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INVERSE DISTANCE VARIATIONS FOR THE FLOW OF CRIME IN URBAN AREAS*

By

Thomas Spence Smith
University of Rochester

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ROCHESTER-MONROE COUNTY CRIMINAL JUSTICE PILOT CITY PROGRAM
UNIVERSITY OF ROCHESTER
GRADUATE SCHOOL OF MANAGEMENT
Room 320, Hopeman
Rochester, New York 14627

Elizabeth Benz Croft, Director

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ABSTRACT

This paper represents an attempt to discover whether sociological models of demographic flow can be useful in "explaining" the flow of crime within a city. A review of some recent research on social gravitation by prominent sociologists and demographers is presented to give the proper perspective on the models to be tested. The relationship of each of these models to local crime data subsequently is examined to see if the theories which apply to population movement within a region also apply to the movement of arrested offenders from their residence to the location of their offense.

The data utilized in this research include: a complete survey of the migration of arrested offenders between census tracts to commit a crime in the City of Rochester in 1972; a measure of the distance between any two census tracts within the City; and demographic data on the socio-economic indicators of all areas within the City.

Among the questions that this research seeks to answer is whether the propensity for criminal movement depends on such things as the population of the destination, the distance traveled to the offense location, the wealth or the racial characteristics of the neighborhoods involved, etc. Although attempts to fit many of the models produced disappointing or inconclusive results, there is evidence that some formulation of the classic gravity model -- in which the attraction between two objects is inversely related to the distance between them -- is an effective predictor of crime flow and deserving of more in-depth examination.

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This paper reports preliminary results of work recently undertaken to model the flow of crime in urban areas. Specific interest is attached to the migration of "offenders" from their places of residence to the locations where they commit their crimes. The "flow of crime," as we define it here, refers to the several streams of criminal migration among urban locations. The surprising absence of past research on this phenomenon, no doubt due in part to the unavailability until recently of suitably comprehensive data, has meant that many questions raised by the mobility of crime for students of urban ecology have thus gone unanswered. Though many of these questions derive from obvious practical and theoretical interests, our focus here might best be described as demographic. Our purpose is not to inquire into the social psychology of criminal migration, nor into the specific social and economic matters that influence particular decisions to commit crimes at given locations within urban areas, but instead to ask whether the gross demographic phenomenon itself--the sheer flow of crime within the city--conforms to known rules describing other forms of demographic gravitation.

The most successful models employed in social research to study demographic flows are based on the physical laws of gravity. By analogy to the rules of Newtonian mechanics, gravity models postulate a gross "force of attraction" operating along a straight line between two points; the magnitude of this attraction, as seen in the exchange of population, is directly proportional to the product of their respective populations, P_1 and P_2, and inversely proportional to the distance between them, D_{12}. The form of the resulting predictor, P_1P_2/D_{12}, thus equates population with the classical notion of mass. Despite a widely appreciated theoretical ambiguity--a variety of mathematical assumptions are compatible with the same models (cf. Ginsberg, 1971)--efforts to fit these models to hitherto unexamined flows will continue to attract attention. Whether crime "behaves" like the migration of families in Cleveland (Stouffer, 1959), the flow of Federal Reserve funds among regional cities (Duncan, et al., 1959), tides of pedestrian or airline traffic, the stream of telephone conversations between cities in Southern Michigan (Carrole, 1955), or the exchange of marriages among areas of Philadelphia (Bossard, 1940) is, of course, a question whose answer will have about the same "surprise value" as another paper relating suicide to anomie. After all that has fallen under these models before, we can only expect crime to appear as another field of steel shavings organized by the same magnets.

