)\omega;\ E = \frac{\dot{m}}{g_c}\left(\overrightarrow{V_{u1}}r_1\omega - \overrightarrow{V_{u2}}r_2\omega\right);\ E = \frac{\dot{m}}{g_c}\left(\overrightarrow{V_{u1}}r_1\frac{2\pi N}{60} - \overrightarrow{V_{u2}}r_2\frac{2\pi N}{60}\right)$

$E = \frac{\dot{m}}{g_c}\left(\overrightarrow{V_{u1}}\frac{\pi D_1 N}{60} - \overrightarrow{V_{u2}}\frac{\pi D_2 N}{60}\right);\ E = \frac{\dot{m}}{g_c}(\overrightarrow{V_{u1}}\,U_1 - \overrightarrow{V_{u2}}\,U_2)$

$\frac{E}{\dot{m}} = \frac{\overrightarrow{V_{u1}}\,U_1 - \overrightarrow{V_{u2}}\,U_2}{g_c}$ — — — called as Eulers turbine equation

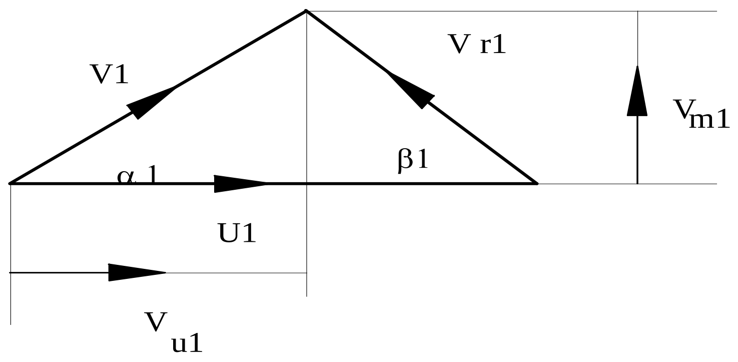
$\frac{E}{\dot{m}}$ is also equal to change in stagnation enthalpy (Δh_o) =C_p (T_{o1} –T_{02}) = $C_p\ \Delta T_o$

$\frac{E}{\dot{m}} = \frac{\overrightarrow{V_{u1}}\,U_1 - \overrightarrow{V_{u2}}\,U_2}{g_c}$

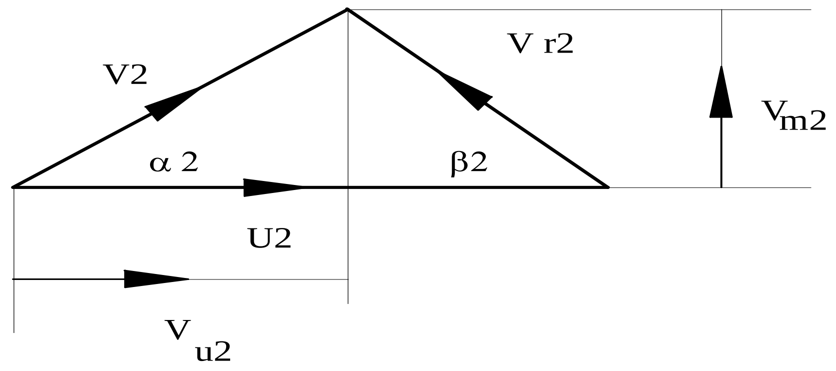
If V_{u1} and V_{u2} are in the opposite direction ie

Hence $\frac{E}{\dot{m}} = \frac{1}{g_c}(\overrightarrow{V_{u1}}\,U_1 + \overleftarrow{V_{u2}}U_2)$

Inlet Velocity triangle



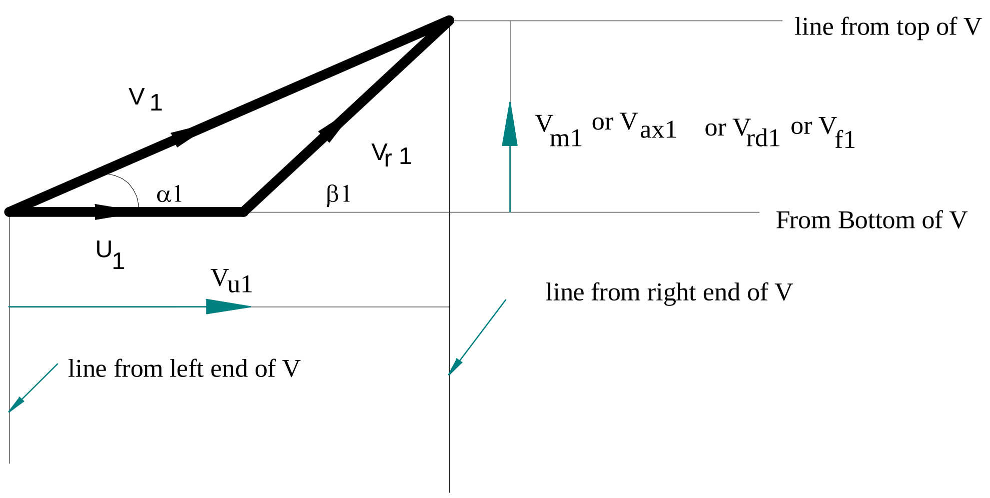
Outlet velocity triangle:



$$\frac{E}{\dot{m}} = \frac{\overrightarrow{V_{u1}}\, U_1 \; - \; \overrightarrow{V_{u2}}\, U_2 \;)}{g_c}$$

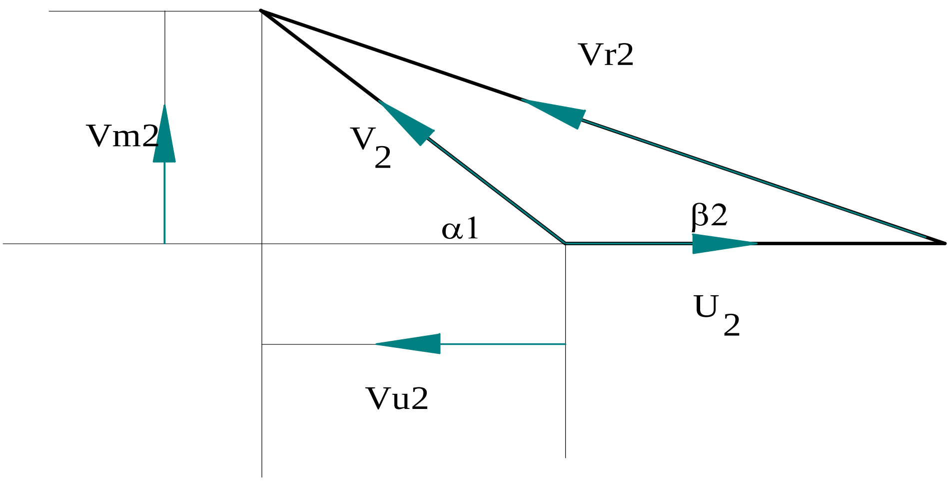
<u>Inlet and outlet velocity triangle if V_{u1} and V_{u2} are in the opposite direction</u>

Inlet Velocity triangle



V_{u1} is +ve

Outlet velocity triangle



$\frac{E}{\dot{m}} = \frac{(\overrightarrow{V_{u1}}\,U_1 + \overleftarrow{V_{u2}}U_2\,)}{g_c}$

In axial flow turbo machines$U_1 = U_2 = U$ since $D_1 = D_2$

And generally (unless stated) $V_{m1} = V_{m2} = V_m$(ie flow velocity is constant)

In radial flow turbomachine U_1 ≠(not equal) U_2 since $D_1 \neq D_2$ (centrifugal compressor, Francis turbine)

All impulse turbine are axial flow machines ie $U_1 = U_2 = U$

In an Impulse turbine generally (unless stated) $V_{r1} = V_{r2}$;

If blade friction coefficient K is given in the problem $V_{r2} = KV_{r1}$

In reaction turbine $V_{r2} > V_{r1}$ and generally $V_{m1} = V_{m2}$ (ie flow velocity is constant)

If blades are equiangular means $\beta_1 = \beta_2$

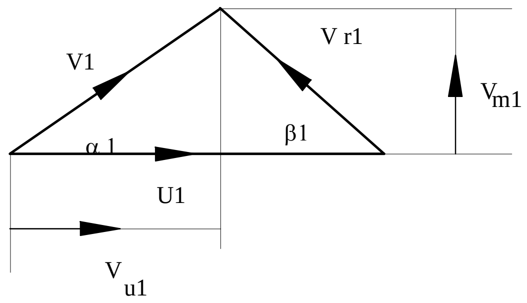
If outlet velocity triangle is 3° is less than inlet blade angle then $\beta_2 = \beta_1 - 3$

Other Important point is

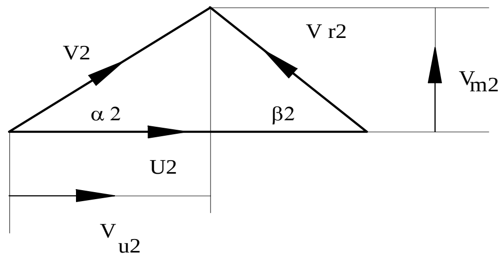
- For the power developing turbomachine ie turbine $\frac{E}{\dot{m}}$is +ve (ie $\overrightarrow{V_{u1}}\,U_1 > \overrightarrow{V_{u2}}\,U_2$)
- For the power absorbing turbomachine ie pump or compressor $\frac{E}{\dot{m}}$is − ve (ie $\overrightarrow{V_{u2}}\,U_2 > \overrightarrow{V_{u1}}\,U_1$)

Alternative form of Eulers turbine equation:

Inlet Velocity triangle



Outlet Velocity triangle



$\frac{E}{\dot{m}} = \frac{(\overrightarrow{V_{u1}}\, U_1 - \overrightarrow{V_{u2}}\, U_2\,)}{g_c}$ --------------------- A

From Inlet Velocity triangle

$V_1^2 = V_{u1}^2 + V_{m1}^2;\quad V_{m1}^2 = V_1^2 - V_{u1}^2$-- 1

$V_{r1}^2 = (U_1 - V_{u1})^2 + V_{m1}^2;\quad V_{m1}^2 = V_{r1}^2 - \ (U_1 - V_{u1})^2;\ \ V_{m1}^2 = V_{r1}^2 - (U_1^2 + V_{u1}^2 - 2U_1V_{u1})$ eqn-2

Eqn 1 =Eqn2; $\quad V_1^2 - V_{u1}^2 = V_{r1}^2 - (U_1^2 + V_{u1}^2 - 2U_1V_{u1})\ ;\qquad V_1^2 = V_{r1}^2 - (U_1^2 - \ 2U_1V_{u1})$

$2U_1\overrightarrow{V_{u1}} = V_1^2 + U_1^2 - V_{r1}^2;\qquad U_1V_{u1} = \frac{V_1^2 + U_1^2 - V_{r1}^2}{2}$ -----------------3

Similarly from outlet velcocity triangle

$U_2\overrightarrow{V_{u2}} = \frac{V_2^2 + U_2^2 - V_{r2}^2}{2}$ ----------------4

Substituting 3 and 4 in equation A

$\frac{E}{\dot{m}} = \ \frac{V_1^2 + U_1^2 - V_{r1}^2}{2g_c} - \frac{V_2^2 + U_2^2 - V_{r2}^2}{2g_c}$

$$\frac{E}{\dot{m}} = \frac{V_1^2-V_2^2}{2g_c} + \frac{U_1^2-U_2^2}{2g_c} - \frac{V_{r1}^2-V_{r2}^2}{2g_c}$$

I[st] term is the change in KE of the fluid due to change in absolute velocity of the fluid

2[nd] term is the change in KE of the fluid due to change in tangential speed of the rotor

3[rd] term is the change in KE of the fluid due to change in relative velocity of the rotor

Hence

$$\frac{E}{\dot{m}} = \frac{\dot{m}}{g_c}\left(\overrightarrow{V_{u1}}\,U_1 - \overrightarrow{V_{u2}}\,U_2\right) = \frac{V_1^2-V_2^2}{2g_c} + \frac{U_1^2-U_2^2}{2g_c} - \frac{V_{r1}^2-V_{r2}^2}{2g_c} = C_p\,\Delta T_o$$

- **Tangential force = F_u = $\frac{\dot{m}}{g_c}\left(\overrightarrow{V_{u1}} - \overrightarrow{V_{u2}}\right)$ Newton**
- **Torque = T = $\frac{\dot{m}}{g_c}\left(\overrightarrow{V_{u1}}\,r_1 - \overrightarrow{V_{u2}}\,r_2\right)$ Newton meter**
- **Power $E = \frac{\dot{m}}{g_c}\left(\overrightarrow{V_{u1}}\,U_1 - \overrightarrow{V_{u2}}\,U_2\right)$ Watts**

  Power $E = \frac{\dot{m}}{g_c}\left(\overrightarrow{V_{u1}}\,U_1 + \overleftarrow{V_{u2}}\,U_2\right)$watts
- **Force along a axial/radial direction= mass (kg/s) x change in velocity along axial/radial direction (Axial/radial thrust)**

  F_a = $\frac{\dot{m}}{g_c}\left(V_{m1} - V_{m2}\right)$ Newton

Degree of Reaction: It is defined as the ratio of static enthalpy drop of the fluid to stagnation enthalpy drop of fluid when it passes through the rotor of the turbomachine

$$\text{Degree of Reaction}=\frac{\text{static enthalpy drop of the fluid}}{\text{stagnation enthalpy drop of fluid}}$$

$$R = \frac{h_1-h_2}{h_{01}-h_{02}} \quad \text{------A}$$

$$h_{01} - h_{02} = \frac{V_1^2-V_2^2}{2g_c} + \frac{U_1^2-U_2^2}{2g_c} - \frac{V_{r1}^2-V_{r2}^2}{2g_c}$$

$$\left(h_{01} - \frac{V_1^2}{2g_c}\right) - \left(h_{02} - \frac{V_2^2}{2g_c}\right) = \frac{U_1^2-U_2^2}{2g_c} - \frac{V_{r1}^2-V_{r2}^2}{2g_c}$$

But $h_1 = \left(h_{01} - \frac{V_1^2}{2}\right)$; $h_2 = \left(h_{02} - \frac{V_2^2}{2g_c}\right)$

Hence, $h_1 - h_2 = \dfrac{U_1^2 - U_2^2}{2g_c} - \dfrac{V_{r1}^2 - V_{r2}^2}{2g_c}$

Substituting, $h_1 - h_2$ and $h_{01} - h_{02}$

$$R = \frac{\dfrac{U_1^2 - U_2^2}{2g_c} - \dfrac{V_{r1}^2 - V_{r2}^2}{2g_c}}{\dfrac{V_1^2 - V_2^2}{2g_c} + \dfrac{U_1^2 - U_2^2}{2g_c} - \dfrac{V_{r1}^2 - V_{r2}^2}{2g_c}}$$

$$R = \frac{\left(\boldsymbol{U_1^2 - U_2^2}\right) + \left(\boldsymbol{V_{r1}^2 - V_{r2}^2}\right)}{\left(\boldsymbol{V_1^2 - V_2^2}\right) + \left(\boldsymbol{U_1^2 - U_2^2}\right) + \left(\boldsymbol{V_{r1}^2 - V_{r2}^2}\right)}$$

Another form of Degree of Reaction

$$R = \frac{\dfrac{V_1^2 - V_2^2}{2g_c} + \dfrac{U_1^2 - U_2^2}{2g_c} - \dfrac{V_{r1}^2 - V_{r2}^2}{2g_c} - \dfrac{V_1^2 - V_2^2}{2g_c}}{\dfrac{V_1^2 - V_2^2}{2g_c} + \dfrac{U_1^2 - U_2^2}{2g_c} - \dfrac{V_{r1}^2 - V_{r2}^2}{2g_c}}$$

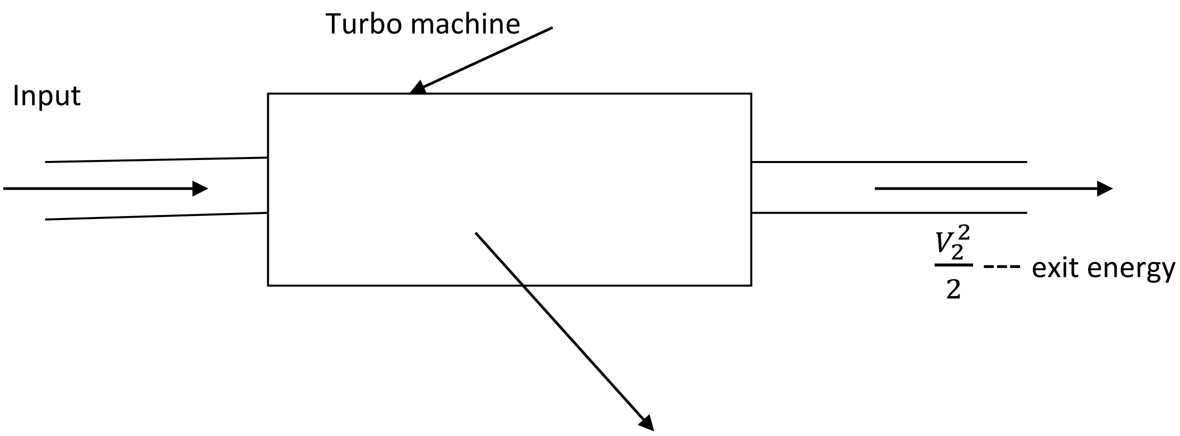
$$R = \frac{\frac{E}{\dot{m}} - \frac{V_1^2 - V_2^2}{2g_c}}{\frac{E}{\dot{m}}}; \qquad R = 1 - \frac{\boldsymbol{V_1^2 - V_2^2}}{\boldsymbol{2}g_c\frac{\boldsymbol{E}}{\boldsymbol{\dot{m}}}}$$

Hence Degree of Reaction can be written in two forms

1. $R = \dfrac{\left(\boldsymbol{U_1^2 - U_2^2}\right) + \left(\boldsymbol{V_{r1}^2 - V_{r2}^2}\right)}{\left(\boldsymbol{V_1^2 - V_2^2}\right) + \left(\boldsymbol{U_1^2 - U_2^2}\right) + \left(\boldsymbol{V_{r1}^2 - V_{r2}^2}\right)}$

2. $R = 1 - \dfrac{\boldsymbol{V_1^2 - V_2^2}}{\boldsymbol{2}g_c\frac{\boldsymbol{E}}{\boldsymbol{\dot{m}}}}$

Utilization factor: is defined as the ratio of ideal work done by the turbomachine to the energy supplied at the inlet of turbine



Output $= \frac{E}{\dot{m}} = \quad \frac{V_1^2 - V_2^2}{2g_c} + \frac{U_1^2 - U_2^2}{2g_c} - \frac{V_{r1}^2 - V_{r2}^2}{2g_c}$

Input = Output + exit fluid KE

Input $= \frac{E}{\dot{m}} + \frac{V_2^2}{2g_c}$

Hence utilization factor, $\epsilon = \frac{\frac{E}{\dot{m}}}{\frac{E}{\dot{m}} + \frac{V_2^2}{2g_c}}$

$$\epsilon = \frac{\frac{V_1^2 - V_2^2}{2g_c} + \frac{U_1^2 - U_2^2}{2g_c} - \frac{V_{r1}^2 - V_{r2}^2}{2g_c}}{\frac{V_1^2 - V_2^2}{2g_c} + \frac{U_1^2 - U_2^2}{2g_c} - \frac{V_{r1}^2 - V_{r2}^2}{2g_c} + \frac{V_2^2}{2g_c}}$$

Hence, $\epsilon = \frac{\frac{V_1^2 - V_2^2}{2g_c} + \frac{U_1^2 - U_2^2}{2g_c} - \frac{V_{r1}^2 - V_{r2}^2}{2g_c}}{\frac{V_1^2}{2g_c} + \frac{U_1^2 - U_2^2}{2g_c} - \frac{V_{r1}^2 - V_{r2}^2}{2g_c}}$

After simplifying ,

$$\epsilon = \frac{\left(\boldsymbol{V_1^2 - V_2^2}\right) + \left(\boldsymbol{U_1^2 - U_2^2}\right) + \left(\boldsymbol{V_{r1}^2 - V_{r2}^2}\right)}{\boldsymbol{V_1^2} + \left(\boldsymbol{U_1^2 - U_2^2}\right) + \left(\boldsymbol{V_{r1}^2 - V_{r2}^2}\right)}$$

Establish the relation between utilization factor and degree of reaction (or prove that

$\epsilon = \frac{\boldsymbol{V_1^2 - V_2^2}}{\boldsymbol{V_1^2 - RV_2^2}}$

$$R = \frac{\left(U_1^2 - U_2^2\right) + \left(V_{r1}^2 - V_{r2}^2\right)}{\left(V_1^2 - V_2^2\right) + \left(U_1^2 - U_2^2\right) + \left(V_{r1}^2 - V_{r2}^2\right)}$$

$$R\,[(V_1^2 - V_2^2) + (U_1^2 - U_2^2) + (V_{r1}^2 - V_{r2}^2)] = (U_1^2 - U_2^2) + (V_{r1}^2 - V_{r2}^2)$$

$$R\,(V_1^2 - V_2^2) + R\,[(U_1^2 - U_2^2) + (V_{r1}^2 - V_{r2}^2)] = (U_1^2 - U_2^2) + (V_{r1}^2 - V_{r2}^2)$$

$$R\,(V_1^2 - V_2^2) = [\,(U_1^2 - U_2^2) + (V_{r1}^2 - V_{r2}^2)]\,(1 - R)$$

$$(U_1^2 - U_2^2) + (V_{r1}^2 - V_{r2}^2) = \frac{R}{1-R}\,(V_1^2 - V_2^2) \text{ ------------------ } 1$$

$$\epsilon = \frac{\left(V_1^2 - V_2^2\right) + \left(U_1^2 - U_2^2\right) + \left(V_{r1}^2 - V_{r2}^2\right)}{V_1^2 + \left(U_1^2 - U_2^2\right) + \left(V_{r1}^2 - V_{r2}^2\right)}$$

substituting 1 in above equation

$$\epsilon = \frac{\left(V_1^2 - V_2^2\right) + \frac{R}{1-R}\left(V_1^2 - V_2^2\right)}{V_1^2 + \frac{R}{1-R}\left(V_1^2 - V_2^2\right)}$$

$$\epsilon = \frac{\frac{(1-R)\left(V_1^2 - V_2^2\right) + R\left(V_1^2 - V_2^2\right)}{(1-R)}}{\frac{(1-R)\left(V_1^2\right) + R\left(V_1^2 - V_2^2\right)}{(1-R)}}$$

after simplification

$\epsilon = \dfrac{V_1^2 - V_2^2}{V_1^2 - RV_2^2}$ Hence proved

For Maximum utilization,

From the expression $\epsilon = \dfrac{V_1^2 - V_2^2}{V_1^2 - RV_2^2}$ it can be understand V_2 is to be minimum

For V_2 to be minimum, V_2 to be in the axial / radial direction ie $\alpha_2 = 90^o$ and $V_{u2} = 0$

Numericals

1. Air enters in an axial flow turbine with a tangential component of the absolute velocity 600m/s in the direction of rotation. At the rotor exit, the tangential component of the absolute velocity is 100m/s in a direction opposite to that of rotational speed. The tangential blade speed is 250m/s. Evaluate (i) The change in total enthalpy of air between the inlet and outlet of the rotor (ii) The power in kW if the mass flow rate is 10kg/s (iii) The change in total temperature across the rotor.(4c, 8, June/July18)

- Axial flow turbine ie $U_1 = U_2 = U$
- Air enters in an axial flow turbine tangential component of the absolute velocity 600m/s in the direction of rotation

$\overrightarrow{V_{u1}} = 600$m/s in the direction to that of rotational speed

At the rotor exit, the tangential component of the absolute velocity is 100m/s in a direction opposite to that of rotational speed.

$\overleftarrow{V_{u2}}$=100m/s opposite to the direction to that of rotational speed ie. Direction of u and V_{u2} are opposite to each other ie V_{u2} direction is negative V_{u2} $\longleftarrow$

- Tangential blade speed $U = 250$m/s

To determine

i)Δh_o=? ii)P=? if m=10kg/s iii) ΔT_o=?

$\frac{E}{\dot{m}} = \frac{1}{g_c}(\overrightarrow{V_{u1}}\, U_1 - \overrightarrow{V_{u2}}\, U_2)$; $\frac{E}{\dot{m}} = \frac{1}{g_c}(\overrightarrow{V_{u1}} + \overleftarrow{V_{u2}})U$ as $U_1 = U_2 = U$

$\frac{E}{\dot{m}} = (600 + 100)250$ = 175000J/kg (175kJ/kg)

i) $\Delta h_0 = \frac{E}{\dot{m}} = 175kJ/kg$

ii) $E = \dot{m}\frac{E}{\dot{m}} = 10 x 175 = 1750kW$ (kW since $\frac{E}{\dot{m}}$ is in kJ/kg)

iii) $\Delta h_0 = C_p(\Delta T_0)$

$175 = 1.005\ \Delta T_0$ as C_p for air is $1.005kJ/kg$

$\Delta T_0 = 174.13$°C

2. Air enters in an axial flow turbine with a tangential component of the absolute velocity 600m/s in the direction of rotation. At the rotor exit, the tangential component of the absolute velocity is 100m/s in a direction same to that of rotational speed. The tangential blade speed is 250m/s. Evaluate (i) The change in total enthalpy of air between the inlet and outlet of the rotor (ii) The power in kW if the mass flow rate is 10kg/s (iii) The change in total temperature across the rotor

3. In a certain turbo machine the fluid enters the rotor with the absolute velocity having an axial component of 10m/s and a tangential component, in the direction of

the rotors motion is 16m/s. The tangential speed of the rotor at inlet is 33m/s. At the outlet of the rotor, the tangential speed of the rotor is 8m/s and absolute velocity of the fluid is 16m/s in axial direction. Evaluate the energy transfer between the fluid and rotor. Is this turbo machine power absorbing and power generating? What is the change in total pressure if the process is loss free and fluid is water Also calculate the blade angles

Given Data

Axial component of absolute of velocity at inlet V_{m1}=10m/s,

Tangential component at inlet, $\overrightarrow{V_{u1}} = 16m/s$,

Tangential speed of the rotor at inlet,$U_1 = 33m/s$

Tangential speed of the rotor at outlet=$U_1 = 8m/s$,

Absolute velocity of the fluid is 16m/s in axial direction ie $V_2 = 16m/s$ and $\alpha_2 = 90^0$

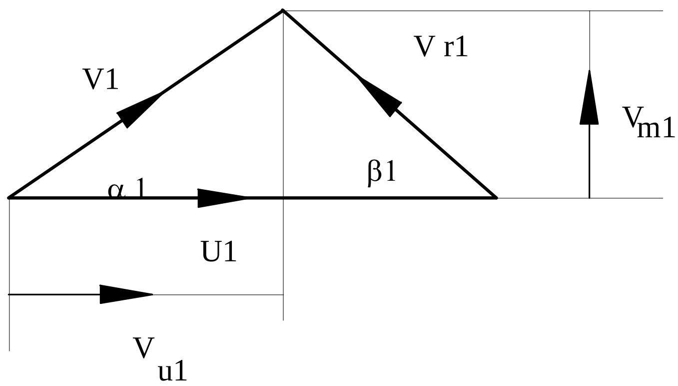
To determine

$\frac{E}{\dot{m}}$ =?, Is the turbomachine is power absorbing or power generating? Δp=? if fluid is water

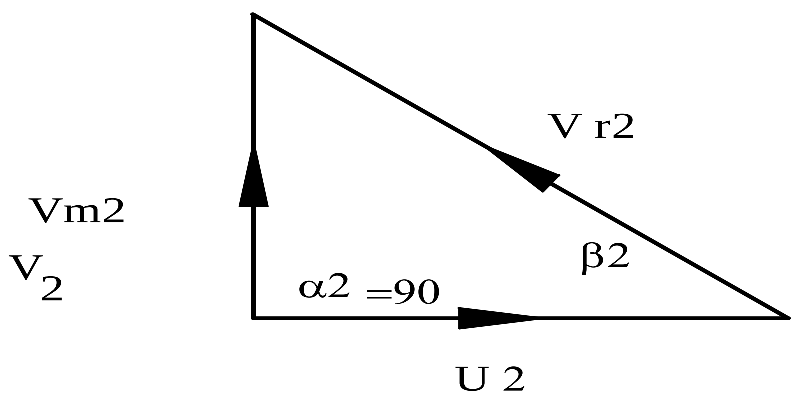
Solution:

Note that $U_1 = 33m/s > \overrightarrow{V_{u1}} = 16m/s$

Hence



α_2=90°



$\overrightarrow{V_{u2}} = 0$

$\frac{E}{\dot{m}} = \frac{1}{g_c}(\overrightarrow{V_{u1}}\, U_1\ - \overrightarrow{V_{u2}}\, U_2)$; $\frac{E}{\dot{m}} = (16 * 33 -\ 0)$; $\frac{E}{\dot{m}} = 528\ \ J/kg$

Since $\frac{E}{\dot{m}}$ = is + ve This machine is power developing machine

ii) For compressible fluid , $\Delta h_0 = \frac{\Delta P_0}{\rho}$

$528\ = \frac{\Delta P_0}{1000}$; $\Delta P_0 = 528000$ N/m² ($\Delta P_0\ is$ N/m² since Δh_0 is in J/kg);

$\Delta P_0 = 528\ kPa$

iii) **Blade angles**

Inlet blade angle

From Inlet velocity triangle, $tan\beta_1 = \frac{V_{m1}}{U_1 - \overrightarrow{V_{u1}}}$; $tan\beta_1 = \frac{10}{33-16}$; $\beta_1 = 30.465^{o}$

Outlet blade angle

From outlet velocity triangle, $tan\beta_2 = \frac{V_{m2}}{U_2}$; $tan\beta_2 = \frac{16}{8}$; $\beta_2 = 63.43^{o}$

Inlet velocity of the fluid

$V_1 = \sqrt{V_{u1}^2 + V_{m1}^2}$; $V_1 = \sqrt{16^2 + 10^2}$; $V_1 = 18.867 m/s$

Outlet guide blade angle α_1

$tan\alpha_1 = \frac{V_{m1}}{V_{u1}}$; $tan\alpha_1 = \frac{10}{16}$; $\alpha_1 = 32^0$

Axial thrust

$F_a = \frac{\dot{m}}{g_c}\,(V_{m1} - V_{m2})$ Newton; $F_a = 1\,(10 - 16)$; $F_a = -6\,N$

Tangential Thrust

$F_u = \frac{\dot{m}}{g_c}\,(\overrightarrow{V_{u1}} - \overrightarrow{V_{u2}})$ Newton ; $F_u = 1\,(16 - 0)\ N$; $F_u = 16\,N$

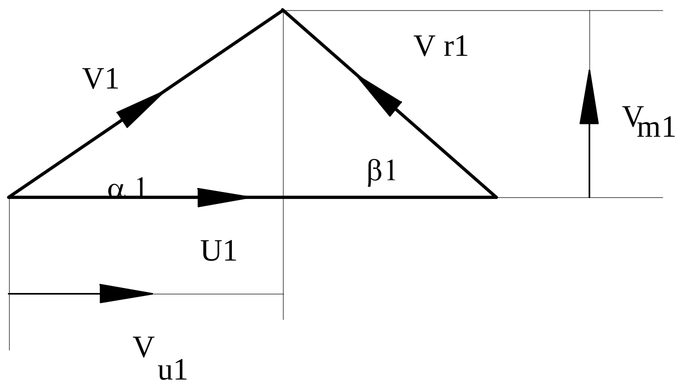
4. The following data refers to a turbo-machine. Inlet velocity of whirl =16m/s, velocity of flow =10m/s, blade speed =33m/s, outlet blade speed =8m/sDischarge is radial with an absolute velocity of 16m/s. If water is the working fluid flowing at the rate of 1 m³/s. Calculate the following i) Power in kW ii) Change in total pressure in kN/m² iii) Degree of reaction iv) Utilization factor (3b, 08, June/July18 15 scheme)

Inlet velocity of whirl =16m/s, ie $\overrightarrow{V_{u1}} = 16 m/s$,

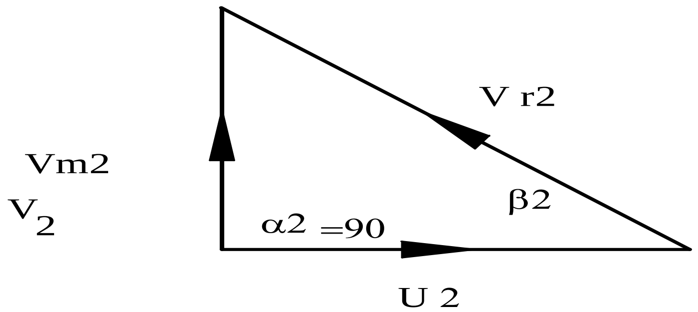
velocity of flow =10m/s, $V_{m1} = 10m/s$,
blade speed =33m/s at inlet, $U_1 = 33m/s$
outlet blade speed =8m/s =$U_2 = 8m/s$,
Discharge is radial with an absolute velocity of 16m/s ie $V_2 = \frac{16m}{s}$ and $\alpha_2 = 90^0$
If water is the working fluid flowing at the rate of 1 m^3/s. $Q = 1m^3/s$
To determine

i)Power in kW ie E=? Ii)Change in total pressure in kN/m^2 ΔP_0 =? iii) R=? Iv) ε =?

Solution:
Note that $U_1 = 33m/s > \overrightarrow{V_{u1}}$=16m/s
Hence



α$_2$=90^o



V_{u2} =0

i) <u>Power in kW</u>

$\frac{E}{\dot{m}} = \frac{1}{g_c}\left(\overrightarrow{V_{u1}}\,U_1 - \overrightarrow{V_{u2}}\,U_2\right)$;
$\frac{E}{\dot{m}} = (16 * 33 - 0)$; $\frac{E}{\dot{m}} = 528\ J/kg$

Since $\frac{E}{\dot{m}}$ = is + ve This machine is power developing machine

$\dot{m} = \rho Q$; $\dot{m} = 1000 * 1$; $\dot{m} = 1000kg/s$

$E = \dot{m}\frac{E}{\dot{m}}$; $E = 1000 * 528\, J/s$; $E = \frac{528000J}{s} = 528kW$

ii) **<u>Change in total pressure in kN/m²</u>**

For compressible fluid , $\Delta h_0 = \frac{\Delta P_0}{\rho}$; $528 = \frac{\Delta P_0}{1000}$; ΔP_0=528000N/m² ;

(ΔP_0 *is* N/m² since Δh_0 is in J/kg); ΔP_0 =528 kPa

iii) <u>Degree of reaction</u>

$$R = \frac{\frac{E}{\dot{m}} - \left(\frac{V_1^2 - V_2^2}{2g_c}\right)}{\frac{E}{\dot{m}}}; \quad R = 1 - \frac{(V_1^2 - V_2^2)}{2g_c\frac{E}{\dot{m}}};$$

$$V_1^2 = V_{u1}^2 + V_{m1}^2; \qquad V_1^2 = 16^2 + 10^2; \qquad V_1^2 = 356; \;\; V_2^2 = 16^2; \;\; V_2^2 = 256$$

$$R = 1 - \frac{(356 - 256)}{2 * 528}; \;\; R = 0.91$$

iV) **<u>Utilization factor</u>**

$$\varepsilon = \frac{\frac{E}{\dot{m}}}{\frac{E}{\dot{m}} + \left(\frac{V_2^2}{2g_c}\right)}; \qquad \varepsilon = \frac{528}{528 + \left(\frac{256}{2}\right)}; \qquad \varepsilon = 0.804$$

Determine blade angles, Axial and tangential thrust in the above problem

v) **<u>Blade angles</u>**

Inlet blade angle

From Inlet velocity triangle, $tan\beta_1 = \frac{V_{m1}}{U_1 - \overrightarrow{V_{u1}}}$; $tan\beta_1 = \frac{10}{33-16}$; $\beta_1 = 30.465°$

Outlet blade angle

From outlet velocity triangle, $tan\beta_2 = \frac{V_{m2}}{U_2}$; $tan\beta_2 = \frac{16}{8}$; $\beta_2 = 63.43°$

Outlet guide blade angle α_1

$tan\alpha_1 = \frac{V_{m1}}{\overrightarrow{V_{u1}}}$; $tan\alpha_1 = \frac{10}{16}$; $\alpha_1 = 32^0$

<u>vi)Axial thrust</u>

$F_a = \frac{\dot{m}}{g_c}(V_{m1} - V_{m2})$ Newton

$F_a = 1\,(10 - 16)$; $F_a = -6\,N$

Tangential Thrust

$F_U = \frac{\dot{m}}{g_c}\left(\overrightarrow{V_{u1}} - \overrightarrow{V_{u2}}\right)$ Newton ; F_U = 1 (16 – 0) N ; F_U =16 N

5. Water approaches the impeller of a mixed flow pump with an absolute velocity having tangential and axial components each of 17m/s. At the rotor exit the radial and tangential components of the absolute velocity are 13m/s and 25m/s respectively. The tangential blade speed at inlet and exit are 12m/s and 47m/s Find

   i) Change in enthalpy across the rotor
   ii) Total change in pressure across the rotor
   iii) Change in static pressure
   iv) Degree of reaction (2b. 10 Dec/Jan 17)*

   Water approaches the impeller of a mixed flow pump absolute velocity having tangential and axial components each of 17m/s. ie $\overrightarrow{V_{u1}} = 17m/s; V_{m1} = 17m/s;$

   At the rotor exit the radial and tangential components of the absolute velocity are 13m/s and 25m/s$\overrightarrow{V_{u2}} = 25m/s; V_{m2} = 13m/s$

   The tangential blade speed at inlet and exit are 12m/s and 47m/s

   $U_1 = 12m/s; U_2 = 47m/s$

   i) Change in enthalpy across the rotor

   $\frac{E}{\dot{m}} = \frac{1}{g_c}\left(\overrightarrow{V_{u1}}\,U_1 - \overrightarrow{V_{u2}}\,U_2\right); \frac{E}{\dot{m}} = (17*12) - (25*47); \frac{E}{\dot{m}} = -971J/kg$

   ii) Total change in pressure across the rotor

   , $\Delta h_0 = \frac{\Delta P_0}{\rho}; 971 = \frac{\Delta P_0}{1000};$ $\quad \Delta P_0 = 971000N/m^2$

   iii) <u>Change in static enthalpy</u>

   $\Delta h_0 = h_{02} - h_{01}; \Delta h_0 = \left(h_2 + \frac{V_2^2}{2g_c}\right) - \left(h_1 + \frac{V_1^2}{2g_c}\right); \Delta h_0 = (h_2 - h_1) + \left(\frac{V_2^2 - V_1^2}{2g_c}\right)$

   $V_1^2 = V_{u1}^2 + V_{m1}^2;\ V_1^2 = 17^2 + 17^2; V_1^2 = 578;\ \ V_2^2 = V_{u2}^2 + V_{m2}^2;$

   $V_2^2 = 25^2 + 13^2;$ $\qquad V_2^2 = 794$

   $971 = (h_2 - h_1) + \left(\frac{794-578}{2}\right);$ $\qquad (h_2 - h_1) = 863J/kg$

   iv) <u>Change in static pressure</u>

   $p_o = p + \frac{\rho V^2}{2g_c};\ \Delta p_o = p_{o2} - p_{o1}; \Delta p_o = \left(p_2 + \frac{\rho V_2^2}{2g_c}\right) - \left(p_1 + \frac{\rho V_1^2}{2g_c}\right)$

$$\Delta p_o = (p_2 - p_1) + \frac{\rho(V_2^2 - V_1^2)}{2g_c}; \; 971000 = (p_2 - p_1) + \frac{1000(794-578)}{2};$$

Change in static pressure $(p_2 - p_1) = 863000 N/m^2$

<u>Degree of reaction</u>

$$R = \frac{\frac{E}{m} - \left(\frac{V_1^2 - V_2^2}{2g_c}\right)}{\frac{E}{m}}; \quad R = 1 - \frac{(V_1^2 - V_2^2)}{2g_c \frac{E}{m}}; \; R = 1 - \frac{(578-794)}{2*(-971)}; \; R = 0.796$$

6. In an inward flow radial hydraulic turbine for maximum utilisation factor show that , $\alpha_1 = cot^{-1}\sqrt{\frac{1-R}{1-\varepsilon}\varepsilon}$ where α_1= nozzle angle, R=Degree of reaction, ε is the utilization factor Assuming the radial velocity component is constant through out and there is no tangential component absolute velocity component at outlet (3a,10 , Dec12) (4b, 8, June/July18)

Given Data :

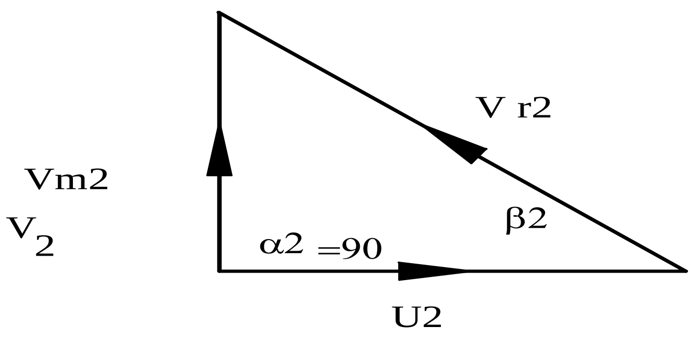
- Radial turbine ie $U_1 \neq U_1$
- Assuming the radial velocity component is constant through out

$V_{m1} = V_{m2} = V_m$

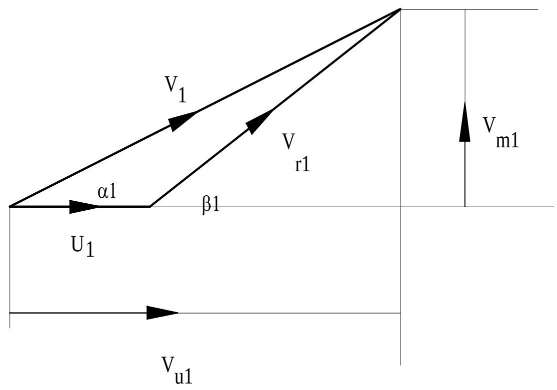
there is no tangential component absolute velocity component at outlet

$\overrightarrow{V_{u2}} = 0$

<u>outlet velocity triangle</u>



<u>Inlet velocity triangle</u>



$$\epsilon = \frac{V_1^2 - V_2^2}{V_1^2 - RV_2^2}; \qquad \epsilon = \frac{V_1^2 - V_m^2}{V_1^2 - RV_m^2}$$

$$sin\alpha_1 = \frac{V_m}{V_1}; \qquad V_1 = V_m cosec\alpha_1$$

$$\epsilon = \frac{V_m^2 cosec^2\alpha_1 - V_m^2}{V_m^2 cosec^2\alpha_1 - RV_m^2} \quad ; \quad \epsilon = \frac{cosec^2\alpha_1 - 1}{cosec^2\alpha_1 - R}$$

$$\mathrm{cosec}^2\alpha_1 = 1 + \cot^2\alpha_1$$

$$\epsilon = \frac{1 + \cot^2\alpha_1 - \mathbf{1}}{1 + \cot^2\alpha_1 - \boldsymbol{R}}; \quad \epsilon = \frac{\cot^2\alpha_1}{1 - \boldsymbol{R} + \cot^2\alpha_1}$$

$$\epsilon(1 - R) + \epsilon\cot^2\alpha_1 = \cot^2\alpha_1; \quad \epsilon(1 - R) = \cot^2\alpha_1(1 - \epsilon)$$

$$\frac{\epsilon(1 - R)}{(1 - \epsilon)} = \cot^2\alpha_1; \qquad \alpha_1 = cot^{-1}\sqrt{\frac{(1 - R)\epsilon}{(1 - \epsilon)}}$$

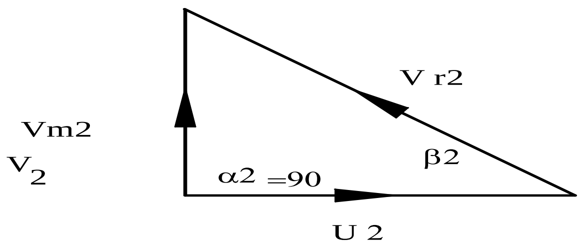
7. In an slow speed inward flow radial hydraulic turbine, degree of reaction is R and utlilization factor is ε. Assuming the radial velocity component is constant through out and there is no tangential component absolute velocity component at outlet, show that the inlet nozzle angle is given by $\alpha_1 = cot^{-1}\sqrt{\frac{(1 - R)\epsilon}{(1 - \epsilon)}}$

8. Show that for an axial flow turbine under maximum utilization factor condition , the speed ratio is Ø is given by $\frac{U}{V_1} = \frac{2}{3}\ cos\alpha_1$ where U is the tangential speed of the rotor and V_1 is the tangential jet velocity of the fluid . Assume flow velocity is to remain constant and $\alpha_1$is the Take degree of reaction =1/4, (3b. 10 Dec/Jan 2016)*

Axial flow turbine ---- $U_1 = U_2 = U$

Utillization factor is maximum ie α_2=90°

Outlet velocity triangle

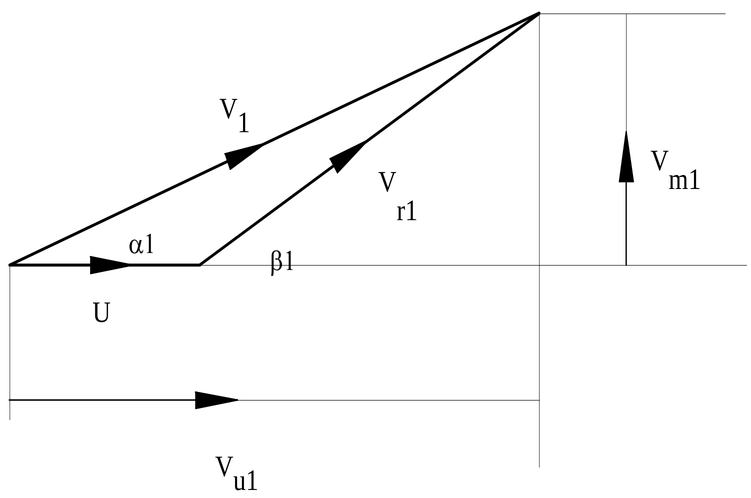


Degree of reaction =1/4

Assume flow velocity is constant from inlet to outlet ie $V_{m1} = V_{m2}$

Prove $\frac{U}{V_1} = \frac{2}{3}\cos\alpha_1$

Inlet Velocity triangle



$$R = 1 - \frac{V_1^2 - V_2^2}{2g_c\frac{E}{\dot{m}}} - - -1$$

$\frac{E}{\dot{m}} = \frac{1}{g_c}\left(\overrightarrow{V_{u1}} - \overrightarrow{V_{u2}}\right)U$; $\frac{E}{\dot{m}} = \frac{1}{g_c}\overrightarrow{V_{u1}}U$ ---------2 as $\overrightarrow{V_{u2}} = 0$

$V_2 = V_{m2}$ (From outlet velocity triangle as $\alpha_2 = 90°$)

$V_2 = V_{m1} \quad - - - eqn\ 3$ (as $V_{m1} = V_{m2}$)

Substituting 2 and 3 in 1

$$R = 1 - \frac{V_1^2 - V_{m1}^2}{2g_c \frac{\overrightarrow{V_{u1}}U}{g_c}}$$

From inlet velocity triangle $\quad V_{u1}^2 = V_1^2 - V_{m1}^2$

Hence, $R = 1 - \frac{V_{u1}^2}{2V_{u1}U}$; $R = 1 - \frac{\overrightarrow{V_{u1}}}{2U}$; $R = 1 - \frac{V_1 cos\alpha_1}{2U}$ as $\overrightarrow{V_{u1}} = V_1 cos\alpha_1$

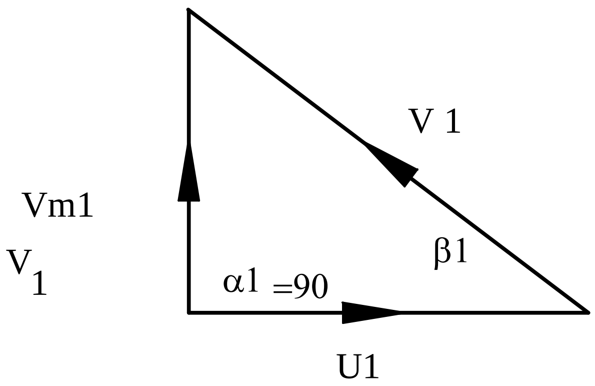
$\frac{1}{4} = 1 - \frac{V_1 cos\alpha_1}{2U}$; $\frac{3}{4} = \frac{V_1 cos\alpha_1}{2U}$; $\frac{U}{V_1} = \frac{2}{3}\cos\alpha_1$

9. A radial outward flow turbomachine has no inlet whirl. The blade speed at the exit is twice at inlet. Radial velocity is constant throughout. Taking the inlet blade angle as 45°, show that the degree of reaction,
$R = \frac{2+cot\beta_2}{4}$ Where β_2 is the blade speed at exit wrt tangential direction
(3a,10June/July 16,) (4b,10June/July 17) (4b,10June/July 13) (4a. 10, Dec12)

Given Data:

A radial outward flow turbomachine has no inlet whirl ie $\overrightarrow{V_{u1}} = 0$

Hence Inlet velocity triangle



The blade speed at the exit is twice at inlet.

$U_2 = 2U_1$;

the inlet blade angle as 45°, ie $\beta_1 = 45°$

The radial component of absolute velocity remains constant throughout ie $V_{m1} = V_{m2} = V_m$

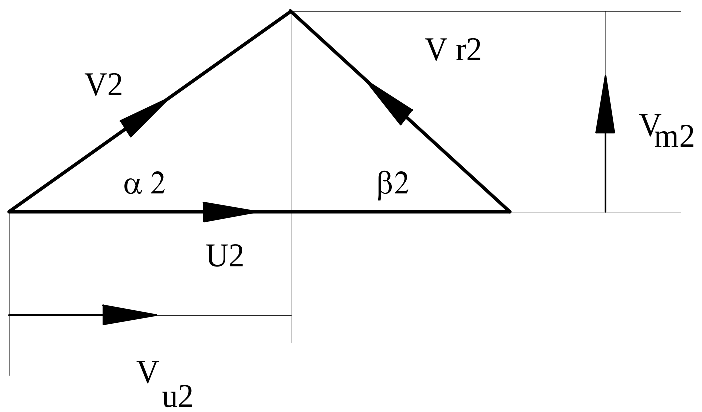
$R = 1 - \frac{V_1^2 - V_2^2}{2g_c \frac{E}{m}}$ ---------A

$V_1 = V_{m1} = V_m$ ----------1 f

rom Inlet velocity triangle asType equation here. α_1=90^0)

Substituting 1,in A $R = 1 - \frac{V_1^2 - V_2^2}{2g_c\frac{E}{\dot{m}}}$; $R = 1 - \frac{V_m^2 - V_2^2}{2g_c\frac{E}{\dot{m}}}$

Outlet velocity triangle



From outlet velocity triangle $V_2^2 = V_{u2}^2 + V_m^2$; $V_m^2 - V_2^2 = V_{u2}^2$

And $\frac{E}{\dot{m}} = \frac{1}{g_c}\left(\overrightarrow{V_{u1}} - \overrightarrow{V_{u2}}\right)U$; $\frac{E}{\dot{m}} = -\frac{1}{g_c}\overrightarrow{V_{u2}}U_2$ as as $\overrightarrow{V_{u1}} = 0$

Hence, $R = 1 - \frac{-V_{u2}^2}{2\left(-U_2\overrightarrow{V_{u2}}\right)}$; $R = 1 + \frac{-\overrightarrow{V_{u2}}}{2(u_2)}$ -----------------------2

From Inlet velocity triangle

$tan\beta_1 = \frac{V_{m1}}{U_1}$; $tan45 = \frac{V_{m1}}{U_1}$; $1 = \frac{V_m}{U_1}$; $U_1 = V_m$

$U_2 = 2U_1$; $U_2 = 2V_m$ -----------3

From outlet velocity triangle

$tan\beta_2 = \frac{V_{m2}}{U_2 - \overrightarrow{V_{u2}}}$; $U_2 - \overrightarrow{V_{u2}} = V_{m2}\,Cot\,\beta_2$;

$\overrightarrow{V_{u2}} = U_2 - V_{m2}\,Cot\,\beta_2$; $\overrightarrow{V_{u2}} = 2V_m - V_m\,Cot\,\beta_2$--------------------4

Substituting 3 and 4 in 2

$R = 1 + \frac{-(2V_m - V_m\,Cot\,\beta_2)}{2(2V_m)}$; $R = 1 + \frac{-(2 - Cot\,\beta_2)}{2(2)}$

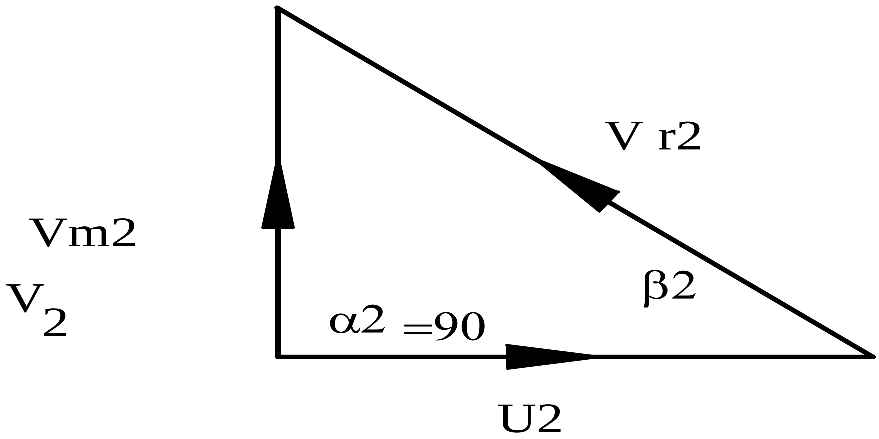
$R = 1 + \frac{-2 + Cot\,\beta_2}{4}$; $R = \frac{4 - 2 + Cot\,\beta_2}{4}$ $R = \frac{2 + cot\beta_2}{4}$

10. An Inward radial flow reaction turbine has radial discharge at outlet The outer blade angle is 45°. The radial component of absolute velocity remains constant. Assuming the the tangential speed of the rotor at inlet to be twice the tangential speed of rotor at exit., determine the energy transfer per unit flow depending on mass and degree of reaction . Assume $V_m = \sqrt{2g_c}$ If the values of degree of reaction respectively are 0 and 1 , what are the corresponding values of energy transfer per unit mass of the fluid (4b,10 Dec15/Jan16)

Data Given

Radial flow turbine $U_1 \neq U_2$, radial discharge at outlet ie $\alpha_2 = 90°$

Outlet velocity Triangle



outlet blade angle of 45°.ie β_2=45°

The radial component of absolute velocity remains constant throughout

$ie\ V_{m1} = V_{m2} = V_m$

$V_{m1} = V_{m2} = V_m = \sqrt{2g_c};$

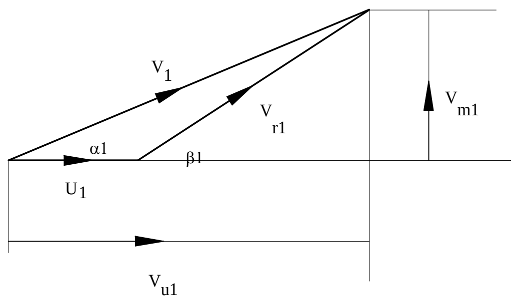
The blade speed at inlet is twice that at outlet U_1=2U_2

To Express $\frac{E}{m} = f(\alpha 1)$ R= $f(\alpha 1)$

α_1=? When R=0, $\frac{E}{m}$ =? $for\ that$ α_1

α_1=? When R=1, $\frac{E}{m}$ =? $for\ that$ α_1

Inlet Velocity Triangle



$\frac{E}{\dot{m}} = \frac{\overrightarrow{V_{u1}} U_1}{g_c}$ as $\overrightarrow{V_{u2}}$ =0 ------------------------------A

Note that here g_c is included in above equation since $V_{m1} = V_{m2} = V_m = \sqrt{2g_c}$

From Outlet velocity triangle

$tan\ \beta_2 = \frac{V_{m2}}{U_2}$; $tan45 = \frac{V_{m2}}{U_2}$; $1 = \frac{V_m}{U_2}$; $U_2 = V_m$

$U_1 = 2U_2$; U_1 =$2V_m ---- eqn1$

$tan\alpha_1 = \frac{V_m}{\overrightarrow{V_{u1}}}$; $\overrightarrow{V_{u1}} = V_m$ cotα_1 – ======= eqn 2

Substituting 1 and 2 in A

$\frac{E}{\dot{m}} = \frac{V_m\ cot\alpha_1\ \ (2V_m)}{g_c}$; $\frac{E}{\dot{m}} = \frac{2V_m^2\ cot\alpha_1}{g_c}$; $\frac{E}{\dot{m}} = \frac{2*\ 2g_c\ cot\alpha_1}{g_c}$;

$\boldsymbol{\frac{E}{\dot{m}}} = \mathbf{4\ cot\alpha_1}$

$R = 1 - \frac{V_1^2 - V_2^2}{2g_c\frac{E}{\dot{m}}}$ ----------------B

From outlet velocity triangle, V_2=V_{m2} =V_{m1}

$R = 1 - \frac{V_1^2 - V_{m1}^2}{2g_c\frac{E}{\dot{m}}}$; $R = 1 - \frac{V_{u1}^2}{2g_c\frac{E}{\dot{m}}}$; $R = 1 - \frac{V_{u1}^2}{2U_1V_{u1}}$; $R = 1 - \frac{V_{u1}}{2\ U_1}$;

$R = 1 - \frac{V_m\ \mathrm{cot}\alpha1}{2\ V_m}$; $R = 1 - \frac{cot\alpha_1}{4}$; $R = \boldsymbol{\frac{4 - cot\alpha_1}{4}}$

At what value of α_1, will the degree of reaction be zero

$0 = \frac{4 - cot\alpha_1}{4}$; $cot\alpha_1$ =4; α_1=

the corresponding values of energy transfer per unit mass

$\frac{\boldsymbol{E}}{\dot{\boldsymbol{m}}} = \mathbf{4\ cot\alpha_1}$; $\frac{E}{\dot{m}} = 4 * 4$; $\frac{E}{\dot{m}} = 16J/kg$

At what value of α_1, will the degree of reaction be 1

$1 = \frac{4 - cot\alpha_1}{4}$; $cot\alpha_1 = 0$;

$tan\alpha_1 = \frac{1}{0}$ $tan\alpha_1 = \infty$; $\alpha_1 = 90^o$

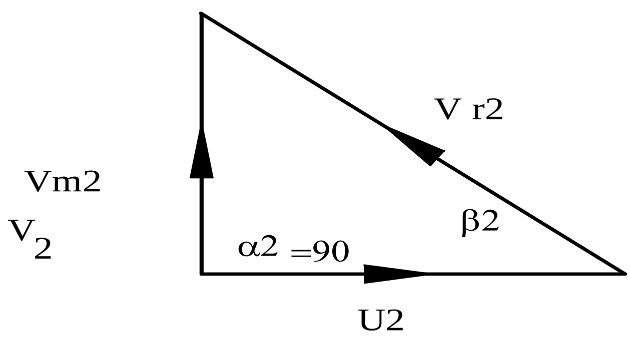
the corresponding values of energy transfer per unit mass

$\frac{E}{\dot{m}} = 4\ cot\alpha_1$; $\frac{E}{\dot{m}} = 4 * 0$; $\frac{E}{\dot{m}} = 0\ J/kg$

11. An Inward radial flow reaction turbine has radial discharge at outlet with outlet blade angle of 45°. The radial component of absolute velocity remains constant throughout and equal to $\sqrt{2gH}$ where g is the acceleration due to gravity and H is the constant head. The blade speed at inlet is twice that at outlet. Express the energy transfer per unit mass and the degree of reaction in terms of α_1, where α_1 is the direction of the absolute velocity at inlet with respect to the blade velocity at inlet. At what value α_1 will be the degree of reaction zero and unity? What are the corresponding values of energy transfer per unit mass

Radial flow turbine $U_1 \neq U_2$, radial discharge at outlet ie α_2=90°

Outlet velocityy triangle



outlet blade angle of 45°.ie $\beta_2 = 45°$

The radial component of absolute velocity remains constant throughout and equal to $\sqrt{2gH}$ ie $V_{m1} = V_{m2} = V_m = \sqrt{2gH}$

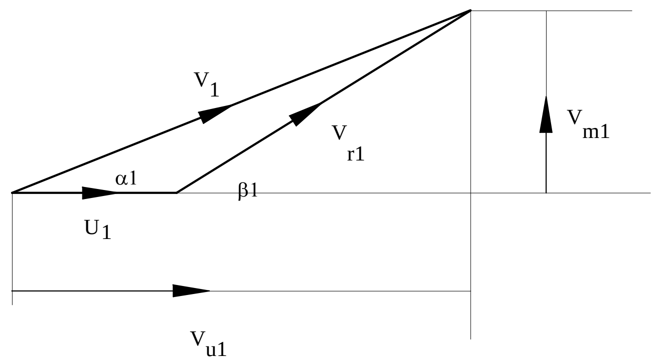
The blade speed at inlet is twice that at outlet $U_1=2U_2$

To Express $\frac{E}{m} = f(\alpha_1)$ R= $f(\alpha 1)$

α_1 =? When $R = 0$, $\frac{E}{m}$ =? $for\ that\ \alpha_1$

α_1 =? When $R = 1$, $\frac{E}{m}$ =? $for\ that\ \alpha_1$

Inlet Velocity Triangle



$\frac{E}{\dot{m}} = \frac{1}{g_c}\overrightarrow{V_{u1}}U_1$ as as $\overrightarrow{V_{u2}}$ =0

From Outlet velocity triangle

$tan\beta_2 \ = \frac{V_{m2}}{U_2}$; $tan45 = \frac{V_{m2}}{U_2}$; $1 = \frac{V_m}{U_2}$; $U_2 = V_m$

$U_1 = 2U_2$; $U_1 = 2V_m$ -------------1

$tan\alpha_1$ = $\frac{V_m}{\overrightarrow{V_{u1}}}$ ie $\overrightarrow{V_{u1}}$ =$V_m\ cot\alpha_1$ -----------------2

Substituting 1 and 2 in A

$\frac{E}{\dot{m}} = \frac{1}{g_c}V_m cot\alpha_1 * V_m$; $\frac{E}{\dot{m}}$ =$\frac{1}{g_c}V_m^2\ cot\alpha_1$; $\frac{E}{\dot{m}} = \frac{1}{g_c}2gH\ cot\alpha_1$;

$\frac{E}{\dot{m}} = \frac{1}{g_c}2gH\ cot\alpha_1$

R =1− $\frac{V_1^2 - V_2^2}{2g_c\frac{E}{\dot{m}}}$ ----------------B

From outlet velocity triangle, V_2=V_{m2} ; V_2=V_{m1} as V_{m2}=V_{m1}

$R \ = 1 - \ \frac{V_1^2 - V_{m1}^2}{2g_c\frac{E}{\dot{m}}}$; $R = 1 - \ \frac{V_{u1}^2}{2g_c\frac{E}{\dot{m}}}$; $R = 1 - \ \frac{V_{u1}^2}{2g_c\frac{1}{g_c}\overrightarrow{V_{u1}}U_1}$; $R = 1 - \ \frac{\overrightarrow{V_{u1}}}{2\,U_1}$

$R = 1 - \frac{V_m cot\alpha_1}{2\,V_m}$; $\quad R = 1 - \frac{cot\alpha_1}{4}$; $\quad \boldsymbol{R} = \frac{\boldsymbol{4 - cot\alpha_1}}{\boldsymbol{4}}$

$\boldsymbol{\alpha_1}$ =? When $R = 0$; $\quad \frac{E}{m}$ =? $for\ that$ α_1

$0 = \frac{4 - cot\alpha_1}{4}$; $\quad cot\alpha_1 = 4$; $\quad \alpha_1 =$

the corresponding values of energy transfer per unit mass

$\frac{E}{\dot{m}} = \frac{1}{g_c} 2gH\ cot\alpha_1$; $\quad \frac{E}{\dot{m}} = \frac{1}{1} 2gH * 4$; $\quad \frac{E}{\dot{m}} = 8gH$

At what value of α_1, will the degree of reaction be 1

$1 = \frac{4 - cot\alpha_1}{4}$; $\quad cot\alpha_1 = 0$;

$tan\alpha_1 = \frac{1}{0}$ $\quad tan\alpha_1 = \infty$; $\quad \alpha_1 = 90^o$

the corresponding values of energy transfer per unit mass

$\frac{E}{\dot{m}} = \frac{1}{g_c} 2gH\ cot\alpha_1$; $\quad \frac{E}{\dot{m}} = 4gH * 0$; $\quad \frac{E}{\dot{m}} = 0\,J/kg$

12. An inward flow radial turbine has nozzle angle α and rotor blades are radial entry. The radial velocity is constant and there is no whirl velocity at discharge. Show that the utilization factor is equal to $\varepsilon = \dfrac{2\cos^2\alpha_1}{1+\cos^2\alpha_1}$
13. An inward flow radial turbine has nozzle angle α and rotor blades are radial entry. The radial velocity is constant and there is no whirl velocity at discharge. Show that the utilization factor is equal to $\varepsilon = \dfrac{2\cos^2\alpha_1}{1+\cos^2\alpha_1}$
14. In an axial flow turbine, for maximum utilization factor, prove that speed ratio is given by $\emptyset = \dfrac{\cos\alpha_1}{2(1-R)}$
15. The velocity of steam in a Delaval turbine is 1200m/s. The nozzle angle being 22°. and rotor blades are equiangular. Assuming the relative velocity of fluid at inlet and exit to be equal and the tangential speed is 400m/s. Determine (i) the blade angles at inlet and exit (ii) the tangential force on the blade ring and (iii) power developed in kW , if mass flow rate is 1kg/s ,iv) . the utilization factor (3a. 10 Dec/Jan 2016)* Assume $V_{r1} = V_{r2}$(3b. 10 Dec17/Jan 2018)

    Delaval turbine is Impulse turbine

    Ie $R = 0$ **and** $U_1 = U_2 = U$

    - Velocity of steam from nozzle= $V_1 = 1200m/s$, nozzle angle , $\alpha_1 = 22^o$
    - the rotor blades are equiangular ie $\beta_1 = \beta_2$

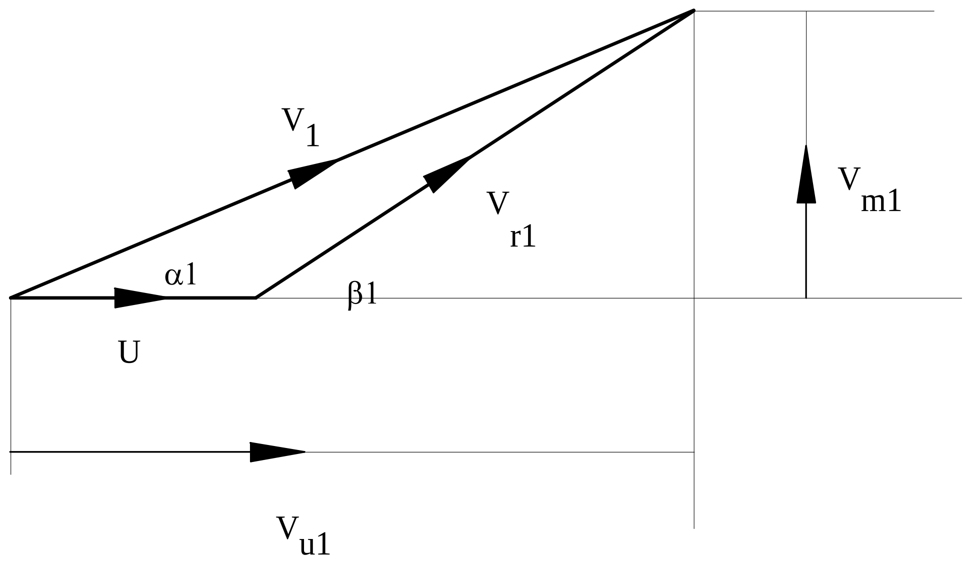
- Tangential speed $U = 400m/s$,
- V_{r1} equals to V_{r2} ie $V_{r1} = V_{r2}$

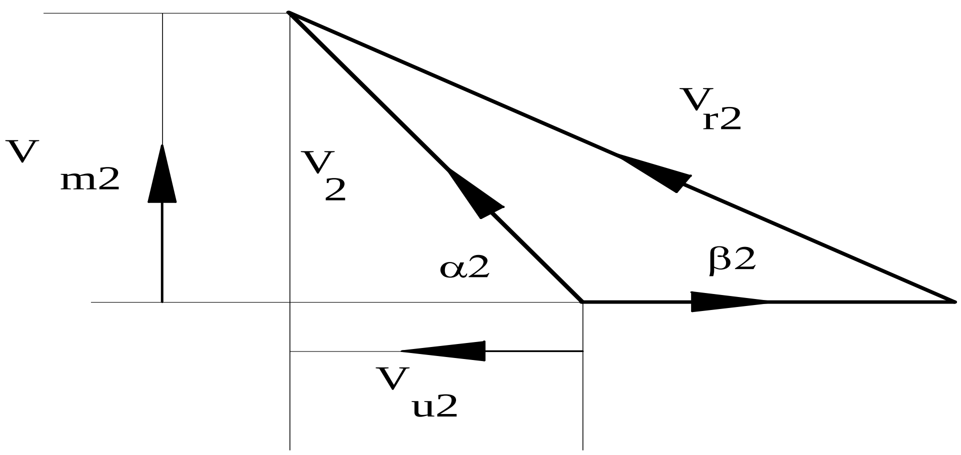
To determine

i)$Rotor\ blade\ angle\ \beta_1 = ?, \beta_2 = ?.$, ii) $tangential\ force\ F_u = ?$ iii) $P = ?$

iv) $Utilization\ factor = \varepsilon = ?$

Inlet Velocity Triangle





Outlet velocity triangle

From inlet velocity triangle

$\overrightarrow{V_{u1}} = V_1 \cos \alpha_{11}$; $\overrightarrow{V_{u1}}$ $=1200Cos22$; $\overrightarrow{V_{u1}} = 1112.62m/s$;

$V_{m1} = V_1\ sin\ \alpha_1$; $V_{m1} = 1200sin22$; V_{m1} =449.527m/s

$tan\ \beta_1 = \frac{V_{m1}}{\overrightarrow{V_{u1}} - U_1}$; $tan\ \beta_1 = \frac{449.527}{111.2.62-400}$; β_1 =32.24°

$Sin\ \beta_1 = \frac{V_{m1}}{V_{r1}}$; $Sin 32.24 = \frac{449.527}{V_{r1}}$; V_{r1} =842.65m/s

$\beta_1 = \beta_2$ (blades are equiangular); $\beta_1 = \mathbf{32.24^o}$; V_{r1} equals to V_{r2} ie $V_{r1} = V_{r2}$

Hence $V_{r2} = 842.65m/s$

From outlet velocity triangle

$\overleftarrow{V_{u2}} = V_{r2}\cos\beta_2 - U$; $\overleftarrow{V_{u2}} = 842.65 \cos 32.24 - 400$; $\overleftarrow{V_{u2}} = 312.732m/s$

ii) Tangential force

Tangential force = $F_u = \frac{\dot{m}}{g_c}(\overrightarrow{V_{u1}} + \overleftarrow{V_{u2}})$

Note that + sign since direction of Vu2 is opposite to the direction of V_{u1}

$F_u = \frac{1}{1}(1112.62 + 312.732)$ assuming m =1kg/s

$F_u = 1425.35\ N/kg/s$

Power:

$\frac{E}{\dot{m}} = \frac{1}{g_c}(\overrightarrow{V_{u1}} + \overleftarrow{V_{u2}})U$; $\frac{E}{\dot{m}} = \frac{1}{1}(1112.62 + 312.732)400$; $\frac{E}{\dot{m}} = 570140.8\ J/kg$

$E = \dot{m}\frac{E}{\dot{m}}$; $E = 1\ x\ 570140.8$; $E = 570140.8\ W$

Utilization factor

$\epsilon = \frac{\boldsymbol{V_1^2 - V_2^2}}{\boldsymbol{V_1^2 - RV_2^2}}$; $\epsilon = \frac{\boldsymbol{V_1^2 - V_2^2}}{\boldsymbol{V_1^2}}\ as\ R = 0$

$sin\ \beta_2 = \frac{V_{m2}}{V_{r2}}$; $sin 32.24 = \frac{V_{m2}}{842.65}$; $V_{m2} = 449.526m/s$

$V_2 = \sqrt{V_{u2}^2 + V_{m2}^2}$; $V_2 = \sqrt{312.732^2 + 449.526^2}$; $V_2 = 369.06m/s$

$\epsilon = \frac{1200^2 - 369.06^2}{1200^2}$; $\epsilon = 0.905$

16. At a nozzle exit of a steam turbine, the absolute steam velocity is 300m/s.The rotor speed is 150m/s at a point where the nozzle angle is 18°. If the outlet rotor blade angle is 3.5° less than the inlet blade angle, find the power output from the stage, for a steam flow rate of 8.5kg/s. Assuming $V_{r1} = V_{r2}$ find utilization factor. Specify how you would alter the blade design so that utilization may become maximum under the given circumstances

17. In a delaval steam turbine nozzle angle at inlet is 18°. The relative velocity is reduced to the exit at 6% when steam flows over the moving blades. The output of the turbine is 120kJ/kg of steam. If the blades are equiangular, find i) speed ratio ii) velocity of steam from nozzle iii) blade speed for maximum utlization
18. At a stage of an impulse turbine the mean blade dia is 0.75m, is rotational speed being 3500rpm. The absolute velocity of fluid discharging form a nozzle inclined at 20° to the plane of the wheel is 275m/s. If the utilization factor is 0.9 and the relative velocity at rotor exit is 0.9 times that at the inlet, find the inlet and exit rotor angle.Also find the power output from stage for mass flow rate of 2 kg/s and axial thrust on the shaft
19. At a stage of an impulse turbine, the mean blade dia is 80cm, its rpm 3000rpm. The absolute velocity of fluid discharging form a nozzle inclined at 20° to the plane of the wheel is 300m/s. If the utilization factor is 0.85 and the relative velocity at rotor exit is equals at inlet, find the inlet and exit rotor angle. Also find the power output from stage for mass flow rate of 1 kg/s (4b,10, Dec 18/Jan19)
20. An impulse turbine the mean blade dia is 0.75m, with a speed of 2800rpm. The absolute velocity of jet leaving a nozzle inclined at 18° to the plane of the wheel is 280m/s. If the utilization factor is 0.88 and the relative velocity at rotor exit at inlet remains same, Determine i) the inlet and outlet blade angles ii) work done iii) power output for a mass flow rate of 10kg/s (3b. 10, June/July18)
21. The following data refer to an axial flow impulse steam turbine: Steam flow rate =20kg/s, blade speed ratio=0.5, blade velocity coefficient V_{r1}/V_{r2}=0.9, the nozzle angle at the rotor inlet = 30° such as to make the whirl velocity at inlet is positive, rotor speed =4000rpm, mean diameter of the rotor = 60cm. Find the rotor blade angles if the rotor blades are equiangular. Find also the power output, axial thrust and the utilization factor. Sketch the velocity triangles
22. In an axial flow turbine , the discharge blade angle are 20°each, for both the stator and the rotor. The steam speed at the exit of the fixed blade is 140m/s. The ratio of $\frac{V_a}{u} = 0.7$ at the entry and 0.76 at the exit of the rotor blade. Find i) the inlet rotor blade angle, ii) the power developed by the blade ring for mass flow rate of 2.6kg/s iii) Degree of reaction (3b. 10 June/July 16) (3b. 10 June/July 13)

axial flow turbine $U_1 = U_2 = U$

the discharge blade angle are 20°each, for both the stator and the rotor.

ie $\propto_1 = 20^o; \beta_2 = 20^0;$

The steam speed at the exit of the fixed blade is 140m/s ie $V_1 = 140$m/s

The ratio of $\frac{V_a}{u} = 0.7$ at the entry and 0.76 at the exit of the rotor blade.

