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Is Piketty's "Second Law of Capitalism" Fundamental?

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I. Introduction

Thomas Piketty's recent book *Capital in the Twenty-First Century* (2014) is a timely and important contribution that turns our attention to striking long-run trends in economic inequality. A large part of the book is thus a documentation of historical data, going further back in time, and focusing more on the very richest in society, than have most existing economic studies. This work is bound to remain influential.

A central theme in the book also goes beyond mere documentation: as the title of the book suggests, it makes predictions about the future. Here, Piketty argues forcefully that future declines in economic growth—stemming from slowdowns in technology or drops in population growth—will likely lead to dramatic concentrations of economic and political power through the accumulation of capital (or wealth) by the very richest. These predictions are the subject of the present note.

Piketty advances two main theories in the book; although they have some overlap, there are very distinct elements to these two theories. The

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first theory is presented in the form of two "fundamental laws of capitalism." These are used for predictions about how an aggregate—the capital-to-output ratio, k/y—will evolve under different growth scenarios. The evolution of this aggregate statistic, Piketty argues, is of importance for inequality because it is closely related—if the return to capital is rather independent of the capital-to-output ratio—to the share of total income paid to the owners of capital, rk/y.[1] The second theory Piketty advances, the "$r > g$ theory," is at its core different in that it speaks directly to inequality. This theory, which is rather mathematical in nature and is developed in detail in Piketty and Zucman (2015), predicts that inequality, appropriately measured, will increase with the difference between the interest rate, r, and the aggregate growth rate of the economy, g.

The point of the present paper is to discuss Piketty's first theory in some detail, in particular his second law. We argue that this law, which embeds a theory of saving, is rather implausible. First, we demonstrate that it implies saving behavior that, as the growth rate falls, requires the aggregate economy to save a higher and higher percentage of GDP. In particular, with zero growth, a possibility that is close to that entertained by Piketty, it implies a 100 percent saving rate. Such behavior is clearly hard to square with any standard theories of how individuals save; these standard theories, moreover, have their roots in an empirical literature studying how individuals actually save. Second, we look at aggregate data to try to compare Piketty's assumption to standard, alternative theories, and we find that the data speak rather clearly against Piketty's theory. Equipped with theories that we find more plausible, we then show that if the rate of economic growth were, say, to fall by half, the capital-to-output ratio would increase only modestly rather than dramatically as the second law would predict.

Piketty's second law says that if the economy keeps the saving rate, s, constant over time, then the capital-to-income ratio k/y must, in the long run, become equal to s/g, where g is the economy's growth rate.[2] In particular, were the economy's growth rate to decline toward zero, the capital-output ratio would rise considerably and in the limit explode.

This argument about the behavior of k/y as growth slows, in its disarming simplicity, does not fully resonate with those of us who have studied basic growth theory based either on the assumption of a constant saving rate—such as in the undergraduate textbook version of Solow's classical model—or on optimizing growth, along the lines of Cass (1965) or its counterpart in modern macroeconomic theory. Why? Because we do not quite recog-

[1] Piketty also argues that because capital income is far more concentrated than labor income, income inequality is likely to increase when k/y rises; see, e.g., the discussion on p. 275 of his book.

[2] It is perhaps relevant to note that the second law does not have any specific connection to capitalism. It is a statement about saving, and saving presumably occurs both in centrally planned economies and in market economies. The first law, in contrast, does make a connection with markets, because it contains a price.

nize the second law, $k/y = s/g$; in particular, we do not recognize the critical role of g. Did we miss something important, even fundamental, that has been right in front of us all along?

Those of you with standard modern training, even at an (advanced) undergraduate level, have probably already noticed the difference between Piketty's equation and the textbook version that we are used to. In the textbook model, the capital-to-income ratio is not s/g but rather $s/(g + \delta)$, where δ is the rate at which capital depreciates. With the textbook formula, growth approaching zero would increase the capital-output ratio much more modestly; when growth falls all the way to zero, the denominator would not go to zero but instead would go from, say, 0.08—with g around 0.03 and $\delta = 0.05$ as reasonable estimates—to 0.05.[3]

As it turns out, however, the two formulas are not inconsistent because Piketty defines his variables, such as income, y, not as the *gross* income (i.e., GDP) that appears in the textbook model but rather as *net* income, that is, income net of depreciation. Similarly, the saving rate that appears in Piketty's second law is not the gross saving rate—gross saving divided by gross income—as in the textbook model but instead the net saving rate: the ratio of net saving (the increase in the capital stock) to net income. On a balanced growth path, with g constant, one can compute the gross or the net saving rate. That is, to describe a given such growth path, one can equivalently use the gross saving rate or a corresponding net saving rate: given a g, they are related to each other via a simple equation. But how, then, is the distinction between gross and net relevant?

The key is that the difference between gross and net is relevant only when one considers a change in a parameter, such as g: it is only then that these formulations are distinct theories. One obtains different predictions about k/y as g changes depending on whether the gross or net saving rate stays constant as g changes. These are thus two theories to be confronted with data and also, possibly, with other theories of saving. The analysis in this paper leads us to conclude that the assumption that the gross saving rate is constant is much to be preferred. The gross saving rate does not, however, appear to be entirely independent of g in the data—s seems to comove positively with g—and such a dependence is instead a natural outcome of standard theories of saving based on optimizing behavior and widely used in macroeconomics.

Piketty's assumption that the net saving rate is constant is actually the same assumption made in the very earliest formulations of the neoclassical growth model, including the formulation by Solow (1956) in his original paper. Interestingly, however, at some point the profession switched from that formulation to one in which the gross saving rate is constant, and today all textbook models of which we are aware use the gross formulation. We have tried to identify the origins of the modern formulation, which criti-

[3] See, e.g., the calibration that Cooley and Prescott (1995) perform.

cally involves explicit treatment of capital depreciation, but we are still unsure of when it appeared. In Solow's (1963) lectures on capital theory and the rate of interest, he does incorporate depreciation explicitly, but we are not sure whether that study was the catalyst.[4] One possibility is that the early work on optimal saving turned attention toward the modern formulation; the formulations in Uzawa (1964) and Cass (1965), for example, both incorporate depreciation in the description of the physical environment within which consumers optimize and have predictions closer in line with the textbook theory.[5]

The paper is organized as follows. In Section II, we present the core distinction between the gross and the net theories of saving and explain how they interrelate. In Section III, we then use each theory to predict the future, on the basis of a falling growth rate. Given the rather dramatic differences in predictions between the two theories, we then attempt to evaluate the theories in Section IV. That section has three parts. In Section IV.A, we show that Piketty's theory generates implausibly high gross saving rates for low growth rates. Section IV.B looks at the predictions coming from the benchmark model used in the empirical microeconomic literature, namely, the setting based on intertemporal utility maximization. In Section IV.C, finally, we look at aggregate data from the United States as well as other countries from the perspective of the textbook Solow theory, Piketty's theory, and that based on optimizing saving. Although our paper can be viewed as a study of different theories of aggregate saving, it is also a comment on Piketty's book, and in Section V, we discuss whether perhaps there could be other interpretations of Piketty's analysis: is our description of the second fundamental theorem here not a fair description of what is in the book? Section VI makes some concluding remarks.

