

Investigative Task – Geometry (Stage 3)

Teachers: Background information

Students have been:

- learning how to measure angles using a protractor
- viewing and comparing maps and 2D images of different scales
- converting measurements on maps using scale

Students are given two aerial images of the same suburban area [Figures 1 and 2]. The photo was taken in 1943; the map in 2016. The old photo has a scale of 1:2 000. The new map has a scale of 1: 1 000. Both images are marked with a scale, and contain grid markings on the outer edges. Some landmarks in the old photo are visible in the new map, such as buildings A, B and C. In the old photo is a building marked D. It is not on the new map because it was demolished in 1960. Students must use both images to identify and explain the exact location of Building D on the new map. Students have multiple copies of both images for rough working. They work in mixed-ability groups of three to solve the problem.



Figure 1

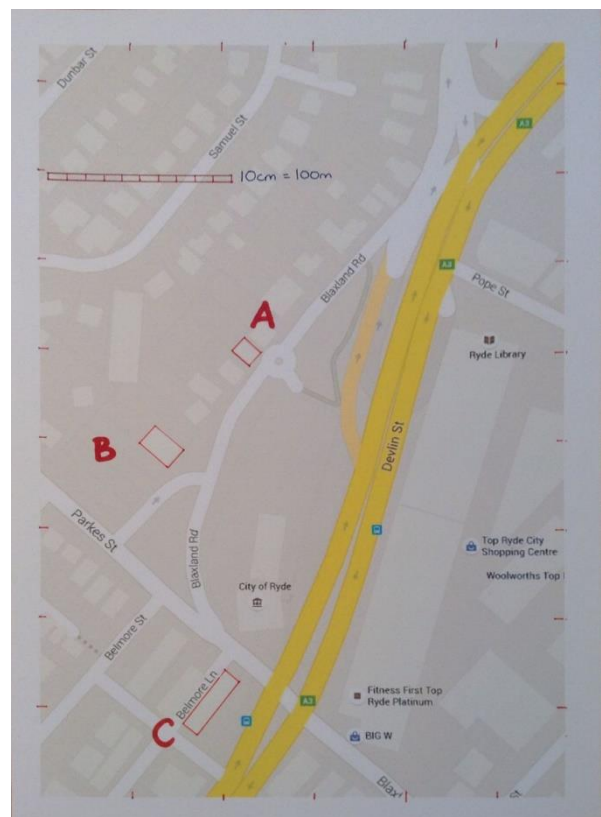


Figure 2

Students: Investigative Task

You are the Town Planner for the City of Ryde. You have two aerial images showing the same part of the City. The old photo was taken in 1943. The new map was created in 2016. You will see that Buildings labelled A, B and C are in both images. Building D, marked on the old photo, was demolished in 1960 and is not on the new map. The building was an important landmark at the time, and now the City Council wants to rebuild it on the same spot. Your job, as Town Planner, is to use the information available in the two aerial images to identify the exact location of Building D.

You will need to:

- Explain how to get to Building D from Buildings A, B and C. Your explanation must be clear and precise enough for someone else to follow. You will need to provide the distance between the buildings as well as the angles. Remember, you need two rays to measure an angle. How will you do this?
- Measure the length of the four outside walls of Building D. Pay careful attention to scale.
- Draw a scaled 'bird's eye' view of Building D, then cut it out and glue it accurately onto the new map. This needs to show its exact location. What scale will you use? How will you know how to position your drawing at the correct angle?
- Determine if it would be possible to reconstruct Building D on its old site. Why or why not?
- Work in groups of three.
- Consider the *Starting question* and *Thinking questions*.
- Record thoughts, questions, findings and progress in your Maths journal.
- Explain and justify your findings to the class at the conclusion of the investigation.

Starting question

- What are the first three things you should do to get started?

Thinking questions

- How might the grid markings and the scale on each image help you?

- When you give directions to Building D, will you give directions to the centre of the building, the northern corner of the building, or some other spot? Why should you be very specific about your start and end points?
- How can the protractor help you?
- When you have measured the three distances and the three angles, how will you know that they are all correct?

Australian Curriculum links (Year 5)

- Apply the enlargement transformation to familiar two dimensional shapes and explore the properties of the resulting image compared with the original (ACMMG115) (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2016a)
- Estimate, measure and compare angles using degrees. Construct angles using a protractor (ACMMG112) (ACARA, 2016a)
- Choose appropriate units of measurement for length, area, volume, capacity and mass (ACMMG108) (ACARA, 2016a)
- Use a grid reference system to describe locations. Describe routes using landmarks and directional language (ACMMG113) (ACARA, 2016a)

First Steps in Mathematics Key Understandings

- Represent transformation 3 (Department of Education WA [DEWA], 2013a, p. 130)
- Represent location 1 (DEWA, 2013a, p. 12)
- Represent location 2 (DEWA, 2013a, p. 26)
- Represent location 3 (DEWA, 2013a, p. 38)

Student demonstration during the task

- Determine a specific 'start' point on Buildings A, B and C, and a specific 'end' point for Building D
- Measure the three distances using a ruler, convert the measurements to metres, and convert the measurements to the scale on the map
- Discuss and decide upon how to measure the three angles, including the consistent selection of either a vertical or horizontal ray
- Correct placement of the vertex for each angle measurement (Wolfram Research Incorporated, 2016)

- Consistently place the protractor along the horizontal or vertical ray to correctly measure each angle
- Explain the route from the 'start' points at Buildings A, B and C to the 'end' point on Building D, using precise measurements (in appropriate units) and angles
- Measure the perimeter of Building D, convert the measurements to the scale of the map, draw and cut out Building D to its correct scale, and glue it onto the new map to show its precise location
- Observe Building D's location on the new map and decide if it would be possible to reconstruct the building on its former site based on the position of any obstructions such as other buildings or roads
- Keep coherent and accurate notes in their Maths journals throughout the investigation
- Explain and justify their answer