Ie $\frac{V_{a1}}{U} = 0.7$ and $\frac{V_{a2}}{U} = 0.76$

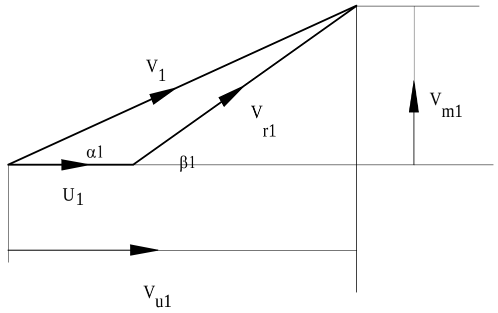
$\overrightarrow{V_{u1}} = V_1 cos\alpha_1;$ $\quad \overrightarrow{V_{u1}} = 140cos20\ ;$ $\quad \overrightarrow{V_{u1}}$=131.56m/s ;

$V_{a1} = V_1 sin\alpha_1;$ $\quad V_{a1} = 140sin20;$ $\quad V_{a1} = 47.88$m/s

$\frac{V_{a1}}{U} = 0.7;$ $\frac{47.88}{U} = 0.7;$ $U = 68.40m/s$

$\frac{V_{a2}}{U} = 0.76\,;$ $\frac{V_{a2}}{68.40} = 0.76;$ $V_{a2} = 51.98m/s$

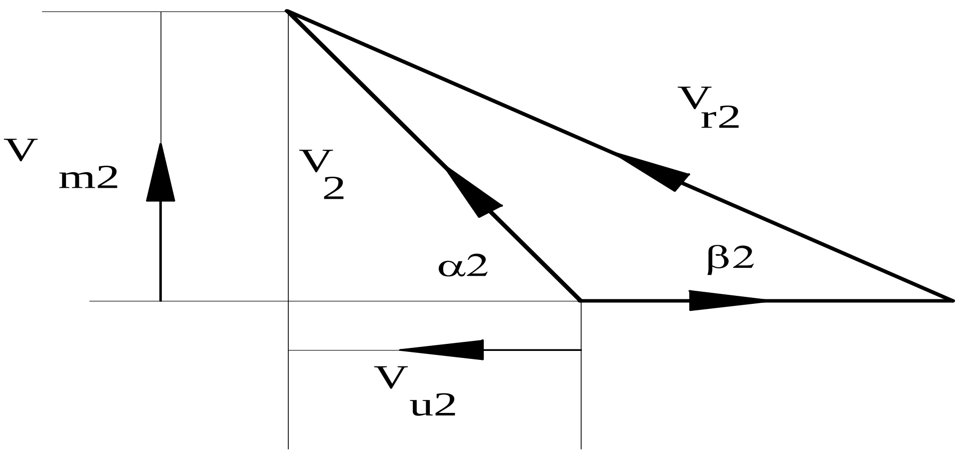
$\overrightarrow{V_{u1}} > U$, Hence Inlet velocity triangle is as follows



$tan\beta_1 = \frac{V_{a1}}{\overrightarrow{V_{u1}} - \text{U}}\,;$ $tan\beta_1 = \frac{47.88}{131.56-68.40}\,;$ $\beta_1 = 37.16^o$

$\frac{V_{a2}}{X} = tan\beta_2;$ $\frac{51.98}{X} = tan20;$ $X = 142.81;$ $V_{r2}\,cos\beta_2 = 142.81$

$V_{r2}\,cos\beta_2 > U$



$\overleftarrow{V_{u2}} = V_{r2}cos\beta_2 - U;$ $\overleftarrow{V_{u2}} = 74.40m/s$

$\frac{E}{\dot{m}} = \frac{(\overrightarrow{V_{u1}}\ + \overleftarrow{V_{u2}})U}{g_c};$ $\frac{E}{\dot{m}} = \frac{(131.56\ \ + 74.40)68.40}{1}\,;$ $\frac{E}{\dot{m}} = 14.08 * 10^3$J/kg;

$E = \dot{m}\frac{E}{\dot{m}};$ $E = 2.6 * 14.08 * 10^3$ $E = 36.62 * 10^3$W

$R\ = 1- \frac{V_1^2 - V_2^2}{2g_c\frac{E}{\dot{m}}};$ $V_1^2 = 140^2;$

$V_2^2 = V_{u2}^2 + V_{a2}^2\ ;$ $V_2^2 = 74.40^2 + 51.98^2\,;$ $V_2^2 = 8237.28$

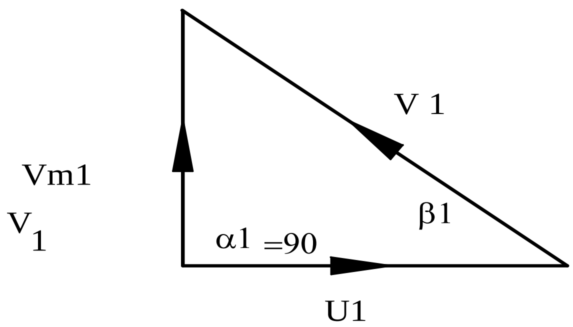
$R = 1 - \frac{140^2 - 8237.28}{2*14.08*10^3};$ $R = 0.5964$

23. Air flows axially through a axial flow turbine at a mean radius of 0.2m. If the tangential component of absolute velocity reduced by 20m/s during passage through the rotor, find the power developed by the turbine for a flow rate 100m^3/s at a point, where the pressure and temperature are 1 bar and 27°C. The rotational speed of the rotor is 3000rpm

24. Liquid water lows at a rate of 31.5kg/s through a rotor of an axial flow turbine, where inlet and outlet mean diameters are 18.5cm and 20cm respectively. The other data are: speed =6000rpm, V_1=35m/s and is directed axially. V_2=160m/s such that $\alpha_2 = 30^o$. Using mean inlet and outlet diameter find i) Torque exerted ii) V_{r1} and V_{r2} (3c, 06, Dec18/Jan19,10scheme)

$\dot{m} = \frac{31.5kg}{s}$; axial $D_1 = 18.5cm = 0.185m$; $D_2 = 20cm = 0.2m$; $N = 6000rpm$;

V_1=35m/s and is directed axially V_1=35m/s; $\alpha_1 = 90^o$; V_2=160m/s; $\alpha_2 = 30^o$



$U_1 = \frac{\pi D_1 N}{60}$; $\quad U_1 = \frac{\pi x 0.185 x 6000}{60}$; $\quad U_1 = 58.12$m/s

$U_2 = \frac{\pi D_2 N}{60}$; $\quad U_2 = \frac{\pi x 0.2 x 6000}{60}$; $\quad U_2 = 62.83$m/s

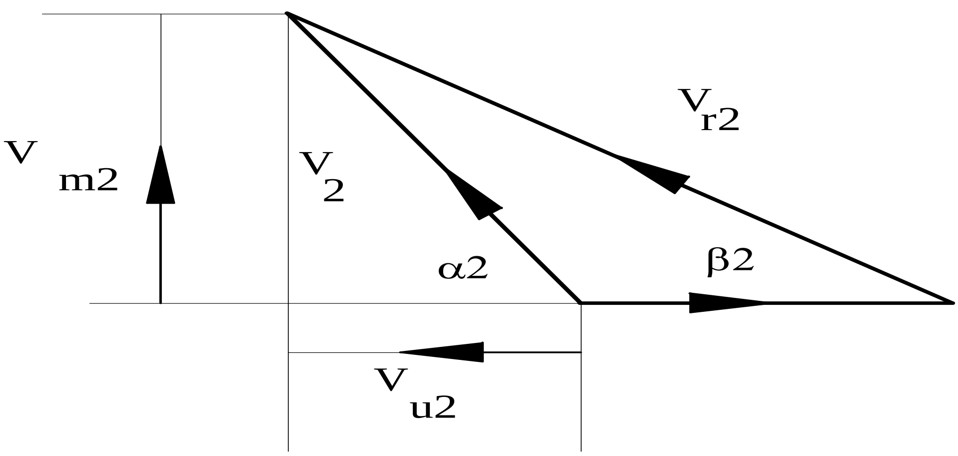
$\overrightarrow{V_{u1}} = V_1 cos\alpha_1$; $\quad \overrightarrow{V_{u1}} = 35 \cos 90$ $\quad \overrightarrow{V_{u1}} = 0$

$V_{m1} = V_1 sin\alpha_1$; $\quad V_{m1} = 35 \sin 90$ $\quad V_{m1} = 35m/s$

$\overrightarrow{V_{u2}} = V_2 cos\alpha_2$; $\quad \overrightarrow{V_{u2}} = 160 \cos 30$ $\quad \overrightarrow{V_{u2}} = 138.56m/s$

$V_{m2} = V_2 sin\alpha_2 = 160 \sin 30 = 80m/s$

$U_2 < \overrightarrow{V_{u2}}$, Hence outlet velocity triangle as given below



$Torque\ exerted = \frac{\dot{m}}{g_c}(\overrightarrow{V_{u1}}R_1 - \overrightarrow{V_{u2}}R_2);$

$T = \frac{31.5}{1}\left(0 - 138.56 * \frac{0.2}{2}\right);\ \ T = -13.856Nm$

From Inlet velocity triangle ; $V_{r1} = U_1;\ V_{r1} = 62.83m/s;$

From outlet velocity triangle : $tan\beta_2 = \frac{V_{m2}}{\overrightarrow{V_{u2}} - U_2}$

Assuming flow velocity is constant $V_{m2} = V_{m1};\ V_{m2} = 35m/s$

$tan\beta_2 = \frac{V_{m2}}{\overrightarrow{V_{u2}} - U_2};$ $tan\beta_2 = \frac{35}{138.56 - 62.83};$ $\beta_2 = 24.8^0$

$sin\beta_2 = \frac{V_{m2}}{V_{r2}};$ $sin24.8 = \frac{35}{V_{r2}};$ $V_{r2} = 83.42m/s$

25. An inward radial flow hydraulic turbine water enters with an absolute velocity of 15m/s with a nozzle angle of 15°. The speed of the rotor is 400rpm. Diameter of the rotor at inlet and outlet are 75cm and 50cm respectively. The fluid leaves the rotor radially with an absolute velocity of 5m/s. Determine i) The blade angles ii) workdone iii) utilization factor (3b,08, Dec18/Jan19,15 scheme)

$V_1 = 15m/s; \alpha_1 = 15^0; N = 400rpm; D_1 = 0.75m;\ D_2 = 0.5m$

The fluid leaves the rotor radially with an absolute velocity of 5m/s. $\alpha_2 = 90^0; V_2 = 5m/s;$

$\beta_1 = ?;\ \beta_2 = ? \frac{E}{\dot{m}} = ?; \epsilon = ?$

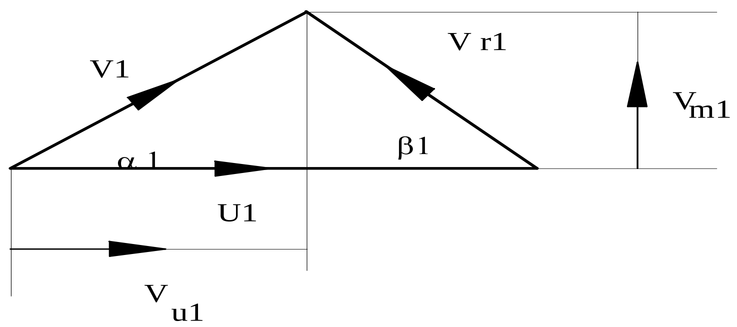
$U_1 = \frac{\pi D_1 N}{60};$ $U_1 = \frac{\pi * 0.75 * 400}{60};$ $U_1 = 15.70m/s;$

$U_2 = \frac{\pi D_2 N}{60};$ $U_2 = \frac{\pi * 0.5 * 400}{60};$ $U_2 = 10.47m/s;$

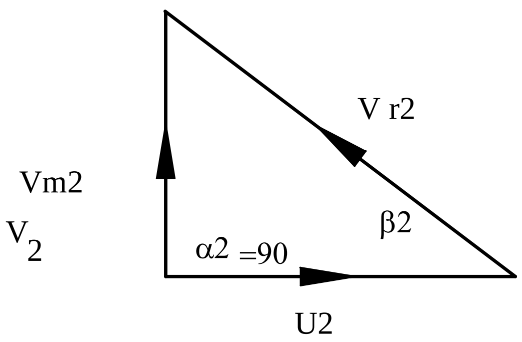
$\overrightarrow{V_{u1}} = V_1 cos\alpha_1;$ $\overrightarrow{V_{u1}} = 15cos15;$ $\overrightarrow{V_{u1}} = 14.49m/s;$

$V_{m1} = V_1 sin\alpha_1;$ $V_{m1} = 15sin15;$ $V_{m1} = 3.88m/s$

$\overrightarrow{V_{u1}} < U_1;$



$tan\beta_1 = \frac{V_{m1}}{U_1 - \overrightarrow{V_{u1}}}$; $tan\beta_1 = \frac{3.88}{15.70-14.49}$; β_1 =72.67°



$$V_{m2} = V_2$$

$tan\beta_2 = \frac{V_{m2}}{U_2}$; $tan\beta_2 = \frac{3.88}{5}$; $\beta_2 = 37.81^o$

$\frac{E}{\dot{m}} = \frac{(\overrightarrow{V_{u1}}\ U_1 - \overrightarrow{V_{u2}} U_2)}{g_c}$; $\overrightarrow{V_{u1}} = 14.49m/s$; $\overrightarrow{V_{u2}} = 0$

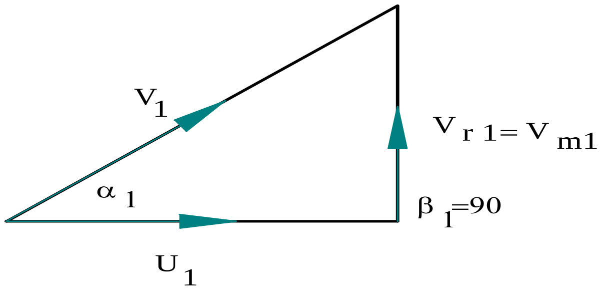
$\frac{E}{\dot{m}} = \frac{(14.49 * \ 15.70 + 0)}{1}$; $\frac{E}{\dot{m}} = 227.49J/kg$

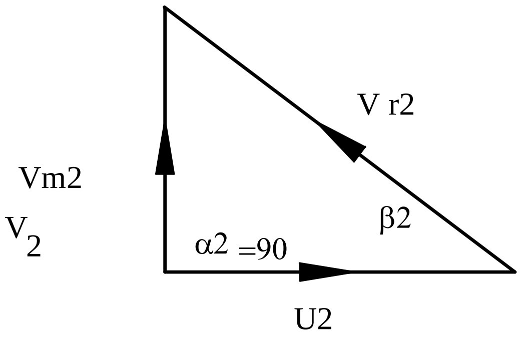
$Utilization\ factor$

$\in = \frac{\frac{E}{\dot{m}}}{\frac{E}{\dot{m}} + \frac{V_2^2}{2g_c}}$; $\in = \frac{227.49}{227.49 + \frac{5^2}{2*1}}$; $\in = 0.9479$

26. An inward flow reaction turbine has outer and inner diameter of the wheel as 1m and 0.5m respectively.. The vanes are radial at inlet , and discharge is radial at outlet and water enters the blade at an angle of 10°. Assume the velocity of flow is constant and equal to 3m/s. Find i) Speed of the wheel ii) outlet blade angle iii) Degree of reaction (2c. 10 June/July 17))(4b,10,June/July14)*

Inward flow turbine; Inner Diameter= 1m ie $D_1 = 1m$; $Outer\ Diameter\ D_2 = 0.5m$ $\alpha_1 = 10^0$. Assume the velocity of flow is constant and equal to 3m/s. $V_{f1} = V_{f2} = 3m/s$; $N = ?$; $R = ?$





From inlet velocity triangle $tan\alpha_1 = \frac{V_{m1}}{U_1}$; $\quad tan10 = \frac{3}{U_1}$; $\quad U_1 = 17.01m/s$;

$U_1 = \frac{\pi D_1 N}{60}$; $\quad 17.01 = \frac{\pi*1*N}{60}$; $\quad N = 324.94rpm$;

$U_2 = \frac{\pi D_2 N}{60}$; $\quad U_2 = \frac{\pi*0.5*324.94}{60}$; $\quad U_2 = 8.5m/s$;

$tan\beta_2 = \frac{V_{m2}}{U_2}$; $\quad tan\beta_2 = \frac{3}{8.5}$; $\quad \beta_2 = 19.424$

$$R = 1 - \frac{V_1^2 - V_2^2}{2g_c\frac{E}{\dot{m}}};$$

$\frac{E}{\dot{m}} = \frac{(\overrightarrow{V_{u1}}\ U_1 - \overrightarrow{V_{u2}} U_2)}{g_c}$;

From Inlet and outlet triangles

$\overrightarrow{V_{u1}} = U_1$; $\overrightarrow{V_{u2}} = 0$

$\frac{E}{\dot{m}} = \frac{(U_1\ U_1 + 0)}{g_c}$; $\quad \frac{E}{\dot{m}} = \frac{17.01*17.01+0}{1}$; $\quad \frac{E}{\dot{m}} = 289.34J/kg$;

$V_1^2 = V_{u1}^2 + V_{m1}^2$; $\quad V_1^2 = 17.01^2 + 3^2$; $\quad V_1^2 = 298.34$; $\quad V_2^2 = 3^2$

$R = 1 - \frac{298.34-9}{2*289.34}$; $\quad R = 0.5$

27. The mean diameter of axial flow steam turbine is 50cm. The maximum utilisation factor is 0.9 and degree of reaction is 0.5. The mass flow rate of steam is 10kg/s. The

speed of the blade is 2000rpm . Calculate i) Inlet and exit absolute velocities ii) Power developed (3c. 08, Dec12)

Axial : $D = D_1 = D_2 = 50cm = 0.5m$; $\in_{max}= 0.9$; $R = 0.5$; $\dot{m} = 10kg/s$; $N = 2000rpm$

$U = \frac{\pi DN}{60}$; $U = \frac{\pi x0.5x2000}{60}$ $U = 52.36m/s$

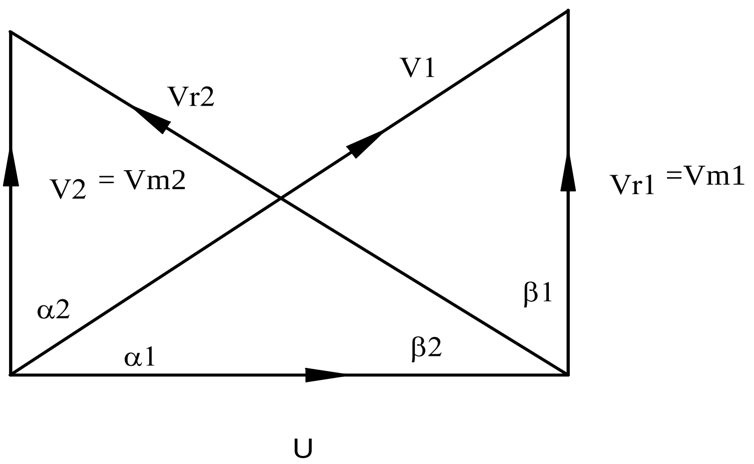
$\epsilon = \frac{V_1^2 - V_2^2}{V_1^2 - RV_2^2}$; $0.9 = \frac{V_1^2 - V_2^2}{V_1^2 - 0.5V_2^2}$

$0.9V_1^2 - (0.9 * 0.5V_2^2) = V_1^2 - V_2^2$; $V_1^2 = 5.5V_2^2$--------------1

$R = 50\%$, ie $V_{r1} = V_2, V_{r2} = V_1, \alpha_1 = \beta_2, \alpha_2 = \beta_1$

α_2 =90° since turbine is for maximum utilization

$\alpha_2 = \beta_1$ for 50% R Hence β_1 =90°



Power Developed

$E = \frac{\dot{m}}{g_c}(\overrightarrow{V_{u1}} + \overleftarrow{V_{u2}})U$

From inlet velocity triangle$\overrightarrow{V_{u1}}$ = U = 52.36 ; From outlet velocity triangle $\overrightarrow{V_{u2}}$=0

Hence $E = \frac{\dot{2}}{1}(52.36 + 0)\ 52.36$; $E = 5.483 * 10^3 Watts$

Exit absolute velocity

$\frac{E}{\dot{m}} = \frac{5.483*10^3}{2}$; $\frac{E}{\dot{m}} = 2741.5J/kg$

$\in = \frac{\frac{E}{\dot{m}}}{\frac{E}{\dot{m}}+\frac{V_2^2}{2g_c}};$ $0.9 = \frac{2741.5}{2741.5+\frac{V_2^2}{2*1}};$ $27415 + \frac{V_2^2}{2*1} = 3046.11;$

$V_2^2 = 609.22;$ $V_2 = \mathbf{24.68}\boldsymbol{m/s}$

<u>Inlet absolute velocity</u>

From outlet velocity triangle

$V_{r2}^2 = V_2^2 + U^2;$ $V_{r2}^2 = 609.22 + 52.36^2$; $V_{r2}^2 = 3350.67$ $V_{r2} = 57.88m/s$

$V_1 = V_{r2}$ since 50%R $V_1 = 57.88m/s$

28. At a 50% reaction stage axial flow turbine, the mean blasé diameter is 0.6mtr. The maximum utilization factor is 0.85 and steam flow rate is 12kg/s. Calculate the inlet and outlet absolute velocities and power developed if the speed is 2500rpm (3b. 10, June/July14)

Axial : $D = D_1 = D_2 = 60cm = 0.6m;$ $\in_{max} = 0.85;$ $R = 0.5$; $\dot{m} = 10kg/s;$ $N = 2000rpm$

$$U = \frac{\pi DN}{60}; U = \frac{\pi x0.6x2000}{60} = 62.83\text{m/s}$$

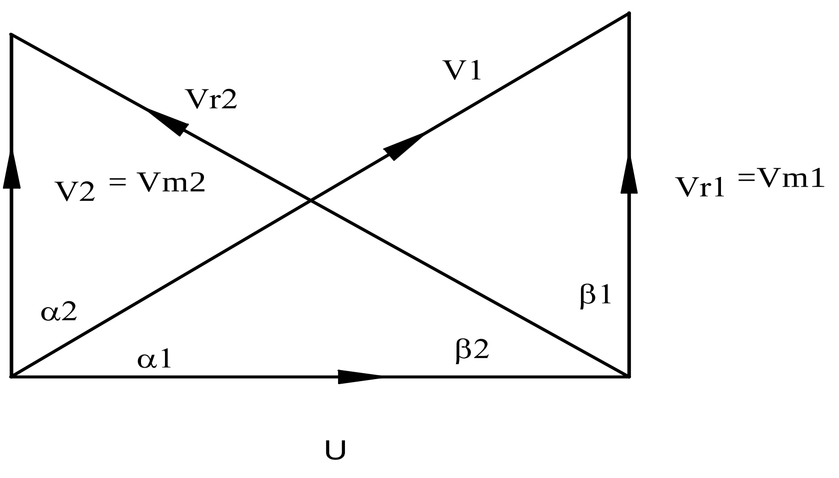
$\in = \frac{V_1^2 - V_2^2}{V_1^2 - RV_2^2};$ $0.9 = \frac{V_1^2 - V_2^2}{V_1^2 - 0.5V_2^2}$

$0.9V_1^2 - 0.9\text{x}0.5V_2^2 = V_1^2 - V_2^2;$ $V_1^2 = 5.5V_2^2$--------------1

$R = 50\%$, ie $V_{r1} = V_2, V_{r2} = V_1, \alpha_1 = \beta_2, \alpha_2 = \beta_1$

α_2=90° since turbine is for maximum utilization

$\alpha_2 = \beta_1$ for 50% R Hence β_1 =90°



Power Developed

$$E = \frac{\dot{m}}{g_c}(\overrightarrow{V_{u1}} \ + \overleftarrow{V_{u2}})U$$

From inlet velocity triangle $\overrightarrow{V_{u1}} \ = U = 62.83$; From outlet velocity triangle V_{u1}=0

Hence $E = \frac{2}{1}(62.83 + 0)62.83$; $\qquad E = 7.895 * 10^3 Watts$

Exit absolute velocity

$\frac{E}{\dot{m}} = \frac{7.895*10^3}{2}$; $\qquad \frac{E}{\dot{m}} = 3947.5 J/kg$

$\in = \frac{\frac{E}{\dot{m}}}{\frac{E}{\dot{m}} + \frac{V_2^2}{2g_c}}$; $\qquad 0.9 = \frac{3947.5}{3947.5 + \frac{V_2^2}{2*1}}$; $\qquad 3947.5 + \frac{V_2^2}{2*1} = 4386.11$;

$V_2^2 = 877.22$; $\qquad V_2 = \mathbf{29.61 m/s}$

Inlet absolute velocity

From outlet velocity triangle

$V_{r2}^2 = V_2^2 + U^2$; $\qquad V_{r2}^2 = 877.22 + 62.83^2$; $V_{r2}^2 = 4824.83$ $\quad V_{r2} = 69.46 m/s$

$V_1 = V_{r2}$ since 50%R $\qquad V_1 = 69.46 m/s$

29. At a 50% reaction stage axial flow turbine, the mean blade diameter is 60cm. The maximum utilization factor is 0.9. Steam flow rate is 10kg/s. Calculate the inlet and outlet absolute velocities and power developed if the speed is 2000rpm
30. The mean rotor blade speed of an axial speed of an axial flow turbine stage with a degree of reaction of 50% is 210m/s. The steam emerges from nozzle inclined at 28° to the wheel plane with an axial velocity component which is equal to blade speed . Assuming symmetric inlet and outlet velocity triangles. Find the rotor blade angles and utilization factor. Find also the degree of reaction to make the utilization maximum, if the axial velocity and the blade speed as well as the nozzle remain the same above(3b. 10, Dec14/Jan 15)*

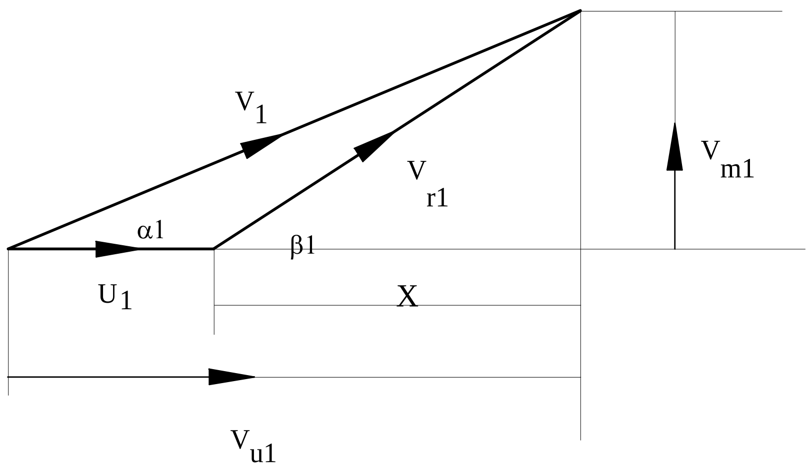
The mean rotor blade speed of axial flow turbine stage with 50% reaction is 210m/s ie U=210m/s, R=0.5 ie $\alpha_1 = \beta_2, \alpha_2 = \beta_1$

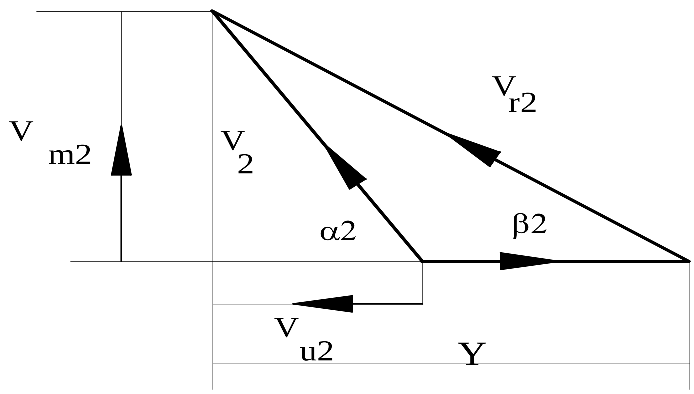
Steam emerges from the nozzle inclined at 28° to the plane of the wheel with axial component equal to the blade speed ie α_1 =28°, V_{m1}=U=210

To determine

ε =?; $\qquad \beta_1$ =? $\quad \beta_2$ =?

R=? for maximum utilization if the axial velocity, blade speed and nozzle angle remain the same.





Rotor Blade angles

$tan\alpha_1 = \frac{V_{m1}}{\overrightarrow{V_{u1}}}$; $tan28 = \frac{210}{\overrightarrow{V_{u1}}}$ $\overrightarrow{V_{u1}} = 395m/s$

$X = \overrightarrow{V_{u1}} - U$ $X = 395 - 210$; X =185m/s

$tan\beta_1 = \frac{V_{m1}}{X}$; $tan\beta_1 = \frac{210}{195}$; $\beta_1 = 48.62°$

$\alpha_2 = \beta_1$; α_1=48.62°

$\beta_2 = \alpha_1$; β_2 =28°

Utilisation factor

$Sin\beta_1 = \frac{V_{m1}}{V_{r1}}$; $Sin\ 48.62 = \frac{210}{V_{r1}}$; V_{r1} = 279.87m/s

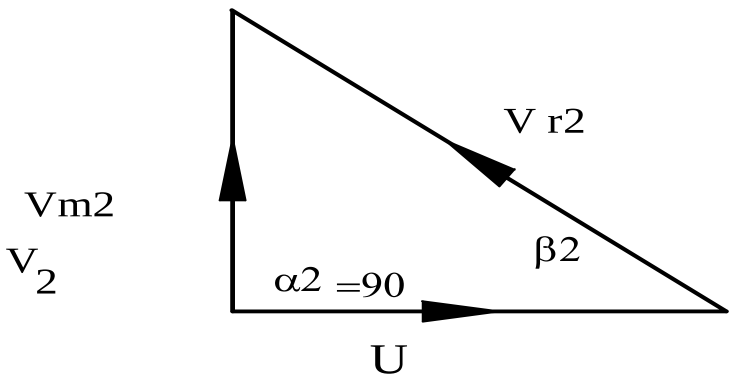
$V_2 = V_{r1}$ (50% R) ie V_2 =279.87m/s

$Sin\alpha_1 = \frac{V_{m1}}{V_1}$; $\qquad Sin\ 28 = \frac{210}{V_1} \qquad V_1 = 447.9 m/s$

$\epsilon\ =\ \frac{V_1^2 - V_2^2}{V_1^2 - RV_2^2}$; $\qquad \epsilon = \frac{447.9^2 - 279.87^2}{447.9^2 - 0.5x279.87^2}$; $\qquad \epsilon$ =0.757

Find also the degree of reaction to make the utilization ma maximum, if the axial velocity and the blade speed as well as the nozzle remain the same above

For maximum utilization outlet velocity triangle



$V_{m1} = U = 210 m/s$; V_{m2}=210m/s

V_{m2} =V_2 as α_2=90°

$R = 1 - \frac{V_1^2 - V_2^2}{2\frac{E}{\dot{m}}}$;

$\frac{E}{\dot{m}} = \frac{\overrightarrow{V_{u1}}U}{g_c}$ as $\qquad \overrightarrow{V_{u2}} = 0$

$\frac{E}{\dot{m}} = \frac{395*210}{1}$; $\qquad \frac{E}{\dot{m}} = 82950\ J/kg$

$R = 1 - \frac{447.9^2 - 210^2}{2x82950}$; $\qquad R = 0.0565$

ϵ = $\frac{V_1^2 - V_2^2}{V_1^2 - RV_2^2}$; $\qquad \epsilon = \frac{447.9^2 - 210^2}{447.9^2 - 0.0565x210^2} \qquad \epsilon$ =0.79

31. The following data refer to a 50% degree of reaction axial flow turbomachine. Inlet fluid velocity =230m/s, inlet rotor angle =60°, Inlet guide angle =30°, outlet rotor angle 25°, Find utilization factor , axial thrust and power output per unit mass flow. (3b,10,Dec13/Jan14)

$R = 0.5$;   axial flow turbomachine $U_1 = U_2 = U$

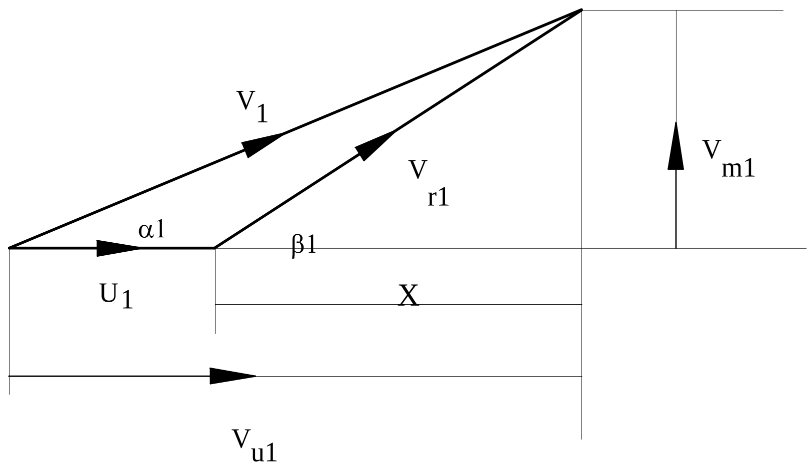
Inlet fluid velocity =230m/s, $V_1 = 230m/s$;

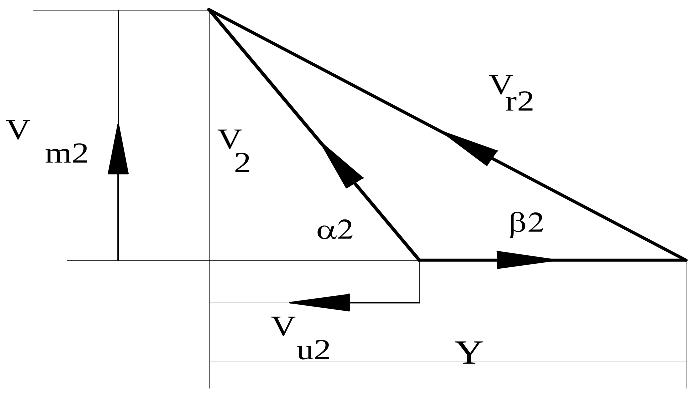
inlet rotor angle =60° , $\beta_1 = 60^0$;

Inlet guide angle =30° please note that this is exit guide blade angle Hence this is not α_1

outlet rotor angle 25° ie $\beta_2 = 25^0$

$\epsilon = ?$; $F_a = ?$; $E = ?$ if $\dot{m} = 1kg/s$





utilization factor

Since 50%R $\alpha_1 = \beta_2$; $\alpha_1 = 25^0$; $\alpha_2 = \beta_1$; $\alpha_2 = 60^0$

$\overrightarrow{V_{u1}} = V_1 \cos\alpha_1$; $\overrightarrow{V_{u1}} = 230\cos 25$ $\overrightarrow{V_{u1}} = 208.45\text{m/s}$

$V_{m1} = V_1 \text{Sin}\alpha_1$; $V_{m1} = 230\sin 25$ $V_{m1} = 97.2\text{m/s}$

$Sin\beta_1 = \frac{V_{m1}}{V_{r1}}$; $\quad Sin60 = \frac{97.2}{V_{r1}}$; $\quad V_{r1} = 112.24m/s$

$V_2 = V_{r1}$; $\quad V_2 = 112.24m/s$; $\quad \alpha_2 = \beta_1$; $\alpha_2 = 60^0$ for 50%R

$\overleftarrow{V_{u2}} = V_2\cos\alpha_2$; $\quad \overleftarrow{V_{u2}} = 112.24\cos60$ $\quad \overleftarrow{V_{u2}} = 56.11\mathrm{m/s}$

$V_{m2} = V_2\mathrm{Sin}\alpha_2$; $\quad V_{m2} = 112.24\sin60$ $\quad V_{m2} = 97.2\mathrm{m/s}$

$U = \overrightarrow{V_{u1}} - \mathrm{X}$; $\quad U = \overrightarrow{V_{u1}} - V_{r1}cos\beta_1$; $U = 208.45 - 112.24\cos60$; $U = 152.33m/s$

$\frac{E}{\dot{m}} = \frac{1}{g_c}(\overrightarrow{V_{u1}} + \overleftarrow{V_{u2}})U$; $\quad \frac{E}{\dot{m}} = \frac{1}{1}(208.45 + 56.11)152.33$; $\frac{E}{\dot{m}} = 40300.42J/kg$

$\in = \frac{\frac{E}{\dot{m}}}{\frac{E}{\dot{m}} + \frac{V_2^2}{2g_c}}$; $\quad \in = \frac{40300.42}{40300.42 + \frac{112.24^2}{2*1}}$; $\quad \in = 0.865$

axial thrust

$F_a = \frac{\dot{m}}{g_c}(V_{m1} - V_{m2})$; $\quad F_a = 0$ since $V_{m1} = V_{m2}$

Power:

$E = \dot{m}\frac{E}{\dot{m}}$; $\quad E = 1 * 40300.42$ $\quad E = 40300.42W$

32. A mixed flow turbine handling water operates under a static head of 65m. In steady flow the static pressure at the rotor inlet is is 3.5atm (guage). The absolute at the rotor inlet is directed at an angle of 25° to the tangent so that whirl velocity is positive. The absolute velocity at the exit is purely axial. If the degree of reaction for the machine is 0.47 and the utilisation factor is 0.896. Compute the tangential blade speed as well as the inlet blade angle. Find the work output per unit mass flow of water. (4b. 10, Dec12)

$p_o = 0$; $V_o = 0$; $Z_o = 65m$; $p_o = 3.5\ atm = 3.5 * 1.03bar$; $Z_1 = 0$

$\alpha_1 = 25^0$; The absolute velocity at the exit is purely axial ie $\alpha_1 = 90^0$; $R = 0.47$;

$\varepsilon = 0.896$

$U_1 =?$; $\beta_1 =?$; $\frac{E}{\dot{m}} =?$

Tangential speed of rotor

Applying burnollis equation

$\frac{p_o}{\omega} + \frac{V_o^2}{2g} + Z_o = \frac{p_1}{\omega} + \frac{V_1^2}{2g} + Z_1$; $\quad 0 + 0 + 65 = \frac{3.5*1.03*10^5}{9810} + \frac{V_1^2}{2*9.81} + 0$;

$V_1^2 = 575.52$;

$\varepsilon = \frac{V_1^2 - V_2^2}{V_1^2 - RV_2^2};$ $0.896 = \frac{575.52 - V_2^2}{575.52 - 0.47 * V_2^2};$ $V_2^2 = 102.01$

$R = 1 - \frac{V_1^2 - V_2^2}{2g_c\frac{E}{\dot{m}}};$ $0.47 = 1 - \frac{575.52 - 102.01}{2 * 21.74 * U_1};$ $U_1 = 20.30m/s$

Inlet Blade angle

$\frac{E}{\dot{m}} = \frac{(\overrightarrow{V_{u1}}\ U_1 - \overrightarrow{V_{u2}} U_2)}{g_c};$ $\overrightarrow{V_{u2}} = 0$ as $\alpha_2 = 0$ hence $\frac{E}{\dot{m}} = \frac{V_{u1}U_1}{g_c}$

$V_{u1} = V_1 cos\alpha_1;$ $V_{u1} = 23.99cos25;$ $V_{u1} = 21.74m/s$

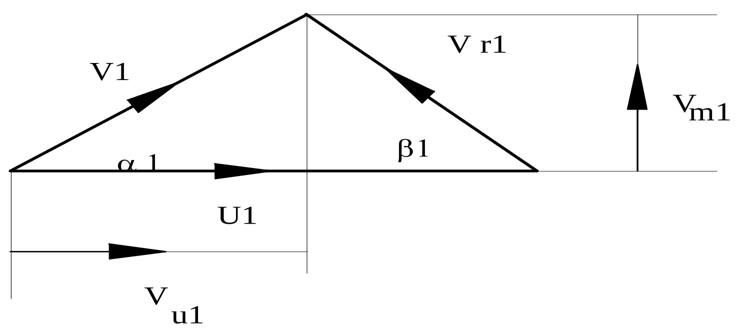
$\frac{E}{\dot{m}} = \frac{V_{u1}U_1}{g_c};$ $\frac{E}{\dot{m}} = \frac{21.74 * 20.30}{1};$ $\frac{E}{\dot{m}} = 437.62J/kg$

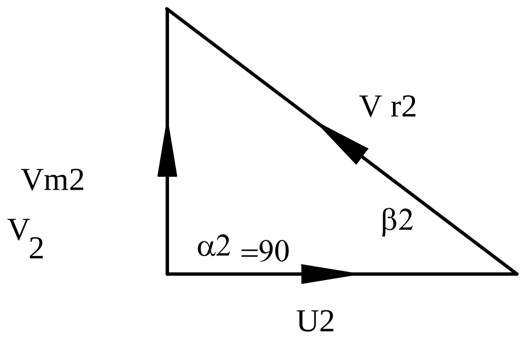
$tan\beta_1 = \frac{V_{m1}}{\overrightarrow{V_{u1}} - U};$ $V_{m1} = V_1 sin\alpha_1;$ $V_{m1} = 23.99sin25;$ $V_{m1} = 10.14m/s$

$tan\beta_1 = \frac{10.14}{21.74 - 20.30};$ $\beta_1 = 82.90^0$

33. A hydraulic reaction turbine of the radial inward flow type works under a head of 160m of water. At the point of fluid entry, the rotor blade angle is 119° and diameter of the runner is 3.65m. At the exit , the runner diameter is 2.45m . If the absolute velocity of the wheel outlet is radially directed with a magnitude of 15.5m/s and the radial component of velocity at the inlet is 10.3m/s. Find the power developed by the machine , assuming that the 88% of the available head of the machine is converted into work and the flow rate is 110m^3/s. Find also the degree of reaction and utilization factor (4b,08, June/July 18 , 15 scheme)

$H = 160m; \beta_1 = 119^o;\ D_1 = 3.65m\ D_2 = 2.45m;$
the absolute velocity of the wheel outlet is radially directed with a magnitude of 15.5m/s
$\propto_2 = 90^o;\ V_2 = 15.5m/s;$
The radial component of velocity at the inlet is 10.3m/s. $V_{m2} = 10.3m/s$
E =? ; the 88% of the available head of the machine is converted into work $\eta_h = 88\%$
flow rate is 110m^3/s $Q = 110m^3/s; R = ?\ \epsilon = ?$





$\eta_h = \frac{\frac{E}{\dot{m}}}{gH}$; $\frac{E}{\dot{m}} = \frac{(\overrightarrow{V_{u1}}\ U_1 - \overrightarrow{V_{u2}} U_2)}{g_c}$; $V_{u2} = 0$ as $\alpha_2 = 90^0$; $\frac{E}{\dot{m}} = \frac{V_{u1}U_1}{g_c}$

$\overrightarrow{V_{u1}} = U_1 - X$; $\frac{V_{m1}}{X} = \tan \beta_1$; $\frac{10.3}{X} = \tan(180 - 119)$; $X = 5.709 m/s$

$\overrightarrow{V_{u1}} = U_1 - 5.709$

$\frac{E}{\dot{m}} = \frac{V_{u1}U_1}{g_c}$; $\frac{E}{\dot{m}} = \frac{(U_1 - 5.709)U_1}{1}$

$\eta_h = \frac{\frac{E}{\dot{m}}}{gH}$; $0.88 = \frac{(U_1 - 5.709)U_1}{9.81*160}$; $1381.248 = U_1^2 - 5.709U_1$;

$U_1^2 - 5.709U_1 - 1381.248 = 0$; $U_1 = 40.13 m/s$;

$\overrightarrow{V_{u1}} = U_1 - 5.709$; $\overrightarrow{V_{u1}} = 40.13 - 5.709$; $\overrightarrow{V_{u1}} = 34.4 m/s$

$\frac{E}{\dot{m}} = \frac{34.4*40.13}{1}$; $\frac{E}{\dot{m}} = 1380.47\ J/kg$

$V_1^2 = V_{u1}^2 + V_{m1}^2$; $V_1^2 = V_{u1}^2 + V_{m1}^2$; $V_1^2 = 34.4^2 + 10.3^2$; $V_1^2 =$; $V_2^2 = 15.5^2$

$R = 1 - \frac{V_1^2 - V_2^2}{2g_c\frac{E}{\dot{m}}}$; $R = 1 - \frac{34.4^2 + 10.3^2 - 15.5^2}{2*1380.47}$; $R = 0.62$

$\epsilon = \frac{\frac{E}{\dot{m}}}{\frac{E}{\dot{m}} + \frac{V_2^2}{2g_c}}$; $\epsilon = \frac{1380.47}{1380.47 + \frac{15.5^2}{2*1}}$; $\epsilon = 0.92$

Power Absorbing machine

According to the direction fluid flow power absorbing turbo machine can be classified into axial and radial flow power absorbing turbo machine

All are centrifugal turbo machines are radial flow power absorbing turbo machines

In axial flow power absorbing turbo machine $U_1 = U_2 = U$

In radial flow power absorbing turbo machine $U_1 \neq U_2$

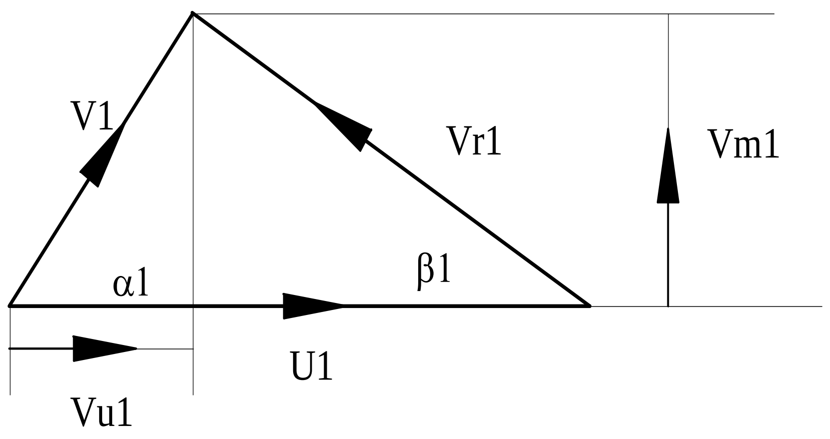
In power absorbing turbo machine

$\frac{E}{\dot{m}} = (\overrightarrow{V_{u1}}U_1 - \overrightarrow{V_{u2}}U_2)$ is negative ie $\overrightarrow{V_{u2}}U_2 > \overrightarrow{V_{u1}}U_1$

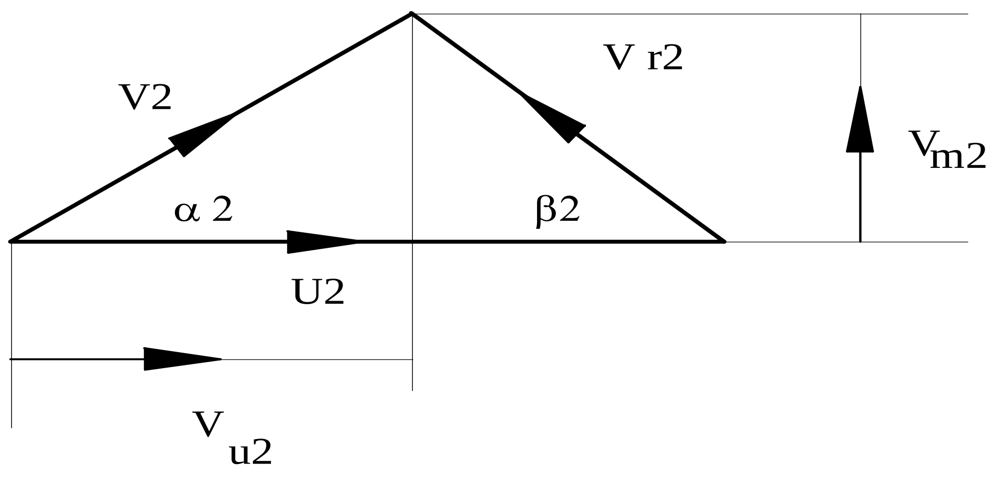
Turning angle of fluid from inlet to outlet is small

Axial flow Compressor

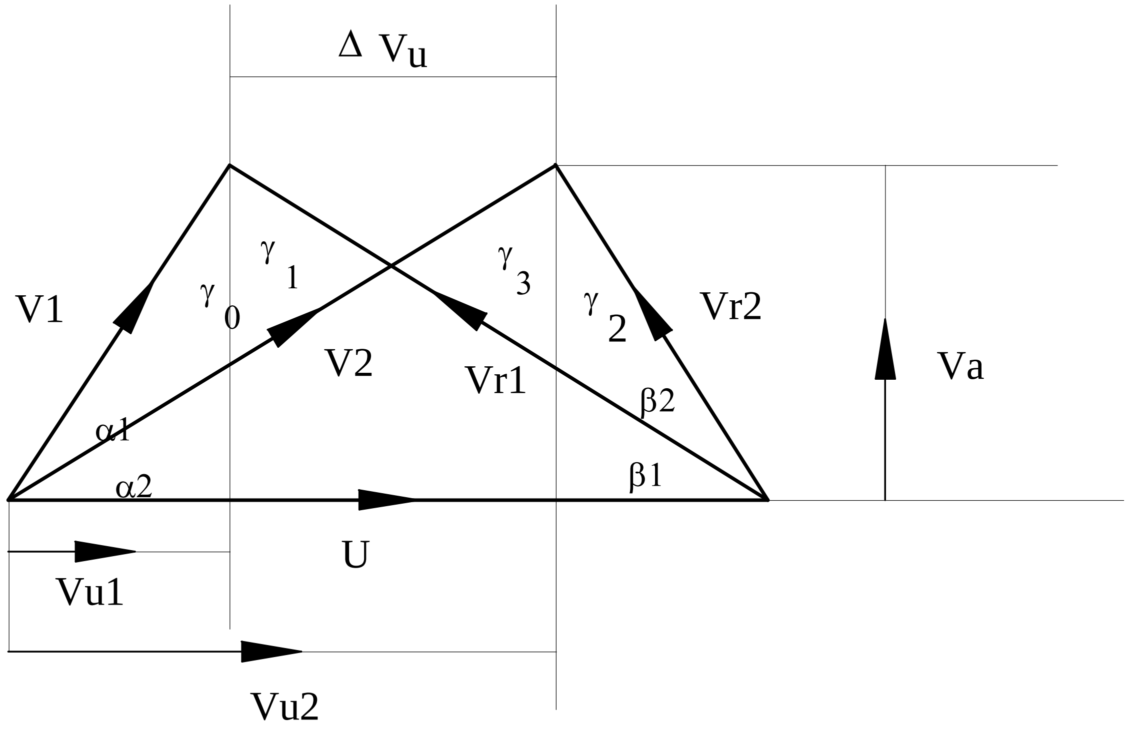
General Inlet velocity triangle



General outlet velocity triangle



In axial flow power absorbing turbomachine, since $U_1 = U_2$ outlet and inlet velocity triangles can be drawn with common base



γ are called air angles

γo is called air angle at inlet, γ_1 is called as air angle at outlet

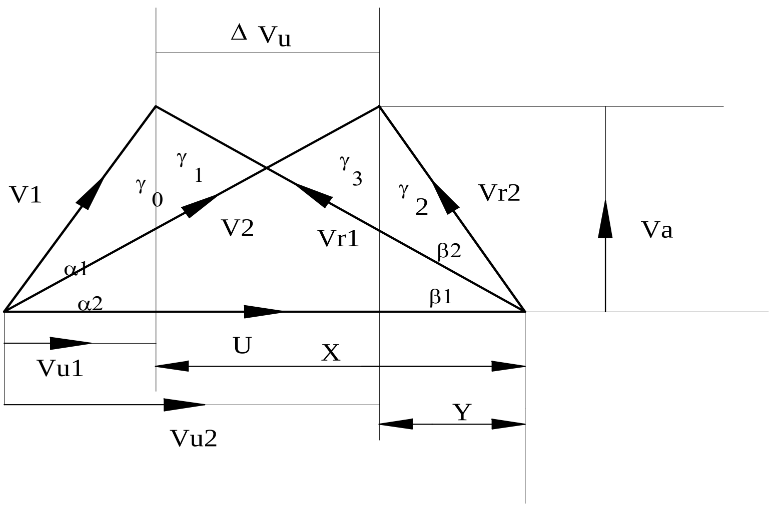
output per unit mass $\frac{E}{\dot{m}} = (V_{u1}U_1 - V_{u2}U_2)$ This expression will have negative value in power absorbing machine

therefore, generally In power absorbing we express Input/per unit mass

Input/per unit mass ie $-\frac{E}{\dot{m}} = (\overrightarrow{V_{u1}}U_1 - \overrightarrow{V_{u2}}U_2)$

Ie negative of output =Input

1. Define degree of reaction for an axial flow machine. Prove that degree of reaction for an axial flow device (assuming constant velocity of flow) is given by $R = \frac{V_f}{2U}\left(\frac{tan\beta_1+tan\beta_2}{tan\beta_1*tan\beta_2}\right)$ where β_1 and β_2 are the angles made with tangent to the blades (4a. 10, Dec13/Jan 14)(4a. 10, Dec18/Jan 19) (4a. 10 Dec17/Jan 2018)



$$R = \frac{\frac{U_1^2-U_2^2}{2} - \frac{V_{r1}^2-V_{r2}^2}{2}}{\frac{V_1^2-V_2^2}{2} + \frac{U_1^2-U_2^2}{2} - \frac{V_{r1}^2-V_{r2}^2}{2}} = \frac{\frac{U_1^2-U_2^2}{2} - \frac{V_{r1}^2-V_{r2}^2}{2}}{\frac{E}{\dot{m}}}$$

$$U_1 = U_1 = U;\ \frac{E}{\dot{m}} = (V_{u1} - V_{u2})\,\mathrm{U}$$

Hence, $R = \dfrac{-\left(\frac{V_{r1}^2-V_{r2}^2}{2}\right)}{(\overrightarrow{V_{u1}} - \overrightarrow{V_{u2}})\,\mathrm{U}};\ R = \dfrac{-\left(V_{r1}^2-V_{r2}^2\right)}{2(\overrightarrow{V_{u1}} - \overrightarrow{V_{u2}})\,\mathrm{U}}$

$$V_{r1}^2 = V_a^2 + X^2;\quad V_{r1}^2 = V_a^2 + (V_a \tan\gamma_1)^2$$

$$V_{r2}^2 = V_a^2 + Y^2;\quad V_{r2}^2 = V_a^2 + (V_a \tan\gamma_2)^2$$

$$R = \frac{-\left(\left(V_a^2+(V_a\tan\gamma_1)^2\right)-\left(V_a^2+(V_a\tan\gamma_2)^2\right)\right)}{2(V_a\tan\gamma_0 - V_a\tan\gamma_3)\,\mathrm{U}};\qquad \mathrm{R}= \frac{V_a^2\left(tan^2\gamma_2-tan^2\gamma_1\right)}{2\mathrm{V_a}(tan\,\gamma_0 - tan\,\gamma_3)\,U}$$

$$\mathrm{R}= \frac{V_a\left(tan^2\gamma_2-tan^2\gamma_1\right)}{2(tan\,\gamma_2 - tan\,\gamma_1)\,U}$$ since, $\tan\gamma_1 - \tan\gamma_2 = \tan\gamma_3 - \tan\gamma_0$

$$\mathrm{R}= \frac{V_a(\tan\gamma_1 + \tan\gamma_2)}{2\,\mathrm{U}}$$

$$\gamma_1 = 180 - \beta_1;\ tan\gamma_1 = tan(180-\beta_1);\ tan\gamma_1 = cot\beta_1$$

$$\gamma_2 = 180 - \beta_1;\ tan\gamma_2 = tan(180-\beta_2);\ tan\gamma_2 = cot\beta_2$$

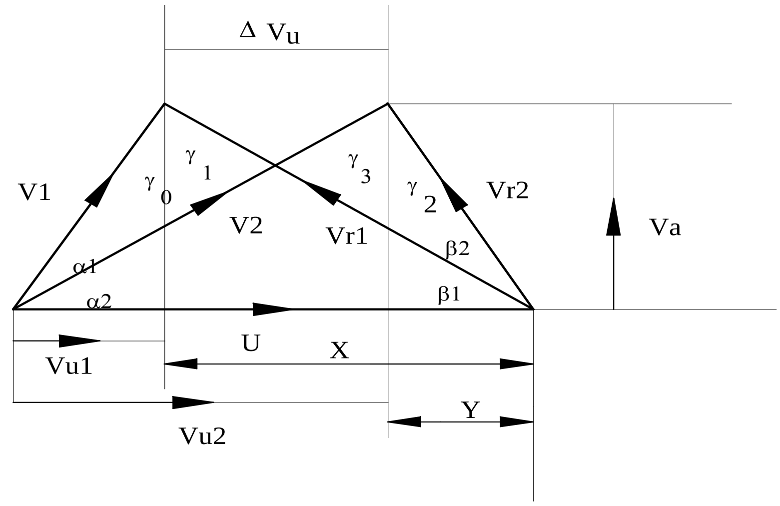
Hence , $R = \dfrac{V_a(cot\beta_1 + cot\beta_2)}{2\,\mathrm{U}}$;

$$R = \frac{V_a\left(\frac{1}{tan\beta_1}+\frac{1}{tan\beta_2}\right)}{2\,\mathrm{U}};\ \ ;\ R = \frac{V_a(tan\beta_2+tan\beta_1)}{2\,\mathrm{U}(tan\beta_1 * tan\beta_2)}$$

2. Draw the velocity triangles for axial flow compressor. From the triangles show that degree of reaction for axial flow compressor is given by $R = \frac{V_a}{2U}(cot\beta_1 + cot\beta_2)$ refer previous problem
3. With the help of inlet and outlet velocity diagrams , show that the degree of reaction for an axial flow compressor is given by
   $R = \frac{V_{ax}}{2U}(tan\gamma_1 + tan\gamma_2)$ Assume axial velocity to remain constant. γ_1 *and* γ_2 are angles made by relative velocities with the axial direction (4a,10, June/July13) refer solution 38
4. Draw velocity triangles for the following types of vanes of centrifugal pumps and compressors i) Backward curved vane ii) Radial vane iii) Forward curved vane (3b. 06, Dec12)
5. The total power input at a stage in an axial –flow compressor with symmetric inlet and outlet velocity triangles (R=0.5) is 27.85kJ/kg of air flow. If the blade speed is 180m/s throughout the rotor, draw the velocity triangles and compute the inlet and outlet rotor blade angles Do you recommend the use of such compressors? Comment on the results you have obtained. Assume axial velocity component to be 120m/s (4a,10, Dec15/Jan16)

   $R = 0.5; \frac{E}{\dot{m}} = 27.85kJ/kg\,; \frac{E}{\dot{m}} = 27850J/kg; U = 180m/s; \beta_1 = ?; \beta_2 = ?;$

   $V_{m1} = V_{m2} = 120m/s$



$$\Delta V_u = (\overrightarrow{V_{u2}} - \overrightarrow{V_{u1}})$$

$$-\frac{E}{\dot{m}} = \frac{(\overrightarrow{V_{u2}} - \overrightarrow{V_{u1}})U}{g_c};\ 27850 = \frac{(\overrightarrow{V_{u2}} - \overrightarrow{V_{u1}})180}{1};\ (\overrightarrow{V_{u2}} - \overrightarrow{V_{u1}}) = 154.72\text{m/s}$$

$$(\overrightarrow{V_{u2}} - \overrightarrow{V_{u1}}) + \text{X} + \text{X} = \text{U}; \qquad 154.72 + 2\text{X} = 180;\ \text{X} = 12.64\text{m/s}$$

$$\tan\alpha_1 = \frac{V_{m1}}{X};\quad \tan\alpha_1 = \frac{120}{12.64};\ \alpha_1 = 83.98^o$$

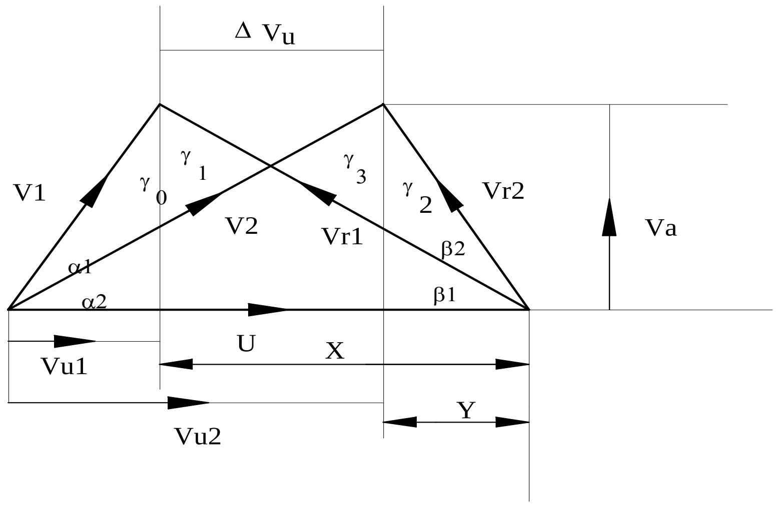
$$\tan\beta_1 = \frac{V_{m1}}{U-X};\quad \tan\beta_1 = \frac{120}{180-12.64};\ \beta_1 = 35.64^o$$

$$\beta_2 = \alpha_1 = 83.98^o$$

6. Draw the velocity triangles at inlet and outlet of an axial flow compressor form the following data. Degree of reaction 0.5., inlet blade angle 45°, axial velocity of flow which is constant throughout 120m/s, speed of rotation 6500rpm, radius of rotation 20cm, blade speed at inlet is equal to blade speed at outlet. Calculate angles at inlet and outlet. Also calculate power needed to handle 1.5kg/s (4b. 10, Dec14/Jan 15)

$R = 0.5; \beta_1 = 45^0; V_{m1} = V_{m2}$ =120m/s;$N = 6500rpm; R = \frac{D}{2} = 0.2m;$

$U_1 = U_2; \beta_2 =? \; \alpha_1 =? \; \alpha_2 =? \; E =? \; ; \dot{m} = 1.5kg/s$



$U = \frac{\pi DN}{60}; U = \frac{\pi x 0.4 x 6500}{60} = 136.13m/s;$

$\tan\beta_1 = \frac{V_{m1}}{U-X}; \tan 45 = \frac{120}{136.13-X}; \quad X = 16.13m/s$

$\tan\alpha_1 = \frac{V_{m1}}{X}; \quad \tan\alpha_1 = \frac{120}{16.13}; \; \alpha_1 = 82.34^o$

$\beta_2 = \alpha_1 = 82.34^o$

$-\frac{E}{\dot{m}} = \frac{(\overrightarrow{V_{u2}} - \overrightarrow{V_{u1}})\mathrm{U}}{g_c}$

$(\overrightarrow{V_{u2}} - \overrightarrow{V_{u1}}) = \mathrm{U} - 2\mathrm{X}; \; (\overrightarrow{V_{u2}} - \overrightarrow{V_{u1}}) = 136.13 - (2 * 16.13);$

$(\overrightarrow{V_{u2}} - \overrightarrow{V_{u1}}) = 103.87\mathrm{m/s}$

$\frac{E}{1.5} = \frac{(103.87)136.13}{1}; \; E = 21.20 * 10^3 W$

7. An axial flow compressor of 50% reaction design has blades with inlet and outlet angles of 44° and 13° respectively. The compressor is to produce a pressure ratio 5:1 with an overall isentropic efficiency of 87% when the inlet temperature is 290K. The mean blade speed and axial velocity are constant throughout the compressor.

Assume that blade velocity is 180m/s and work input factor is 0.85, Find the number of stages required and the change of entropy (4b. 10 Dec17/Jan 2018)

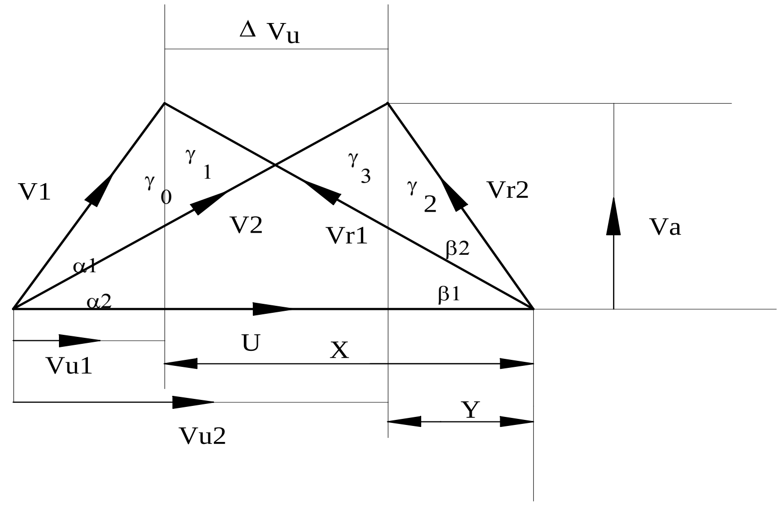
$R = 50\%; \beta_2 = 44^o; \beta_1 = 13^o$;

The compressor is to produce a pressure ratio of 6:1 ie $\frac{p_{k+1}}{p_1} = 5$; $\eta_0 = 0.87; T_{01} = 290K$

The blade speed and axial velocity are constant throughout the compressor.

ie $U_1 = U_2 = U$ and $V_{a1} = V_{a2} = V_{a2}$

$U = 180m/s$; $k = ?$ when $\Omega = 0.85$



the number of stages for work done factor is unity

$R = \frac{V_a(tan\beta_2+tan\beta_1)}{2\,U(tan\beta_1*tan\beta_2)}$; $\quad 0.5 = \frac{V_a(tan44+tan13)}{2*180*(tan44*tan13)}$; $\quad V_a = 33.54m/s$;

$tan\beta_2 = \frac{V_a}{Y}$; $\quad Y = \overrightarrow{V_{u1}}$; $\quad tan\beta_2 = \frac{V_a}{\overrightarrow{V_{u1}}}$;; $\quad tan44 = \frac{33.54}{\overrightarrow{V_{u1}}}$; $\quad \overrightarrow{V_{u1}} = 34.73m/s$

$\overrightarrow{V_{u2}} = U - Y$; $\quad \overrightarrow{V_{u2}} = 180 - 34.73$ $\quad \overrightarrow{V_{u2}} = 145.26m/s$

$-\frac{E}{\dot{m}} = \frac{\Omega(\overrightarrow{V_{u2}}-\overrightarrow{V_{u1}})U}{g_c}$; $\quad$ Increase in entalphy $\Delta h_o/stage = \frac{0.85(145.26-34.73)180}{1}$,

$\Delta h_o/stage = 16911.09J/kg$

$\eta_0 = \frac{\Delta h_{0s}}{\Delta h_0}$; $\quad \eta_0 = \frac{C_p(T_{osk+1}-T_{01})}{\Delta h_0}$; $\quad \eta_0 = \frac{C_pT_{01}\left(\frac{T_{osk+1}}{T_{01}}-1\right)}{\Delta h_0}$

$\eta_0 = \frac{C_pT_{01}\left(\left(\frac{p_{k+1}}{p_1}\right)^{\frac{\gamma-1}{\gamma}}-1\right)}{\Delta h_0}$; $\quad \Delta h_0 = \frac{C_pT_{01}\left(\left(\frac{p_{k+1}}{p_1}\right)^{\frac{\gamma-1}{\gamma}}-1\right)}{\eta_0}$;

$\Delta h_0 = \frac{C_pT_{01}\left(\left(\frac{p_{k+1}}{p_1}\right)^{\frac{\gamma-1}{\gamma}}-1\right)}{\eta_0}$; $\quad (\Delta h_0)_{\text{total}} = \frac{1005*290[(5)^{0.286}-1]}{0.85}$; $\quad (\Delta h_0)_{\text{total}} = 200431.22J/kg$

Number of stages, k, $= \frac{(\Delta h_0)_{\text{total}}}{\Delta h_o/stage}$; $\quad k = \frac{200431.22}{16911.09}$; $\quad k = 11.85$ $\quad$ say 12 stages

8. A single stage axial blower with no inlet guide vanes is running at 3600 rpm. The mean diameter of the rotor is 16cm and the mass flow rate of air through the blower is 0.45kg/s. In the rotor the air is turned such that the absolute velocity of air at exit makes angle of 20° with respect to the axis . Assuming that the axial component of fluid remains constant, determine power input and degree of reaction. Assume that the density of air is constant at 1.185kg/m³ and area of flow is 0.02m² (4b,10 Dec 13/Jan14)

axial blower ie $U_1 = U_2 = U$; no inlet guide vanes $\alpha_1 = 90^0$

$N = 3600rpm$; The mean diameter of the rotor is 16cm, $D_1 = D_2 = D = 0.16m$

$\dot{m} = 0.45kg/s$;

In the rotor the air is turned such that the absolute velocity of air at exit makes angle of 20° with respect to the axis $\gamma_3 = 20^0$;

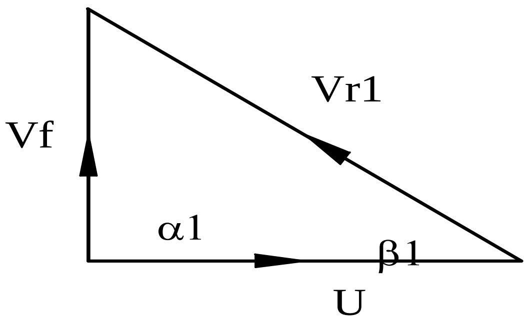
Assuming that the axial component of fluid remains constant $V_{f1} = V_{f2} = V_f$

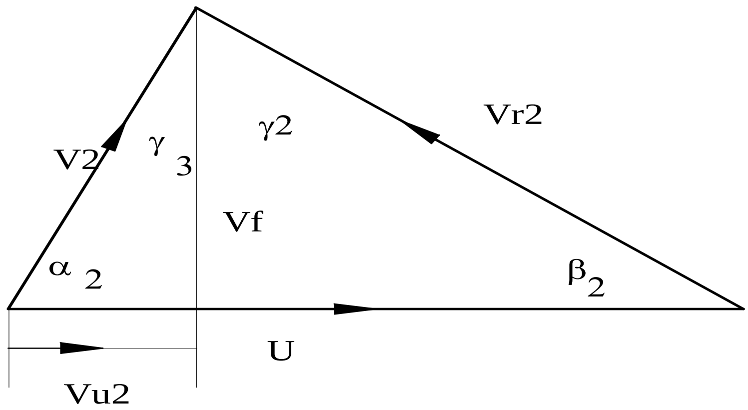
$E = ?$; $R = ?$

$\rho = 1.185kg/m^3$; area of flow is 0.02m² ie $A_f = 0.02m^2$

$U = \frac{\pi DN}{60}$; $U = \frac{\pi * 0.16 * 3600}{60}$ $U = 30.15m/s$

$\dot{m} = \rho A_f V_f$; $0.45 = 1.185 * 0.02 * V_f$; $V_f = 18.98m/s$





Power Input

$tan\gamma_3 = \frac{\overrightarrow{V_{u2}}}{V_f}$; $tan20 = \frac{\overrightarrow{V_{u2}}}{18.98}$; $\overrightarrow{V_{u2}} = 6.91m/s$

$-\frac{E}{\dot{m}} = \frac{(\overrightarrow{V_{u2}} - \overrightarrow{V_{u1}})\text{U}}{g_c};$ $\quad -\frac{E}{\dot{m}} = \frac{(6.91 - 0)30.15}{1}$ $\quad -\frac{E}{\dot{m}} = 208.28J/kg$

$-E = \dot{m} * \left(-\frac{E}{\dot{m}}\right);$ $\quad -E = 0.45 * 208.28$ $\quad -E = 93.72W$

Power Input= $93.72W$

Degree of reaction

$cos\gamma_3 = \frac{V_f}{V_2};$ $\quad cos20 = \frac{18.98}{V_2}$ $\quad V_2 = 20.19m/s$

$V_1 = V_f;$ $\quad V_1 = 18.98m/s$

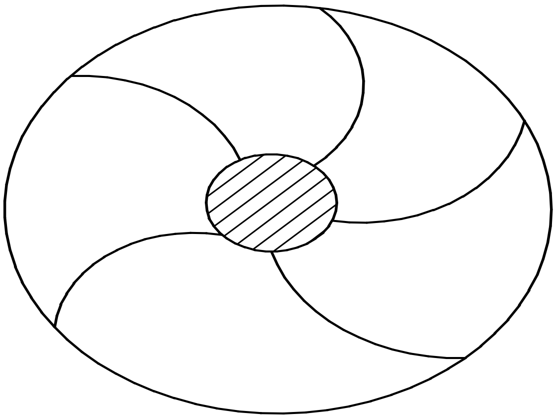
$-\frac{E}{\dot{m}} = 208.28J/kg;$ $\quad \frac{E}{\dot{m}} = -208.28J/kg$

$R = 1 - \frac{V_1^2 - V_2^2}{2g_c\frac{E}{\dot{m}}};$ $\quad R = 1 - \frac{18.98^2 - 20.19^2}{2*1*(-208.28)};$ $\quad R = 0.886$

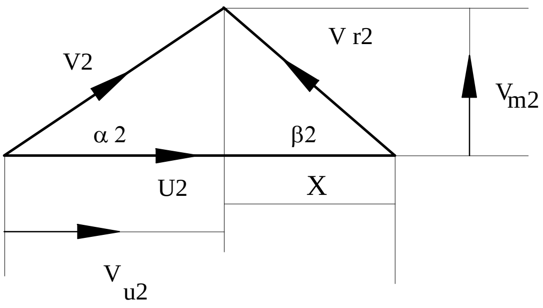
Radial flow Power absorbing machine

Types of vanes in centrifugal pump:

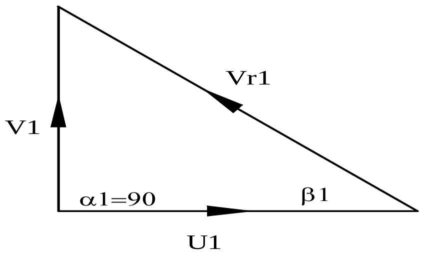
Backward curved vane: $\beta_2 < 90^o$



Outlet velocity triangle

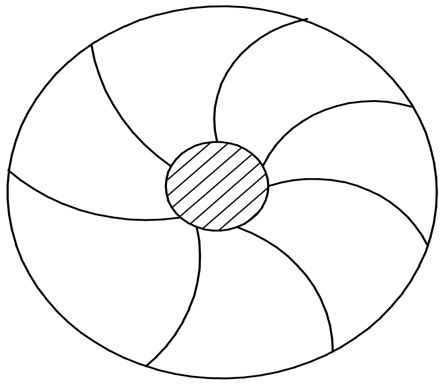


Inlet velocity triangle

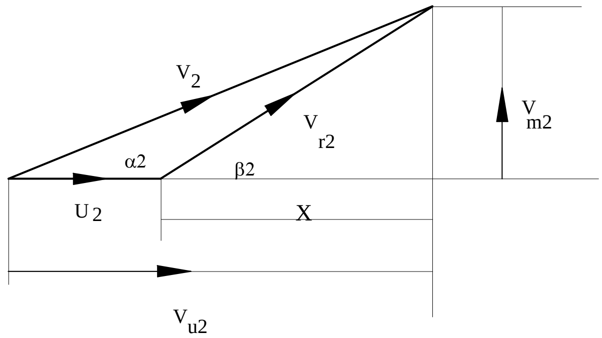


As $\beta_2 < 90^0$ $\cot\beta_2$ is positive. Therefore as flow rate Q increases head H_e decreases. Most preferable design

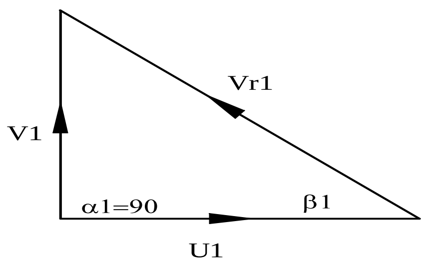
Forward curved vane: $\boldsymbol{\beta_2 > 90^o}$



Outlet velocity triangle



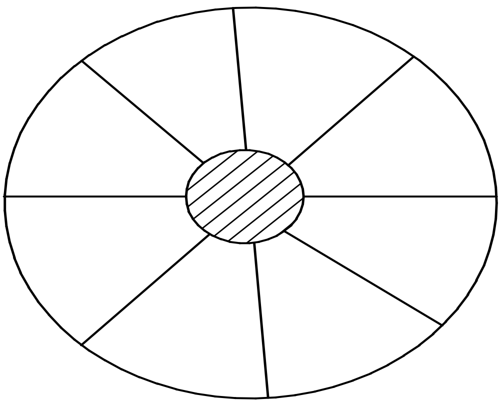
Inlet velocity triangle



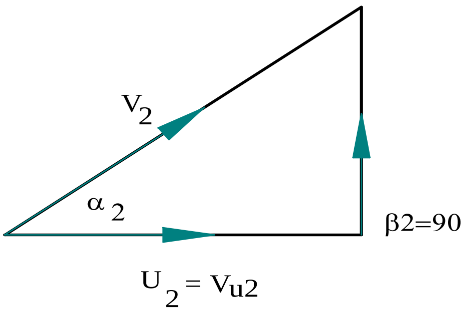
If $\beta_2 > 90^o$ K_2 becomes negative as cotβ_2 is negative. Therefore as Q increases H_e increases

This design is unstable since head goes on increases as head increases

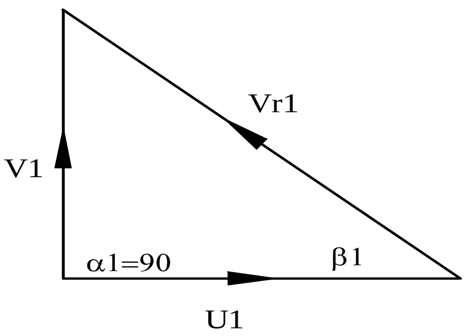
Radial vanes: $\boldsymbol{\beta_2 = 90^o}$



Outlet velocity triangle



Inlet velocity triangle

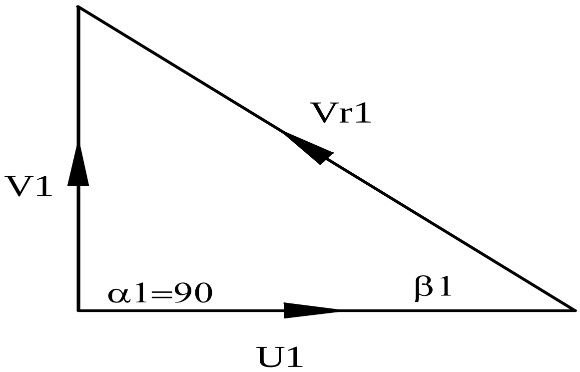


9. Draw the velocity diagram for a power absorbing radial flow turbo machine and show that
$R = \frac{1}{2}\left(1 + \frac{V_{m2} cot\beta_2}{U_2}\right)$ (4a. 10, Dec14/Jan 15)

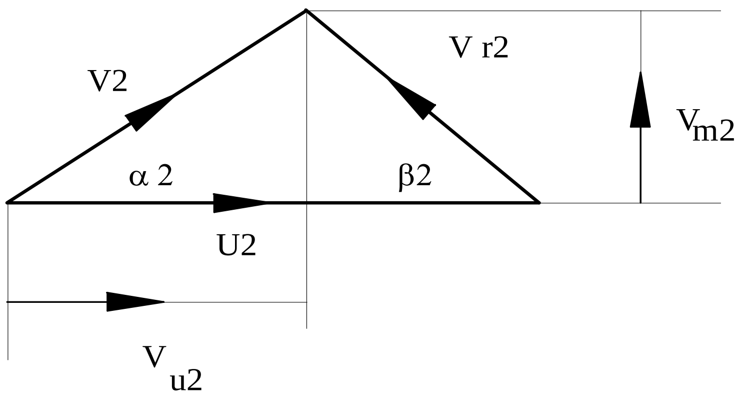
Radial flow compressors

Generally $\overrightarrow{V_{u1}}$ =0 (whirl velocity (tangential component) at inlet is zero)

Hence Inlet velocity triangle



Outlet velocity triangle



Input/per unit mass $\left(-\frac{E}{\dot{m}}\right) = (\overrightarrow{V_{u2}}\ U_2 - \overrightarrow{V_{u1}}U_1)$

Input/per unit mass $\left(-\frac{E}{\dot{m}}\right) = \overrightarrow{V_{u2}}\ U_2$ as $V_{u1} = 0$

$U_2 = \overrightarrow{V_{u2}} + Y$

$\tan\beta_2 = \frac{V_{m2}}{Y}$; $Y = V_{m2}cot\beta_2$; $U_2 = \overrightarrow{V_{u2}} + V_a cot\beta_2$

$\overrightarrow{V_{u2}} = U_2 - V_{m2}cot\beta_2$

Input , $\left(-\frac{E}{\dot{m}}\right) = (U_2 - V_{m2}cot\beta_2)U_2$

Degree of Reaction

$$R = 1 - \frac{V_1^2 - V_2^2}{2g_c\frac{E}{\dot{m}}}$$ -----------------A

$V_1^2 = V_{m1}^2 = V_{m2}^2;$ $\quad V_1^2 - V_2^2 = V_{m2}^2 - V_2^2;$ $\quad V_2^2 = V_{u2}^2 + V_{m2}^2;$ $\quad V_{m2}^2 - V_2^2 = -V_{u2}^2$

$V_1^2 - V_2^2 = -V_{u2}^2$-------------1

$\frac{E}{\dot{m}} = -U_2\overrightarrow{V_{u2}}$ --------------------2 as $V_{u1} = 0$

$\frac{E}{\dot{m}} = -U_2(U_2 - V_{m2}cot\beta_2)$

Substituting 1 and 2 in A

$R = 1 - \frac{-V_{u2}^2}{2(-U_2V_{u2})};$ $\quad R = 1 - \frac{\overrightarrow{V_{u2}}}{2(U_2)};$ $\quad R = 1 - \frac{U_2 - V_{m2}cot\beta_2}{2(U_2)};$

$R = 1 - \frac{1}{2} + \frac{V_{m2}cot\beta_2}{2(U_2)};$ $\quad R = \frac{1}{2}\left(1 + \frac{V_{m2}cot\beta_2}{U_2}\right)$

10. Derive an expression for degree of reaction for radial outward flow machine and explain briefly the effect of β_2, balde angle at exit with repect to tangential direction (4a, 10 ,june/July 17)

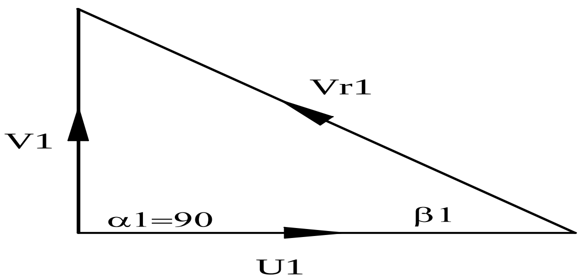
    Solution same as Problem number 9

11. Derive theoretical head capacity relation in case of radial flow pump (Centrifugal)

    $H = U_2^2 - \frac{U_2^2Qcot\beta_2}{A_2}$ β_2 =Discharge blade angle with respect to tangential direction.

    Explain the effect of discharge angle on it $\left(4b.08, \frac{Dec18}{Jan19}, 15\text{ scheme}\right)$

$$H = \frac{U_2^2}{g_c} - \frac{U_2^2Qcot\beta_2}{A_2g_c}(4a.08, \text{June/July18}, 15\text{ scheme})$$



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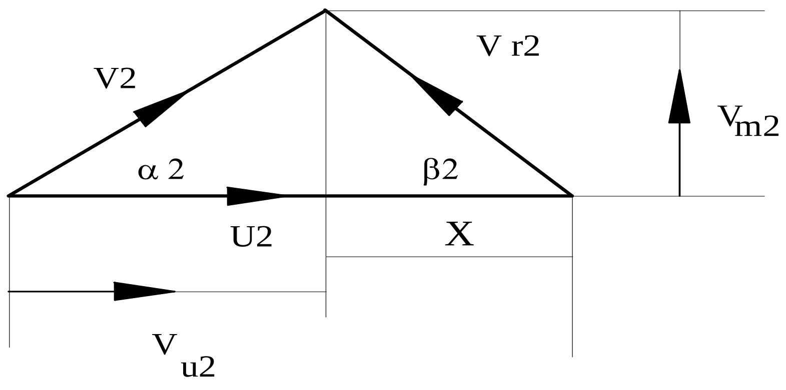
Inlet velocity triangle

V_{u1}=0 ; V_{m1}=V_1; $tan\beta_1 = \frac{V_1}{U_1}$



Outlet velocity triangle

$\overrightarrow{V_{u2}} = U_2 - X; \quad tan\beta_2 = \frac{V_{m2}}{X}; \; X = V_{m2}cot\beta_2; \quad \overrightarrow{V_{u2}} = U_2 - V_{m2}cot\beta_2$

$\frac{E}{\dot{m}} = (\overrightarrow{V_{u1}}U_1 \; - \overrightarrow{V_{u2}}\,U_2\,)$

$\frac{E}{\dot{m}} = -\overrightarrow{V_{u2}}U_2$ as $\overrightarrow{V_{u1}} = 0$

Output = $-\overrightarrow{V_{u2}}U_2$; − Output = $\overrightarrow{V_{u2}}U_2$; Input = $\overrightarrow{V_{u2}}U_2$;

gH_e = $V_{u2}U_2$; $H_e = \frac{V_{u2}U_2}{g}$

Substituting $\overrightarrow{V_{u2}} = U_2 - V_{m2}cot\beta_2$ in above equation

$H_e = \frac{(U_2 - V_{m2}cot\beta_2)U_2}{g}; \; H_e = \frac{U_2^2}{g} - \frac{V_{m2}U_2cot\beta_2}{g} \quad --eqn\,1$

$Q = A_f\,V_f; \qquad Q = \pi D_2B_2V_{m2}; \qquad V_{m2} = \frac{Q}{\pi D_2B_2} - eqn2$

Substituting 2 in 1

$H_e = \frac{U_2^2}{g} - \left(\frac{Q}{\pi D_2B_2} * \frac{U_2cot\beta_2}{g}\right)$

$H_e = K_1 - K_2Q$ where $K_1 = \frac{U_2^2}{g}$ and $K_2 = \frac{U_2cot\beta_2}{\pi D_2B_2g}$

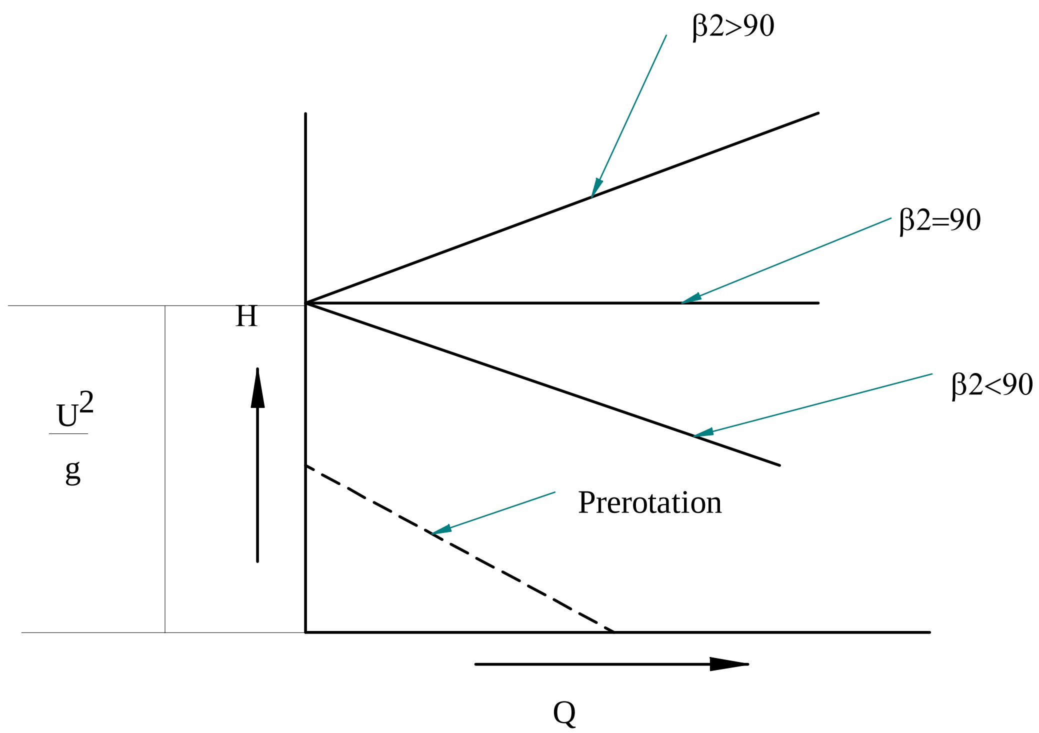
Above equation is called as H-Q characteristic equation

If $\beta_2 < 90^o$ K_2 becomes positive as cotβ_2 is positive. Therefore as Q increases H_e decreases

If $\beta_2 > 90^o$ K_2 becomes negative as cotβ_2 is negative. Therefore as Q increases H_e increases

If $\beta_2 = 90^o$ K_2 becomes zero as cotβ_2 is zero. Therefore as Q increases H_e remains constant

Above characteristics can be plotted as shown in fig



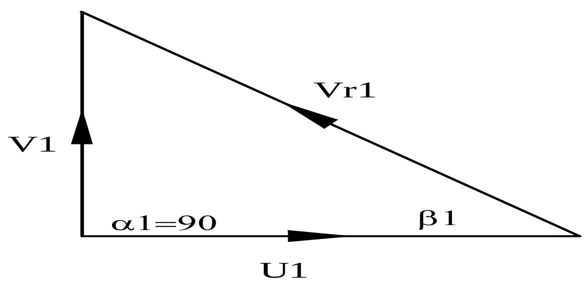
12. Derive head – capacity relationship for centrifugal pumps and explain the effect of discharge angle on it (4b,10, Dec16/Jan17)(4a,10,June/July14)
    Solution is same as above question 11
13. The internal and external diameters of the impeller of a centrifugal pump are 20cm and 40cm respectively. The pump is running at 1200rpm. The vane angle of impeller at inlet is 20°. The water enters the impeller radially and velocity of flow is constant. Calculate work done by the impeller /kg of water for the following two cases
    i) When vane angle at outlet is 90°
    ii) When vane angle at outlet is 100° (4b,10 Dec 16/Jan17)

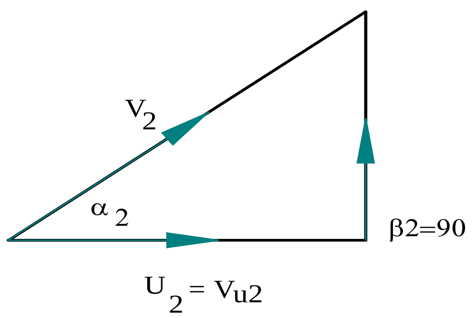
$D_1 = 0.2m;\ D_2 = 0.4m; N = 1200rpm; \beta_1 = 20^o;\ \beta_2 = 90^o;\ V_{f1} = V_{f2}; \frac{E}{m} = ?$

i) When $\beta_2 = 90^o$ ii) when $\beta_2 = 100^o$

When $\beta_2 = 90^o$

$U_1 = \frac{\pi D_1 N}{60}; U_1 = \frac{\pi * 0.2 * 1200}{60} = 12.56m/s; U_2 = \frac{\pi D_2 N}{60}; U_1 = \frac{\pi * 0.4 * 1200}{60} = 25.13m/s;$

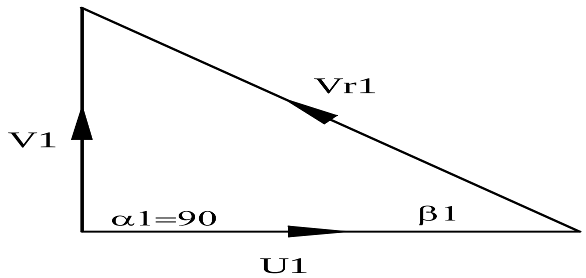


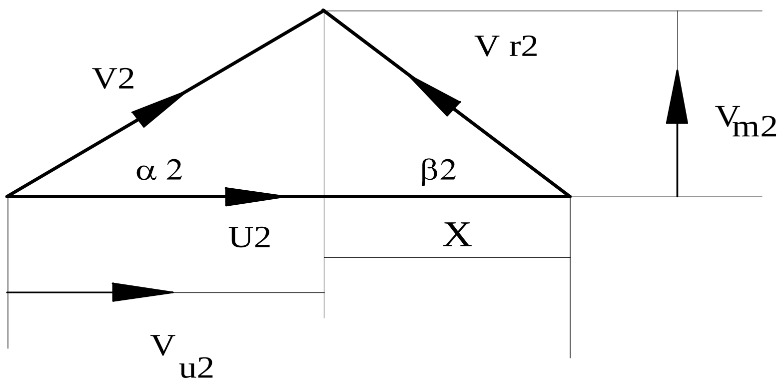


$\frac{E}{m} = \frac{(\overrightarrow{V_{u1}}\,U_1 - \overrightarrow{V_{u2}}U_2)}{g_c}$; $\overrightarrow{V_{u1}} = 0$; $\overrightarrow{V_{u2}} = U_2$; $\frac{E}{m} = \frac{-U_2U_2}{g_c}$; $\frac{E}{m} = \frac{-25.13*25.13}{1}$

$\frac{E}{m} = -631.51 \;\; J/kg$; −sign indicates the input to the pump

ii) When $\beta_2 = 110^o$





$\tan\beta_1 = \frac{V_{m1}}{U_1}$; $\tan 20 = \frac{V_{m1}}{12.56}$; $V_{m1} = 4.57m/s$; $V_{m2} = V_{m1} = 4.57m/s$;

$\overrightarrow{V_{u1}} = 0$; $\overrightarrow{V_{u2}} = U_2 - X$; $X = V_{m2}cot(180-100)$; $X = 4.57 * cot\, 80$; $X = 0.805m$

$\overrightarrow{V_{u2}} = 25.13 - 0.805$; $\overrightarrow{V_{u2}} = 24.325m/s$;

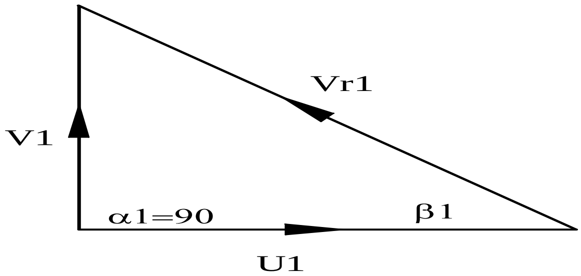
$\frac{E}{m} = \frac{(\overrightarrow{V_{u1}}\,U_1 - \overrightarrow{V_{u2}}U_2)}{g_c}$; $\overrightarrow{V_{u1}} = 0$; ; $\frac{E}{m} = \frac{-\overrightarrow{V_{u2}}U_2}{g_c}$; $\frac{E}{m} = \frac{-24.325*25.13}{1}$; $\frac{E}{m} = -611.28J/kg$

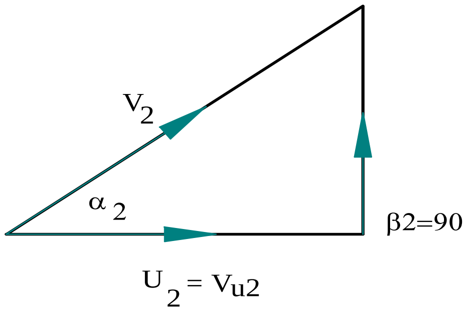
−sign indicates the input to the pump

14. A centrifugal pump delivers water against a head of 25m. The radial velocity of flow is 3.5m/s and is constant ., the flow rate of water is 0.05m^3/s. The blades are radial at tip and pump runs at 1500rpm. Determine i) Diameter at tip ii) width of blade at tip iii) inlet diffuser angle at impeller exit (4b,10, June/July16)
$H = 25m;\ V_{m2} = V_{m1} = 3.5m/s;\ Q = 0.05m^3/s;\ \beta_2 = 90^o;\ N = 1500rpm;$

Assuming Hydrulaic efficiency is 100%

$$-\frac{E}{\dot{m}} = gH; \frac{E}{\dot{m}} = -9.81 * 25\ ; \frac{E}{\dot{m}} = -245.25J/kg$$





$$\frac{E}{m} = \frac{(\overrightarrow{V_{u1}}\,U_1 - \overrightarrow{V_{u2}}U_2)}{g_c}; \qquad \overrightarrow{V_{u1}} = 0; \qquad \overrightarrow{V_{u2}} = U_2; \qquad \frac{E}{m} = \frac{-\,U_2U_2}{g_c};$$

$$-245.25 = -\frac{\mathrm{U}_2^2}{1}; \qquad U_2 = 15.66m/s$$

$$15.66 = \frac{\pi D_2 * 1500}{60}; \qquad D_2 = 0.199m$$

$$Q = \pi D_2 B_2 V_{m2};\ 0.05 = \ \pi * 0.199 * B_2 * 3.5;\ B_2 = 0.022m$$

UNIT 3

STEAM TURBINE

1. Define steam turbine. List the differences between Impulse and reaction steam turbines (5a. 08, Dec15/Jan16)(5a, 06, June/July18)
2. Briefly explain velocity compounding (5b. 08, Dec15/Jan16)
3. Derive the condition for maximum utilisation factor for impulse turbine (5a,10, June/July17)
4. Draw the inlet and exit velocity triangles for a single stage steam turbine. Further prove that maximum blade efficiency is given by
$$\eta_{max} = cos^2\alpha_1$$
$Assume\ V_{r1} = V_{r2}$ and $\beta_1 = \beta_2$ (7a. 10, June/July13) (5a. 08, June/July18,15CBCS)
5. Write a note on compounding of steam turbines and explain any two types of compounding with neat sketches (5b. 10, Dec16/Jan17) Show the velocity and pressure variations across the turbine (5a,10, Dec13/Jan14)
6. Define compounding .List different types of compounding. Explain any one method of compounding with neat sketch showing variations of pressure and velocity of steam (5a, 8,June/July18 15CBCS)
7. What is necessity of compounding of steam turbines and Discuss two types of compounding with neat sketches (5a. 10, Dec17/Jan18)what is compounding 5a. 08, Dec18/Jan19)
8. What is compounding or staging? Name the different compounding methods (5a,04, June/July14)
9. With neat sketch , explain the pressure –velocity compounding of steam turbine (5a,08, June/July16)(5b, 06, June/July18)
10. Show that the maximum blade efficiency $\eta_{blade\ max} = \frac{2cos^2\alpha_1}{1+cos^2\alpha_1}$ for a 50% reaction Parsons turbine (4a,10,Dec18/19) (6a,08,CBCS 15,Dec18/19)
11. For a 50% reaction turbine show that $\alpha_1 = \beta_2$and $\alpha_2 = \beta_1$, where α_1 and $\beta_1$are the inlet angles of fixed and moving blades , α_2 and $\beta_2$are the outlet angles of fixed and moving blades (5a, 08 , Dec 12)

Definition of Steam turbine(*)

Steam Turbine is is a power generating machine in which pressure energy of the steam is converted into mechanical energy due to dynamic action

Classification of steam turbine:

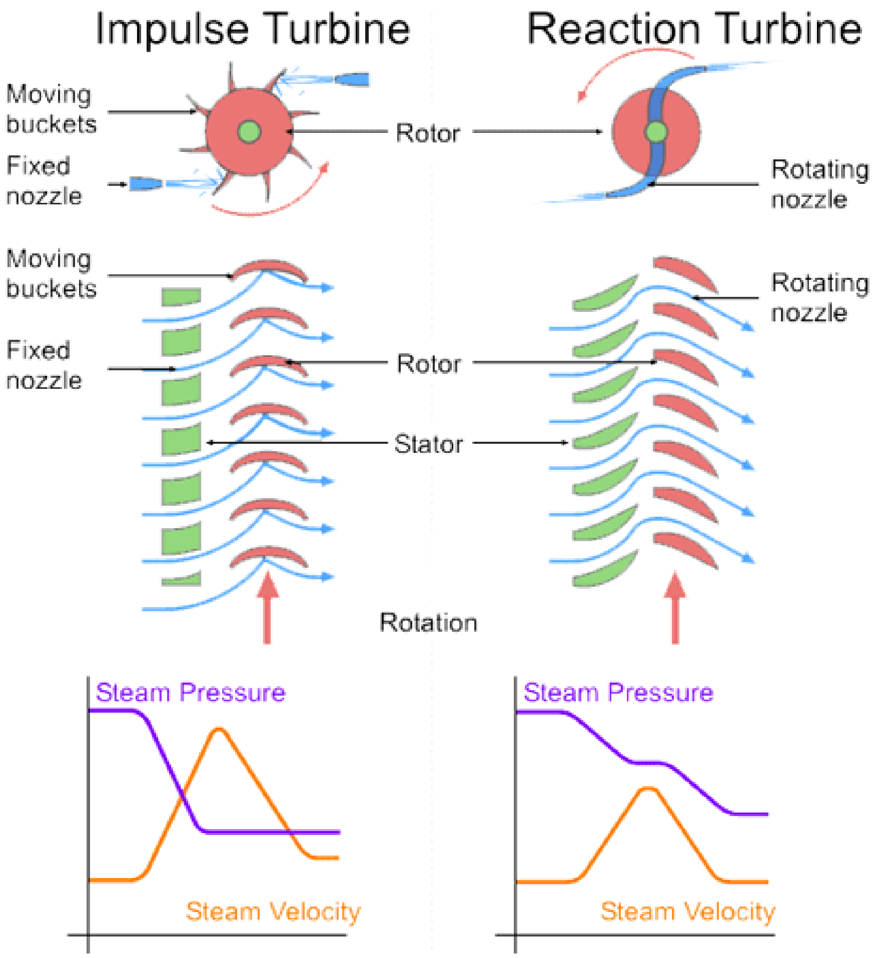
1. Based on working principle : a. Impulse Turbine b. Reaction turbine
2. Based on staging :a. Single stage b. Multi stage

Working Principle of Impulse turbine : The high pressure and high temperature steam generated in the steam generator is expanded in a steam nozzle or fixed blade passages and expanded steam with high velocity made to pass through the moving blades which is mounted on the shaft. In moving blades decrease in velocity and pressure of steam takes place which results in force impart on the moving blades . The resulting force rotates the rotor.

Steam turbine generally axial flow turbine $U_1 = U_2 = U$

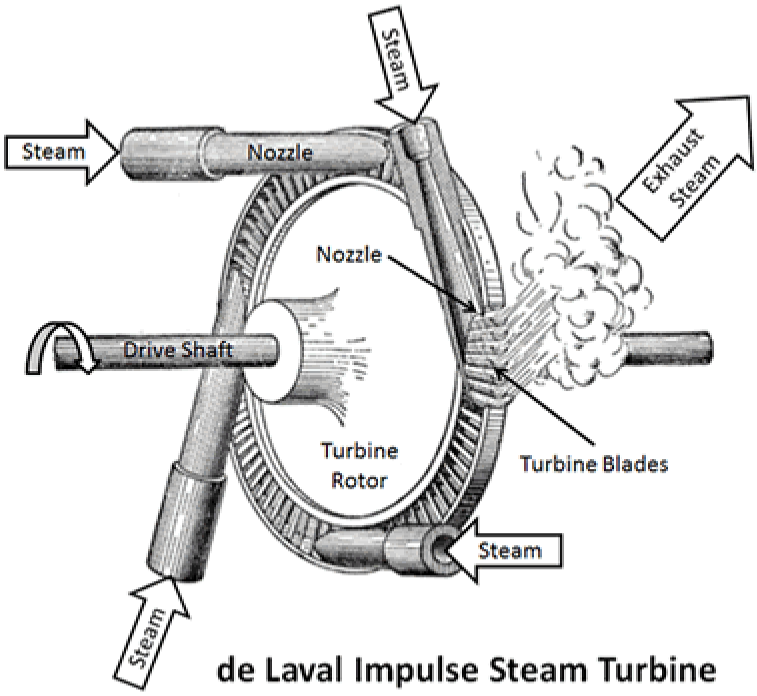
Differences between Impulse and Reaction turbines(***)

Sl no	Impulse	Reaction
1.	High Pressure and High temperature steam is expanded in set of nozzle and pressure energy is completely converted into kinetic energy and steam with high velocity directed to set of moving blades where kinetic energy absorbed in blades and converted into impulse This impulse set the blade into motion	High pressure stem is directly passed into the blades and pressure of the steam continuously drops and velocity increases . The steam leaving the blades will exert reactive force in the backward direction of flow and reactive force set the blade in motion
2.	Blades are symmetrical in shape	Blades are aerofoil in shape
3.	The pressure of steam remains constant when it flows through the moving blades	The pressure of steam continuously drops when it flows through the moving blades
4.	Impulsive force is converted into work	Reactive force is converted into work
5.	Low efficiency	Relatively high efficiency
6	High speed	Relatively low speed
7.	Compact	Bulky
8.	Less stages required	More stages required for the same power generation
9.	Used for small power generation	Used for medium and large power generation

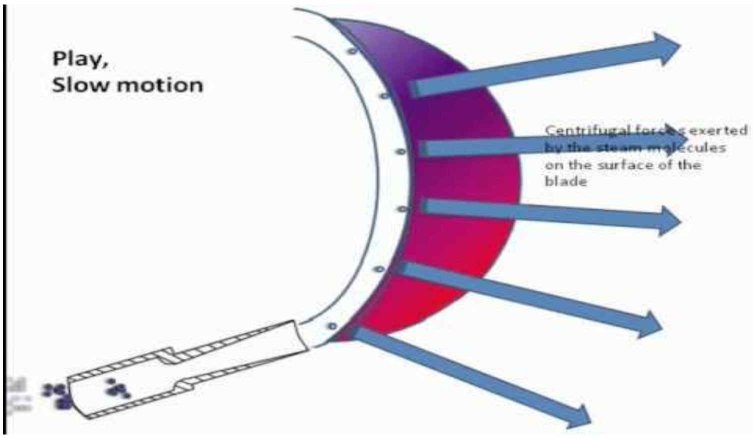


Single Stage Impulse Turbine : De Laval Turbine

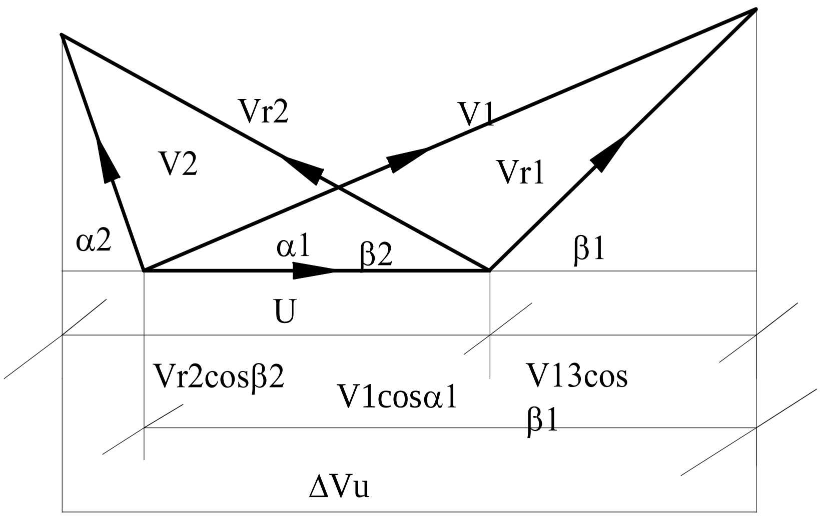
Working Principle: In a single stage impulse turbine high pressure steam enters set of nozzle (part of stator or casing) and expands completely in nozzle which results in conversion of pressure energy to kinetic energy. The steam with Kinetic energy made to flow through moving blades mounted on the rotor wherein change in velocity takes place which results in change in momentum takes place. This results in the rotation of rotor. There is no pressure drop as the stem flow through the passages of moving blades. Hence the relative velocity between steam and moving blades remains constant over the blades. Hence the degree of reaction is zero



de Laval Impulse Steam Turbine



Analysis on single stage Impulse turbine: (***)



Forces on the blade:

Tangential force: (***)

$F_t = \frac{m(\overrightarrow{V_{u1}} - \overrightarrow{V_{u2}})}{g_c}$ or $F_t = \frac{m(\overrightarrow{V_{u1}} + \overleftarrow{V_{u2}})}{g_c}$

Axial thrust:

$F_a = \frac{m(V_{f1} - V_{f2})}{g_c}$ Newton

Energy Per unit mass

$\frac{E}{m} = \frac{m(\overrightarrow{V_{u1}} - \overrightarrow{V_{u2}})U}{g_c}$ J/kg Or $\frac{E}{m} = \frac{m(\overrightarrow{V_{u1}} + \overleftarrow{V_{u2}})U}{g_c}$

Power

$E = \frac{m(\overrightarrow{V_{u1}} - \overrightarrow{V_{u2}})U}{g_c}$ Watts

Blade efficiency(***): It is defined as the ratio of workdone per kg of steam by the rotor to the energy available at the inlet per kg of steam

$$\eta_b = \frac{\text{workdone per kg of steam by the rotor}}{Energy\ available\ at\ the\ inlet\ per\ kg\ of\ stam}$$

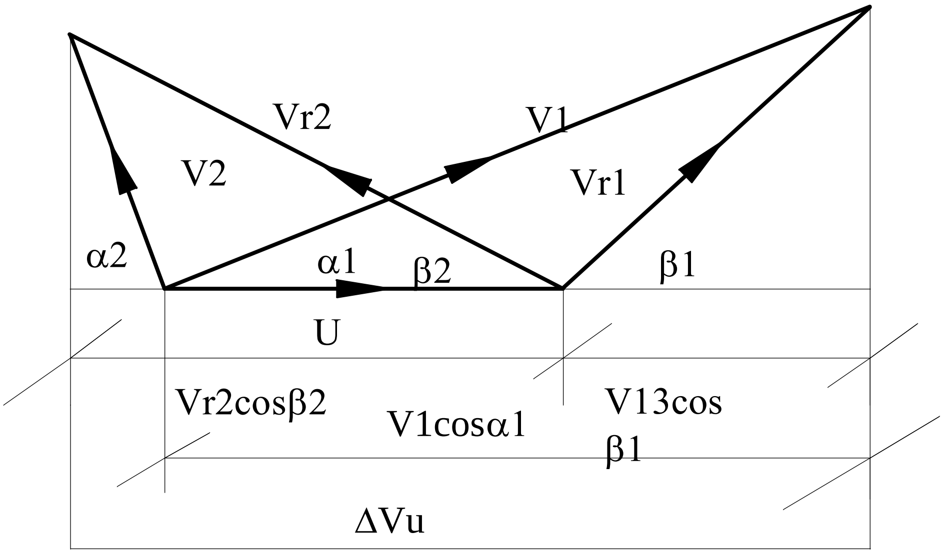
$\eta_b = \frac{\frac{E}{m}}{\frac{V_1^2}{2g_c}}$; $\eta_b = \frac{\frac{m(\overrightarrow{V_{u1}} - \overrightarrow{V_{u2}})U}{g_c}}{\frac{V_1^2}{2g_c}}$; $\eta_b = \frac{2(\overrightarrow{V_{u1}} - \overrightarrow{V_{u2}})U}{V_1^2}$ Also $\eta_b = \frac{2(\overrightarrow{V_{u1}} + \overleftarrow{V_{u2}})U}{V_1^2}$

Stage Efficiency:(***)

It is the ratio of work done per kg of steam by the rotor to the isentropic enthalpy change per kg of steam in the nozzle

$$\eta_{stage} = \frac{\frac{E}{m}}{\Delta h_{0s}} = \frac{(\overrightarrow{V_{u1}} + \overleftarrow{V_{u2}})U}{\Delta h_{0s}} = \frac{(\overrightarrow{V_{u1}} + \overleftarrow{V_{u2}})U}{\frac{V_1^2}{2}} * \frac{\frac{V_1^2}{2}}{\Delta h_{0s}} = \eta_b * \eta_{nozzle}$$

Condition for maximum Efficiency (******)



$\frac{E}{m} = (\overrightarrow{V_{u1}} + \overleftarrow{V_{u2}})U$; $\frac{E}{m} = U\Delta V_u$;

From velocity triangle, $\Delta V_u = V_{r1}cos\beta_1 + V_{r2}cos\beta_2$

Hence, $\frac{E}{m} = \frac{U\,(V_{r1}Cos\beta_1 + V_{r2}Cos\beta_2)}{g_c}$; $\frac{E}{m} = \frac{UV_{r1}Cos\beta_1\left(1+\frac{V_{r2}Cos\beta_2}{V_{r1}Cos\beta_1}\right)}{g_c}$

$\frac{V_{r2}}{V_{r1}} = C_b$ (blade friction coefficient) ; $\frac{Cos\beta_2}{Cos\beta_1} = K$, constant

$\frac{E}{m} = \frac{UV_{r1}Cos\beta_1(1+C_bK)}{g_c}$; But from Inlet velocity triangle, $V_{r1}Cos\beta_1 = V_1Cos\alpha_1 - U$

$$\frac{E}{m} = \frac{U\,(V_1Cos\alpha_1 - U)\,(1 + C_bK)}{g_c}$$

$\eta_b = \frac{\frac{E}{m}}{\frac{V_1^2}{2g_c}}$; $\eta_b = \frac{\frac{U\,(V_1Cos\alpha_1 - U)\,(1+C_bK)}{g_c}}{\frac{V_1^2}{2g_c}}$; $\eta_b = \frac{2U\,(V_1Cos\alpha_1 - U)\,(1+C_bK)}{V_1^2}$

$\eta_b = 2\frac{U}{V_1}x\frac{(V_1Cos\alpha_1 - U)}{V_1}x(1 + C_bK)$; $\eta_b = 2\phi\,(cos\alpha_1 - \emptyset)(1 + C_bK)$ where $\phi = \frac{U}{V_1}$

For max efficiency

$\frac{\partial\eta}{\partial\emptyset} = 0$; $\frac{\partial}{\partial\emptyset}\left(2\emptyset\,(cos\alpha_1 - \emptyset)(1 + C_bK)\right) = 0$; $\frac{\partial}{\partial\emptyset}\left(2\emptyset\,(cos\alpha_1 - \emptyset)\right) = 0$

$\frac{\partial}{\partial\emptyset}(\emptyset\,cos\alpha_1 - \Phi^2) = 0$; $cos\alpha_1 - 2\emptyset = 0$; $\emptyset = \frac{cos\alpha_1}{2}$

Substituting $\emptyset = \frac{cos\alpha_1}{2}$ in $\eta_b = 2\phi\,(cos\alpha_1 - \emptyset)(1 + C_bK)$ will give max efficiency

$$\eta_b = 2\frac{cos\alpha_1}{2}\left(cos\alpha_1 - \frac{cos\alpha_1}{2}\right)(1 + C_bK)$$

$$\eta_b = \frac{cos^2\alpha_1}{2}(1 + C_b K)$$

If rotor blade angles are equiangular

If $V_{r1} = V_{r2}$; $\beta_1 = \beta_2$

$\eta_{bmax} = \frac{cos^2\alpha_1}{2} * 2;$ $\eta_{bmax} = cos^2\alpha_1$

Necessity of compounding:(*****)

Single stage impulse turbine operates at very high speed.

High speeds are undesirable for the following reasons

- High speed causes high blade tip stresses due to centrifugal forces acting at the tip of blade
- Large losses due to disc friction
- Low efficiencies due to large exit steam velocity in the turbine
- Gear trains with large efficiencies and high speed ratios must be used to match between the turbine speed and the driven component speed since most driven machines run at speeds around a few thousand RPM at most

Reasonable blade tip speeds are obtained in impulse turbines by the compounding stage.

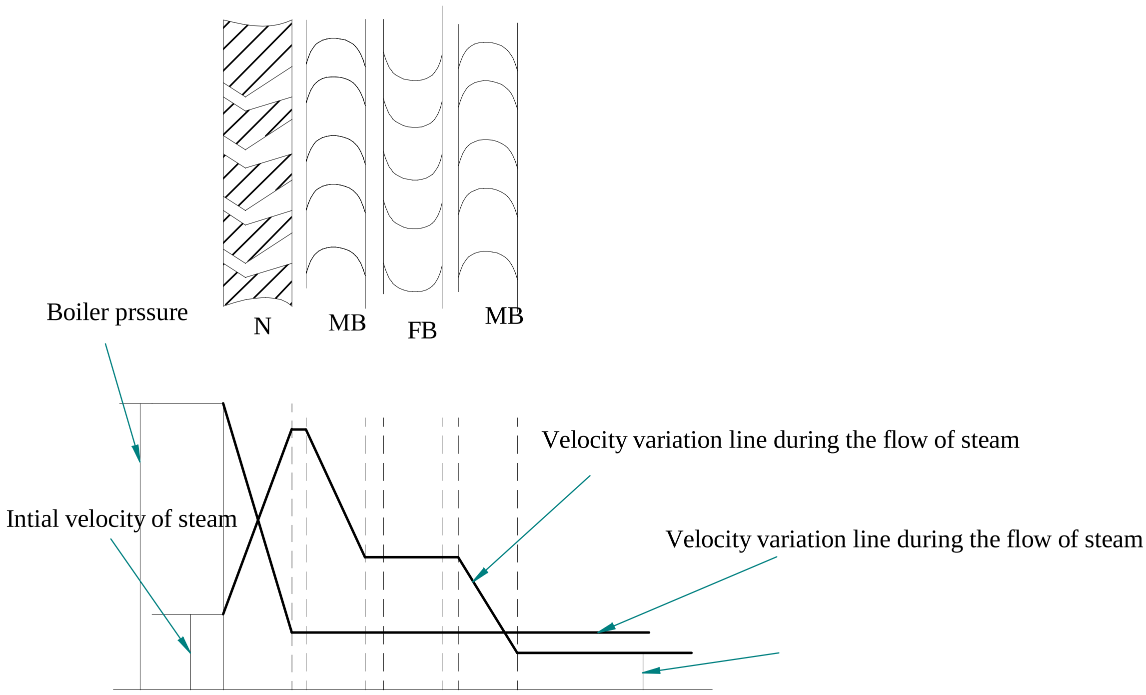
Definition of Compounding(******)

Compounding is the method of reducing blade speed for a given overall pressure drop. Multiple rotors are mounted on common shaft in series and velocity is obsorbed in stages as it flows over the blades

Types of Compounding(*******)

1. Velocity compounded turbine
2. Pressure compounded turbine
3. Velocity and pressure compounded turbine

Velocity compounded turbine: compounding involves in which the whole pressure drop occurs in one set of stationary blades or nozzles where as all the kinetic energy is absorbed in usually two, three or even four rows of moving blades with a row of stationary blades between every pair of them. The total energy of the stream can be absorbed by all the rows in succession until the kinetic energy at the end of last row becomes negligible.



When the steam flows through the nozzle steam expands nearly to atmospheric pressure and velocity increases.

While steam flows through moving blades velocity decreases while steam pressure remains constant

While steam flows through the stationary (fixed) blades both pressure and velocity remains constant

Both stationary blades and moving blades are symmetrical

The first row of the moving blades absorbs most of jet energy while latter absorbs comparatively less

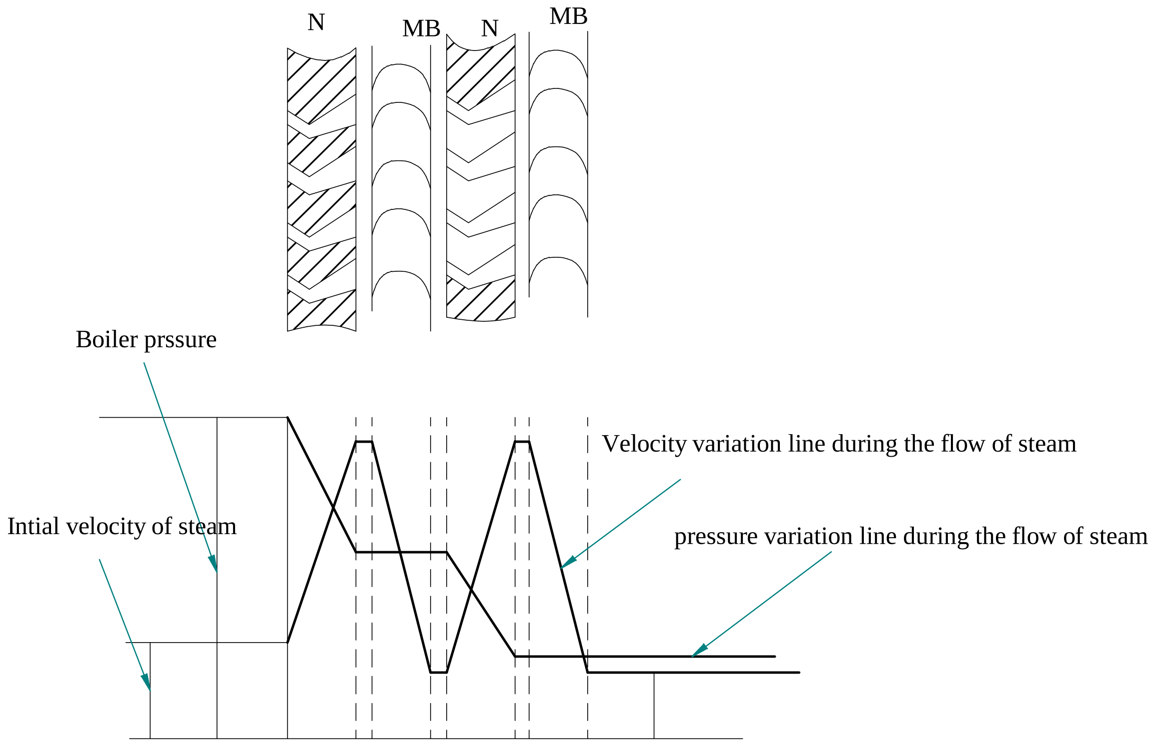
Advantages:

1. Maximum possible pressure energy is converted into kinetic energy in nozzles of first stage and there is no pressure drop in stages and hence the stress in the turbine is less
2. Fewer stages are sufficient due large kinetic energy drop compared to pressure compounding
3. Compact compared to pressure compounding

Disadvantage:

1. The friction losses are more due to high velocity of steam

Pressure Compounding



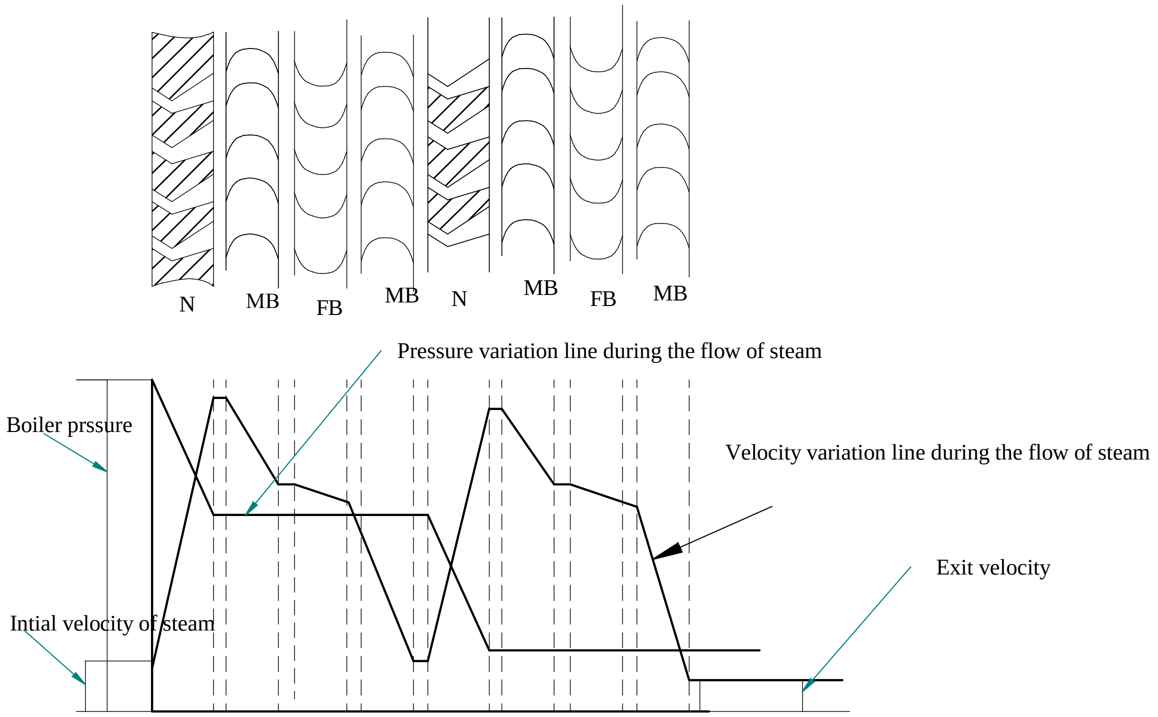
It is equivalent to a number of simple impulse stages put together. It is the type of compounding in which pressure drop occur in each stator row. Between the two moving blade rows there is a row of nozzles are often referred to as diaphragms.

In the row of nozzle pressure decreases and velocity increases. In rows of moving blade velocity decreases while pressure remains constant.

Advantages: High efficiency because very high velocities are avoided.

Disadvantages: Leakage loss is higher compared to velocity compounding

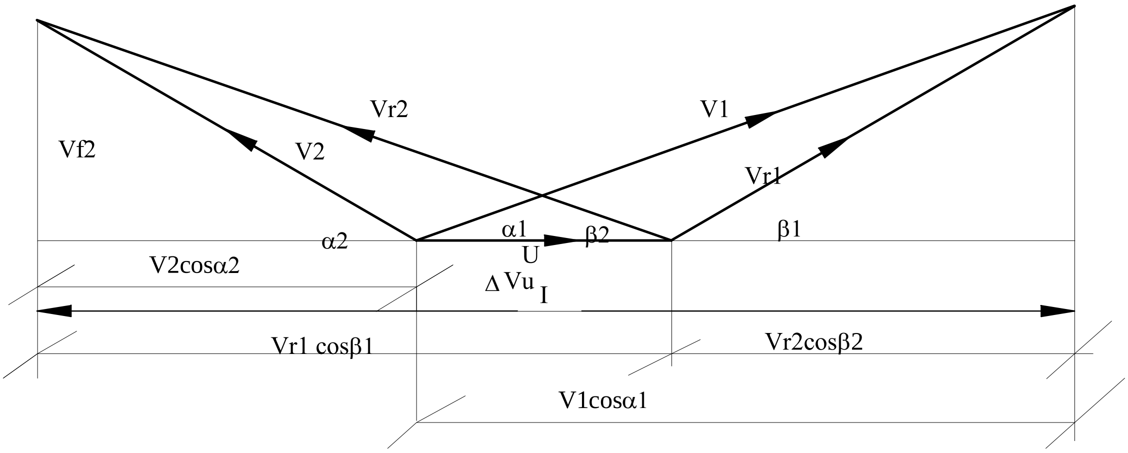
Velocity and pressure compounded steam turbine

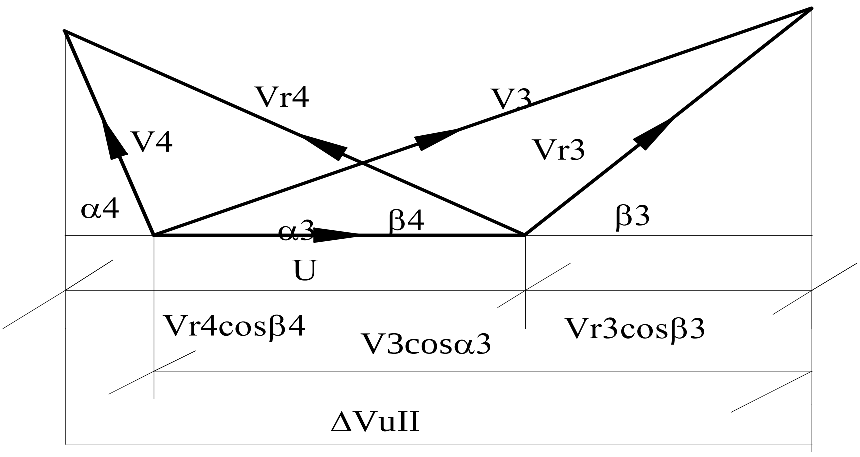


In this method high rotor speeds are reduced without sacrificing the efficiency or the output. Pressure drop from boiler pressure to the condenser pressure occurs in two stages

First and second stage taken separately are identical to a velocity compounding consists of set of nozzle and rows of moving blade fixed to the shaft and rows of fixed blades to casing in which pressure decreases and velocity increases in the nozzle. While moving through the moving blades velocity decreases while pressure remains constant where as while the steam flows through the fixed blade pressure and velocity remains constant

Analysis of maximum efficiency for velocity compounded steam turbine





$\left(\frac{E}{m}\right)_I = \frac{U(\overrightarrow{V_{u1}}+\overleftarrow{V_{u2}})}{g_c} = \frac{U*(\Delta V_u)_I}{g_c}$ ----------A

From velocity triangle for I stage, $(\Delta V_u)_I = (V_{r1}Cos\beta_1 + V_{r2}Cos\beta_2)$

Hence $\left(\frac{E}{m}\right)_I = \frac{U\ (V_{r1}Cos\beta_1+V_{r2}Cos\beta_2)}{g_c}$

If $V_{r1} = V_{r2}$ and $\beta_1 = \beta_2$, $\left(\frac{E}{m}\right)_I = \frac{U*2V_{r1}Cos\beta_1}{g_c}$

But from Inlet velocity triangle of Ist stage, $V_{r1}Cos\beta_1 = V_1Cos\alpha_1 - U$

$\left(\frac{E}{m}\right)_I = \frac{U*2(V_1Cos\alpha_1-U)}{g_c}$ ----------------1

$\left(\frac{E}{m}\right)_{II} = \frac{U(\overrightarrow{V_{u3}}+\overleftarrow{V_{u4}})}{g_c} = \frac{U*(\Delta V_u)_{II}}{g_c}$

From triangle for II stage, $(\Delta V_u)_{Ii} = (V_{r3}Cos\beta_3 + V_{r4}Cos\beta_4)$

If $V_{r3} = V_{r4}$ and $\beta_3 = \beta_4$, $\left(\frac{E}{m}\right)_I = \frac{U*2V_{r3}Cos\beta_3}{g_c}$

But from Inlet velocity triangle of II stage, $V_{r3}Cos\beta_3 = V_3Cos\alpha_3 - U$

$$\left(\frac{E}{m}\right)_{II} = \frac{2U(V_3Cos\alpha_3 - U)}{g_c}$$

If $V_3 = V_2$ *and* $\alpha_3 = \alpha_2$, $\left(\frac{E}{m}\right)_{II} = \frac{2U(V_2Cos\alpha_2 - U)}{g_c}$

But from outlet velocity triangle of Ist stage, $V_2Cos\alpha_2 = V_{r2}Cos\beta_2 - U$

$V_{r2} = V_{r2}$ and $\beta_2 = \beta_1$; Hence, $V_2Cos\alpha_2 = V_{r1}Cos\beta_1 - U$

Substituting above relation in $\left(\frac{E}{m}\right)_{II}$

$\left(\frac{E}{m}\right)_{II} = \frac{2U(V_{r1}Cos\beta_1 - U - U)}{g_c}$; $\left(\frac{E}{m}\right)_{II} = \frac{2U(V_{r1}Cos\beta_1 - 2U)}{g_c}$

But from Inlet velocity triangle of Ist stage, $V_{r1}Cos\beta_1 = V_1Cos\alpha_1 - U$

$\left(\frac{E}{m}\right)_{II} = \frac{2U(V_1Cos\alpha_1 - U - 2U)}{g_c}$; $\left(\frac{E}{m}\right)_{II} = \frac{2U(V_1Cos\alpha_1 - 3U)}{g_c}$

$\left(\frac{E}{m}\right)_T = \left(\frac{E}{m}\right)_I + \left(\frac{E}{m}\right)_{II}$;

$\left(\frac{E}{m}\right)_T = \frac{2U(V_1Cos\alpha_1 - U)}{g_c} + \frac{2U(V_1Cos\alpha_1 - 3U)}{g_c}$; $\left(\frac{E}{m}\right)_T = \frac{2U(V_1Cos\alpha_1 - U + V_1Cos\alpha_1 - 3U)}{g_c}$

$\left(\frac{E}{m}\right)_T = \frac{2U(2V_1Cos\alpha_1 - 4U)}{g_c}$; $\left(\frac{E}{m}\right)_T = \frac{4U(V_1Cos\alpha_1 - 2U)}{g_c}$ for 2 stages

In general form

$\left(\frac{E}{m}\right)_T = \frac{2nU(V_1Cos\alpha_1 - nU)}{g_c}$ for n stages

Blade efficiency

$\eta_b = \frac{\left(\frac{E}{m}\right)_T}{\frac{V_1^2}{2g_c}}$; $\eta_b = \frac{\frac{4U(V_1Cos\alpha_1 - 2U)}{g_c}}{\frac{V_1^2}{2g_c}}$; $\eta_b = \frac{8U(V_1 cos\alpha_1 - 2U)}{V_1^2}$

$\eta_b = \frac{8U}{V_1}\, x\, \frac{(V_1 cos\alpha_1 - 2U)}{V_1}$; $\eta_b = 8\Phi\, x\, (cos\alpha_1 - 2\Phi)$

For max efficiency

$\frac{\partial \eta}{\partial \phi} = 0$; $\frac{\partial}{\partial \phi}\left(8\Phi\, x\, (cos\alpha_1 - 2\Phi)\right) = 0$; $\frac{\partial}{\partial \phi}(\Phi cos\alpha_1 - 2\Phi^2) = 0$

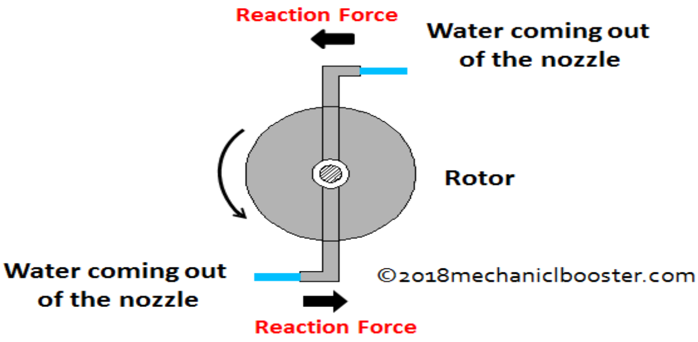
$cos\alpha_1 - 4\Phi = 0$; $\Phi = \frac{cos\alpha_1}{4}$ condition for max efficiency for 2 stages

Substituting $\Phi = \frac{cos\alpha_1}{4}$ in η_b $= 8\Phi\, x\, (cos\alpha_1 - 2\Phi)$ will give max efficiency

$\eta_{bmax} = 8\frac{\cos\alpha_1}{4}\left(\cos\alpha_1 - 2\frac{\cos\alpha_1}{4}\right)$; $\eta_{bmax} = cos^2\alpha_1$

Reaction Turbine

In the case of reaction turbine, the moving blades of a turbine are shaped in such a way that the steam expands and drops in pressure as it passes through them. As a result of pressure decrease in the moving blade, a reaction force will be produced. This force will make the



blades to rotate.

Impulse Reaction turbine

In the impulse reaction turbine, power is generated by the combination of impulse action (Impulse force) and reaction (Reactive force) by expanding the steam in both fixed blades and moving blades. In fixed blades as velocity increases as steam expands where as in moving blades both pressure and velocity decreases. In other words in Impulse reaction turbine both pressure energy is converted into work

Degree of Reaction

Degree of reaction defined as ratio of static enthalpy drop to stagnation enthalpy drop in the stage

$$R = \frac{\frac{U_1^2-U_2^2}{2g_c} - \frac{V_{r1}^2-V_{r2}^2}{2g_c}}{\frac{V_1^2-V_2^2}{2g_c} + \frac{U_1^2-U_2^2}{2g_c} - \frac{V_{r1}^2-V_{r2}^2}{2g_c}}; \qquad \frac{E}{m} = \frac{V_1^2-V_2^2}{2g_c} + \frac{U_1^2-U_2^2}{2g_c} - \frac{V_{r1}^2-V_{r2}^2}{2g_c}$$

For Axial flow turbine U_1=U_2

$$R = \frac{-\frac{V_{r1}^2-V_{r2}^2}{2g_c}}{\frac{E}{m}}; \qquad R = \frac{V_{r2}^2-V_{r1}^2}{2\frac{E}{m}}$$

Also

$$R = \frac{(\Delta h_0)_{moving\ blade}}{(\Delta h_0)_{fixed\ blade} + (\Delta h_0)_{moving\ blade}}$$

$(\Delta h_0)_{fixed\ blade}=\dfrac{V_1^2-\varphi V_2^2}{2\ \eta_n}$; $(\Delta h_0)_{moving\ blade}=\dfrac{V_{r2}^2-\varphi V_{r1}^2}{2\ \eta_n}$

φ= carry over factor ; η_n= nozzle efficiency

Derive an expression for Degree of Reaction for axial reaction turbine(***)

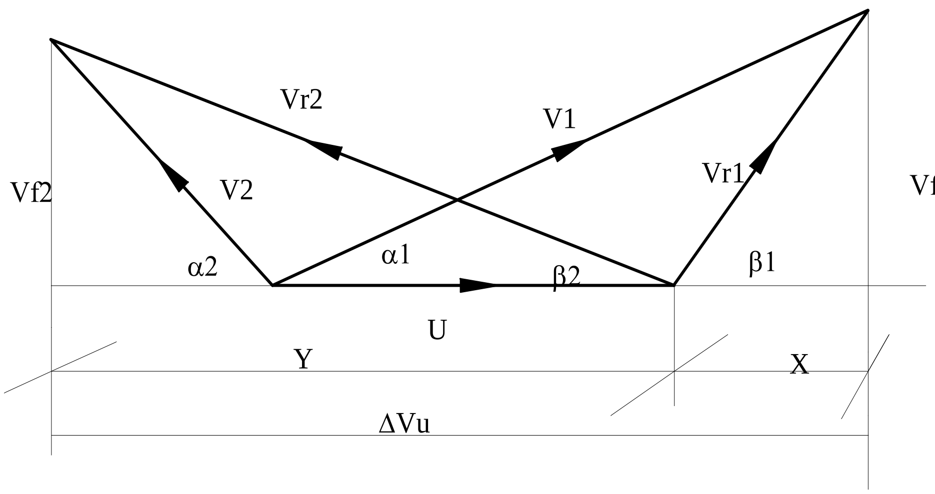
$$R=\frac{\frac{U_1^2-U_2^2}{2g_c}-\frac{V_{r1}^2-V_{r2}^2}{2g_c}}{\frac{V_1^2-V_2^2}{2g_c}+\frac{U_1^2-U_2^2}{2g_c}-\frac{V_{r1}^2-V_{r2}^2}{2g_c}};\qquad \frac{E}{m}=\frac{V_1^2-V_2^2}{2g_c}+\frac{U_1^2-U_2^2}{2g_c}-\frac{V_{r1}^2-V_{r2}^2}{2g_c}$$

For Axial flow turbine $U_1=U_2$

$$R=\frac{-\frac{V_{r1}^2-V_{r2}^2}{2g_c}}{\frac{E}{m}}$$

$$R=\frac{V_{r2}^2-V_{r1}^2}{2g_c\frac{E}{m}}\text{ -------------A}$$

$$\left(\frac{E}{m}\right)=U(\Delta V_u)$$



$\Delta V_u = X+Y$

$\frac{V_f}{X}=tan\beta_1$; $\frac{V_f}{Y}=tan\ \beta_2$

$X=V_f\ cot\beta_1$; $Y=V_f\ cot\beta_2$

$\Delta V_u=V_f\ cot\beta_1+V_f\ cot\beta_2$

$\left(\frac{E}{m}\right)=\frac{U(V_f\ cot\beta_1+V_f\ cot\beta_2)}{g_c}$; $\qquad \left(\frac{E}{m}\right)=\frac{U(\ cot\beta_1+\ cot\beta_2)\ V_f}{g_c}$ --------------1

$\frac{V_f}{V_{r1}}$ = sin β_1 ; $\quad V_{r1} = V_f \text{cosec } \beta_1 \qquad \frac{V_f}{V_{r2}}$ = sin $\beta_2 \quad V_{r2} = V_f \text{cosec } \beta_2$

Substituting 1 and 2 in equation A

$$R = \frac{(V_f cosec\ \beta_2)^2 - (V_f cosec\ \beta_1)^2}{2\,\mathrm{U}(\,cot\beta_1 + cot\beta_2)\,V_f};$$

$$R = \frac{\mathrm{V_f^2}(\mathrm{cosec^2\beta_2 - cosec^2\beta_1})}{2\,\mathrm{U}(\,\mathrm{cot\beta_1 + cot\beta_2})\,\mathrm{V_f}}; \qquad R = \frac{V_f\left(1+cot^2\beta_2 - (1+cot^2\beta_1)\right)}{2\,\mathrm{U}(\,cot\beta_1 + cot\beta_2)}; \qquad R = \frac{V_f(\mathrm{cot^2}\beta_2 - \mathrm{cot^2}\beta_2)}{2\,\mathrm{U}(\,cot\beta_1 + cot\beta_2)}$$

$$R = \frac{V_f(\,cot\beta_1 + cot\beta_2)(\,cot\beta_2 - cot\beta_1)}{2\,U(\,cot\beta_1 + cot\beta_2)}; \qquad R = \frac{V_f(\,cot\beta_2 - cot\beta_1)}{2\,\mathrm{U}}; \quad R = \frac{V_f}{2\,\mathrm{U}}(\,cot\beta_2 - \,cot\beta_1)$$

$$R = \frac{V_f}{2\,\mathrm{U}}\left(\frac{1}{tan\beta_2} - \frac{1}{tan\beta_1}\right); \qquad R = \frac{V_f}{2\,\mathrm{U}}\left(\frac{tan\beta_1 - tan\beta_2}{tan\beta_1.tan\beta_2}\right)$$

Condition for 50% reaction turbine(******)

$$R = \frac{V_f}{2\,\mathrm{U}}(\,cot\beta_2 - \,cot\beta_1)$$

For 50% reaction $R = \frac{1}{2}$

$\frac{1}{2} = \frac{V_f}{2\,\mathrm{U}}(\,cot\beta_2 - \,cot\beta_1); \qquad U \; = \; V_f(\,cot\beta_2 - \,cot\beta_1)$ --------------1

From Inlet velocity triangle

$R = \mathrm{V_{u1}} - \mathrm{X}$

But $\quad tan\beta_1 = \frac{V_f}{X}$; $\quad X = V_f\ cot\beta_1$

$tan\alpha_1 = \frac{V_f}{V_{u1}}$; $\; V_{u1} = V_f\ cot\alpha_1$

Hence, U $= V_f\ cot\alpha_1 - V_f\ cot\beta_1$

$U \; = V_f(cot\alpha_1 - \ cot\beta_1)$ ------------2

1=2; $\quad V_f(\,cot\beta_2 - \,cot\beta_1) = V_f(cot\alpha_1 - \,cot\beta_1); \quad cot\beta_2 = \,cot\alpha_1; \qquad \underline{\boldsymbol{\beta_2 = \alpha_1}}$

From outlet velocity triangle

$U \; = \; Y - \overrightarrow{V_{u1}}$

But $\quad tan\beta_2 = \frac{V_f}{Y}$; $\quad Y = V_f\ cot\beta_2; \qquad tan\alpha_2 = \frac{V_f}{V_{u2}}$; $\; \overleftarrow{V_{u2}} = V_f\ cot\alpha_2$

Hence, $U \; = V_f\ cot\beta_2 - V_f\ cot\alpha_2; \qquad U \; = V_f(cot\beta_2 - \ cot\alpha_2)$ ------------3

1=3; $\qquad V_f(\,cot\beta_2 - \,cot\beta_1) = V_f(cot\beta_2 - \,cot\alpha_2); \quad cot\beta_1 = \,cot\alpha_2; \quad \underline{\boldsymbol{\beta_1 = \alpha_2}}$

Hence in 50% reaction turbine

$\boldsymbol{\beta_2 = \alpha_1\ ;\quad \beta_1 = \alpha_2}$

Hence both Inlet and Outlet triangle in 50% Reaction turbine are symmetrical

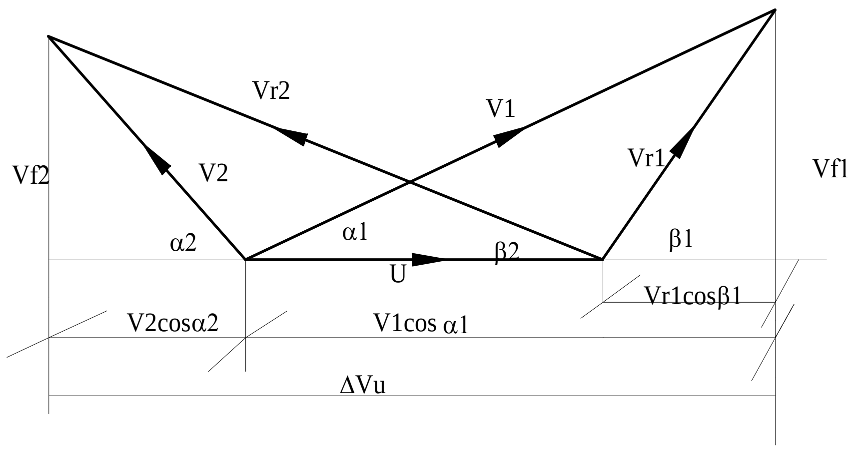
Hence $\mathbf{V_{r1} = V_2;\ V_{r2} = V_1}$

Efficiency of 50% reaction turbine(***)

$$\eta = \frac{\frac{V_1^2 - V_2^2}{2g_c} + \frac{U_1^2 - U_2^2}{2g_c} - \frac{V_{r1}^2 - V_{r2}^2}{2g_c}}{\frac{V_1^2}{2g_c} + \frac{U_1^2 - U_2^2}{2g_c} - \frac{V_{r1}^2 - V_{r2}^2}{2g_c}} = \frac{\frac{E}{m}}{\frac{V_1^2}{2g_c} - \frac{V_{r1}^2 - V_{r2}^2}{2g_c}}$$

$$\eta = \frac{\frac{E}{m}}{\frac{V_1^2}{2g_c} - \frac{V_{r1}^2 - V_{r2}^2}{2g_c}} \quad --eqn$$ A as $U_1 = U_2 = U$ for axial flow turbine

$$\frac{E}{m} = \frac{U(\overrightarrow{V_{u1}} + \overleftarrow{V_{u2}})}{g_c}$$



$$\overrightarrow{V_{u1}} + \overleftarrow{V_{u2}} = V_1 cos\alpha_1 + V_2 cos\alpha_2$$

$$\overrightarrow{V_{u1}} + \overleftarrow{V_{u2}} = V_1 cos\alpha_1 + V_{r1} cos\beta_1$$