II. The Two Models, Assuming Balanced Growth

The accounting framework is the typical one for a closed economy:

$$c_t + i_t = y_t,$$
$$k_{t+1} = (1 - \delta)k_t + i_t,$$

where c_t, i_t, y_t, and k_t are consumption, (gross) investment, output, and the capital stock, respectively, in period t. Let us now introduce the textbook model of saving, along with Piketty's alternative:

- In the textbook model, $i_t = sy_t$. That is, gross investment is a constant fraction (s) of gross output.

[4] Depreciation plays little to no role in Solow's (1970) lectures on growth theory but does appear explicitly in Uzawa (1961).
[5] But Koopmans (1965) does not.

- In the Piketty model, $k_{t+1} - k_t = i_t - \delta k_t = \tilde{s}(y_t - \delta k_t)$. That is, net investment (or the increase in the capital stock) is a constant fraction ($\tilde{s}$) of net output ($y_t - \delta k_t$).

A neoclassical model—the textbook one or that used by Piketty—also includes a production function with some properties along with specific assumptions on technological change. With appropriate such assumptions, consumption, output, capital, and investment converge to a balanced growth path: all these variables then grow at rate g.[6] We can easily derive the capital-output ratio on such a balanced growth path.

For the textbook model, first, we obtain

$$k_{t+1} = (1 - \delta)k_t + sy_t. \tag{1}$$

Dividing both sides by y_t and assuming that both y and k grow at rate g between t and $t + 1$, we can solve for $k_t/y_t = k_{t+1}/y_{t+1}$ on a balanced growth path:

$$\frac{k_t}{y_t} = \frac{s}{g + \delta}. \tag{2}$$

This is the familiar formula.

For Piketty's model, let us first define net output: $\tilde{y}_t = y_t - \delta k_t$. We then obtain

$$k_{t+1} = k_t + \tilde{s}\tilde{y}_t. \tag{3}$$

Thus, (3) differs from (1) in two ways: the depreciation rate does not appear and output is expressed in net terms. Along a balanced growth path we obtain, after dividing by $\tilde{y}_t$ and again assuming that both $\tilde{y}$ and k grow at the rate g,

$$\frac{k_t}{\tilde{y}_t} = \frac{\tilde{s}}{g}. \tag{4}$$

This is Piketty's second fundamental law of capitalism.

On a given balanced growth path, these two formulations are, in fact, equivalent. In the textbook model the ratio of capital to net output on a balanced growth path is

$$\frac{k}{y - \delta k} = \frac{1}{(g + \delta)/s - \delta} = \frac{s}{g + \delta(1 - s)}. \tag{5}$$

[6] For example, with a production function that exhibits constant returns, one could assume labor-augmenting technological progress at a fixed rate as well as the population growth rate at a fixed rate; it is straightforward to show then that on a balanced growth path, all variables will grow at the sum of these two rates.

Similarly, one can show that in the textbook model, the (implied) net saving rate on a balanced growth path is

$$\tilde{s} = \frac{sg}{g + \delta(1 - s)}. \tag{6}$$

In other words, in the textbook model, $k/\tilde{y} = \tilde{s}/g$ on a balanced growth path, as in Piketty's second law. Thus, for any given g, one can think of the observed ratio of capital to output (or capital to net output) as arising either from a gross saving rate s or from a corresponding net saving rate $\tilde{s}$ given by equation (6).

III. Using the Models to Predict the Future

Up until this point, thus, the two frameworks for saving look entirely consistent with each other. But let us now interpret the two frameworks for what they are: theories of saving. We will, in particular, demonstrate that they have different implications for capital-output ratios when parameters of the model change. The only parameters of the model, so far, are g and δ, and we will focus on g because in Piketty's book it is the main force driving changes in capital-output ratios and in inequality. The two theories thus differ in that they hold different notions of the saving rate constant as g changes. Piketty argues that g is poised to fall significantly, and his second law then implies that the capital-output ratio will rise quite drastically. So what does the textbook model say, and is there a way of comparing how reasonable the two theories are? We will deal with the first question first. The discussion about reasonableness is contained in Section IV below.

To this end, let us first simply use the expressions we already derived. We note that a lower g leads to a higher capital-output ratio also for the textbook model; in addition, it leads to a higher ratio of capital to net income, as shown in equation (5). The question is what the quantitative differences are. Table 1 gives the answer, for two different values of δ.[7]

The table shows that the two models yield very different quantitative predictions for how the capital-to-output (k/y) and capital-to-net-output ($k/\tilde{y}$) ratios vary when g falls. Halving g from 0.026 to 0.013 when $\delta = 0.032$ leads to a 29 percent increase in k/y in the gross model, as compared to an 80 percent increase in the net model. Similarly, $k/\tilde{y}$ increases by 33 per-

[7] For each value of δ, the gross saving rate is chosen to generate a k/y ratio equal to 3.35 when $g = 0.026$, so $s = 0.194$ when $\delta = 0.032$ and $s = 0.3$ when $\delta = 0.064$. On a balanced growth path, the choices for s imply values for $\tilde{s}$ according to eq. (6); in this case these values are 0.097 and 0.11, respectively. The entries for the gross model hold s fixed as g drops, whereas the entries for the model hold $\tilde{s}$ fixed. Note that for the international data discussed in Sec. IV.C.2, the average gross and net saving rates across all observations are 0.194 and 0.097, respectively, and the average growth rate of GDP is 0.026; the first line of table 1 therefore replicates these averages.

TABLE 1
QUANTITATIVE IMPLICATIONS OF THE MODELS

		Gross Model		Net Model		Net/Gross	
δ	g	k/y	$k/\tilde{y}$	k/y	$k/\tilde{y}$	k/y	$k/\tilde{y}$
.032	.026	3.35	3.75	3.35	3.75	1.00	1.00
.032	.013	4.31	5.00	6.04	7.49	1.40	1.50
.032	.000	6.06	7.52	31.25	∞	5.16	∞
.064	.026	3.35	4.24	3.35	4.24	1.00	1.00
.064	.013	3.90	5.19	5.50	8.48	1.41	1.63
.064	.000	4.69	6.70	15.63	∞	3.33	∞

cent in the gross model, as compared to a 100 percent increase in the net model. For the case in which $\delta = 0.064$, k/y increases by 16 percent in the gross model when g halves, as compared to 64 percent in the net model. Similarly, in this case $k/\tilde{y}$ increases by 22 percent in the gross model, as compared to a 100 percent increase in the net model. In sum, the gross model predicts modest increases in the capital-to-output ratio when g halves, whereas the net model predicts rather more dramatic increases. When g drops all the way to zero, the differences between the two models are even starker: for example, when $\delta = 0.032$ and $g = 0$, k/y is more than five times as large in the net model as in the gross model.

