Inventory Management Theory: a Critical Review

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Abstract

It was carried out by studying available Czech and foreign scientific references, which serve as the basis for mapping the approaches to inventory management as well as research into more complex inventory theory models. The paper deals with the inventory theory itself and describes the formation of quantitative models as tools for optimizing inventory under certain conditions the models have been derived from. The paper also discusses an issue most frequently raised in terms of the inventory theory, i.e. optimization of costs related to the costs of holding stock, which follows the analysis of demand for commodities the selected stock is produced from. Thus, the issues of inventory theory and demand for final product are closely related. Results suggest the use of the "Mathematica" platform and modelling demand of the stochastic nature.

Keywords: supply chain, inventory theory, demand, deterministic models, stochastic models

Introduction

Since the 20th century, an emphasis has been put on continuous increase in efficiency of business activities. The development of operational research methods and their implementation with the use of modern information technologies has contributed to reducing corporate costs. One of the ways of how to achieve their reduction is the optimization of logistics activities, which also includes inventory management theory (Bowerson and Closs 1996).

The current market situation could be described as a competitive environment resulting from frequent economic changes and intensive relationship networking within the supply chains (Bazan, Jaber and Zanoni 2016). The concept of Supply Chain Management first appeared in the mid-1990s (Sixta and Žižka 2009). Kuncová (2009) states that the supply chain management is currently one of the most developing areas. For most companies, entering these supply chains has been becoming a routine, gradually imposed by the market and the conditions of such companies. An essential condition in this context is to use the latest technologies for a quick information and data interchange between customers and suppliers. Mutual cooperation and collaboration of the supply chain members, including sharing information, are the driving factors ensuring a higher competitiveness in the market as well as a larger scope of activities in the market both for the companies and the whole chains (Lee, Chan and Langevin 2016). Due to the continuously growing customers´ demands, the limiting and decisive factor is not only the price or quality of the product, but also lead time, services provided by the producer and/or the customers´ relationship to the company and the product(s). As a result of these requirements, companies currently engage more in the supply chains and their mutual cooperation can generate far more benefits than their mutual competitiveness (Basl, Majer, Šmíra 2003). The main objective of the supply chain management is to stay one step ahead of the competitors and a great emphasis is also put on the information involving demand (the number of items sold, sellers´ expectations, predicting customers´ behaviour, various marketing actions, competition). An important role of the inventory theory is to satisfy the demand and determine its further development as well as to ensure an adequate quantity of the goods (Daněk and Plevný 2005).

In the supply chain, inventory (or stock) has an important role from a commercial point of view. There is a wide range of factors affecting the supply chain, such as the stock level throughout the whole supply chain (and not only in a particular entity), costs related to storing and maintaining the inventory, in particular their minimization in the whole supply chain, and at the same time, the pursuit of maximum demand satisfaction (Samal and Pratihar 2014). Inventory management is an important element both in the management of individual companies and the supply chains as such. Regardless of whether the inventory refers to raw materials, material, semi-finished or finished products, it is an element which influences the operations of companies and supply chains, and should therefore be given adequate attention. Owing to frequent uncertainties in the market development, fluctuating demand or production and changes in lead times, the inventory management as such may be very complicated (Emmett 2008). There is no universal model, with a wide range of factors affecting the inventory stock, and thus the situation here is closely related to the ability to predict the future consumption induced by future demand (Bartmann and Beckamann 1992). The future demand varies in the course of time and is covered by the production, which, by contrast, remains constant, and any changes that do occur are step changes rather than incremental ones. In this case, the inventory serves to compensate for the differences between the productive potential and the demand volume (Lukáš 2009). The goal of the paper is to critically review methods of Inventory management theory. The scope of this paper is based on collection of methods which were derived form a comprehensive work of Lukáš (2005a, 2005b, 2009, 2012, and 2016) who deals with probability models in detailed fashion. The value added is in comparison of the method which is based on the comparison of practical usefulness and academic value measured by publications.

Materials and Methods

A detailed mathematical analysis of the deterministic and stochastic models is conducted to classify and identify the methods. Current usability of the methods is measured by their publication frequency in databases, such as "Web of Science, ProQuest, Scopus, and ScienceDirect". Based on the classification of such models, the study examines the application of cost models, stochastic demand and also the approaches within the deterministic models. The authors focus on the models concerning the field of operational research, specifically the inventory theory. The operational research can be characterized as a set of relatively independent areas aimed at the analysis of various types of decision problems. The inventory theory as such is one of the operational research areas.

The authors have used the methods known as mathematical programming methods (mainly linear and nonlinear programming, stochastic programming and dynamic programming), which are essential for the research and analysis here. The methods used also include simulation procedures and techniques.

Other objective is a detailed mathematical analysis of the deterministic and stochastic models and the identification of their current usability by means of publication outcomes in scientific databases, such as "Web of Science, ProQuest, Scopus, Web of Knowledge, ScienceDirect". Those models are predominantly the outputs of specific platforms, such as "Matlab, Mathematica", more specifically by means of m-files, or the programmes for Inventory Control or Stochastic Inventory Control.

In order to carry out this study, the authors used a method of analysis and synthesis of selected articles and papers, available from the aforementioned databases. The authors mainly drew their attention to the works by Lukáš (2005a, 2005b, 2009, 2012, and 2016), and by Bowersox, Closse (1996), Bazana, Jabera and Zazoni (2016), and by Lee, Chan and Langevin (2016) .

Results

In this part classification of the inventory theory models is presented. The business sector representatives will be able to use this classification to optimize their inventories as well as their subsequent warehouse inventory management.

Tab. 1: Publications of models of inventory theory

	Deterministic models		Stochastic models	
	Constant	Combined	Single-product	Multi-product
Publications in databases	-	- +	+	++
Applicability in practice	-	- +	+	++
Mathematica – software	-	-+	+++	+++
Matlab – software	-	-+	+	++

Source: Authors´ own compilation

Table 1 lists the models of inventory management theory, which can be modelled using mathematical tools and the necessary software. The authors present two basic descriptions of demand, which also separate and refer to the two models of inventory theory - "Deterministic models“ and “Stochastic models." The authors focused on the publications of each model in various scientific databases to verify the availability of scientific studies by individual authors. Based on this analysis, they discovered that the highest number of published sources, while also applicable to business practice, is represented by the models of stochastic demand description, with multiple-product models being the most usable in terms of their variability as well as being the most customizable to modelling capabilities by responding to different situations that may arise on the market in various sectors. Essentially, this is demand for certain goods available on the market for given manufacturing companies, which thus ensured continuity of production even over a period with certain unforeseen situations, e.g. traffic problems during shipment and transport of goods necessary for the production cycle and production procurement.

Within a practical context, there are only a few situations that would correspond to a purely deterministic model. If they do occur, it is either in combination with another model of stochastic nature, or as an approximation of the stochastic model with very little dispersion. Also, the deterministic models usually serve as the fundamental basis of complex, stochastic models of inventory management. The above table also shows that on analysing the scientific studies, the authors most frequently came across the use of modelling platforms (software) that the studies´ authors applied to modelling appropriate situations in the form of the "Mathematica" platform. In comparison with the "Matlab" platform, this software is easier to use and is characterized by its variability. Additionally, models describing the stochastic demand (specifically the multi-product ones) are the most common variant of modelling in this platform. These aspects of applying individual platforms are quite the measure of usability (applicability) of modelled situations in the form of a particular model in practice, where the situations are modelled on data of specific companies and capture their desired optimization conditions.

Discussion

The main contribution of this part should be the analysis, characterisation and derivation of the inventory theory models, which the business sector representatives will be able to use to optimize their inventories as well as their subsequent warehouse inventory management.

Inventory theory models

The knowledge and character of demand are very important for the whole inventory management. There are basically three demand modelling methods. One of them is the deterministic demand model, where its explicit expression is known. The demand function may not only be a linear demand function, but also the polynomial of general degree n function, or any other known function. It merely depends on the real situation to be modelled (Lukáš 2005).

As for the deterministic inventory models, the demand function may either be estimated or approximated. These inventory models are based on the assumptions that both the demand volume and procurement cycle are predetermined. Knowing the exact volume of needs to be met out of inventories, even notwithstanding the random variations in deliveries from suppliers, it is pointless to create any safety stock. Therefore, all of the deterministic models only optimize certain inventory turnover ratio and a cost optimum is searched only by means of storage costs and one-off costs to replenish inventory (Jablonský 2007, Lukáš 2005, Kořenář 2002, Lysina 2000).

Deterministic demand function as a model

In order to solve these models, (rather than the simulation methods) other optimization techniques are preferably used. For example, to solve the deterministic model with a constant need, the Harris-Wilson formula may be applied for determining (calculating) the optimal supply. Other deterministic inventory models include the EOQ (Economic Order Quantity) model, which is based on periodic replenishment at uniform demand and constant rate of supply (Lukáš 2012):

$$CN(q) = VN + FN = c_1 * \frac{q}{2} + c_2 * \frac{Q}{q} \qquad (1)$$

Here, an order size, or order quantity (q), is the model variable. Therefore, to find out the function´s minimum value, the first derivative (according to q) may equal to zero. Calculating (determining) the optimal order quantity (q*) formula may then be obtained as follows:

$$q^* = \sqrt{\frac{2 * Q * c_1}{c_2}} \qquad (2)$$

complemented by the following formula for determining the total minimum cost:

$$CN * = \sqrt{2 * Q * c_1 * c_2} \qquad (3)$$

Based on these data, it is also possible to obtain the optimal replenishment cycle length (or time) (t*):

$$t* = \sqrt{\frac{2* c_2}{Q* c_1}} \quad (4)$$

and the reorder point (rp*), which indicates at which level of items in stock an order is to be placed to replenish that particular stock at a given (or desired) point – i.e. when running out of inventory:

$$r_p = (Q * L) mod\ q * \quad (5)$$

Modifications of this model include the "Model of delayed demand satisfaction", which allows even for a temporary shortage of stock and delayed demand satisfaction, the "EPL (Economic Production Lot-Size)" model, where the make-to-stock production phase and the use-up-the-stock phase (or only the latter) follow each other, and the "Model of quantitative rebate", when the calculation also includes quantitative rebates from suppliers (Giri, Chaudhuri 1998):

$$VN_t = c_1 * \frac{q-z}{2} * t_1 + c_3 * \frac{z}{2} * t_2 \quad (6)$$

Here, the number of order cycles is given again by the ratio of total demand to the order quantity, i.e. Q/q. Fixed costs may then be obtained by multiplying the number of orders with the c2 constant (cost per order). Consequently, the total annual cost of inventory may be obtained in the following manner:

$$CN = VN + FN = c_2 * \frac{Q}{q} + \left(c_1 * \frac{q-z}{2} * t_1 + c_3 * \frac{z}{2} * t_2\right) * \frac{Q}{q} = \left(c_1 * \frac{q-z}{2} * t_1 + c_2 + c_3 * \frac{z}{2} * t_2\right) * \frac{Q}{q} \quad (7)$$

The above function is of four variables, which through mutual substitution may be reduced down to two, i.e. the order quantity (q) and the unsatisfied inventory size or quantity (z). Partial derivative (Ter-Manuelianc 1980, Jablonský 2007) will result in the optimal order quantity and the unsatisfied demand size (or quantity):

$$q* = \sqrt{\frac{2*Q* c_2}{c_1}} * \sqrt{\frac{c_1 + c_3}{c_3}} \quad (8)$$

$$z^* = q^* * \frac{c_1 + c_3}{c_3} \quad (9)$$

The optimal order quantity is therefore derived from the value obtained in the basic EOQ model by multiplying both of the constants dependent on the storage costs (c1) and the costs associated with shortage of stock (c3). However, if the c3 costs are disproportionately higher than the storage costs (which could often be anticipated, particularly when recognizing the customer's potential loss), then this constant approximately equals one and the shortage of stock is basically not considered. As a result, it is the return to the basic EOQ model.

Stochastic inventory models differ from the deterministic ones only in the character of demand. Whilst demand is fixed in the deterministic models, demand in the former is of stochastic (probabilistic) nature, which means it is a random variable with a probability distribution. The stochastic models represent a certain demand, where its explicit expression is known. The demand function may include not only a linear function, but also a function with a polynomial of general degree n, or any other known function. It is merely dependent on the real situation to be modelled (Kořenář 2010).

As for the static stochastic modelling, the main prerequisite here is the impossibility of further inventory replenishment. Therefore, these are situations, where over a certain period it is necessary to satisfy the needs from the stock that can be created only once. If the generated stock is lower than the actual need, certain costs from a shortage of stock will emerge. On the contrary, provided that the generated stock is higher than the actual need, some additional costs will be incurred again, for after the end of the period, the stock will not be usable (e.g. a Christmas tree retailer).