But $V_{r1} cos\beta_1 = V_1 cos\alpha_1 - U$

$$\overrightarrow{V_{u1}} + \overleftarrow{V_{u2}} = V_1 cos\alpha_1 + V_1 \cos \alpha_1 - U$$

$$\overrightarrow{V_{u1}} + \overleftarrow{V_{u2}} = 2V_1 cos\alpha_1 - U$$

$$\frac{E}{m} = \frac{U(2V_1 cos\alpha_1 - U)}{g_c}$$

$$\frac{E}{m} = \frac{2V_1 U cos\alpha_1 - U^2}{g_c}$$

$$\frac{E}{m} = \frac{V_1^2 \left(\frac{2U cos\alpha_1}{V_1} - \frac{U^2}{V_1^2}\right)}{g_c}$$

But $\frac{U}{V_1}$ speed ratio Φ

$\frac{E}{m} = \frac{V_1^2 \left(2\Phi cos\alpha_1 - \Phi^2\right)}{g_c}$ ---------------B

$\frac{V_1^2}{2g_c} - \frac{V_{r1}^2 - V_{r2}^2}{2g_c}$; $\frac{V_1^2}{2g_c} + \frac{V_{r2}^2}{2g_c} - \frac{V_{r1}^2}{2g_c}$

$V_{r2} = V_1$ (50%R)

$\frac{V_1^2}{2g_c} + \frac{V_1^2}{2g_c} - \frac{V_{r1}^2}{2g_c}$; $\frac{V_1^2}{g_c} - \frac{V_{r1}^2}{2g_c} - - - 1$

From outlet velocity triangle and applying cosine rule $V_{r1}^2 = V_1^2 + U^2 - 2V_1 U cos\alpha_1$

Hence, eqn 1 becomes $\frac{V_1^2}{g_c} - \frac{V_1^2 + U^2 - 2V_1 U cos\alpha_1}{2g_c}$;

$\frac{V_1^2}{2g_c} + \frac{V_{r2}^2}{2g_c} - \frac{V_{r1}^2}{2g_c} = \frac{V_1^2}{g_c} - \frac{V_1^2 + U^2 - 2V_1 U cos\alpha_1}{2g_c}$; $\frac{V_1^2}{2g_c} + \frac{V_{r2}^2}{2g_c} - \frac{V_{r1}^2}{2g_c} = \frac{V_1^2}{g_c} - \frac{V_1^2}{2g_c} - \frac{U^2}{2g_c} + \frac{2V_1 U cos\alpha_1}{2g_c}$

$\frac{V_1^2}{2g_c} + \frac{V_{r2}^2}{2g_c} - \frac{V_{r1}^2}{2g_c} = \frac{V_1^2}{2} - \frac{U^2}{2} + \frac{2V_1 U cos\alpha_1}{2}$

$$\frac{V_1^2}{2g_c} + \frac{V_{r2}^2}{2g_c} - \frac{V_{r1}^2}{2} = \frac{V_1^2}{2g_c}\left(1 - \frac{U^2}{V_1^2} + \frac{2V_1 U cos\alpha_1}{V_1^2}\right)$$

$\frac{V_1^2}{2g_c} + \frac{V_{r2}^2}{2g_c} - \frac{V_{r1}^2}{2g_c} = \frac{V_1^2}{2g_c}(1 - \Phi^2 + 2\Phi cos\alpha_1)$--------------C

$\eta_b = \frac{\frac{V_1^2 \left(2\Phi cos\alpha_1 - \Phi^2\right)}{g_c}}{\frac{V_1^2}{2g_c}(1 - \Phi^2 + 2\Phi cos\alpha_1)}$; $\eta_b = \frac{2\left(2\Phi cos\alpha_1 - \Phi^2\right)}{(1 - \Phi^2 + 2\Phi cos\alpha_1)}$

$\eta_b = \frac{2\left(2\Phi cos\alpha_1 - \Phi^2 + 1 - 1\right)}{(1 - \Phi^2 + 2\Phi cos\alpha_1)}$; $\eta_b = \frac{2\left(2\Phi cos\alpha_1 - \Phi^2 + 1\right) - 2}{(1 - \Phi^2 + 2\Phi cos\alpha_1)}$; $\eta_b = 2 - \frac{2}{(1 - \Phi^2 + 2\Phi cos\alpha_1)}$

for max efficiency

$\frac{\partial \eta_b}{\partial \emptyset} = 0$

$\frac{\partial}{\partial \emptyset}\left(2 - \frac{2}{(1 - \Phi^2 + 2\Phi cos\alpha_1)}\right) = 0$; $\frac{\partial}{\partial \emptyset}\left(\frac{2}{(1 - \Phi^2 + 2\Phi cos\alpha_1)}\right) = 0$

$\frac{\partial}{\partial \emptyset}(1 - \Phi^2 + 2\Phi cos\alpha_1) = 0$; $0 - 2\Phi + 2\cos\alpha_1 = 0$; $2\Phi = 2\cos\alpha_1$; $\Phi = \cos\alpha_1$

$\Phi = \cos\alpha_1$ condition for max efficiency for 2 stages

Substituting

$\Phi = \cos\alpha_1$ in $\eta_b = 2 - \frac{2}{(1-\Phi^2+2\Phi cos\alpha_1)}$

will give max efficiency

$$\eta_{bmax} = 2 - \frac{2}{(1-cos^2\alpha_1+2\cos\alpha_1 cos\alpha_1)}$$

$$\eta_{bmax} = 2 - \frac{2}{(1+cos^2\alpha_1)}$$

$$\eta_{bmax} = \frac{2(1+cos^2\alpha_1)-2}{(1+cos^2\alpha_1)}$$

$$\eta_{bmax} = \frac{2\,cos^2\alpha_1}{(1+cos^2\alpha_1)}$$

Numerical Problems

1. The following data refers to Delaval turbine. Velocity of steam at exit of the nozzle is 1000m/s with a nozzle angle 20^0. The blade velocity is 400m/s and blades are equiangular. Assume a mass flow rate of 1000kg/hr, friction coefficient 0.8, nozzle efficiency is 0.95, calculate i) blade angles ii) Work done /per of steam iii) Power developed iv) Blade efficiency v) Stage efficiency (5b. 10, June/July17) (5b. 10, Dec14/Jan15)

Given Data:

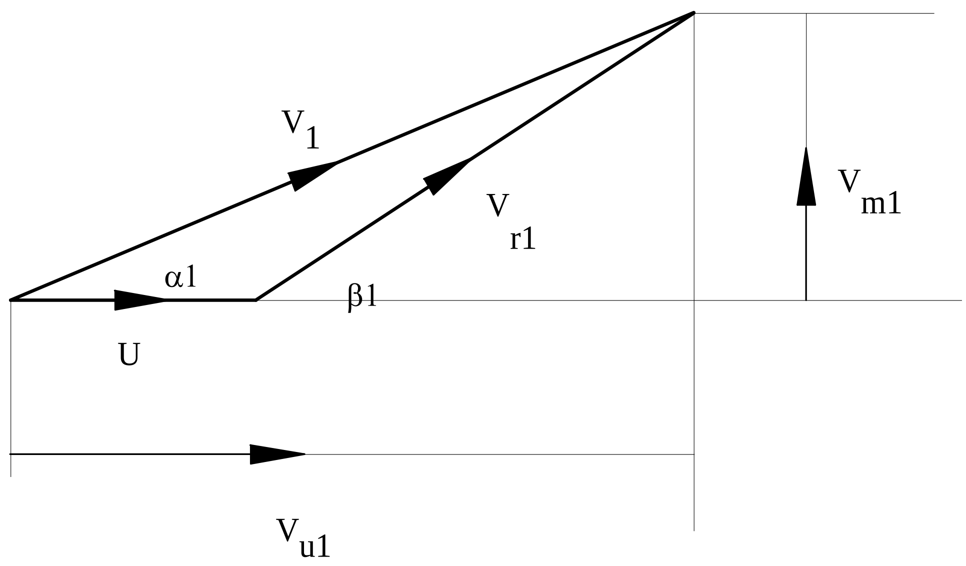
Delaval turbine is Impulse turbine

Ie $R = 0$ **and** $U_1 = U_2 = U$

- Velocity of steam from nozzle= V_1=1000m/s, nozzle angle = $\alpha_1=20^o$
- the rotor blades are equiangular ie $\beta_1 = \beta_2$
- Tangential speed $= U = 400m/s$,
- $\dot{m} = 1000kg/hr;\ \ \dot{m} = 0.28kg/s$
- friction coefficient 0.8 ie $\frac{V_{r2}}{V_{r1}} = 0.8$
- $\eta_{nozzle} = 0.95$
- **To determine**

i)Rotor blade angle β_1=?, β_2=?., ii) E =?iii) η_{blade}=? iv) $\eta_{stage=?}$

Inlet Velocity Triangle



Blade angles

From inlet velocity triangle

$\overrightarrow{V_{u1}} = V_1 cos\alpha_1$; $\overrightarrow{V_{u1}} = 1000\ cos20$; $\overrightarrow{V_{u1}} = 939.69m/s$

$V_{m1} = V_1 Sin\alpha_1$ $V_{m1} = 1000\ sin20;$ $V_{m1} = 342.02m/s$

$tan\beta_1 = \frac{V_{m1}}{\overrightarrow{V_{u1}}-U}$; $tan\beta_1 = \frac{342.02}{939.69-400}$; $\beta_1 = 32.36^o$

$\beta_1 = \beta_2$ (blades are equiangular**);** $\beta_2 = 32.36^o$

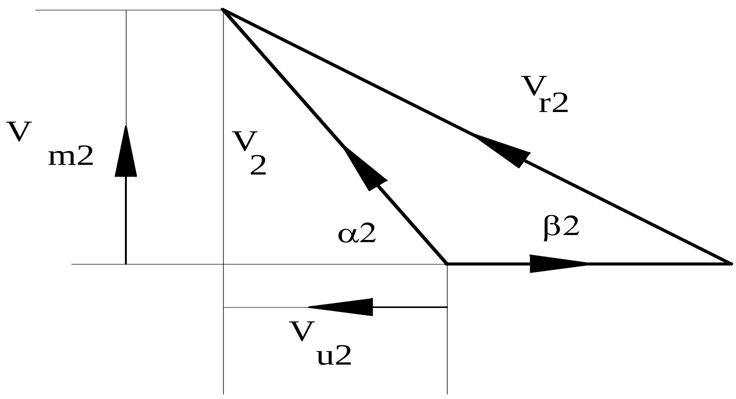
<u>Work done /per of steam</u>

$sin\beta_1 = \frac{V_{m1}}{V_{r1}}$; $sin 32.36 = \frac{342.02}{V_{r1}}$; $V_{r1} = 639 m/s$

$\frac{V_{r2}}{V_{r1}} = 0.8$; $\frac{V_{r2}}{639} = 0.8$; $V_{r2} = 511.2 m/s$

$V_{r2}\, cos\, \beta_2 = 511.2\, cos\, 32.36$; $V_{r2}\, cos\, \beta_2 = 431.81$; $U = 400 m/s$

$V_{r2}\, cos\, \beta_2 > U$,   Hence , Outlet velocity triangle as given below



<u>From outlet velocity triangle</u>

$\overleftarrow{V_{u2}} = V_{r2}\, cos\, \beta_2 - U$; $\overleftarrow{V_{u2}} = 431.81 - 400$; $\overleftarrow{V_{u2}} = 31.81$ m/s

<u>Power:</u>

$\frac{E}{\dot{m}} = \frac{(\overrightarrow{V_{u1}}+\overleftarrow{V_{u2}})U}{g_c}$; $\frac{E}{\dot{m}} = \frac{(939.69+31.81)400}{1}$; $\frac{E}{\dot{m}} = 388600 J/kg$

$E = \dot{m}\frac{E}{\dot{m}}$; $E = 0.28 * 388600$; $E = 108808 W$; $E = 108.808 kW$

<u>Blade efficiency</u>

$\eta_b = \frac{\frac{E}{\dot{m}}}{\frac{V_1^2}{2g_c}}$; $\eta_b = \frac{388600}{\frac{1000^2}{2}}$; $\eta_b = 0.772$

<u>Stage Efficiency:</u>

$\eta_{stage} = \eta_b \eta_{nozzle}$; $\eta_{stage} = 0.7772 * 0.95$ $\eta_{stage} = 0.7383$

ii) Tangential force

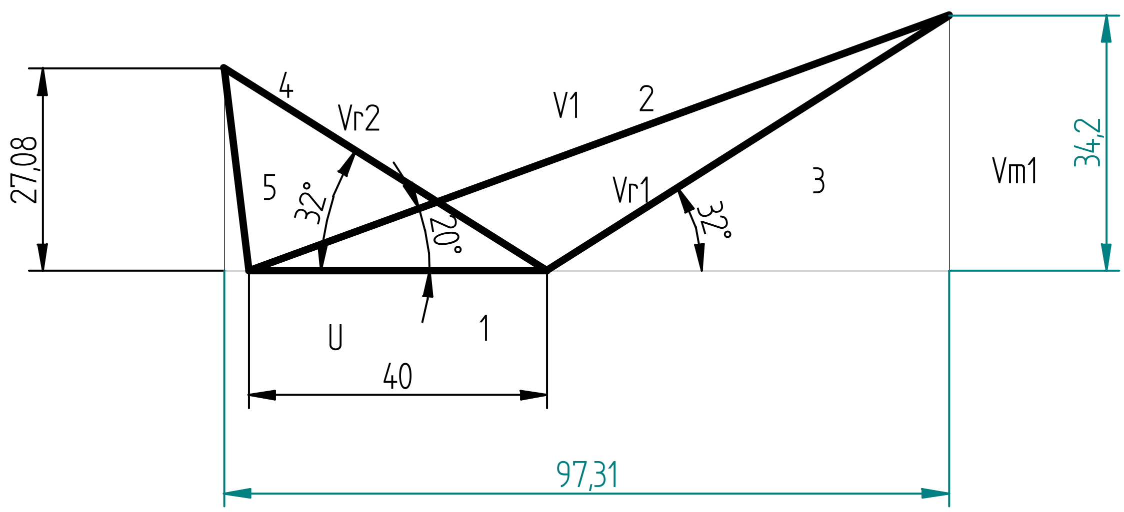
Tangential force ,$F_u = \dot{m}\,(\overrightarrow{V_{u1}} + \overleftarrow{V_{u2}})$; $F_u = 0.28(939.69 + 31.81)$; F_u =272.02N

Axial thrust : $F_a = \dot{m}\,(V_{m1} - V_{m2})$

$Sin\beta_2 = \frac{V_{m2}}{V_{r2}}$; $Sin32.36 = \frac{V_{m2}}{511.2}$; $V_{m2} = 273.61m/s$

$F_a = \dot{m}\,(V_{m1} - V_{m2})$; $F_a = 0.28(342.02 - 273.61)$ $F_a = 19.15N$

Graphical solution



Scale 1cm = 100m/s

1 . $U = 400m/s$; $U = 4cm$; 2. $V_1 = 1000m/s$; $V_1 = 10cm$ $\alpha_1 = 20^0$

3. Complete triangle (right side) 4. Measure V_{r1} and β_1 ie $V_{r1} = 63.89mm$ $\beta_1 = 32^0$

4. $V_{r2} = 0.8V_{r1}$ $V_{r2} = 0.8 * 63.89 = 51.11mm$; $\beta_2 = \beta_1$ $(blades\ are\ equianglular)$ $\beta_2 = 32^0$

5. Draw $V_{r2} = 51.11mm$ at an angle $\beta_2 = 32^0$ measuring from right and complete right triangle

6. Measure ΔV_u ie $\Delta V_u = 9.731cm$ ie $\Delta V_u = 973.1m/s$; $V_{m1} = 3.42cm$ ie $V_{m1} = \frac{342m}{s}$

$V_{m2} = 2.708cm = 270.8m/s$

Calculation

Power

$\frac{E}{\dot{m}} = \frac{(\overrightarrow{V_{u1}}+\overleftarrow{V_{u2}})U}{g_c}$; $\frac{E}{\dot{m}} = \frac{\Delta V_u U}{g_c}$ $\frac{E}{\dot{m}} = \frac{973.1*400}{1}$; $\frac{E}{\dot{m}} = 389240J/kg$

$E = \dot{m}\frac{E}{\dot{m}}$; $E = 0.28 * 389240$; $E = 108987W$; $E = 108.987kW$

Blade efficiency

$$\eta_b = \frac{\frac{E}{\dot{m}}}{\frac{V_1^2}{2g_c}}; \quad \eta_b = \frac{389240}{\frac{1000^2}{2}}; \quad \eta_b = 0.778$$

Stage Efficiency:

$$\eta_{stage} = \eta_b \eta_{nozzle}; \quad \eta_{stage} = 0.778 * 0.95 \quad \eta_{stage} = 0.739$$

10% Difference between theoretical and graphical solution is permitted

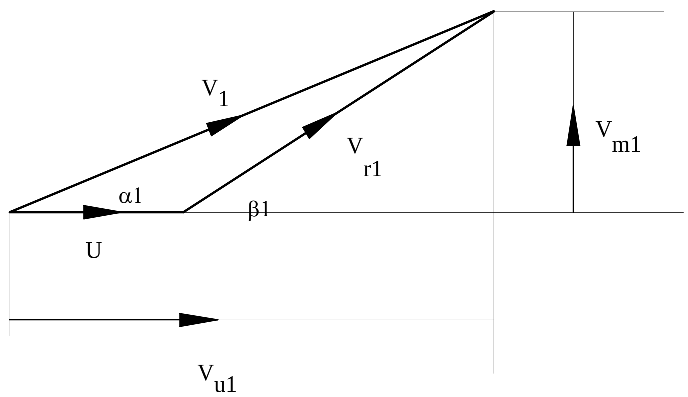
2. Steam issues from a nozzle of a Delaval turbine with a of 1200m/s. The nozzle angle is 20^0. The blade speedy is 400m/s. The inlet and outlet blades are equal. Assume a mass flow rate of 1000kg/hr, calculate i) blade angles ii) relative velocities of blade entering ii) Axial thrust iii) Power developed iv) Blade efficiency Assume K=0.8 (5b. 10, Dec18/Jan19)
3. Steam issues from a nozzle of a Delaval turbine with a of 1000m/s. The nozzle angle is 20^0. The blade speedy is 400m/s. The inlet and outlet blades are equal. Assume a mass flow rate of 900kg/hr, calculate i) blade angles ii) relative velocities if blade velocity coefficient is 0.8 ii) Tangential force on the blades iii) Power developed iv) Blade efficiency (5b. 10, Dec17/Jan18)
4. The data pertaining to an impulse turbine is as follows: Steam velocity =500m/s, blade speed =200m/s, exit angle at moving blade =25° measured from tangential direction, nozzle angle=20°. Neglecting the effect of friction when passing through blade passages, calculate : i) Inlet angle of moving blade ii) Exit velocity and direction iii) Work done per kg of steam iv) Power developed v) Diagram efficiency (5b,16, June/July14)

   $V_1 = 500m/s$; $U = 200m/s$; $\beta_2 = 25^0$; Nozzle angle= 20^0 ie $\alpha_1 = 20^0$

   Neglecting the effect of friction when passing through blade passages, $V_{r1} = V_{r2}$

Inlet blade angle β_1

$$\overrightarrow{V_{u1}} = V_1 cos\alpha_1 ; \quad \overrightarrow{V_{u1}} = 500\ cos20 ; \quad \overrightarrow{V_{u1}} = 469.85m/s$$



$$V_{m1} = V_1 Sin\alpha_1 \quad V_{m1} = 500\ sin20; \quad V_{m1} = 171.01m/s$$

$$tan\beta_1 = \frac{V_{m1}}{\overrightarrow{V_{u1}} - U}; \quad tan\beta_1 = \frac{171.01}{469.85 - 200} \quad ; \quad \beta_1 = 32.36^o$$

Exit velocity and direction

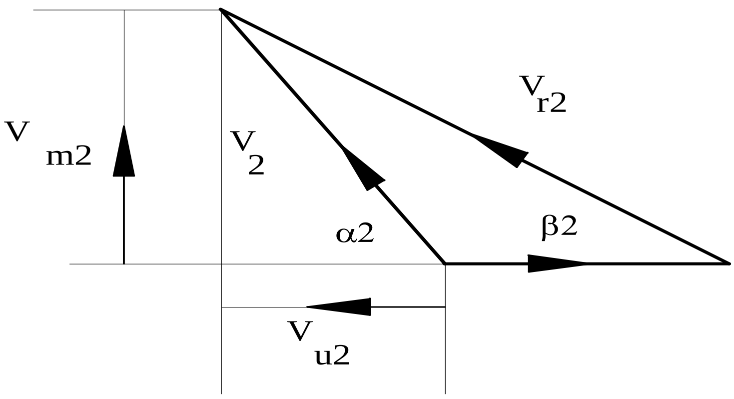
$\beta_2 = 25^o$ (given)

$sin\beta_1 = \frac{V_{m1}}{V_{r1}}$; $sin32.36 = \frac{171.01}{V_{r1}}$; $V_{r1} = 319.50m/s$

$V_{r2} = V_{r1}$; ; $V_{r2} = 319.50m/s$

$V_{r2}\ cos\ \beta_2 = 319.50cos\ 25$; $V_{r2}\ cos\ \beta_2 = 289.56$; $U = 200m/s$

$V_{r2}\ cos\ \beta_2 > U$, Hence , Outlet velocity triangle as given below



From outlet velocity triangle

$\overleftarrow{V_{u2}} = V_{r2}\ cos\ \beta_2 - U$; $\overleftarrow{V_{u2}} = 289.56 - 200$; $\overleftarrow{V_{u2}} = 89.56$ m/s

$Sin\beta_2 = \frac{V_{m2}}{V_{r2}}$; $Sin25 = \frac{V_{m2}}{319.50}$; $V_{m2} = 135.02m/s$

$V_2^2 = V_{u2}^2 + V_{m2}^2$; $V_2 = \sqrt{V_{u2}^2 + V_{m2}^2}$; $V_2 = \sqrt{89.56^2 + 135.02^2}$; $V_2 = 162.02m/s$

$tan\alpha_2 = \frac{V_{m2}}{V_{u2}}$; $tan\alpha_2 = \frac{135.02}{89.56}$; $\alpha_2 = 56.44^o$

Work done /kg of steam

$\frac{E}{\dot{m}} = \frac{(\overrightarrow{V_{u1}} + \overleftarrow{V_{u2}})U}{g_c}$; $\frac{E}{\dot{m}} = \frac{(469.85+89.56)200}{1}$; $\frac{E}{\dot{m}} = 111882\ J/kg$

Power

Assume $\dot{m} = 1kg/s$

$E = \dot{m}\frac{E}{\dot{m}}$; $E = 1 * 111882$; $E = 111882W$; $E = 111.882kW$

Diagram efficiency

$\eta_b = \frac{\frac{E}{\dot{m}}}{\frac{V_1^2}{2g_c}}$; $\eta_b = \frac{111882}{\frac{500^2}{2}}$; $\eta_b = 0.895$

Tangential force

Tangential force ,$F_u = \dot{m}\,(\overrightarrow{V_{u1}} + \overleftarrow{V_{u2}})$; $F_u = 1(469.85 + 89.56)$; F_u =559.41N

Axial thrust : $F_a = \dot{m}\,(V_{m1} - V_{m2})$

$F_a = \dot{m}\,(V_{m1} - V_{m2})$; $F_a = 1(171.01 - 135.02)$ $F_a = 35.99N$

5. The following particulars refer to a single impulse turbine. Mean diameter of blade ring =2.5m, speed =3000rpm. Nozzle angle 20°, ratio of blade velocity to steam =0.4, blade friction factor =0.8, blade angle at exit is 3° less than that at inlet. Steam flow rate 36000kg/hr. Draw the velocity diagram and calculate i) power developed ii) blade efficiency (5b. 08, Dec18/Jan19 CBCS)

Mean diameter of blade ring =2.5m $D_1 = D_1 = D = 2.5m;\ N = 3000rpm;\ \alpha_1 = 20^0$

ratio of blade velocity to steam =0.4 ie $\frac{U}{V_1} = 0.4$; blade friction factor =0.8 ie $\frac{V_{r2}}{V_{r1}} = 0.8$

blade angle at exit is 3° less than that at inlet ie $\beta_2 = \beta_1 - 3$; $\dot{m} = 36000kg/hr, \dot{m} = 10kg/s$

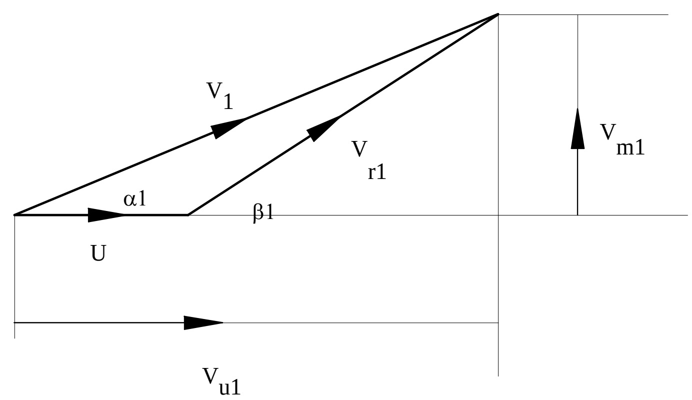
E =?; η_b =?

Power

$U = \frac{\pi DN}{60}$; $U = \frac{\pi * 2.5 * 3000}{60}$; $U = 392.7m/s$

$\frac{U}{V_1} = 0.4$; $\frac{392.69}{V_1} = 0.4$; $V_1 = 981.75m/s$

$\overrightarrow{V_{u1}} = V_1 cos\alpha_1$; $\overrightarrow{V_{u1}} = 981.75\ cos20$; $\overrightarrow{V_{u1}} = 922.54m/s$



$V_{m1} = V_1 Sin\alpha_1$ $V_{m1} = 981.75\ sin20$; $V_{m1} = 335.78m/s$

$tan\beta_1 = \frac{V_{m1}}{\overrightarrow{V_{u1}}-U}$; $tan\beta_1 = \frac{335.78}{922.54-392.7}$; $\beta_1 = 32.36^o$

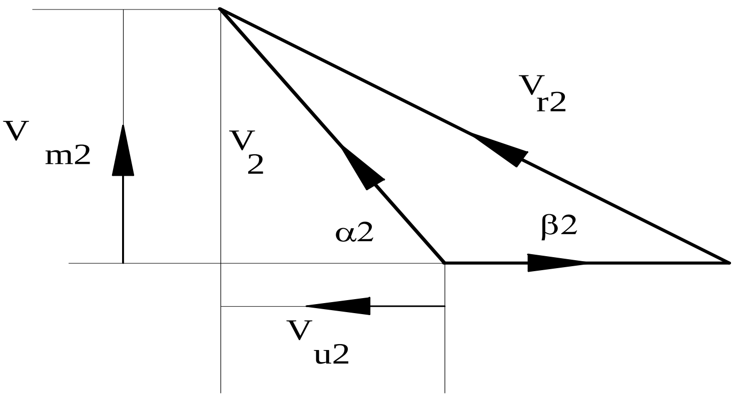
$\beta_2 = \beta_1 - 3$; $\beta_2 = 32.36 - 3$; $\beta_2 = 29.36$

$sin\beta_1 = \frac{V_{m1}}{V_{r1}}$; $sin32.36 = \frac{335.78}{V_{r1}}$; $V_{r1} = 627.34m/s$

$\frac{V_{r2}}{V_{r1}} = 0.8$; $\frac{V_{r2}}{627.35} = 0.8$; $V_{r2} = 501.88m/s$

$V_{r2}\cos\beta_2 = 501.88\cos 29.36$; $V_{r2}\cos\beta_2 = 437.42$; $U = 392.7m/s$

$V_{r2}\cos\beta_2 > U$, Hence , Outlet velocity triangle as given below



$\overleftarrow{V_{u2}} = V_{r2}\cos\beta_2 - U$; $\overleftarrow{V_{u2}} = 437.42 - 392.7$; $\overleftarrow{V_{u2}} = 44.71m/s$

$\frac{E}{\dot{m}} = \frac{(\overrightarrow{V_{u1}}+\overleftarrow{V_{u2}})U}{g_c}$; $\frac{E}{\dot{m}} = \frac{(922.54+44.71)392.7}{1}$; $\frac{E}{\dot{m}} = 379841.15\, J/kg$

$\dot{m} = 10kg/s$; $E = \dot{m}\frac{E}{\dot{m}}$; $E = 10 * 379841.15$; $E = 3798411.5\, W$; $E = 3798.41kW$

Blade Efficiency

$\eta_b = \frac{\frac{E}{\dot{m}}}{\frac{V_1^2}{2g_c}}$; $\eta_b = \frac{379841.15}{\frac{981.75^2}{2}}$; $\eta_b = 0.788$

6. In a single stage impulse turbine the mean diameter of blades is 1m. It runs at 3000rpm. The steam is supplied from a nozzle at a velocity of 350m/s and the nozzle angle is 20°. The rotor blades are equianglular. The blade friction factor is 0.86. Draw the velocity diagram and calculate the power developed if the axial thrust is 117.72 Newtons (5b,10, Dec13/Jan14) (5c,08, June/July18)

7. In a single stage impulse turbine the mean diameter of blades is 80cm. It runs at 3000rpm. The steam issues from a nozzle with a velocity of 300m/s and the nozzle angle is 20°. The rotor blades are equiangular. The blade velocity coefficient is 0.85. what is the power developed when the axial thrust is 140 Newtons (6b,08, June/July,15,18CBCS)

Mean diameter of blades $= 80cm \quad D_1 = D_1 = D = 0.8m; N = 3000rpm; \; V_1 = 300m/s; \; \alpha_1 = 20^0$

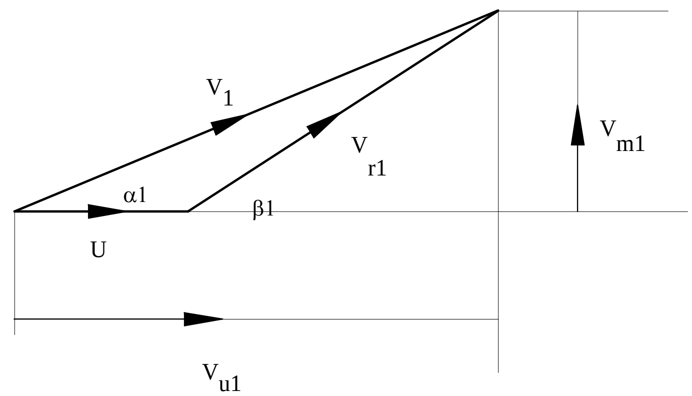
The rotor blades are equiangular. $\beta_2 = \beta_1$; blade friction factor =0.85 ie $\frac{V_{r2}}{V_{r1}} = 0.85$

$E = ?\,; \quad The\ axial\ thrust\ is\ 140\ Newtons$ ie $F_a = 140\ N$

Inlet blade angle β_1

$\overrightarrow{V_{u1}} = V_1 cos\alpha_1\,; \quad \overrightarrow{V_{u1}} = 300\ cos20\,; \qquad \overrightarrow{V_{u1}} = 281.91m/s$

$U = \frac{\pi DN}{60}; \qquad U = \frac{\pi*0.8*3000}{60}; \qquad U = 125.66m/s$



$V_{m1} = V_1 Sin\alpha_1 \quad V_{m1} = 300\ sin20; \qquad V_{m1} = 102.61m/s$

$tan\beta_1 = \frac{V_{m1}}{\overrightarrow{V_{u1}} - U}; \quad tan\beta_1 = \frac{102.61}{281.91 - 125.66} \quad ; \quad \beta_1 = 33.29^o$

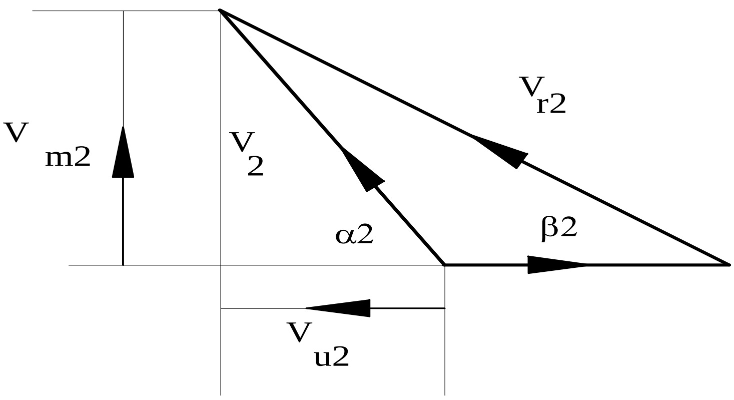
$\beta_2 = \beta_1; \quad \beta_2 = 33.29^o$

$sin\beta_1 = \frac{V_{m1}}{V_{r1}}; \qquad sin33.29 = \frac{102.61}{V_{r1}}; \quad V_{r1} = 186.95m/s$

$\frac{V_{r2}}{V_{r1}} = 0.85; \quad \frac{V_{r2}}{186.95} = 0.85; \quad V_{r2} = 158.9m/s$

$V_{r2}\ cos\ \beta_2 = 158.9\ cos\ 33.29$; $V_{r2}\ cos\ \beta_2 = 132.83m/s\,; \quad U = 125.66m/s$

$V_{r2}\ cos\ \beta_2 > U$, Hence , Outlet velocity triangle as given below



From outlet velocity triangle

$\overleftarrow{V_{u2}} = V_{r2} \cos \beta_2 - U$; $\overleftarrow{V_{u2}} = 132.82 - 125.66;$ $\overleftarrow{V_{u2}} = 7.17$ m/s

$\frac{E}{\dot{m}} = \frac{(\overrightarrow{V_{u1}} + \overleftarrow{V_{u2}})U}{g_c};$ $\frac{E}{\dot{m}} = \frac{(281.91+7.17)125.66}{1};$ $\frac{E}{\dot{m}} = 36325.54\, J/kg$

$Sin\beta_2 = \frac{V_{m2}}{V_{r2}}$; $Sin 33.29 = \frac{V_{m2}}{158.9};$ $V_{m2} = 87.22 m/s$

$F_a = \dot{m}(V_{m1} - V_{m2})$; $140 = \dot{m}(102.61 - 87.22);$ $\dot{m} = 9.09 kg/s$

$E = \dot{m}\frac{E}{\dot{m}}$; $E = 9.09 * 36325.54;$ $E = 330372.6W$; $E = 330.37kW$

8. The simple impulse turbine has a mean blade speed of 200m/s. The nozzles are inclined at 20^o to the planes of rotation of the blades. The steam velocity from nozzles is 600m/s. The turbine uses 3500kg/hr of steam. The absolute velocity at exit is along the axis of turbine Determine i) Inlet and exit angles of blades ii) Power output of turbine iii) Diagram efficiency (5a. 10, Dec16/Jan17)

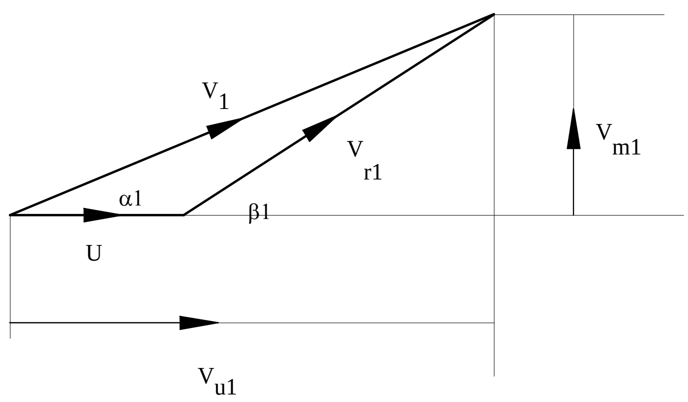
Inlet and exit angles of blades

$U = 200 m/s; \alpha_1 = 20^o$; $V_1 = 600 m/s;$ $\dot{m} = 3500 kg/hr;$ $\dot{m} = 0.972 kg/s$

The absolute velocity at exit is along the axis of turbine ie V_2 is axial $\alpha_2 = 90^o$

$\overrightarrow{V_{u1}} = V_1 cos\alpha_1$; $\overrightarrow{V_{u1}} = 600\, cos 20$; $\overrightarrow{V_{u1}} = 563.82 m/s$

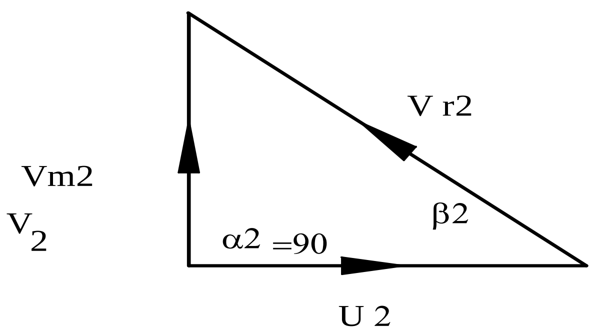
$V_{m1} = V_1 Sin\alpha_1;$ $V_{m1} = 600\, sin 20;$ $V_{m1} = 205.21\, m/s$



$tan\beta_1 = \frac{V_{m1}}{\overrightarrow{V_{u1}}-U}$; $tan\beta_1 = \frac{205.21}{563.82-200}$; $\beta_1 = 29.42^o$

$sin\beta_1 = \frac{V_{m1}}{V_{r1}}$; $sin29.42 = \frac{205.21}{V_{r1}}$; $V_{r1} = 418.17m/s$

, Hence outlet velocity triangle is as given below



Assume $V_{r2} = V_{r1}$; $V_{r2} = 418.17m/s$

$cos\beta_2 = \frac{U}{V_{r2}}$; $cos\beta_2 = \frac{200}{418.17}$; $\beta_2 = 61.42^o$

Power output of turbine

$\frac{E}{\dot{m}} = \frac{(\overrightarrow{V_{u1}}-\overrightarrow{V_{u2}})U}{g_c}$; $\overrightarrow{V_{u2}} = 0$ as $\alpha_2 = 90^o$; $\frac{E}{\dot{m}} = \frac{(563.82-0)200}{1}$; $\frac{E}{\dot{m}} = 112764\ J/kg$

$E = \dot{m}\frac{E}{\dot{m}}$; $E = 0.972 * 112764$; $E = 109606.6W$; $E = 109.606kW$

Diagram efficiency

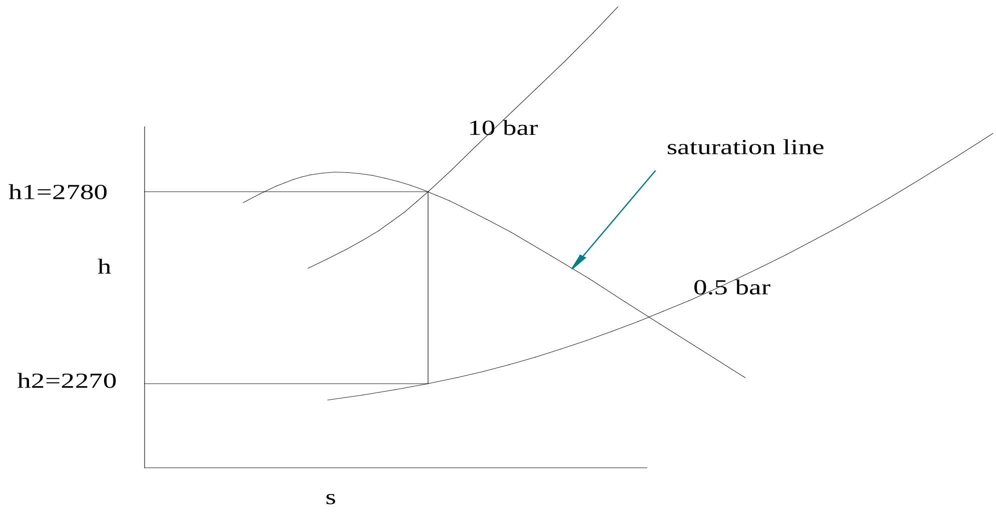
$\eta_b = \frac{\frac{E}{\dot{m}}}{\frac{V_1^2}{2g_c}}$; $\eta_b = \frac{109606.6}{\frac{600^2}{2}}$; $\eta_b = 0.608$

9. One stage of an impulse turbine consists of a nozzle and one ring of moving blades. The nozzle is inclined a 22^o to the tangential speed of blades and the blade tip angles are equiangular and equal to 35^o (a) Find the blade speed, diagram efficiency by neglecting losses, if the velocity of steam at the exit of the nozzle is 660m/s (b) If the relative velocity of steam is reduced by 15% in passing through the blade ring. Find the blade speed, diagram efficiency and end thrust on the shaft when the blade ring develops 1745kW
10. Steam flows through the nozzle with a velocity of 450m/s at a direction which is inclined at an angle of 16^o to the wheel tangent. Steam comes out of the moving blades with a velocity of 100m/s in the direction of 110^o with the direction of blade motion. The blades are equiangular and the steam flow rate is 10kg/s. Find (i) Power developed (ii) the power loss due to friction (iii) Axial thrust (iv) blade efficiency and (v) blade coefficient
11. Dry saturated steam at 10atmospheric pressure is supplied to single rotor impulse wheel, the condenser pressure being 0.5 atmosphere with the nozzle efficiency of 0.94 and the nozzle angle at the rotor inlet is 18^o to the wheel plane. The rotor blades which move with the speed of 450m/s are equiangular. If the coefficient velocity for the rotor blades is 0.92. find (i) the specific power output (ii) the rotor efficiency (iii) the stage efficiency (iv) axial thrust (v) Velocity and the direction of exit steam (5b,12, Dec 12)

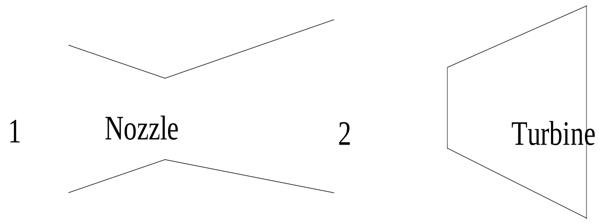
- Dry saturated steam at 10 atmospheric pressure is supplied to single rotor impulse wheel the steam enters nozzle at 10 atmospheric pressure and dry
- the nozzle efficiency of 0.94 ; η_n=0.94
- nozzle angle at the rotor inlet is 18^o to the wheel plane $\alpha_1=18^o$
- The rotor blades which move with the speed of 450m/s are equiangular ie U=450m/s and $\beta_1=\beta_2$
- If the coefficient velocity for the rotor blades is 0.92 ie $\frac{V_{r2}}{V_{r1}}$ =0.92

find (i) the specific power output ie E=? (ii) the rotor efficiency η_b=? (iii) the stage efficiency η_s=? (iv) axial thrust F_a =? (v) the direction of exit steam α_2=?

$h_1 = h_g at\ 10bar = 2780kJ/kg$
$h_2 = 2270\ kJ/kg$ from mollier diagram

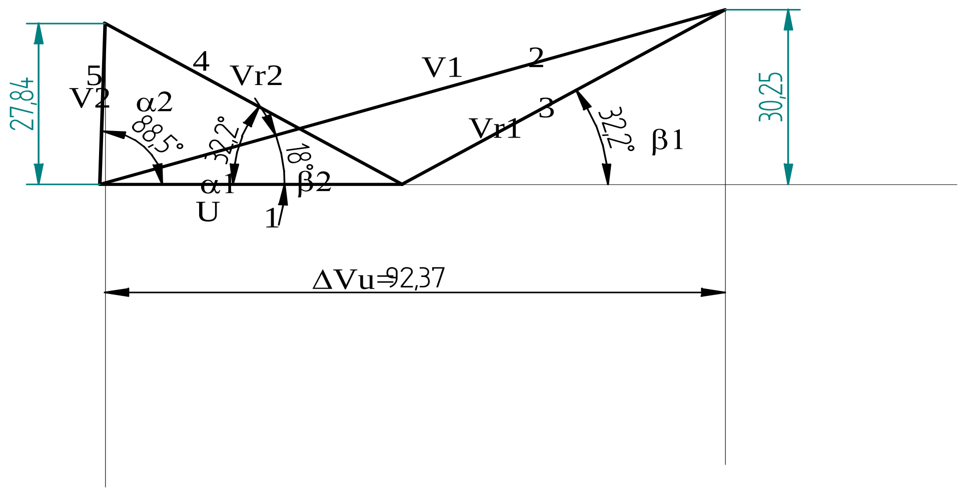


s



$V_1 = \sqrt{2(h_1 - h_2)10^3\eta_n}$; $V_1 = \sqrt{2(2780 - 2270)10^3x0.94}$; $V_1 = 979.2m/s$

Take scale $1cm = 100m/s$



Measure $\Delta V_u = 9.237cm$; $\Delta V_u = 9.237 * 100 = 92.37m/s$

$\frac{E}{m} = \frac{\Delta V_u U}{g_c}$; $\frac{E}{m} = \frac{(923.7)\ x\ 450}{1}$; $\frac{E}{m} = 415665J/kg$

ii) Rotor efficiency

Blade $\eta = \frac{\frac{E}{m}}{\frac{V_1^2}{2}}$; $\eta = \frac{415665}{\frac{979.2^2}{2}}$; $\eta = 0.867$

iii) stage efficiency $\eta_s = \eta_n \eta_b$

$\eta_s = 0.867x0.94$ $\eta_s = 0.814$

iv) Axial thrust

$V_{m1} = 3.025cm_;$ $V_{m1} = 3.025 * 100m/s_;$ $V_{m1} = 302.5m/s$

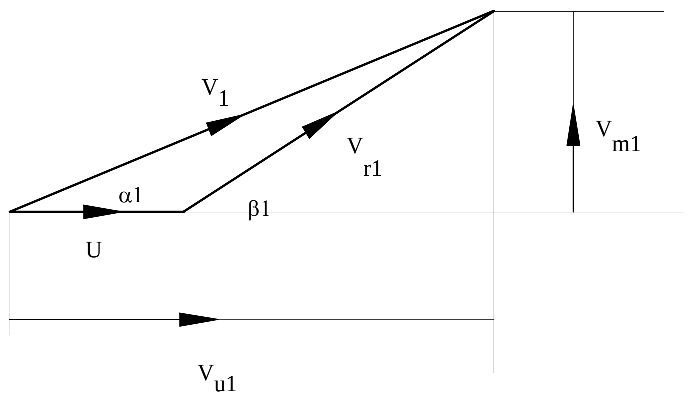
$V_{m2} = 2.784cm_;$ $V_{m2} = 2.784 * 100m/s_;$ $V_{m2} = 278.4m/s$ Assume $\dot{m} = 1kg/s$

$F_a = \dot{m}\,(V_{m1} - V_{m2})$; $F_a = 1\,(302.5 - 278.4)$ $F_a = 24.1N$

Analytical method

$\overrightarrow{V_{u1}} = V_1 cos\alpha_1$; $\overrightarrow{V_{u1}} = 979.2\ cos18$; $\overrightarrow{V_{u1}} = 931.27m/s$

$V_{m1} = V_1 Sin\alpha_1$; $V_{m1} = 979.2\ sin18$; $V_{m1} = 302.58\ m/s$



$tan\beta_1 = \frac{V_{m1}}{\overrightarrow{V_{u1}} - U}$; $tan\beta_1 = \frac{302.58}{931.27 - 450}$; $\beta_1 = 32.15^o$

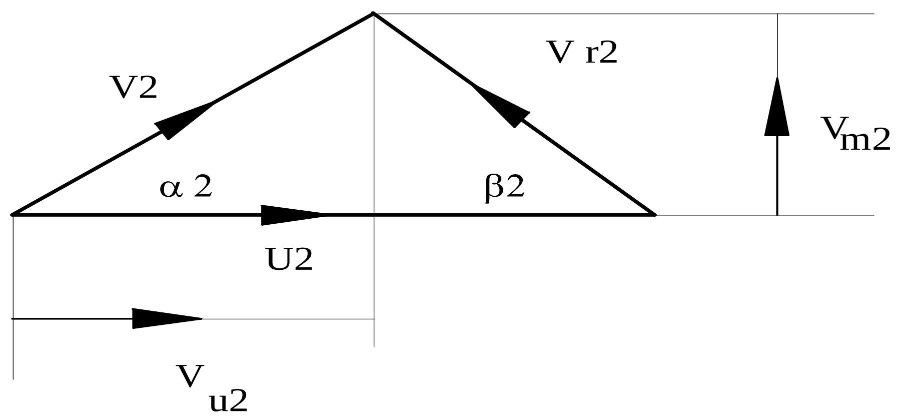
$\beta_2 = \beta_1$; $\beta_2 = 32.15^o$

$sin\beta_1 = \frac{V_{m1}}{V_{r1}}$; $sin32.15 = \frac{302.58}{V_{r1}}$; $V_{r1} = 568.61m/s$

$\frac{V_{r2}}{V_{r1}} = 0.92$; $\frac{V_{r2}}{568.61} = 0.92$; $V_{r2} = 523.12m/s$

$V_{r2}\ cos\ \beta_2 = 523.12\ cos\ 32.15$; $V_{r2}\ cos\ \beta_2 = 442.92m/s$; $U = 450m/s$

$V_{r2}\ cos\ \beta_2 < U$, Hence , Outlet velocity triangle as given below



$\overrightarrow{V_{u2}} = U - V_{r2}\ cos\ \beta_2$; $\overrightarrow{V_{u2}} = 450 - 442.92$; $\overrightarrow{V_{u2}} = 7.08$ m/s

$\frac{E}{\dot{m}} = \frac{(\overrightarrow{V_{u1}} - \overrightarrow{V_{u2}})U}{g_c}$; $\frac{E}{\dot{m}} = \frac{(931.27 - 7.08)450}{1}$; $\frac{E}{\dot{m}} = 415885.5\, J/kg$

ii) Rotor efficiency

Blade $\eta = \frac{\frac{E}{m}}{\frac{V_1^2}{2}}$; $\eta = \frac{415885.5}{\frac{979.2^2}{2}}$; $\eta = 0.867$

iii) stage efficiency $\eta_s = \eta_n \eta_b$

$\eta_s = 0.867x0.94$ $\eta_s = 0.814$

iv) Axial thrust

$Sin\beta_2 = \frac{V_{m2}}{V_{r2}}$; $Sin32.15 = \frac{V_{m2}}{523.12}$; $V_{m2} = 278.37\, m/s$; Assume $\dot{m} = 1kg/s$

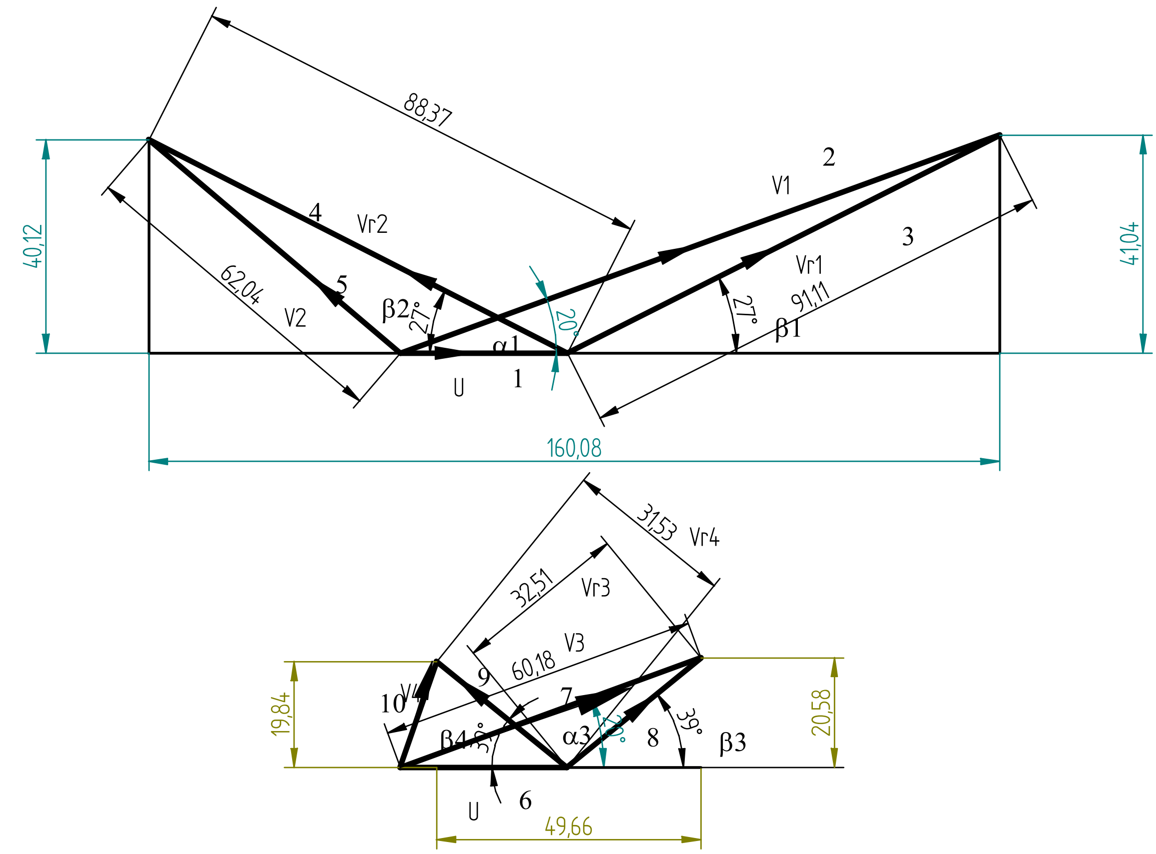
$F_a = \dot{m}(V_{m1} - V_{m2})$; $F_a = 1(302.5 - 278.37)$; $F_a = 24.12N$

(v) Velocity and the direction of exit steam

$V_2^2 = V_{u2}^2 + V_{m2}^2$; $V_2 = \sqrt{V_{u2}^2 + V_{m2}^2}$; $V_2 = \sqrt{7.08^{\,2} + 278.37^{\,2}}$; $V_2 = 278.46m/s$

$tan\alpha_2 = \frac{V_{m2}}{V_{u2}}$; $tan\alpha_2 = \frac{278.37}{7.08}$; $\alpha_2 = 88.54^o$

12. In a two row velocity compounded impulse steam turbine, the steam from the nozzle issues at a velocity of 600m/s. The nozzle angle is 20^o to the plane of rotation of the wheel. The mean diameter of rotor is 1m and the speed is 3000rpm. Both rows of moving blades have equiangular blades. The intermediate row of fixed guide blades makes the steam flow again at 20^0 to the second moving blade ring . The frictional losses in each row are 3%. Find i) The inlet and outlet angles of moving blades of each row ii) The inlet blade angle of the guide blade iii) The power output of first and second moving blade rings for unit mass flow rate iv) the blade efficiency v) the stage efficiency (assume nozzle efficiency =0.95)
    the steam from the nozzle issues at a velocity of 600m/s ie $V_1 = 600m/s$
    $\alpha_1 = 20^o$; The mean diameter of rotor is 1m ie $D_1 = 1m$; $N = 3000rpm$
    Both rows of moving blades have equiangular blades. Ie $\beta_1 = \beta_2$ and $\beta_3 = \beta_4$
    The intermediate row of fixed guide blades makes the steam flow again at 20^0 to the second moving blade ring . ie $\alpha_3 = 20^o$
    The frictional losses in each row are 3%. Ie $\frac{V_{r2}}{V_{r1}} = \frac{V_3}{V_2} = \frac{V_{r4}}{V_{r3}} = 1 - 0.03$;
    $\frac{V_{r2}}{V_{r1}} = \frac{V_3}{V_2} = \frac{V_{r4}}{V_{r3}} = 0.97$
    $i)\ \beta_1 = \beta_2 =?$; $\beta_3 = \beta_4 =?$ ii) $\left(\frac{E}{\dot{m}}\right)_I =?$ and $\left(\frac{E}{\dot{m}}\right)_{II} =?$ iii)$\eta_b =?$ iv) $\eta_s =?$ if $\eta_n = 0.95$
    $U = \frac{\pi DN}{60}$; $U = \frac{\pi * 1 * 3000}{60}$ $U = 157.07m/s$



1. Draw U =3,14cm ie$\frac{157.07}{50}$
2. Draw line $V_1 = 600m/s$ ie 12cm at an angle $\alpha_1 = 20^o$
3. Join end of U and V_1 ie

13. In a Curtis stage with two rows of moving blades the rotor are equiangular. The first rotor has angle of 29° each while second rotor has angle of 32° each. The velocity of steam at the exit of nozzle is 530m/s and the blade coefficients are 0.9 in the first , 0.95 in the stator and in the second rotor. If the absolute velocity at the stage exit should be axial, find i) Mean blade speed ii) The rotor efficiency iii) The power output for a flow rate of 32kg/s (5b,12, June/July16)

Two rows of moving blades the rotor are equiangular ie $\beta_1 = \beta_2; \quad \beta_3 = \beta_4$

The first rotor has angle of 29° each $\beta_1 = \beta_2 = 29^0; \beta_3 = \beta_4 = 32^0$

The velocity of steam at the exit of nozzle is 530m/s $V_1 = 530m/s$

Blade coefficients are 0.9 in the first ie $\frac{V_{r2}}{V_{r1}} = 0.9;$

Blade coefficients 0.95 in the stator ie $\frac{V_3}{V_2} = 0.95;$

Blade coefficients 0.95 in the second rotor ie $\frac{V_{r4}}{V_{r3}} = 0.95$

If the absolute velocity at the stage exit should be axial ie $\alpha_4 = 90^0$

Assume $U = 3cm$

Start from last triangle (4)

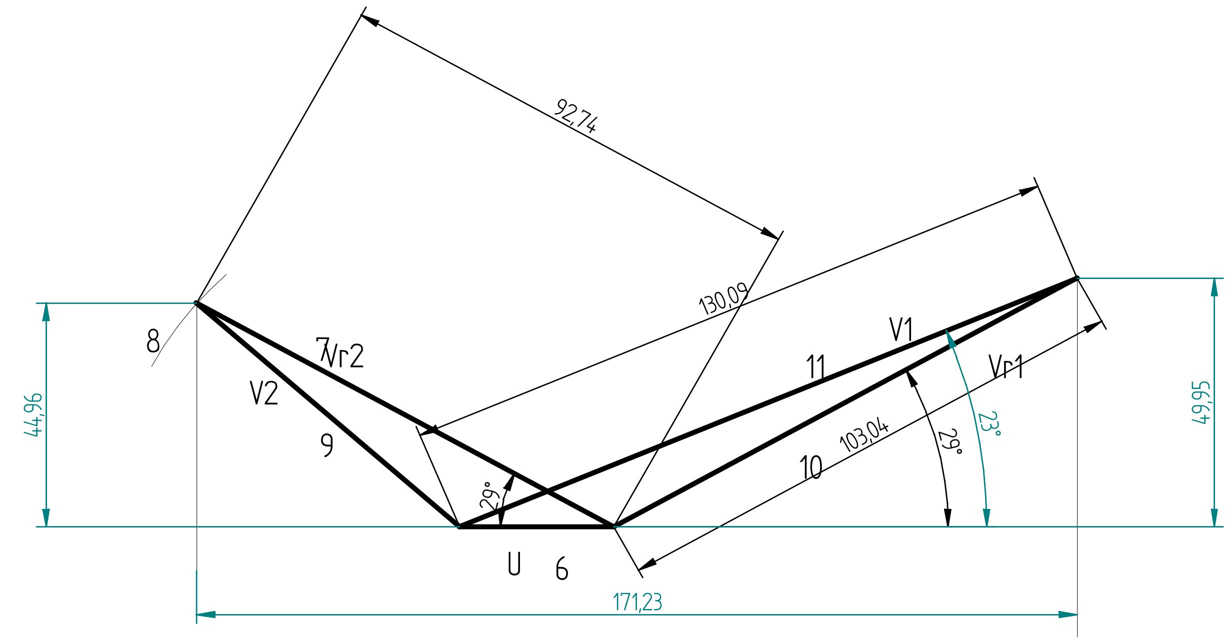
1. Draw $U = 30mm$ for 2 stage
2. Draw V_4 line vertical since $\alpha_4 = 90^0$

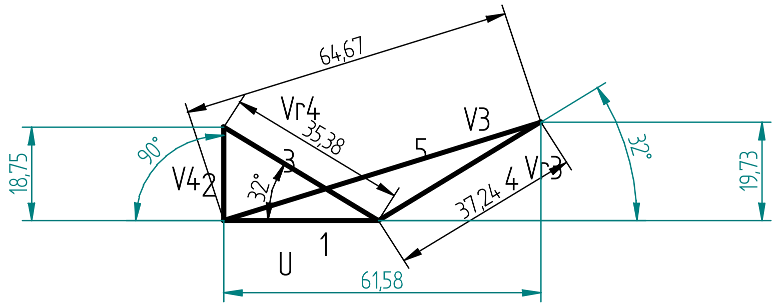
3. Draw V_{r4} line at an angle 32^0, V_4 Line and V_{r4} line intersect each other ,

measure $V_{r4} = 35.38mm$; $\frac{V_{r4}}{V_{r3}} = 0.95; \frac{35.38}{V_{r3}} = 0.95;\ V_{r3} = 37.24mm;$

4. Draw V_{r3} line equal to $37.24mm$
5. Draw V_3 line joining right end of U and end of V_{r3} line measure V_3 ie 37.24mm
6. Draw $U = 30mm$ for first stage ,
7. Draw V_{r2} line from right end of U of first stage at an angle $\beta_2 = 29^0$
8. Left end of the U of first stage velocity triangle as centre draw an arc of radius V_3 =37.24 mm to cut V_{r2} line
9. Join left end of U of First stage velocity triangle to intersection point of arc and V_{r2} line ie V_2 line, Measure V_{r2} ie 92.74mm

   $\frac{V_{r2}}{V_{r1}} = 0.9;\ \frac{92.74}{V_{r1}} = 0.9;\ V_{r1} = 103.04mm$
10. Draw V_{r1} line from right end of U of the first stage velocity triangle measuring $103.04mm$ at an angle $\beta_1 = 29^0$
11. Join end of V_{r1} line with left end of U of first stage velocity triangle ie V_1 $line$
12. Measure V_1 ie$130.09mm$ at $\alpha_1 = 23^0$





$V_1 \ = 130.09mm$ on the drawing

Actual $V_1 \ = 530m/s$

Hence $13.009cm = 530m/s$ ie $1cm = \frac{530}{13.009} m/s; \quad 1cm = 40.74m/s$

Mean Blade speed

Mean blade speed $U = (3cm * 40.74)m/s; \ U = 122.22m/s$

Rotor Efficiency

$(\Delta V_u)_I = 17.123 * 40.74m/s; \ (\Delta V_u)_I = 697.59m/s$

$(\Delta V_u)_{II} = 6.158 * 40.74m/s; \ (\Delta V_u)_{II} = 250.87m/s$

$\Delta V_u = (\Delta V_u)_I + (\Delta V_u)_{II}; \ \Delta V_u = 697.59 + 250.87; \ \ \Delta V_u = 948.47m/s$

$$\frac{E}{m} = \frac{\Delta V_u U}{g_c}; \quad \frac{E}{m} = \frac{948.47*122.22}{1}; \quad \frac{E}{m} = 115920.91\, J/kg$$

$$\eta_b = \frac{\frac{E}{m}}{\frac{V_1^2}{2g_c}}; \quad \eta_b = \frac{115920.91}{\frac{530^2}{2*1}}; \ \ \eta_b = 0.825$$

The power output for a flow rate of 32kg/s

$$E = \dot{m}\frac{E}{\dot{m}} \ ; \quad E = 32 * 115920.91\, ; \ \ E = 3709469.12W\, ; E = 3.709MW$$

14. In a curtis stage turbine steam enters the first row of moving blades at 700m/s. The outlet angles of the first rotor blade, the stator blade and the last rotor blade are respectively, 23^o, 19^o and 37^o. If the mean bade speed is 160m/s and the blade coefficient is 0.93 for all blades and steam flow rate is 162kg/min, Discharge is axial. estimate: i) Power developed in the stage ii) rotor efficiency iii) axial thrust and iv) tangential force on the blades
15. In a curtis stage turbine steam enters the first row of moving blades at 700m/s. The outlet angles of the firstrotor blade, the stator blade and the last rotor bade are respectively, 23^o, 19^o and 37^o.and the blade coefficient is 0.93 for all blades and steam flow rate is 162kg/min, Discharge is axial. estimate: i) Power developed in the stage ii) rotor efficiency iii) axial thrust and iv) tangential force on the blades
16. A curtis impulse stage has two rotors moving with an average tangential speed of 250m/s. The fluid relative velocity is reduced 10% in its passage over every blade, whether fixed or moving. The nozzle is inclined at an angle of 17^o to the wheel tangent, has an efficiency of 0.92. The inlet angle of the first rotor blade is 22^o. **The intermediate stator inlet and exit angles are respectively 31.5^o and 20^o**. Assuming that the second rotor is of equiangular. Find: i) the absolute velocity V_1 and the speed ratio ii) the ratio of work output from the second rotor to that of the first rotor iii) stage efficiency and iv) the power output and axial thrust for a flow of 5kg/s of steam over the blade
17. The mean rotor blade speed of an axial flow turbine stage with 50% reaction is 200m/s. The steam emerges from the nozzle at 28^o to the wheel plane with axial velocity component equal to the blade speed. Assuming symmetric inlet and outlet velocity triangles, find the rotor blade angles and utilization factor

18. A Parsons turbine is running at 1200rpm. The mean rotor diameter is 1m. Blade outlet angle is 23^o, speed ratio is 0.75 , stage efficiency is 0.8. Find the isentropic enthalpy drop in this stage (6b, 08 Deec18/Jan19, 15 CBCS)

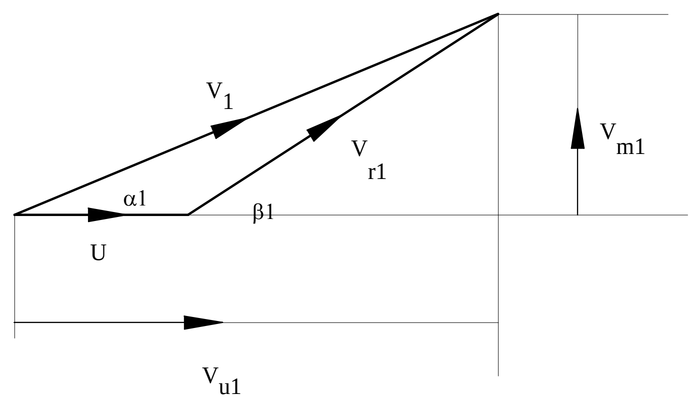
Parson turbine ie 50% Reaction turbine; $N = 1200rpm; D = 1m;\ \ \beta_2 = 23^o;$

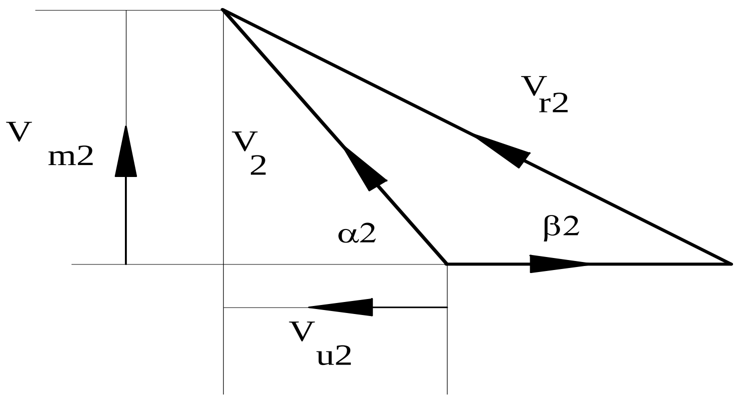
Speed ratio $= 0.75$ ie $\frac{U}{V_1} = 0.75; \eta_s = 0.8$

For $50\%R$ turbine $\alpha_1 = \beta_2;\ \ \alpha_1 = 23^o$

$U = \frac{\pi DN}{60};$ $\quad U = \frac{\pi*1*1200}{60};$ $\quad U = 62.83m/s$

$\frac{U}{V_1} = 0.75;$ $\quad \frac{62.83}{V_1} = 0.75;$ $\quad V_1 = 83.78m/s$





$\overrightarrow{V_{u1}} = V_1 cos\alpha_1$; $\overrightarrow{V_{u1}} = 83.78\ cos23$; $\overrightarrow{V_{u1}} = 77.12\ m/s$

$V_{m1} = V_1 Sin\alpha_1$; $V_{m1} = 83.78\ Sin\ 23$ $V_{m1} = 32.73m/s$

For 50%R turbine, $V_{r2} = V_1$; $V_{r2} = 83.78m/s$

$\overleftarrow{V_{u2}} = V_{r2}\ cos\ \beta_2 - U$; $\overleftarrow{V_{u2}} = 83.78\ cos\ 23 - 62.83$; $\overleftarrow{V_{u2}} = 14.28m/s$

$\frac{E}{\dot{m}} = \frac{(\overrightarrow{V_{u1}}+\overleftarrow{V_{u2}})U}{g_c}$; $\frac{E}{\dot{m}} = \frac{(77.12\ +14.29\ )62.83}{1}$; $\frac{E}{\dot{m}} = 5743.29\ J/kg$;

$\Delta h_0 = \frac{E}{\dot{m}}$; $\Delta h_0 = 5743.29\ J/kg$

$\eta_s = \frac{\Delta h_0}{\Delta h_{0s}}$; $0.8 = \frac{5743.29}{\Delta h_{0s}}$; $\Delta h_{0s} = 7179.11J/kg$

Graphical Solution

Parson turbine ie 50% Reaction turbine; $N = 1200rpm$; $D = 1m$; $\beta_2 = 23^o$;

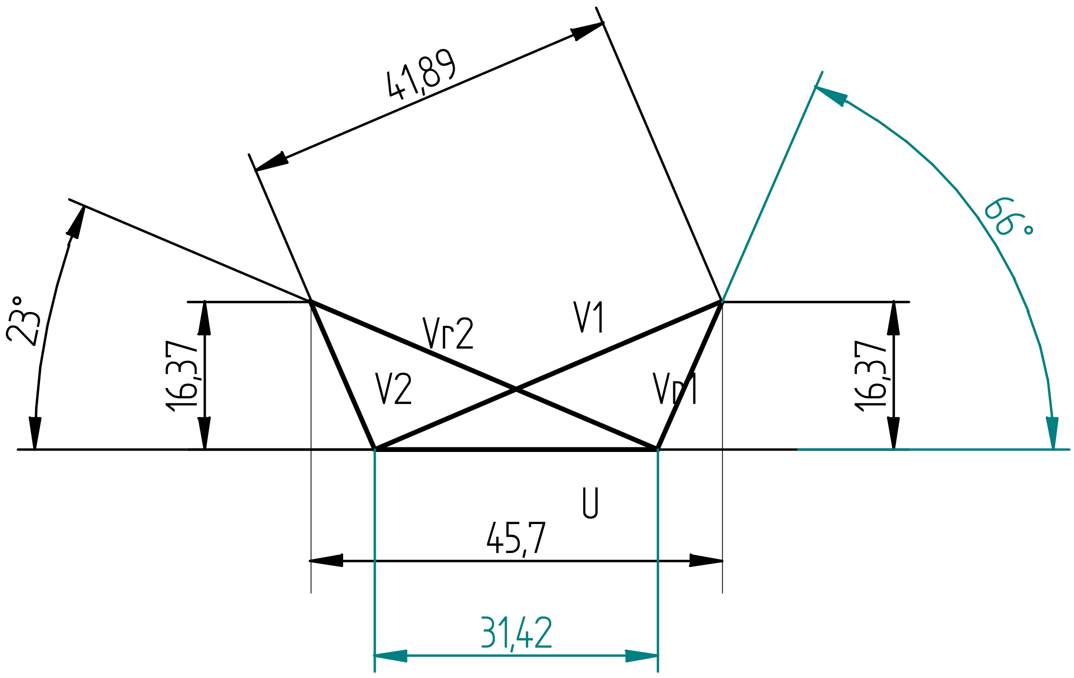
Speed ratio = 0.75 ie $\frac{U}{V_1} = 0.75$; $\eta_s = 0.8$

For 50%R turbine $\alpha_1 = \beta_2$; $\alpha_1 = 23^o$

$U = \frac{\pi DN}{60}$; $U = \frac{\pi*1*1200}{60}$; $U = 62.83m/s$

$\frac{U}{V_1} = 0.75$; $\frac{62.83}{V_1} = 0.75$; $V_1 = 83.78m/s$

$Scale\ 1cm = 20m/s;\ \ U = \frac{62.83}{20}cm\ ;\ U = 3.142cm;\ V_1 = \frac{83.78}{20}cm;\ V_1 = 4.189cm$



$\Delta V_u = 4.57cm;\ \ \Delta V_u = 4.57 * 20;\ \ \ \Delta V_u = 91.4m/s$

$\frac{E}{\dot{m}} = \frac{(\overrightarrow{V_{u1}}+\overleftarrow{V_{u2}})U}{g_c};\ \ \ \frac{E}{\dot{m}} = \frac{(\Delta V_u)U}{g_c}\ \ \frac{E}{\dot{m}} = \frac{91.4*62.83}{1};\ \ \frac{E}{\dot{m}} = 5742.62\ J/kg;$

$\Delta h_0 = \frac{E}{\dot{m}};\ \ \ \Delta h_0 = 5742.62\ J/kg$

$\eta_s = \frac{\Delta h_0}{\Delta h_{0s}};\ \ \ \ \ 0.8 = \frac{5742.62}{\Delta h_{0s}};\ \ \ \Delta h_{0s} = 7178.33\ J/kg$

19. The following particulars refer to a stage of a parsons steam turbine. Mean diameter of blade ring =70cm, steam velocity at inlet of moving blades=160m/s, outlet blade angles of moving blades $\beta_2 = 20^0$, steam flow through the blades =7kg/s and speed 1500rpm ,$\eta = 0.8$. Draw the velocity diagram and find the following: i) Blade inlet angle ii) Power developed in the stage iii) Available isentropic enthalpy drop (5b, 08,June/July1815CBCS)

Parson Turbine- 50%; $D = 70cm\ D = 0.7m;$

steam velocity at inlet of moving blades=160m/s, $V_1 = 160m/s;$

outlet blade angles of moving blades $\beta_2 = 20^0$,

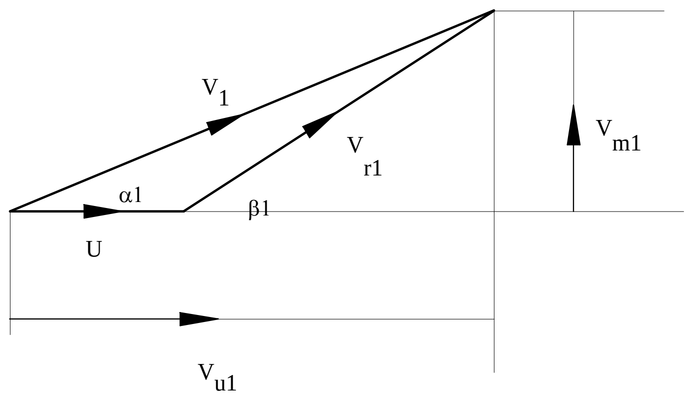
steam flow through the blades =7kg/s $\dot{m} = 7kg/s;$ $N = 1500rpm;$ $\eta = 0.8$

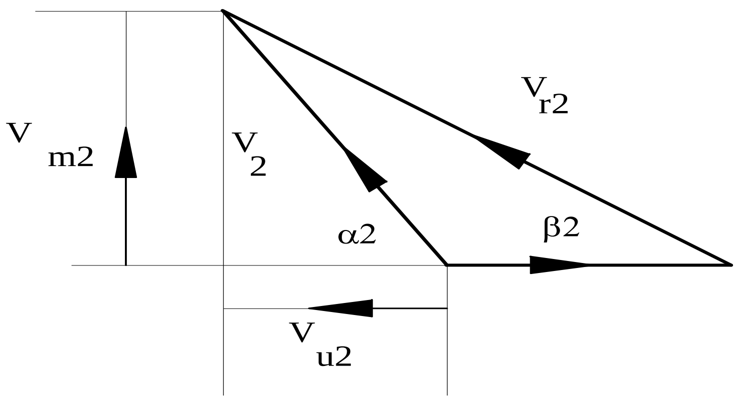
i) Blade inlet angle β_1 =? ii) Power developed in the stage E =?

iii) Available isentropic enthalpy drop Δh_{os} =?

$U = \frac{\pi DN}{60}$; $\quad U = \frac{\pi * 0.7 * 1500}{60}$; $\quad U = 54.98 m/s$

For 50%R turbine $\alpha_1 = \beta_2$; $\alpha_1 = 20^o$





$\overrightarrow{V_{u1}} = V_1 cos\alpha_1$; $\quad \overrightarrow{V_{u1}} = 160\ cos20$; $\quad \overrightarrow{V_{u1}} = 150.35\ m/s$

$V_{m1} = V_1 Sin\alpha_1$; $\quad V_{m1} = 160\ Sin20 \quad V_{m1} = 54.72 m/s$

For 50%R turbine, $\quad V_{r2} = V_1$; $\quad V_{r2} = 160 m/s$

$\overleftarrow{V_{u2}} = V_{r2}\ cos\ \beta_2 - U$; $\quad \overleftarrow{V_{u2}} = 160\ cos\ 20 - 54.98$; $\quad \overleftarrow{V_{u2}} = 95.37 m/s$

$\frac{E}{\dot{m}} = \frac{(\overrightarrow{V_{u1}} + \overleftarrow{V_{u2}})U}{g_c}$; $\quad \frac{E}{\dot{m}} = \frac{(150.35 + 95.37)54.98}{1}$; $\quad \frac{E}{\dot{m}} = 13509.68\ J/kg$;

$\Delta h_0 = \frac{E}{\dot{m}}$; $\quad \Delta h_0 = 13509.68 J/kg$

$\eta_s = \frac{\Delta h_0}{\Delta h_{0s}}$; $\quad 0.8 = \frac{13509.68}{\Delta h_{0s}}$; $\quad \Delta h_{0s} = 16887.10 J/kg$

Graphical Solution

Parson Turbine- 50%; $D = 70cm\ D = 0.7m$;

steam velocity at inlet of moving blades=160m/s, $V_1 = 160 m/s$;

outlet blade angles of moving blades $\beta_2 = 20^0$,

steam flow through the blades =7kg/s $\quad \dot{m} = 7kg/s$; $\quad N = 1500rpm$; $\eta = 0.8$

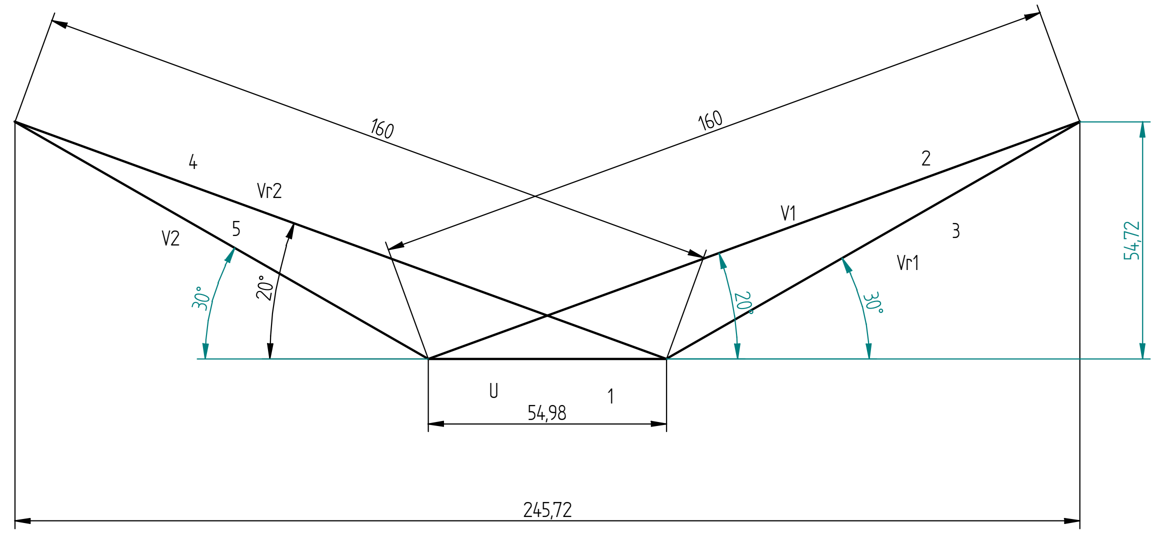
i) Blade inlet angle $\beta_1 =?$ ii) Power developed in the stage $E =?$

iii) Available isentropic enthalpy drop $\Delta h_{os} =?$

$U = \frac{\pi DN}{60}$; $U = \frac{\pi * 0.7 * 1500}{60}$; $U = 54.98 m/s$

For 50%R turbine $\alpha_1 = \beta_2$; $\alpha_1 = 20^o$

Scale : 1cm=10m/s; $U = \frac{54.98}{10} cm$; $U = 5.498\ cm$; $V_1 = \frac{160}{20} cm$; $V_1 = 16cm$



$\Delta V_u = 24.57cm$; $\Delta V_u = 24.57 * 10$; $\Delta V_u = 245.7 m/s$

$\frac{E}{\dot{m}} = \frac{(\overrightarrow{V_{u1}} + \overleftarrow{V_{u2}})U}{g_c}$; $\frac{E}{\dot{m}} = \frac{(\Delta V_u)U}{g_c}$ $\frac{E}{\dot{m}} = \frac{245.72 * 54.98}{1}$; $\frac{E}{\dot{m}} = 13509.68\ J/kg$;

$\Delta h_0 = \frac{E}{\dot{m}}$; $\Delta h_0 = 13509.68\ J/kg$

$\eta_s = \frac{\Delta h_0}{\Delta h_{0s}}$; $0.8 = \frac{13509.68}{\Delta h_{0s}}$; $\Delta h_{0s} = 16887.11\ J/kg$

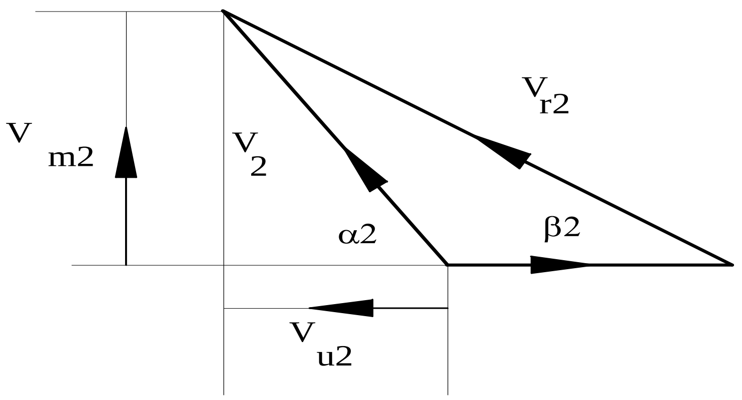
20. The following data refer to a 50% reaction turbine D=1.5m $\rho = \frac{U}{V_1} = 0.72$; $\beta_2 = 20^o$, $N = 3000 rpm$ find $i)$ blade efficiency ii) Determine percentage increase in the blade efficiency and rotor speed if the rotor is designed to run at its best theoretical speed , the exit angle $\alpha_1$is 20°. Blade efficiency for the best speed is given by $\frac{2cos^2\alpha_1}{1+cos^2\alpha_1}$ (7b, 10, June/July 13)

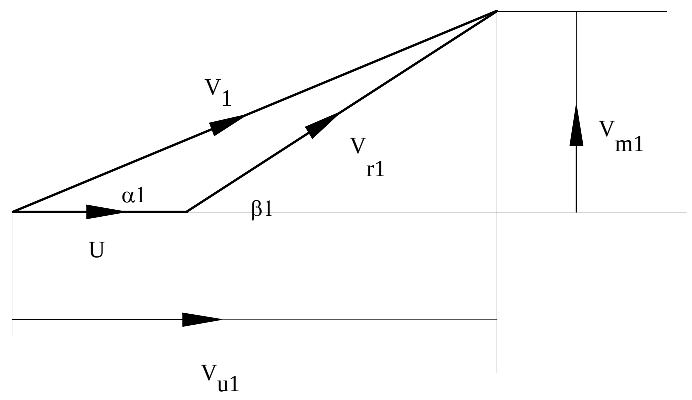
50% Reaction turbine; ; $D = 1.5m$; $\rho = \frac{U}{V_1} = 0.72$; $\beta_2 = 20^o$; $N = 3000rpm$;

$\eta_b =?$ ii) Determine percentage increase in the blade efficiency and rotor speed if the rotor is designed to run at its best theoretical speed , the exit angle $\alpha_1$is 20°. Blade efficiency for the best speed is given by $\frac{2cos^2\alpha_1}{1+cos^2\alpha_1}$

For 50%R turbine $\alpha_1 = \beta_2$; $\alpha_1 = 20^o$

$U = \frac{\pi DN}{60}$; $U = \frac{\pi * 1.5 * 3000}{60}$; $U = 235.61 m/s$





$\frac{U}{V_1} = 0.72$; $\frac{235.62}{V_1} = 0.72$; $V_1 = 327.25 m/s$

$\overrightarrow{V_{u1}} = V_1 cos\alpha_1$; $\overrightarrow{V_{u1}} = 327.25\ cos 20$; $\overrightarrow{V_{u1}} = 307.51\ m/s$

$V_{m1} = V_1 Sin\alpha_1$; $V_{m1} = 327.25\ Sin\ 20$ $V_{m1} = 119.27 m/s$

For 50%R turbine, $V_{r2} = V_1$; $V_{r2} = 327.25 m/s$

$\overleftarrow{V_{u2}} = V_{r2}\ cos\ \beta_2 - U$; $\overleftarrow{V_{u2}} = 327.25\ cos\ 20 - 235.61$; $\overleftarrow{V_{u2}} = 71.90 m/s$

$\frac{E}{\dot{m}} = \frac{(\overrightarrow{V_{u1}} + \overleftarrow{V_{u2}})U}{g_c}$; $\frac{E}{\dot{m}} = \frac{(307.51 + 71.90)235.61}{1}$; $\frac{E}{\dot{m}} = 89393.82\ J/kg$;

$V_{m2} = V_{m1}$; $V_{m2} = 119.27 m/s$; $V_2^2 = V_{u2}^2 + V_{m2}^2$

$V_2^2 = 71.90^2 + 119.27^2$; $V_2^2 = 19394.94$

$\eta_b = \frac{\frac{E}{\dot{m}}}{\frac{E}{\dot{m}} + \frac{V_2^2}{2g_c}}$; $\eta_b = \frac{89393.82}{89393.82 + \frac{19394.94}{2*1}}$; η_b =0.902

For best theoretical speed $\eta_{b1} = \frac{2cos^2\alpha_1}{1+cos^2\alpha_1}$

$\eta_{b1} = \frac{2cos^2 20}{1+cos^2 20}$; $\eta_{b1} = 0.937$

Percentage increase in efficiency $\frac{\eta_{b1}-\eta_b}{\eta_b} * 100$

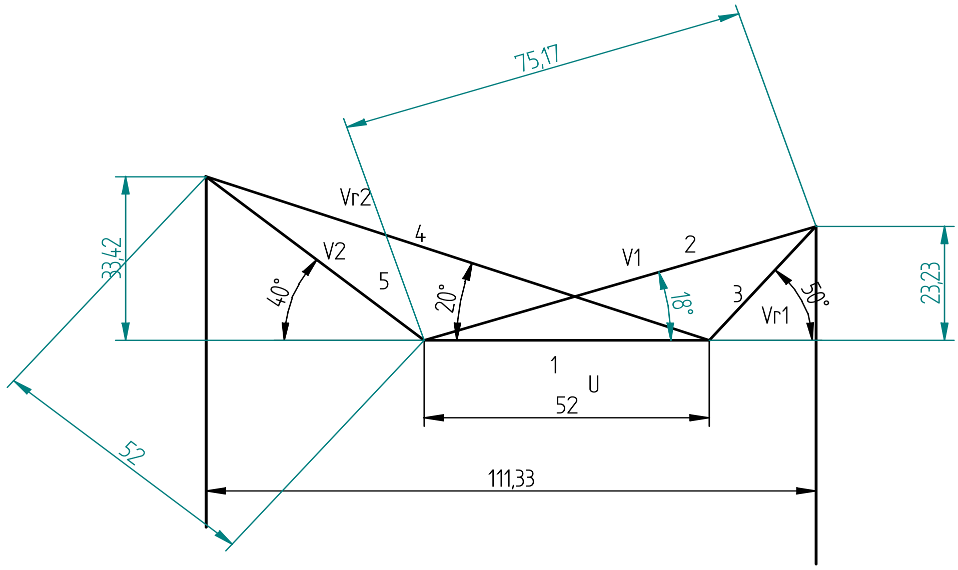
$\frac{93.7-90.2}{90.2} x100$;   3.88%

$optimum\ speed\ ratio\ for\ maximum\ efficiency\ \emptyset = cos\alpha_1$

$\frac{U}{V_1} = cos\alpha_1$; $\frac{U}{V_1} = cos20$; $\frac{U}{V_1} = 0.94$; $\frac{U}{327.25} = 0.94$; $U = 307.51 m/s$

$U = \frac{\pi DN}{60}$; $307.51 = \frac{\pi * 1.5 * N}{60}$; $N = 3915.33 rpm$

21. In a reaction turbine , the inlet and outlet blade angles are 50° and 20° respectively. Steam enters at 18° to the plane wheel and leaves at 40° . The rotor speed is 260m/s. Calculate the speed ratio , specific work and degree of reaction (5c. 08, Dec15/Jan16)
    Reaction turbine
    the inlet and outlet blade angles are 50° and 20° respectively ie $\beta_1 = 50^0$; $\beta_2 = 20^0$
    Steam enters at 18° to the plane wheel and leaves at 40° ie $\alpha_1 = 18^0$; $\alpha_2 = 40^0$
    The rotor speed is 260m/s ie $U = 260 m/s$
    Scale 1cm =50m/s



1. Draw line U
2. Draw V_1 line at an angle at $\alpha_1 = 18^0$
3. Draw V_{r1} line at an angle $\beta_1 = 50^0$; V_1line and V_{r1} line intersect each other ie end of V_1 and V_{r1}
4. Draw V_{r2} line at an angle $\beta_1 = 20^0$
5. Draw V_2 line at an angle at $\alpha_1 = 40^0$; V_{r2} line and V_2 line intersect each other ie is the end of V_2 and V_{r2}

Speed ratio

Measure V_1ie 7.517cm ie $V_1 = 7.517 * 50$ $V_1 = 375.85 m/s$

Speed ratio $= \frac{U}{V_1}$ ie Speed ratio $= \frac{260}{375.85}$ Speed ratio= 0.69

Specific work output

Measure $\Delta V_u = 11.133cm;$ $\Delta V_u = 11.133 * 50$ $\Delta V_u = 555.65m/s$

$\frac{E}{\dot{m}} = \frac{\Delta V_u U}{g_c};$ $\frac{E}{\dot{m}} = \frac{555.65*260}{1};$ $\frac{E}{\dot{m}} = \frac{144729J}{kg}$

Degree of Reaction

Measure V_2 ie 5.2cm cm ie $V_1 = 5.2 * 50$ $V_1 = 260m/s$

$$R = \frac{\frac{E}{m} - \frac{(V_1^2 - V_2^2)}{2g_c}}{\frac{E}{m}}; \qquad R = 1 - \frac{(V_1^2 - V_2^2)}{\frac{E}{m}2g_c}; \qquad R = 1 - \frac{375.85^2 - 260^2}{144729*2*1} \qquad R = 0.254$$

22. At a stage in reaction turbine the mean blade ring diameter is 1m and the turbine runs at a speed of 50rev/s. The blades are designed for 50% reaction with exit angles 60^o and inlet angles 40^o with respect to the axial direction. The turbine is supplied with steam at the rate of 60000 kg/hr and the stage efficiency is 85%. Determine a) the power output of the stage, (b) the ideal specific enthalpy drop in the stage in kJ/kg and c) the percentage increase in relative velocity
23. One stage of an impulse turbine consists of a nozzle and one ring of moving blades. The nozzle is inclined a 22^o to the tangential speed of blades and the blade tip angles are equiangular and equal to 35^o (a) Find the blade speed, diagram efficiency by neglecting losses, if the velocity of steam at the exit of the nozzle is 660m/s (b) If the relative velocity of steam is reduced by 15% in passing through the blade ring. Find the blade speed, diagram efficiency and end thrust on the shaft when the blade ring develops 1745kW
24. A 50% reaction steam turbine, running at 7445rpm develops 5MW and has a steam mass flow rate of 6.5kg/kWhr. At a particular stage in the expansion the absolute pressure is 85kPa at a steam dryness fraction of 0.94. If the exit angle of the blade is 70^o measured from the axial flow direction, and the outlet relative velocity of the steam is 1.3 times the mean blade speed, find the blade height if the ratio of rotor hub diameter to blade height is 14
25. In a reaction turbine, the blade tips are inclined at 35^o and 20^o in the direction of rotor. The stator blades are the same shape as the moving blades, but reversed in direction. At a certain place in the turbine, the drum is 1m diameter and the blades are 10cm high. At this place, the steam has pressure of 1.75 bar and dryness is 0.935. If the speed of the turbine is 250rpm and the steam passes through the blades without shock find the mass of steam flow and power developed in the ring of moving blades
26. Consider a 2 stage velocity compounded axial flow impulse steam turbine. The absolute velocity of steam entering the first row of moving blades=450m/s. The steam leaves the last row of moving blades axially. The blade angles at inlet and outlet of both the rotors are the same and equal to 30^o . Sketch the velocity triangle at inlet and outlet of each stage separately. Find the blade speed
27. The velocity of steam at the exit of a nozzle is 440m/s which is compounded in an impulse turbine by passing successively through moving, fixed, and finally through a second ring of moving blades, The tip angles of moving blades through out the turbine are 30^o.Assume loss of 10% in velocity due to friction when the steam passes over a blade ring. Find the velocity of moving blades in order to have a final discharge of steam as axial. Also determine the diagram efficiency

28. Given data for a two-wheel velocity compounded Curtis steam turbine stage are as follows: mean rotor speed=450m/s. Rotor exit angles are 22^o and 33^o respectively for the 2 rotors, stator blade exit angle=20^o. Blade velocity coefficient for each blade (stator and rotors) = 0.95. Assume axial discharge (assume rotors are to be equiangular). Draw velocity triangles to a scale. Find nozzle angle at inlet and the rotor efficiency
29. The following details refers to a Curtis turbine: i) both rotors are equiangular (ii) rotor blade angles are 29^o in first rotor and 32^o in second rotor (iii) absolute velocity of steam entering the first row of blades is 530m/s iv) blade coefficients are 0.9 in first rotor, 0.91 in stator and 0.93 in second rotor v) the absolute velocity is axial from the second rotor. Find graphically or otherwise a) mean blade speed b) power output for a flow rate of 3.2kg/s

HYDRAULIC TURBINES

1. Classify Hydraulic turbines with example (6a, 05, Dec13/Jan14)
2. With mathematical expression , define the following i) Hydraulic efficiency ii) Mechanical efficiency iii) overall efficiency iv) Volumetric efficiency $(8a, 08 Dec18/Jan19, 15\ scheme)$ $(6a, 08, Dec12)$
3. Obtain an expression for the workdone per second by water on the runner a pelton wheel and hydraulic efficiency (6a, 10,June/July14)
4. Show that for maximum efficiency of pelton wheel the bucket velocity is equal to half of the jet velocity $U = \frac{V_1}{2}$ $(7a, 08 Dec18/Jan19, 15\ scheme)(7a, 08, June/July18, 15\ scheme)$
5. Show that for a Pelton turbine the maximum hydraulic efficiency is given by $\eta_{max} = \frac{1+C_b cos\beta_2}{2}$ where C_b is the blade velocity coefficient and β_2 is the blade discharge angle (6a, 08, June/July 16) (6a, 08, Dec15/Jan 16) Draw the inlet and exit velocity triangles for a pelton wheel turbine (5a, 10, Dec13/Jan 14) (6b, 08,Dec17/Jan18)
6. Derive an expression for maximum hydraulic efficiency of a Pelton wheel in terms of runner tip angle and bucket velocity coefficient (6a, 10,June/July18)
7. Draw an neat sketch of a Francis turbine and draw velocity triangles at inlet and outlet (6c, 05, Dec13/Jan14)
8. Define the draft tubes with neat sketch. Explain different type of draft tubes (6a, 05,June/July 17) (6b, 05,June/July 17)
9. Explain the function of draft tubes and mention its types (6b, 04, June/July 16) (6b, 04, Dec15/Jan 16) (6a, 05,Dec17/Jan18)
10. Write short note on draft tubes in a reaction hydraulic turbine (6a, 04,Dec14/Jan15)
11. Show that pressure at the exit of the reaction turbine with draft tube is less than atmospheric pressure (8a,08,June/July18, 15 scheme)
12. With a meat sketch , explain the working of Kaplan turbine , Mention the functions of draft tube (6a, 10, Dec16/Jan17)
13. Draw the cross sectional view of a Kaplan turbine and explains its working . Also sketch the velocity triangles at inlet and outlet (6a, 10, Dec18/Jan19)
14. Define mechanical efficiency and overall efficiency of turbines

Hydraulic turbine is a turbomachine which converts Hydraulic energy into mechanical energy by dynamic action of water flowing from a high level . Hydrualic energy is in the fom of potential and kinetic

Classifiacation

1. Based on the type of hydraulic enrgy at the inlet of turbine:
   i) Impulse turbine in which inlet energy is in kinetic form ex: Pelton wheel , Turgo wheel
   ii) Reaction turbine in which inlet energy is in the form of kinetic and pressure Ex : Francis , Propeller, Kaplan, tubular Bulb
2. Based on the direction of flow of water through the runner
   i) Tangential flow in which water flows in a direction to path of the rotation example : Pelton wheel
   ii) Radial flow –a) radial inward b) radial outward
   In radial inward water flows along radius of runner from outer diameter to inner diameter ex Francis
   In radial outward flow turbine water flows from inner diameter to outer diameter ex: Forneyron
   iii) Axial flow: water flows parallel to the axis of the turbine Ex Kaplan turbine
   iv) Mixed flow : water enters radially at outer periphery and leaves axially example : modern Francis turbine
3. Based on the pressure head under which turbine work
   i) High Head Impulse turbine example: Pelton wheel
   ii) Medium Head reaction turbine Ex : Frnacis
   iii) Low Head reaction turbine Ex: Kaplan , Propeller
4. Based on the specific speed of the turbine
   i) Low specific speed, Impulse turbine Ex Pelton wheel
   ii) Medium specific reaction turbine Ex Francis turbine
   iii) High specific speed reaction turbine Ex Kaplan and propeller turbine

Efficiencies in Hydraulic turbine

1. Hydraulic Efficiency is defined as the ratio of Power developed by the runner to water power available at the inlet of turbine
   $\eta_h = \frac{P_R}{P_W}$;
   $\eta_h = \frac{E}{\omega Q H}$ where E is calculated from Eulers turbine equation and Q is the discharge in m^3/s, H is the net pressure head available at the inlet of turbine
   In Pelton wheel at the inlet of turbine water power is in the form of Kinetic energy
   Hence , $\eta_h = \frac{E}{\frac{1}{2}\dot{m}V_1^2}$ where $\dot{m}$ is the mass flow rate of water flowing through the runner V_1 is the velocity of water enters the turbine

2. Mechanical Efficiency: is the ratio of Power developed at the shaft of turbine to the power developed at the runner

$\eta_{mech} = \frac{P_s}{P_R}; \quad \eta_{mech} = \frac{P_s}{E}$

3. Overall efficiency : is the ratio of power developed at the shaft to the water power available at the inlet of turbine

$\eta_0 = \frac{P_s}{P_w}; \quad \eta_0 = \frac{P_R}{P_w} * \frac{P_s}{P_R}; \quad \eta_0 = \eta_h * \eta_{mech}$

4. Volumetric Efficiency: is the ratio of actual volume of water used in the energy transfer to the volume of water supplied to the turbine

$$\eta_V = \frac{Q_a}{Q_{th}}$$

If the volumetric efficiency is given in the problem, then actual hydraulic efficiency is

$\eta_{ha} = \eta_h * \eta_V$

Pressure Heads:

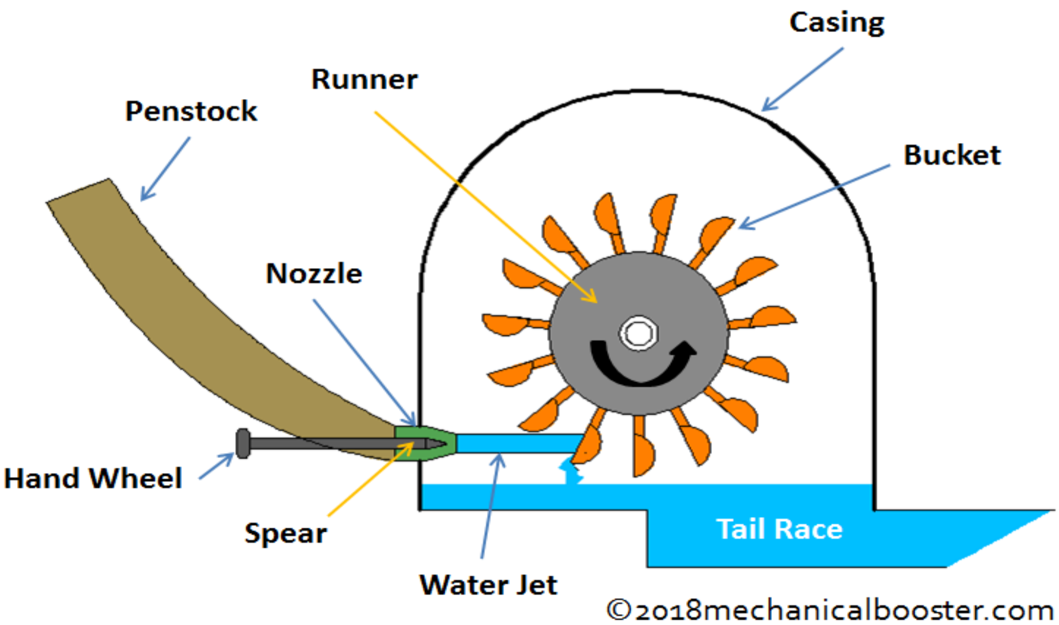
Gross Head, H_g : Pressure head of water available at the Dam

Friction Head H_f : Pressure head of water lost due to friction in the penstock which supplies the water from Dam to turbine

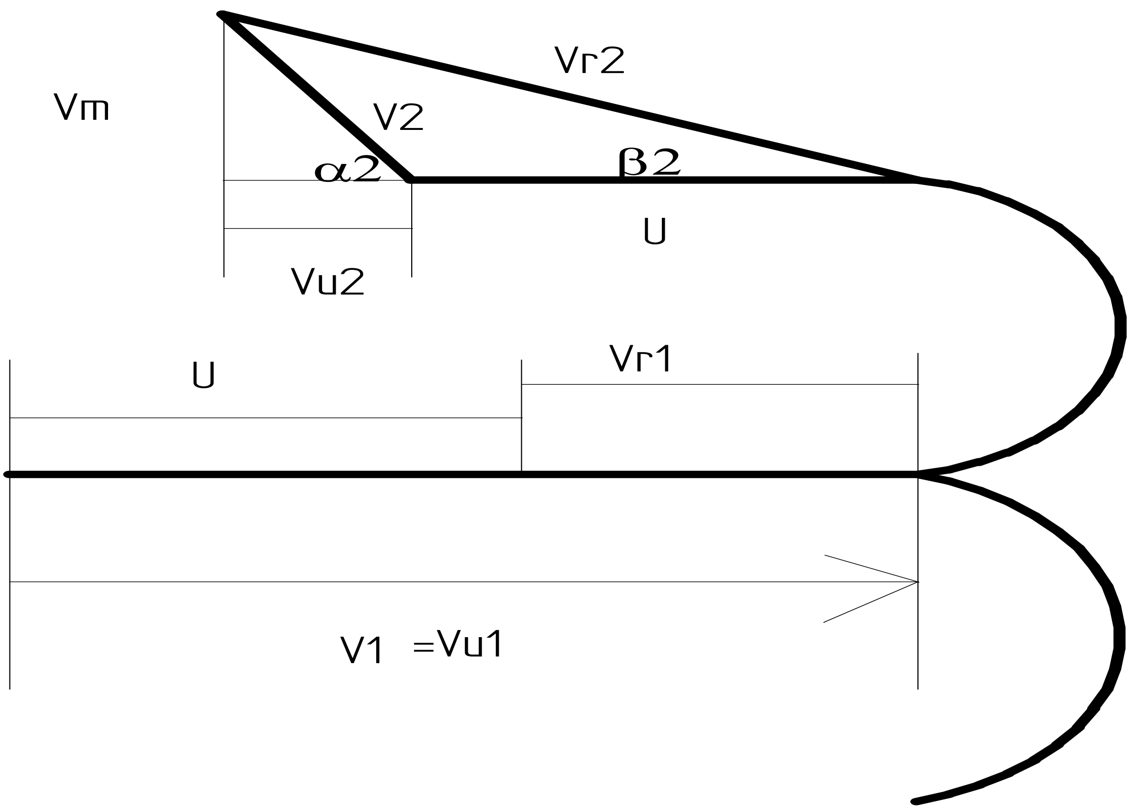
Net Head H: Pressure Head available at the inlet of turbine $H = H_g - H_f$

Pelton wheel

The water from the reservoir at higher position flows down through penstocks end of which is fitted with nozzle. In a nozzle potential energy is converted into kinetic energy. The high velocity from the exit of nozzle strikes the buckets fitted at the periphery of the rotor at centre. Water while passing through the buckets there is change in velocity and direction which results in change in momentum. The tangential force induced due to change in tangential component of fluid in buckets sets the rotor in rotary motion. Thus kinetic energy of fluid converted into work.



Main Parts of Impulse Turbine



The tangential force =mass flow rate ∗ change in velocity of fluid in tangential direction

$F_u = \frac{\dot{m}}{g_c}(\overrightarrow{V_{u1}} - \overrightarrow{V_{u2}})$; $F_u = \frac{\dot{m}}{g_c}(\overrightarrow{V_{u1}} + \overleftarrow{V_{u2}})$

Torque $T = \frac{\dot{m}}{g_c}\left(\overrightarrow{V_{u1}} + \overleftarrow{V_{u2}}\right) * R;$ $\qquad T = \frac{\dot{m}}{g_c}\left(\overrightarrow{V_{u1}}R + \overleftarrow{V_{u2}}R\right)$

Power : $E = \frac{\dot{m}}{g_c}\left(\overrightarrow{V_{u1}} + \overleftarrow{V_{u2}}\right)U$

Axial thrust: $F_a = \frac{\dot{m}}{g_c}\left(V_{f1} - V_{f2}\right)N$

Important design parameters for Pelton wheel

- Jet velocity emerging from nozzle $V_1 = C_V\sqrt{2gH};$ where C_V is coefficient of velocity ie 0.96 to 0.98, H is the net head available ie $H_g - H_f$, where H_g is the gross head available at the Dam, H_f is the friction loss in penstocks
- Speed ratio $\phi = \frac{U}{\sqrt{2gH}}$ ie 0.43 to 0.46
- Jet ratio m : $\frac{D}{d}$ where D is the runner diameter , d is the jet diameter ie 14 to 16
- No of Buckets $Z = \frac{m}{2}$ ie $Z = \frac{D}{2d}$

Hydraulic Efficiency (Theoritical)

Blade efficiency or Diagram efficiency or Utilization factor is given by is difined as the ratio of power developed by the runner to the water power available in the turbine

$$\eta_b = \frac{Power\ developed\ by\ the\ runner}{water\ power}$$

$$\eta_b = \frac{m\Delta V_u\, U}{\omega QH}; \qquad \eta_b = \frac{\rho Q\Delta V_u\, U}{\rho gQH} \qquad \eta_b = \frac{\Delta V_u\, U}{gH}$$

Water power =ωQH where ω is specific weight of the fluid (Since here fluid is water ω is 9810 N/m3, Q is the rate of flow of water in m^3/s and H is the net head available at the inlet of turbine

Above equation is holds good for all the hydraulic turbine

For Impulse turbine (Pelton wheel) water power also equal to $m\frac{V_1^2}{2}$

$$\eta_b = \frac{m\left(\left(\overrightarrow{V_{u1}}+\overleftarrow{V_{u2}}\right)\right)U}{\frac{mV_1^2}{2}} \Big/ \eta_b = \frac{m\left(\overrightarrow{V_{u1}}-\overrightarrow{V_{u2}}\right)U}{\frac{mV_1^2}{2}}\ ; \qquad \eta_b = \frac{2\Delta V_u\, U}{V_1^2}$$

Above equation is holds good only for impulse turbine

II) **Volumetric Efficiency**: It is the ratio of quantity of water actually striking the runner to the quantity of water supplied to the runner

$\eta_v = \frac{Q_a}{Q_{th}} = \frac{Q-\Delta Q}{Q}$ where ΔQ is the amount of water that slips directly to the tail race without striking or it is known as leakage or loss

Actual Hydraulic efficiency is the product of theoretical hydraulic efficiency and Volumetric efficiency

$\eta_h = \eta_b \eta_v$

iii) Mechanical Efficiency: is the ratio of shaft power output by the turbine to the power developed by the runner

$$\eta_m = \frac{Shaft\ Power\ out\ put}{Power\ developed\ by\ the\ runnerwater\ power}$$

$$\boldsymbol{\eta_m = \frac{P_s}{\rho Q \Delta V_u\ U}}$$

iv) Overall efficiency: is defined as ratio of shaft output power by the turbine to the water power available at inlet of the turbine

$\eta_o = \frac{Shaft\ Power\ out\ put}{water\ power}$; $\eta_o = \frac{\boldsymbol{P_s}}{\boldsymbol{\omega Q H}}$

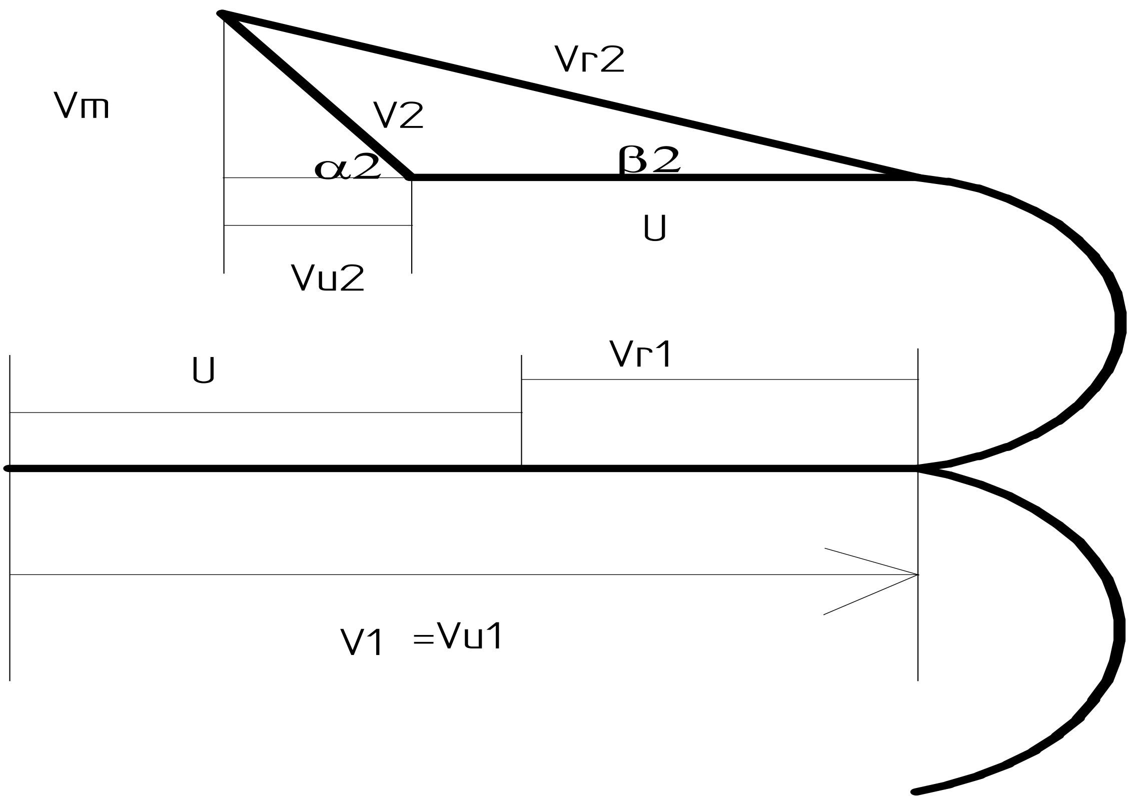
$\eta_o = \eta_{hact} * \eta_m$ $\eta_o = \eta_{htheortical} x \eta_v\ x \eta_m$

Generator Efficiency : is the ratio of Generator output to Power at the shaft

$$\eta_o = \frac{P_g}{P_s}$$

It converts large portion of velocity energy rejected from the runner into useful pressure energy

Work done by the Pelton wheel



$\frac{E}{\dot{m}} = \frac{1}{g_c}(\overrightarrow{V_{u1}} + \overleftarrow{V_{u2}})U;$

$\overrightarrow{V_{u1}} = V_1;$ $\quad V_{r1} = V_1 - U;$ $\quad$ Blade friction coefficient $K = \frac{V_{r2}}{V_{r1}}$

$V_{r2} = KV_{r1};$ $\quad V_{r2} = K(V_1 - U)$

From outlet velocity triangle; $\overleftarrow{V_{u2}} = V_{r2}cos\beta_2 - U;$ $\overleftarrow{V_{u2}} = K(V_1 - U)cos\beta_2 - U$

$\dot{m} = \rho Q;$

$$\frac{E}{\dot{m}} = \frac{1}{g_c}(V_1 + (K(V_1 - U)cos\beta_2 - U))U$$

$\frac{E}{\dot{m}} = \frac{1}{g_c}[V_1 - U + K(V_1 - U)cos\beta_2]U;$ $\quad \frac{E}{\dot{m}} = \frac{1}{g_c}(V_1 - U)(1 + Kcos\beta_2)U$

$\frac{E}{\rho Q} = \frac{1}{g_c}(V_1 - U)(1 + Kcos\beta_2)U;$ $\quad E = \frac{1}{g_c}(V_1 - U)(1 + Kcos\beta_2)U$

Blade efficiency/Hydraulic efficiency

$\eta_b = \frac{\frac{E}{\dot{m}}}{\frac{V_1^2}{2g_c}}$; $\eta_b = \frac{\frac{1}{g_c}(V_1 - U)(1 + K\cos\beta_2)U}{\frac{V_1^2}{2g_c}}$;

$\eta_b = \frac{(V_1 - U)(1 + K\cos\beta_2)U}{\frac{V_1^2}{2}}$; $\eta_b = \frac{2(V_1 - U)}{V_1} * \frac{U}{V_1} * (1 + K\cos\beta_2)$

Speed ratio $\Phi = \frac{U}{V_1}$

$\eta_b = 2(1 - \Phi)\Phi(1 + K\cos\beta_2)$

For Maximum efficiency

$\frac{d\eta_b}{d\Phi} = 0$; $\frac{d(2(1-\Phi)\Phi(1+K\cos\beta_2))}{d\Phi} = 0$; $2(1 + K\cos\beta_2)\frac{d((1-\Phi)\Phi)}{d\Phi} = 0$

$\frac{d(1-\Phi)\Phi}{d\Phi} = 0$; $\frac{d(\Phi - \Phi^2)}{d\Phi} = 0$; $1 - 2\Phi = 0$; $\Phi = \frac{1}{2}$;

$\frac{U}{V_1} = \frac{1}{2}$; $U = \frac{V_1}{2}$

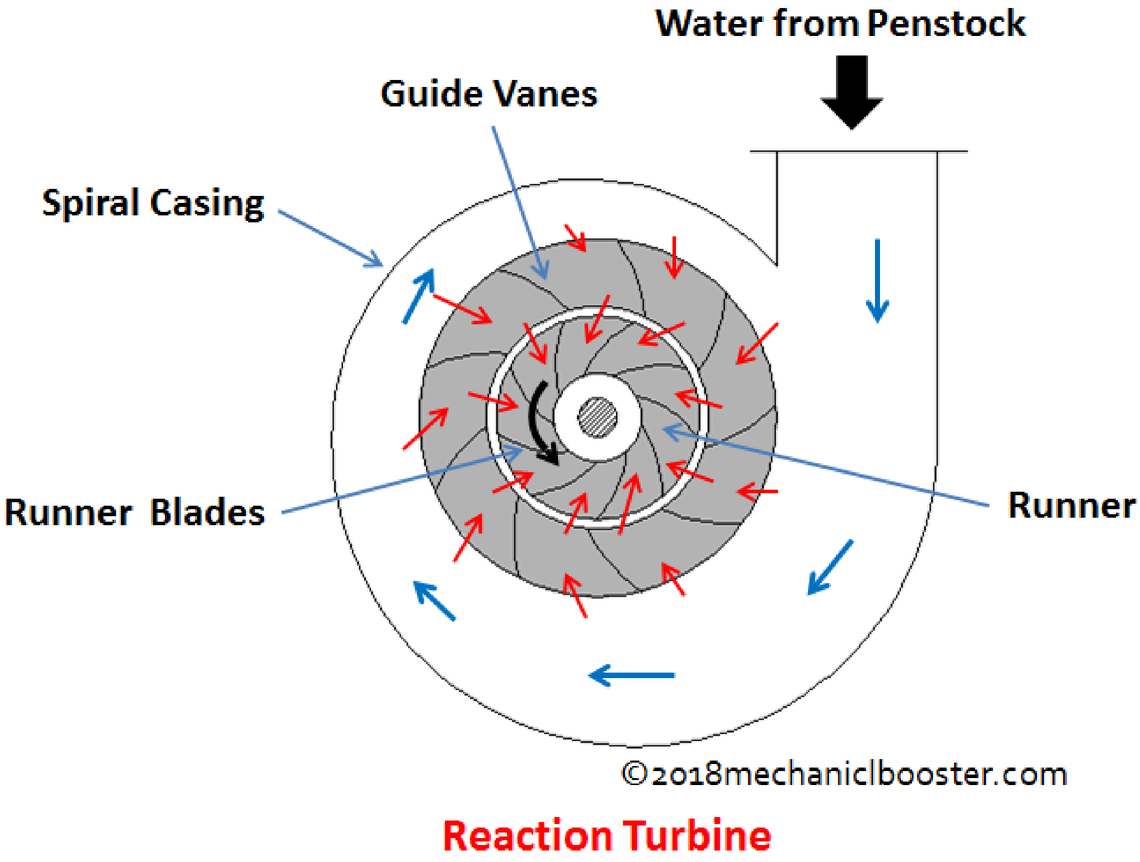
Hence blade speed = 50% of jet velocity for maximum efficiency

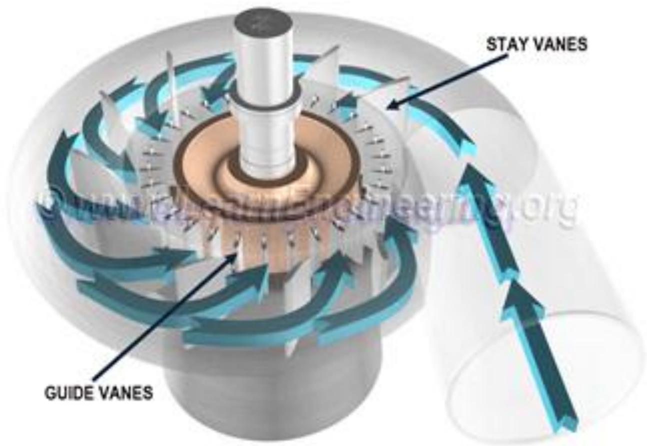
Max Efficiency $\eta_{bmax} = 2\left(1 - \frac{1}{2}\right)\frac{1}{2}(1 + K\cos\beta_2)$; $\eta_{bmax} = 2 * \frac{1}{2} * \frac{1}{2}(1 + K\cos\beta_2)$

$\eta_{bmax} = \frac{(1 + K\cos\beta_2)}{2}$

FRANCIS TURBINE

Main Components of Francis Turbine





The major components of Francis turbine are

1. Spiral Casing

Spiral casing is the inlet medium of water to the turbine. The water flowing from the reservoir or dam is made to pass through this pipe with high pressure.

To maintain the same pressure the diameter of the casing is gradually reduced, so as to maintain the pressure uniform, thus uniform momentum or velocity striking the runner blades.

2. Stay Vanes
Stay vanes and guide vanes guides the water to the runner blades. Stay vanes remain stationary at their position and reduces the swirling of water due to radial flow, as it enters the runner blades. Thus making turbine more efficient.

3. Guide Vanes
Guide vanes change their angle as per the requirement to control the angle of striking of water to turbine blades to increase the efficiency. They also regulate the flow rate of water into the runner blades thus controlling the power output of a turbine according to the load on the turbine.

4. Runner Blades
The performance and efficiency of the turbine is dependent on the design of the runner blades. In a Francis turbine, runner blades are divided into 2 parts. The lower half is made in the shape of small bucket so that it uses the impulse action of water to rotate the turbine.

The upper part of the blades use the reaction force of water flowing through it. These two forces together makes the runner to rotate.

5. Draft Tube
The pressure at the exit of the runner of Reaction Turbine is generally less than atmospheric pressure. The water at exit cannot be directly discharged to the tail race. A tube or pipe of gradually increasing area is used for discharging water from the exit of turbine to the tail race.

This tube of increasing area is called Draft Tube. One end of the tube is connected to the outlet of runner while the other end is sub-merged below the level of water in the tail-race.

Working Principle

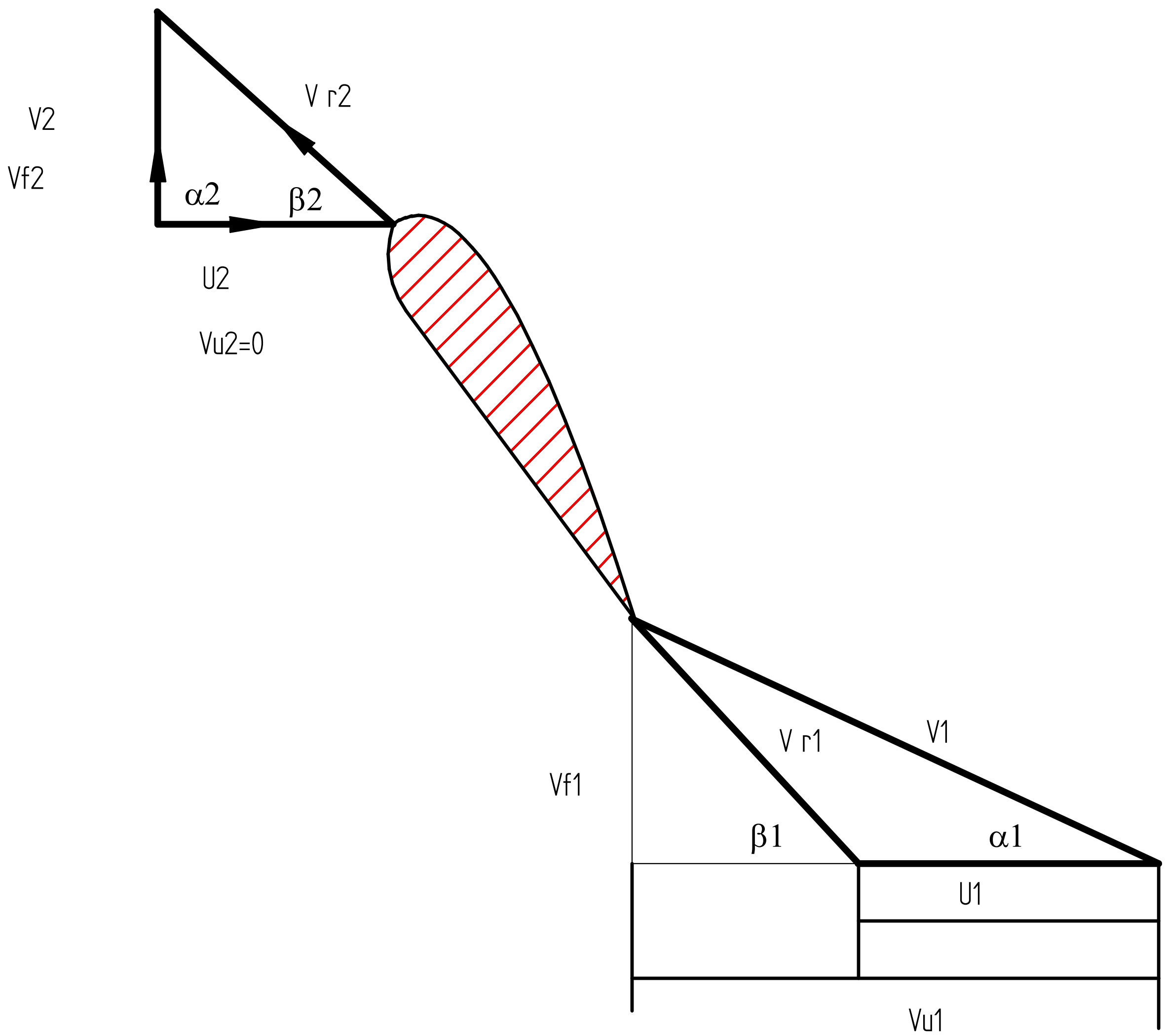
Francis Turbines are generally installed with their axis vertical. Water with high head (pressure) enters the turbine through the spiral casing surrounding the guide vanes. The water looses a part of its pressure in the volute (spiral casing) to maintain its speed. Then water passes through guide vanes where it is directed to strike the blades on the runner at optimum angles. As the water flows through the runner its pressure and angular momentum reduces. This reduction imparts reaction on the runner and power is transferred to the turbine shaft.

If the turbine is operating at the design conditions the water leaves the runner in axial direction. Water exits the turbine through the draft tube, which acts as a diffuser and reduces the exit velocity of the flow to recover maximum energy from the flowing water

In Francis turbine the pressure and velocity of the fluid decreases as it flows through the moving blades. Hence it converts both the kinetic energy and pressure energy is converted into work

The water coming out of runner blades would lack both the kinetic energy and pressure energy, so we use the draft tube to recover the pressure as it advances towards tail race, but still we cannot recover the pressure to that extent that we can stop air to enter into the runner housing thus causing cavitation.

Velocity triangle



Applications of Francis Turbine

- Francis turbine is the most widely used turbine in hydro-power plants to generate electricity.
- Mixed flow turbine is also used in irrigation water pumping sets to pump water from ground for irrigation.
- It is efficient over a wide range of water head and flow rate.
- It is most efficient hydro-turbine we have till date.

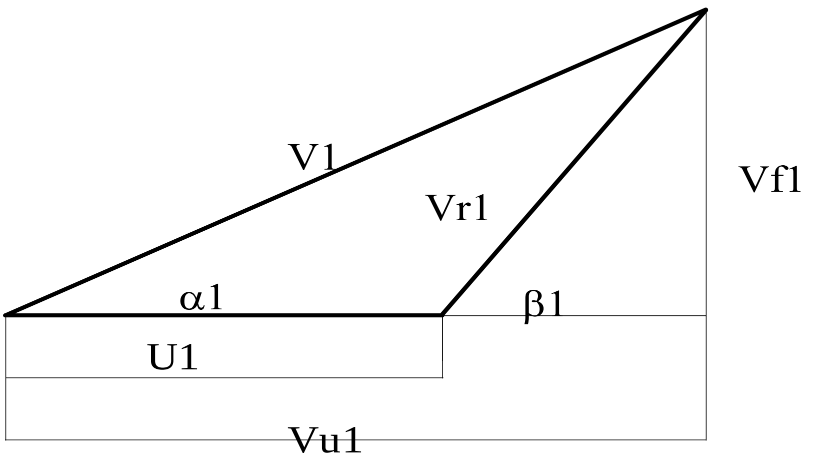
Analysis of Francis Turbine:

$\frac{E}{m} = \overrightarrow{V_{u1}} U_1$ **as** $\overrightarrow{V_{u2}}$**=0** where m =ρQ

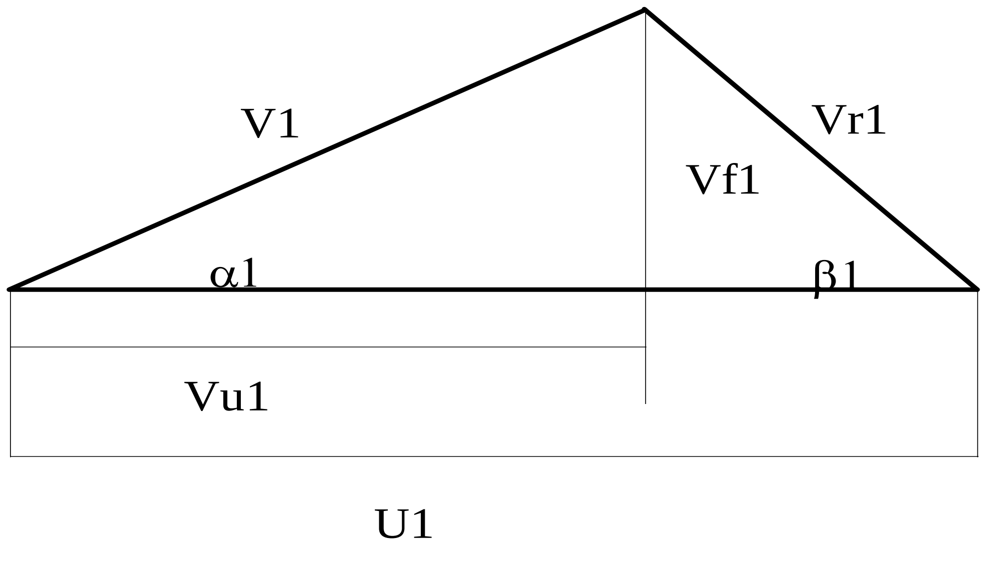
Q=(1−%of Blockage) $\boldsymbol{\pi}$D$_1$B$_1$V$_{f1}$ = (1−%of Blockage) $\boldsymbol{\pi}$D$_2$B$_2$V$_{f2}$

Inlet Velocity Triangle

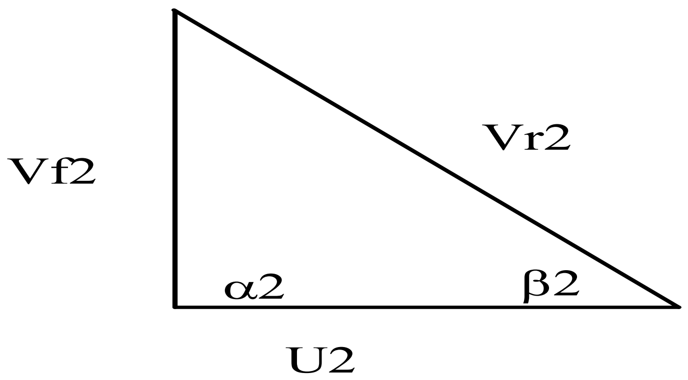
If $V_{u1} > U_1$



If U$_1$>V$_{u1}$



Outlet Velocity Triangle



V_{u2}=0

Hydraulic Efficiency:

Blade efficiency or Diagram efficiency or Utilization factor is given by is difined as the ratio of power developed by the runner to the water power available in the turbine

$$\eta_b = \frac{Power\ developed\ by\ the\ runner}{water\ power}$$

$$\eta_{h(th)} = \frac{\dot{m}(\overrightarrow{V_{u1}}U_1 - \overrightarrow{V_{u2}}U_2)}{\omega QH} \ ; \ \eta_{h(th)} = \frac{\rho Q(\overrightarrow{V_{u1}}U_1)}{\rho g QH} \qquad \overrightarrow{V_{u2}} = 0 \ ; \dot{m} = \rho Q \ ; \omega = \rho g$$

$$\boldsymbol{\eta_{h(th)}} = \frac{\overrightarrow{V_{u1}}U_1}{\boldsymbol{gH}}$$

II) Volumetric Efficiency: It is the ratio of quantity of water actually striking the runner to the quantity of water supplied to the runner

$\eta_v = \frac{Q_a}{Q_{th}} = \frac{Q-\Delta Q}{Q}$ where ΔQ is the amount of water that slips directly to the tail race without striking or it is known as leakage or loss

Actual Hydraulic efficiency is the product of theoretical hydraulic efficiency and Volumetric efficiency

$\eta_h = \eta_b \eta_v$

iii) Mechanical Efficiency: is the ratio of shaft power output by the turbine to the power developed by the runner

$$\eta_m = \frac{Shaft\ Power\ out\ put}{Power\ developed\ by\ the\ runnerwater\ power}$$

$$\eta_m = \frac{P_s}{\rho Q \Delta V_u\, U}$$

iv) Overall efficiency: is defined as ratio of shaft output power by the turbine to the water power available at inlet of the turbine

$$\eta_o = \frac{Shaft\ Power\ out\ put}{water\ power}; \qquad \eta_o = \frac{P_s}{\omega Q H}$$

$$\eta_o = \eta_{hact} * \eta_m \qquad \eta_o = \eta_{htheortical} x \eta_v\, x \eta_m$$

Generator Efficiency : is the ratio of Generator output to Power at the shaft

$$\eta_o = \frac{P_g}{P_s}$$

Working Proportions of Frnancis turbine

Speed ratio: $\phi = \frac{U_1}{\sqrt{2gH}}$

Flow ratio $\psi = \frac{V_{f1}}{\sqrt{2gH}}$

Area of flow=(1-Blockage fraction) $\boldsymbol{\pi} \mathbf{D_1 B_1}$

Generally Radial flow velocity is constant $V_{f1}= V_{f2}$ and V_{r2} is greater than V_{r1}

Overall efficiency and Mechanical efficiency is same as Pelton wheel

Important points

$U_1 \neq U_2$ (U_1 is not equal to U_2 ie $U_1=\pi D_1 N/60$ and $U_2=\pi D_2 N/60$

α_2 is 90^o and $V_{u2}=0$

Area of flow=(1-Blockage fraction) $\boldsymbol{\pi} \mathbf{D_1 B_1}$

Kaplan Turbine

Schematic Diagram of Kaplan Turbine and working principle:

Scroll casing: The water from the penstock enters the scroll casing. The main function of spiral casing is to provide an uniform distribution of water around the runner and hence to provide constant flow velocity

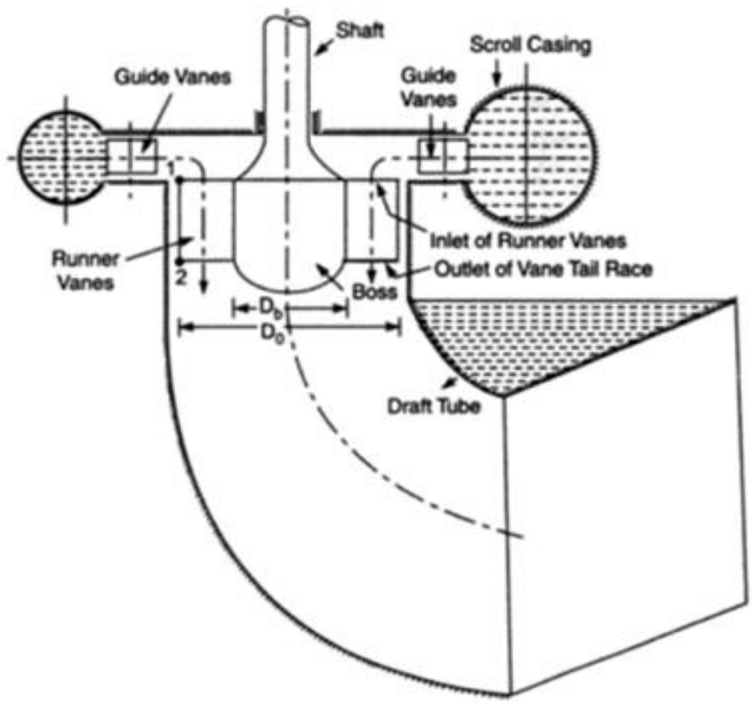
Guide vanes: Water from the Scroll casing into guide vanes. Main function is i) to regulate the quantity of water entering the runner and ii) to direct the water on to the runner

Runner: This houses the moving vanes or runner blades usually 4 to 6. From the guide vane directed to the moving vanes. As the water flows through the moving vanes both pressure and velocity decreases. Here both pressure energy and kinetic energy converted into power which is is responsible for the rotation of the shaft.

Draft Tube: The water from the runner flows to the tail race through the draft tube. The shape of draft tube is in diverging in nature.

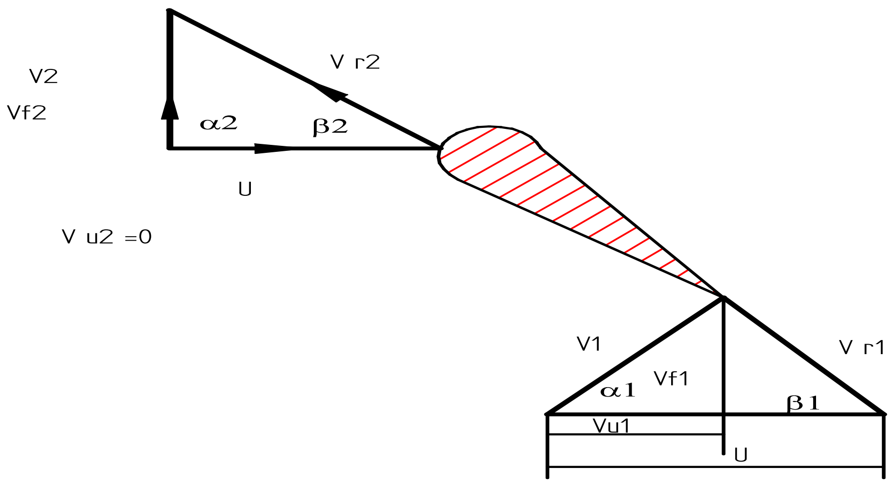
The main function of draft tube is i) It permits a negative or suction head established at the runner exit

It converts large portion of velocity energy rejected from the runner into useful pressure energy



Working Principle

Velocity triangle



The water from the pen-stock enters into the scroll casing. The water moves into the scroll casing and the guide vanes directs the water from the casing to the blades of the runner. The vanes are adjustable and can adjust itself according to the requirement of flow rate. As the water moves over the blades it starts rotating due to reaction force of the water. The blades in the Kaplan turbine is also adjustable. From the runner blades, the water enters into the draft tube where its pressure energy and kinetic energy decreases. Actually here the K.E. is gets converted into pressure energy results in increased pressure of the water. Finally the water discharged to the trail race. The rotation of the turbine is used to rotate the shaft of generator for electricity production and for some other mechanical work.

Advantages

- It can work more efficiently at low water head and high flow rates as compared with other types of turbines.
- It is smaller in size.
- It is easy to construct and space requirement is less.
- The Efficiency of Kaplan turbine is very high as compares with other hydraulic turbine.

Disadvantages

Cavitation is the major problem in this turbine. Use of draft tube and proper material generally stainless steel for the runner blades may reduce the cavitation problem to a greater extent.

Application

This turbine is used in power generation (mostly electricity) where water is available at low head and at higher flow rates.

This is the all about Kaplan Turbine. If you find anything missing or incorrect than lets us notify through your valuable comments. And if this article has enhanced some knowledge in you than don't forget to like and share it on Facebook , Google+, Twitter and on other social medial networks.

It is an axial flow reaction turbine Here $U_1=U_2=U$

U is based on Outer rim Diameter ie D_o U= $\pi D_o N/60$

Q = $\frac{\pi\left(D_o^2-D_h^2\right)}{4}V_{f1}$

Speed ratio: $\phi = \frac{U_1}{\sqrt{2gH}}$

Flow ratio $\psi = \frac{V_{f1}}{\sqrt{2gH}}$

$V_{f1} = V_{f2}$

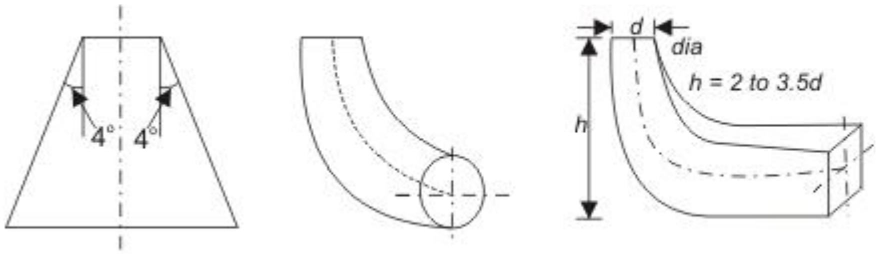
Draft tubes

In power turbines like reaction turbines, Kaplan turbines, or Francis turbines, a diffuser tube is installed at the exit of the turbine, known as **draft tube**.[1]

This draft tube at the end of the turbine increases the pressure of the exiting fluid at the expense of its velocity. This means that the turbine can reduce pressure to a higher extent without fear of back flow from the tail race.