When the third and sixth rows of table 1 are compared, it is clear that in the textbook model a drop in g to zero can increase the capital-output ratio substantially if the rate of depreciation is small enough. As explained in footnote 7, the first line of table 1 replicates the long-run averages in the international data, some of which go back to the early nineteenth century. Carefully measuring the rate of depreciation is fraught with both theoretical and empirical difficulties and is certainly well beyond the scope of this paper; but with the growing importance of information technology and its very high rates of economic obsolescence (a critical component of depreciation), we think that the calibration in the bottom half of the table, with a higher rate of depreciation, comes closer to matching the modern economy.

IV. Which Model Makes More Sense?

In the previous section we used the two models to obtain predictions for the object of interest: the capital-output ratio. We now turn to comparing the reasonableness of the models. The comparison proceeds along three lines. First, we discuss the saving rate predictions of the two models, because they turn out to be quite informative. Second, on the basis of the literature examining consumption in the microeconomic data, we study optimal savings. Third, we examine historic aggregate data.

A. *Predictions for Saving Rates*

To use a model with an assumption about saving to obtain predictions about saving sounds circular, but there are, recall, two notions of the saving rate in play here: the gross one and the net one. In the textbook model, the gross saving rate is fixed but the net saving rate is not: it is endogenous. In fact, equation (6) shows how the net saving rate in the textbook model depends on the gross saving rate, as well as g and δ, on a balanced growth path. An important implication of this equation is that the net saving rate implied by the textbook model goes to zero as growth goes to zero: this is simply the usual steady-state condition in the textbook model that k_t is constant in the absence of growth, so net saving, $k_{t+1} - k_t$, equals zero.

Conversely, Piketty's model assumes that the net saving rate is fixed. Now gross saving is endogenous, and so Piketty's model makes predictions for how the gross saving rate responds, in particular, to changes in g, just as the textbook model makes predictions for how the net saving rate responds to such changes. The gross saving rate implied by Piketty's model on a balanced growth path is simply the inverse of the relation derived in (6):

$$s(g) = \frac{\tilde{s}(g + \delta)}{g + \tilde{s}\delta}, \tag{7}$$

where the notation $s(g)$ makes clear that the gross saving rate in Piketty's model depends on the growth rate, as long as the net saving rate is positive.

Note first that the gross saving rate in Piketty's model is decreasing in g: saving behavior is increasingly aggressive as growth falls. Moreover, as growth goes to zero, the implied gross saving rate goes to one! Put differently, in this limit, the economy consumes a fraction zero of its total output. This limit case is very useful for understanding what it means to maintain a constant (and positive) net saving rate. Without growth, to require a positive net saving rate means that the capital stock must go up in every period by a fraction of net output. This is the sense in which saving is particularly aggressive: the capital stock is forced to grow until net output is zero, that is, until the depreciation of the capital stock is as large as output itself.[8] Then consumption is literally zero, assuming a standard production function in which the marginal product of capital goes to zero as the capital stock goes to infinity.[9] At that point, capital accumulation has been so aggressive that all of output is used to replace

[8] James Hamilton makes a similar point in a numerical example on his blog at http://www.econbrowser.com/archives/2014/05/criticisms-of-piketty.

[9] Piketty actually considers a nonstandard production function without strongly decreasing returns, in which consumption does not go to zero. However, the consumption-output ratio does go to zero. He assumes that $F(k, \cdot) - \delta k$ is positive and increasing in k and satisfies an Inada condition: $F_1(k, \cdot) - \delta \to 0$ as $k \to \infty$. When g is exactly zero, there is no balanced growth path in Piketty's model; but in that case, one can show that as $t \to \infty$,

the depreciation of the large stock, and there are no resources left for consumption. Net output is zero, and hence capital accumulation has stopped at that point. This is why $k/\tilde{y} = \infty$. On the other hand, k/y is finite but is at its largest possible stationary value—because the saving rate is one.

That market economies would accumulate as aggressively as implied by Piketty's theory of saving as growth falls seems implausible. Moreover, though the case $g = 0$ is extreme in some sense (but not necessarily so unrealistic?), for growth rates close to zero—indeed at rates close to those considered in Piketty's predictions—similar results apply since the function $s(g)$ in equation (7) is continuous.

B. *Standard Saving Theory Based on Intertemporal Optimization*

Though the empirical literature on consumption and saving has developed rather sophisticated settings (see Attanasio and Weber [2010] for a survey), at its core these models all rely on intertemporal utility maximization. What do such settings imply for saving rates? In particular, how do saving rates depend on growth rates? There are many possible structures one could adopt here, but the most commonly used setting is one with infinitely lived dynasties, and that is the one we will use as well. Since our focus is on illustration, using the limiting case in which $g = 0$, we formally describe optimization for that case only. But we also report results from a model with $g > 0$ as well. We look first at the simplest possible optimal-saving problem: one in partial equilibrium in which the interest rate and wage rate are constant. We then look at a general equilibrium economy in which the production technology is that assumed by Piketty. In both cases we find that optimal behavior entails setting the net saving rate $\tilde{s}$, as defined by Piketty, equal to zero when $g = 0$. More generally, optimizing theory predicts that as g falls, so do both the gross and net saving rates, though the gross rate is of course still positive when $g = 0$.

1. A Single Consumer

We assume that the consumer has preferences given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t),$$

---

the gross saving rate still converges to one. The gross saving rate in Piketty's model equals $[\tilde{s} + \delta(k_t/\tilde{y}_t)]/[1 + \delta(k_t/\tilde{y}_t)]$. We will show that the ratio $k_t/\tilde{y}_t \to \infty$ as $t \to \infty$. First, note that the difference $k_{t+1} - k_t = \tilde{s}\tilde{F}(k_t, \cdot)$ is positive for all t (provided $\tilde{s} > 0$) and increases over time because $\tilde{F}(k, \cdot)$ is increasing in k, implying that $k_t \to \infty$ as $t \to \infty$. Second, by l'Hôpital's rule,

$$\lim_{k\to\infty}[\tilde{F}(k, \cdot)/k] = \lim_{k\to\infty} \tilde{F}_1(k, \cdot).$$

By the Inada condition on $\tilde{F}$, this limit is zero, so $k_t/\tilde{y}_t \to \infty$.

where u is an increasing and strictly concave function; concavity here implies consumption smoothing, for which there appears to be strong support in countless empirical studies of individual consumption behavior. The consumer's budget constraint reads

$$c_t + k_{t+1} = (1 + r - \delta)k_t + w,$$

where w is the (constant) wage, $1 + r$ is the (constant) gross return from capital, and δ, as above, is the depreciation rate.[10] We take as implicit a condition preventing the consumer from pyramid scheme borrowing but otherwise assume no constraints on either saving or borrowing. The consumer thus starts out with some capital k_0 and, given a wage and a net return that are equal to w and $r - \delta$ at all times, we ask: how will he save?