Specific maths outcomes

By the end of this investigative task the students will be able to:

- Accurately convert measurements between two different scales
- Use a protractor to measure angles between objects
- Combine distance and angle measurements to accurately locate objects on a map (DEWA, 2013a, p. 51)
- Use measurement terminology to explain the location of a place on a map in relation to objects on the same map
- Justify their answers (Jones, 2003, p. 93)

Likely range of outcomes

Satisfactory response

- Gives approximate measurements from Buildings A, B and C
- Converts millimetres to metres
- Converts measurements according to scales
- Includes angles for each measurement

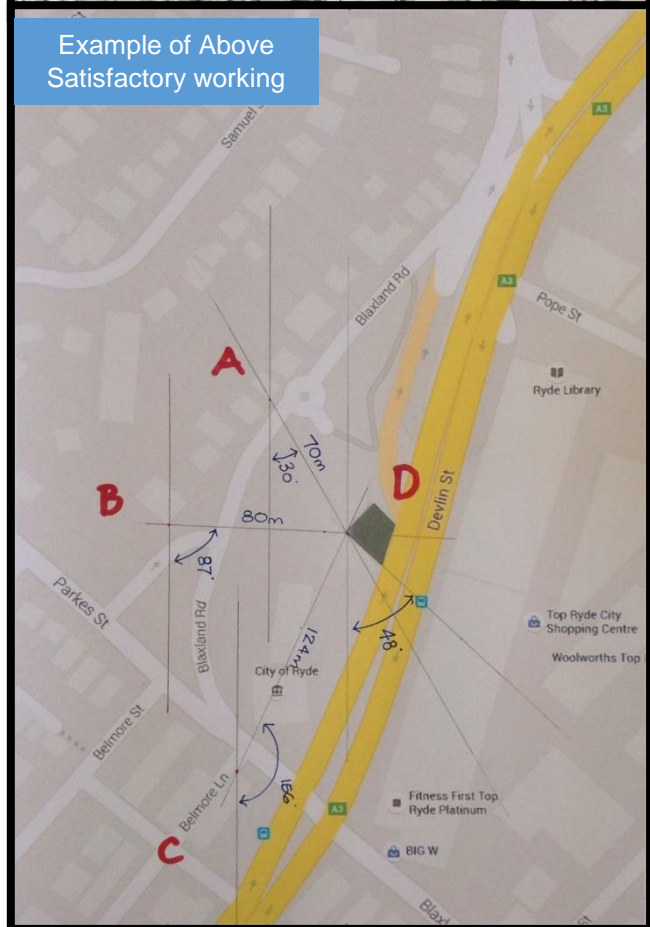
- States start and end points for the three explanations
- Draws Building D to correct scale
- Shows Building D in the correct location
- Makes notes in Maths journal
- Justifies decisions

Above Satisfactory response

- Gives precise measurements from Buildings A, B and C
- Converts millimetres to metres
- Converts measurements according to scales
- Includes precise angles from each starting point
- States start and end points clearly and consistently for the three explanations
- Draws Building D to correct scale
- Shows Building D in the correct location
- Maths journal notes are detailed and coherent
- Findings are justified using notes and measurements

Below Satisfactory response

- Gives imprecise or inaccurate measurements from Buildings A, B and C
- Dimensions incorrectly or inconsistently converted



- Angles inconsistently measured and/or incorrect
- Directions from A, B and/or C lead to different end points
- Start and end points vague or not stated
- Building D is not drawn to scale
- Building D in incorrect location

Correct answer

Buildings A, B and C were all measured for their distance from the western corner of Building D.

The distance from the southern corner of Building A to the end point of Building D is 70 metres, measured at an angle of 30 degrees from vertical.

The distance from the southern corner of Building B to the end point of Building D is 80 metres, measured at an angle of 87 degrees from vertical.

The distance from the northern corner of Building C to the end point of Building D is 124 metres, measured at an angle of 156 degrees from vertical.

The outside walls of Building D were measured on the old photo. The western wall is 9mm, the southern wall is 11 mm, the eastern wall is 10 mm and the northern wall is 7mm. The scale of the old photo is 1cm (10mm) is equal to 20m. This means that 1mm is equal to 2m. The full scale measurements of the outside walls of Building D were calculated. The western wall is 18m, the southern wall is 22m, the eastern wall is 20m, and the northern wall is 14m.

A bird's eye view of Building D was drawn to the scale of the new map. The scale of that image is 1cm is equal to 10m. To position the scale drawing accurately on the new map, the angle of Building D was measured on the old photo. The southern wall of the building was measured as 48 degrees from horizontal. This angle was transferred to the 'end point' on the new map, and the scale drawing was glued in place.

Building D can be reconstructed on the old site because the land is free of buildings, roads or other structures.

Misconceptions and potential points of difficulty

- Interpreting the scale on each image, and making allowances for the different scales (Reys et al., 2012, p. 312).
- Converting measurements from millimetres to metres (Reys et al., 2012, p. 420).

- Using consistent start and end points for the three measurements. This could mean Building D is placed incorrectly on the map.
- Using a protractor (Booker, Bond, Sparrow & Swan, 2014, p. 431) to accurately determine angles from each building, particularly deciding upon the position of the second ray against which to measure the angle.
- If students do not measure angles and distances successfully on the old photo, this might result in multiple 'end points' when drawn onto the new map.
- Drawing Building D to the correct scale. If students use the photo scale, Building D will be too small for the map and be in an inaccurate position.