It is of interest to recognize about the problem of criminal migration, however, that its spatial anatomy is rather more complex than that attaching to telephone conversations or household migration. Crime often involves more than two (usually three) locations--two origins (the places of residence of offenders and their victims) and a single destination (the location of the crime). The problem here is not that of a gravity field occupied by more than two masses--a matter that resolves itself into vector sums--but that crime often has a location which is the result of the common attraction of two origins and a single destination. Specifically, for a non-negligible fraction of crimes in urban areas, the destinations of criminal migration are not locations with large residential

populations (e.g., shopping centers, central business districts, etc.), but rather, disproportionate concentrations of "opportunities" for crime (commercial density, considerable daytime use, etc.). Thus, the equating of mass and population in the usual formulations of the gravity rule is, for the case of crime, an inadequate operational convention, one that we may anticipate will require revision. Students of migration will recognize in this difficulty a problem somewhat analogous to that treated by Stouffer (1959; cf. Galle and Taeuber, 1966), who sought to formulate a migration rule having the mathematical form of gravity models but including variables other than population and distance as components of the attraction between two points.

In the present paper, our objective is to examine the adaptability of classic gravity rules to the problem of intra-urban criminal migration. The question we shall deal with is whether any of several of these so-called "inverse distance relations" are adequate to describing crime flows in a gravity field encompassing an origin at the offender's residence and a destination at the location of his crime. In the course of fitting a number of models to the data at hand, comparisons will be presented based on the most successful formulations employed in previous research, with the intention of reporting the best fit achieved empirically by introducing specific changes demanded by the nature of crime.

INTERVENING OPPORTUNITIES AND INVERSE DISTANCE VARIATIONS

Our problem finds a starting point in a contrast between the two most widely used and discussed migration rules found in the sociological literature. These are the simple "gravity model" generalized by J. Q. Stewart (1941; 1948) and G. Zipf (1946) from work originating in the 19th century

(Carey, 1858-1859; Ravenstein, 1885), and the model of so-called "intervening opportunities" proposed by Stouffer (1959) as a more suitable representation of the migration process. (A review of the important early work on "gravity" and "potential" models, complete up to about 1956, is contained in Carrothers [1956].)

The general form of the simple gravity model states a relationship between a migratory flow connecting two areal units, their respective populations, and the distance separating them. The model is thus usually written as $M_{ij} = KP_iP_j/D_{ij}$, where K is a constant of proportionality, P_i and P_j are the population sizes of an origin i and a destination j, and D_{ij} is the distance separating the two locations. Controversy surrounds whether this function should be written with variable exponents for population and distance, though in our view there seems to be no a priori reason to assume a fixed power of unity for either. Thus, a number of variations on the basic gravity model have been written allowing an empirical determination to be made of exponents for population and distance. For instance, Duncan, et al. (1959) examined the following two generalizations of the basic model: Model I, $M_{ij} = A(P_iP_j/D_{ij})^b$, and Model II, $M_{ij}/P_iP_j = C/D_{ij}{}^d$, both of which are subject to logarithmic transformations into simple linear equations that may be estimated with OLS regression techniques. We shall have occasion to return to these models later.

Stouffer's theory of intervening opportunities was put forward as an alternative to the inverse distance variations represented in the gravity models applied to the movement of people. Stouffer believed the gravity models to be oversimplified, and proposed the concept of "intervening opportunities" as a substitute for the distance variable. The classic gravity

models were flawed, in his view, by discontinuities in the distance variable and, in any case, involved concepts operating merely as surrogates for the things that really moved individuals, namely, opportunities. Thus, in the second version of his theory, worked out in the early 1950's, he proposed a model which suggested that the population movement between two areas was directly proportional to the opportunities at the destination and inversely proportional to the product of two terms he called "intervening opportunities" and "competing migrants." Intervening opportunities were defined as the opportunities an individual would have to pass up to get to any point j, at distance D_{ij} from his origin, i.e., all of the opportunities within the circle with center i and radius equal to D_{ij}. Competing migrants he defined as the number of people moving to j from all points within the circle having j as its center and radius equal to D_{ij}. Letting opportunities be proportional to the product of the migrant population from i and to j, the function can be written as $M_{ij} = k\, X_m/(X_bX_c)^b$, where $X_m = M_i.M._j$. X_b and X_c are intervening opportunities and competing migrants respectively, their product raised as the power b, determined empirically. X_m, opportunities, is decomposed as the product of the outmigrants from i, $M_i.$, and the inmigrants to j, $M._j$. K is a constant of proportionality.

These two models differ basically only in the terms they would include in their denominators. It has remained an empirical question whether, in fact, distance may be replaced by intervening opportunities, or whether intervening opportunities is the refinement over the distance variable Stouffer believed it to be. Both variables have been of significant value as predictors, though doubts still attach to the opportunities formulation among students of migration (e.g. Anderson, 1955; Ikle, 1955 etc.).

DATA AND OPERATIONAL CONSIDERATIONS

Given the crudeness of the rationales behind each of these models--both, as Ikle (1955) has argued, should be regarded as unrealistic in some of their assumptions--we should not feel confident that mere comparison of the two constitutes in any sense a "test" of one against the other or each by itself. Nonetheless, comparison does suggest points at which the models fail to yield comparable results, if any, and also indicates where improvements can be made in their formulation. If on no grounds other than those of empirical "fit" to the sample of data at hand, therefore, some judgments may be reached about these models. Accordingly, we shall offer below a variety of formulations of these basic functions, constructed on the basis of our own reasoning about crime flows in urban areas.

Source of Data

Crime data in a form amenable to gravitational analysis will probably become increasingly common in the United States in the next few years, primarily because of the steady introduction of computerized criminal information systems in urban police departments. Up to now, the data have been available but inaccessible for technical or policy reasons. The data we analyze in this paper have been extracted from the records of the Rochester (New York) Police Department, and constitute a complete file of arrests and reported offenses, geocoded by census tract, for the year 1972. Disregarding for the moment the sources of error and bias in these data, it is quite clear that they constitute the most complete record of crime, available and suitable for our purposes, for the Rochester area. (The city itself has a population of about 300,000, the metropolitan area about 800,000.)

The analysis below is based on all offenses that led to arrest. Thus, we have followed the procedure of pairing the addresses of the locations of crimes with the addresses of the offenders arrested for those crimes. This procedure produces a matrix (M_{ij}) of flows within and between the 91 census tracts into which those addresses can be geocoded. Rather than computing the actual distance of each move, the centoid of each census tract was estimated on a grid system and distances between it and all other tracts subsequently calculated. The distance associated with a given move was this inter-centroid estimate. The mean d_{ij} for the total enumeration of moves was 5,282 feet, the standard deviation 7,692 feet, indicating that the majority of all migration of crime is within two miles of the offenders' residences.

Our analysis does not ignore the main diagonal of the flow matrix, as is conventional. Because of irregularities in population density, the tracts vary considerably in size, a fact which would bias estimates of the effects of distance and population if intra-tract crime were to be ignored. Instead, for all M_{ij} where $i = j$ we have assigned distances equal to one-half of the least inter-centroid distance between a tract of origin and all tracts adjacent to it.[1] And finally, since each move was geocoded into census tracts, characteristics from the 1970 Census were available to the analysis, and served as the source of population estimates.

Definitions of Crime Variables

Other than distance and population, the principal variables operating in our models are constructed from records of all known arrests (A) and offenses (O) in Rochester during 1972.

1. Migration flow, M_{ij}. The flow of crime is defined as the total

number of arrests in i for offenses in j.

2. Opportunities, X_m. The opportunities variable has been operationalized in a variety of ways, as discussed below. In Stouffer's formulation, opportunities were equated with the product $M_{i.}M_{.j}$, the migratory stream out of i to all j and the migratory stream into j from all i. We have translated this expression into various terms based on the number of arrests in i and j and the number of offenses in i and j, and have also considered an unusual operationalization based on the ratio of in-migrating to out-migrating arrests in j. Because of the ambiguous nature of these expressions, we leave their discussion to the subsequent analysis.

3. Intervening opportunities, X_b. Following Stouffer's original definitions as precisely as possible, intervening opportunities were defined as the number of arrests in i, for all crimes committed by residents of i in tracts falling within a circle described by the radius D_{ij} with a center at i (including M_{ij}). In this procedure, tracts with inter-centroid distances from i less than or equal to D_{ij} were considered to fall within the circle centered at i.

4. Competing criminals, X_c. Again, following Stouffer as closely as possible, a second circle was defined with radius D_{ij}, but in this case centered at j. Competing criminals were then defined as those individuals who were arrested for crimes committed in j but whose addresses fell within any tracts encompassed by the circle centered at j.