Substituting c_t into the objective function and taking derivatives with respect to k_{t+1}, we obtain

$$u'(c_t) = \beta u'(c_{t+1})(1 + r - \delta).$$

Consumption behavior here depends critically on whether $\beta(1 + r - \delta)$ is above, below, or equal to one. Assuming first that it is equal to one, because this is the only case that allows an exact steady state, we obtain a solution with constant consumption, $c_t = c_{t+1}$, since u' is monotone. This implies, from the budget constraint, that for all t,

$$c_t = (r - \delta)k_t + w$$

and

$$k_t = k_0.$$

This is classic "permanent-income behavior": the consumer keeps his asset holding constant and consumes the return on his assets plus his wage income. Here the consumer's net, or "disposable," income $\tilde{y}_t$ is $k_t(r - \delta) + w$. Hence, writing

$$c_t = (1 - \tilde{s})\tilde{y}_t$$

and

$$k_{t+1} - k_t = \tilde{s}\tilde{y}_t,$$

we see that net saving as a fraction of disposable income, that is, $\tilde{s}$, is zero, as we just showed the textbook model also implies.

One can, of course, depart from $\beta(1 + r - \delta) = 1$. Any such departure would imply that $\tilde{s}$ would depend on time and would either begin

[10] We can equivalently think of this as an open economy: the interest rate is the world interest rate and $rk_t + w$ is GNP.

positive and eventually turn negative or the other way around; loosely speaking, the rate would be around zero. Moreover, small departures from $\beta(1 + r - \delta) = 1$ would produce only small departures from $\tilde{s}$ in finite time. Thus, we conclude that this model robustly predicts $\tilde{s} = 0$, along with a bounded value of $k/\tilde{y}$.

The permanent-income model, thus, suggests that it is not immaterial whether one expresses saving behavior in the "textbook way" or in the "Piketty way." The former is consistent with this model but the latter is not.[11] Or, rather, it is consistent only if the relevant saving rate is zero, but this is precisely the rate that makes Piketty's main argument—that the ratio of capital to net income explodes at $g = 0$—break down.

2. General Equilibrium

It is easy to verify that the results in the previous section generalize to a general equilibrium perspective given the kinds of production functions traditionally used in the macroeconomic literature. We will now also show that, even using the production function without strongly decreasing returns that Piketty entertains (see n. 9), one cannot rationalize his assumed saving behavior. Piketty's assumption is that net production, $\tilde{F}(k, l)$, is always positive, so let us then use the production function $F(k, l) = Ak^{\alpha} l^{1-\alpha} + \delta k$.[12] This specification implies that $\tilde{F}(k, l) = Ak^{\alpha}$ and hence satisfies his assumption (Cobb-Douglas is not essential here). This makes the economy's resource constraint read

$$c_t + k_{t+1} = Ak_t^{\alpha} + \delta k_t + (1 - \delta)k_t = Ak_t^{\alpha} + k_t.$$

So we essentially have a model with no depreciation, and clearly (as demonstrated above) this model allows unbounded growth. What is, however, reasonable saving behavior for such a model? Let us again use the dynastic setup, and let us for simplicity focus on the planner's problem, as it delivers quantities that coincide with those of the competitive equilibrium:

$$\max_{\{k_{t+1}\}} \sum_{t=0}^{\infty} \beta^t u(Ak_t^{\alpha} + k_t - k_{t+1}),$$

[11] The textbook saving rate would be defined by $c_t = (1 - s)(rk_t + w)$ and $k_{t+1} - k_t(1 - \delta) = s(rk_t + w)$, implying

$$s = \delta \frac{k}{rk + w} = \delta \frac{k}{y}.$$

Depending on the initial capital stock, national income and capital will have different values but the capital-output ratio will be s/δ.

[12] Jones and Manuelli (1990) conduct a more general analysis of production technologies like this one in which the marginal product of capital is bounded away from zero. Under some conditions, unbounded growth is optimal in such settings. For the specific technology that we consider, however, we show that optimal behavior implies convergence to a steady state.

a problem that is concave and has a unique solution characterized by the usual Euler equation

$$\frac{1}{Ak_t^{\alpha} + k_t - k_{t+1}} = \beta \frac{\alpha A k_{t+1}^{\alpha-1} + 1}{Ak_{t+1}^{\alpha} + k_{t+1} - k_{t+2}}$$

along with a transversality condition. The Euler equation admits a steady state k defined uniquely by the condition

$$1 = \beta(\alpha A k^{\alpha-1} + 1),$$

and one can show, using standard methods, that there is convergence to this steady state (with an accompanying convergence of consumption to a constant number).[13]

Put differently, even with the (unusual) production function used by Piketty (and by Solow before him)—one that admits unbounded growth without technical change—standard assumptions on behavior (i.e., the optimization of a reasonable-looking utility function) deliver a steady state, quite in contrast with Piketty's assumption on saving. His assumption on saving is that $\tilde{s} > 0$, but the above analysis shows instead that rather $\tilde{s} = 0$ is optimal in the long run. This, again, must obviously hold since net saving, $k_{t+1} - k_t$, is zero whenever the economy reaches a steady state.

Optimal-savings theory implies, more generally, that on a balanced growth path, both the gross and net saving rates depend positively on g. Figure 1 provides a quantitative illustration for the case of a standard production function, that is, one that is Cobb-Douglas.[14] Both the gross and net saving rates increase with g, though the net saving rate increases more rapidly than the gross saving rate. As in the other models of optimizing behavior discussed in this section, the net saving rate is zero when $g = 0$.[15]

C. *Long-Run Data on Saving Rates and Growth Rates*

In this section, we use data on saving rates and growth rates over long periods of time in a variety of countries to study the empirical relationship between long-run growth rates and net and gross saving rates. The goal is not to carry out a definitive analysis of the patterns in the data but rather to evaluate and compare the different models in light of the

[13] Clearly, in a steady state the transversality condition is met, too.

[14] To generate fig. 1, we use a calibrated model in which one period corresponds to 1 year, utility is logarithmic, labor-augmenting technology grows at rate g, capital's share equals 0.36, the discount factor equals 0.96, and the depreciation rate equals 0.05.