By placing a draft tube (also called a diffuser tube or pipe) at the exit of the turbine, the turbine pressure head is increased by decreasing the exit velocity, and both the overall efficiency and the output of the turbine can be improved. The draft tube works by converting some of the kinetic energy at the exit of the turbine runner into the useful pressure energy

Types of Draft tubes



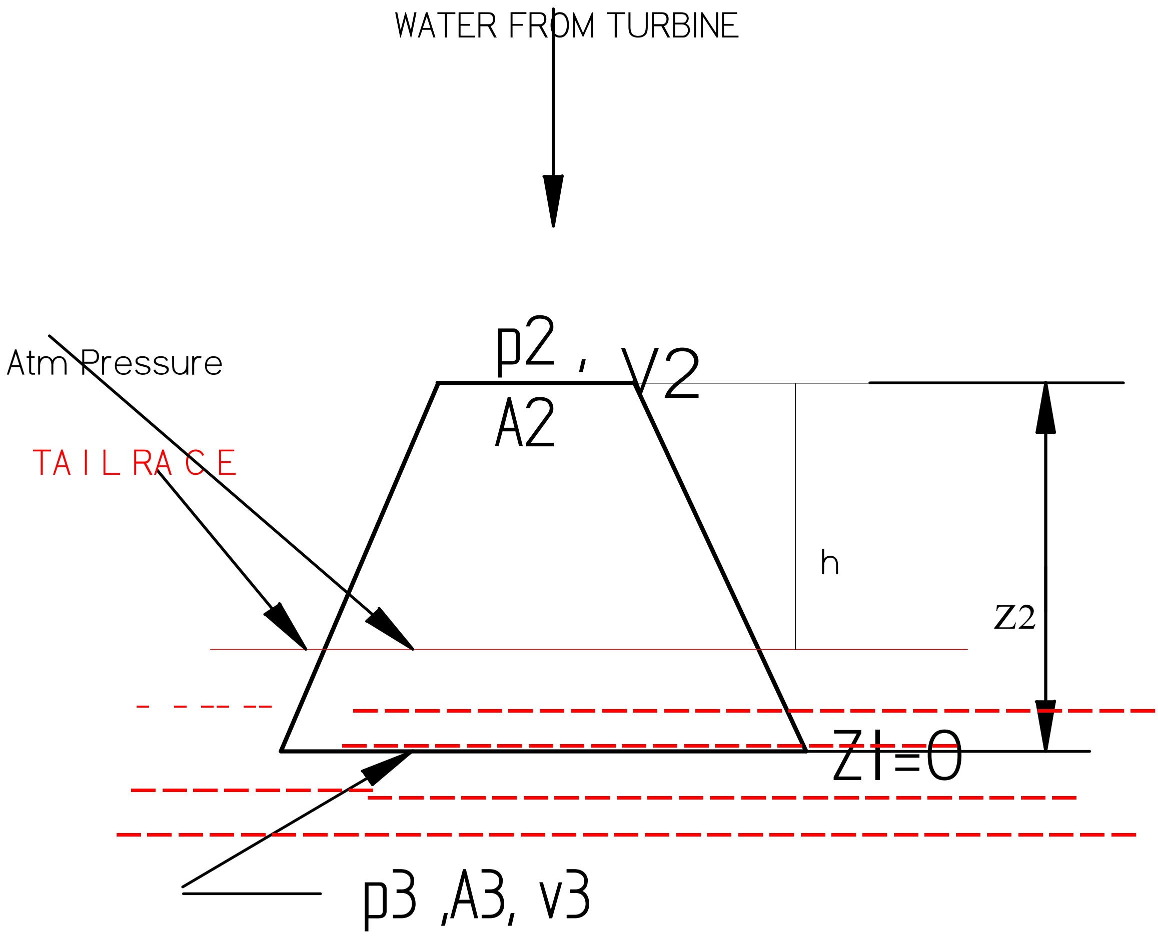
(a) Straight type (b) Simple elbow type (c) Elbow type with varying cross-section

1. **Conical diffuser or straight divergent tube**-This type of draft tube consists of a conical diffuser with half angle generally less than equal to 10° to prevent flow separation. It is usually employed for low specific speed,vertical shaft francis turbine. Efficiency of this type of draft tube is 90%

2. **Simple elbow type draft Tube**-It consists of an extended elbow type tube. Generally, used when turbine has to be placed close to the tail-race. It helps to cut down the cost of excavation and the exit diameter should be as large as possible to recover kinetic energy at the outlet of runner. Efficiency of this kind of draft tube is less almost 60%

3. **Elbow with varying cross section**-It is similar to the Bent Draft tube except the bent part is of varying cross section with rectangular outlet.the horizontal portion of draft tube is generally inclined upwards to prevent entry of air from the exit end

Draft tube analysis



$$\frac{p_2}{\omega} + \frac{V_2^2}{2g} + Z_2 = \frac{p_3}{\omega} + \frac{V_3^2}{2g} + Z_3 + hydraulic\ losses\ in\ Draft\ tube\ h_L$$

Draft tube efficiency $\eta_{\mathrm{d}} = \frac{Actual\ regain\ of\ pressure\ Head}{Velocity\ head\ at\ entrance\ to\ draft\ tube}$

$$\eta_{\mathrm{d}} = \frac{\frac{V_3^2 - V_2^2}{2g} - h_L}{\frac{V_2^2}{2g}}$$

Important Note:

If Draft tube is fitted at the exit of Francis and Kaplan turbine

Then theoretical Efficiency is

$$\eta_{h(theoritical)} = \frac{H - \frac{V_3^2}{2g}}{H}$$

Numerical Problems :

Pelton wheel

1. A pelton wheel turbine is required to develop 10MW of power when working under a head of 200m. The runner is having a speed of 650rpm. Assuming overall efficiency of 88%. Determine i) quantity of water required ii) Diameter of the wheel. Take C_V=0.98 and value of ϕ=0.48
2. A pelton wheel produces 15500 kW under a head of 350m at 500rpm. If overall efficiency of the wheel is 84% Find: i) Required number of jets and diameter of each jet ii) number of buckets iii) Tangential force exerted

3. In a power station single jet Pelton wheel produces 23110kW under a head of 1770m while running at 750rpm. Estimate i) jet diameter ii) Mean diameter of the runner iii) Number of buckets Assume $C_v = 0.97$; $\phi = 0.46$; $\eta_t = 0.85$ (6b, 06,June/July18)
$P_s = 23110kW; H = 1770m; N = 750rpm;$ i) $d = ?$ $ii)$ $D = ?$ iii) $n = ?$

i) Diameter of jet

Overall efficiency is not given Hence Assume overall efficiency = 88%

$\eta_o = \frac{P_s}{\omega Q_T H};$ $\quad 0.85 = \frac{23110*10^3}{9810*Q_T*1770};$ $\quad Q_T = 1.566m^3/s$

$V_1 = C_v\sqrt{2gH};$ $\quad V_1 = 0.97\sqrt{2*9.81*1770}$ $\quad V_1 = 180.76m/s$

$Q_T = n\frac{\pi d^2}{4}V_1;$ $\quad 1.566 = 1*\frac{\pi d^2}{4}*180.76;$ $\quad d = 0.105m$

ii) Mean diameter of runner

Assume $\phi = 0.46$

$\phi = \frac{U}{\sqrt{2gH}};$ $\quad 0.46 = \frac{U}{\sqrt{2*9.81*1770}}$ $\quad U = 85.72m/s$

$U = \frac{\pi DN}{60};$ $\quad 85.72 = \frac{\pi*D*750}{60}$ $\quad D = 2.18m$

iii) Number of buckets

$Z = \frac{m}{2} + 15;$ $\quad Z = \frac{m}{2} + 15;$ $\quad Z = \frac{D}{2d} + 15;$

$Z = \frac{2.18}{2*0.105} + 15;$ $\quad Z = 25.38;$ $\quad Z = 26$

4. In a power station single jet Pelton wheel produces 23110kW under a head of 1770m while running at 750rpm. Estimate i) jet diameter ii) Mean diameter of the runner iii) Number of buckets Assume the necessary data suitably. (6b, 06,Dec14/Jan15)
   $P_s = 23110kW; H = 1770m; N = 750rpm;$ i) $d = ?\ ii)\ D = ?$ iii) $n = ?$**<u>iv)</u>Diameter of jet=?**
   Overall efficiency is not given Hence Assume overall efficiency $= 88\%$

$\eta_o = \frac{P_s}{\omega Q_T H};$ $\quad 0.88 = \frac{23110*10^3}{9810*Q_T*1770};$ $\quad Q_T = 1.512m^3/s$

Assume C_v=1 as it has not been given in the data

$V_1 = C_v\sqrt{2gH};$ $\quad V_1 = \sqrt{2*9.81*1770}$ $\quad V_1 = 186.35m/s$

$Q_T = n\frac{\pi d^2}{4}V_1;$ $\quad 1.512 = 1*\frac{\pi d^2}{4}*186.35;$ $\quad d = 0.1016m$

iv) <u>Mean diameter of runner</u>

Assume $\phi = 0.46$

$\phi = \frac{U}{\sqrt{2gH}};$ $\quad 0.46 = \frac{U}{\sqrt{2*9.81*1770}}$ $\quad U = 85.72m/s$

$U = \frac{\pi DN}{60};$ $\quad 85.72 = \frac{\pi*D*750}{60}$ $\quad D = 2.18m$

v) <u>Number of buckets</u>

$Z = \frac{m}{2} + 15;$ $\quad Z = \frac{m}{2} + 15;$ $\quad Z = \frac{D}{2d} + 15;$

$Z = \frac{2.18}{2*0.1016} + 15;$ $\quad Z = 25.72;$ $\quad Z = 26$

5. A pelton wheel is required to develop 12000kW with a head of 400m. The wheel speed is 720rpm. Assuming ϕ=0.45, C_V=0.98,η_o=86% and approximate jet ratio =8, Design the machine and specify a) bucket circle diameter b) number of buckets and 3) Number of jet
6. A pelton wheel produces 15500kW under a head of 350m at 500rpm. IF the overall efficiency of the wheel is 84% . Find i) Required number of jets and diameter of each jet ii) Number of buckets iii) Tangential force exerted Assume : jet ratio $= 9.5$; $Q = 160^o;\ \phi = 0.46$ (6b,10, Dec18/Jan19)

$P_s = 15500kW; H = 350m;$ $\quad N = 500rpm;\ \eta_o = 0.84;\ i)\ n = ?; d = ?$ ii) $Z = ?$ iii) $F_t = ?$

$jet\ ratio\ m = 9.5\ \ ie\ \frac{D}{d} = 9.5;\ Q = 160^o\ ie$ The angle of deflection of the jet is 160°;

$\phi = 0.46$

i) Required number of jets and diameter of each jet

$\beta_2 = 180 - Defelction\ angle;$ $\quad \beta_2 = 180 - 160$ $\quad \beta_2 = 20^0$

Assume $C_v = 1$

$V_1 = C_v\sqrt{2gH};$ $\quad V_1 = 1\sqrt{2*9.81*350}$ $\quad V_1 = 82.86m/s$

$\phi = \frac{U}{\sqrt{2gH}}$; $\quad 0.46 = \frac{U}{\sqrt{2*9.81*350}}$ $\quad U = 38.12m/s$

$U = \frac{\pi DN}{60}$; $\quad 38.12 = \frac{\pi * D*500}{60}$ $\quad D = 1.46m$

Dia meter of jet

$\frac{D}{d} = 9.5$; $\quad \frac{1.46}{d} = 9.5$; $\quad d = 0.154m$

Dia meter of jet $d = 0.154m$

$\eta_o = \frac{P_s}{\omega Q_T H}$; $\quad 0.85 = \frac{15500*10^3}{9810*Q_T*350}$; $\quad Q_T = 5.311m^3/s$

$Q_T = n * \frac{\pi d^2}{4} * V_1$; $\quad 5.31 = n * \frac{\pi 0.154^2}{4} * 82.86$ $\quad n = 3.44$ jets

$n = 4\ jets$

Number of jets $n = 4$

ii) Number of bucket

$Z = \frac{m}{2} + 15$; $\quad Z = \frac{9.5}{2} + 15$; $\quad Z = 19.75$; $\quad Z = 20$

iii) Tangential force exerted

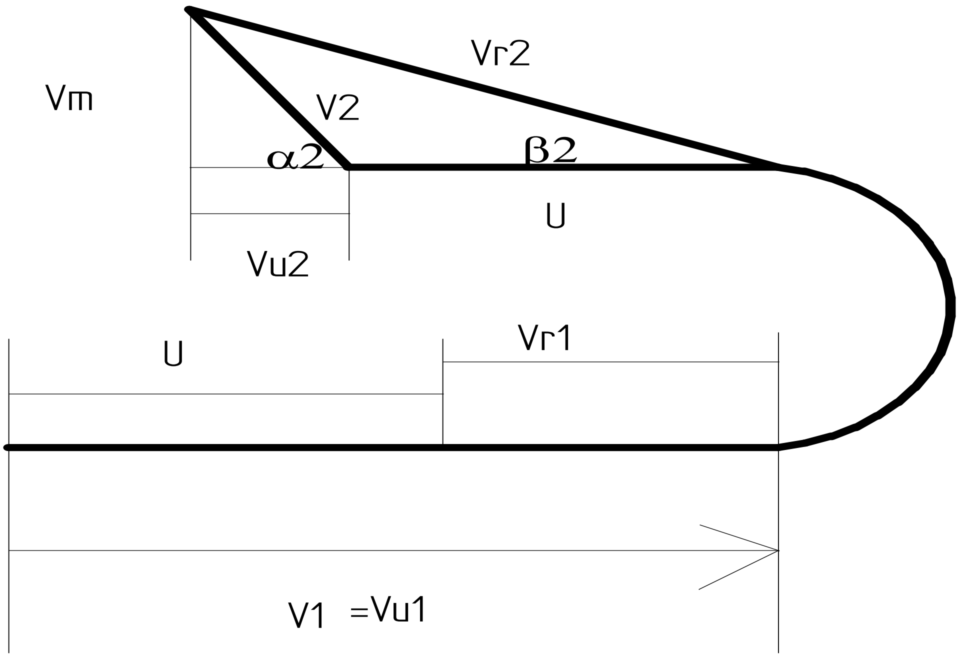
$\overrightarrow{V_{u1}} = V_1$; $\quad \overrightarrow{V_{u1}} = 82.86m/s$

$V_{r1} = V_1 - U$; $\quad V_{r1} = 82.86 - 38.12$ $\quad V_{r1} = 44.74m/s$

Since blade coefficient K is not given consider it is 1 ie $V_{r2} = V_{r1}$; $\quad V_{r2} = 44.74m/s$

$V_{r2} cos\beta_2 = 44.74cos20$; $\quad V_{r2} cos\beta_2 = 42.04m/s$

$V_{r2} cos\beta_2 > U$;



$\overleftarrow{V_{u2}} = V_{r2} cos\beta_2 - U;$ $\overleftarrow{V_{u2}} = 42.04 - 38.12;$ $\overleftarrow{V_{u2}} = 3.92 m/s$

$\dot{m} = \rho Q_T;$ $\dot{m} = 1000 * 5.31$ $5311 kg/m^3$

$F_t = \dot{m}\left(\overrightarrow{V_{u1}} + \overleftarrow{V_{u2}}\right);$ $F_t = 5311(82.86 + 3.92)$ $F_t = 460.889 * 10^3 N$

7. Two jet strike at buckets of a Pelton wheel , which is having shaft power as 14715kW. The diameter of each jet is given as 150mm. If the net head on the turbine is 500m, find the overall efficiency , take $C_v = 1.0$ and speed ratio $= 0.46$. If the blade angle at outlet is 15^0 and reduction in relative velocity over the bucket is 5% , find the hydraulic efficiency (6b, 10,Dec 13/14)

$n = 2;$ $P_s = 14715 kW;$ $d = 150mm; d = 0.15m;$ $H = 500m;$ $\eta_o = ?$ $C_v = 1.0$

speed ratio $= 0.46$ ie $\phi = 0.46;$ $\beta_2 = 15^0;$

reduction in relative velocity over the bucket is 5% ie $K = 1 - 0.05$ ie $\frac{V_{r2}}{V_{r1}} = 0.95;$ $\eta_h = ?$

i) overall efficiency

$V_1 = C_v \sqrt{2gH};$ $V_1 = 1\sqrt{2 * 9.81 * 500}$ $V_1 = 99.04 m/s$

$Q_T = n\frac{\pi d^2}{4} V_1$ $Q_T = 2 * \frac{\pi * 0.15^2}{4} * 99.04;$ $Q_T = 3.5 m^3/s$

$\eta_o = \frac{P_s}{\omega Q_T H};$ $\eta_o = \frac{14715 * 10^3}{9810 * 3.5 * 500}$ $\eta_o = 0.8256$

ii) Hydraulic Efficiency

$\phi = \frac{U}{\sqrt{2gH}};$ $0.46 = \frac{U}{\sqrt{2 * 9.81 * 500}}$ $U = 45.56 m/s$

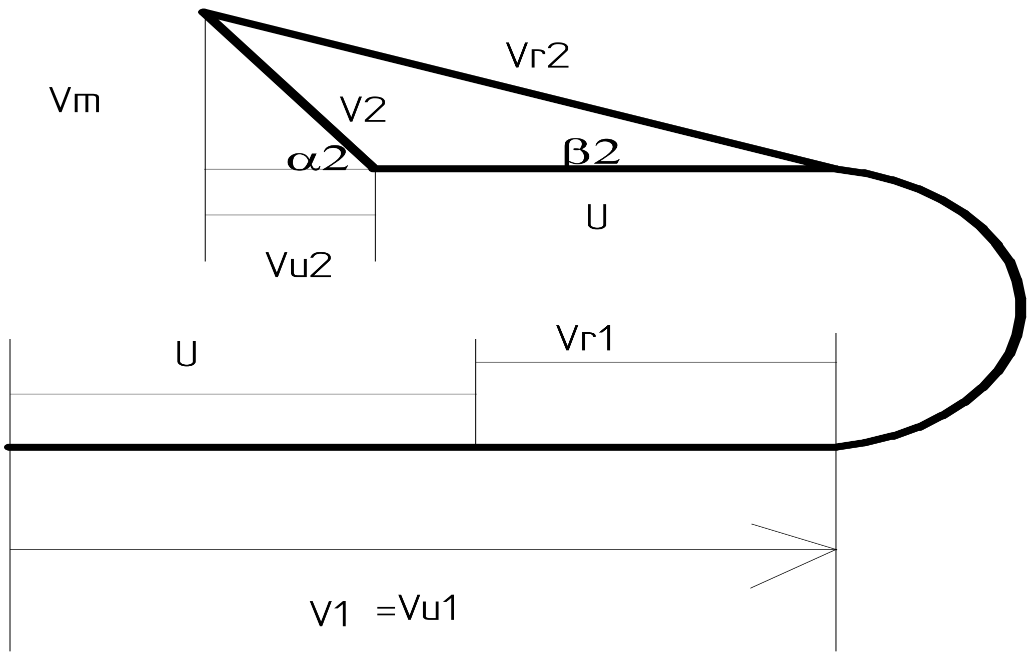
$\overrightarrow{V_{u1}} = V_1;$ $\overrightarrow{V_{u1}} = 99.04m/s$

$V_{r1} = V_1 - U;$ $V_{r1} = 99.04 - 45.56$ $V_{r1} = 53.48m/s$

$\frac{V_{r2}}{V_{r1}} = 0.95;$ $\frac{V_{r2}}{53.48} = 0.95$ $V_{r2} = 50.81m/s$

$V_{r2}cos\beta_2 = 50.81cos15;$ $V_{r2}cos\beta_2 = 49.07m/s$

$V_{r2}cos\beta_2 > U;$



$\overleftarrow{V_{u2}} = V_{r2}cos\beta_2 - U;$ $\overleftarrow{V_{u2}} = 49.07 - 45.56;$ $\overleftarrow{V_{u2}} = 3.51m/s$

$\eta_b = \frac{(\overrightarrow{V_{u1}} + \overleftarrow{V_{u2}})U}{\frac{1}{2}V_1^2};$ $\eta_b = \frac{(99.04 + 3.51)45.56}{\frac{1}{2} * 99.04^2}$ $\eta_b = 0.9526$

8. A three jet Pelton wheel is required to generate 10000kW under a head of 400m. The blade angle at outlet is 15° and reduction in relative velocity over the buckets is 5%. If overall efficiency is 80%, C_V=0.98 and speed ratio =0.46 Find i) Diameter of jet ii) Total flow in m^3/s iii) Force exerted by a jet on the buckets
9. The penstock supplies water from a reservoir to the Pelton wheel with a gross head of 500m. One third is lost in friction in the penstock. The rate of flow of water through the nozzle fitted at the end of penstock is 2m^3/s. The angle of deflection of the jet is 165°. Determine the power given by the water to the runner and also hydraulic efficiency of the Pelton wheel. Take speed ratio 0.45 and C_v =1.0 (6b, 10, Dec16/Jan17)
gross head of 500m. Ie $H_g = 500m;$

One third is lost in friction in the penstock ie $H_f = \frac{1}{3}H_g$ ie $H_f = \frac{1}{3} * 500\ ;\ H_f = 166.66m$

$Q = 2m^3/s;$ The angle of deflection of the jet is 165°;

speed ratio 0.45 ie $\phi = 0.45$; and C_v =1.0

Power given by the water to the runner

Net Head available ie $H = H_g - H_f$; $H = 500 - 166.67$ $H = 333.33$m

$\beta_2 = 180 - Defelction\ angle$; $\beta_2 = 180 - 165$ $\beta_2 = 15^0$

$V_1 = C_v\sqrt{2gH}$; $V_1 = 1\sqrt{2*9.81*333.33}$ $V_1 = 80.87m/s$

$\phi = \frac{U}{\sqrt{2gH}}$; $0.45 = \frac{U}{\sqrt{2*9.81*333.33}}$ $U = 36.39m/s$

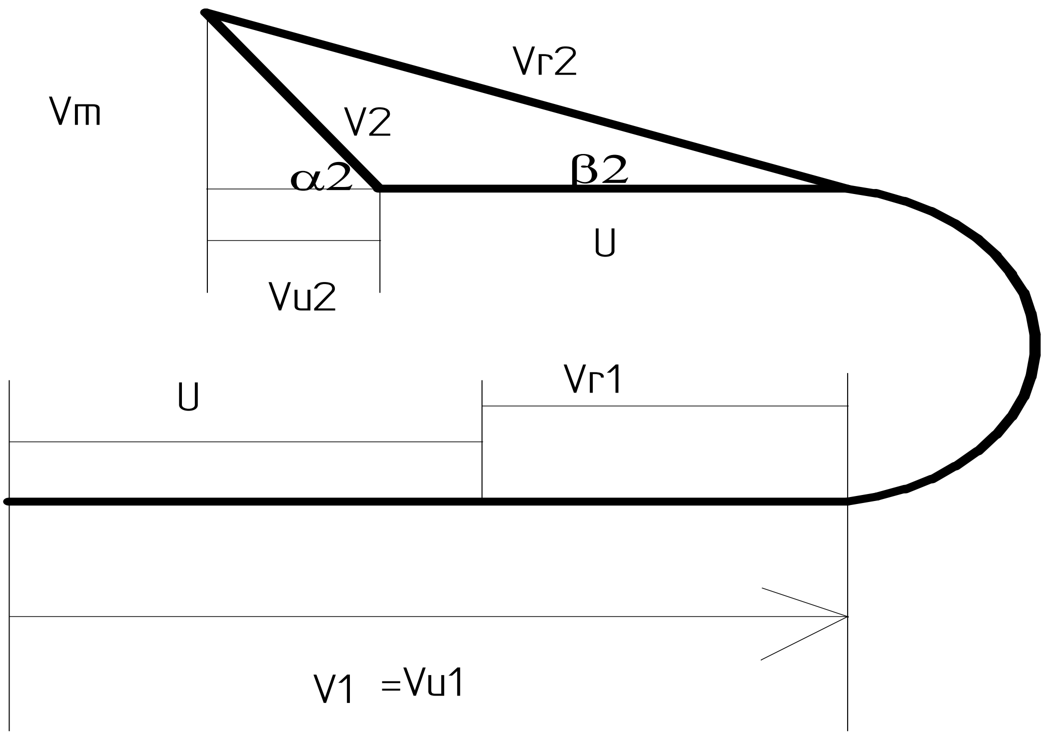
$\overrightarrow{V_{u1}} = V_1$; $\overrightarrow{V_{u1}} = 80.87m/s$

$V_{r1} = V_1 - U$; $V_{r1} = 80.87 - 36.39$ V_{r1} =44.48m/s

Since blade coefficient K is not given consider it is 1 ie $V_{r2} = V_{r1}$; $V_{r2} = 44.48m/s$

$V_{r2}cos\beta_2 = 44.48cos15$; $V_{r2}cos\beta_2 = 42.96m/s$

$V_{r2}cos\beta_2 > U$;



$\overleftarrow{V_{u2}} = V_{r2}cos\beta_2 - U$; $\overleftarrow{V_{u2}} = 42.96 - 36.39$; $\overleftarrow{V_{u2}} = 6.57m/s$

$\frac{E}{\dot{m}} = \frac{(\overrightarrow{V_{u1}}+\overleftarrow{V_{u2}})U}{g_c}$; $E = \frac{\dot{m}(\overrightarrow{V_{u1}}+\overleftarrow{V_{u2}})U}{g_c}$; $E = \frac{\rho\ Q_T(\overrightarrow{V_{u1}}+\overleftarrow{V_{u2}})U}{g_c}$

$E = \frac{1000*2*(80.87+6.57)*36.39}{1}$; $E = 6.363 * 10^6 W$

$E = \frac{1000*2*(80.87+6.57)36.39}{1}$;

ii) Hydraulic efficiency of the Pelton wheel

$\eta_b = \frac{(\overrightarrow{V_{u1}} + \overrightarrow{V_{u2}})U}{\frac{1}{2}V_1^2}$; $\eta_b = \frac{(80.87+6.57)36.39}{\frac{1}{2}*80.87^2}$ $\eta_b = 0.973$

10. A double jet pelton wheel develops 1200MHP with an overall efficiency of 82% and head is 60m. The speed ratio =12 and nozzle coefficient =0.97. Find the diameter of jet, wheel diameter, and wheel speed in rpm

11. A double over hung pelton wheel unit is to produce 30000kW at the generator under an effective head of 300m at base of the nozzle. Find the size of the jet , mean diameter of the runner, speed and specific speed of each pelton turbine. Assume generator efficiency $= 93\%$, Pelton wheel efficiency $= 85\%$, speed ratio $= 0.46$, jet velocity coefficient $= 0.97$, and jet ratio 12 (6b, 8,June/July18 15 scheme)

Double over hung wheel

$P_g = 30000kW; H = 300m;\ d = ?; D = ?; N = ?\ N_s = ?\ \eta_g = 93\%; \eta_0 = 85\%;\ \phi = 0.46;$
$C_v = 0.97\ ; m = \frac{D}{d} = 12$

i) Size of jet

$\eta_g = \frac{P_s}{P_g}$; $0.93 = \frac{P_s}{30000}$; $P_s = 27900kW$

Since there are two pelton wheel connected

Power at the shaft of each Pelton wheel $= \frac{27900}{2}$ $P_{s-each} = 13950kW$

$\eta_o = \frac{P_{s-each}}{\omega Q_T H}$; $0.85 = \frac{13950*10^3}{9810*Q_{T-each}*300}$ $Q_{T-each} = 5.58m^3/s$

$V_1 = C_v\sqrt{2gH}$; $V_1 = 0.97\sqrt{2*9.81*300}$ $V_1 = 74.42m/s$

$Q_{T-each} = n\frac{\pi d^2}{4}V_1$ $5.58 = 1*\frac{\pi*d^2}{4}*74.42$; $d = 0.309m$

ii) Mean Diameter

$\phi = \frac{U}{\sqrt{2gH}}$; $0.46 = \frac{U}{\sqrt{2*9.81*300}}$ $U = 35.29m/s$

$m = \frac{D}{d} = 12$; $\frac{D}{0.309} = 12$ $D = 3.708m$

$U = \frac{\pi DN}{60}$; $35.29 = \frac{\pi*3.708*N}{60}$ $N = 181.76rpm$

iii) Specific speed

$N_s = \frac{N\sqrt{P}}{H^{5/4}}$; $N_s = \frac{181.76\sqrt{13950}}{300^{5/4}}$ $N_s = 17.19$

12. A pelton wheel is designed to develop 12000kW of power at an overall efficiency of 86%. The speed is 0.46 times the jet velocity. Assuming a nozzle coefficient of 0.975 and an approximate jet ratio of 10, calculate the wheel diameter, number of jets diameter of each jet and number of buckets

13. In a power station . a pelton wheel produces 15000kW under a head of 350m while running at 500rpm. Assume a turbine efficiency of 0.84, coefficient of velocity for nozzle as 0.98 , speed ratio 0.46 and bucket velocity coefficient 0.86, Calculate i) number of jet ii) Diameter of each jet iii) Tangential force on the buckets if the bucket deflect the jet through 165^0 (7b, 8,Dec18/Jan19, 15 scheme)

$P_s = 15000kW; H = 350m;\ N = 500rpm;\ \eta_0 = 0.84;\ C_v = 0.98;\ \phi = 0.46; K = 0.86;$

$i)\ n = ?\ ii)\ d = ?\ iii)\ F_t = ?$ bucket deflect the jet through 165^0 ie $\beta_2 = 180 - 165; \beta_2 = 15^o$

i) number of jet

$\eta_o = \frac{P_s}{\omega Q_T H};$ $\quad 0.84 = \frac{15000*10^3}{9810*Q_T*350}$ $\quad Q_T = 5.20m^3/s$

$V_1 = C_v\sqrt{2gH};$ $\quad V_1 = 0.98\sqrt{2*9.81*350}$ $\quad V_1 = 81.21\ m/s$

$\phi = \frac{U}{\sqrt{2gH}};$ $\quad 0.46 = \frac{U}{\sqrt{2*9.81*350}}$ $\quad U = 38.12m/s$

$U = \frac{\pi DN}{60};$ $\quad 38.12 = \frac{\pi * D*500}{60}$ $\quad D = 1.46m$

Assuming jet ratio 12 (not given)

$m = \frac{D}{d} = 12;$ $\quad \frac{1.46}{d} = 12$ $\quad d = 0.1213m$

$Q_T = n\frac{\pi d^2}{4}V_1$ $\quad 5.20 = n * \frac{\pi*0.1213^2}{4}*81.21;$ $\quad n = 5.54$ say 6

Diameter of each jet ie $d = 0.1213m$

iii) Tangential force on the buckets if the bucket deflect the jet through 165^0

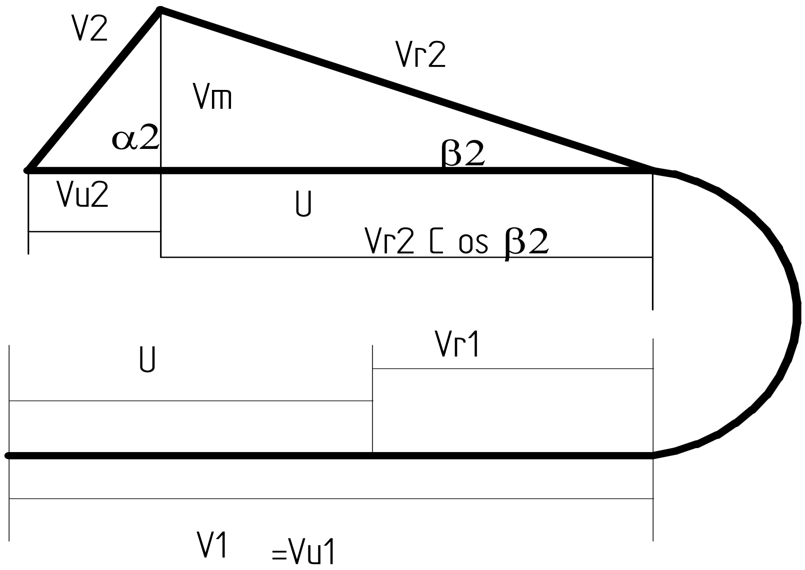
$\overrightarrow{V_{u1}} = V_1;$ $\quad \overrightarrow{V_{u1}} = 81.21m/s$

$V_{r1} = V_1 - U;$ $\quad V_{r1} = 81.21 - 38.12$ $\quad V_{r1} = 43.09m/s$

$K = 0.86;$ $\quad \frac{V_{r2}}{V_{r1}} = 0.86$ $\quad \frac{V_{r2}}{43.09} = 0.86$ $\quad V_{r2} = 37.06m/s$

$V_{r2}cos\beta_2 = 37.06\ cos15;$ $\quad V_{r2}cos\beta_2 = 35.79m/s$

$V_{r2}cos\beta_2 < U;$



$\overrightarrow{V_{u2}} = U - V_{r2}cos\beta_2;$ $\overrightarrow{V_{u2}} = 38.12 - 35.79;$ $\overrightarrow{V_{u2}} = 2.33m/s$

$\dot{m} = \rho Q_T;$ $\dot{m} = 1000 * 5.20$ $\dot{m} = 5200kg/m^3$

$F_t = \dot{m}(\overrightarrow{V_{u1}} - \overrightarrow{V_{u2}});$ $F_t = 5200(81.21 - 2.33)$ $F_t = 410.176 * 10^3 N$

14. A double jet Pelton wheel is supplied with water through a pipeline 1600m long from reservoir in which the level of the water is 350m above that of the Pelton wheel. The turbine runs at 500rpm and develops an output of 6800kW. If the pipe losses are 10% of the gross head and friction coefficient f=0.005. Determine i) the diameter of the pipe ii) The cross section of each jet iii) Mean diameter of the bucket circle. Assume that C_j of jet is equal to 0.98, bucket speed is equal to 0.45 times the jet velocity and turbine efficiency is 0.86

15. The supply to a single jet Pelton wheel is from a reservoir 310m above the nozzle center through a pipe 67.5cm diameter, 5.6km long. The friction coefficient for pipe is 0.008. The jet has a diameter of 9cm and its velocity coefficient 0.97, The blade speed ratio is 0.47 and the buckets deflect the water through 170^o. The relative velocity is of water is reduced by 15% in passing over the buckets. Determine the hydraulic and overall efficiency of system if mechanical efficiency is 88%
16. A pelton wheel turbine has 2 wheels and 2 jets (one jet for one wheel) producing 42.5MW per wheel under a head of 730m. Estimate the speed of the wheel if runner diameter is 4m. Draw the inlet and outlet velocity triangles and calculate the hydraulic efficiency. If the volumetric efficiency is 98%, find the discharge through the nozzle and jet diameter. Take C_V=0.98; $\frac{U}{V_1} = 0.47$; β_2=15^o
17. A Francis turbine works under a head of 260m and develops 16.2MW at a speed of 600r/min. The volume flow rate through the machine is $7m^3/s$. If outside wheel diameter is 1.5m and axial wheel width at inlet is 135mm, find overall efficiency , hydraulic efficiency, and inlet angles of guide blades and rotor angles. Assume a volumetric efficiency of 0.98 and

velocity at draft tube exit to be 17.7m/s. The whirl velocity component at the wheel exit is zero

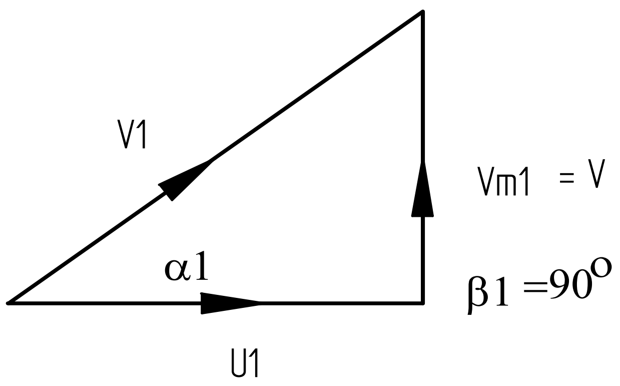
18. The internal and external diameters of an inward flow reaction turbine are 1.2m and 0.6m respectively. The head on the turbine is 22m and velocity of flow through the runner is constant and is equal to 2.5m/s. The guide blade angle is 10^o and runner vanes are radial at inlet. If the discharge at outlet is radial. Find i) speed of turbine ii) Vane angle at outlet iii) Hydraulic efficiency iv) Draw velocity triangles (5b,10, June/July2013)

$D_1 = 1.2m$; $D_2 = 0.6m$; $H = 22m$; $V_{f1} = V_{f2} = 2.5m/s$; The guide blade angle is 10^o ie $\alpha_1 = 10^o$

runner vanes are radial at inlet ie $\beta_1 = 90^0$ If the discharge at outlet is radial $\alpha_2 = 90^o$;i) $N =?$;

$ii)\ \beta_2 =?$ iii)$\eta_h =?$

Speed of the turbine

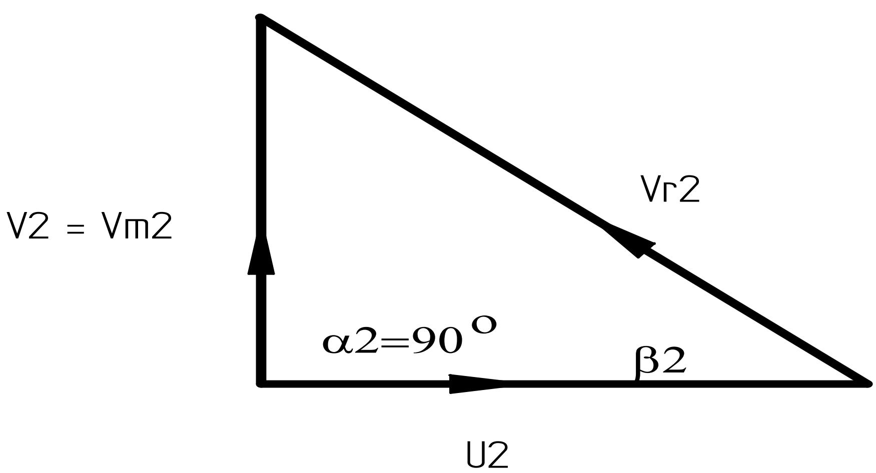


$tan\alpha_1 = \frac{V_{f1}}{V_{u1}}$; $tan10 = \frac{2.5}{\overrightarrow{V_{u1}}}$; $\overrightarrow{V_{u1}} = 14.18m/s$;

From Inlet velocity triangle $U_1 = \overrightarrow{V_{u1}}$; $U_1 = 14.18m/s$

$U_1 = \frac{\pi D_1 N}{60}$; $14.18 = \frac{\pi * 1.2 * N}{60}$ $N = 225.65rpm$

Vane angle at outlet



$U_2 = \frac{\pi D_2 N}{60}$; $U_2 = \frac{\pi * 0.6 * 225.65}{60}$ $U_2 = 7.09 m/s$

$From\ outlet\ velocity\ traingle\ \ tan\beta_2 = \frac{V_{f2}}{U_2}$; $tan\beta_2 = \frac{2.5}{7.09}$; $\beta_2 = 19.42^0$

Hydraulic efficiency

$\eta_h = \frac{\overrightarrow{V_{u1}} U_1}{gH}$; $\eta_h = \frac{14.18 * 14.18}{9.81 * 22}$ $\eta_h = 0.9317$

19. The following data is given for a Francis turbine: net head =70m, speed=600rpm, shaft power=367.5 kW, Overall efficiency =85%, Hydraulic efficiency =95%, flow ratio=0.25, width ratio =0.1, outer to inner diameter ratio = 2.0, the thickness of vanes occupies 10% of the circumferential area of the runner, Velocity of flow is constant at inlet and outlet, and discharge is radial at outlet. Determine 1) Guide blade angle ii) Runner vane angles at inlet and outlet and iii) Diameter of runner at inlet and outlet and iv) Width of the wheel at inlet (6c,10,Dec12)

$H = 70m$; $N = 600rpm$; $P_s = 367.5kW$; $\eta_0 = 85\%$; $\eta_h = 95\%$,

flow ratio=0.25 ie $\varphi = \frac{V_{f1}}{\sqrt{2gH}} = 0.25$; breadth ratio $= 0.1$ ie $\frac{B_1}{D_1} = 0.1$

outer diameter of runner = 2 times inner diameter of runner ,ie $D_1 = 2D_2$

velocity of flow is constant at inlet and outlet ie $V_{f1} = V_{f2}$

the thickness of vanes occupies 10% of the circumferential area of the runner $C = 1 - 0.1 = 0.9$

discharge is radial at outlet $\alpha_2 = 90^o$; $i)\ \alpha_1 = ?$

ii) Runner vane angles at inlet and outlet ie $\beta_1 = ?$; $\beta_2 = ?$

iii) $D_1 = ?$; $D_2 = ?$

Width of the wheel at inlet ie B_1=?

Diameter of runner at inlet and outlet

$\varphi = \frac{V_{f1}}{\sqrt{2gH}} = 0.25$; $V_{f1} = 0.25\sqrt{2x9.81x70}$ $V_{f1} = 9.26 m/s$

$\eta_0 = \frac{P_s}{\omega Q H}$; $0.85 = \frac{367.5 * 10^3}{9810 * Q * 70}$ $Q = 0.63 m^3/s$

$Q = C\pi D_1 B_1 V_{f1}$; $0.63 = 0.9\pi D_1 * 0.12 D_1 * 9.26$ $D_1 = 0.448m$

Diameter at inlet $D_1 = 0.448m$

$D_1 = 2D_2$; $0.448 = 2D_2$; $D_2 = 0.224m$ Diameter at outlet $= 0.224m$

Diameter at outlet D_2=0.2225m

Guide blade angle

$U_1 = \frac{\pi D_1 N}{60}$; $U_1 = \frac{\pi x 0.448 x 600}{60}$ $U_1 = 14.07 m/s$

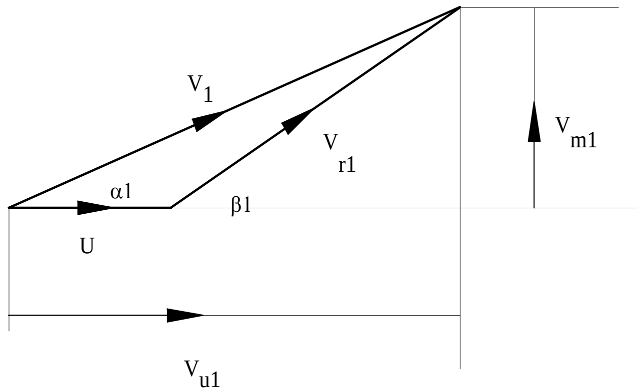
$\eta_h = \frac{\overrightarrow{V_{u1}}\, U_1}{gH};$ $\quad 0.95 = \frac{\overrightarrow{V_{u1}} x 14.07}{9.81 x 70}$ $\quad \overrightarrow{V_{u1}} = 46.36 m/s$

$tan\alpha_1 = \frac{V_{f1}}{\overrightarrow{V_{u1}}};$ $\quad tan\alpha_1 = \frac{9.26}{46.36};$ $\quad \alpha_1 = 11.29^o$

Guide blade angle=11.29^o

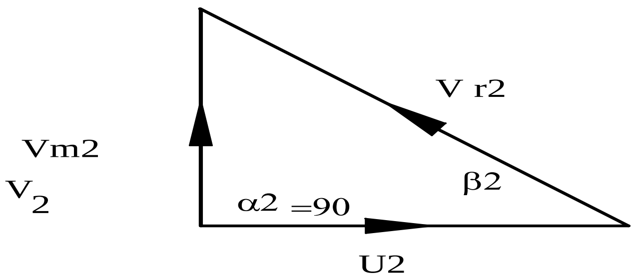
<u>Runner vane angles at inlet and outlet</u> ie β_1 =?; β_2 =?

$U_1 < \overrightarrow{V_{u1}}$



$tan\beta_1 = \frac{V_{f1}}{V_{u1} - U_1};$ $\quad tan\beta_1 = \frac{9.26}{46.36 - 14.07}$ $\quad \beta_1 = 16^o$

Runner blade angle at inlet=18.14°



$U_2 = \frac{\pi D_2 N}{60};$ $\quad U_2 = \frac{\pi x 0.224 x 600}{60};$ $\quad U_2 = 7.037 m/s$

$V_{f2} = V_{f1};$ $\quad V_{f2} = 9.26 m/s$

From outlet velocity triangle

$tan\beta_2 = \frac{V_{f2}}{U_2};$ $\quad tan\beta_2 = \frac{9.26}{7.037}$ $\quad \beta_2 = 52.76^o$

Runner blade angle at outlet =52.76^o

<u>Width of the wheel at inlet</u>

20. The following data is given for a Francis turbine: net head =70m, speed=600rpm, shaft power=368kW,η_o=86%, η_H=95%, flow ratio=0.25, breadth to diameter ratio =0.12, outer diameter of runner = 2 times inner diameter of runner , velocity of flow is constant at inlet

and outlet, the thickness of vanes occupies 10% of the circumferential area of the runner and discharge is radial at outlet. Determine 1) Guide blade angle ii) Runner vane angles at inlet and outlet and iii) Diameter of runner at inlet and outlet and iv) Width of the wheel at inlet

H=70m ; N=600rpm; shaft power=368kW ie P_s=368kW; η_o=86%, η_H=95%,

flow ratio=0.25 ie $\varphi = \frac{V_{f1}}{\sqrt{2gH}}$=0.25; breadth ratio =0.12 ie $\frac{B_1}{D_1}$=0.12

outer diameter of runner = 2 times inner diameter of runner ,ie $D_1=2D_2$

velocity of flow is constant at inlet and outlet ie $V_{f1}=V_{f2}$

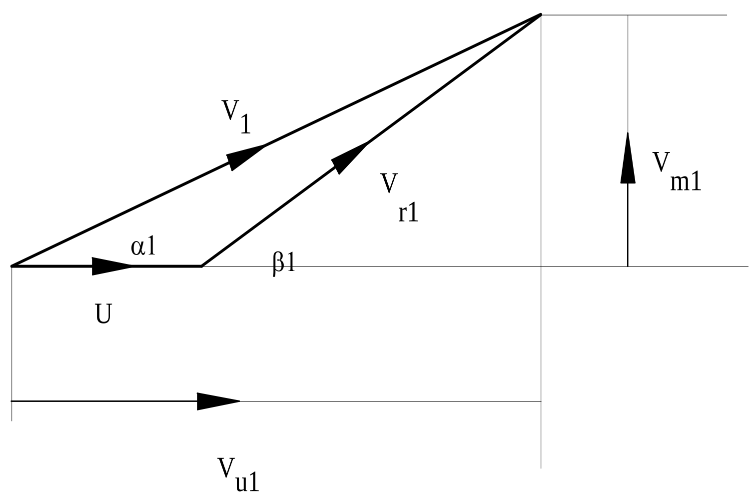
the thickness of vanes occupies 10% of the circumferential area of the runner C=$1-0.1 = 0.9$

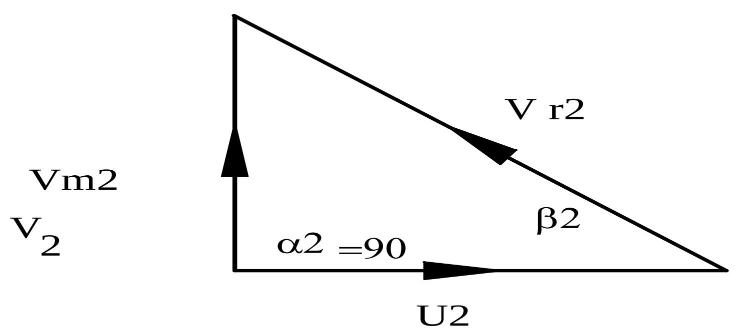
discharge is radial at outlet $\alpha_2=90^o$

α_1=?;

Runner vane angles at inlet and outlet ie β_1=?; β_2=?

Width of the wheel at inlet ie B_1=?





Flow ratio=$\frac{V_{f1}}{\sqrt{2gH}}$=0.25; V_{f1} =$0.25\sqrt{2x9.81x70}$ =9.26m/s

η_o=86%, $\eta_o= \frac{P}{\omega QH}$; $0.86 = \frac{368x10^3}{9810xQx70}$

Q =0.623m^3/s; $Q = C\pi D_1 B_1 V_{f1}$; 0.623=0.9x$\pi x D_1 x 0.12 D_1$x9.26

Diameter at inlet D_1=0.445m

$D_1=2D_2$; $0.445=2D_2$; Diameter at outlet D_2=0.2225m

$U_1 = \frac{\pi D_1 N}{60}$; $U_1 = \frac{\pi x 0.445 x 600}{60}$ $U_1 = 13.98$m/s

$\eta_h = \frac{V_{u1}\,U_1}{gH}$; $0.86 = \frac{\overrightarrow{V_{u1}} x 13.98}{9.81 x 70}$; $\overrightarrow{V_{u1}}$=42.24m/s

$tan\alpha_1 = \frac{V_f}{V_{u1}}$

$tan\alpha_1 = \frac{9.26}{42.24}$ ie $\alpha_1 = 12.36^o$

Guide blade angle=12.36^o

$tan\beta_1 = \frac{V_{f1}}{V_{u1}-U_1}$; $tan\beta_1 = \frac{9.26}{42.24-13.98}$; β_1 =18.14^o

Runner blade angle at inlet=18.14^o

$\frac{U_1}{U_2} = \frac{\frac{\pi D_1 N}{60}}{\frac{\pi D_2 N}{60}}$; $\frac{U_1}{U_2} = \frac{D_1}{D_2}$; $\frac{13.98}{U_2} = \frac{2}{1}$; $U_2 = 6.99m/s$

$tan\beta_2 = \frac{V_{f2}}{U_2}$; $tan\beta_2 = \frac{9.26}{6.99}$; $\beta_2 = 52.95^0$

Runner blade angle at outlet =52.95^o

Width of the blade at inlet = 0.12D_1=0.12x0.445=0.534m

21. The following data is given for a Francis turbine: net head =70m, speed=600rpm, shaft power=368kW,η_o=86%, η_H=95%, flow ratio=0.25, breadth to diameter ratio =0.12, outer diameter of runner = 2 times inner diameter of runner , velocity of flow is constant at inlet and outlet, the thickness of vanes occupies 10% of the circumferential area of the runner and discharge is radial at outlet. Determine 1) Guide blade angle ii) Runner vane angles at inlet and outlet and iii) Diameter of runner at inlet and outlet and iv) Width of the wheel at inlet
22. In a Francis turbine , the discharge is radial, the blade speed at inlet is 25m/s. At the inlet tangential component of velocity is 18m/s. The radial velocity of flow is constant and equal to 2.5m/s. Water flows at the rate of 0.8m^3/s. The utilization factor is 0.82, Find i) Eulers head ii) Power developed iii) Inlet blade angle iv) Degree of reaction. Draw the velocity triangles (6c,08, June/July16)
    the discharge is radial ie $\alpha_2 = 90^o$; , the blade speed at inlet is 25m/s $U_1 = 25m/s$
    At the inlet tangential component of velocity is 18m/s. $\overrightarrow{V_{u1}} = 18m/s$
    The radial velocity of flow is constant and equal to 2.5m/s. $V_{f2} = V_{f1} = 2.5m/s$
    Water flows at the rate of 0.8m^3/s $Q = 0.8m^3/s$
    The utilization factor is 0.82, $\varepsilon = 0.82$
    i) Eulers head=? $\frac{\overrightarrow{V_{u1}}\,U_1}{g}$ =? ii) E =? iii) β_1 =? iv) R =?

i) Eulers head

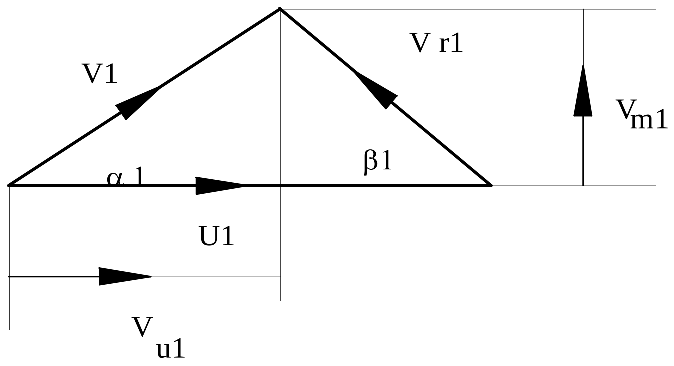
$H_e = \frac{\overrightarrow{V_{u1}}\,U_1}{g}$; $H_e = \frac{18*25}{9.81}$; $H_e = 45.87m$

ii) Power developed

$\frac{E}{m} = \frac{\overrightarrow{V_{u1}}\,U_1}{g_c}$; $E = m\frac{\overrightarrow{V_{u1}}\,U_1}{g_c}$; $E = \rho Q\frac{\overrightarrow{V_{u1}}\,U_1}{g_c}$; $E = 1000 * 0.8\frac{18*25}{1}$ $E = 360000W$

iii) Inlet blade angle

$U_1 > \overrightarrow{V_{u1}}$



$tan\beta_1 = \frac{V_{f1}}{U_1 - \overrightarrow{V_{u1}}}$; $tan\beta_1 = \frac{2.5}{25-18}$ $\beta_1 = 19.65^o$

iv)Degree of reaction

$R = \frac{\frac{E}{m}}{\frac{E}{m}+\frac{V_2^2}{2g_c}}$; $\frac{E}{m} = \frac{\overrightarrow{V_{u1}}\,U_1}{g_c}$ $\frac{E}{m} = \frac{18*25}{1}$ $\frac{E}{m} = 450J/kg$; $V_2 = V_{f2} = 2.5m/s$

$R = \frac{450}{450+\frac{2.5^2}{2*1}}$; $R = 0.993$

15. An inward flow reaction turbine with radial discharge having overall efficiency 80% when power developed is 147kW.The head is 8m. The peripheral velocity of the fluid is $0.96\sqrt{2gH}$.The flow velocity of the fluid is $0.36\sqrt{2gH}$.The speed of the rotor is 1500rpm and hydraulic losses is 22% of available energy. Determine the following : i) Inlet guide vane and blade angles ii) Diameter of the rotor iii) width of the rotor (6c, 08, Dec15/Jan 16)

$\eta_0 = 80\%; P_s = 147kW\;\; H = 8m$;

The peripheral velocity of the fluid is $0.96\sqrt{2gH}$ ie $U_1 = 0.96\sqrt{2gH}$;

The flow velocity of the fluid is $0.36\sqrt{2gH}$ ie $V_{f1} = 0.36\sqrt{2gH}$; $N = 1500rpm$

Hydrulic losses 22% available energy $H_{losses} = 0.22H$

$\eta_h = \frac{H-H_{losses}}{H}$; $\eta_h = \frac{H-0.22H}{H}$; $\eta_h = 0.78$

i) Inlet guide vane and blade angles $\alpha_1 = ?$; $\beta_1 = ?$; $\beta_2 = ?$

ii) Diameter of the rotor $D_2 = ?$

iii) width of the rotor B_1 =?;

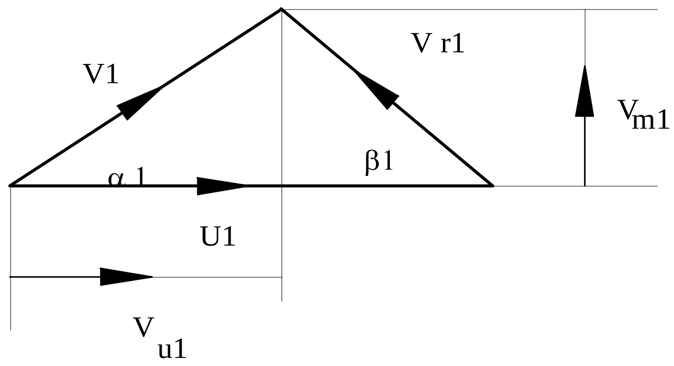
Inlet guide vane and blade angles

$U_1 = 0.96\sqrt{2gH};$ $U_1 = 0.96\sqrt{2 * 9.81 * 8}$ $U_1 = 12.03m/s$

$V_{f1} = 0.36\sqrt{2gH};$ $V_{f1} = 0.36\sqrt{2 * 9.81 * 8}$ V_{f1} =4.51m/s

$\eta_h = \frac{\overrightarrow{V_{u1}}U_1}{gH};$ $0.78 = \frac{\overrightarrow{V_{u1}} * 12.03}{9.81*8}$ $\overrightarrow{V_{u1}} = 5.09m/s$

$U_1 > \overrightarrow{V_{u1}}$ Hence, Inlet triangle as given below



$tan\alpha_1 = \frac{V_{f1}}{\overrightarrow{V_{u1}}};$ $tan\alpha_1 = \frac{4.51}{5.09};$ $\alpha_1 = 41.54^o$

$tan\beta_1 = \frac{V_{f1}}{U_1 - \overrightarrow{V_{u1}}};$ $tan\beta_1 = \frac{4.51}{12.03-5.09}$ $\beta_1 = 33.02^o$

Diameter of the rotor

$U_1 = \frac{\pi D_1 N}{60};$ $12.03 = \frac{\pi x D_1 x 1500}{60}$ $D_1 = 0.1531m$

width of the rotor

$\eta_0 = \frac{P_s}{\omega Q H};$ $0.8 = \frac{147*10^3}{9810*Q*8}$ $Q = 2.34m^3/s$

$Q = C\pi D_1 B_1 V_{f1}$ Take C=1 since blockage is not given

$2.34 = 1 * \pi * 0.1531 * B_1 * 4.51;$ $B_1 = 1.07m$

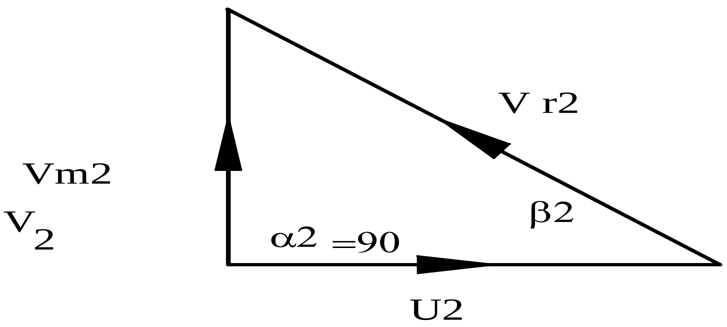
23. An Inward flow reaction turbine works under a head of 110m. The inlet and outlet diameters of the runner are 1.5m and 1.0m respectively. The width of the runner is constant throughout as 150mm. The blade angle at outlet is 15°. The hydraulic efficiency is 0.9. Calculate i) The speed of the turbine ii) The blade angles iii) The power developed when the discharge velocity is 6m/s(6c, 10,Dec14/Jan15)

    $H = 110m;$ $D_1 = 1.5m;$ $D_2 = 1.0m;$ $B_1 = B_2 = 150mm$ $B_1 = B_2 = 0.15m;$ $\beta_2 = 15^o$

    $\eta_h = 0.9;$ i) N =? ii) β_1 =? β_2 =? iii) E =? $V_2 = 6m/s$

    i) The speed of the turbine

$V_{f2} = V_2 = 6m/s;$ $\qquad V_{f2} = 6m/s$



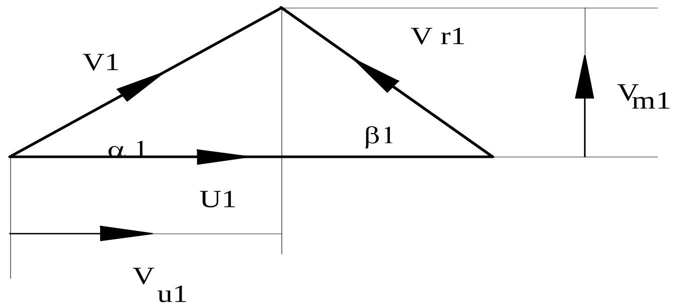
$tan\beta_2 = \frac{V_{f2}}{U_2};$ $\qquad tan15 = \frac{6}{U_2};$ $\qquad U_2 = 22.39m/s$

$U_2 = \frac{\pi D_2 N}{60}$ $\qquad 22.39 = \frac{\pi * 1.0 * N}{60};$ $\qquad N = 427.62rpm$

$U_1 = \frac{\pi D_1 N}{60};$ $\qquad U_1 = \frac{\pi x 1,5 x 427.62}{60}$ $\qquad U_1 = 33.59m/s$

$\eta_h = \frac{\overrightarrow{V_{u1}}\, U_1}{gH};$ $\qquad 0.9 = \frac{\overrightarrow{V_{u1}} * 33.59}{9.81 * 110}$ $\qquad \overrightarrow{V_{u1}} = 28.91m/s$

$U_1 > \overrightarrow{V_{u1}}$ Hence, Inlet triangle as given below



$Assuming\ \ V_{f1} = V_{f2} = 6m/s$

$tan\beta_1 = \frac{V_{f1}}{U_1 - \overrightarrow{V_{u1}}};$ $\qquad tan\beta_1 = \frac{6}{33.59 - 28.91}$ $\qquad \beta_1 = 52.04^o$

$tan\alpha_1 = \frac{V_{f1}}{\overrightarrow{V_{u1}}};$ $\qquad tan\alpha_1 = \frac{6}{28.91};$ $\qquad \alpha_1 = 11.72^o$

The power developed

$Q = C\pi D_1 B_1 V_{f1}$ Take C=1 since blockage is not given

$Q = 1 * \pi * 1.5 * 0.15 * 6\,;$ $\qquad Q = 4.24m^3/s$

$\frac{E}{m} = \frac{\overrightarrow{V_{u1}}\, U_1}{g_c};$ $\qquad E = m\frac{\overrightarrow{V_{u1}}\, U_1}{g_c};$ $\qquad E = \rho Q \frac{\overrightarrow{V_{u1}}\, U_1}{g_c}$

$E = 1000 * 4.24 * \frac{28.91 * 33.59}{1};$ $\qquad E = 4.11 * 10^6\, W$

24. Design an inward flow Francis turbine whose power output is 330kW under a head of 70m running at 750rpm., η_h=94%,η_o=85%. The flow ratio at inlet is 0.15, the breadth ratio is 0.1. The outer diameter of the runner is twice the inner diameter of runner. The thickness of the vanes occupy 6% the circumferential area of the runner. Flow velocity is constant and discharge is radial at outlet
25. Two inwards flow reaction turbine have the same runner diameter of 0.6m and the same hydraulic efficiency. They work under the same head and have the same velocity of flow of 6m/s. One runner A revolves at 520rpm and has an inlet vane angle of 65^o. If the other runner B has an inlet vane angle of 110^o , at what speed should it run?. Assume for both the turbine, the discharge is radial at outlet
26. A Francis turbine works a head of 180m while running at 750rpm. The outer and inner diameter of the runner is 1.4m and 0.85m. The water enters the runner with a velocity of 30m/s. The outlet angle of the guide blade is 10^o. Estimate the runner blade angles at inlet and outlet if the discharge is radial and velocity of flow is constant through the runner. Also, calculate the hydraulic efficiency
27. A medium Francis turbine has diameter 75cm and width 10cm. Water leaves the guide vanes at a velocity of 16m/s inclined at 25^o with the runner periphery. The net head is 20m. The overall and hydraulic efficiencies are 80% and 90% respectively. Assuming that 8% of the flow area is lost due to the runner vanes thickness, calculate the runner vane angle at inlet and outlet, Power output by the runner,speed and specific speed of machine and mechanical efficiency
28. A Francis turbine required to develop a power of 330kW under a head of 30m while running at 350rpm, if η_0=85%,η_h=88%,φ=0.75,ψ=0.25 and diameter ratio of outer to inner diameter =2 , calculate the stator and rotor angles and the dimensions of the runner.
29. A Francis turbine has to be designed to give an output of 500MHP under a head of 80m. The rotational speed is 700rpm. Determine the main dimensions of the runner and the guide vane and runner blade angles assuming the following data: Hydraulic losses 10%; flow ratio =0.15; ratio of inner to outer diameters=0.5; ratio of width to diameter at inlet =0.1; overall efficiency=82%; Area blocked by thickness of runner vane=15%
30. A Francis turbine is working under a head of 100m and the discharge $5m^3/s$. The velocity of flow is assumed constant through the runner is 16m/s. The runner blade angle at inlet is 90^o. The width of the blades at inlet is 0.15 times at the inlet diameter, the outer diameter is 0.6 times the inlet diameter. Find the hydraulic efficiency when the wheel is rotating at 500rpm and discharge is axial. Assume that 10% of the flow area is blocked by the thickness of the blades
31. Francis turbine has a cylindrical draft tube 2.5m in diameter. The velocity of water at inlet to the draft tube is 5m/s. Calculate the percentage gain in power output if the outlet diameter is changed to 3.5m. The draft tube efficiency is 75%. Assume the available head at inlet is 5m

KAPLAN TURBINE

32. A Kaplan turbine develops 10MW under an effective head of 8m. The overall efficiency is 0.86, the speed ratio is 2.0 and the flow ratio is 0.6. The hub diameter is 0.35 times the outside diameter of the wheel. Find the diameter and speed of the turbine(6c, 08,Dec17/Jan18)

$P_s = 10MW = 10000kW;\ ;H = 8;\ \eta_0 = 0.86\ ;\ \ speed\ ratio\ \phi = 2.0$; Flow ratio $\varphi = 0.6$

The hub diameter is 0.35 times the outside diameter of the wheel ie $D_h = 0.35D_o;\ \ D_o = ?\ N = ?$

$speed\ ratio\ \phi = 2.0;$ $\frac{U}{\sqrt{2gH}} = 2.0;$ $\frac{U}{\sqrt{2*9.81*8}} = 2.0$; $U = 25.05m/s$

$\frac{V_{f1}}{\sqrt{2gH}} = 0.6;$ $\frac{V_{f1}}{\sqrt{2*9.81*8}} = 0.6$; $V_{f1} = 12.52m/s$

Diameter of the turbine

$\eta_0 = \frac{P_s}{\omega QH};$ $0.86 = \frac{10*10^6}{9810*Q*8}$ $Q = 148.16m^3/s$

$D_h = 0.35D_o;$ $D_h = 0.35D_0$

$Q = \frac{\pi(D_0^2 - D_h^2)}{4} * V_{f1};$ $148.16 = \frac{\pi(D_0^2 - 0.35^2 D_0^2)}{4} * 12.52;$ $148.16 = \frac{\pi D_0^2(1 - 0.35^2)}{4} * 12.52$

$D_0^2 = 17.17m^2;$ $\boldsymbol{D_o = 4.14m}$

Speed of the turbine

$U = \frac{\pi D_0 N}{60};$ $25.05 = \frac{\pi*4.14*N}{60};$ $\boldsymbol{N = 115.56rpm}$

33. A Kaplan turbine produces 30000kW under a head of 9.6m, while running at 65.2rpm. The discharge through the turbine is $350m^3/s$. The diameter of the runner is 7.4m . The hub diameter is 0.432 times the tip diameter. Calculate i) Turbine efficiency ii) Specific speed of the turbine iii) Speed ratio (based on tip diameter) iv) Flow ratio (8b,08,June/July18,15 scheme)

$P_s = 30000kW;\ ;H = 9.6m;\ N = 80rpm\ \ Q = 350m^3/s;\ D_o = 7.4m;\ D_h = 0.432D_o$

$i)\eta_0 = ?\ \ ii)N_s = ?;\ \ iii)\phi = ?$

i) Turbine efficiency

$\eta_o = \frac{P_s}{\omega QH};$ $\frac{30000*10^3}{9810*350*9.6};$ $\eta_o = 0.91$

Specific speed of the turbine

$N_s = \frac{N\sqrt{P_s}}{H^{5/4}};$ $N_s = \frac{80\sqrt{30000}}{9.6^{5/4}}$ $N_s = 820$

Speed ratio (based on tip diameter)

$U = \frac{\pi D_o N}{60};$ $U = \frac{\pi*7.4*80}{60}$ $U = 31m/s$

$\emptyset = \frac{U}{\sqrt{2gH}}$; $\qquad \emptyset = \frac{31}{\sqrt{2*9.81*9.6}}$ $\qquad \emptyset = 2.25$

Flow ratio

$D_h = 0.432D_o$; $\qquad D_h = 0.432 * 7.4$ $\qquad D_h = 3.2m/s$

$Q = \frac{\pi(D_0^2 - D_h^2)}{4}V_f$; $\qquad 350 = \frac{\pi(7.4^2-3.2^2)}{4}V_f$ $\qquad V_f = 10m/s$

$\varphi = \frac{V_f}{\sqrt{2gH}}$; $\qquad \varphi = \frac{10}{\sqrt{2*9.81*9.6}}$ $\qquad \varphi = 0.73$

34. A Kaplan turbine has an outer dia of 8m and inner diameter 3m and develops 30000kW at 80rpm under a head of 12m. The discharge through the runner is $300m^3/s$. If the hydraulic efficiency is 95% , determine i) Inlet and outlet blade angels ii) Mechanical efficiency iii) Overall efficiency (6b,10, June/July14)
$D_o = 8m;\ D_h = 3m;\ P_s = 30000kW;\ N = 80rpm\,; H = 12m;\ Q = 300m^3/s; \eta_h = 0.95$
$i)\ \beta_1 = ?;\ \beta_2 = ?\ \ ii)\eta_{mech} = ?;\ \ iii)\eta_0 = ?$

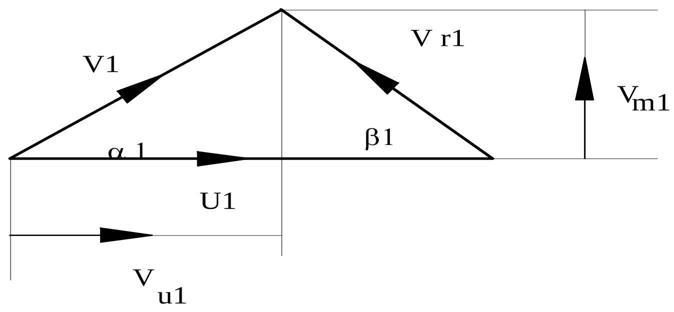
Inlet and outlet blade angels

$U = \frac{\pi D_o N}{60}$; $\qquad U = \frac{\pi*8*80}{60}$ $\qquad U = 33.51m/s$

$U_1 = U_2 = U$

$\eta_h = \frac{\overrightarrow{V_{u1}}\ U}{gH}$; $\qquad 0.95 = \frac{\overrightarrow{V_{u1}}x33.51}{9.81x12}$ $\qquad \overrightarrow{V_{u1}} = 3.337m/s$

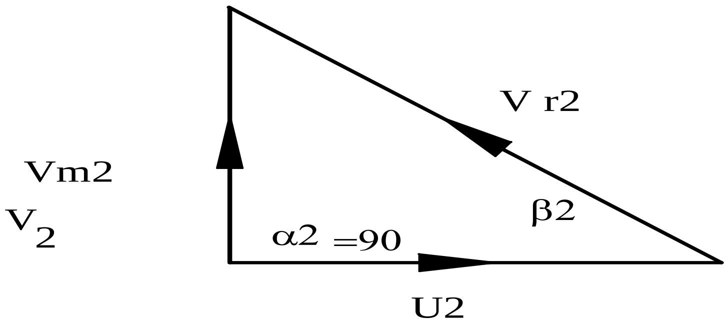
$U_1 > \overrightarrow{V_{u1}}$ Hence, Inlet triangle as given below



$Q = \frac{\pi(D_0^2 - D_h^2)}{4}V_f$; $\qquad 300 = \frac{\pi(8^2-3^2)}{4}V_f$ $\qquad V_f = 6.94m/s$

$tan\beta_1 = \frac{V_f}{U - \overrightarrow{V_{u1}}}$; $\qquad tan\beta_1 = \frac{6.94}{33.51-3.337}$ $\qquad \beta_1 = 12.96^o$

$tan\alpha_1 = \frac{V_f}{\overrightarrow{V_{u1}}}$; $\qquad tan\alpha_1 = \frac{6.94}{3.337}$; $\qquad \alpha_1 = 64.32^o$



$tan\beta_2 = \frac{V_f}{U}$; $tan\beta_2 = \frac{6.94}{33.51}$ $\beta_2 = 11.7^o$

Overall efficiency

$\eta_o = \frac{P_s}{\omega QH}$; $\eta_o = \frac{30000*10^3}{9810*300*12}$; $\eta_o = 0.849$

$\eta_o = \eta_h\eta_{mech}$ $0.849 = 0.95 * \eta_{mech}$ $\eta_{mech} = 0.8936$

35. A Kaplan turbine generates 45MW under a head of 22m. The overall efficiency is 90% and ratio of outlet to hub diameters is 2.85. Calculate the speed, specific speed and diameters of the runner. Assume ϕ=2.2 ; ψ=0.8
36. A Kaplan turbine develops 9000kW under a head of 10m. Overall efficiency of the turbine is 85%. The speed ratio based on outer diameter is 2.2 and flow ratio is 0.66. Diameter of the boss is 0.4 times the outer diameter of the runner. Determine the diameter of the runner, boss diameter and specific speed of the runner
37. A Kaplan turbine develops 2 MW at a head of 30m. The flow and speed ratio are 0.5 and 2.0m respectively. The hub diameter is 0.3 times the outer diameter of the runner. Calculate the runner diameter and speed of the turbine when the overall efficiency is 85%
38. A Kaplan turbine working under a head of 15m develops 7350kW. The outer diameter of the runner is 4m and hub diameter=2m. The guide blade angle at the extreme edge of the runner is 30°. The hydraulic and the overall efficiency of the turbine are 90% and 85% respectively. If the velocity of whirl is zero at outlet determine i) runner vane angle at inlet and outlet at the extreme edge of the runner ii) speed of the turbine
39. A Kaplan turbine working under a head of 20m develops 11772kW of shaft power. The outer diameter of runner is 3.5m and hub diameter is 1.75m. The guide blade angle at the extreme edge of the runner is 35°. The hydraulic and overall efficiencies of the turbines are 88% and 84% respectively. If the velocity of whirl is zero at outlet, determine i) Runner vane angle at the inlet and outlet at the extreme edge of the runner ii) speed of the turbine(8b, 08,Dec18)

$H = 20m$; $P_s = 11772kW$; $D_o = 3.5m$; $D_h = 1.75m$; $\eta_h = 0.88$; $\eta_0 = 0.84$;

The guide blade angle at the extreme edge of the runner is 35° $\alpha_1 = 35^o$;

If the velocity of whirl is zero at outlet ie $\overrightarrow{V_{u2}} = 0$; i) $\beta_1 = ?$ $\beta_2 = ?$ ii) $N = ?$

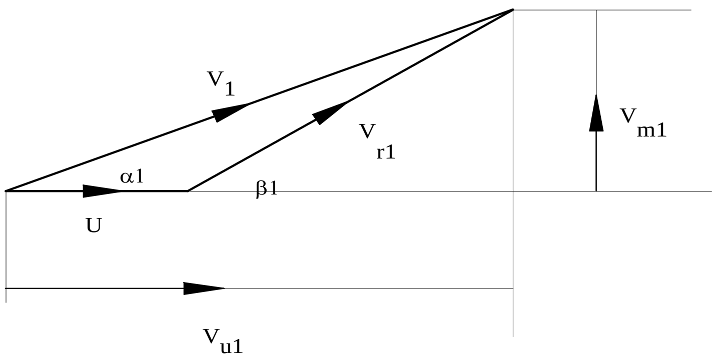
$\eta_0 = \frac{P_s}{\omega QH}$; $0.84 = \frac{11772*10^3}{9810*Q*20}$ $Q = 71.42\ m^3/s$

$Q = \frac{\pi(D_0^2 - D_h^2)}{4} * V_{f1}$; $71.42 = \frac{\pi(3.5^2 - 1.75^2)}{4} * V_{f1}$ $V_{f1} = 9.89m/s$

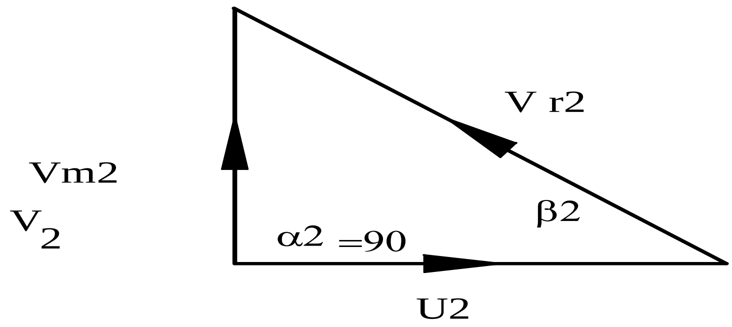
$tan\alpha_1 = \frac{V_{f1}}{\overrightarrow{V_{u1}}}$; $\quad tan35 = \frac{9.89}{\overrightarrow{V_{u1}}}$; $\quad \overrightarrow{V_{u1}} = 14.12m/s$

$\eta_h = \frac{\overrightarrow{V_{u1}}\ U}{gH}$; $\quad 0.88 = \frac{14.12*\ U}{9.81*20}$; $\quad U = 12.23m/s$

$\overrightarrow{V_{u1}} > U$, hence velocity triangle as given below



$tan\beta_1 = \frac{V_f}{\overrightarrow{V_{u1}} - U}$; $\quad tan\beta_1 = \frac{9.89}{14.12-12.23}$ $\quad$ $\boldsymbol{\beta_1 = 79.16^o}$



$U_2 = U_1 = U$; $\quad V_{f2} = V_{f1} = V_f$

$tan\beta_2 = \frac{V_f}{U}$; $\quad tan\beta_2 = \frac{9.89}{12.23}$; $\quad$ $\boldsymbol{\beta_2 = 38.96^o}$

<u>speed of the turbine</u>

$U = \frac{\pi D_0 N}{60}$; $\quad 12.23 = \frac{\pi*1.75*N}{60}$; $\quad$ $\boldsymbol{N = 133.47rpm}$

40. A Kaplan turbine produces 10MW at head of 25m. The runner and hub diameters are 3m and 1.2m respectively. The inlet and outlet are right angled triangles. Calculate the speed , and outlet angles of the guide and runner blades if the hydraulic and overall efficiencies are 96 and 85 percent respectively.
41. Determine the efficiency of a Kaplan turbine developing 2940kW under a head of 5m. It is provided with a draft tube with its inlet diameter 3m set at 1.6m above the tailrace level. A vacuum pressure gauge is connected to draft tube inlet indicates a reading of 5m of water. Assume that draft tube efficiency is 78%
42. A Kaplan turbine model built to a reduced scale of 1:10 develops 25MHP when run at 400rpm under the head of 6m. If its overall efficiency is 85%. What flow rate should be supplied to the model? If the prototype machine works under a head of 40m, compute the

speed, power output and discharge of the machine. Assume same overall efficiency for the model and prototype

29 A propeller turbine has outer diameter of 4.5m and inner diameter 2m. It develops 20,605kW under a head of 20m at 137rpm, the hydraulic efficiency is 0.94, overall efficiency is 0.88 Find i) the Runner blade angles ii) Discharge through the runner (6c, 10,June/July 17)

$D_o = 4.5m$; $D_h = 2m$; $P_s = 20605kW$; $H = 20m$; $N = 137rpm$; $\eta_h = 0.94$; $\eta_o = 0.88$

$i)$ $\beta_1 =?$ $\beta_2 =?$ ii) $Q =?$

ii) Discharge through the runner

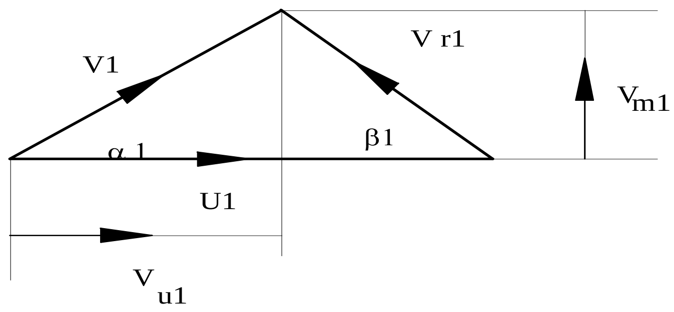
$\eta_0 = \frac{P_s}{\omega QH}$; $0.88 = \frac{20605*10^3}{9810*Q*20}$ $Q = 119.34\ m^3/s$

i) the Runner blade angles

$U = \frac{\pi D_0 N}{60}$; $U = \frac{\pi*4.5*137}{60}$; $U = 32.28m/s$

$\eta_h = \frac{\overrightarrow{V_{u1}}\ U}{gH}$; $0.94 = \frac{\overrightarrow{V_{u1}}*\ 32.28}{9.81*20}$; $\overrightarrow{V_{u1}} = 5.71m/s$

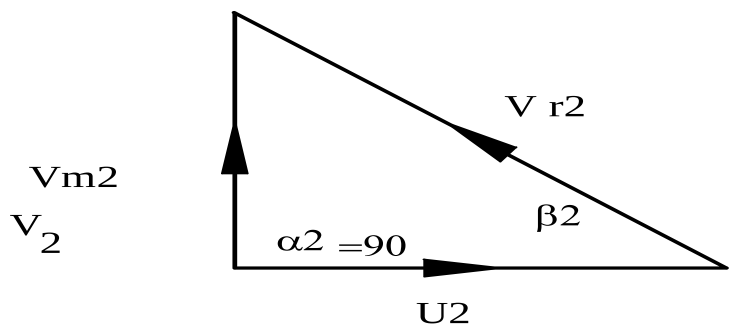
$U_1 > \overrightarrow{V_{u1}}$ Hence, Inlet triangle as given below



$Q = \frac{\pi(D_0^2-D_h^2)}{4} V_f$; $119.34 = \frac{\pi(4.5^2-2^2)}{4} V_f$ $V_f = 9.35\ m/s$

$tan\beta_1 = \frac{V_f}{U-\overrightarrow{V_{u1}}}$; $tan\beta_1 = \frac{9.35}{32.28-5.71}$ $\beta_1 = 19.38^o$

$tan\alpha_1 = \frac{V_f}{\overrightarrow{V_{u1}}}$; $tan\alpha_1 = \frac{9.35}{5.71}$; $\alpha_1 = 58.58^o$



$U_2 = U_1 = U;\quad V_{f2} = V_{f1} = V_f$

$tan\beta_2 = \frac{V_f}{U};\qquad tan\beta_2 = \frac{9.35}{32.28};\qquad \boldsymbol{\beta_2 = 16.15^o}$

MODULE 5

Centrifugal Pumps

1. Define a Centrifugal pump. With usual notations, derive theortical head –capacity relationship for a centrifugal pump (7a,8,Dec15/Jan16)
2. Define the following with respect to centrifugal pumps: i) Manometric head ii) Manometric efficiency iii) Overall efficiency (7b, 6,Dec16/Jan 17)
3. What is priming ? How priming will be done in centrifugal pumps ? (7b, 04, Dec12)
4. Explain the following with reference to centrifugal pump ; i) Manometric efficiency with expression ii) Cavitation in pumps iii) Need for priming iv) Pumps in series (6a, 10,June July 13)(7a, 10,June/July 18)
5. Define the following :i) Suction head ii) Delivery Head iii) Manometric Head iv) Net positive suction head (9a, 8, Dec18/Jan19,15 scheme)
6. With reference to the centrifugal pump explain what do you mean by i) Net positive section Head (NPSH) (7a, 6,Dec13/Jan 14)
7. Explain the phenomenon of cavitation in centrifugal pump (7b, 4,June/July16) (7c, 4,Dec13/Jan 14)
8. What is Cavitation? What are the causes for cavitation? Explain the steps to be taken to avoid cavitiation (7a, 6, Dec18/19)
9. What is Cavitation? What are its effects (7b,4,Dec15/Jan16)
10. Explain the following , with reference to the centrifugal pump: i) Slip and it effects ii) Cavitaiton , its effect and remedies to it iii) Difference between manometric head and NPSH (7a, 10,Dec17/Jan18)
11. What are the applications of multi-stage centrifugal pump? With a sketch, explain centrifugal pumps in series and parallel (7a, 8,June/July16)(7a,08,Dec12)
12. Explain with a neat sketch , multistage centrifugal compressor (7a, 5June July 17)
13. Explain with neat sketch , different casings of pump and label the parts (7b, 5June July 17) (7b, 6, Dec18/19)
14. What is minimum starting speed of a centrifugal pump? Derive an expression for minimum starting speed of a centrifugal pump (7a, 6,Dec16/Jan 17) (7a,6,Dec14/Jan15) (7a,12,June/July14)
15. Show that the pressure rise in the impeller of a centrifugal pump when the frictional and other losses in the impeller are neglected is given
    $\frac{1}{2g}\left(V_{f1}^2 + U_2^2 - V_{f2}^2 cosec^2\beta_2\right)$
    Where V_{f1} and V_{f2} are the flow velocities at inlet and outlet of the impeller, U_2 =Tangential speed of the impeller at the exit,β_2=Exit blade angle (7c,8,Dec15/Jan16) (9a, 8,June/July18 15 sheme)

Centrifugal pump is defined as a power absorbing machine in which the dynamic pressure generated by the forced vertex motion of the blades lifts the water from a low level to high level at the expense of mechanical energy. In other words it is defined as a turbomachine in which mechanical energy is converted into pressure energy. It is radial inward power absorbing machine

Classification of centrifugal pump

1. According to shape of impeller and casing
   a) Volute or spiral casing type b) Vortex (whirlpool) casing c) Diffuser type
2. According to type of impeller
   a) Closed or shrouded impeller b) Semi- open impeller c) open impeller
3. According to working head
   a) Low head centrifugal pump b) Medium head centrifugal pump iii) High Head centrifugal pump

Definitions with respect to centrifugal pump

1. Suction Head (h_s) : It is the vertical distance between the centre line of the pump and the water surface in the sump
2. Delivery Head(h_d) : It is the vertical distance between the centre line of the pump and the water surface at the delivery tank
3. Static Head (h) : It is the vertical distance between the liquid level in the sump and the delivery tank ie $h = h_s + h_d$
4. Total Head: The net work done by the pump on the water should be enough to overcome the static head and also total loss in the system due to friction, turbulence, foot valves and bends, while providing the kinetic energy of water at the delivery tank $\frac{V_d^2}{2g}$

   Thus $h_e = h + h_f + \frac{V_d^2}{2g}$

$$h_f = h_{fs} + h_{fd}$$

$h_{fs} = \frac{4fL_s V_s^2}{2gd_s}$; $Q = \frac{\pi d_s^2}{4} * V_s$; $V_s^2 = \frac{16Q}{(\pi d_s^2)^2}$; $h_{fs} = \frac{4fL_s}{2gd_s} x \frac{16Q^2}{\pi^2 d_s^4}$; $h_{fs} = \frac{64fL_s Q^2}{2g\pi^2 d_s^5}$

$h_{fd} = \frac{4fL_d V_d^2}{2gd_d}$; $Q = \frac{\pi d_d^2}{4} * V_d$; $V_d^2 = \frac{16Q}{(\pi d_d^2)^2}$ $h_{fd} = \frac{4fL_d}{2gd_d} x \frac{16Q^2}{\pi^2 d_d^4}$; $h_{fd} = \frac{64fL_d Q^2}{2g\pi^2 d_d^5}$

$\frac{V_d^2}{2g} = \frac{16Q^2}{\pi^2 d_d^4}$ x $\frac{1}{2g}$

5. Manometric Head: It is pressure head against which pump has to work

   $H_m = \frac{p_d}{\omega} - \frac{p_s}{\omega} = h + h_f + \frac{V_s^2}{2g}$

   $\frac{p_s}{\omega} = \frac{p_a}{\omega} - \frac{V_s^2}{2g} - h_s - h_{fs}$;

$= h+h_f + \frac{V_d^2}{2g} - \frac{V_d^2}{2g} + \frac{V_s^2}{2g}$

$= H_e + \frac{V_s^2}{2g} - \frac{V_d^2}{2g}$

If diameter of suction and delivery pipe are same then $V_s=V_d$

Hence $H_m = H_e$

Even otherwise also V_s and V_d are very small such that kinetic energy associated with them are negligible.

Hence generally $H_m=H_e$ if manometric efficiency is not given

If losses are given

$H_m = H_e -$losses (ie loss of head in the impellor and casing)

4. A centrifugal pump lifts water under a static head of 36m of water of which 4m is suction lift. Suction and delivery pipes have both 150mm in diameter. The head loss in suction pipe is 1.8m and 7m in delivery pipes. The impeller is 380mm in diameter and 25mm wide at mouth and revolves at 1200rpm . The exit blade angle is 35° . If the manometric efficiency of the pump is 82% , find the discharge and pressure at the suction and delivery branches of the pump. (7c,08, Dec12)

Discharge

$h = 36m;\quad h_s = 4m;\quad h_{fs} = 1.8m;\quad h_{fd} = 7m\ D_2 = 380mm = 0.38m;$

$B_2 = 25mm = 0.025m;\ N = 1200rpm;\quad \beta_2 = 35^o;\quad \eta_m = 82\%;\quad Q = ?;\ p_s = ?;\ p_d = ?;$

$H_m = h + h_f + \frac{V_s^2}{2g};\qquad H_m = 36 + (1.8 + 7)\qquad H_m = 44.8m$

$U_2 = \frac{\pi D_2 N}{60};\qquad U_2 = \frac{\pi * 0.38 * 1200}{60};\qquad U_2 = 23.88m/s$

$\eta_m = \frac{gH_m}{\overrightarrow{V_{u2}} U_2};\qquad 0.82 = \frac{9.81*44.8}{\overrightarrow{V_{u2}}*23.88};\qquad \overrightarrow{V_{u2}} = 22.44m/s$

$\overrightarrow{V_{u2}} = U_2 - V_{f2} cot\beta_2;\qquad 22.44 = 23.88 - V_{f2} cot35\qquad V_{f2} = 1m/s$

$Q = C\pi D_2 B_2 V_{f2}$ where $C = 1$ since there is no blockage , Hence $Q = \pi D_2 B_2 V_{f2}$

$Q = \pi * 0.38 * 025 * 1;\qquad Q = 0.029845m^3/s$

Suction pressure:

$Q = \frac{\pi d_s^2}{4} * V_s;\qquad 0.029845 = \frac{\pi * 0.15^2}{4} * V_s\qquad V_s = 1.69m/s$

$\frac{p_s}{\omega} = \frac{p_a}{\omega} - \frac{V_s^2}{2g} - h_s - h_{fs};$ $\frac{p_s}{\omega} = 10.3 - \frac{1.69^2}{2*9.81} - 4 - 1.8$ $\frac{p_s}{\omega} = 4.35\, of\ water$

delivery pressure

$\frac{p_d}{\omega} = \frac{p_a}{\omega} + h_d + h_{fd}$ $\frac{p_d}{\omega} = 10.3 + 32 + 7$ $\frac{p_d}{\omega} = 49.3m\ of\ water$

5. A centrifugal pump impeller has radial vanes from inner radius of 8cm to outer radius 24cm. The width of the impeller is constant and is 6cm between the shrouds. If the speed is 1500rpm and the discharge is 25litres/s. Find i) Change in enthalpy ii) The outlet pressure if inlet pressure is 0.8kPa and flow is outward (7c, 8,June/July16)

radial vanes, $\beta_2 = 90^0$; $R_1 = 8cm$; $D_1 = 16cm = 0.16m$; $R_2 = 24\ cm$;

$D_2 = 48cm = 0.48m$ $B_1 = B_2 = 6cm = 0.06m$; $N = 1500rpm$; $Q = 25litrs/s$ $=0.025m^3/s$

$U_2 = \frac{\pi D_2 N}{60}$; $U_2 = \frac{\pi*0.48*1500}{60}$; $U_2 = 37.7m/s$

$Q = C\pi D_2 B_2 V_{f2}$ where $C = 1$ since there is no blockage , Hence $Q = \pi D_2 B_2 V_{f2}$

$0.025 = \pi * 0.48 * 0.06 * V_{f2}$; $V_{f2} = 0.2763m/s$

Change in enthalpy= $\frac{E}{\dot{m}}$;

radial vanes, $\beta_2 = 90^0$

$\overrightarrow{V_{u2}} = U_2$; $\frac{E}{\dot{m}} = \frac{\overrightarrow{V_{u2}} U_2}{g_c}$ $\frac{E}{\dot{m}} = \frac{U_2 U_2}{g_c}$; $\frac{E}{\dot{m}} = \frac{37.7*37.7}{1}$

$\frac{E}{\dot{m}} = 1421.29J/kg$

The outlet pressure if inlet pressure is 0.8kPa and flow is outward

$\frac{p_d}{\omega} = \frac{p_s}{\omega} + H_e$

$H_e = \frac{\overrightarrow{V_{u2}} U_2}{g}$; $H_e = \frac{U_2 U_2}{g}$ $H_e = \frac{37.7^2}{9.81}$; $H_e = 144.88\ m\ of\ water$

$\frac{p_d}{\omega} = \frac{800}{9810} + 144.88$; $\frac{p_d}{\omega} = 144.96\ m\ of\ water$ $\frac{p_d}{9810} = 144.96$

$p_d = 144.96 * 9810$; $p_d = 1422057.6N/m^2$ $p_d = 1.422bar$

6. A centrifugal pump is to discharge 0.118m^3/s of water at a speed of 1450rpm against a head of 25m. The impeller diameter is 25cm and its width at the outlet is 5cm and manometric efficiency is 75% Calculate the vane angle at outlet (7b,6,Dec14/Jan15)

$Q = 0.118m^3/s$; $N = 1450rpm$; $H_m = 25m$; $D_2 = 25cm = 0.25m$; $B_2 = 5cm = 0.05m$;

$\eta_m = 75\%$; $\beta_2 = ?$

vane angle at outlet β_2

$Q = C\pi D_2 B_2 V_{f2}$ where $C = 1$ since there is no blockage , Hence $Q = \pi D_2 B_2 V_{f2}$

$0.118 = \pi * 0.25 * 0.05 * V_{f2}$; $V_{f2} = 3m/s$

$U_2 = \frac{\pi D_2 N}{60}$; $U_2 = \frac{\pi * 0.25 * 1450}{60}$; $U_2 = 18.98\ m/s$

$\eta_m = \frac{gH_m}{\overrightarrow{V_{u2}} U_2}$; $0.82 = \frac{9.81 * 25}{\overrightarrow{V_{u2}} * 18.98}$; $\overrightarrow{V_{u2}} = 15.76 m/s$

$\overrightarrow{V_{u2}} = U_2 - V_{f2} cot\beta_2$; $15.76 = 18.98 - 3cot\beta_2$; $cot\beta_2 = 1.074$

$tan\beta_2 = 0.93$ $\beta_2 = 42.98^o$

7. A centrifugal pump working in dock pumps 1565lit/s against ahead (mean lift) of 6.1m when the impeller rotates at 200rpm. The impeller diameter is 122cm and the area at outlet periphery is 6450cm^2. If the vanes are set back at an angle of 26^o at the outlet, find i) hydraulic efficiency ii) Pump required to drive the pump. If the ratio of external to internal diameter is 2, find the minimum speed to start pumping (9b, 8,Dec18/Jan19 15 scheme)

$Q = 1565 lit/s = 1565 * 10^{-3} m^3/s; H_m = 6.1m;$ $N = 200rpm;$; $D_2 = 122cm = 1.22m;$

$A_{f2} = 6450cm^2 = 6450 * 100^{-2};$ $\beta_2 = 26^o$ $\eta_m = ?;$

Pump required to drive the pump $E = ?$

If the ratio of external to internal diameter is 2 ie $\frac{D_2}{D_1} = 2$ $N_{min} = ?$

<u>hydraulic efficiency</u>

$Q = A_{f2} V_{f2};$ $1565 * 10^{-3} = 6450 * 10^{-4} * V_{f2};$ $V_{f2} = 2.43 m/s$

$U_2 = \frac{\pi D_2 N}{60}$; $U_2 = \frac{\pi * 1.22 * 200}{60}$; $U_2 = 12.78 m/s$

$\overrightarrow{V_{u2}} = U_2 - V_{f2} cot\beta_2$; $\overrightarrow{V_{u2}} = 12.78 - 2.43 cot 26$; $\overrightarrow{V_{u2}} = 7.8 m/s$

$\eta_m = \frac{gH_m}{\overrightarrow{V_{u2}}U_2}$; $\eta_m = \frac{9.81*6.1}{7.8*12.78}$; $\eta_m = 0.6$

minimum speed to start pumping

For minimum starting speed

$\frac{D_2}{D_1} = 2$; $\frac{1.22}{D_1} = 2$ $D_1 = 0.61m$

$N_{min}^2 = \left(\frac{60}{\pi}\right)^2 \left(\frac{2gH_m}{D_2^2 - D_1^2}\right)$; $N_{min}^2 = \left(\frac{60}{\pi}\right)^2 \left(\frac{2*9.81*6.1}{1.22^2 - 0.61^2}\right)$; $N_{min}^2 = 39106.65$

$N_{min} = 197.75rpm$

8. A centrifugal pump with 1.2 diameter runs at 200rpm and pumps 1.88m^3/s, the average lift being 6m. The angle which the vane makes at exit with the tangent to the impeller is 26^o and radial velocity of flow is 2.5m/s. Find manometric efficiency and the least speed to start pumping if the inner diameter being 0.6m(7c,8,Dec14/Jan15)
9. A centrifugal pump has its impeller diameter 30cm and a constant area of flow 210cm^2. The pump runs at 1440rpm and delivers 90LPS against a head of 25m. If there is no whirl velocity at entry, compute the rise in pressure head across the impeller and hydraulic efficiency of pump (9b, 08,June/July18)

Rise in pressure head across the impeller

$D_2 = 30cm = 0.3m$; constant area of flow 210cm^2. $A_{f1} = A_{f2} = 210\text{cm}^2$;

$A_{f1} = A_{f2} = 210 * 10^{-4}\ \text{m}^2$; $N = 1440rpm$; $Q = 90lit/s = 90 * 10^{-3}m^3/s$; $H_m = 25m$

$Q = A_{f2}V_{f2}$; $90 * 10^{-3} = 210 * 10^{-4} * V_{f2}$; $V_{f2} = 4.28\ m/s$

$U_2 = \frac{\pi D_2 N}{60}$; $U_2 = \frac{\pi*0.3*1440}{60}$; $U_2 = 22.61m/s$

Blade angle at outlet is missing Hence assume $\beta_2 = 30^o$

$\overrightarrow{V_{u2}} = U_2 - V_{f2}cot\beta_2$; $\overrightarrow{V_{u2}} = 22.61 - 4.28cot30$; $\overrightarrow{V_{u2}} = 15.19m/s$

$V_2^2 = V_{u2}^2 + V_{f2}^2$; $V_2^2 = 15.19^2 + 4.28^2$; $V_2^2 = 249.05$

Assuming flow velocity is constant $V_{f1} = V_{f2}$; ie $V_{f1} = 4.28m/s$; $V_1 = V_{f1} = 4.28m/s$

$H_e = \frac{V_{u2}U_2}{g}$; $H_e = \frac{15.19*22.61}{9.81}$ $H_e = 35\ m$ of water

$\frac{p_1}{\omega} + \frac{V_1^2}{2g} + H_e = \frac{p_2}{\omega} + \frac{V_2^2}{2g}$; $\frac{p_1}{\omega} + \frac{V_1^2}{2g} + \frac{V_{u2}U_2}{g} = \frac{p_2}{\omega} + \frac{V_2^2}{2g}$; $\frac{p_2}{\omega} - \frac{p_1}{\omega} = \frac{V_{u2}U_2}{g} - \left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g}\right)$

$\frac{p_2}{\omega} - \frac{p_1}{\omega} = 35 - \left(\frac{249.05}{2*9.81} - \frac{4.28^2}{2*9.81}\right)$; $\frac{p_2}{\omega} - \frac{p_1}{\omega} = 23.24\ m\ of\ water$

<u>hydraulic efficiency of pump</u>

$\eta_m = \frac{gH_m}{\overrightarrow{V_{u2}}U_2}$; $\eta_m = \frac{9.81*25}{15.19*22.61}$; $\eta_m = 0.7140$

10. A centrifugal designed to run at 1450rpm , with a maximum discharge of 1800litrres/min against a total head of 20m. The suction and delivery pipes are designed such that they are equal in size of 100mm. If the inner diameter and outer diameters of the impeller are 12cm and 24cm respectively. Determine the blade angles $\beta_1$and $\beta_2$for radial entry. Neglect friction and other losses (7c,10,Dec13/Jan14)

$N = 1450rpm;\ Q = 1800lit/min = 1800 * 10^{-3}m^3/min$

$Q = \frac{1800}{60}10^{-3}m^3/s$; $Q = 30 * 10^{-3}\ m^3/s$; $H_m = 30m$; $d_s = d_d = 100mm\ \ d_s = d_d = 0.1m$

$D_1 = 12cm = 0.12m$; $D_2 = 24cm = 0.24m$

Neglect friction and other losses ie $\eta_m = 1$

i) β_1=? Ii) β_2=?

$Q = \frac{\pi d_s^2}{4}V_{f1}$; Also $Q = \frac{\pi d_d^2}{4}V_{f2}$

$Q = \frac{\pi d_s^2}{4}V_{f1}$; $30 * 10^{-3} = \frac{\pi*0.1^2}{4}V_{f1}$ $V_{f1} = 3.82m/s$

$Q = \frac{\pi d_d^2}{4}V_{f2}$; $30 * 10^{-3} = \frac{\pi*0.1^2}{4}V_{f2}$ $V_{f2} = 3.82m/s$

$U_1 = \frac{\pi D_1 N}{60}$; $U_2 = \frac{\pi*0.12*1450}{60}$; $U_1 = 9.11m/s$

$U_2 = \frac{\pi D_2 N}{60}$; $U_2 = \frac{\pi*0.24*1440}{60}$; $U_2 = 18.22m/s$

$\eta_m = \frac{gH_m}{\overrightarrow{V_{u2}}U_2}$; $1 = \frac{9.81*30}{\overrightarrow{V_{u2}}*18.22}$; $\overrightarrow{V_{u2}} = 16.15m/s$

$\overrightarrow{V_{u2}} = U_2 - V_{f2}cot\beta_2$; $16.15 = 18.22 - 3.82cot\beta_2$; $cot\beta_2 = 0.54$

$tan\beta_2 = 1.85$; $\beta_2 = 61.54^o$

$tan\beta_1 = \frac{V_{f1}}{U_1}$; $tan\beta_1 = \frac{3.82}{9.11}$; $\beta_1 = 22.74^o$

11. A 3 stage centrifugal pump has impeller each of 38cm diameter and 1.9cm wide at outlet. The vane are curved at anangle is 45° at the outlet and reduced the circumferential area by 10%. The manometric efficiency is 90% and overall efficiency is 0.8%. Find the total head generated by the pump when running at 1000 rpm delivering 50litres /s. Also calculate the power required to drive the pump(7c, 10, June July 17)*

12. A centrifugal pump having outer diameter equal to two times the inner diameter and running at 1000rpm works against a total head of 40m. The velocity of flow through the impeller is constant and equal to 2.5m/s. The vanes are set back at an angle of 40° at outlet. Of the outer diameter of the impeller is 500mm and width at outlet is 50mm, determine i) vane angle at inlet ii) Work done by impeller on water /s iii) manometric efficiency (7c, 8,Dec16/Jan 17)

   outer diameter equal to two times the inner diameter ie $D_2 = 2D_1$;

   $N = 1000rpm$; $H_m = 40m$; $V_{f1} = V_{f2} = 2.5m/s$; $\beta_2 = 40^0$; $D_2 = 500mm = 0.5m$;

   $B_2 = 50mm = 0.05m$; i) $\beta_1 =?$ ii) $E =?$ iii) $\eta_m =?$

   <u>vane angle at inlet</u>

$D_2 = 2D_1$; $0.5m = 2D_1$ $D_1 = 0.25m$

$U_1 = \frac{\pi D_1 N}{60}$; $U_2 = \frac{\pi * 0.25 * 1000}{60}$; $U_1 = 13.09m/s$

$tan\beta_1 = \frac{V_{f1}}{U_1}$; $tan\beta_1 = \frac{2.5}{13.09}$; $\beta_1 = 10.81^o$

<u>Work done by impeller on water /s</u>

$Q = C\pi D_2 B_2 V_{f2}$ where $C = 1$ since there is no blockage , Hence $Q = \pi D_2 B_2 V_{f2}$

$Q = \pi * 0.5 * 0.05 * 2.5;$ $Q = 0.1963\ m^3/s$

$\dot{m} = \rho Q;$ $\dot{m} = 1000 * 0.1963$ $\dot{m} = 196.3 kg/s$

$U_2 = \frac{\pi D_2 N}{60};$ $U_2 = \frac{\pi*0.5*1000}{60};$ $U_2 = 26.17 m/s$

$\overrightarrow{V_{u2}} = U_2 - V_{f2} cot\beta_2;$ $\overrightarrow{V_{u2}} = 26.17 - 2.5cot40;$ $\overrightarrow{V_{u2}} = 23.19 m/s$

$\frac{E}{\dot{m}} = \frac{V_{u2}U_2}{g_c};$ $\frac{E}{196.3} = \frac{23.19*26.17}{1}$ $E = 119134.16 Watts$

13. A centrifugal pump having outer diameter equal to two times the inner diameter and running at 1200rpm works against a total head of 75m. The velocity of flow through the impeller is constant and equal to 3m/s. The vanes are set back at an angle of 30° at outlet. Of the outer diameter of the impeller is 60cm and width at outlet is 5cm, determine i) vane angle at inlet ii) Work done by impeller on water /s iii) manometric efficiency (7b,08,June/July14)

14. The outer diameter of a pump is 50cm and inner diameter is 25cm and runs at 1000rpm against a head of 40m. Velocity of flow is constant and is equal to 2.5m/s. Vanes are set back an angle 40°at the outlet . Width at outlet is 5cm Find i) Vane angle at inlet ii) Work done by impeller iii) Manometric efficiency (6b, 10,June July 13)

15. The outer diameter of the impeller of a centrifugal pump is 40cm and Width of the impeller at outlet is 5cm The pump is running at 800 rpm and working against a total head of 1.5m.The Vanes angle at outlet 40° and manometric efficiency is 75%. Determine i) velocity of flow at outlet ii) velocity of water leaving the vane iii) Angle made by the absolute velocity at outlet with the direction of motion at outlet iv) Discharge (7b, 08,Dec18/Jan19) (7b, 10,Dec17/Jan18)

$D_2 = 40cm = 0.4m; B_2 = 5cm = 0.05m;\ \ N = 800rpm; H_m = 15m;\ \beta_2 = 40^0; \eta_m = 75\%$

$i) V_{f2} = ?\ \ ii)\ V_2 = ?$ ii) $\alpha_2 = ?$ iii) $Q = ?$

i) **<u>velocity of flow at outlet</u>**

$U_2 = \frac{\pi D_2 N}{60};$ $U_2 = \frac{\pi*0.4*800}{60};$ $U_2 = 16.76 m/s$

$\eta_m = \frac{gH_m}{\overrightarrow{V_{u2}}U_2};$ $0.75 = \frac{9.81*15}{\overrightarrow{V_{u2}}*16.76};$ $\overrightarrow{V_{u2}} = 11.71 m/s$

$\overrightarrow{V_{u2}} = U_2 - V_{f2}cot\beta_2;$ $11.71 = 16.76 - V_{f2}cot40\ ;$ $V_{f2} = 4.23m/s$

ii) **<u>velocity of water leaving the vane</u>**

$V_2^2 = V_{u2}^2 + V_{f2}^2;$ $V_2^2 = 11.71^2 + 4.23^2;$ $V_2^2 = 155.08$

$V_2 = 12.45m/s$

iii) **<u>Angle made by the absolute velocity at outlet with the direction of motion at outlet</u>**

$tan\alpha_2 = \frac{V_{f2}}{U_2};$ $tan\alpha_2 = \frac{4.23}{16.76}$ $\alpha_2 = 14.16^o$

iv) **<u>Discharge</u>**

$Q = C\pi D_2 B_2 V_{f2}$ where $C = 1$ since there is no blockage , Hence $Q = \pi D_2 B_2 V_{f2}$

$Q = \pi * 0.4 * 0.05 * 4.23;$ $Q = 0.266m^3/s$

16. A three stage centrifugal pump has impeller of 40cm diameter and 2.5cm wide at the outlet. The vanes are curved back at outlet at 30° and reduce the circumferential area by 15%. The manometric efficiency is 85% and overall efficiency is 75%. Determine the head generated by the pump when running at 12000rpm, and discharging the water at 0.06m^3/s. Find the shaft power also
17. A 4 stage centrifugal pump has impellers each of 38cms diameter and 1.9cm wide at outlet. The outlet vane angle is 49° and vanes occupy 8% of outlet area. The manometric efficiency is 84% and overall efficiency is75%. Determine the head generated by the pump when running at 900rpm discharging 59litres per sec
18. A centrifugal pump is running at 1000rpm, the output vane angle of the impeller is 45° and the velocity of flow at outlet is 2.5m/s. The discharge through the pump is 200lit/s, when the pump is working against the total head of 20m. If the manometric efficiency of the pump is 80%, determine : i) Diameter of the impeller ii) width of the impeller at outlet
19. The diameters of the impeller of a centrifugal pump are 30cm and 60cm respectively. The velocity of flow at outlet is 2m/s and the vanes are set back at an angle of 45° at outlet. Determine the minimum starting speed of the pump if its manometric efficiency is 70%
20. A centrifugal pump with impeller outer diameter 1.5m runs at 180rpm and pumps 1.9m^3/s. The average lift is 8.0m. The angle which the vane makes at exit with the tangent to the impeller is

24^o and the radial velocity is 2.8m/s. Determine the manometric efficiency of the centrifugal pump

21. A centrifugal pump delivers 50lit/s against a total head of 24m when running at 1500rpm. The velocity of the flow is maintained constant at 2.4m/s. and blades are curved back at 30^o to the tangent at outlet. The inner diameter is half the outer diameter. If the manometric efficiency is 80%, determine i) Blade angle at inlet ii) Power required to drive the pump)(7b, 10,June/July 18)*

    $Q = 50lit/s; H_m = 24m;\ N = 1500rpm;\ \ V_{f1} = V_{f2} = 2.4m/s; \beta_2 = 30^0$;

    The inner diameter is half the outer diameter ie $D_1 = \frac{1}{2}D_2$; $\eta_m = 80\%$ i) $\beta_1 = ?$; ii) $E = ?$

i) **Blade angle at inlet**

$\overrightarrow{V_{u2}} = U_2 - V_{f2}cot\beta_2;$ $\quad \overrightarrow{V_{u2}} = U_2 - 2.4cot40$ $\quad \overrightarrow{V_{u2}} = U_2 - 2.86$

$\eta_m = \frac{gH_m}{\overrightarrow{V_{u2}}U_2};$ $\quad 0.8 = \frac{9.81*24}{(U_2-2.86)U_2};$ $\quad 0.8(U_2 - 2.86)U_2 = 235.44$

$(U_2 - 2.86)U_2 = 294.3;$ $\quad U_2^2 - 2.86U_2 = 294.3$ $\quad U_2^2 - 2.86U_2 - 294.3 = 0$

$U_2 = \frac{-b\pm\sqrt{b^2-4ac}}{2a};$ $\quad U_2 = \frac{+2.86\pm\sqrt{2.86^2-(4*1*[-294.3])}}{2*1};$ $\quad U_2 = 18.64m/s$

$U_2 = \frac{\pi D_2 N}{60};$ $\quad 18.64 = \frac{\pi * D_2 * 1500}{60};$ $\quad D_2 = 0.237m$

$D_1 = \frac{1}{2}D_2;$ $\quad D_1 = \frac{1}{2} * 0.237;$ $\quad D_1 = 0.119m$

$U_1 = \frac{\pi D_1 N}{60};$ $\quad U_2 = \frac{\pi * 0.119 * 1500}{60};$ $\quad U_1 = 9.32m/s$

$tan\beta_1 = \frac{V_{f1}}{U_1};$ $\quad tan\beta_1 = \frac{2.4}{9.32};$ $\quad \beta_1 = 13.43^o$

<u>Power required to drive the pump</u>

$\overrightarrow{V_{u2}} = U_2 - 2.86;$ $\quad \overrightarrow{V_{u2}} = 18.64 - 2.86;$ $\quad \overrightarrow{V_{u2}} = 15.78m/s$

$Q = C\pi D_2 B_2 V_{f2}$ where $C = 1$ since there is no blockage , Hence $Q = \pi D_2 B_2 V_{f2}$

$Q = \pi * 0.237 * 0.05 * 2.4;$ $\quad Q = 0.0893m/s$

$\dot{m} = \rho Q;$ $\quad \dot{m} = 1000 * 0.0893$ $\quad \dot{m} = 89.3kg/s$

$\frac{E}{\dot{m}} = \frac{V_{u2}U_2}{g_c};$ $\quad \frac{E}{89.3} = \frac{15.78*26.17}{1}$ $\quad E = 36877.56Watts$

22. The outer diameter of the impeller of a centrifugal pump is 40cm and width of the impeller at outlet is 5cm. The pump is running at 800rpm and is working against a total head of 15m. The vane angle at outlet is 40^o and manometric efficiency is 75%. Determine i) Velocity of flow at outlet ii) Velocity of water leaving the vane iii) Angle made by the absolute velocity at outlet with the direction of motion at outlet iv) Discharge
23. A backward swept centrifugal fan develops a pressure of 75mm W.G. It has an impeller diameter of 89cm and runs at 720rpm. The blade angle at the tip is 39^o and the width of the impeller 10cm. Assuming a constant radial velocity of 9.15m/s and density of $1.2kg/m^3$, determine the fan efficiency , discharge , power required , stage reaction and pressure coefficient
24. A centrifugal pump discharges $0.15m^3/s$ of water against a head of 12.5m, the speed of the impeller being 600rpm. The outer and inner diameter of the impeller are 50cm and 20cm respectively and the vanes are bent back at 35^o to the tangential at the exit. If the area of flow remains $0.07m^2$from inlet to outlet, calculate i) manometric efficiency of the pump ii) vane angle at inlet iii) loss of head at inlet to the impeller when the discharge is reduced by 40% without changing the speed
25. The following data refers to a Centrifugal pump: Pump being single stage radial bladed. i)impeller diameter =120mm ii) Discharge gauge reading =1.5 bar iii) Suction gauge reading =150mm of Hg below atmosphere iv) RPM =1440 v) Flow rate =240lit/min vi) Power required =1kW vii) Impeller width at tip= 10mm. Find Overall efficiency and specific speed
26. A centrifugal pump has its impeller diameter 30cm and a constant area of flow $210cm^2$. The pump runs at 1440 rpm and delivers 90lps against a head of 25m. If there is no whirl velocity at entry, compute the rise in pressure head across the impeller and hydraulic efficiency of pump. The vanes at exit are bent back at 22^o wrt tangential speed
27. A centrifugal pump with an impeller outer diameter of 1.05m runs at 1000RPM. The blades are backward curved and they makes an angle of 20^o with the wheel tangent at the blade tip. If the radial velocity of flow at the tip is 8m/s and the slip coefficient is 0.86. Find i) The actual work input /kg of water flow ii) the absolute velocity of fluid at the impeller tip and iii) hydraulic efficiency.If the pump is fitted with diffusion chamber with an efficiency of 0.75, so that the exit velocity is reduced to 5m/s. Find the new efficiency
28. A single stage centrifugal pump with a impeller of diameter of 30cm rotates at 2000rpm and lifts $3m^3/s$ water to a height of 40m with a manometric efficiency of 75%. Find the number of stages

and diameter of each impeller of a multistage pump to lift 5 m^3/s of water to height of 200m when rotating at 1500rpm

29. Explain the phenomenon of surging , stalling and choking in centrifugal compressor stage (7a, 5June July 17)

Module 5 compressors

1. With neat schematic diagram, explain an axial flow compressor, Also sketch the general velocity triangles for an axial flow compressor(8a, 10, Dec18/19)
2. With a neat sketch, explain the axial flow compressor (8b, 6, Dec16/Jan17)
3. What are the types of diffuser ? Explain any two $(10b, 08\ Dec18/Jan19CBCS)$
4. What is a function of diffuser? Name the different types of diffusers and explain them with neat sketch (8a,10,June/July 18)
5. Draw the velocity triangles at the entry and exit for the axial compressor stage (8b,6,June/July14) (
6. Explain the working principle of axial flow compressor along with a neat sketch of compressor (10a, 10,June/July18)
7. For axial flow compressor show that $E = V_f U \left(\frac{tan\beta_2 - tan\beta_1}{tan\beta_1 tan\beta_2}\right)$ $(10a, 08\ Dec18/Jan19CBCS)$
8. Derive an expression for overall pressure ratio for a centrifugal compressor in terms of impeller tip speed , slip . power input factor and isentropic efficiency of compressor(8a,12,Dec 13/Jan14)
9. With the help of H-Q plot explain the phenomenon of surging in centrifugal compressor(8a, 10,June/July13)
10. Define the following terms of centrifugal compressor i) slip factor , ii) Power factor and iii) Pressure coefficient (8a,6,June/July 16)
11. Explain slip and slip coefficient (8a,5,June/July17)and slip factor (8a, 06,Dec14/Jan15)
12. Define the following terms of centrifugal compressor:i) overall pressure ratio i) slip factor , ii) Power factor and iii) Pressure coefficient (8a, 08,Dec12)
13. Explain surging and choking of compressor (8b,5June/July17) (8b,4,June/July 16)(8b, 04,Dec14/Jan15) (8b, 08,Dec12)
14. Explain surging , stalling and Slip factor with reference a compressor (8a, 8,Dec15/Jan16)(8a,6,June/July14)
15. Explain the phenomenon of surging and stalling in centrifugal compressor(8a, 6, Dec16/Jan17)and Choking(8a,6,June/July14)
16. What is radial equilibrium in an axial flow compressor? Derive an expression for radial equilibrium in terms of flow velocity and whirl velocity of fluid (8a,10, Dec17/Jan18)
17. What is a function of diffuser? Name the different types of diffusers and explain them with neat sketch (8a,10,June/July 18)

Axial flow compressor

An axial flow compressor is essentially an axial flow turbine driven the reverse direction. The turning angle is very small preferably lower than 30^o to avoid flow separation.

It consists of number of fixed blades which are attached to the casing and alternative rows of moving blades on to the shaft which is mounted on bearings. Air progresses from one blade row to the next blade rows guided through the fixed blades. Fixed blades serves the function of diffuser and hence the pressure of air increases when it comes out of it. Air flows in the compressor parallel to the shaft.

The usual type of compressor is of 50% degree of reaction in which static enthalpy change in rotor is half the stage static enthalpy change (total head). In 50% Reaction compressor velocity triangle at inlet and outlet are symmetrical as in the case of 50% reaction turbine. It is necessary to achieve prewhirl at inlet so as to maintain the Mach number below 0.9 as it required for high efficiency.

Because of low turning angle the pressure rise per stage is very small. However, the large axial velocity of flow at the exit can cause more losses and hence it leads to a low total to static efficiency.

For ideal condition diffuser inlet angle α_1 is same as the fluid angle at inlet of the rotor

If the fluid enters axially, no guide vane blades are required ie $\alpha_1=90^o$, the static pressure rise in the rotor blades is much larger than in the stator blades. Because of the large relative velocities compared to 50% reaction, the stage efficiency is lower than that of symmetrical stage.

If the fluid leaves axially $\alpha_2=90^o$ the static pressure rise occurs entirely in the rotor blades, the stator blades causing only a small pressure rise. The degree of reaction is therefore greater than 100%. The energy transfer per stage is low and hence stage efficiency also . But due to low exit velocity, the overall efficiency is likely to be larger than the other types of compressors with R<0.5, R=0.5,R<1

With neat schematic diagram, explain an axial flow compressor, Also sketch the general velocity triangles for an axial flow compressor(8a, 10, Dec18/19)

A single stage axial flow compressor consisting one row of inlet guide vanes, one row of rotor blades (moving blades) and one row of diffusers (fixed blades) as shown in figure 4.1(a). The main function of the inlet guide vane is to control the direction of fluid flow at the rotor inlet. The rotor blades exert a torque on the fluid, its pressure and velocity increases. The diffuser blades increase the fluid pressure further by decreasing fluid velocity. The pressure and velocity variations through an axial flow compressor stage are shown in the figure

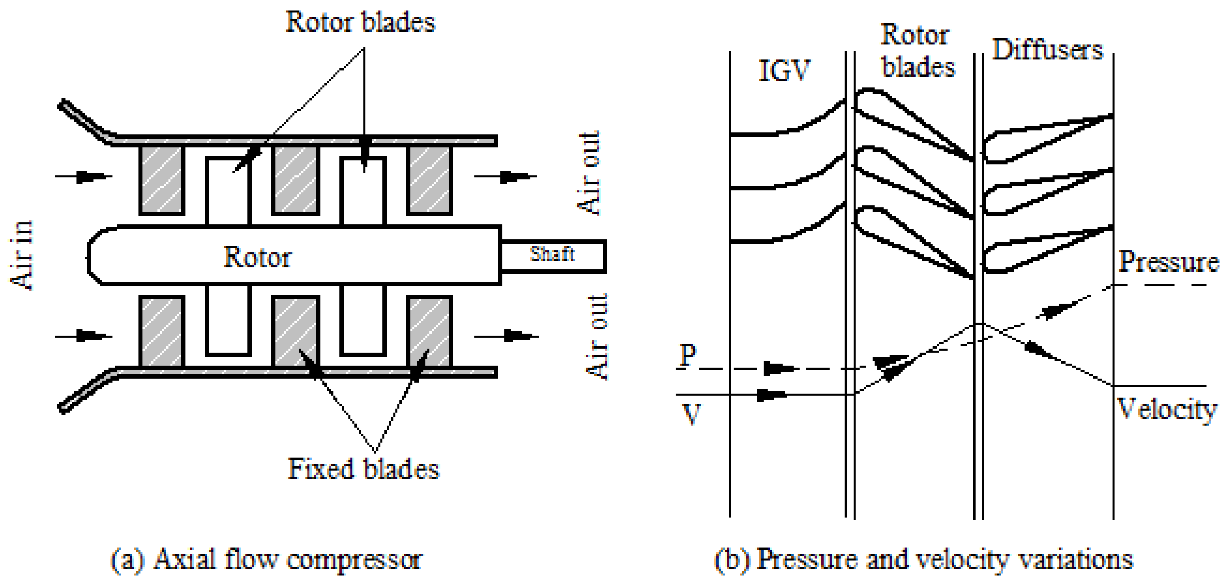
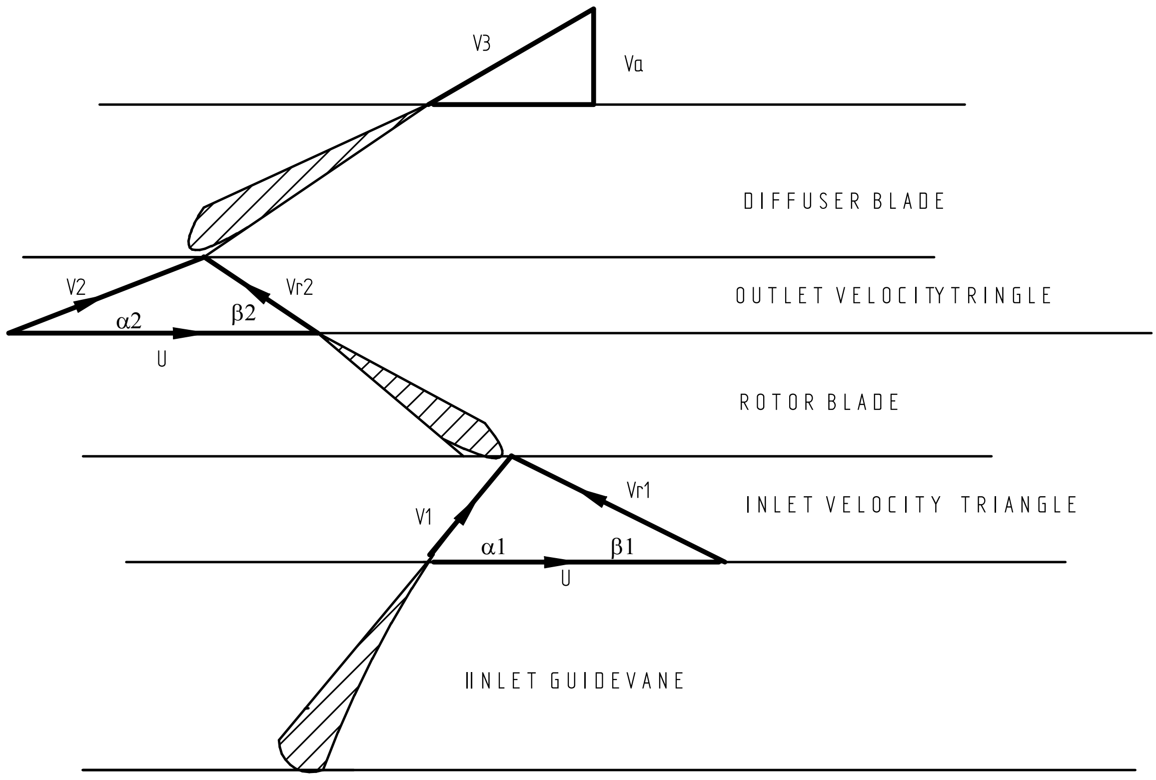


Fig. 4.1 The pressure and velocity variations through an axial flow compressor stage

The general velocity triangles for an axial flow compressor are shown in the figure For axial flow compressors the mean tangential rotor velocity remains constant ($U_1=U_2=U$). If the flow is repeated in another stage of axial flow compressor, then $V_1=V_3$ and $\alpha_1=\alpha_3$.



Velocity triangles for an axial flow compressor

Work Input and Efficiencies in Compressor

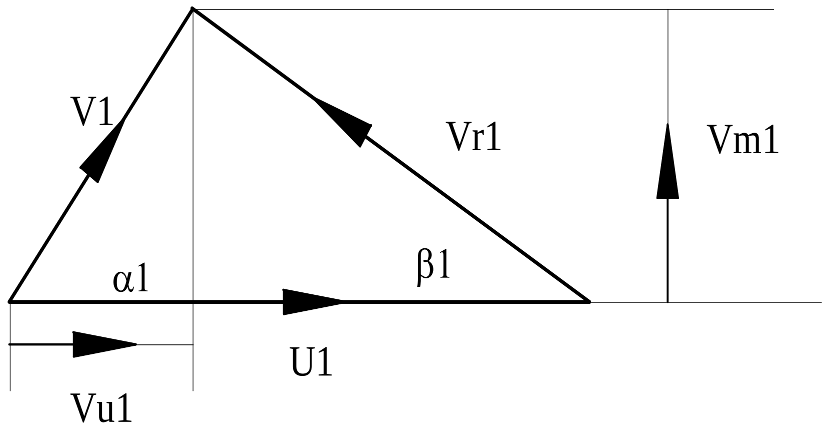
All are centrifugal turbo machines are radial flow power absorbing turbo machines

In axial flow power absorbing turbo machine $U_1 = U_2 = U$

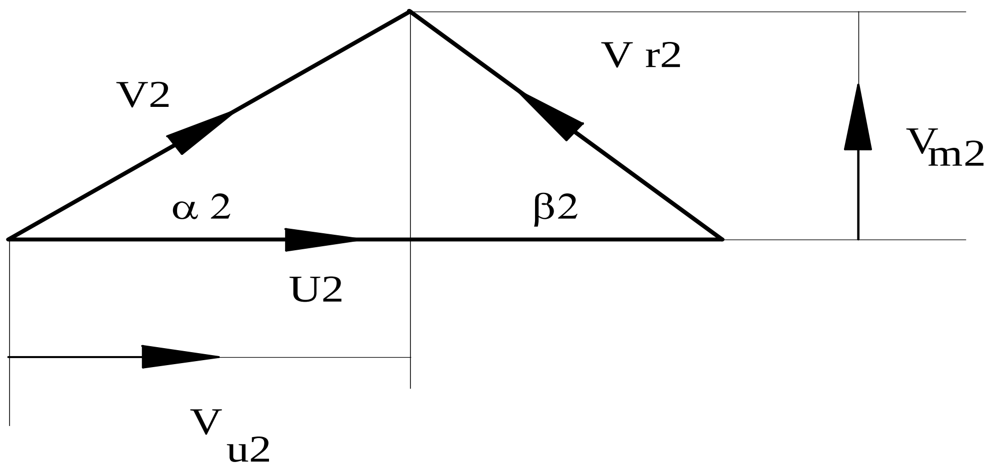
In radial flow power absorbing turbo machine $U_1 \neq U_2$

In power absorbing turbo machine
$\frac{E}{\dot{m}} = (\overrightarrow{V_{u1}} U_1 - \overrightarrow{V_{u2}} U_2)$ is negative ie $\overrightarrow{V_{u2}} U_2 > \overrightarrow{V_{u1}} U_1$
Turning angle of fluid from inlet to outlet is small

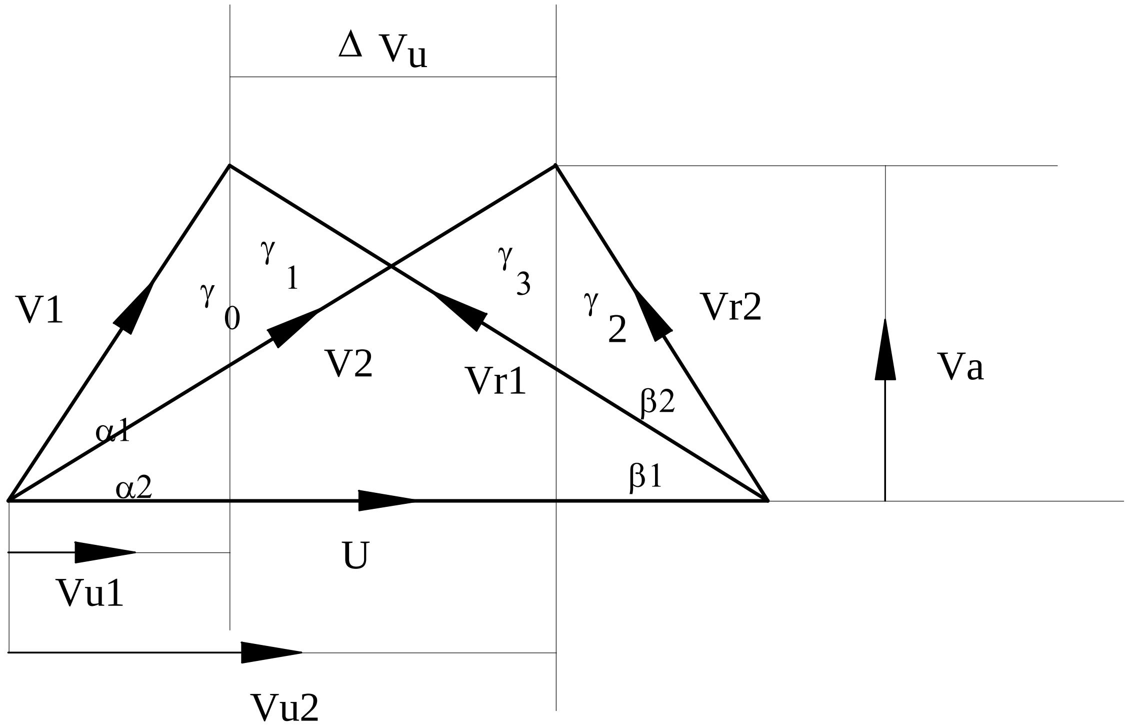
Axial flow Compressor
General Inlet velocity triangle



General outlet velocity triangle



In axial flow power absorbing turbomachine, since $U_1 = U_2$ outlet and inlet velocity triangles can be drawn with common base



γ are called air angles

γ_1 is called air angle at inlet, γ_2 is called as air angle at outlet

output per unit mass $\frac{E}{\dot{m}} = \frac{1}{g_c}(\overrightarrow{V_{u1}}U_1 - \overrightarrow{V_{u2}}U_2)$

output per unit mass $\frac{E}{\dot{m}} = \frac{1}{g_c}\left(\overrightarrow{V_{u1}} - \overrightarrow{V_{u2}}\right)U$ since $U_1 = U_2 = U$

This expression will have negative value in power absorbing machine since $\overrightarrow{V_{u2}} > \overrightarrow{V_{u1}}$

therefore, generally In power absorbing we express Input/per unit mass

Input/per unit mass ie $-\frac{E}{\dot{m}} = \frac{1}{g_c}\left(\overrightarrow{V_{u2}} - \overrightarrow{V_{u1}}\right)U$

Ie negative of output =Input

<u>When the kinetic energy is neglected</u>

The actual work Input , $W_a = h_2 - h_1$; $W_a = C_P(T_2 - T_1)$

The Ideal work, $W_s = h_{2s} - h_1$ $W_s = C_P(T_{2s} - T_1)$

$\eta_{s-s} = \frac{W_s}{W_a}$; $\eta_{s-s} = \frac{C_P(T_{2s}-T_1)}{C_P(T_2-T_1)}$; $\eta_{s-s} = \frac{(T_{2s}-T_1)}{(T_2-T_1)}$

$$\eta_{s-s} = \frac{T_1\left(\frac{T_{2s}}{T_1}-1\right)}{T_2-T_1}$$

But $\frac{T_{2s}}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$; $\frac{T_{2s}}{T_1} = (p_r)^{\frac{\gamma-1}{\gamma}}$ where p_r is the static pressure ratio

$$\eta_{stage\ s-s} = \frac{T_1\left(p_r^{\gamma-1}-1\right)}{T_2-T_1};$$

$\eta_s = \frac{W_S}{W_a}$; $W_a = \frac{C_P(T_{2s}-T_1)}{\eta_{s-s}}$; $W_a = \frac{C_P\ T_1\left(\frac{T_{2s}}{T_1}-1\right)}{\eta_{s-s}}$

$W_a = \frac{C_P\ T_1\left(\frac{T_{2s}}{T_1}-1\right)}{\eta_{s-s}}$; $W_a = \frac{C_P\ T_1\left(\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}-1\right)}{\eta_{s-s}}$; $W_a = \frac{C_P\ T_1\left(p_r^{\frac{\gamma-1}{\gamma}}-1\right)}{\eta_s}$ where p_r is the static pressure ratio

For more general case, when the kinetic energy is significant then, the actual work required is

$\eta_s = \frac{W_S}{W_a}$; $\eta_{stage\ t-t} = \frac{W_{S\ t-t}}{W_{a\ t-t}}$; $W_{a\ stage\ t-t} = \frac{W_{S\ t-t}}{\eta_{t-t}}$

$W_{astage\ t-t} = \frac{C_P(T_{02s}-T_{01})}{\eta_{s-s}}$; $W_{astage\ t-t} = \frac{C_P\ T_{01}\left(\frac{T_{02s}}{T_{01}}-1\right)}{\eta_{t-t}}$; $W_{at-t} = \frac{C_P\ T_{01}\left(\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}}-1\right)}{\eta_{t-t}}$

$W_{astage\ t-t} = \frac{C_P\ T_{01}\left(p_r^{\frac{\gamma-1}{\gamma}}-1\right)}{\eta_{t-t}}$; $p_r^{\frac{\gamma-1}{\gamma}} - 1 = \frac{\eta_{t-t}W_{at-t}}{C_P\ T_{01}}$; $p_{r0} = \left(\frac{\eta_{t-t}W_{at-t}}{C_P\ T_{01}} + 1\right)^{\frac{\gamma}{\gamma-1}}$

$p_{r0} = \left(\frac{\eta_{t-t}C_P(T_{02}-T_{01})}{C_P\ T_{01}} + 1\right)^{\frac{\gamma}{\gamma-1}}$; $p_{r0} = \left(\frac{\eta_{t-t}(T_{02}-T_{01})}{T_{01}} + 1\right)^{\frac{\gamma}{\gamma-1}}$;

η_{t-t} and η_{s-s} are not differ much for compressor and generally both are same. Hence, isentropic efficiency of compressor is generally based on static values

Work Done factor:

Due to the growth of boundary layers on the hub and casing of the axial flow compressor, the axial velocity along the blade height is not uniform. This effect is not so significant in the first stage of a multi-stage machine but is quite significant in the subsequent stages.

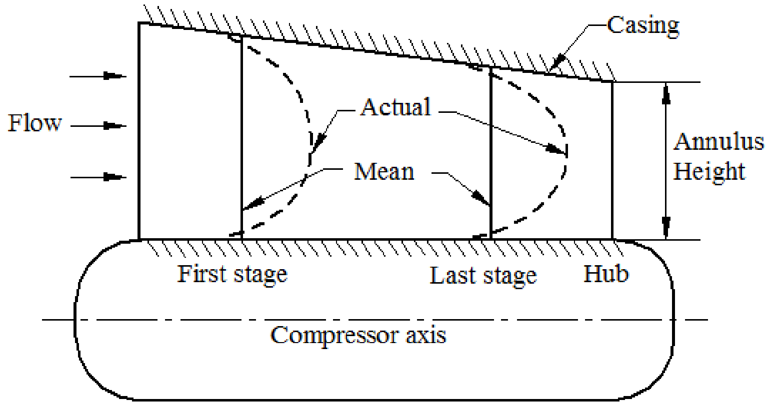


Fig :Axial velocity profile along blade height in an axial flow compressor

Figure shows the axial velocity distribution in the first and last stages of a multi-stage axial flow compressor. The degree of distortion in the axial velocity distribution will depend on the number of the stages. On account of this, axial velocity in the hub and tip regions is much less than the mean value, whereas in the central region its value is higher than the mean.

In cascade design, generally the value of α_1, $\beta_1$and U will be kept constant. It may be the work absorption capacity decreases with an increase in the axial velocity and vice versa. Therefore, the work absorbing capacity of the stage is reduced in the central region of the annulus and increased in the hub and tip region. However, the expected increase in the work at the tip and hub is not obtained in actual practice on account of higher losses. Therefore stage work is less than that given by the Euler's equation based on a constant value of the axial velocity along the blade height. This reduction in the work absorbing capacity of the stage is taken into account by a "work done factor".

The work done factor (Ω) is defined as the ratio of stage work to Euler's work. It can also be defined as the ratio of actual work absorbing capacity to ideal work absorbing capacity

Diffuser

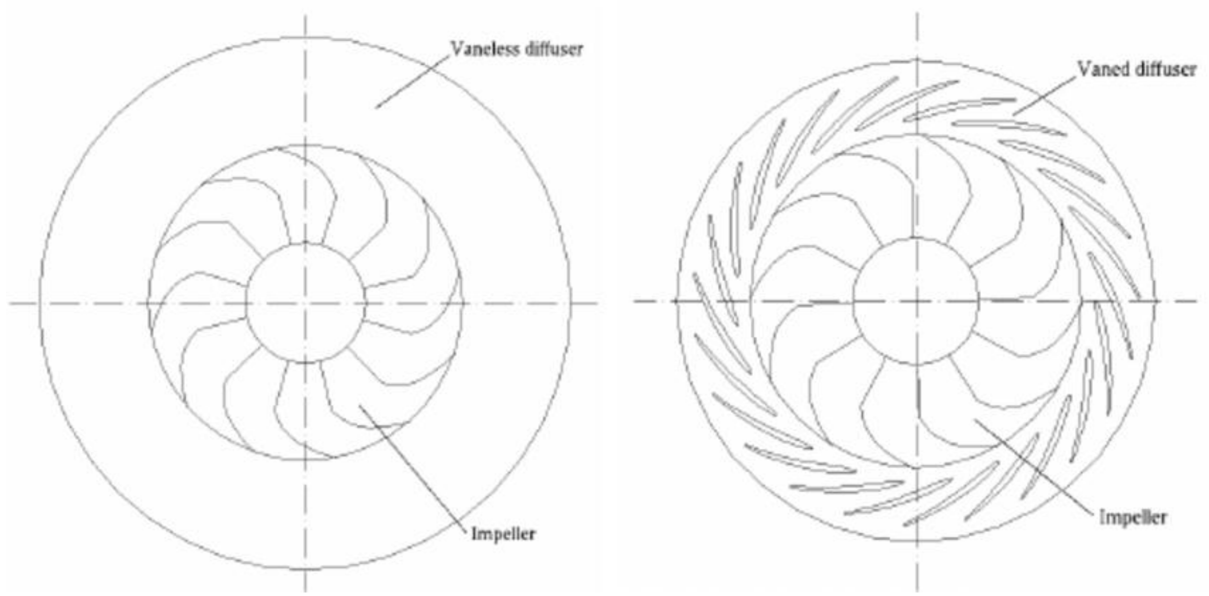
Diffuser is a set of stationary vanes that surround the impeller. The purpose of diffuser is to increase the efficiency by allowing a more gradual expansion and less turbulent area for the liquid **and** convert the velocity energy at the exit of the the rotor to pressure energy,

Diffusers can be vaneless, vaned or an alternating combination. High efficiency vaned diffusers are also designed over a wide range of solidities from less than 1 to over 4. Hybrid versions of vaned diffusers include: wedge, channel, and pipe diffusers. Some turbochargers have no diffuser.

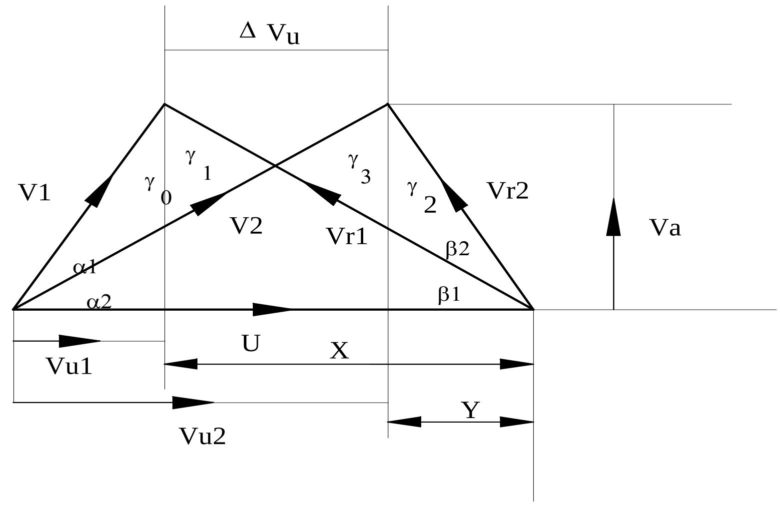
Bernoulli's fluid dynamic principle plays an important role in understanding diffuser perform

Vaneless diffusers have a wider flow range but lower pressure recovery and efficiency, whereas vaned **diffusers** have higher pressure recovery and efficiency, but narrower flow range. ...

The **diffuser** with the constant area **diffuser** has a slightly lower efficiency but the operation range was larger.



1. For axial flow compressor show that $E = V_f U\left(\frac{tan\beta_2 - tan\beta_1}{tan\beta_1 tan\beta_2}\right)$ $(10a, 08\ Dec18/Jan19CBCS)$



$\frac{E}{\dot{m}} = \frac{1}{g_c}\left(\overrightarrow{V_{u1}} - \overrightarrow{V_{u2}}\right)\mathrm{U}$; $\qquad -\frac{E}{\dot{m}} = \frac{1}{g_c}\left(\overrightarrow{V_{u1}} - \overrightarrow{V_{u2}}\right)\mathrm{U}$

$tan\,\gamma_0 = \frac{\overrightarrow{V_{u1}}}{V_a}$; $\quad \overrightarrow{V_{u1}} = V_a\,tan\,\gamma_o$; $\quad tan\,\gamma_3 = \frac{\overrightarrow{V_{u2}}}{V_a}$; $\quad \overrightarrow{V_{u2}} = V_a\,tan\,\gamma_3$

$$-\frac{E}{\dot{m}} = \frac{1}{g_c}\left(\overrightarrow{V_{u2}} - \overrightarrow{V_{u1}}\right)\mathrm{U}$$

$-\frac{E}{\dot{m}} = \frac{1}{g_c}(V_a\,tan\,\gamma_3 - V_a\,tan\,\gamma_o)\mathrm{U}$; $\qquad -\frac{E}{\dot{m}} = \frac{V_a}{g_c}(tan\,\gamma_3 - tan\,\gamma_o)\mathrm{U}$

$U = V_a(tan\,\gamma_0 + tan\,\gamma_1)$; $\qquad$ also $U = V_a(tan\,\gamma_2 + tan\,\gamma_3)$

$V_a(tan\,\gamma_0 + tan\,\gamma_1) = V_a(tan\,\gamma_2 + tan\,\gamma_3)$; $\qquad tan\,\gamma_0 + tan\,\gamma_1 = tan\,\gamma_2 + tan\,\gamma_3$

$$\tan\gamma_3 - \tan\gamma_o = \tan\gamma_1 - \tan\gamma_2$$

Hence, $-\frac{E}{\dot{m}} = \frac{V_a}{g_c}(\tan\gamma_1 - \tan\gamma_2)U$

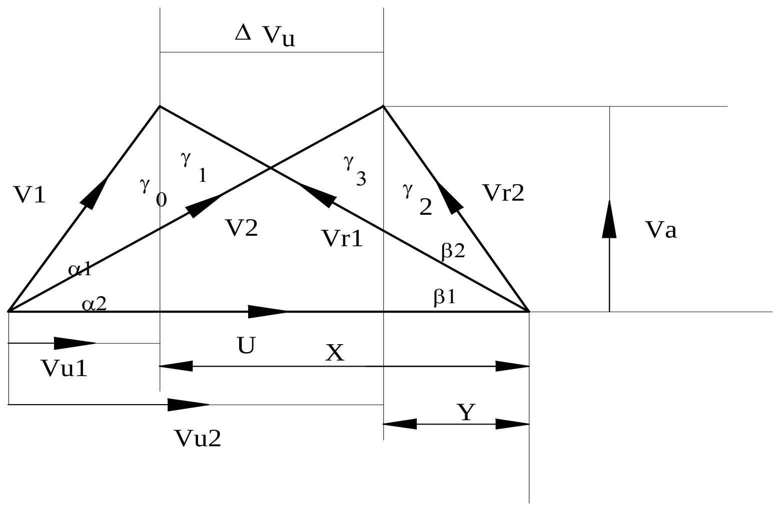
$$\gamma_1 = 180 - \beta_1;\ tan\gamma_1 = tan(180 - \beta_1);\ tan\gamma_1 = cot\beta_1$$

$$\gamma_2 = 180 - \beta_1;\ tan\gamma_2 = tan(180 - \beta_2);\ tan\gamma_2 = cot\beta_2$$

$$-\frac{E}{\dot{m}} = \frac{V_a}{g_c}(cot\beta_1 - cot\beta_2)U; \quad -\frac{E}{\dot{m}} = \frac{V_a}{g_c}\left(\frac{1}{tan\beta_1} - \frac{1}{tan\beta_2}\right)U; \quad -\frac{E}{\dot{m}} = \frac{V_a}{g_c}U\left(\frac{tan\beta_2 - tan\beta_1}{tan\beta_1 tan\beta_2}\right)$$

Power Input in axial flow compressor is $\frac{V_a}{g_c}\left(\frac{tan\beta_2 - tan\beta_1}{tan\beta_1 tan\beta_2}\right)U$

2. Define degree of reaction for an axial flow machine. Prove that degree of reaction for an axial flow device (assuming constant velocity of flow) is given by $R = \frac{V_f}{2U}\left(\frac{tan\beta_1 + tan\beta_2}{tan\beta_1 * tan\beta_2}\right)$ where β_1 *and* β_2 are the angles made with tangent to the blades (4a. 10, Dec13/Jan 14)(4a. 10, Dec18/Jan 19) (4a. 10 Dec17/Jan 2018)



$$R = \frac{\frac{U_1^2 - U_2^2}{2} - \frac{V_{r1}^2 - V_{r2}^2}{2}}{\frac{V_1^2 - V_2^2}{2} + \frac{U_1^2 - U_2^2}{2} - \frac{V_{r1}^2 - V_{r2}^2}{2}} = \frac{\frac{U_1^2 - U_2^2}{2} - \frac{V_{r1}^2 - V_{r2}^2}{2}}{\frac{E}{\dot{m}}}$$

$$U_1 = U_1 = U; \qquad \frac{E}{\dot{m}} = \frac{1}{g_c}(\overrightarrow{V_{u1}} - \overrightarrow{V_{u2}})U$$

Hence, $R = \frac{-\left(\frac{V_{r1}^2 - V_{r2}^2}{2g_c}\right)}{\frac{1}{g_c}(\overrightarrow{V_{u1}} - \overrightarrow{V_{u2}})U};$ $\qquad R = \frac{-(V_{r1}^2 - V_{r2}^2)}{2(\overrightarrow{V_{u1}} - \overrightarrow{V_{u2}})U}$

$$V_{r1}^2 = V_a^2 + X^2; \quad V_{r1}^2 = V_a^2 + (V_a \tan\gamma_1)^2$$

$V_{r2}^2 = V_a^2 + Y^2; \quad V_{r2}^2 = V_a^2 + (V_a \tan\gamma_2)^2$

$tan\,\gamma_0 = \frac{\overrightarrow{V_{u1}}}{V_a}; \qquad \overrightarrow{V_{u1}} = V_{a1}\,tan\,\gamma_o; \qquad tan\,\gamma_3 = \frac{\overrightarrow{V_{u2}}}{V_a}; \qquad \overrightarrow{V_{u2}} = V_a\,tan\,\gamma_3$

$R = \frac{-\left(\left(\ V_a^2+(V_a\tan\gamma_1)^2\right)-\left(V_a^2+(V_a\tan\gamma_2)^2\right)\right)}{2(V_a\tan\gamma_0 - V_a\tan\gamma_3)\ \mathrm{U}}; \qquad R = \frac{V_a^2\left(tan^2\gamma_2 - tan^2\gamma_1\right)}{2V_a(tan\,\gamma_0 - tan\,\gamma_3)\ U}$

$R = \frac{V_a\left(tan^2\gamma_2 - tan^2\gamma_1\right)}{2(tan\,\gamma_2 - tan\,\gamma_1)\ U}$ since, $\tan\gamma_1 - \tan\gamma_2 = \tan\gamma_3 - \tan\gamma_0$

$R = \frac{V_a(\tan\gamma_1 + \tan\gamma_2)}{2\ \mathrm{U}}$

$\gamma_1 = 180 - \beta_1;\ tan\gamma_1 = tan(180 - \beta_1); tan\gamma_1 = cot\beta_1$

$\gamma_2 = 180 - \beta_1;\ tan\gamma_2 = tan(180 - \beta_2); tan\gamma_2 = cot\beta_2$

Hence , $R = \frac{V_a(cot\beta_1 + cot\beta_2)}{2\ \mathrm{U}}$;

$R = \frac{V_a\left(\frac{1}{tan\beta_1}+\frac{1}{tan\beta_2}\right)}{2\ \mathrm{U}}; \quad ; \ R = \frac{V_a(tan\beta_2 + tan\beta_1)}{2\ \mathrm{U}(tan\beta_1 * tan\beta_2)}$

The effect of axial velocity:

$$\frac{U}{g_c}(V_{u2} - V_{u1})$$

From Inlet and outlet velocity triangle

$V_{u2} = U - V_a cot\beta_2$ and $V_{u1} = U - V_a cot\beta_1$

Hence, W= $U((U - V_a cot\beta_2) - (U - V_a cot\beta_1))$

W= $U(V_a cot\beta_1 - V_a cot\beta_2)$

Adding and subtracting $cot\alpha_1$ to the above equation

W= $U\left(V_a(cot\beta_1 + cot\alpha_1) - V_a(cot\beta_2 + cot\alpha_1)\right)$

But U= $cot\beta_1 + cot\alpha_1$

Hence, $W = U\left(U - V_a(cot\beta_2 + cot\alpha_1)\right)$

From above equation fit is seen that if U, $\beta_2$and $\alpha_1$are kept constant in axial flow compressor Work absorbed by the compressor to raise the pressure depends only axial velocity

Variation of V_a alters U

As V_a increases U decreases Hence W decreases as U is +ve and V_a have –ve sign in Work absorbing equation Hence Work absorption capacity decreases. Therefore, the work absorbing capacity of the stage is reduced in the central region of the annulus and increased in the work at the hub and tip region

Therefore stage work is less than that given by the Eulers equation based on a constant value of the axial velocity along the blade height. This reduction in the work absorbing capacity of the stage is taken into account by **Work Done Factor**

Work done factor is defined as the ratio of stage work input to Eulers work Input

Also degined as the ratio of actual absorbing capacity to Ideal work absorbing capacity

Radial Equilibrium Condition:

Assumptions:

1) The rate of radial pressure gradient $\frac{dp}{dr}$ is assumed
2) There is no flow in radial direction ie flow in axial direction only
3) Stream lines do not experience any shift in radial direction and therefore lie on cylindrical surface coaxially

In axial flow machines the stream lines in a flow do not experience any radial shift and therefore flow is assumed along coaxial cylinders. Such a flow in an annulus is known as radial equilibrium.

