Inverse trigonometric functions (Sect. 7.6)

Today: Definitions and properties.

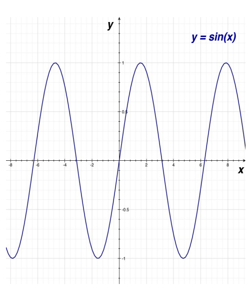
- Domains restrictions and inverse trigs.
- Evaluating inverse trigs at simple values.
- Few identities for inverse trigs.

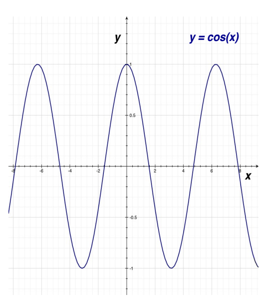
Next class: Derivatives and integrals.

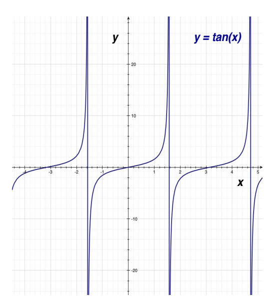
- Derivatives.
- Anti-derivatives.
- Usual substitutions

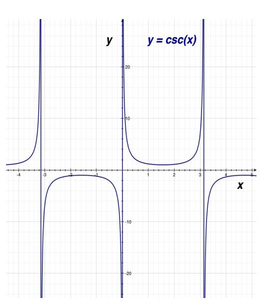
Domains restrictions and inverse trigs

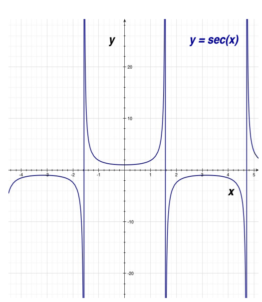
Remark: The trigonometric functions defined on their biggest domain are not invertible.

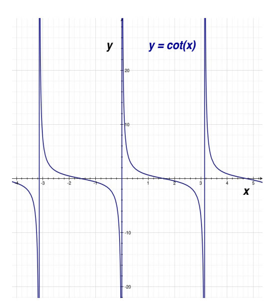






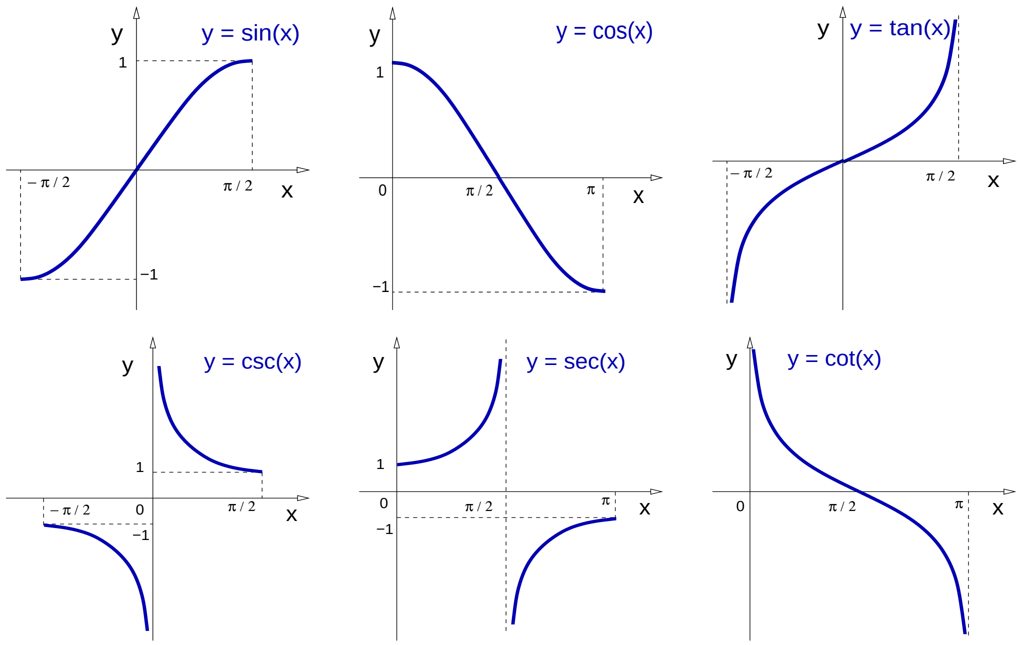






Domains restrictions and inverse trigs

Remark: On certain domains the trigonometric functions are invertible.