[15] Homburg (2014) makes a related point, using a two-period overlapping generations model to argue, as we do, that the net saving rate "is not exogenous but an increasing function of the growth rate . . . running through the origin" (5).

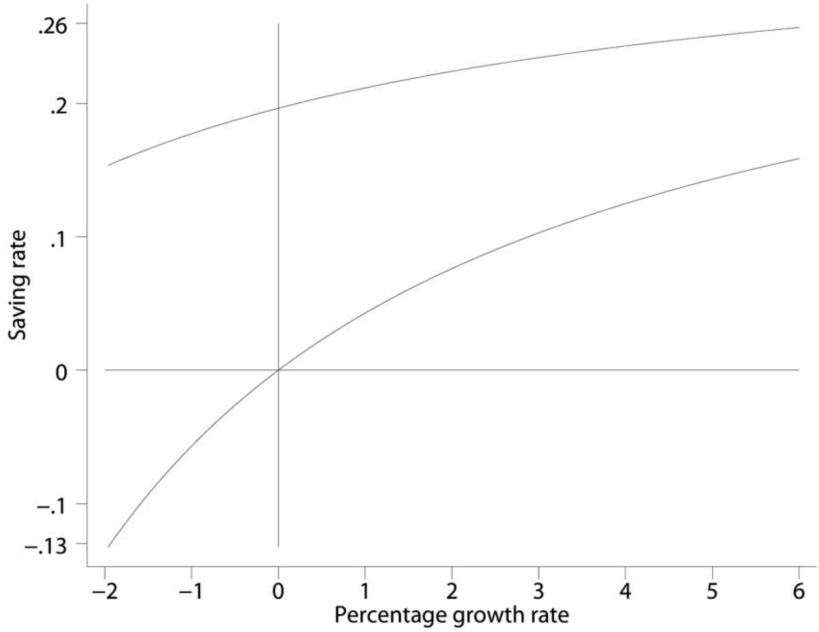

FIG. 1.—Gross and net saving rates versus the growth rate in a standard optimizing growth model. The upper line is the gross saving rate on a balanced growth path and the lower line is the net saving rate on a balanced growth path.

data. Recall that in Piketty's model the net saving rate is constant over time and independent of the growth rate g, implying that as growth increases the implied gross saving rate declines (eq. [7] above). In contrast, in the textbook version of the Solow model the gross saving rate is constant, with the implied net saving rate responding positively to the growth rate (eq. [6]). Finally, in the usual optimizing growth model with a standard production function, both saving rates (net and gross) are increasing in g.

1. US Data

We look first at US annual time series for output and net and gross saving rates since 1930.[16]

Figure 2 plots the annual gross and net saving rates since 1930. In percentage terms, fluctuations in $\tilde{s}$ are substantially larger than those in s (the coefficient of variation for the $\tilde{s}$ series is about 0.7, whereas it is only

[16] Specifically, from the FRED database, we use series A023RX1A020NBEA on real gross national income, series W206RC1A156NBEA on gross saving as a percentage of gross national income, and series W207RC1A156NBEA on net saving as a percentage of gross national income. These series all come from the Bureau of Economic Analysis; with them, it is straightforward to construct a series for net saving as a percentage of net national income.

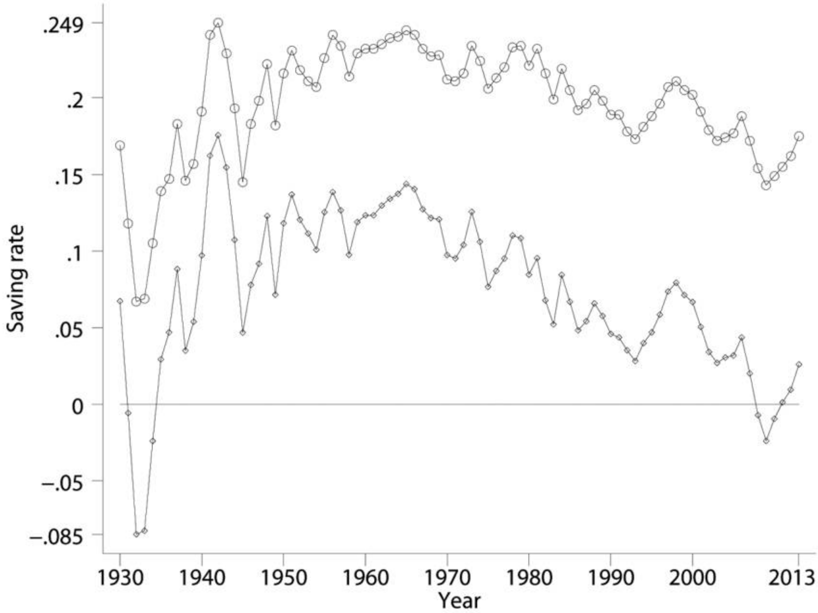

FIG. 2.—Annual gross and net savings rates: United States, 1930–2013. The upper line is the gross saving rate and the lower line is the net saving rate.

about 0.2 for the s series). In this sense, if one were to choose between making one of them constant over time, it would make more sense to assume s constant: the textbook version of the Solow model. We see also that $\tilde{s}$ has fallen gradually toward zero; it was below zero during the recent recession and over the last 5 or so years is well approximated by zero. Thus, that $\tilde{s}$ will remain constant and positive in the twenty-first century does not appear like a good assumption at all.

We turn next to how saving rates vary with growth rates in the observed data. As an approximation to behavior on a balanced growth path, we calculate 10-year averages of saving rates and growth rates for eight decades starting in 1930.[17] Figure 3 plots these averages, revealing a strong positive relationship between both growth rates and saving rates.

Although it is difficult to make truly long-run evaluations without much longer time series, the data are certainly consistent in this respect with optimal savings theory (e.g., compare fig. 3 to fig. 1). A regression of the 10-year net saving rates on the 10-year growth rate yields an intercept very close to zero—again consistent with the optimizing model—and a slope of 0.025 (so that an additional percentage point of "long-run" growth increases the net saving rate by 2.5 percentage points). Thus,

[17] Evaluating the dynamics of the various models is far more involved and is best left for future study.

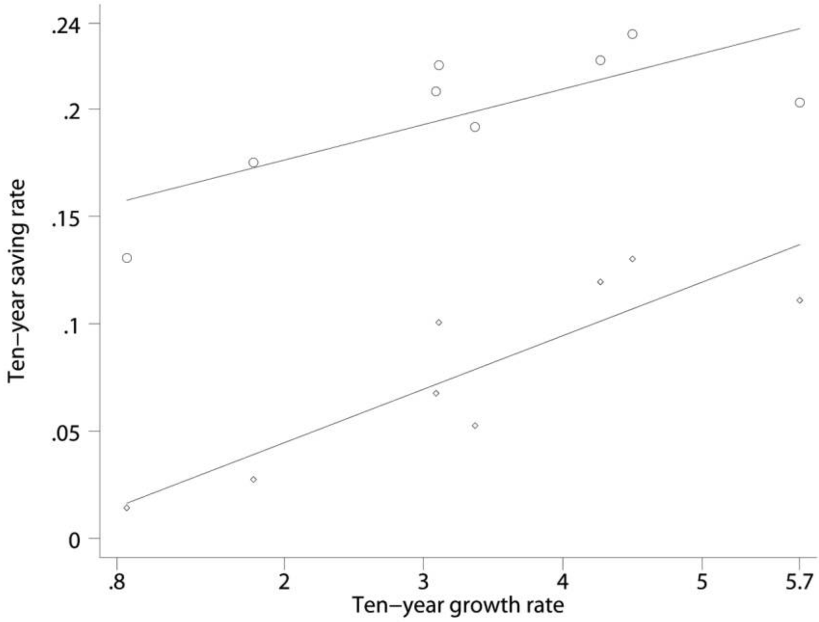

FIG. 3.—Gross and net savings rates: United States, 10-year averages, 1930–2009. The circles are observations on the gross saving rate and the diamonds are observations on the net saving rate. The coefficients for the two regression lines are reported in the text.

predicting the future using Piketty's second fundamental law, with $\tilde{s}$ remaining positive as g goes to zero, appears unwise. Finally, a regression of the 10-year gross saving rate on the 10-year growth rate yields an intercept of 0.14 and a slope of 0.017. An increase in g therefore increases the net saving rate by more than it increases the gross saving rate, and it is straightforward to show that this finding is at least qualitatively consistent with the optimizing model. By contrast, neither Piketty's model nor the textbook model can match this finding (since each holds one of the saving rates constant as g changes).

2. Piketty and Zucman's Data

We turn next to data documented in Piketty and Zucman (2014) for eight countries, in some cases over very long horizons, as far back as 1831 for France and 1871 for the United States and the United Kingdom.[18] The advantage of these data is that we can construct averages over even longer

[18] Specifically, we use the spreadsheets available on Zucman's website in support of Piketty and Saez (2014); the URL is http://gabriel-zucman.eu/capitalisback/. From these we extract annual series for net and gross saving rates and growth rates of GDP going back to 1831 for France, 1871 for the United States and the United Kingdom, 1951 for Germany, and 1971 for Australia, Canada, Italy, and Japan.

time periods, in this case 20 years, to approximate better the notion of a balanced growth path. In total we have 34 observations on average net and gross saving rates and average growth rates: nine for France, seven each for the United States and the United Kingdom, three for Germany, and two for each of the remaining four countries.[19] Figure 4 plots the data.

The basic message is the same as that for the US data displayed in figure 3: both the net and gross saving rates increase with g, the net saving rate changes more rapidly than the gross rate as g increases, and the net saving rate is close to zero when g is zero. More formally, we regressed the net and gross saving rates on the growth rate and a set of country dummy variables, and we obtained coefficients very similar to the ones reported in Section IV.C.1 for the US data obtained from the Bureau of Economic Analysis: a regression of the gross rate on the growth rate yields a (statistically significant) slope of 0.018 and an R^2 of .80; and a regression of the net rate on the growth rate yields a (statistically significant) slope of 0.024 and an R^2 of .62.[20]

To sum up the conclusions from both data sets, the data speak quite strongly against Piketty's model: of the three savings models considered here, it conforms most closely with the optimizing model.

Let us finally briefly comment on Piketty's point of view; clearly, since his assumptions on saving are nonstandard, relative to the applied economics literature, a comparison with the standard model ought to be a main concern in his works, where he does appear to claim that his model allows an accurate account of the historical data. Piketty and Zucman (2014) do study capital accumulation in a cross section of countries from the perspective of his formulation of aggregate saving but do not address, to the best of our knowledge, the central question of how net saving rates vary with growth rates. Instead, this paper uses the growth model to perform an accounting exercise: changes in Piketty and Zucman's broad measure of wealth that cannot be accounted for by the accumulation of savings (given the observed saving rates) are attributed instead to capital gains, that is, to changes in the market value of capital. We find this accounting exercise interesting, but it is not, as far as we can see, a test that can discriminate between different ways of formulating the growth model.

V. Have We Misinterpreted the Second Law?

This paper can be viewed both as a comment on Piketty's book and as a note on different theories of aggregate saving that have played central

[19] We also studied 10-year averages with Piketty and Zucman's data, obtaining very similar results.

[20] The country dummies are all large and statistically significant in the first regression but close to zero and statistically insignificant in the second regression (with the exception of Japan, for which in any event there are only two observations).

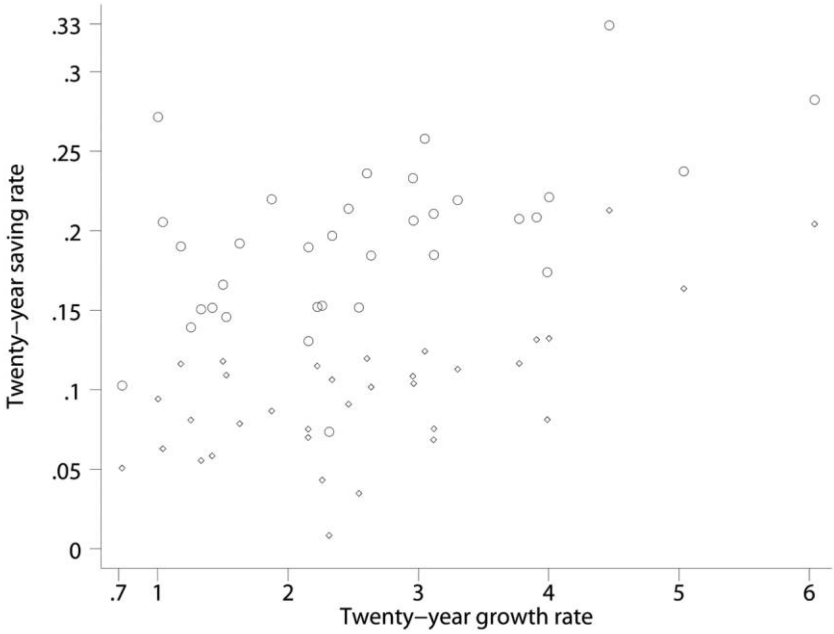

FIG. 4.—Gross and net savings rates: eight countries, 20-year averages. The circles are observations on the gross saving rate and the diamonds are observations on the net saving rate.

roles in the development of growth theory, starting with Solow's seminal work. Until now we have mostly elaborated on the latter. Turning again to our commentary on the book, we have argued that Piketty uses the second fundamental law to predict that the capital-income ratio will rise substantially in the twenty-first century as growth slows and that this prediction is flawed because it relies on an implausible theory of saving in which the economy's net saving rate remains constant as growth falls. But does Piketty in fact use the second law in the way that we have argued? Is our interpretation of Piketty's second law perhaps too strict? In this section, we use quotations from Piketty's book and other related writings to answer these questions.

In Piketty's book and in the papers that its online appendix cites, one can find numerous examples of comparative statics exercises that hold the net saving rate constant as the growth rate changes. This passage from the book (on p. 167), just after Piketty has formally introduced the second law, is representative:

> The basic point is that small variations in the rate of growth can have very large effects on the capital/income ratio over the long run.
>
> For example, given a savings rate of 12 percent, if the rate of growth falls to 1.5 percent a year (instead of 2 percent), then the

> long-term capital/income ratio . . . will rise to eight years of national income (instead of six). If the growth rate falls to 1 percent, then [the long-term capital/income ratio] will rise to twelve years, indicative of a society twice as capital intensive as when the growth rate was 2 percent. . . .
>
> On the other hand if the growth rate increases to 3 percent then [the long-term capital/income ratio] will fall to just four years of national income. If the savings rate simultaneously decreases slightly to . . . 9 percent, then the long-run capital/income ratio will decline to 3.
>
> These effects are all the more significant because the growth rate that figures in the [second fundamental] law . . . is the overall rate of growth of national income, that is, the sum of the per capita growth rate and the population growth rate. In other words, for a savings rate on the order of 10–12 percent and a growth rate of national income per capita on the order of 1.5–2 percent a year, it follows immediately that a country that has near-zero demographic growth and therefore a total growth rate close to 1.5–2 percent, as in Europe, can expect to accumulate a capital stock worth six to eight years of national income, whereas a country with demographic growth on the order of 1 percent a year and therefore a total growth rate of 2.5–3 percent, as in the United States, will accumulate a capital stock worth only three to four years of national income. And if the latter country tends to save a little less than the former, perhaps because its population is not aging as rapidly, this mechanism will be further reinforced as a result. In other words, countries with similar growth rates of income per capita can end up with very different capital/income ratios simply because their demographic growth rates are not the same.

In the main calculations in this passage, Piketty clearly keeps the net saving rate constant as growth is lowered. He also entertains the possibility that this saving rate might fall a little when population growth increases, but we view this as a robustness check on the main calculations, whose overall thrust is clear: small changes in growth rates lead to large changes in capital-income ratios.

In a later passage in the book (on pp. 195–96), Piketty writes that

> global output will gradually decline from the current 3 percent a year to just 1.5 percent in the second half of the twenty-first century. I also assume that the savings rate will stabilize at about 10 percent in the long run. With these assumptions, the [second fundamental] law . . . implies that the global capital/income ratio will quite logically continue to rise and could approach 700 percent before the end of the twenty-first century. . . . Obviously,

> this is just one possibility among others. As noted, these growth predictions are extremely uncertain, as is the prediction of the rate of saving. These simulations are nevertheless plausible and valuable as a way of illustrating the crucial role of slower growth in the accumulation of capital.

Here again we see that declines in growth rates play a central role in Piketty's prediction for the capital-income ratio. Piketty provides no motivation for the 10 percent (net) saving rate, but it is close to the average net saving rate in the cross-country data plotted in figure 4.[21] Piketty qualifies his prediction by noting that his predicted saving rate is very uncertain but gives no indication that it might vary systematically with the growth rate, as it does both in the data and in the two canonical models of aggregate saving that we have discussed here. A few pages later (on p. 199) Piketty asserts that "these two macrosocial parameters," that is, the (net) saving rate and the growth rate, are "influenced by any number of social, economic, cultural, psychological, and demographic factors" and are "largely independent of each other." Clearly here he treats the net saving rate as a free parameter, analogously to how the textbook Solow model treats the gross saving rate.

In their 2014 article in *Science* elucidating the main arguments in the book, Piketty and Saez illustrate again (on p. 840) the use of the second law in doing comparative statics with respect to the growth rate (in this passage s refers to the net saving rate and Y refers to net income):

> In the long-run . . . one can show that the wealth-to-income (or capital-to-income) ratio . . . converges toward $\beta = s/g$, where s is the long-run annual saving rate and g is the long-run annual total growth rate. The growth rate g is the sum of the population growth rate . . . and the productivity growth rate. . . .
>
> That is, with a saving rate $s = 10\%$ and a growth rate $g = 3\%$, then $\beta \approx 300\%$. But if the growth rate drops to $g = 1.5\%$, then $\beta \approx 600\%$. In short: Capital is back because low growth is back. . . .
>
> In the extreme case of a society with zero population and productivity growth, income Y is fixed. As long as there is a positive net saving rate $s > 0$, the quantity of accumulated capital K will go to infinity. Therefore, the wealth-income ratio $\beta = K/Y$ would rise indefinitely (at some point, people in such a society would probably stop saving, as additional capital units become almost useless). With positive but small growth, the process is not as extreme: The rise of β stops at some finite level. But this finite level can be very high.

[21] But note that the average growth rate in these data is 2.6 percent; the regression results reported in Secs. IV.C.1 and IV.C.2 suggest that the net saving rate would be lower than 10 percent when the growth rate is instead 1.5 percent.

This passage also explicitly considers the case of zero growth, backing off slightly from the extreme implication of an indefinite rise in the capital-income ratio but nonetheless giving the clear impression that the second law implies a "very high" capital-income ratio with zero growth. This passage again does not recognize that in both the textbook Solow model and the optimizing model the net saving rate goes to zero as growth goes to zero, rendering the second law, now reading $k/y = 0/0$, difficult to interpret. Instead the net saving rate is kept constant at a positive value as growth falls.

The case of (near) zero growth in fact plays a central role in the book, where Piketty draws strong connections between the second law with zero growth and Karl Marx's views on the accumulation of capital. This quotation is drawn from the book's introduction (pp. 9–11), before the second law is formally introduced:

> [Marx's] principal conclusion was what one might call the "principle of infinite accumulation," that is, the inexorable tendency for capital to accumulate and become concentrated in ever fewer hands, with no natural limit to the process. . . .
>
> The principle of infinite accumulation that Marx proposed contains a key insight, as valid for the study of the twenty-first century as it was for the nineteenth century. . . . If the rates of population and productivity growth are relatively low, then accumulated wealth naturally takes on considerable importance, especially if it grows to extreme proportions and becomes socially destabilizing. In other words, low growth cannot adequately counterbalance the Marxist principle of infinite accumulation: the resulting equilibrium is not as apocalyptic as the one predicted by Marx but is nevertheless quite disturbing. Accumulation ends at a finite level, but that level may be high enough to be destabilizing. In particular, the very high level of private wealth that has been attained since the 1980s and 1990s in the wealthy countries of Europe and in Japan, measured in years of national income, directly reflects the Marxian logic.

This passage, which stresses positive and significant net saving rates even when there is very little growth—in contrast to our arguments that these should smoothly approach zero—appears before the second law is presented. Later, after the law is presented, Piketty returns to these themes in the following passage (on pp. 227–29), where he ties the logic together and again discusses the limiting case in which g is zero:

> For Marx, the central mechanism by which "the bourgeoisie digs its own grave" corresponded to what I referred to in the Introduction as "the principle of infinite accumulation": capi-

> talists accumulate ever increasing quantities of capital. . . . Marx did not use mathematical models . . . , so it is difficult to be sure what he had in mind. But one logically consistent way of interpreting his thought is to consider the [second fundamental] law . . . in the special case where the growth rate g is zero or very close to zero. . . .
>
> Where there is no structural growth, and the productivity and population growth rate g is zero, we run up against a logical contradiction very close to what Marx described. If the savings rate . . . is positive . . . then the capital/income ratio will increase indefinitely. More generally, if g is close to zero, the long-term capital/income ratio . . . tends towards infinity. . . .
>
> The dynamic inconsistency that Marx pointed out thus corresponds to a real difficulty, from which the only logical exit is structural growth, which is the only way of balancing the process of capital accumulation (to a certain extent). Only permanent growth of productivity and population can compensate for the permanent addition of new units of capital, as the [second fundamental] law . . . makes clear. Otherwise, capitalists do indeed dig their own grave: either they tear each other apart in a desperate attempt to combat the falling rate of profit . . . or they force labor to accept a smaller and smaller share of national income, which ultimately leads to proletarian revolution and general expropriation. In any event, capital is undermined by its internal contradictions.

These "internal contradictions"—the notion that capitalism naturally and inevitably generates extreme wealth inequality—are arguably the central theme of Piketty's book, and as this passage makes clear, they are intimately related to his second law as we have portrayed it here.

These two passages also point to another central issue, largely absent from Piketty's book, namely, the role of depreciation in macroeconomics.[22] Depreciation is clearly a threat to "capitalists": it eats up their capital and limits their ability to build it up. But it is also a threat to Piketty's vision of capitalism's purported internal contradictions: depreciation destroys capital and forces capitalists to devote resources not to its accu-

[22] The phrase "rate of depreciation" (or "rate of annual depreciation") appears in the book only three times, and one of those times is in a footnote (fn. 12 in chap. 5); the same footnote contains the only occurrence of the phrase "depreciation rate." In this footnote Piketty notes that the saving rate could alternatively be defined as the "total" or gross saving rate, and he states that the second law could then be written alternatively as $\beta = s/(g + \delta)$. But this alternative formulation plays no role in the book (and in fact is not, strictly speaking, correct unless β is redefined to be the ratio of capital to gross output rather than the ratio of capital to net output). This footnote is also the only place in the book where Piketty offers a value for δ, namely 2 percent, surely much lower than any reasonable estimate.

mulation but rather simply to its maintenance. The phrase "permanent addition of new units of capital" in the just-quoted passage is telling: it is in fact difficult to conceive of such additions, because whether through physical decay or economic obsolescence, all capital depreciates. Throughout his book Piketty rails against the "disproportionate importance" of "wealth accumulated in the past" (see, e.g., p. 166 in his book), the latter phrase appearing no fewer than nine times in his book.[23] But depreciation erodes wealth, and in our view it is critical to incorporate this corrosive force explicitly in any analysis of wealth accumulation.

We close this section with another possible interpretation of the second law: that it is not stated as a theory but rather as just another definition or accounting identity. The second law would then simply point out that in the long run, k/y depends mechanically on s and g when these are constant. In fact, as we have argued above, we do not think that this mechanical interpretation of the second law represents well Piketty's own views. But were one to take this interpretation, then we must insist that the two fundamental laws would be void of content. In particular, they would not be useful at all for making statements about how k/y might change in the future if, say, g were to fall. To make predictions, one would need to use a specific theory of saving (and our favored theory is one that would not predict sharp increases in k/y in response to a fall in g). The way the book is written—with the fundamental laws presented early in the book and figuring prominently in Piketty's predictions for the twenty-first century—it is hard, we think, for any reader not to get the impression that the second law is stated as something to build an argument on, and not as a mere identity. Moreover, if the laws are in fact just accounting tools, then we maintain that, so as not to mislead readers, the book should have made very clear that the fundamental laws have no relevance at all for thinking about the future and that any predictions must come from somewhere else. It would obviously also be important, in that case, to defend such predictions on some other grounds.

VI. Conclusions

We have argued in this paper that Piketty's predictions for the twenty-first century depend critically on the saving theory that one employs and that the theory he uses—comparative statics exercises based on his second law of capitalism, hence keeping the net saving rate fixed at a positive level—is a poor theory, especially for the low values of growth that Piketty foresees. The textbook Solow model, which maintains a constant gross saving rate, does a better job of matching past data, but mod-

[23] Piketty contrasts "wealth accumulated in the past" with saving, i.e., "wealth accumulated in the present"; see p. 378 of his book.

els based on standard intertemporal utility maximization provide an even better match, since these predict falling (net and gross) saving rates as g falls, as has been observed in long-run data. These models are also firmly grounded on empirical work documenting how households save.

Our conclusion is not to sanction complacency about the future developments of wealth inequality. To the contrary, we consider the topic very important from both a positive and a normative perspective, and we particularly perceive a major need for theory in trying to interpret past movements in wealth inequality. Without Piketty's impressive data work, these movements would have been neither emphasized nor quantified. Looking forward, what are reasonable theories that might undergird quantitative predictions for the evolution of inequality? In his book, Piketty proposes another theory, one we have not reviewed here, that stresses the comparison between the rate of return on capital and the growth rate. To make his arguments, he uses a rather abstract mathematical model (see Piketty and Zucman 2015) showing how, under certain mild conditions, a wealth distribution with a realistic Pareto-shaped right tail emerges as an equilibrium outcome. (The thickness of this tail is then shown to vary directly with the difference between the real return and the growth rate.) Quantitatively restricting models to match this and other features of the wealth distribution will be very important going forward, in our view, and we look forward to further developments along these lines.[24] Without such theory development, we will be sorely short not only of predictions for the twenty-first century but also, and perhaps more importantly, of coherent arguments about the welfare consequences of different policy suggestions aimed at containing future wealth inequality.

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[24] In ongoing work—joint with Joachim Hubmer—we show that the model in Krusell and Smith (1998) with random discount rates in fact generates a wealth distribution with a Pareto right tail and that Piketty and Zucman's setting, grounded instead in random savings rates, can be viewed as a reduced form of this model.

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