Static stochastic demand description as a model

These are models often used for a single replenishment, mostly with sporadic demand (i.e. Lumpy Demand). The aim of these models is to find the lowest value of the total cost for the delivery of q = xi volume. The lowest value may be obtained by searching progressive values:

$$E(N(q = x_{i-1})) \geq E(N(q = x_i = q_0)) \leq E(N(q = x_{i+1})) \tag{10}$$

By determining three values in $E(N(q))\, proq = \{x_{i-1}, x_i, x_{i+1}\}$, a local minimum is obtained, thus the lowest actual cost for q = xi. The mean value for $E(N(q))$ may be determined through (Lukáš, 2005):

$$E\big(N(q)\big) = C_1 \sum_{j=0}^{i-1}(q - x_j)\ p\left(x_j\right) + C_2 \sum_{j=i+1}^{n}\left(x_j - q\right) p\ (x_j) \tag{11}$$

Dynamic stochastic demand description as a model

The most common assumption is that the demand distribution in a given period follows a normal distribution with a mean value (μQ) and a standard deviation (σQ). Likewise, demand during a particular lead time (L) is normally distributed with a mean value (μL) and a variance (σL), where:

$$\mu_L = L * \mu_Q \tag{12}$$

$$\sigma_L = L * \sigma_Q \tag{13}$$

Owing to the probabilistic nature of demand, all of the available inventories may not necessarily be used up during the lead time, but two cases may occur - upon the arrival of the ordered goods, there are still some items in stock, or there is a shortage of stock during the lead time. One way to find out is to determine the level of safety stock.

The safety stock level (or quantity) can be determined in many ways (Kutiš, 2004), with the majority of them working with the assumption of stochastic lead time. The following text merely indicates determining the minimum safety stock based on demand during a constant lead time (L). Assuming that during the acquisition (lead) time the demand has a normal distribution with a mean value (μL) and a variance (σL) and knowing a probability (γ), which corresponds to a value of the distribution function of a standard normal distribution (zγ), then the following should apply:

$$w \geq z_\gamma * \sigma_L \tag{14}$$

The new reorder point (rw) can then be obtained by increasing the original point (rp*) by a value of safety stock (w), thus reading:

$$r_w = w + (Q * L\ (mod\ q)) \tag{15}$$

where the *mod* function represents a particular remainder after the whole number division.

Another option is to exactly determine the safety stock level with the help of lead time, but this procedure is not suitable for a lead time shorter than one period. The functions then take the following form:

$$w = z_\gamma * \sigma_Q * \sqrt{L} \tag{16}$$

$$r_w = L * \mu_Q + w \tag{17}$$

In this case it is possible to obtain the optimal order quantity in a similar way to the deterministic EOQ model, which is:

$$q^* = \sqrt{\frac{2 * \mu_Q * c_2}{c_1}} \tag{18}$$

The total costs are also similar. However, it is necessary to extend them by the costs of safety stock holding:

$$CN^* = \sqrt{2 * Q * c_1 * c_2} + c_1 * w \tag{19}$$

Conclusion

This study described selected issues of supply chains and summarized the basic management concepts that have evolved over time and have had an influence on forming the methods for supply chain management in the view of the inventory theory. Inventory management may use either the already developed mathematical models or some of the managerial control concepts mentioned here, and last but not least, the simulation models as well. The most familiar mathematical model is the EOQ (Economic Order Quantity) model, which defines the methods for determining both the order quantity and lead time in order to minimize the inventory costs. It is also possible to use a variety of methods to estimate demand. Most frequently, they encompass certain predictions based on the trends and developments in the previous period (i.e. using statistical methods and estimates), or on the econometric models, which include the most important factors affecting the demand at given time and region. Additionally, this paper points out the possibility of using simulation models, thus generating a particular demand on the basis of pre-selected probability distributions and parameters for selected periods. For the real use of this approach, it is necessary to choose the distributions whose parameters are comprehensible to those having any impact on inventory management.

The most covered inventory theory methods (models) in scientific studies are multiple-product models with stochastic demand description. To name but a few Czech authors engaged in this area, doc. Lukáš and dr. Hofman and their scientific studies deserve to be mentioned, as listed in the references. When considering the foreign authors, who present their results in scientific studies discussing the stochastic demand description models, there is a large number of them, e. g, Lee, Chan, Langevin "Integrated inventory-transportation model by synchronizing delivery and production cycles", Samal and Pratihar – "Optimization of variable demand fuzzy economic order quantity inventory models without and with back ordering".

According to their assessment of the above analysis, the authors recommend the use of the "Mathematica" platform, which is not difficult to control and serves as the basis for modelling various situations in terms of a product demand. This platform is certainly suitable for companies that will resolve certain issues via this platform. The authors suggest modelling demand of the stochastic nature, where usability is at a higher level and it is possible to model different situations that the market may offer with regards to demand, which may not be exactly determined, and thus can react to the actual conditions possibly occurring on the market, irrespective of their susceptibility. Using these models, the issues can be solved not only in businesses operating in the Czech Republic, but also in multinational companies worldwide.

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