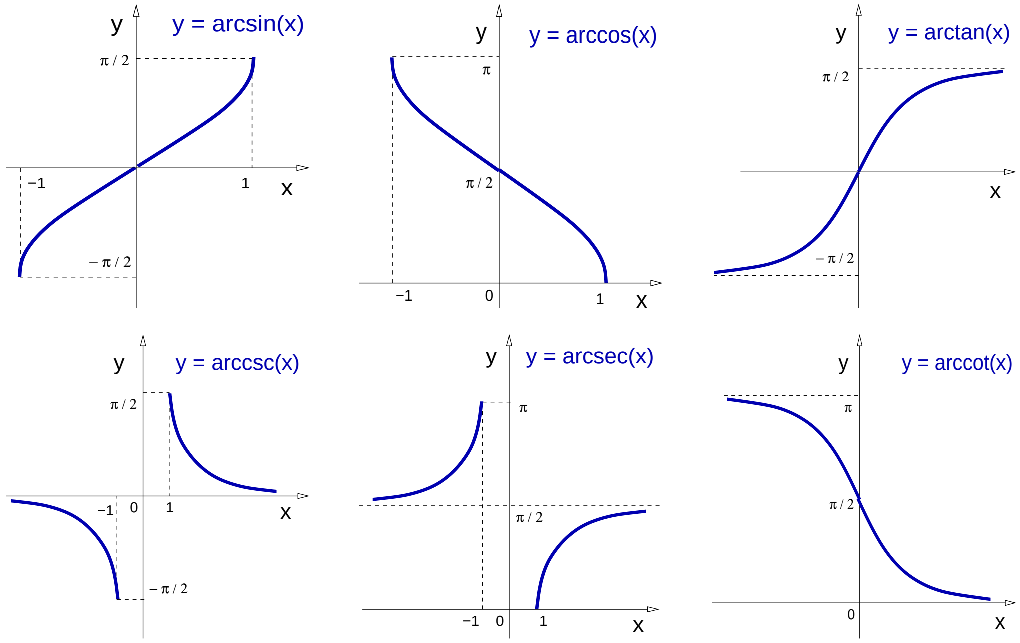
Domains restrictions and inverse trigs

Definition (arc{trig} is the inverse of {trig})

- The function arcsin : $[-1, 1] \to [-\pi/2, \pi/2]$ is the inverse of sin : $[-\pi/2, \pi/2] \to [-1, 1]$.
- The function arccos : $[-1, 1] \to [0, \pi]$ is the inverse of cos : $[0, \pi] \to [-1, 1]$.
- The function arctan : $(-\infty, \infty) \to [-\pi/2, \pi/2]$ is the inverse of tan : $[-\pi/2, \pi/2] \to (-\infty, \infty)$.
- The function arccsc : $(-\infty, -1] \cup [1, \infty) \to [-\pi/2, \pi/2] - \{0\}$ is the inverse of csc : $[-\pi/2, \pi/2] - \{0\} \to (-\infty, -1] \cup [1, \infty)$.
- The function arcsec : $(-\infty, -1] \cup [1, \infty) \to [0, \pi] - \{\pi/2\}$ is the inverse of sec : $[0, \pi] - \{\pi/2\} \to (-\infty, -1] \cup [1, \infty)$.
- The function arccot : $(-\infty, \infty) \to [0, \pi]$ is the inverse of cot : $[0, \pi] \to (-\infty, \infty)$.

Domains restrictions and inverse trigs

Remark: The graph of the inverse function is a reflection of the original function graph about the $y = x$ axis.



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Evaluating inverse trigs at simple values

Notation: In the literature is common the notation $\sin^{-1} = \arcsin$, and similar for the rest of the trigonometric functions.

Do not confuse $\dfrac{1}{\sin(x)} \neq \sin^{-1}(x) = \arcsin(x)$.

Remark: sin, cos have simple values at particular angles.

θ	$\sin(\theta)$	$\cos(\theta)$
0	0	1
$\pi/6$	$1/2$	$\sqrt{3}/2$
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$
$\pi/3$	$\sqrt{3}/2$	$1/2$
$\pi/2$	1	0

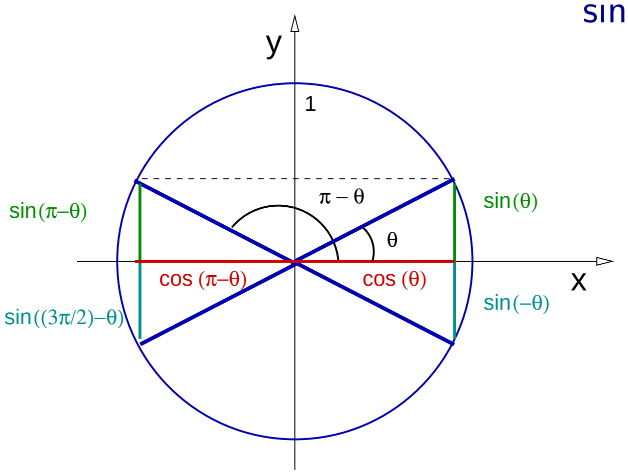
Evaluating inverse trigs at simple values

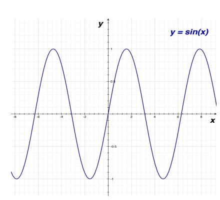
Remark: the symmetry properties of the sine and cosine can be used to evaluate them at a bigger set of angles.