Equation of motion for three dimensional flow in cylindrical co-ordinate is therefore given as

$$V_{rd}\frac{dV_{rd}}{dr} + V_a\frac{dV_a}{dr} - \frac{V_u^2}{r} = -\frac{1}{\rho}\frac{dp}{dr}$$

There is no flow in radial direction $V_{rd} = 0$

Flow velocity is constant $V_a = constant$ ie $\frac{dV_a}{dr} = 0$

Hence above equation becomes

$0+0-\frac{V_u^2}{dr} = -\frac{1}{\rho}\frac{dp}{dr}$

$\frac{V_u^2}{r} = \frac{1}{\rho}\frac{dp}{dr}$ -----------------1

Stagnation pressure $p_0 = p + \rho\frac{1}{2}V^2$

But $V^2 = V_{rd}^2 + V_a^2 + V_u^2$; $V^2 = V_a^2 + V_u^2$ as $V_{rd} = 0$

Hence, $p_0 = p + \rho\frac{1}{2}(V_a^2 + V_u^2)$

$$\frac{dp_0}{dr} = \frac{dp}{dr} + \rho \frac{1}{2} \frac{d(V_a^2 + V_u^2)}{dr}$$

$$\frac{1}{\rho} \frac{dp_0}{dr} = \frac{1}{\rho} \frac{dp}{dr} + \frac{1}{2} 2V_u \frac{dV_u}{dr} + \frac{1}{2} 2V_a \frac{dV_a}{dr}$$

$$\frac{1}{\rho} \frac{dp}{dr} = \frac{1}{\rho} \frac{dp_0}{dr} - V_u \frac{dV_u}{dr} - V_a \frac{dV_a}{dr} \qquad \text{------------2}$$

From 1 $\frac{V_u^2}{r} = \frac{1}{\rho} \frac{dp}{dr}$

$$\frac{dp}{dr} = \rho \frac{V_u^2}{r} \qquad \text{------------3}$$

Substituting 3 in 2

$$\frac{1}{\rho} \rho \frac{V_u^2}{r} = \frac{1}{\rho} \frac{dp_0}{dr} - V_u \frac{dV_u}{dr} - V_a \frac{dV_a}{dr}$$

$$\frac{V_u^2}{r} = \frac{1}{\rho} \frac{dp_0}{dr} - V_u \frac{dV_u}{dr} - V_a \frac{dV_a}{dr}$$

This is known as radial equilibrium equation for axisymmetric unsteady flow in turbomachine

The pressure coefficient φ_p is defined is the ratio of the actual stagnation enthalpy change to kinetic energy of a fluid which has the same speed as the blades.

$$\varphi_p = \frac{\Delta h_o}{\frac{U^2}{2}}; \qquad \varphi_p = \frac{\Delta V_u U}{\frac{U^2}{2}}; \qquad \varphi_p = \frac{2\Delta V_u U}{U^2}$$

If Fluid is incompressible $\Delta h_o = \frac{\Delta p_o}{\rho}$

Then for Incompressible fluid , $\varphi_p = \frac{2\Delta p_o}{\rho U^2}$

Pressure coefficient usually ranges between 0.4 and 0.7

The flow coefficient ψ is defined as is the ratio of flow velocity to the blade speed

$$\psi = \frac{V_{ax}}{U}$$

Numericals

1. An axial flow compressor of 50% reaction design has blades with inlet and outlet angles with respect of axial directions of 45^o and 10^orepectively . The compressor is to produce a pressure ratio of 6:1 with a overall isentropic efficiency of 0.85. When the inlet static temperature of 37^oC. The blade speed and axial velocity are constant throughout the compressor. Assuming a value of 200m/s for blade speed. Find the number of stages required if the work done factor is i) unity and ii) 0.87 for all stages (8c,10, June/July17)(8b,08,Dec13/Jan14)

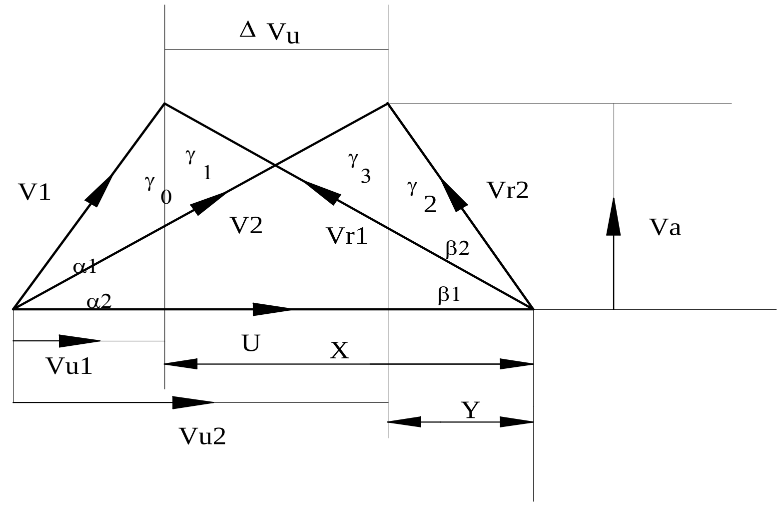
$R = 50\%; \gamma_2 = 45^o; \gamma_3 = 10^o$;

The compressor is to produce a pressure ratio of 6:1 ie $\frac{p_{k+1}}{p_1} = 6;\ \ \eta_0 = 0.85; T_1 = 37^o$C

The blade speed and axial velocity are constant throughout the compressor.

ie $U_1 = U_2 = U$ and $V_{a1} = V_{a2} = V_{a2}$

$U = 200m/s;\ k = ?$ when i) $\Omega = 1$ ii) $\Omega = 0.87$



the number of stages for work done factor is unity

$R = \frac{V_a(tan\gamma_1 + tan\gamma_2)}{2U}$; $\quad 0.5 = \frac{V_a(tan45 + tan10)}{2*200}$; $\quad V_a = 170m/s$;

$tan\gamma_2 = \frac{Y}{V_a}$; $\quad Y = \overrightarrow{V_{u1}}$; $\quad tan\gamma_2 = \frac{V_{u1}}{V_a}$; $\quad tan10 = \frac{\overrightarrow{V_{u1}}}{170}$, $\quad \overrightarrow{V_{u1}} = 29.97m/s$

$\overrightarrow{V_{u2}} = U - Y$; $\quad \overrightarrow{V_{u2}} = 200 - 29.97$ $\quad \overrightarrow{V_{u2}} = 170.03m/s$

$-\frac{E}{\dot{m}} = \frac{\Omega(\overrightarrow{V_{u2}} - \overrightarrow{V_{u1}})U}{g_c}$; Increase in entalphy $\Delta h_o/stage = \frac{1(170.03-29.97)200}{1}$,

$\Delta h_o/stage = 28012$J/kg

$\eta_0 = \frac{\Delta h_{0s}}{\Delta h_0}$; $\quad \eta_0 = \frac{C_p(T_{osk+1} - T_{01})}{\Delta h_0}$; $\quad \eta_0 = \frac{C_pT_{01}\left(\frac{T_{osk+1}}{T_{01}} - 1\right)}{\Delta h_0}$

$\eta_0 = \frac{C_pT_{01}\left(\left(\frac{p_{k+1}}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right)}{\Delta h_0}$; $\quad \Delta h_0 = \frac{C_pT_{01}\left(\left(\frac{p_{k+1}}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right)}{\eta_0}$;

$V_1^2 = V_{u1}^2 + V_a^2$; $\quad V_1^2 = 29.97^2 + 170^2$; $\quad V_1^2 = 29798.2$

$T_{01} = T_1 + \frac{V_1^2}{2*C_p}$; $\quad T_{01} = 310 + \frac{29798.2}{2*1005}$; $\quad T_{01} = 324.82K$

$\Delta h_0 = \frac{C_pT_{01}\left(\left(\frac{p_{k+1}}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right)}{\eta_0}$; $\quad (\Delta h_0)_{\text{total}} = \frac{1005*324.82[(6)^{0.286} - 1]}{0.85}$; $\quad (\Delta h_0)_{\text{total}} = 257070.82J/kg$

Number of stages, $k, = \frac{(\Delta h_0)_{\text{total}}}{\Delta h_o/stage}$; $\quad k = \frac{257070.82}{28012}$; $\quad k = 9.18$ say 10 stages

the number of stages for work done factor is 0.87

$-\frac{E}{\dot{m}} = \frac{\Omega(\overrightarrow{V_{u2}} - \overrightarrow{V_{u1}})U}{g_c}$; Increase in entalphy $\Delta h_o/stage = \frac{0.87(170.03-29.97)200}{1}$,

$\Delta h_o/stage = 23810.2J/kg$;

$$\eta_0 = \frac{C_pT_{01}\left(\left(\frac{p_{k+1}}{p_1}\right)^{\frac{\gamma-1}{\gamma}}-1\right)}{\Delta h_0}; \qquad \Delta h_0 = \frac{C_pT_{01}\left(\left(\frac{p_{k+1}}{p_1}\right)^{\frac{\gamma-1}{\gamma}}-1\right)}{\eta_0}; \qquad (\Delta h_0)_{\text{total}} = \frac{1005*324.82[(6)^{0.286}-1]}{0.85}$$

$(\Delta h_0)_{\text{total}} = 257070.82 J/kg$

Number of stages, $k, = \frac{(\Delta h_0)_{\text{total}}}{\Delta h_o/stage}$; $\quad k = \frac{257070.82}{23810.2}$; $\quad k = 10.79$ $\quad$ say 11 stages

2. The axial flow compressor with 50% reaction is having a flow coefficient of 0.54. Air enters the compressor at stagnation condition of 1 bar and 30^oC. The total to total efficiency across the rotor is 0.88. The total to pressure ratio across the rotor is 1.26. The pressure coefficient is 0.45 and workdone factor is 0.88. The mass flow rate is 15kgs. Calculate i) The mean rotor blade speed ii) Rotor blade angles at inlet and exit iii) Power input to the system (8b,10,Dec 12)

$R = 0.5$; flow coefficient $\psi = 0.54$; $p_{01} = 1\ bar$; $T_{01} = 30^oC$; $\eta_{tt} = 0.88$; $p_{ro} = 1.26$;
$Pressure\ coefficient\ \varphi_p = 0.45$; work done factor $\Omega = 0.88$; $\dot{m} = 15kg/s$

i) $U = ?$ ii) $\beta_1 = ?$, $\beta_2 = ?$; iii) Power input $E = ?$

$$\psi = \frac{V_a}{U} = 0.54; \ \varphi_p = \frac{\Delta h_0}{\frac{u^2}{2}} = 0.45$$

<u>i) The mean rotor blade speed</u>

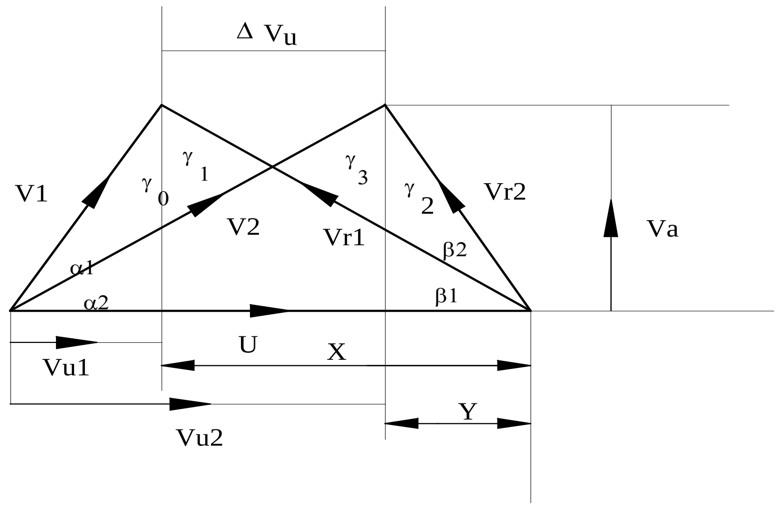
$$\eta_{tt} = \frac{\Delta h_{0s}}{\Delta h_0}; \qquad \eta_0 = \frac{C_p(T_{osk+1}-T_{01})}{\Delta h_0}; \qquad \eta_0 = \frac{C_pT_{01}\left(\frac{T_{osk+1}}{T_{01}}-1\right)}{\Delta h_0}$$

$$\eta_0 = \frac{C_pT_{01}\left(\left(\frac{p_{k+1}}{p_1}\right)^{\frac{\gamma-1}{\gamma}}-1\right)}{\Delta h_0}; \qquad \Delta h_0 = \frac{1005*303((1.26)^{0.286}-1)}{0.88}; \qquad \Delta h_0 = 23645.37\ J/kg$$

$$\varphi_p = \frac{2\Delta h_0}{u^2} = 0.45; \qquad 0.45 = \frac{2*23645.37}{u^2}; \qquad u^2 = 105.090.52 \qquad U = 324.17m/s$$

<u>ii) Rotor blade angles at inlet and exit</u>

$$\psi = \frac{V_a}{U} = 0.54; \qquad \frac{V_a}{324.17} = 0.54; \qquad V_a = 175.05m/s$$



$-\frac{E}{\dot{m}} = \frac{\Omega(\overrightarrow{V_{u2}} - \overrightarrow{V_{u1}})U}{g_c}$; $\Delta h_0 = \frac{\Omega \Delta \mathrm{V_u}\ U}{g_c}$; $23645.37 = \frac{0.88 * \Delta \mathrm{V_u} * 324.17}{1}$ $\Delta \mathrm{V_u} = 82.89\mathrm{m/s}$

$Y = \overrightarrow{V_{u1}}$ since $R = 0.5$ ie triangles are symmetrical

$U = \Delta \mathrm{V_u} + \overrightarrow{V_{u1}} + Y$; $U = \Delta \mathrm{V_u} + 2\overrightarrow{V_{u1}}$; $324.17 = 82.89 + 2\overrightarrow{V_{u1}}$; $\overrightarrow{V_{u1}} = 120.64$

$tan\alpha_1 = \frac{V_a}{\overrightarrow{V_{u1}}}$; $tan\alpha_1 = \frac{175.05}{120.64}$; $\alpha_1 = 55.42^0$;

For $R = 0.5$ $\beta_2 = \alpha_1$; $\beta_2 = 55.42^0$

$\Delta \mathrm{V_u} = \overrightarrow{V_{u2}} - \overrightarrow{V_{u1}}$; $82,89 = \overrightarrow{V_{u2}} - 120.64$ $\overrightarrow{V_{u2}} = 203.53m/s$

$tan\alpha_2 = \frac{V_a}{\overrightarrow{V_{u2}}}$; $tan\alpha_2 = \frac{175.05}{203.53}$; $\alpha_2 = 40.69^0$

For $R = 0.5$ $\beta_1 = \alpha_2$; $\beta_1 = 40.69^0$

iii) Power input to the system

$-\frac{E}{\dot{m}} = \frac{\Omega(\overrightarrow{V_{u2}} - \overrightarrow{V_{u1}})U}{g_c}$; $-\frac{E}{\dot{m}} = \Delta h_0$; $\frac{-E}{15} = 23645.37$ $-E = 354680.55\ W$

3. An axial flow compressor has the following data entry condition 1 bar, 20^oC,degree of reaction 50% mean blade ring diameter 36cm. Rotational speed 18000rpm bade height at entry 6cm. Blade angle at rotor and stator exit 65^o axial velocity 180m/s mechanical efficiency 0.967. Find i) Guide blade angle at outlet ii) Power required to drive the compressor (8c, 8, Dec16/Jan17)

$p_{01} = 1\ bar$; $T_{01} = 20^oC = 293K$; $R = 0.5$; $D = 36cm = 0.36m$; $N = 18000\mathrm{rpm}$;

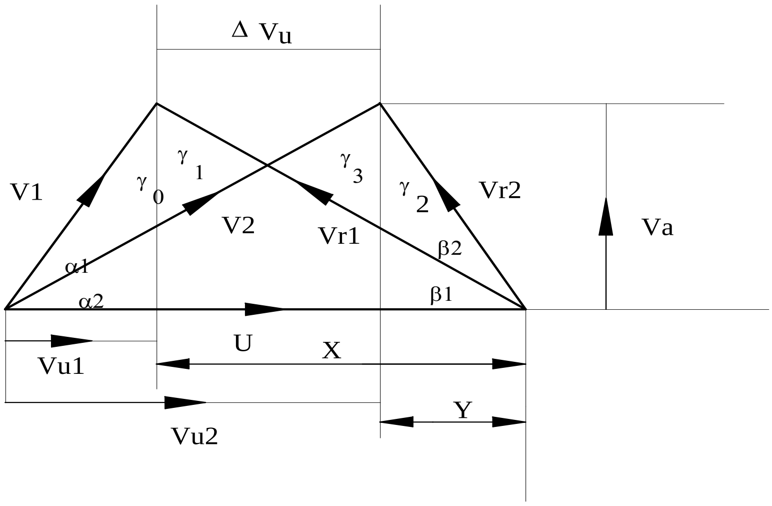
bade height at entry 6cm $h = 6cm = 0.06m$;

Blade angle at rotor and stator exit 65^o axial velocity 180m/s ; $\alpha_1 = \beta_2 = 65^o$

$V_{a1} = V_{a2} = V_a = 180m/s$; $\eta_{mech} = 0.967$

i) $\alpha_2 =?$; ii) $E =?$

$U = \frac{\pi DN}{60}$; $U = \frac{\pi * 0.36 * 18000}{60}$ $U = 339.3m/s$



$tan\alpha_1 = \frac{V_a}{\overrightarrow{V_{u1}}};$ $tan65 = \frac{180}{\overrightarrow{V_{u1}}}$ $\overrightarrow{V_{u1}} = 83.93m/s$

Since, $R = 0.5,$ $Y = \overrightarrow{V_{u1}}$ $Y = 83.93m/s$

$\overrightarrow{V_{u2}} = U - Y;$ $\overrightarrow{V_{u2}} = 339.3 - 83.94$ $\overrightarrow{V_{u2}} = 255.36m/s$

$tan\alpha_2 = \frac{V_a}{\overrightarrow{V_{u2}}};$ $tan\alpha_2 = \frac{180}{255.36};$ $\alpha_2 = 35.17^0$

For $R = 0.5$ $\beta_1 = \alpha_2;$ $\beta_1 = 35.17^0$

ii) Power required to drive the compressor

$-\frac{E}{\dot{m}} = \frac{\Omega(\overrightarrow{V_{u2}} - \overrightarrow{V_{u1}})U}{g_c};$

$\dot{m} = \rho A_f V_a;$ $\rho = \frac{p_1}{RT}$ $\rho = \frac{1*10^5}{287*293};$ $\rho = 1.189kg/m^3$

$A_f = \pi Dh;$ $A_f = \pi * 0.36 * 0.06$ $A_f = 0.068m^2$

$\dot{m} = \rho A_f V_a;$ $\dot{m} = 1.189 * 0.067 * 180$ $\dot{m} = 14.34kg/s$

$-\frac{E}{\dot{m}} = \frac{\Omega(\overrightarrow{V_{u2}} - \overrightarrow{V_{u1}})U}{g_c};$ $\frac{-E}{14.34} = \frac{1(255.36-83.93)339.3}{1};$ $-E = 834103.3W$

Power input to the impeller $= 834103.3W$

Power Required to drive the shaft of Impeller $= \frac{Power\ Input\ to\ blades}{\eta_{mech}}$

Power Required to drive the shaft of Impeller $= \frac{834103.3}{0.967}$ ie $862568W$ $P = 862568W$

4. The speed of an axial flow compressor is $15000rpm$. The mean diameter is $0.6m$. The axial velocity is constant and is $225m/s$. The velocity of whirl at inlet is 85m/s. The work done is $45\ kJ/kg\ of\ air$. The inlet conditions are 1 bar and $300K$. Assume a stage efficiency of

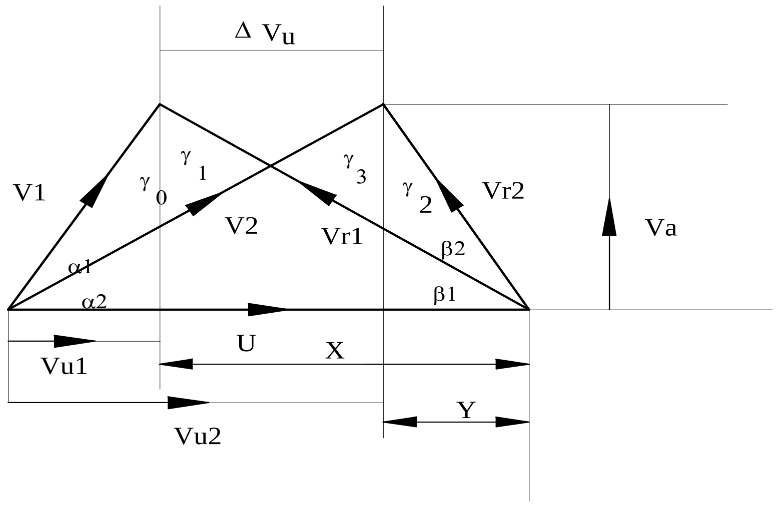
0.89. Calculate i) Fluid deflection angle ii) Pressure ratio iii) Degree of reaction iv) Mass flow rate of air . Power developed is $425kW$ (8c,10,June/July 16)

$N = 15000rpm; D = 0.6m; V_a = 225m/s; \overrightarrow{V_{u1}} = 85m/s\ ; \ -\frac{E}{\dot{m}} = 45kJ/kg\ ;$

$p_1 = 1\ bar;\ T_1 = 300K;\ \eta_s = 0.89$

i) Fluid deflection angle ii) Pressure ratio $\frac{p_2}{p_1} =?;$ iii) $R =?$ iv) $\dot{m} =?$ $E = 425kW$

$U = \frac{\pi DN}{60};$ $U = \frac{\pi*0.6*15000}{60}$ $U = 471.24m/s$



$-\frac{E}{\dot{m}} = \frac{\Omega(\overrightarrow{V_{u2}}-\overrightarrow{V_{u1}})U}{g_c};$ Assume $\Omega = 1$ $45000 = \frac{1(\overrightarrow{V_{u2}}-85)471.24}{1};$

$\overrightarrow{V_{u2}} - 85 = 95.49;$ $\overrightarrow{V_{u2}} = 180.49m/s;$

$tan\alpha_1 = \frac{V_a}{\overrightarrow{V_{u1}}};$ $tan\alpha_1 = \frac{225}{85};$ $\alpha_1 = 69.3^0$

$tan\alpha_2 = \frac{V_a}{\overrightarrow{V_{u2}}};$ $tan\alpha_2 = \frac{225}{180.49};$ $\alpha_2 = 51.26^0$

$tan\beta_1 = \frac{V_a}{U-\overrightarrow{V_{u1}}};$ $tan\beta_1 = \frac{225}{471.21-85}$ $\beta_1 = 30.22^o$

$tan\beta_2 = \frac{V_a}{U-\overrightarrow{V_{u2}}};$ $tan\beta_2 = \frac{225}{471.21-180.49}$ $\beta_2 = 37.73^o$

Pressure ratio $\frac{p_2}{p_1}$

$\eta_s = \frac{\Delta h_{0s}}{\Delta h_0};$ $\eta_0 = \frac{C_p(T_{o2s}-T_{01})}{\Delta h_0}$; $\eta_0 = \frac{C_pT_{01}\left(\frac{T_{O2S}}{T_{01}}-1\right)}{\Delta h_0}$

$\eta_s = \frac{C_pT_{01}\left(\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}}-1\right)}{\Delta h_0};$ $\Delta h_0 = -\frac{E}{\dot{m}} = 45kJ/kg;$ $\Delta h_0 = 45000J/kg$

$0.89 = \frac{1005*300\left(\left(\frac{p_{02}}{p_{01}}\right)^{0.286}-1\right)}{45000};$ $0.133 = \left(\frac{p_{02}}{p_{01}}\right)^{0.286} - 1;$ $\frac{p_{02}}{p_{01}} = 1.546$

iii) Degree of reaction

$R = \frac{\frac{E}{\dot{m}} - \left(\frac{V_1^2 - V_2^2}{2g_c}\right)}{\frac{E}{\dot{m}}}$; $R = 1 - \frac{(V_1^2 - V_2^2)}{2g_c\frac{E}{\dot{m}}}$;

$V_1^2 = V_{u1}^2 + V_a^2$; $V_1^2 = 85^2 + 225^2$; $V_1^2 = 57850$

$V_2^2 = V_{u2}^2 + V_a^2$; $V_2^2 = 180.49^2 + 225^2$; $V_2^2 = 83201.64$

$-\frac{E}{\dot{m}} = 45kJ/kg$; $\frac{E}{\dot{m}} = -45000J/kg$

$R = 1 - \frac{(V_1^2 - V_2^2)}{2g_c\frac{E}{\dot{m}}}$; $R = 1 - \frac{(57850 - 83201.64)}{2*1*(-45000)}$ $R = 1 - 0.282$ $R = 0.718$

$R = \frac{V_a}{2U}\left(\frac{tan\beta_1 + tan\beta_2}{tan\beta_1 * tan\beta_2}\right)$; $R = \frac{225}{2*471.24}\left(\frac{tan30.22 + tan37.73}{tan30.22 * tan37.73}\right)$; $R = 0.718$

$R = \frac{V_a}{2U}\left(\frac{tan\beta_1 + tan\beta_2}{tan\beta_1 * tan\beta_2}\right)$ —this formula can be used only when workdone factor is 1

iv) **<u>Mass flow rate of air .</u>**

$-E = 425kW$

$-\frac{E}{\dot{m}} = 45kJ/kg$; $\frac{-E}{\dot{m}} = 45kJ/kg$; $\frac{425kW}{\dot{m}} = 45kJ/kg$

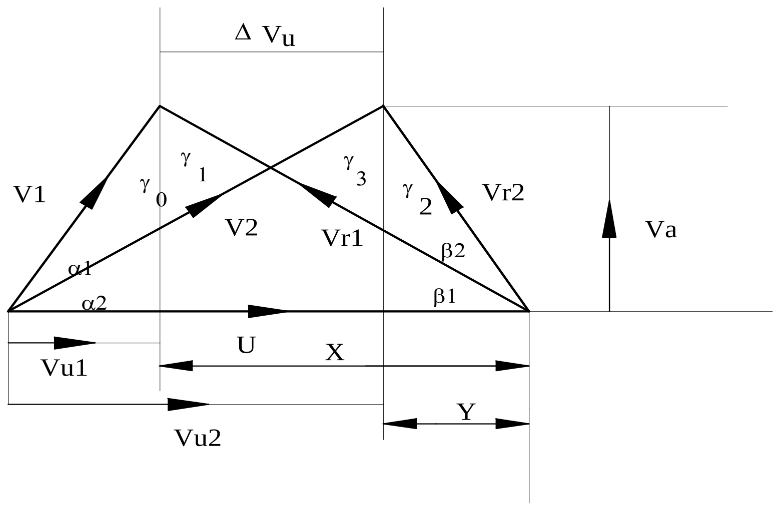
$\dot{m} = 9.44kg/s$

5. The mean diameter of rotor of an axial flow compressor is 0.5m, and it rotates at 15000 rpm. The velocity of flow 220m/s, is constant and the velocity of whirl at the inlet is 80m/s. The inlet pressure and temperature are 1 bar and 300K. The stage efficiency is 0.88. The pressure ratio through the stage is 1.5. Calculate i) Fluid deflection angle ii) The degree of reaction if work done factor is 0.8 (8c, 10,Dec14/Jan15)

$D = 0.5m$; $N = 15000rpm$; $V_a = 220m/s$; $\overrightarrow{V_{u1}} = 80m/s$; $p_1 = 1\ bar$; $T_1 = 300K$

$\eta_s = 0.88$; $\frac{p_2}{p_1} = 1.5$; work done factor is 0.8 $\Omega = 0.8$

$U = \frac{\pi DN}{60}$; $U = \frac{\pi*0.5*15000}{60}$ $U = 392.69m/s$



$\eta_s = \frac{\Delta h_{0s}}{\Delta h_0}$; $\eta_s = \frac{C_p(T_{02s}-T_{01})}{\Delta h_0}$; $\eta_s = \frac{C_pT_{01}\left(\frac{T_{02s}}{T_{01}}-1\right)}{\Delta h_0}$

$\eta_s = \frac{C_pT_{01}\left(\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}}-1\right)}{\Delta h_0}$; $0.88 = \frac{1005*300(1.5^{0.286}-1)}{\Delta h_0}$; $\Delta h_0 = 42125.83 J/kg$

$\frac{-E}{\dot{m}} = \Delta h_0$; $\frac{-E}{\dot{m}} = 42125.83 J/kg$

$-\frac{E}{\dot{m}} = \frac{\Omega(\overrightarrow{V_{u2}}-\overrightarrow{V_{u1}})U}{g_c}$; $42125.83 = \frac{0.8(\overrightarrow{V_{u2}}-80)392.69}{1}$; $\overrightarrow{V_{u2}} = 214.09 m/s$

$tan\alpha_1 = \frac{V_a}{\overrightarrow{V_{u1}}}$; $tan\alpha_1 = \frac{220}{80}$; $\alpha_1 = 70^0$

$tan\alpha_2 = \frac{V_a}{\overrightarrow{V_{u2}}}$; $tan\alpha_2 = \frac{220}{214.09}$; $\alpha_2 = 45.78^0$

$tan\beta_1 = \frac{V_a}{U-\overrightarrow{V_{u1}}}$; $tan\beta_1 = \frac{220}{392.69-80}$ $\beta_1 = 35.12^o$

$tan\beta_2 = \frac{V_a}{U-\overrightarrow{V_{u2}}}$; $tan\beta_2 = \frac{220}{392.69-214.09}$ $\beta_2 = 50.92^o$

iii) Degree of reaction

$R = \frac{\frac{E}{\dot{m}}-\left(\frac{V_1^2-V_2^2}{2g_c}\right)}{\frac{E}{\dot{m}}}$; $R = 1-\frac{(V_1^2-V_2^2)}{2g_c\frac{E}{\dot{m}}}$;

$V_1^2 = V_{u1}^2 + V_a^2$; $V_1^2 = 80^2 + 220^2$; $V_1^2 = 54800$

$V_2^2 = V_{u2}^2 + V_a^2$; $V_2^2 = 214.09^2 + 220^2$; V_2^2 =94234.53

$-\frac{E}{\dot{m}} = 42125.83\ J/kg$; $\frac{E}{\dot{m}} = -42125.83 J/kg$

$R = 1-\frac{(V_1^2-V_2^2)}{2g_c\frac{E}{\dot{m}}}$; $R = 1-\frac{(54800-94234.53)}{2*1*(-42125.83)}$ $R = 1-0.468$ $R = 0.531$

$R = \frac{V_a}{2U}\left(\frac{tan\beta_1 + tan\beta_2}{tan\beta_1 * tan\beta_2}\right)$; $-$ this formula cannot be used since work done factor is not 1

Centrifugal Power absorbing machine

Power absorbing tubomachines are classified into

i) Fans, ii) Blowers and iii) compressors

Difference between fan , blower and compressor

A fan consists of single rotor with or without a stator which causes only small pressure rise as low as few centimeters of water column (70cm of water). In analysis of fan fluid considered as incompressible fluid as the density change is very small due to small pressure rise.

Blowers may consist of one or multistage of compression with rotor mounted on a common shaft. The air is compressed in a series of successive stages and passed through a diffuser located near the exit to recover the pressure energy from the large kinetic energy. The overall pressure rise may be in the range of 1.5 bars to 2. Bars. Blowers are used in ventilation, power station, workshops etc

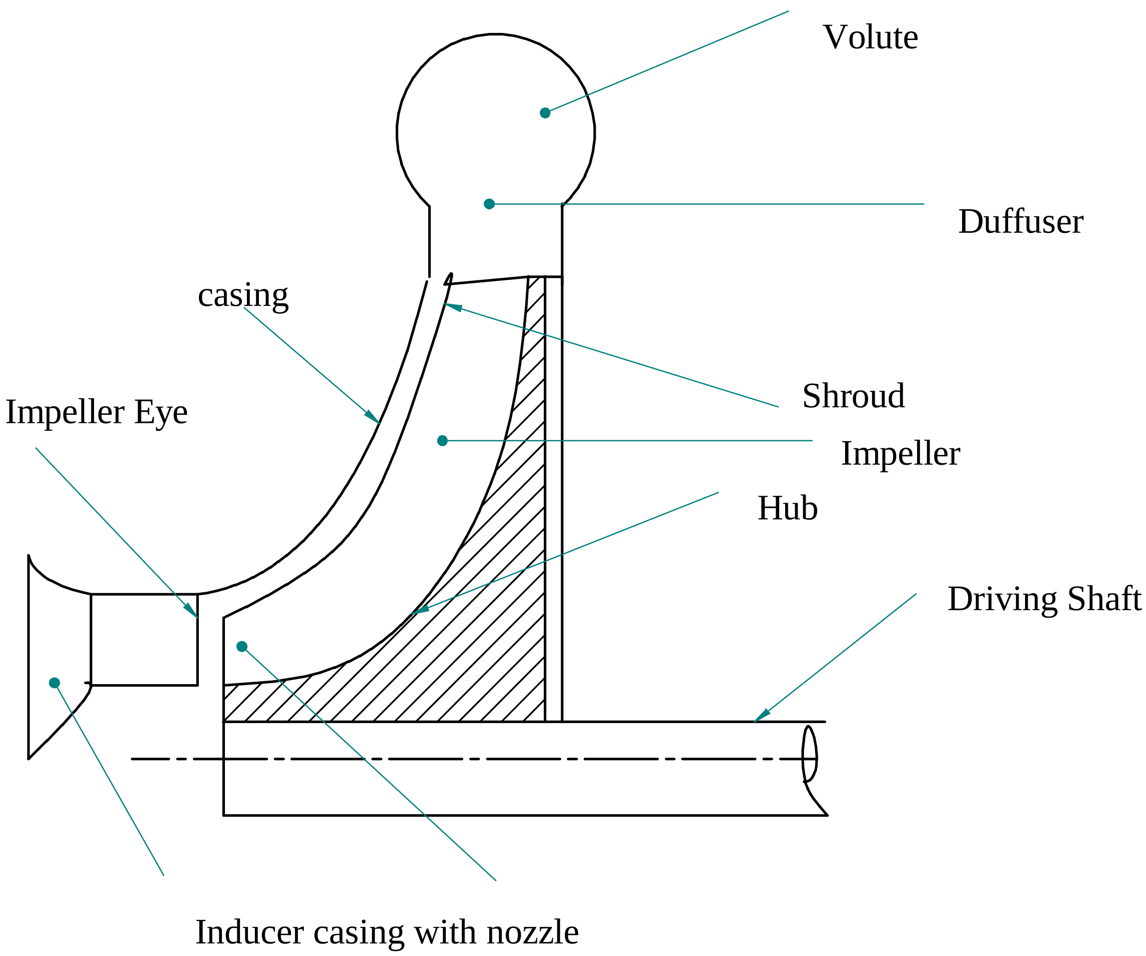
Compressors is used to produce large pressure rise ranging from 2.5 bars to 10bars or more. A single stage compressor can generally produce a pressure rise upto 4 bar.

Important elements of centrifugal compressor

i) Nozzle ii) Impeller iii) diffuser iv) casing

Functions

i) Inlet casing with convergent nozzle: is to accelerate the entering fluid to the impeller inlet which direct the flow in the desired direction at the inlet of the impeller from state 0 to 1
ii) Impeller converts the supplied mechanical energy into fluid energy wherein the fluid kinetic energy converted into static pressure rise. Impeller consists of radial vane fitted to shrouds and has two portion inducer and a large radial portion. The inducer receives the flow between hub and tip diameter of the impeller eye and passes onto the radial portion of the impeller blades. The impeller may be single sided or double sided. Double sided impeller may be used where for the given size the compressor has to handle more flow
iii) The diffuser receives the flow from the impeller through a vane less space and raises the static pressure of the fluid further an account of conversion of exit high energy to pressure energy
iv) Spiral casing: The flow at the outer periphery of the diffuser is collected by a spiral casing known as volute, which discharges the flow into the delivery pipe.



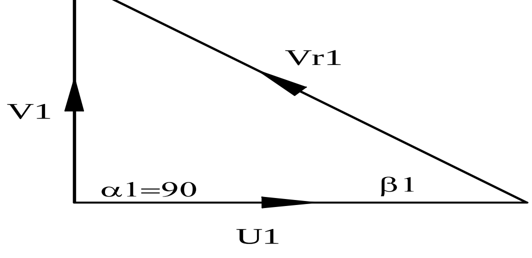
Variation of Pressure and Velocity.

As the fluid approaches the impeller, it is subjected to centrifugal effect thereby the kinetic energy and pressures of the fluid both increases along the radial direction.

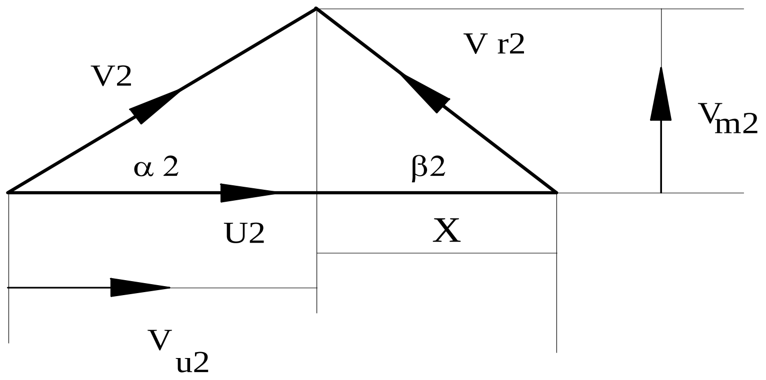
When the impeller discharges the fluid into the diffuser, the static pressure of the fluid rises due to the deceleration of the flow results in reduction in velocity and increase in pressure further due to conversion of kinetic energy into pressure energy.

Energy Transfer

Velocity triangle at Inlet



Velocity triangle at Outlet



$\frac{E}{m} = \frac{V_{u1}U_1 - V_{u2}U_2}{g_c}$; $\frac{E}{m} = \frac{-V_{u2}U_2}{g_c}$ $as\ V_{u1} = 0;$ $-\frac{E}{m} = \frac{+V_{u2}U_2}{g_c}$

Energy Input == $\frac{+V_{u2}U_2}{g_c}$ where $g_c = 1$

Eulers Head

For maximum Efficiency vanes are assumed to be radial $\beta_2 = 90^0$

Ie $V_{u2} = U_2$

Outlet velocity triangle

Energy Input $= \frac{U_2 U_2}{g_c}$;   vvEnergy Input $= \frac{U_2^2}{g_c}$

Slip and Slip coefficienct:

In Euler's equation , it is assumed that the velocities in turbomachine is constant across the given area. But in actual machine, velocities vary across the given area. This results in change of pressure across the vane, with high pressure at the leading face and a low pressure at the trailing face. As a result, the fluid leaves tangentially only at the high pressure face and nowhere else. The lower pressure at the trailing face results in a lower speed of fluid flow compared with that of leading face and the mean vane exit angle β_2 is less than the vane exit angle β_2'

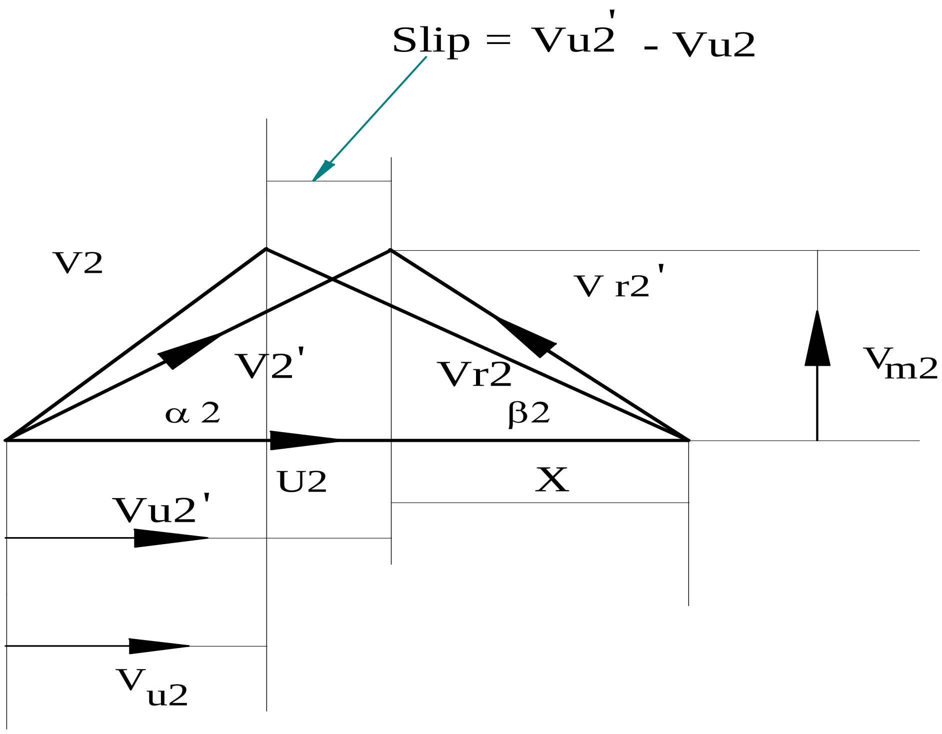
Hence the actual tangential component of absolute velocity at the exit (V_{u2})will be less than the tangential component of absolute velocity for the ideal condition (V_{u2}') . Hence, Eulers head developed in actual condition is less than the actual condition The difference between Eulers head developed for actual case and ideal condition is called slip

Slip = $H_e' - H_e$

The ratio of actual Eulers head developed (with pre-rotation) to the ideal Eulers head developed (with out pre-rotation) is called as slip coefficient

Slip coefficient $\mu = \frac{H_e}{H_e'} = \frac{V_{u2}}{V_{u2}'}$

Actual Eulers head developed $= \mu \frac{V_{u2}U_2}{g}$



With slip for radial curved vane

Energy Input $= \frac{\mu U_2^2}{g_c}$

Power Input Factor or Work done factor

Power Input factor is defined as the ratio of actual power to be supplied to theoretical work supplied

In real fluid, some part of the power supplied by the impeller on the air is used to overcome the losses like windage, disc friction and losses. Therefore the power required is greater than the actual power to be supplied on the air and hence the actual power to be supplied is taken care by the term power Input factor

$$\varphi = \frac{\text{actual power to be supplied}}{theortical\ work\ supplied} = \frac{E}{\frac{\mu U_2^2}{g_c}}$$

$$E = \varphi \frac{\mu U_2^2}{g_c}$$

The typical value of $\varphi\ ranging\ from\ 1.035\ to\ 1.04$

Overall pressure ratio (p_{ro})

$$E = h_{02} - h_{01} = \psi \frac{\mu U_2^2}{g_c}$$

As there is no work transfer in the diffuser, by energy balance we can write,

$h_{03} - h_{02} = h_{02} - h_{01}$ as in diffuser enthalpy remains constant ie $h_{03} = h_{02}$

$$\eta_{tt} = \frac{C_P(T'_{03} - T_{01})}{C_P(T_{03} - T_{01})}; \qquad (T'_{03} - T_{01}) = \eta_{tt}(T_{03} - T_{01})$$

Dividing both sides by T_{01}

$$\frac{T'_{03}}{T_{01}} - 1 = \frac{\eta_{tt}(T_{03} - T_{01})}{T_{01}}; \qquad \left(\frac{p_{03}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1 = \frac{\eta_{tt}(T_{03} - T_{01})}{T_{01}}; \qquad \frac{p_{03}}{p_{01}} = \left(1 + \frac{\eta_{tt}(T_{03} - T_{01})}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{p_{03}}{p_{01}} = \left(1 + \frac{\eta_{tt} C_P(T_{03} - T_{01})}{C_P T_{01}}\right)^{\frac{\gamma}{\gamma-1}}; \qquad \frac{p_{03}}{p_{01}} = \left(1 + \frac{\eta_{tt}\psi\mu U_2^2}{C_P T_{01}}\right)^{\frac{\gamma}{\gamma-1}}$$

Pressure coefficient /Loading Coefficient

Because of compressor losses as well as the kinetic energy, the actual pressure rise is less than theoretical specified by the impeller speed. This is expressed by a quantity is called pressure coefficient

It is defined as the ratio of isentropic work input across the impeller to the Eulers work input

$$\phi_P = \frac{\text{isentropic work input across the impeller}}{\text{Max Eulers work input}}$$

$$\phi_P = \frac{C_P(T'_{02} - T_{01})}{U_2^2}; \qquad \eta_c = \frac{C_P(T'_{02} - T_{01})}{C_P(T_{02} - T_{01})}; \qquad \eta_c C_P(T_{02} - T_{01}) = C_P(T'_{02} - T_{01})$$

Hence, $\phi_P = \frac{\eta_c C_P(T_{02} - T_{01})}{U_2^2}$

$$E = h_{02} - h_{01} \qquad E = \psi \frac{\mu U_2^2}{g_c}$$

$$C_P(T_{02} - T_{01}) = \psi \frac{\mu U_2^2}{g_c}$$

$$\phi_P = \frac{\psi\mu U_2^2}{U_2^2}$$

The loading factor in terms of exit blade angle

$$\phi_P = \frac{\text{isentropic work input across the impeller}}{\text{Max Eulers work input}}$$

$$\phi_P = \frac{V_{u2}U_2}{U_2^2}$$

$$V_{u2} = U_2 - V_{f2}cot\beta_2$$

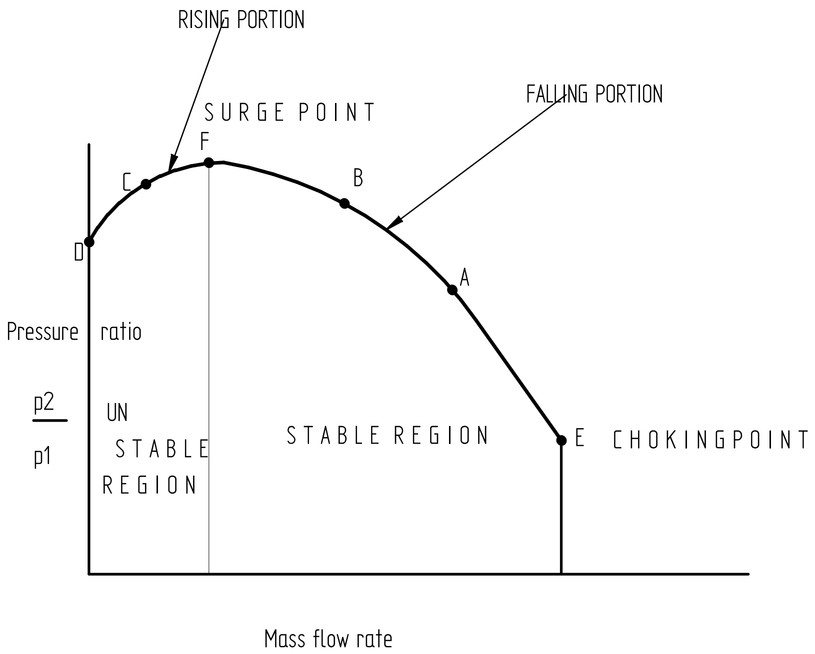
$$\phi_P = \frac{(U_2 - V_{f2}cot\beta_2)U_2}{U_2^2}$$

$$\phi_P = \frac{U_2 - V_{f2}cot\beta_2}{U_2}$$

$$\phi_P = 1 - \frac{V_{f2}}{U_2}\ cot\beta_2$$

Surging

Surge is a characteristic behavior of a centrifugal compressor that can occur when inlet flow is reduced such that the head developed by the compressor is insufficient to overcome the pressure at the discharge of the compressor. Once surge occurs, the output pressure of the compressor is drastically reduced, resulting in flow reversal within the compressor.



At very high flow rates in machines with high pressure ratios , it possible for the flow to be chocked . In that case , the mass flow will be fixed no matter how low the delivery pressure. The charecterstic becomes nearly vertical

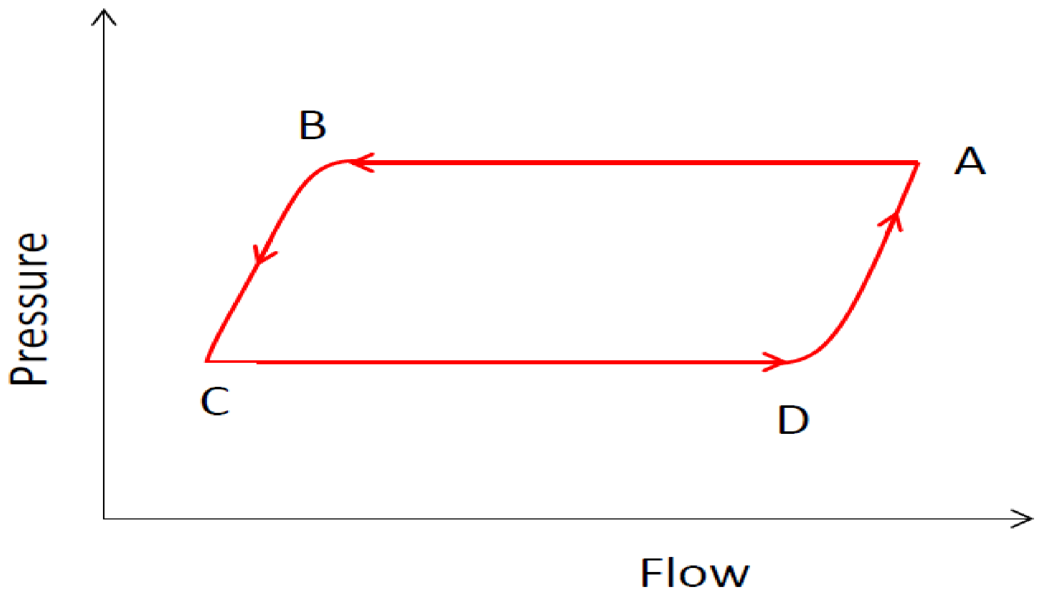
Consider the machine with an actual characteristic D-C-F-B-A. A stable operation at a point such as A implies that head developed by the machine H_A at A equals the losses due to friction and other causes in delivery pipe when the flow rate is Q_A.At A, if due to an instantaneous disturbance is in the operating conditions the frictional losses are slightly increased, the flow rate tends to decrease. The operating point tends to move towards the left (ie, towards B) the delivery head developed by the machine increases. The rising head tends to compensate for the increased frictional losses and increases the flow rate to push the operating point back to A . In a similar way , if instantaneously the exit pipe loss decreased due to small disturbance, the flow rate tend increase and the head developed by the machine tends increase and the head developed by the machine tends to drip as a consequence, trying to reduce the flow and to bring operating point back to its location at A. The machine thus tries to maintain a stable operating point. Thus machine maintain a stable operating point with a constant flow rate at a constant head

If the machine is initially operating at a point such as C on the rising portion of the characteristic, any slight disturbance in the operating conditions tending to increase the frictional losses leads to a decrease in flow. However, since C is on the rising portion of characteristic curve the head decreases (opposite to operating condition at A) Decreased head results in still decrease in flow and continues. The operating point tend move down still further along the curve CD in an unstable manner until the point D is reached. At D fluid cease to flow and loss of head due to friction is tends to zero since there is no flow. Then it starts to deliver suddenly. . Thus operating point moves up and down to cause a recurrence of unstable equilibrium. The periodic on and off operation of the machine in an unstable condition is referred to as surge.

Let C is the point at which compressor is operating. At C if the flow is reduced by gradual closing of control valve operating point will be shifted to B and become stable. Further gradual closing of control valve at point B pressure ratio increases and it reaches maximum pressure ratio at A. If flow decrease by gradual closing of control valve beyond the point A pressure ratio decreases. At this condition, at downstream pressure is higher than the upstream of control valve which results in momentary stop of flow of compressed air and even flow may be reversal. Due to momentary stop or reverse flow pressure in the downstream decreases which causes delivery of compressed air to the downstream of control valve. Again, the pressure at downstream increases, which causes again stoppage of flow or reverse flow. Due to this again after a moment pressure at downstream decreases causing flow from downstream to upstream. Likewise cycle gets repeated with high frequency. This Phenomenon is called Surging or Pumping

Surge Mechanism

When a centrifugal compressor surges, there is an actual reversal of gas flow through the compressor impeller. The surge usually starts in one stage of a multistage compressor and can occur very rapidly.



Now we will see how the surge starts in a compressor. The surge is starting with the instant flow reverses, shown in the above figure. Consider the Point "A" is the actual flow developed by the compressor during normal operation. Due to the decrease in flow the operating point shift from "A" to "B". The compressed gas actually rushes backwards through the impeller from the discharge to the inlet. The release of compressed gas from the discharge side results in the pressure drop from "B" to "C". The reduction in pressure allows the flow to be re-established in the positive direction "C" to "D" and increase the discharge pressure from "D" to "A". If nothing in the system change, then the surge cycle will continuous. The increase in duration of the surge cycle results in damage to the compressor.

How Surge takes place in Centrifugal Compressor

Now we will see the surge phenomena in centrifugal compressor with the example compressor performance curve

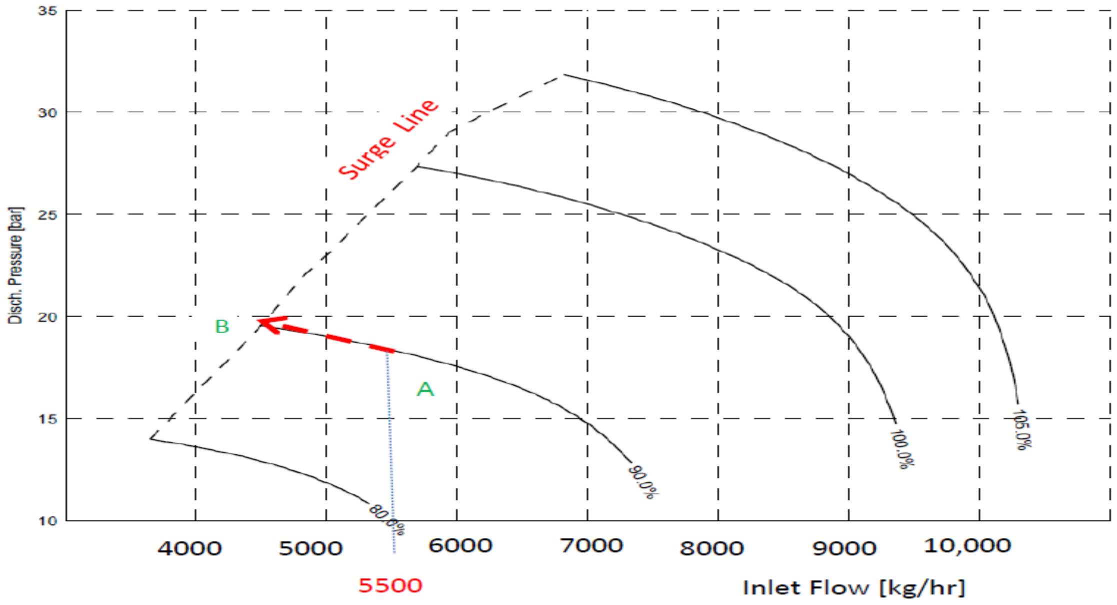
Suction throttling

Now consider the inlet flow decreases due to the suction valve throttling (they are many reasons will cause the compressor inlet flow rate decreases). Consider the compressor operating at the following conditions

Flow rate = 5500 kg/hr

Discharge pressure = 20 bar.

Speed = 6000 rpm



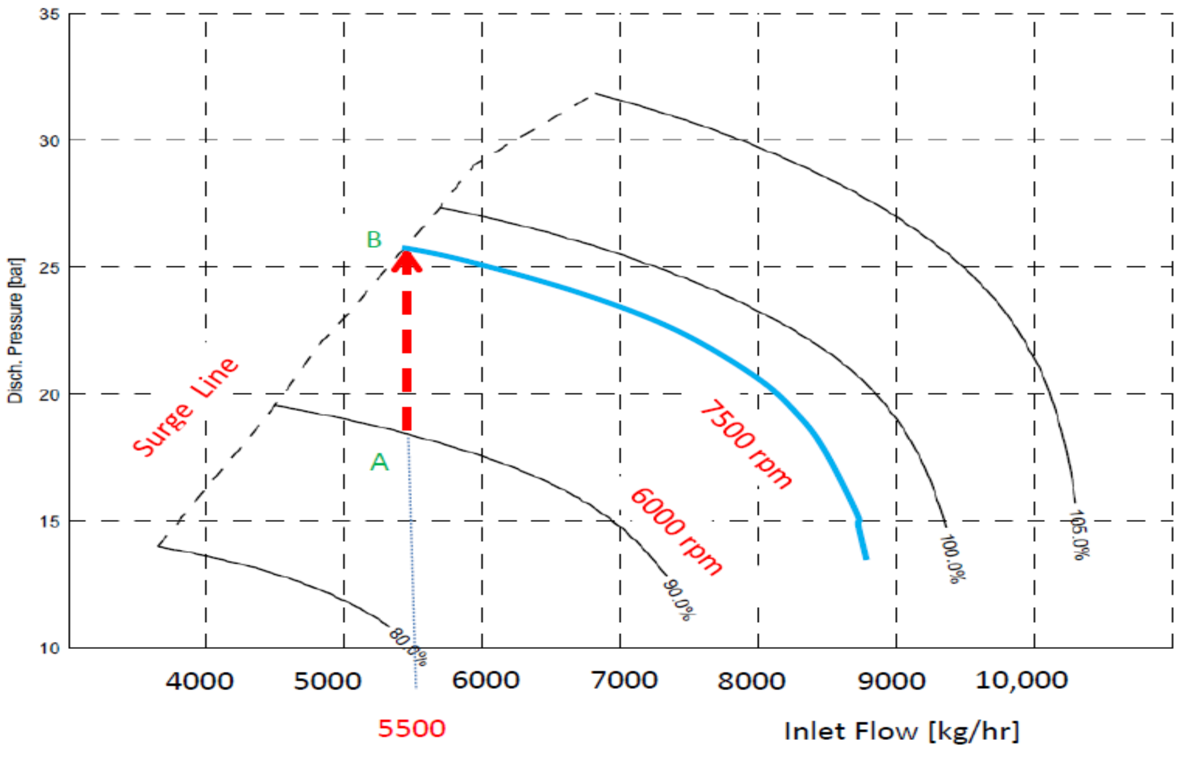
Due to suction valve throttling the inlet flow rate of the compressor is decreased from operating point (A) to the new operating point (B) (Refer above figure). At the new operating point (B) the compressor flow is reduced on the other hand the discharge pressure will rise further. Due to the rise of pressure and decrease flow will cause the Surge cycle.

Discharge valve throttling

Similarly, if the discharge side system resistance will increase due to discharge valve throttling. The pressure developed by the compressor will increase and flow rate will start decreases. The same phenomena will happen, that is the operating point "A" shifted to the new operating point of "B". This will cause the surge in the centrifugal compressor.

Change in Speed

An increase in operating speed of the compressor also causes compressor surge. Now consider the operating speed of the compressor is 6000 rpm. If the speed increase to 7500 rpm. The Operating point of the compressor will shift from "A" to "B". (Refer below figure)



At new operating point "B", the discharge pressure of the compressor will increase. Due to the increase in pressure. The point "B" fall in the surge line. Due to this surge phenomena will occur in the compressor.

Other reasons will cause Surge in Centrifugal Compressor

Inlet Filter Chocking

Due to dirty particles present in inlet filter will decrease the flow rate and reduce the suction pressure. Due to the reduce flow rate, the operating point will move toward the surge line. Once the operating point touches the surge line then Surge occur in the compressor.

Driver Input Speed

In the case of the compressor is driven by Turbine or Variable speed drives. Sometimes the increase in speed may cause operating will shift to surge limit line and surge will occur.

Change in Compressed gas Property

The change in operating gas can cause compressor surge.

Surge result in centrifugal compressor

As we seen the surge is due to the flow reversal in the compressor. As a result of the surge in the compressor, it may lead to damage of compressor or compressor system. The following are some of the resulting due to surging.

- During the surge, a significant mass gas will flow in the reverse direction. As a result of a large dynamic force act on the impeller or blading within the compressor. Due to this the components of the compressor (such as thrust bearings, bearing, casing) exposed to large changes in axial force on the rotor. If the surge is not controlled it may result in fatigue damage to compressor or piping components.
- During the surge, the reversal of flow within the compressor results in hot compressed gas returning to the compressor inlet. If the surge is not controlled, as a result the temperature at compressor inlet will increase and leads to a potential rubbing of close clearance components. Due to the differential thermal expansion of components within the compressor.

6. A centrifugal compressor delivers $20kg/s$ of air with a total head pressure ratio of $4:1$. The speed of the compressor is $12{,}000rpm$. Inlet total temperature is 15^oC stagnation pressure at inlet is $1.0\ bar$, slip factor is 0.9, and power input is 1.04. Efficiency is 80%. Calculate the outer diameter of the impeller (8b,10,June/July 18)

$\dot{m} = 20kg/s;\ \frac{p_{0k+1}}{p_{01}} = 4;\quad N = 12000rpm;\ T_{01} = 15^o = 288K\ ;\ p_{01} = 1\ bar;$

$Slip\ factor\ , \mu = 0.9;$ Power Input factor $\varphi = 1.04; \eta_{tt} = 0.8;\ D_2 = ?$

outer diameter of the impeller

$\eta_s = \frac{\Delta h_{0s}}{\Delta h_0};\qquad \eta_s = \frac{C_p(T_{02}-T_{01})}{\Delta h_0}\ ;\qquad \eta_s = \frac{C_pT_{01}\left(\frac{T_{02}}{T_{01}}-1\right)}{\Delta h_0}$

$\eta_s = \frac{C_pT_{01}\left(\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}}-1\right)}{\Delta h_0};\qquad 0.80 = \frac{1005*288(4^{0.286}-1)}{\Delta h_0};\qquad \Delta h_0 = 176045.72J/kg$

$\frac{-E}{\dot{m}} = \Delta h_0;\qquad \frac{-E}{\dot{m}} = 176045.72J/kg$

$\frac{-E}{\dot{m}} = \varphi\frac{\mu U_2^2}{g_c};\qquad 176045.72 = 1.04 * \frac{0.9*U_2^2}{1};\qquad U_2^2 = 188083.03$

$U_2 = m/s;$

$U_2 = \frac{\pi D_2 N}{60}$; $433.68 = \frac{\pi * D_2 * 12000}{60}$ $D_2 = 0.69m$

7. A centrifugal compressor delivers 18.2kg/s of air with a total pressure ratio of 4:1. Speed is 15000rpm. Inlet total temperature is 15^oC, slip coefficient is 0.9, power input factor is 1.04. Efficiency is 0.8. calculate overall diameter of impeller(8b, 10,June/July13)

$\dot{m} = 18.2kg/s; \frac{p_{0k+1}}{p_{01}} = 4;$ $N = 15000rpm;$ $T_{01} = 15^o = 288K$; $p_{01} = 1\ bar;$

$Slip\ factor\ , \mu = 0.9;$ Power Input factor $\varphi = 1.04; \eta_{tt} = 0.8;$ $D_2 = ?$

<u>outer diameter of the impeller</u>

$\eta_s = \frac{\Delta h_{0s}}{\Delta h_0}$; $\eta_s = \frac{C_p(T_{02s} - T_{01})}{\Delta h_0}$; $\eta_s = \frac{C_p T_{01}\left(\frac{T_{02s}}{T_{01}} - 1\right)}{\Delta h_0}$

$\eta_s = \frac{C_p T_{01}\left(\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right)}{\Delta h_0}$; $0.80 = \frac{1005*288(4^{0.286} - 1)}{\Delta h_0}$; $\Delta h_0 = 176045.72J/kg$

$\frac{-E}{\dot{m}} = \Delta h_0$; $\frac{-E}{\dot{m}} = 176045.72J/kg$

$\frac{-E}{\dot{m}} = \varphi \frac{\mu U_2^2}{g_c}$; $176045.72 = 1.04 * \frac{0.9 * U_2^2}{1}$; $U_2^2 = 188083.03$

$U_2 = m/s;$

$U_2 = \frac{\pi D_2 N}{60}$; $433.68 = \frac{\pi * D_2 * 15000}{60}$ $D_2 = 0.55m$

8. A centrifugal compressor runs at a speed of $15000rpm$ and delivers air at $20kg/s$. Exit radius is $0.35m$, relative velocity and vane angles at exit are $100m/s$ and 75^o respectively . Assuming axial inlet stagnation temperature and stagnation pressure as $300K$ and 1 bar respectively. Calculate: i) the torque ii) the power required to drive the compressor iii) ideal head developed iv) the work done and v) the exit total pressure. Take C_p of air= $1.005kJ/kgK$(8b,10, Dec17/Jan18)

$N = 15000rpm;$ $\dot{m} = 20kg/s;$ $R_2 = 0.35m;$ $V_{r2} = 100m/s;$ $\beta_2 = 75^o;$ $T_{01} = 300K$; $p_{01} = 1\ bar$

i) Torque=? ii) E =? iii) H_{ideal} =? iv)E =? v) p_{02} =? C_p of air= $1.005kJ/kgK = 1005J/kgK$

i) Torque=?

$U_2 = \frac{\pi D_2 N}{60}$; $U_2 = \frac{\pi * 0.7 * 15000}{60}$ $U_2 = 549.77m/s$

$V_{r2} cos\beta_2 = 100 cos 75$; $V_{r2} cos\beta_2 = 25.88m/s$

$\overrightarrow{V_{u2}} = U_2 - V_{r2} cos\beta_2$; $\overrightarrow{V_{u2}} = 549.77 - 25.88$ $\overrightarrow{V_{u2}} = 523.88m/s$

Torque, $T = \dot{m}(\overrightarrow{V_{u2}} R_2 - \overrightarrow{V_{u1}} R_1)$; $T = 20(523.88 * 0.35 - 0)$; $T = 3667.16Nm$

ii) the power required to drive the compressor

$\frac{-E}{\dot{m}} = \frac{(\overrightarrow{V_{u2}}U_2 - \overrightarrow{V_{u1}}U_1)}{g_c}$; $\frac{-E}{20} = \frac{523.88*549.77-0}{1}$; $-E = 5760270.15W$

Power required to drive the compressor$= 5.76 * 10^6 W = 5.76MW$

iii) **ideal head developed**

$H_{ideal} = \frac{\overrightarrow{V_{u2}}U_2}{g}$; $H_{ideal} = \frac{523.88*549.77}{9.81}$ $H_{ideal} = 29.359 * 10^3 m\ of\ air$

iv) the work done

$\frac{-E}{\dot{m}} = \frac{(\overrightarrow{V_{u2}}U_2 - \overrightarrow{V_{u1}}U_1)}{g_c}$; $\frac{-E}{\dot{m}} = \frac{523.88*549.77-0}{1}$ work Input per kg $= 288.013 * 10^3 J/kg$

v) the exit total pressure

$\eta_{tt} = \frac{\Delta h_{0s}}{\Delta h_0}$; $\eta_{tt} = \frac{C_p(T_{02s}-T_{01})}{\Delta h_0}$; $\eta_{tt} = \frac{C_p T_{01}\left(\frac{T_{02s}}{T_{01}}-1\right)}{\Delta h_0}$

$\eta_{tt} = \frac{C_p T_{01}\left(\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}}-1\right)}{\Delta h_0}$; Assuming $\eta_{tt} = 100\%$ $\Delta h_0 = C_p T_{01}\left(\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}}-1\right)$

$\frac{-E}{\dot{m}} = \Delta h_0$; $\Delta h_0 = 288.013 * 10^3 J/kg$

$288.013 * 10^3 = 1005 * 300\left(\left(\frac{p_{02}}{p_{01}}\right)^{0.286}-1\right)$; $\left(\frac{p_{02}}{p_{01}}\right)^{0.286} = 1.955$; $\frac{p_{02}}{p_{01}} = 10.42$

9. The impeller tip speed of a centrifugal compressor is 370m/s, slip factor is 0.9, and the radial component at the exit is 35m/s. If the flow area at the exit is 0.18m^2 and compressor efficiency is 88%. Determine the mass flow rate of air and the absolute Mach number at impeller tip. Assume air density $= 1.57 kg/m^3$ and inlet stagnation temperature 290K. Neglect the work input factor. Also find the overall pressure ratio of the compressor (8b, 12, Dec15/Jan16)

$U_2 = 370 m/s$; slip factor i $\mu = 0.9$;

the radial component at the exit is 35m/s ie $V_{rd2} = 35\text{m/s}$;

flow area at the exit is 0.18m^2 ie $A_{f2} = 0.18 m^2$; $\eta_{tt} = 0.88$

i) $\dot{m}$ =? ii) absolute Mach number at impeller tip M_2 =?; Assume $\rho = 1.57 kg/m^3$; $T_{01} = 290K$

Neglect work input factor ie $\varphi = 1$ iii) $\frac{p_{02}}{p_{01}}$ =?

the mass flow rate of air

$\dot{m} = \rho A_{f2} V_{rd2}$; $\dot{m} = 1.57 * 0.18 * 35$ $\dot{m} = 9.891 kg/s$

Absolute Mach number at impeller tip.

$\mu = \frac{\overrightarrow{V_{u2}}}{V'_{u2}}$; $\overrightarrow{V_{u2}} = \mu \overrightarrow{V'_{u2}}$ assuming radial vane at outlet ie $\overrightarrow{V'_{u2}} = U_2$; $\overrightarrow{V_{u2}} = \mu U_2$

$\overrightarrow{V_{u2}} = 0.9 * 370;$ $\overrightarrow{V_{u2}} = 333m/s$

$V_2^2 = V_{u2}^2 + V_{rd2}^2;$ $V_2^2 = 333^2 + 35^2;$ $V_2^2 = 112114$ $V_2 = 334.83$

$\frac{-E}{\dot{m}} = \frac{\overrightarrow{V_{u2}} U_2}{g_c};$ $\frac{-E}{\dot{m}} = \frac{\mu U_2 U_2}{g_c};$ $\frac{-E}{\dot{m}} = \frac{0.9*370^2}{1};$ $\frac{-E}{\dot{m}} = 123210J/kg;$

$\frac{-E}{\dot{m}} = \Delta h_0;$ $\Delta h_0 = 123210J/kg$

$\Delta h_0 = C_p(T_{02} - T_{01});$ $123210 = 1005(T_{02} - 290)$ $T_{02} = 412.59K$

$T_{02} = T_2 + \frac{V_2^2}{2g_c C_p};$ $412.59 = T_2 + \frac{112114}{2*!*1005};$ $T_2 = 356.81K$

$M_2 = \frac{V_2}{\sqrt{\gamma R T_2}};$ $M_2 = \frac{334.83}{\sqrt{1.4*287*356.81}}$ $M_2 = 0.884$

overall pressure ratio of the compressor

$\eta_{tt} = \frac{\Delta h_{0s}}{\Delta h_0};$ $\eta_{tt} = \frac{C_p(T_{02s} - T_{01})}{\Delta h_0}$; $\eta_{tt} = \frac{C_p T_{01}\left(\frac{T_{02s}}{T_{01}} - 1\right)}{\Delta h_0}$

$$\eta_{tt} = \frac{C_p T_{01}\left(\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right)}{\Delta h_0}$$

$\Delta h_0 = \frac{-E}{\dot{m}};$ $\frac{-E}{\dot{m}} = \frac{\overrightarrow{V_{u2}} U_2}{g_c};$ $\frac{-E}{\dot{m}} = \frac{\mu U_2 U_2}{g_c};$ $\frac{-E}{\dot{m}} = \frac{0.9*370^2}{1}$

$\frac{-E}{\dot{m}} = 123210J/kg;$ $\Delta h_0 = 123210J/kg$

$\eta_{tt} = \frac{C_p T_{01}\left(\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right)}{\Delta h_0};$ $0.88 = \frac{1005*290\left(\left(\frac{p_{02}}{p_{01}}\right)^{0.286} - 1\right)}{123210}$; $\left(\frac{p_{02}}{p_{01}}\right)^{0.286} = 1.372$

$\frac{p_{02}}{p_{01}} = 3.02$

10. Backward swept centrifugal fan develops a pressure of 75mm WG. It has an impeller diameter of 89cm and runs at 720rpm. The blade angle at the tip is 39^o and the width of the impeller is 10cm. Assuming a constant velocity of flow of 9.15m/s and density of $1.2kg/m^3$, determine the fan efficiency , discharge, power required , stage reaction and pressure coefficient (8b, 10, Dec18/19)

pressure of 75mm WG ie $\frac{p_2}{\rho g}$ =75mm WG; $D_2 = 89cm = 0.89m;$ $N = 720rpm;$

The blade angle at the tip is 39^o ie $\beta_2 = 39^o$; width of the impeller is 10cm $H_2 = 0.1m$

constant velocity of flow of 9.15m/s ie V_{f2} =9.15m/s ; $\rho = 1.2kg/m^3$

i) η_{tt} =?; ii) $\dot{m}$ =? iii) $-E$ =?; iv) R =? v) pressure coefficient φ_p =?

<u>fan efficiency</u>

$U_2 = \frac{\pi D_2 N}{60}$; $U_2 = \frac{\pi * 0.89 * 720}{60}$ $U_2 = 33.55m/s$

$V_{f2} cot\beta_2 = 9.15 cot39$; $V_{f2} cot\beta_2 = 11.29m/s$

$\overrightarrow{V_{u2}} = U_2 - V_{f2} cot\beta_2$; $\overrightarrow{V_{u2}} = 33.55 - 11.29$ $\overrightarrow{V_{u2}} = 22.26m/s$

$\frac{-E}{\dot{m}} = \frac{\overrightarrow{V_{u2}} U_2}{g_c}$; $\frac{-E}{\dot{m}} = \frac{22.26 * 33.55}{1}$ $\frac{-E}{\dot{m}} = 746.89J/kg$

$\Delta h_0 = \frac{-E}{\dot{m}}$; $\Delta h_0 = 746.89J/kg$

$\eta_{tt} = \frac{\Delta h_{0s}}{\Delta h_0}$; $\eta_{tt} = \frac{C_p(T_{02s} - T_{01})}{\Delta h_0}$; $\eta_{tt} = \frac{C_p T_{01}\left(\frac{T_{02s}}{T_{01}} - 1\right)}{\Delta h_0}$

$$\eta_{tt} = \frac{C_p T_{01}\left(\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right)}{\Delta h_0}$$

$\frac{p_2}{\omega}$ =75mm WG= $0.075m\ of\ water\ (guage)$;

$\frac{p_{02}}{\omega}(absolute) = guage\ pressure\ head + atm. Pressure$ head (generally 10.3m of water)

$\frac{p_{02(guage)}}{9,81 * 1000} = 0.075m\ of\ water$; $p_{02(guage)} = 735.75N/m^2$

$p_{02(abs)} = p_{02(guage)} + p_{atm}$; Assume $p_{atm} = 1\ bar = 10^5 N/m^2$;

$p_{02(abs)} = (735.75 + 10^5)N/m^2$; $p_{02(abs)} = 1.0074\ bar$

Assume $p_{01} = 1\ bar$; $T_{01} = 300K$

$\eta_{tt} = \frac{1005 * 300\left(\left(\frac{1.0074}{1}\right)^{0.286} - 1\right)}{746.89}$; $\eta_{tt} = 0.852$

<u>Discharge</u>

$Q = A_{f2} V_{f2}$; $Q = (\pi D_2 H_2) V_{f2}$; $Q = \pi * 0.89 * 0.1 * 9.15$; $Q = 2.558m^3/s$

$\dot{m} = \rho Q$; $\dot{m} = 1.2 * 2.558\ kg/s$ $\dot{m} = 3.07kg/s$

<u>power required</u>

$-E = \frac{-E}{\dot{m}} \dot{m}$; $-E = 746.89 * 3.07W$ Power required = $2292.97W$

<u>stage reaction</u>

$R = \frac{\frac{E}{\dot{m}} - \left(\frac{V_1^2 - V_2^2}{2g_c}\right)}{\frac{E}{\dot{m}}}$; $R = 1 - \frac{(V_1^2 - V_2^2)}{2g_c\frac{E}{\dot{m}}}$;

$V_2^2 = V_{u2}^2 + V_{f2}^2$; $V_2^2 = 22.26^2 + 9.15^2$; $V_2^2 = 579.23$

$V_1^2 = V_{u1}^2 + V_{f1}^2$; $V_1^2 = 0 + 9.15^2$; $V_1^2 = 83.72$

$\frac{-E}{\dot{m}} = 746.89 J/kg$; $\frac{E}{\dot{m}} = -\ 746.89 J/kg$

$R = 1 - \frac{(V_1^2 - V_2^2)}{2g_c\frac{E}{\dot{m}}}$; $R = 1 - \frac{(83.72 - 579.23)}{2*1*(-746.89)}$; $R = 0.668$

pressure coefficient $\varphi_p = \frac{\Delta h_0}{U_2^2}$

$\varphi_p = \frac{\Delta h_0}{U_2^2}$; $\varphi_p = \frac{746.89}{33.55^2}$ $\varphi_p = 0.663$

11. An axial compressor / blower supplies air to furnace at the rate of 3kg/s. The atmospheric conditions being 100kPa and 310K , the blower efficiency is 80%. Mechanical efficiency is 85% The power supplied to 30kW. Estimate the overall efficiency and pressure developed in mm WG(8c,8,June/July14)

$\dot{m} = 3kg/s$; $p_{01} = 100kPa$; $T_{01} = 310K$ $\eta_{tt} = 0.8$ $E = 30kW = 3000W$

overall efficiency

$\eta_o = \eta_{tt}\,\eta_{mech}$; $\eta_o = 0.8 * 0.85$ $\eta_o = 0.68$

pressure developed in mm WG

$\frac{-E}{\dot{m}} = \frac{3000}{3}$; $\frac{-E}{\dot{m}} = 1000 J/kg$; $\frac{-E}{\dot{m}} = \Delta h_0$; $1000 J/kg = \Delta h_0$

$\eta_{tt} = \frac{\Delta h_{0s}}{\Delta h_0}$; $\eta_{tt} = \frac{C_p(T_{02s} - T_{01})}{\Delta h_0}$; $\eta_{tt} = \frac{C_pT_{01}\left(\frac{T_{02s}}{T_{01}} - 1\right)}{\Delta h_0}$

$\eta_{tt} = \frac{C_pT_{01}\left(\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right)}{\Delta h_0}$; $0.8 = \frac{1005*310\left(\left(\frac{p_{02}}{p_{01}}\right)^{0.286} - 1\right)}{1000}$ $\left(\frac{p_{02}}{p_{01}}\right)^{0.286} = 1.00257$

$\frac{p_{02}}{p_{01}} = 1.009$; $\frac{p_{02}}{100*10^3} = 1.009$ $p_{02} = 100901.48Pa$

$p_{02}(guage) = 100901.48 - 100000$; $p_{02}(guage) = 901.48Pa$

$\frac{p_{02}}{\omega} = \frac{901.48}{9810}$; $\frac{p_{02}}{\omega} = 0.0918m\ of\ water$ $\frac{p_{02}}{\omega} = 91.8mm\ WG$