ANALYSIS

Prior to fitting the models discussed above, plots were made of the relationships between M_{ij} and each of the predictors to determine whether

transformations would be necessary to eliminate any obvious nonlinearity. Special interest attached here to distance, since it had been the subject of a controversy over exponents in previous research. While no obvious nonlinearity emerged from this effort, separate regressions of M_{ij} on $1/D_{ij}{}^{\alpha}$ were run, with α = 1, 2, and 3. The coefficients of multiple determination of these three equations varied only .0001 percent, the distance variable by itself never explaining more than 4 percent of the variance in M_{ij}. Since there is no theoretical basis for perferring one of these exponents to another, it would seem judicious to prefer the form with α = 1 or to allow the exponent to be determined empirically. The criterion we shall use, however, is the improvement in the fit of the model to the data occasioned by the introduction of an exponent other than unity.

In fitting the gravity models, therefore, we began with the classic form RP_iP_j/D_{ij} and moved on to Models I and II of Duncan, et al. The equation for the classic model produced an R^2 equalling .336, while the R^2 for the Duncan et al. Models I and II were 0.0205 and .2153 respectively. Since, as Duncan et al. point out, it is quite clear that these models are incomplete in failing to include other determinants of the M_{ij} flows, it should be remembered that empirical determination of an exponent for distance will change with the addition of further variables to the equation. In the case of Model I, A = -12.33 and b = .2378 (t = 13.17), while for Model II, C = 1.13 x 10^{-5} and d = -1.25 (t = 47.64). On the basis of fit to the data, however, the classic gravity model is to be preferred.

To determine whether the relatively poor fit of all models tested might have been due to a lack of homogeneity in the effects of distance over its range, the same regressions were run with the restriction that $i \neq j$. Though much economic research on migration has suggested that the "costs" of short distances are trivial and may be recouped by individuals in very short periods (cf. Schwartz, 1974), our results when ignoring the shortest moves reversed rather than improved the fit.

In examining the "intervening opportunities" model, it was not immediately clear how to translate $M_i.M._j$ into the available flow variables, since we were in possession of information on both known offenses (O) and arrests (A) in i and j. We thus formulated a number of models that seemed to be susceptible to interpretation, including A_iA_j/X_bX_c, O_iO_j/X_bX_c and A_iO_j/X_bX_c. Of these three expressions, the one closest to Stouffer's operational judgment involves the product A_iO_j in the numerator. None of the three models successfully predicted M_{ij}, however. The best fit, a disappointing $R^2 = .0142$, was for the A_iO_j/X_bX_c function.

Though these models faithfully represented Stouffer's thinking in terms of the flow of crime and failed to predict with satisfactory accuracy, several other formulations of the intervening opportunities model were examined. Redefining the opportunities variable as the ratio of in-migrating to out-migrating arrests in j, the model was rerun with even worse results (R^2=.0003). Regressing M_{ij} on intervening opportunities itself, $1/X_b$, also produced a poor fit (R^2 = .0034). For every modification of the model examined, in fact, there were consistently disappointing results.

By itself, we felt the original opportunities variable, A_iO_j, seemed to promise a more accurate representation of the allotment of arrests in i among crimes in j. Because it corresponded more closely to the hydraulics involved, we examined revisions of the original gravity model, replacing P_iP_j with A_iO_j. Similarly, we paired the remaining terms of the original

models, substituting X_bX_c for D_{ij} in the original gravity model, to form the prediction, P_iP_j/X_bX_c. As before, the function including the intervening opportunities variable failed to produce an acceptable fit (R^2 = 0.010). But in the case of the modified gravity formulation, A_iO_j/D_{ij}, we achieved a surprisingly satisfying result. In this case, R^2 = .65. The level of this fit more closely parallels the success of the classic gravity model in representing the flow of things other than crime. In this case, the full equation was:

$$\underset{(t\text{ - ratio})}{M_{ij}} = \underset{(5.41)}{-.132851} + \underset{(123.53)}{1.256614} \left(\frac{A_iO_j}{D_{ij}}\right)$$

Why A_iO_j/D_{ij} forms a more satisfying predictor than any other expression we examined is a question open to speculation. The reader will have noted, of course, that the dependent variable M_{ij} -- the number of crimes moving from _i_ to _j_ -- may more reasonably be thought of as constrained by the distribution of offenses among the j than by the distribution of arrests among j, and is defined in terms of the number of arrests in _i_. Thus, of the various predictors examined, A_iO_j/D_{ij} is more closely proportional to the "chances" of arrest in i for an offense in j, given the formulation of the dependent variable, than the other expressions. This does not deny the significance of distance, since by itself A_iO_j predicts M_{ij} very poorly. It is neither of these terms separately but their interaction that is the successful predictor.

In addition, it worth noting that some expression based on the distributions of arrests and offenses among areas was expected to serve us better in modelling the gravitation of crime. The classic formulation of the gravity model, in which population is equated to mass, we doubted would adequately predict crime flows, since much crime occurs at locations with small resident populations. It was reasonable to anticipate, therefore, that a surrogate variable would be required to replace the product of the populations at i and j, and that this surrogate should be some index of the opportunities or chances for crime at various destinations. We thus find the most successful predictor of M_{ij} to be a composite gravity rule based on a numerator which is the equivalent of Stouffer's measure of opportunities and a denominator formed simply by approximations of the linear distance between locations.

FURTHER MODIFICATIONS

1. Numerous previous students of flows among areas have noted that refinements in the elementary gravity model should be possible by reformulating the "force of attraction" variable, i.e., the product P_iP_j. It has been suggested that suitable weights, based on the wealth or income of an area, might be devised, as well as other weights based on unemployment, racial composition, and age composition. We reasoned similarly in the case of crime, and accordingly attempted to improve the fit achieved above by various weightings. The alternative to the formulation, we felt, would be to write a system of simultaneous equations, including one in which we would predict A_iO_j.

The weights examined included i) wealth (the product of the number of families in a tract and the median family income in that tract), ii) income (median income in tract), iii) percent unemployed in i (j), iv) percent Black in i (j), v) percent males aged 15-24 in i (j). By themselves, none of these variables turned out to be successful predictors of M_{ij}. Not surprisingly, therefore, when they were treated as weighting factors in a

variety of formulations, they failed to yield significant improvements in the fit of the basic model. The failure of these weights does not rule out the usefulness of others in future research.

2. Bogue (1959), recognizing the difficulty of improving the study of migration based on the classic gravity model, has proposed turning toward a formulation he calls "relative stream velocity." This is a measure that ignores direction of movement, and expresses a relationship between the total number of migrants in a stream, the population at i and j, and the total population of all potential destinations (including i = i). In Bogue's formulation, V (velocity) = $(M_{ij}/P_i \div P_j/P_t)$ 100 (where P_t = total population of all potential j). This notion, treated as a dependent variable, may be rewritten in terms of our crime variables, as:

$$V = \frac{M_{ijk} \Sigma O_k}{A_i O_j}$$

Bogue suggests regressing this quantity on a variety of variables, including distance. For convenience, a multiplicative function, incorporating variable exponents, was written, that included as predictors distance, intervening opportunities, competing criminals, opportunities, the ratio of income in i to income in j, and unemployment in j.

$$V = AD^{\alpha}X_b^{\beta}X_c^{\gamma}X_m^{\delta}(I_i/I_j)^{\xi}U_j^{\lambda}$$

where I is income, U is unemployment, and the other variables are defined as before. The results of this regression, however, were disappointing, the multiple R^2 equalling only .103. When simple linear regressions of V on each of these predictors were run separately, no predictor "explained" more than two percent of the variance in V.

SOURCES OF ERROR

While these results suggest rather forcibly that the "intervening opportunities" model, as we have operationalized it at least, fares poorly in comparison with the gravity model in predicting the flow of crime in urban areas, there are numerous reasons to regard this comparison as subject to improvement and therefore inconclusive.

1) Operational questions attach to the definition both of "opportunities" and "intervening opportunities" in the case of crime. As Anderson (1955) has pointed out, Stouffer's operationalizations even for the treatment of migration were crude at best. Though Lazarsfeld (1962) has suggested the desirability of fitting the Stouffer model to crime, neither he nor other admirers of Stouffer's original formulations have considered how operationalizing its constructs may require redefinitions in approaching each new gravitational phenomenon. (This matter also concerned Galle and Taeuber, 1966).

The gravitation of crimes considered here is surely a matter which reason and previous research on crime suggest might better be predicted with measures of opportunity defined in terms of such economic indicators as commercial density. These operational extensions of the work reported here await further research, however.

2) Numerous sources of error in our data and in our measurement procedures would also suggest caution to the researcher. We shall merely enumerate the most obvious.

i) Distance. Distance measures used in our calculations are inexact, since we measured distance between the centroids of tracts. This measurement is further compromised because the tract centroids were estimated by averaging the latitudes and longitudes of block grouping within tracts.

ii) Incomplete Criminal Statistics. Our analysis is based not only on an incomplete enumeration of offenses -- only those crimes known to the police -- but is also tied only to those offenses for which an arrest was made. Bias is thus introduced into our data both by the selectivity of reporting crimes -- underestimating, probably, the amount of crime at shorter distances -- and by whatever differentials attach to the probability of police apprehension (clearance of crime) correlated with distance. There are several reasonable but complex arguments which may be made about the direction in these biases, but they are, at this point, based on mere speculation on our part or on the part of the police.

Since we have been dealing with a finite reporting area, our data are also flawed by their failure to include an enumeration of crimes -- at least arrests -- crossing political jurisdictions. The number of extra-urban arrests for intra-urban offenses, and vice versa, is unknown to us. This seems a more serious disturbance for tracts at the periphery of the city, but there is no way to check this hunch.

In addition to these problems, we have arbitrarily included multiple arrests for the same crime into our analysis. This decision was based on our interpretation of the operational requirements of the gravity models, but the decision nonetheless complicates an interpretation of biases attaching to the apprehension of offenders. Beyond this, moreover, are the additional biases introduced by police errors in making arrests. Our data included all arrests, regardless of the disposition reach in court (which is also hardly a criterion of the validity of arrests).

iii) Demographic Change. Measurements of population, income, and other weighting characteristics were based on 1970 census reports, whereas arrest and offense data are for 1972. Though this is a short enough gap not to have seriously distorted our analysis, there is a strong impression that the census information is now seriously out-dated for perhaps ten percent of the tracts in the city. Many of these tracts, unfortunately, are in zones of transition or urban renewal. It is the impression of police officials that the crime statistics for some of these tracts have completely altered in the period since the last census. The impact of this change on the analysis is probably minor, however.

CONCLUSIONS

The results of the research reported above make it clear that crime, like pedestrian traffic, shopping, telephone conversations, migration, and a host of other phenomena of flow, is subject to the general class of inverse distance variations formulated as gravity laws. Though we have not rejected the intervening opportunities "refinements" proposed by Stouffer, we have found no evidence to support them either.

Future research on the migration of crime should seek to go beyond the preliminary work reported here. There are a number of directions which seem most promising. 1) Further efforts should be made to operationalize the opportunities concept in connection with the dynamics of crime; 2) The present work on gravitation of crime should be extended into crime-specific comparisons; 3) Following Bogue's suggestion, the gravity function should be reformulated in terms that make it a dependent variable, including efforts to incorporate the inverse effects of distance into a general structural equation model of crime; and 4) The effects of distance must not be assumed to be homogeneous for entire populations but to vary

as a function of other considerations, such as education and age. Thus, efforts should be made to "interpret" the effects of distance sociologically.

FOOTNOTES

[1] This is a convention in keeping with research on the spacing and density of population, where it has been useful as part of the technique of linear distance mapping (cf. the discussion in Duncan, 1957). Because of the irregular shapes and varying areas of many tracts, this convention recommended itself as avoiding some of the more unrealistic assumptions and biases inherent in other measures, e.g., the mean linear distance to all adjacent tracts, etc.

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