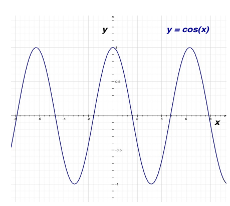
$$\sin(-x) = -\sin(x), \qquad \sin(\pi - x) = \sin(x).$$

$$\cos(-x) = \cos(x), \qquad \cos(\pi - x) = -\cos(x).$$

$$\sin\left(\theta + \frac{\pi}{2}\right) = \cos(\theta).$$







Evaluating inverse trigs at simple values

Example

Find the values: $\tan(\pi/3)$, $\sec(2\pi/3)$, $\csc(-\pi/6)$.

Solution:

$$\tan\left(\frac{\pi}{3}\right) = \frac{\sin(\frac{\pi}{3})}{\cos(\frac{\pi}{3})} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} \quad \Rightarrow \quad \tan\left(\frac{\pi}{3}\right) = \sqrt{3}.$$

$$\sec\left(\frac{2\pi}{3}\right) = \frac{1}{\cos(\frac{2\pi}{3})} = \frac{1}{-\cos(\frac{\pi}{3})} = \frac{1}{-\frac{1}{2}} \quad \Rightarrow \quad \sec\left(\frac{2\pi}{3}\right) = -2.$$

$$\csc\left(-\frac{\pi}{6}\right) = \frac{1}{\sin(-\frac{\pi}{6})} = \frac{1}{-\sin(\frac{\pi}{6})} = \frac{1}{-\frac{1}{2}} \quad \Rightarrow \quad \csc\left(-\frac{\pi}{3}\right) = -2.$$

$\triangleleft$

Evaluating inverse trigs at simple values

Example

Find the values: $\arcsin(\sqrt{3}/2)$, $\arccos(1/\sqrt{2})$.

Solution:

$$\arcsin\left(\frac{\sqrt{3}}{2}\right) = \theta \quad \Leftrightarrow \quad \sin(\theta) = \frac{\sqrt{3}}{2}, \quad \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right].$$

We conclude that $\arcsin\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$.

$$\arccos\left(\frac{1}{\sqrt{2}}\right) = \theta \quad \Leftrightarrow \quad \cos(\theta) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}, \quad \theta \in [0, \pi].$$

We conclude that $\arccos\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$. $\triangleleft$

Evaluating inverse trigs at simple values

Example

Given that $x = \text{arcsec}(-5\sqrt{5})$, find $\sin(x)$.

Solution: Recall:

$$\text{arcsec}(-5\sqrt{5}) = x \quad \Leftrightarrow \quad \sec(x) = -5\sqrt{5}$$

$$\frac{1}{\cos(x)} = -5\sqrt{5}. \quad \Leftrightarrow \quad \cos(x) = -\frac{1}{5\sqrt{5}}.$$

How do we find $\sin(x)$? Use trigonometric identities:

$$\sin(x) = \pm\sqrt{1 - \cos^2(x)} = \pm\sqrt{1 - \frac{1}{125}} \ \Rightarrow \ \sin(x) = \pm\sqrt{\frac{124}{125}}.$$

$\triangleleft$

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Today: Definitions and properties.

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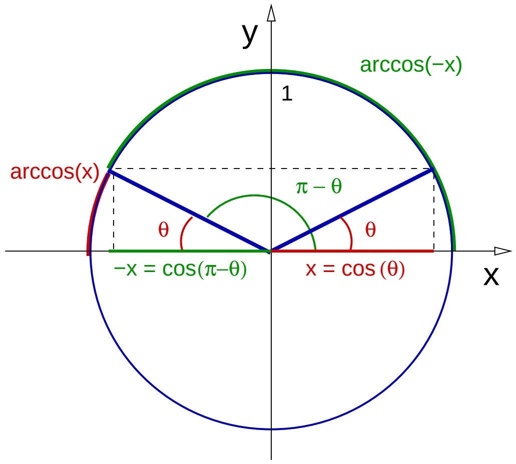
Few identities for inverse trigs

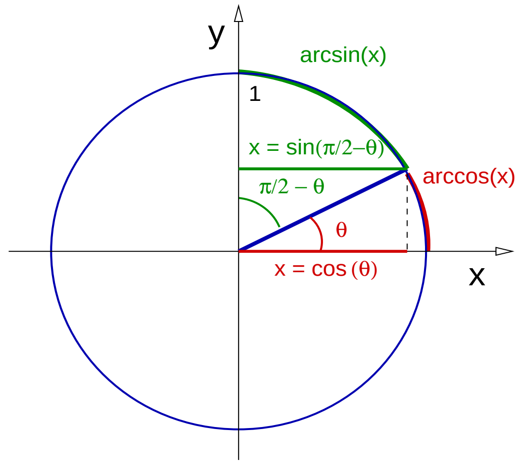
Theorem

For all $x \in [-1, 1]$ the following identities hold,

$$\arccos(x) + \arccos(-x) = \pi, \qquad \arccos(x) + \arcsin(x) = \frac{\pi}{2}.$$

Proof:





Few identities for inverse trigs

Theorem

For all $x \in [-1, 1]$ the following identities hold,

$$\arcsin(-x) = -\arcsin(x),$$
$$\arctan(-x) = -\arctan(x),$$
$$\text{arccsc}(-x) = -\text{arccsc}(x).$$

Proof: